

Integration (Indefinite Integrals)

Q.1)	$I = \int x \cos^3(x^2) \sin(x^2) dx$
Sol.1)	$I = \int x \cos^3(x^2) \sin(x^2) dx$ put $x^2 = t$ $2x dx = dt \Rightarrow x dx = \frac{1}{2} dt$ $\therefore I = \frac{1}{2} \int \cos^3 t \cdot \sin t dt$ put $\cos t = z$ $\sin t dt = -dz$ $\therefore I = -\frac{1}{2} \int z^3 dz$ $= -\frac{1}{2} \frac{z^4}{4} + c$ replacing 'z' $= -\frac{1}{8} \cos^4 t + c$ replacing t $= -\frac{1}{8} \cos^4(x^2) + c$ ans.
Q.2)	(i) $I = \int \frac{1}{\sin^3 x \cdot \cos x} dx$
Sol.2)	(i) $I = \int \frac{1}{\sin^3 x \cdot \cos x} dx$ Divide N & D by $\cos^4 x$ $I = \int \frac{\sec^4 x}{\tan x} dx$ $= \int \frac{\sec^2 x \cdot \sec^2 x}{\tan x} dx$ $= \int \frac{(1 + \tan^2 x) \cdot \sec^2 x}{\tan x} dx$ put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $I = \int \frac{(1 + t^2)}{t} dt$ $= \int \frac{1}{t} + t dt$ $= \log t + \frac{t^2}{2} + c$ $I = \log \tan x + \frac{1}{2} \tan^2 x + c$ ans.
Q.3)	(i) $I = \int \frac{1}{\sin^4 x \cdot \cos^2 x} dx$
Sol.3)	(i) $I = \int \frac{1}{\sin^4 x \cdot \cos^2 x} dx$

	Divide N & D by $\cos^6 x$ $= \int \frac{\sec^6 x}{\tan^2 x} dx$ $= \int \frac{\sec^4 x \cdot \sec^2 x}{\tan^2 x} dx$ $= \int \frac{(1+\tan^2 x)^2 \cdot \sec^2 x}{\tan^2 x} dx$ put $\tan x = t$ $\sec^2 x dx = dt$ $\therefore I = \int \frac{(1+t^2)^2}{t^2} dt$ $= \int \frac{1+t^4+2t^2}{t^2} dt$ $= \int \frac{1}{t^2} + t^2 + 2dt$ $= -\frac{1}{t} + \frac{t^3}{3} + 2t + c$ $\therefore I = -\frac{1}{\tan x} + \frac{\tan^3 x}{3} + 2\tan x + c \quad \text{ans.}$
Q.4)	$I = \int \frac{\cos^9 x}{\sin x} dx$
Sol.4)	$I = \int \frac{\cos^9 x}{\sin x} dx$ $= \int \frac{\cos^8 x \cdot \cos x}{\sin x} dx$ $= \int \frac{(\cos^2 x)^4 \cdot \cos x}{\sin x} dx$ $= \int \frac{(1-\sin^2 x)^4 \cos x dx}{\sin x}$ put $\sin x = t$ $\therefore \cos x dx = dt$ $= \int \frac{(1-t^2)^4}{t} dt$ $= \int \frac{(1+t^4-2t^2)^2}{t} dt$ $= \int \frac{1+t^3+4t^4+2t^4-4t^6-4t^2}{t} dt$ $= \int \frac{t^8-4t^6+4t^4-4t^2+1}{t} dt$ $= \int t^7 - 4t^5 + 4t^3 - 4t + \frac{1}{t} dt$ $= \int \frac{t^8}{8} - \frac{4t^6}{6} + \frac{4t^4}{4} - \frac{4t^2}{2} + \log t + c$ $= \int \frac{\sin^8 x}{8} - \frac{2}{3} \sin^6 x + \sin^4 x - 2\sin^2 x + \log \sin x + c$
→ <u>Sin x and Cos x in multiplication with different Angles :-</u>	
Q.5)	(i) $I = \int \sin(3x)\cos(2x)dx$
Sol.5)	(i) $I = \int \sin(3x)\cos(2x)dx$

	$= \frac{1}{2} \int 2\sin(3x) \cdot \cos(2x) dx$ $= \frac{1}{2} \int \sin(5x) + \sin(x) dx$ $I = \frac{1}{2} \left[\frac{-\cos(5x)}{5} - \cos x \right] + c \quad \text{ans.}$
Q.6)	(i) $I = \int \sin(2x) \cdot \sin(4x) \cdot \sin(6x) dx$ (ii) $I = \int \frac{\sin(4x)}{\sin x} dx$
Sol.6)	(i) $I = \int \sin(2x) \cdot \sin(4x) \cdot \sin(6x) dx$ $= \frac{1}{2} \int [2\sin(2x) \cdot \sin(4x)] \cdot \sin(6x) dx$ $= \frac{1}{2} \int [\cos(2x) - \cos(6x)] \cdot \sin(6x) dx$ $= \frac{1}{2} \int \sin(6x) \cdot \cos(2x) - \sin(6x) \cdot \cos(6x) dx$ $= \frac{1}{4} \int 2\sin(6x)\cos(2x) - 2\sin(6x) \cdot \cos(6x) dx$ $= \frac{1}{4} \int \sin(8x) + \sin(4x) - \sin(12x) dx$ $= \frac{1}{4} \left[\frac{-\cos(8x)}{8} - \frac{\cos(4x)}{4} + \frac{\sin(12x)}{12} \right] + c \quad \text{ans.}$ (ii) $I = \int \frac{\sin(4x)}{\sin x} dx$ $= 2 \int \frac{\sin(2x) \cdot \cos(2x)}{\sin x} dx$ $= 4 \int \frac{\sin x \cdot \cos x \cdot \cos(2x)}{\sin x} dx$ $= \int 2 \cos x \cdot \cos(2x) dx$ $= 2 \int \cos(3x) + \cos x dx \quad \dots \{2 \cos A \cos B = \cos(A+B) + \cos(A-B)\}$ $= 2 \left[\frac{\sin(3x)}{3} + \sin x \right] + c \quad \text{ans.}$
Q.7)	(i) $I = \int \tan x \cdot \sec^4 x dx$ (ii) $I = \int \tan^3 x \cdot \sec^3 x dx$ (iii) $I = \int \sec^n x \cdot \tan x dx$
Sol.7)	(i) $I = \int \tan x \cdot \sec^4 x dx$ $= \int \tan x \cdot (1 + \tan^2 x) \cdot \sec^2 x dx$ put $\tan x = t$ $\therefore \sec^2 x dx = dt$ $I = \int t(1 + t^2) dt$ $= \int t + t^3 dt$ $= \frac{t^2}{2} + \frac{t^4}{4} + c$ $I = \frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c \quad \text{ans.}$ (ii) $I = \int \tan^3 x \cdot \sec^3 x dx$ $= \int \tan^2 x \cdot \sec^2 x \cdot \tan x \cdot \sec x dx$

	$= \int (\sec^2 x - 1) \cdot \sec^2 x - \tan x \cdot \sec x dx$ put $\sec x = t$ $\sec x \tan x dx = dt$ $I = \int (t^2 - 1)t^2 dt$ $= \int t^4 - t^2 dt$ $= \frac{t^5}{5} - \frac{t^3}{3} + c$ $I = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + c \quad \text{ans.}$ (iii) $I = \int \sec^n x \cdot \tan x dx$ $= \int \sin^{n-1} x \cdot \tan x \cdot \sec x dx$ put $\sec x = t$ $\sec x \tan x dx = dt$ $I = \int t^{n-1} dt$ $= \frac{t^n}{n} + c$ $I = \frac{\sec^n x}{n} + c \quad \text{ans.}$
	→ <u>QNS Based On Sin(A ± B) and Cos(A ± B) :-</u>
Q.8)	$(i) I = \int \frac{\sin(x-a)}{\sin x} dx \quad (ii) I = \int \frac{\sin x}{\sin(x+a)} dx \quad (iii) I = \int \frac{\sin(x+a)}{\sin(x+b)} dx$
Sol.8)	(i) $I = \int \frac{\sin(x-a)}{\sin x} dx$ $= \int \frac{\sin x \cos a - \cos x \sin a}{\sin x} dx$ Separate $= \int \cos a - \cot x \cdot \sin a dx$ $I = x \cos a + \log \sin x \cdot \sin a + c$ (ii) $I = \int \frac{\sin x}{\sin(x+a)} dx$ $= \int \frac{\sin(x+a-a)}{\sin(x+a)} dx$ $= \int \frac{\sin(x+a) \cos a - \cos(x+a) \sin a}{\sin(x+a)} dx$ $= \int \cos a - \cot(x+a) \sin a dx$ $I = x \cos a - \log \sin(x+a) \sin a + c \quad \text{ans.}$ (iii) $I = \int \frac{\sin(x+a)}{\sin(x+b)} dx$ $= \int \frac{\sin(x+a+b-b)}{\sin(x+b)} dx$ $= \int \frac{\sin[(x+b)+(a-b)]}{\sin(x+b)} dx$ $= \int \frac{\sin(x+b) \cos(a-b) + \cos(x+b) \sin(a-b)}{\sin(x+b)} dx$

	$= \int \cos(a-b) + \cot(x+b)\sin(a-b)dx$ $I = x \cos(a-b) + \log \sin(x+b) \sin(a-b) + c \text{ ans.}$
Q.9)	(i) $I = \int \frac{\sin(2x)}{\sin(5x)\sin(3x)} dx$
Sol.9)	$(i) I = \int \frac{\sin(2x)}{\sin(5x)\sin(3x)} dx$ $= \int \frac{\sin(5x-3x)}{\sin(5x).\sin(3x)} dx$ $= \int \frac{\sin(3x)\cos(3x)-\cos(5x).\sin(3x)}{\sin(5x).\sin(3x)} dx$ Separate $I = \int \cot(3x) - \cot(5x)dx$ $= \frac{1}{3} \log \sin(3x) - \frac{1}{5} \log \sin(5x) + c \text{ ans.}$
Q.10)	$(i) I = \int \frac{1}{\sin(x-a)\sin(x-b)} dx \quad (ii) I = \int \frac{1}{\cos(x+a)\cos(x+b)} dx$ $(iii) I = \int \frac{1}{\sin(x+b)\cos(x-b)} dx$
Sol.10)	$(i) I = \int \frac{1}{\sin(x-a)\sin(x-b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\sin(x-a)\sin(x-b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b+x-x)}{\sin(x-a)\sin(x-b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin[(x-b)-(x-a)]}{\sin(x-a)\sin(x-b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(x-b)\cos(x-a)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)} dx$ Separate $= \frac{1}{\sin(a-b)} \int \cot(x-a) - \cot(x-b)dx$ $= \frac{1}{\sin(a-b)} \int [\log \sin(x-a) - \log \sin(x-b)] + c$ $I = \frac{1}{\sin(a-b)} \log \left \frac{\sin(x-a)}{\sin(x-b)} \right + c \text{ ans.}$ $(ii) I = \int \frac{1}{\cos(x+a)\cos(x+b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b)}{\cos(x+a).\cos(x+b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(a-b+x-x)}{\cos(x+a).\cos(x+b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin[(x+a)-(x+b)]}{\cos(x+a).\cos(x+b)} dx$ $= \frac{1}{\sin(a-b)} \int \frac{\sin(x+a)\cos(x+b)-\cos(x+a)\sin(x+b)}{\cos(x+a).\cos(x+b)} dx$ Separate: $= \frac{1}{\sin(a-b)} \int \tan(x+a) - \tan(x+b)dx$

	$= \frac{1}{\sin(a-b)} [\log \sec(x+a) - \log \sec(x+b)] + c$ $I = \frac{1}{\sin(a-b)} \log \left \frac{\sec(x+a)}{\sec(x+b)} \right + c \quad \text{ans.}$
	$(iii) I = \int \frac{1}{\sin(x-a)\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b)}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(a-b+x-x)}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos[(x-b)-(x-a)]}{\sin(x-a).\cos(x-b)} dx$ $= \frac{1}{\cos(a-b)} \int \frac{\cos(x-b)\cos(x-a)+\sin(x-b)\sin(x-a)}{\sin(x-a).\cos(x-b)} dx$ Separate $= \frac{1}{\cos(a-b)} \int \cot(x-a) + \tan(x-b) dx$ $I = \frac{1}{\cos(a-b)} [\log \sin(x-a) + \log \sec(x-b)] + c \quad \text{ans.}$