Class 12^{th}
Relations & Functions

Q.1)	Show that the relation R in set Z given by $R\{(a\,,b)\,:\,2\;divides\;\;a-b\}$ is an Equivalence relation.
Sol.1)	We have, $R\;=\{(a\,,b)\,:\,2\;divide\;a-b\}$ Symmetric : let $(a\,,b)\;\in\;R$ $\Rightarrow\;a-b\;is\;divisible\;by\;2$ $\Rightarrow\;a-b\;=2\lambda$ ......$\{\lambda\epsilon Z\}$ $\Rightarrow\;b-a\;=-2\lambda$ which is also divisible by 2 $\Rightarrow\;(b\,,a)\;\in$ $\therefore$ R is Symmetric Reflexive : for each $a\;\in\;Z$ $\Rightarrow\;a-a\;=\;0$ which is divisible by 2 $\Rightarrow\;(a\,,a)\;\in\;R$ $\therefore$ R is Reflexive Transitive : let $(a,b)\;\in\;R$ and $(b\,,c)\;\in\;R$ $\Rightarrow\;a-b\;=2\lambda$ and $b-c\;=\;2\,k$ .....$\{\lambda,k\epsilon Z\}$ Now , $a-c\;=\;(a-b)\;+\;(b-c)$ $\Rightarrow\;a-c\;=2\lambda+2k$ $\Rightarrow\;a-c\;=2(\lambda+k)$which is also divisible by 2 $\Rightarrow\;(a\,,c)\;\in\;R$ $\therefore$ R is transitive since R is Symmetric , Reflexive and transitive $\therefore R$ is an Equivalence relation ans.
Q.2)	Show that the relation R in the set A, $A\;=\{x\;\in\;z\;:\;0\;\leq\;x\;\leq\;12\}$ given by $R\;=\{(a,b)\,:\,(a-b)$ is multiple of 4} is an equivalence relation. Find the set of all the elements in set A which are related to 1.
Sol.2)	We have , $R\;=\{(a\,,b)\,:\,\|a-b\|$ is multiple of 4} Symmetric : let $(a,b)\;\in\;R$ $\Rightarrow\;\|a-b\|$ is multiple of 4 $\Rightarrow\;\|a-b\|\;=4\lambda$ ......$(\lambda\epsilon z)$ $\Rightarrow\;\|b-a\|\;=4\lambda$ which is multiple by 4 $\Rightarrow\;(b,a)\;\in\;R$ $\therefore$ R is Symmetric Reflexive : for each $a\;\in\;A$ we have, $\|a-a\|\;=\;0$ which is multiple of 4 $\Rightarrow\;(a\,,a)\;\in\;R$ $\therefore$ R is Reflexive Transitive :

<table>
<tr><td></td><td>let $(a , b) \in R$ and $(b , c) \in R$
$\Rightarrow |a - b| = 4\lambda$ and $|b - c| = 4k$ .....$\{\lambda, k \epsilon Z\}$
$\Rightarrow (a - b) = \pm 4\lambda$ and $(b - c) = -4k$
Now , $(a - c) = (a - b) + (b - c)$
$\Rightarrow (a - c) = \pm 4\lambda \pm 4k$
$\Rightarrow (a - c) = \pm 4(\lambda + k)$
$\Rightarrow |a - c| = |\lambda + k|$ which is multiple of 4
$\Rightarrow (a , c) \in R$
∴ R is transitive
∴ R is an Equivalence relation
The elements which related to 1 are 1, 5, 9
∴ required set is {1, 5, 9} ans.</td></tr>
<tr><td>Q.3)</td><td>Let R be a relation on the set "A" of ordered pairs defined by $(x , y) R(u , v)$ if and only if $xv = yu$. Show that R is an equivalence relation.</td></tr>
<tr><td>Sol.3)</td><td>Given : A → set of ordered pairs
$(x , y) R(u , v) \Rightarrow xv = yu$
Symmetric :
let $(x , y) R (u , v)$
$\Rightarrow xv = yu$
$\Rightarrow vx = uy$
$\Rightarrow uy = vx \Rightarrow (4 , v) R(x , y)$
(Rough work)
$((u , v) R(x , y))$
$(uy = vx)$
∴ R is Symmetric

Reflexive :
for each (x , y) ∈ A
$\Rightarrow xy = yx$
$\Rightarrow (x , y) R(x , y)$
∴ R is Reflexive
{Rough work}
$\{(x , y) R(xy)\}$
$\{xy = yx\}$

Transitive :
let $(x , y) R(u , v)$ and $(u , v) R(a , b)$
$\Rightarrow xv = yu$ and $ub = va$
$\Rightarrow xv = yu$ and $v = \frac{ub}{a}$ .....$\{Rough\ (x , y) R(a , b) , xb = ya\}$
$\Rightarrow x\left(\frac{ub}{a}\right)$= yu
$\Rightarrow xb = ya$
$\Rightarrow (x , y) R(a , b)$
∴ R is transitive
since R is Symmetric , Reflexive as well as transitive
∴ R is an Equivalence relation ans.</td></tr>
<tr><td>Q.4)</td><td>If R_1 and R_2 are equivalence relations in set A , show that $R_1 \cap R_2$is also on equivalence relation.</td></tr>
<tr><td>Sol.4)</td><td>Given :- R_1 and R_2 are equivalence relations
Symmetric :</td></tr>
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	let $(a, b) \in R_1 \cap R_2$ $\Rightarrow (a, b) \in R_1$ and $(a, b) \in R_2$ $\Rightarrow (b, a) \in R_1$ and $(b, a) \in R_2$ ....{$\because$ R and R are symmetric relations} $\Rightarrow (b, a) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$is Symmetric Reflexive : for each $a \in A$ we have, $(a, a) \in R_1$ and $(a, a) \in R_2$ $\ldots\ldots\ldots\ldots\{R1\ and\ R_2\ are\ reflexive\}$ $\Rightarrow (a, a) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$is Reflexive Transitive : let $(a, b) \in R_1 \cap R_2$ and $(b, c) R_1 \cap R_2$ $\Rightarrow (a, b) \in R_1\ and\ (a, b) \in R_2\ and\ (b, c) \in R_1\ \&\ (b, c) \in R_2$ $\Rightarrow (a, b) \in R_1\ and\ (b, c) \in R_1$ $\mid (a, b)\ R\ and\ (b, c) \in R_2$ $\Rightarrow (a, c) \in R$ $\mid (a, c) \in R_2$ $\ldots.\{R_1\ \&\ R_2\ are\ transitive\}$ $\Rightarrow (a, c) \in R_1 \cap R_2$ $\therefore$ $R_1 \cap R_2$is transitive since $R_1 \cap R_2$ is Symmetric , Reflexive as well as transitive $\therefore R_1 \cap R_2$is an Equivalence relation ans.
Q.5)	R is a relation on set N given by $aRb \leftrightarrow b$ is divisible by $a; a.b \in N$check whether R is Symmetric , reflexive and transitive.
Sol.5)	We have,$aRb \leftrightarrow b$ is divisible by a Symmetric : $2R6 \Rightarrow 6$ is divisible by 2 ....$\left\{\frac{6}{2} = 3\right\}$ but $6R2 \Rightarrow 2$is not div by 6 .....$\left\{\frac{2}{6} = \frac{1}{2}\right\}$ $\therefore$ R is not symmetric Reflexive : for each $a \in N$ a is always divisible by a $\Rightarrow aRa$ $\therefore$ R is Reflexive Transitive : let aRb and bRc $\Rightarrow$ b is divisible by a and c is div by b $\Rightarrow$ $b = a\lambda$ and $c = bk$ ....$\{\lambda, k \epsilon N\}$ $\Rightarrow$ $c = (a\lambda)k$ ....$\{\because b = a\lambda\}$ $\Rightarrow \frac{c}{a} = \lambda k$ clearly c is div by a $\Rightarrow aRc$ $\therefore$ R is transitive ans.
Q.6)	R be relation in P(x) , where x is a non-empty set , given by ARB if only if ACB , where A & B are subsets in $P(x)$. Is R is an equivalence relation on $P(x)$? Justify

	your answer.
Sol.6)	Let ARB $\Rightarrow \ A \subset B$ then it is not necessary that B is a subset of A i.e. $B \not\subset A$ $\Rightarrow$ B R A ∴ R is not symmetric and hence R is not an equivalence relation eg. $x = \{1,2,3\}$ $P(x) = \{\{1\}\{2\}\{3\}\{1,2\}\{2,3\}\{1,3\}\{1,2,3\}\}$ clearly $\{2\} \subset \{1,2\}$ between$\{1,2\} \subset \{2\}$ ∴ R is not symmetric ans.
Q.7)	Show that the relation R defined in the set A of all triangles as R-{(T_1 , T_2) : T_1 is similar to T_2} is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1 , T_2 and T_3 are related ?
Sol.7)	A → set of are triangles $R = \{(T_1, T_2) : T_1 \sim T_2\}$ Symmetric : let $(T_1, T_2) \in R$ $\Rightarrow T_1 \sim T_2$ $\Rightarrow T_2 \sim T_1$ $\Rightarrow (T_2, T_1) \in R$ ∴ R is symmetric Reflexive : for each triangle $T \in A$ $(T, T) \in R$ since every triangle is similar to itself ∴ R is reflexive Transitive : let $(T_1, T_2) \in R$ and $(T_2 \sim T_3) \in R$ $\Rightarrow T_1 \sim T_2 \ and \ T_2 \sim T_3$ $\Rightarrow T_1 \sim T_3$ $\Rightarrow T_1, T_3) \in R$ ∴ R is transitive and hence R is an equivalence relation $T_1 : 3, 4, 5$ $T_2 : 5, 12, 13$ $T_3 : 6, 8, 10$ clearly sides of triangles T_1 and T_3 are in equal proportion i.e $\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$ ∴ $T_1 \sim T_3$ $\Rightarrow$ T_1 and T_3 are related to each other ans.
Q.8)	Check whether the relation R in R (real no's) define by $R = (a, b): a \le b^3$is reflexive, symmetric or transitive.

Sol.8)	Symmetric : $(1,2) \in R$ as $1 \leq 2^3$ but $(2,1) \notin R$ since $2 \nleq 13$ ∴ R is not symmetric Reflexive : $\frac{1}{2} \in R$ but $\left(\frac{1}{2},\frac{1}{2}\right) \notin R$ as $\frac{1}{2} \nleq \left(\frac{1}{2}\right)^3$ ∴ R is not reflexive Transitive : $(9,4) \in R$ and $(4,2) \in R$ as $9 \leq 4^3$ and $4 \leq 2^3$ but $(9,2) \notin R$ since $9 \nleq 2^3$ ∴ R is not transitive ans.
Q.9)	Show that the relation R in the set {1,2,3} given by R = {(1 , 1), (2 , 2), (3 , 3), (1 , 2), (2 , 3)} is reflexive neither symmetric nor transitive.
Sol.9)	We have, $A = \{1,2,3\}$ $R = \{(1,1),(2,2),(3,3),(1,2),(2,3)\}$ since $(1,2) \in R$ but $(2,1) \notin R$ ∴ R is not Symmetric $(1,2) \in R$ and $(2,3) \in R$ but $(1,3) \notin R$ ∴ R is not transitive for each $a \in A$ $(a,a) \in R$ i.e. $(1,1),(2,2),(3,3) \in R$ ∴ R is reflexive ans.
Q.10)	Determine whether each of the following relations are reflexive, symmetric and transitive (i) Relation in set A = {1,2,3,.......... 13,14} defined by $R = (x,y): 3x - y = 0$. (ii) Relation in N defined as $R = (x,y): y = x + 5; x < 4$. (iii) Relation in set A = {1,2,3,4,5,6} defined as $R = (x,y): y$ is divisible by x. (iv) Relation in Z defined as $R = (x,y): x - y$ is an integer. (v) Relation in R (real nos) defined as $R = (a,b): a \leq b^2$.
Sol.10)	(i) $R = \{(1,3),(2,6),(3,9),(4,12)\}$ ......$(y = 3x)$ clearly $(1,3) \in R$ but $(3,1) \notin R$ ∴ not symmetric $1 \in A$ but $(1,1) \notin R$ ∴ not reflexive

	$(1,3) \in R$and $(3,9) \in R$but$(1,9) \notin R$ $\therefore$ not transitive (ii) $R = \{(1,6),(2,7),(3,8)\}$ …..$\{\ldots\ y = x + 5\ and\ x < 4\}$ Do yourself (iii) $R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),(4,4),(5,5),(6,6)\}\ldots\{\ldots y\ is\ divisible\ by\ x\}$ clearly for each $a \in A$ $(a\ ,a) \in R$ i.e. $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \in R$ $\therefore$ R is reflexive $(1\ ,2) \in R$ but$(2,1) \notin R$ since 1 in not divisible by 2 $\therefore$ R is not transitive for each $(a\ ,b)$ and $(b\ ,c) \in R$ clearly $(a\ ,c) \in R$ $\therefore$ R is transitive (iv) Symmetric let $(x,y) \in R$ $\Rightarrow$ $x - y = \lambda$ ….. where $\lambda \rightarrow$ integer $\Rightarrow$ $y - x = -\lambda$ which is also an integer $\Rightarrow (y\ ,x) \in R$ $\therefore$ R is Symmetric Reflexive and transitive (Do yourself) (v) give same examples as in case of $a \leq b^3$ It is neither symmetric, nor reflexive, nor transitive.