Determinants
Class 12th

Q.1)	Find the value of x if the area of Δ is 35 square units with vertices (x, 4), (2, –6) and (5, 4).
Sol.1)	Let vertices are $A(x, 4)$, $B(2, -6)$ and $C(5, 4)$ Area of ΔABC = $\frac{1}{2}\begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$ 35 = $\frac{1}{2}$ $\lvert x(-10) - 4(-3) + 1(38) \rvert$ ⇒ 35 = $\frac{1}{2}$ $\lvert -10x + 12 + 38 \rvert$ ⇒ 70 = $\lvert -10x + 50 \rvert$ $70 = -10x + 50$ | $-70 = -10x + 50$ $10x = -20$ | $10x = 120$ $x = -2$ | $x = 12$ ∴ $x = -2$, $x = 12$ ans.
Q.3)	Find the value of x so that matrix A = $\begin{bmatrix} (x-1) & 1 & 1 \\ 1 & (x-1) & 1 \\ 1 & 1 & (x-1) \end{bmatrix}$ is singular/ Non-Invertible.
Sol.3)	Since matrix A is singular ∴ |A| = 0 $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$ ⇒ $(x-1)\,[(x-1)^2 - 1] - 1[x - 1 - 1] + 1[1 - x + 1] = 0$ ⇒ $(x-1)\,(x^2 - 2x) - 1(x - 2) + (2 - x) = 0$ ⇒ $x^3 - 2x^2 - x^2 + 2x - x + 2 + 2 - x = 0$ ⇒ $x^3 - 3x^2 + 4 = 0$ By trial method $(x + 1)\,(x - 2)\,(x + 1) = 0$ ⇒ $x = -1, x = 2$ ans.
Q.4)	(a) Evaluate the determinant Δ = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$. Also prove $2 \le \Delta \le 4$. (b) Prove that Δ = $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ.
Sol.4)	(a) we have, Δ = $\begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$ ⇒ $\Delta = 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$ ⇒ $\Delta = 1 + \sin^2\theta + 0 + \sin^2\theta + 1$ ⇒ $\Delta = 2 + 2\sin^2\theta$ Now , we know $-1 \le \sin\theta \le 1$ ⇒ $0 \le \sin^2\theta \le 1$ ⇒ $0 \le 2\sin^2\theta \le 2$ (multiply by 2) ⇒ $2 \le 2 + 2\sin^2\theta \le 4$ (adding 2) ⇒ $2 \le \Delta \le 4$ (proved) (b) $\Delta = x(-x^2 - 1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$ $\Delta = -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

	$\Delta = -x^3 - x + x(sin^2\theta + cos^2\theta)$ $\Delta = -x^3 - x + x(1)$ $\Delta = -x^3$ *which is independent* of θ.

Short Questions

Q.5) Order 3×3 , $|A| = 5$. Find $|\text{Adj A}| = ?$

Sol.5) We have $n = 3$, $|A| = 5$

and $|Adj\ A| = |A|^{n-1}$

$= (5)^{3-1} = 25$ ans.

Q.6) Order 3×3 , $|\text{Adj A}| = 81$ find $|A| = ?$

Sol.6) We have n = 3 , $|\text{Adj A}| = 81$

$\Rightarrow |\text{Adj A}| = |A|^{n-1}$

$\Rightarrow 81 = |A|^2$

$\Rightarrow |A| = \pm 9$ ans.

Q.7) Order 3×3 ; $|A| = 3$ find $|4A|$ = ?

Sol.7) We have n = 3 , $|A| = 3$

$|4A| = 4^3\ |A|$ .....{$\because |kA| = k^n\ |A|$}

$= 64 \times 3$

$= 192$ ans.

Q.8) *Order* 3×3 ; $|A| = 5$ *find* $|2Adj\ A| = ?$

Sol.8) $|2Adj\ A| = 2^3|Adj\ A| = 2^3|A|^{3-1}$

$= 8\ (5)^2 = 200$ *ans.*

Q.9) Order 4×4 ; $|3\ \text{Adj A}| = 243$ Find $|A| = ?$

Sol.9) We have $|3\ \text{Adj A}| = 3^4|\text{Adj A}|$

$243 = 3^4\ |A|^{4-1}$

$243 = 81\ |A|^3$

$|A|^3 = 3$

$|A| = (3)^{1/3}$ ans.

Q.10) Order 4×4 ; $|A| = 5$ find $|A'| = ?$

Sol.10) We know $|A'| = |A|$

$\Rightarrow |A'| = 5$ ans.