

Exercise -1.1

1. Is zero a rational number? Can you write it in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$?

Sol:

Yes, zero is a rotational number. It can be written in the form of $\frac{p}{q}$ where q to as such as

$$\frac{0}{3}, \frac{0}{5}, \frac{0}{11}, \text{etc.....}$$

2. Find five rational numbers between 1 and 2.

Sol:

Given to find five rotational numbers between 1 and 2

A rotational number lying between 1 and 2 is

$$(1+2) \div 2 = 3 \div 2 = \frac{3}{2} \quad \text{i.e., } 1 < \frac{3}{2} < 2$$

Now, a rotational number lying between 1 and $\frac{3}{2}$ is

$$\left(1 + \frac{3}{2}\right) \div 2 = \left(\frac{2+3}{2}\right) \div 2 = \frac{5}{2} \div 2 = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$$

$$\text{i.e., } 1 < \frac{5}{4} < \frac{3}{2}$$

Similarly, a rotational number lying between 1 and $\frac{5}{4}$ is

$$\left(1 + \frac{5}{4}\right) \div 2 = \left(\frac{4+5}{4}\right) \div 2 = \frac{9}{4} \div 2 = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$$

$$\text{i.e., } 1 < \frac{9}{8} < \frac{5}{4}$$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(1 + \frac{5}{4}\right) \div 2 = \left(\frac{4+5}{4}\right) \div 2 = \frac{9}{4} \div 2 = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8}$$

$$\text{i.e., } 1 < \frac{9}{8} < \frac{5}{4}$$

Now, a rotational number lying between $\frac{3}{2}$ and 2 is

$$\left(\frac{3}{2} + 2\right) \div 2 = \left(\frac{3+4}{2}\right) \div 2 = \frac{7}{2} \times \frac{1}{2} = \frac{7}{4}$$

$$\text{i.e., } \frac{3}{2} < \frac{7}{4} < 2$$

Similarly, a rotational number lying between $\frac{7}{4}$ and 2 is

$$\left(\frac{7}{4} + 2\right) \div 2 = \left(\frac{7+8}{4}\right) \div 2 = \frac{15}{4} \times \frac{1}{2} = \frac{15}{8}$$

$$\text{i.e., } \frac{7}{4} < \frac{15}{8} < 2$$

$$\therefore 1 < \frac{9}{8} < \frac{5}{4} < \frac{3}{2} < \frac{7}{4} < \frac{15}{8} < 2$$

Recall that to find a rational number between r and s , you can add r and s and divide the sum by 2, that is $\frac{r+s}{2}$ lies between r and s . So, $\frac{3}{2}$ is a number between 1 and 2. you can

proceed in this manner to find four more rational numbers between 1 and 2, These four numbers are, $\frac{5}{4}, \frac{11}{8}, \frac{13}{8}$ and $\frac{7}{4}$

3. Find six rational numbers between 3 and 4.

Sol:

Given to find six rotational number between 3 and 4

We have,

$$3 \times \frac{7}{7} = \frac{21}{7} \text{ and } 4 \times \frac{7}{7} = \frac{28}{7}$$

We know that

$$21 < 22 < 23 < 24 < 25 < 26 < 27 < 28$$

$$\Rightarrow \frac{21}{7} < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < \frac{28}{7}$$

$$\Rightarrow 3 < \frac{22}{7} < \frac{23}{7} < \frac{24}{7} < \frac{25}{7} < \frac{26}{7} < \frac{27}{7} < 4$$

Hence, 6 rotational number between 3 and 4 are

$$\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$$

4. Find five rational numbers between $\frac{3}{4}$ and $\frac{4}{5}$

Sol:

Given to find 5 rotational numbers lying between $\frac{3}{5}$ and $\frac{4}{5}$.

We have,

$$\frac{3}{5} \times \frac{6}{6} = \frac{18}{100} \text{ and } \frac{4}{5} \times \frac{6}{6} = \frac{24}{30}$$

We know that

$$18 < 19 < 20 < 21 < 22 < 23 < 24$$

$$\Rightarrow \frac{18}{30} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30} < \frac{23}{30} < \frac{24}{30}$$

$$\Rightarrow \frac{3}{5} < \frac{19}{30} < \frac{20}{30} < \frac{21}{30} < \frac{22}{30}, \frac{23}{30}, \frac{4}{5}$$

$$\Rightarrow \frac{3}{5} < \frac{19}{30} < \frac{2}{3} < \frac{7}{10} < \frac{11}{15} < \frac{23}{30} < \frac{4}{5}$$

Hence, 5 rotational number between $\frac{3}{5}$ and $\frac{4}{5}$ are

$$\frac{19}{30}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{23}{30}.$$

5. Are the following statements true or false? Give reasons for your answer.

- (i) Every whole number is a rational number.
- (ii) Every integer is a rational number.
- (iii) Every rational number is a integer.
- (iv) Every natural number is a whole number.
- (v) Every integer is a whole number.
- (vi) Evert rational number is a whole number.

Sol:

- (i) False. As whole numbers include zero, whereas natural number does not include zero
- (ii) True. As integers are a part of rotational numbers.
- (iii) False. As integers are a part of rotational numbers.
- (iv) True. As whole numbers include all the natural numbers.
- (v) False. As whole numbers are a part of integers
- (vi) False. As rotational numbers includes all the whole numbers.

Exercise – 1.2

Express the following rational numbers as decimals:

1. (i) $\frac{42}{100}$ (ii) $\frac{327}{500}$ (iii) $\frac{15}{4}$

Sol:

(i) By long division, we have

$$\begin{array}{r} 100 \overline{) 42.00} \quad 0.42 \\ \underline{400} \\ 200 \\ \underline{200} \\ 0 \end{array}$$

$$\therefore \boxed{\frac{42}{100} = 0.42}$$

(ii) By long division, we have

$$\begin{array}{r} 500 \overline{) 327.000} \quad 0.654 \\ \underline{3000} \\ 2700 \\ \underline{2500} \\ 2000 \\ \underline{2000} \\ 0 \end{array}$$

$$\therefore \boxed{\frac{327}{500} = 0.654}$$

(iii) By long division, we have

$$\begin{array}{r} 4 \overline{) 15.00} \quad 3.75 \\ \underline{12} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

$$\therefore \boxed{\frac{15}{4} = 3.75}$$

2. (i) $\frac{2}{3}$ (ii) $-\frac{4}{9}$ (iii) $\frac{-2}{15}$ (iv) $-\frac{22}{13}$ (v) $\frac{437}{999}$

Sol:

- (i) By long division, we have

$$\begin{array}{r}
 3 \overline{) 2.0000} \quad (0.6666 \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 20 \\
 \underline{18} \\
 2
 \end{array}$$

$$\therefore \frac{2}{3} = 0.6666..... = 0.\overline{6}$$

- (ii) By long division, we have

$$\begin{array}{r}
 9 \overline{) 4.0000} \quad (0.4444 \\
 \underline{36} \\
 40 \\
 \underline{36} \\
 40 \\
 \underline{36} \\
 40 \\
 \underline{36} \\
 4
 \end{array}$$

$$\therefore \frac{4}{9} = 0.4444..... = 0.\overline{4}$$

Hence, $-\frac{4}{9} = -0.\overline{4}$

- (iii) By long division, we have

$$\begin{array}{r}
 5 \overline{) 2.0000} \quad (0.1333 \\
 \underline{15} \\
 50 \\
 \underline{45} \\
 50 \\
 \underline{45}
 \end{array}$$

$$\begin{array}{r} 50 \\ 45 \\ \hline 5 \end{array}$$

$$\therefore \frac{2}{15} = 0.1333 \dots = 0.1\bar{3}$$

Hence, $\boxed{\frac{-2}{15} = -0.1\bar{3}}$

(iv) By long division, we have

$$\begin{array}{r} 13 \overline{)22.0000} \quad (1.692307692307 \\ \underline{-13} \\ 90 \\ \underline{-78} \\ 120 \\ \underline{-117} \\ 30 \\ \underline{-26} \\ 40 \\ \underline{-39} \\ 100 \\ \underline{-91} \\ 90 \\ \underline{-78} \\ 120 \\ \underline{-117} \\ 30 \\ 26 \end{array}$$

$$\therefore \frac{22}{13} = 1.692307692307 \dots = 1.\overline{692307} \Rightarrow \boxed{-\frac{22}{13} = -1.\overline{692307}}$$

(v) By long division, we have

$$\begin{array}{r} 999 \overline{)437.000000} \quad (0.437437 \\ \underline{3996} \\ 3740 \\ \underline{2997} \\ 7430 \\ \underline{6993} \end{array}$$

4370

3996

3740

2997

7430

6993

4370

$$\therefore \frac{437}{999} = 0.437437\ldots = 0.\overline{437}$$

(vi) By long division, we have

$$26 \overline{) 33.000000000000} (1.2692307692307$$

26

70

52

180

156

240

234

60

52

80

78

200

182

180

156

240

234

60

52

80

78

200

182

18

$$\therefore \frac{33}{26} = 1.2692307692307\ldots = 1.\overline{2692307}$$

3. Look at several examples of rational numbers in the form $\frac{p}{q}$ ($q \neq 0$), where p and q are integers with no common factors other than 1 and having terminating decimal representations. Can you guess what property q must satisfy?

Sol:

A rational number $\frac{p}{q}$ is a terminating decimal only, when prime factors of q are 2 and 5

only. Therefore, $\frac{p}{q}$ is a terminating decimal only, when prime factorization of q must have only powers of 2 or 5 or both.

Exercise -1.3

1. Express each of the following decimals in the form $\frac{p}{q}$:

- (i) 0.39
- (ii) 0.750
- (iii) 2.15
- (iv) 7.010
- (v) 9.90
- (vi) 1.0001

Sol:

- (i) We have,

$$0.39 = \frac{39}{100}$$

$$\Rightarrow 0.39 = \frac{39}{100}$$

- (ii) We have,

$$0.750 = \frac{750}{1000} = \frac{750 \div 250}{1000 \div 250} = \frac{3}{4}$$

$$\therefore 0.750 = \frac{3}{4}$$

- (iii) We have

$$2.15 = \frac{215}{100} = \frac{215 \div 5}{100 \div 5} = \frac{43}{20}$$

$$\therefore 2.15 = \frac{43}{20}$$

(iv) We have,

$$7.010 = \frac{7010}{1000} = \frac{7010 \div 10}{1000 \div 10} = \frac{701}{100}$$

$$\therefore 7.010 = \frac{701}{100}$$

(v) We have,

$$9.90 = \frac{990}{100} = \frac{990 \div 10}{100 \div 10} = \frac{99}{10}$$

$$\therefore 9.90 = \frac{99}{10}$$

(vi) We have,

$$1.0001 = \frac{10001}{10000}$$

$$\therefore 1.0001 = \frac{10001}{10000}$$

2. Express each of the following decimals in the form $\frac{p}{q}$:

(i) $0.\bar{4}$

(ii) $0.\overline{37}$

Sol:

(i) Let $x = 0.\bar{4}$

Now, $x = 0.\bar{4} = 0.444....$ --- (1)

Multiplying both sides of equation (1) by 10, we get,

$10x = 4.444....$ --- (2)

Subtracting equation (1) by (2)

$\therefore 10x - x = 4.444... - 0.444...$

$\Rightarrow 9x = 4$

$\Rightarrow x = \frac{4}{9}$

Hence, $0.\bar{4} = \frac{4}{9}$

(ii) Let $x = 0.\overline{37}$

Now, $x = 0.3737... ..$ (1)

Multiplying equation (1) by 10.

$\therefore 10x = 3.737....$ --- (2)

$$100x = 37.3737... \quad \text{---(3)}$$

Subtracting equation (1) by equation (3)

$$\therefore 100x - x = 37$$

$$\Rightarrow 99x = 37$$

$$\Rightarrow x = \frac{37}{99}$$

$$\text{Hence, } 0.\overline{37} = \frac{37}{99}$$

Exercise -1.4

1. Define an irrational number.

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number. For example, 1.01001000100001...

2. Explain, how irrational numbers differ from rational numbers?

Sol:

A number which can neither be expressed as a terminating decimal nor as a repeating decimal is called an irrational number, For example, 0.33033003300033...

On the other hand, every rational number is expressible either as a terminating decimal or as a repeating decimal. For examples, $3.\overline{24}$ and 6.2876 are rational numbers

3. Examine, whether the following numbers are rational or irrational:

(i) $\sqrt{7}$

(ii) $\sqrt{4}$

(iii) $2 + \sqrt{3}$

(iv) $\sqrt{3} + \sqrt{2}$

(v) $\sqrt{3} + \sqrt{5}$

(vi) $(\sqrt{2} - 2)^2$

(vii) $(2 - \sqrt{2})(2 + \sqrt{2})$

(viii) $(\sqrt{2} + \sqrt{3})^2$

(ix) $\sqrt{5} - 2$

(x) $\sqrt{23}$

(xi) $\sqrt{225}$

(xii) 0.3796

(xiii) 7.478478.....

(xiv) 1.101001000100001.....

Sol:

$\sqrt{7}$ is not a perfect square root, so it is an irrational number.

We have,

$$\sqrt{4} = 2 = \frac{2}{1}$$

$\therefore \sqrt{4}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

The decimal representation of $\sqrt{4}$ is 2.0.

2 is a rational number, whereas $\sqrt{3}$ is an irrational number.

Because, sum of a rational number and an irrational number is an irrational number, so

$2 + \sqrt{3}$ is an irrational number.

$\sqrt{2}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3} + \sqrt{2}$ is an irrational number.

$\sqrt{5}$ is an irrational number. Also $\sqrt{3}$ is an irrational number.

The sum of two irrational numbers is irrational.

$\therefore \sqrt{3} + \sqrt{5}$ is an irrational number.

We have,

$$\begin{aligned} (\sqrt{2} - 2)^2 &= (\sqrt{2})^2 - 2 \times \sqrt{2} \times 2 + (2)^2 \\ &= 2 - 4\sqrt{2} + 4 \\ &= 6 - 4\sqrt{2} \end{aligned}$$

Now, 6 is a rational number, whereas $4\sqrt{2}$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number.

So, $6 - 4\sqrt{2}$ is an irrational number.

$\therefore (\sqrt{2} - 2)^2$ is an irrational number.

We have,

$$\begin{aligned} (2 - \sqrt{2})(2 + \sqrt{2}) &= (2)^2 - (\sqrt{2})^2 & [\because (a-b)(a+b) = a^2 - b^2] \\ &= 4 - 2 \\ &= 2 = \frac{2}{1} \end{aligned}$$

Since, 2 is a rational number.

$\therefore (2 - \sqrt{2})(2 + \sqrt{2})$ is a rational number.

We have,

$$\begin{aligned} (\sqrt{2} + \sqrt{3})^2 &= (\sqrt{2})^2 + 2 \times \sqrt{2} \times \sqrt{3} + (\sqrt{3})^2 \\ &= 2 + 2\sqrt{6} + 3 \\ &= 5 + 2\sqrt{6} \end{aligned}$$

The sum of a rational number and an irrational number is an irrational number, so $5 + 2\sqrt{6}$ is an irrational number.

$\therefore (\sqrt{2} + \sqrt{3})^2$ is an irrational number.

The difference of a rational number and an irrational number is an irrational number

$\therefore \sqrt{5} - 2$ is an irrational number.

$$\sqrt{23} = 4.79583152331.....$$

$$\sqrt{225} = 15 = \frac{15}{1}$$

Rational number as it can be represented in $\frac{p}{q}$ form.

0.3796

As decimal expansion of this number is terminating, so it is a rational number.

$$7.478478..... = 7.\overline{478}$$

As decimal expansion of this number is non-terminating recurring so it is a rational number.

4. Identify the following as rational numbers. Give the decimal representation of rational numbers:

(i) $\sqrt{4}$

(ii) $3\sqrt{18}$

(iii) $\sqrt{1.44}$

(iv) $\sqrt{\frac{9}{27}}$

(v) $-\sqrt{64}$

(vi) $\sqrt{100}$

Sol:

We have

$$\sqrt{4} = 2 = \frac{2}{1}$$

$\sqrt{4}$ can be written in the form of $\frac{p}{q}$, so it is a rational number.

Its decimal representation is 2.0.

We have,

$$\begin{aligned} 3\sqrt{18} &= 3\sqrt{2 \times 3 \times 3} \\ &= 3 \times 3\sqrt{2} \\ &= 9\sqrt{2} \end{aligned}$$

Since, the product of a rational and an irrational is an irrational number.

$\therefore 9\sqrt{2}$ is an irrational

$\Rightarrow 3\sqrt{18}$ is an irrational number.

We have,

$$\begin{aligned} \sqrt{1.44} &= \sqrt{\frac{144}{100}} \\ &= \frac{12}{10} \\ &= 1.2 \end{aligned}$$

Every terminating decimal is a rational number, so 1.2 is a rational number.

Its decimal representation is 1.2.

We have,

$$\begin{aligned} \sqrt{\frac{9}{27}} &= \frac{3}{\sqrt{27}} = \frac{3}{\sqrt{3 \times 3 \times 3}} \\ &= \frac{3}{3\sqrt{3}} \\ &= \frac{1}{\sqrt{3}} \end{aligned}$$

Quotient of a rational and an irrational number is irrational numbers so $\frac{1}{\sqrt{3}}$ is an irrational number.

$\Rightarrow \sqrt{\frac{9}{27}}$ is an irrational number.

We have,

$$\begin{aligned} -\sqrt{64} &= -\sqrt{8 \times 8} \\ &= -8 \\ &= -\frac{8}{1} \end{aligned}$$

$-\sqrt{64}$ can be expressed in the form of $\frac{p}{q}$, so $-\sqrt{64}$ is a rational number.

Its decimal representation is -8.0.

We have,

$$\sqrt{100} = 10$$

$$= \frac{10}{1}$$

$\sqrt{100}$ can be expressed in the form of $\frac{p}{q}$, so $\sqrt{100}$ is a rational number.

The decimal representation of $\sqrt{100}$ is 10.0.

5. In the following equations, find which variables x, y, z etc. represent rational or irrational numbers:

(i) $x^2 = 5$

(ii) $y^2 = 9$

(iii) $z^2 = 0.04$

(iv) $u^2 = \frac{17}{4}$

(v) $v^2 = 3$

(vi) $w^2 = 27$

(vii) $t^2 = 0.4$

Sol:

- (i) We have

$$x^2 = 5$$

Taking square root on both sides.

$$\Rightarrow \sqrt{x^2} = \sqrt{5}$$

$$\Rightarrow x = \sqrt{5}$$

$\sqrt{5}$ is not a perfect square root, so it is an irrational number.

- (ii) We have

$$y^2 = 9$$

$$\Rightarrow y = \sqrt{9}$$

$$= 3$$

$$= \frac{3}{1}$$

$\sqrt{9}$ can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

- (iii) We have

$$z^2 = 0.04$$

Taking square root on the both sides, we get,

$$\sqrt{z^2} = \sqrt{0.04}$$

$$\begin{aligned}\Rightarrow z &= \sqrt{0.04} \\ &= 0.2 \\ &= \frac{2}{10} \\ &= \frac{1}{5}\end{aligned}$$

z can be expressed in the form of $\frac{p}{q}$, so it is a rational number.

(iv) We have

$$u^2 = \frac{17}{4}$$

Taking square root on both sides, we get,

$$\begin{aligned}\sqrt{u^2} &= \sqrt{\frac{17}{4}} \\ \Rightarrow u &= \sqrt{\frac{17}{2}}\end{aligned}$$

Quotient of an irrational and a rational number is irrational, so u is an irrational number.

(v) We have

$$v^2 = 3$$

Taking square root on both sides, we get,

$$\begin{aligned}\sqrt{v^2} &= \sqrt{3} \\ \Rightarrow v &= \sqrt{3}\end{aligned}$$

$\sqrt{3}$ is not a perfect square root, so v is an irrational number.

(vi) We have

$$w^2 = 27$$

Taking square root on both sides, we get,

$$\begin{aligned}\sqrt{w^2} &= \sqrt{27} \\ \Rightarrow w &= \sqrt{3 \times 3 \times 3} \\ &= 3\sqrt{3}\end{aligned}$$

Product of a rational and an irrational is irrational number, so w is an irrational number.

(vii) We have

$$t^2 = 0.4$$

Taking square root on both sides, we get

$$\sqrt{t^2} = \sqrt{0.4}$$

$$\Rightarrow t = \sqrt{\frac{4}{10}}$$
$$= \frac{2}{\sqrt{10}}$$

Since, quotient of a rational and an irrational number is irrational number, so t is an irrational number.

6. Give an example of each, of two irrational numbers whose:

- (i) difference is a rational number.
- (ii) difference is an irrational number.
- (iii) sum is a rational number.
- (iv) sum is an irrational number.
- (v) product is a rational number.
- (vi) product is an irrational number.
- (vii) quotient is a rational number.
- (viii) quotient is an irrational number.

Sol:

- (i) $\sqrt{3}$ is an irrational number.
Now, $(\sqrt{3}) - (\sqrt{3}) = 0$
0 is the rational number.
- (ii) Let two irrational numbers are $5\sqrt{2}$ and $\sqrt{2}$
Now, $(5\sqrt{2}) - (\sqrt{2}) = 4\sqrt{2}$
 $4\sqrt{2}$ is the irrational number.
- (iii) Let two irrational numbers are $\sqrt{11}$ and $-\sqrt{11}$
Now, $(\sqrt{11}) + (-\sqrt{11}) = 0$
0 is the rational number.
- (iv) Let two irrational numbers are $4\sqrt{6}$ and $\sqrt{6}$
Now, $(4\sqrt{6}) + (\sqrt{6}) = 5\sqrt{6}$
 $5\sqrt{6}$ is the irrational number.
- (v) Let two irrational numbers are $2\sqrt{3}$ and $\sqrt{3}$
Now, $2\sqrt{3} \times \sqrt{3} = 2 \times 3$
 $= 6$
6 is the rational number.
- (vi) Let two irrational numbers are $\sqrt{2}$ and $\sqrt{5}$
Now, $\sqrt{2} \times \sqrt{5} = \sqrt{10}$

$\sqrt{10}$ is the rational number.

(vii) Let two irrational numbers are $3\sqrt{6}$ and $\sqrt{6}$

$$\text{Now, } \frac{3\sqrt{6}}{\sqrt{6}} = 3$$

3 is the rational number.

(viii) Let two irrational numbers are $\sqrt{6}$ and $\sqrt{2}$

$$\text{Now, } \frac{\sqrt{6}}{\sqrt{2}} = \frac{\sqrt{3+2}}{\sqrt{2}}$$

$$= \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2}}$$

$$= \sqrt{3}$$

$\sqrt{3}$ is an irrational number.

7. Give two rational numbers lying between 0.23233233323332 ... and 0.21211211121112.

Sol:

Let, $a = 0.21211211121112$

And, $b = 0.23233233323332...$

Clearly, $a < b$ because in the second decimal place a has digit 1 and b has digit 3

If we consider rational numbers in which the second decimal place has the digit 2, then they will lie between a and b .

Let,

$$x = 0.22$$

$$y = 0.22112211...$$

Then,

$$a < x < y < b$$

Hence, x , and y are required rational numbers.

8. Give two rational numbers lying between 0.51511511151115 ... and 0.5353353335 ...

Sol:

Let, $a = 0.51511511151115...$

And, $b = 0.5353353335...$

We observe that in the second decimal place a has digit 1 and b has digit 3, therefore, $a < b$. So if we consider rational numbers

$$x = 0.52$$

$$y = 0.52052052...$$

We find that,

$$a < x < y < b$$

Hence x , and y are required rational numbers.

9. Find one irrational number between 0.2101 and $0.2222 \dots = 0.\bar{2}$

Sol:

Let, $a = 0.2101$

And, $b = 0.2222\dots$

We observe that in the second decimal place a has digit 1 and b has digit 2, therefore

$a < b$. in the third decimal place a has digit 0. So, if we consider irrational numbers

$x = 0.211011001100011\dots$

We find that

$a < x < b$

Hence, x is required irrational number.

10. Find a rational number and also an irrational number lying between the numbers 0.3030030003 ... and 0.3010010001 ...

Sol:

Let, $a = 0.3010010001$

And, $b = 0.3030030003\dots$

We observe that in the third decimal place a has digit 1 and b has digit 3, therefore $a < b$. in the third decimal place a has digit 1. so, if we consider rational and irrational numbers

$x = 0.302$

$y = 0.302002000200002\dots$

We find that

$a < x < b$

And, $a < y < b$

Hence, x and y are required rational and irrational numbers respectively.

11. Find two irrational numbers between 0.5 and 0.55.

Sol:

Let $a = 0.5 = 0.50$

And, $b = 0.55$

We observe that in the second decimal place a has digit 0 and b has digit 5, therefore $a < b$. so, if we consider irrational numbers

$x = 0.51051005100051\dots$

$y = 0.530535305353530\dots$

We find that

$a < x < y < b$

Hence, x and y are required irrational numbers.

12. Find two irrational numbers lying between 0.1 and 0.12.

Sol:

Let, $a = 0.1 = 0.10$

And, $b = 0.12$

We observe that in the second decimal place a has digit 0 and b has digit 2, Therefore

$a < b$. So, if we consider irrational numbers

$x = 0.11011001100011\dots$

$y = 0.111011110111110\dots$

We find that,

$a < x < y < b$

Hence, x and y are required irrational numbers.

13. Prove that $\sqrt{3} + \sqrt{5}$ is an irrational number.

Sol:

If possible, let $\sqrt{3} + \sqrt{5}$ be a rational number equal to x . Then,

$$x = \sqrt{3} + \sqrt{5}$$

$$\Rightarrow x^2 = (\sqrt{3} + \sqrt{5})^2$$

$$\Rightarrow x^2 = (\sqrt{3})^2 + (\sqrt{5})^2 + 2 \times \sqrt{3} \times \sqrt{5}$$

$$= 3 + 5 + 2\sqrt{15}$$

$$= 8 + 2\sqrt{15}$$

$$\Rightarrow x^2 - 8 = 2\sqrt{15}$$

$$\Rightarrow \frac{x^2 - 8}{2} = \sqrt{15}$$

Now, x is rational

$\Rightarrow x^2$ is rational

$\Rightarrow \frac{x^2 - 8}{2}$ is rational

$\Rightarrow \sqrt{15}$ is rational

But, $\sqrt{15}$ is irrational

Thus, we arrive at a contradiction. So, our supposition that $\sqrt{3} + \sqrt{5}$ is rational is wrong.

Hence, $\sqrt{3} + \sqrt{5}$ is an irrational number.

14. Find three different irrational numbers between the rational numbers $\frac{5}{7}$ and $\frac{9}{11}$

Sol:

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{9}{11} = 0.\overline{81}$$

3 irrational numbers are

0.73073007300073000073.....

0.75075007500075000075.....

0.79079007900079000079.....

Exercise -1.5

1. Complete the following sentences:

- Every point on the number line corresponds to a _____ number which may be either _____ or _____
- The decimal form of an irrational number is neither _____ nor _____
- The decimal representation of a rational number is either _____ or _____
- Every real number is either _____ number or _____ number.

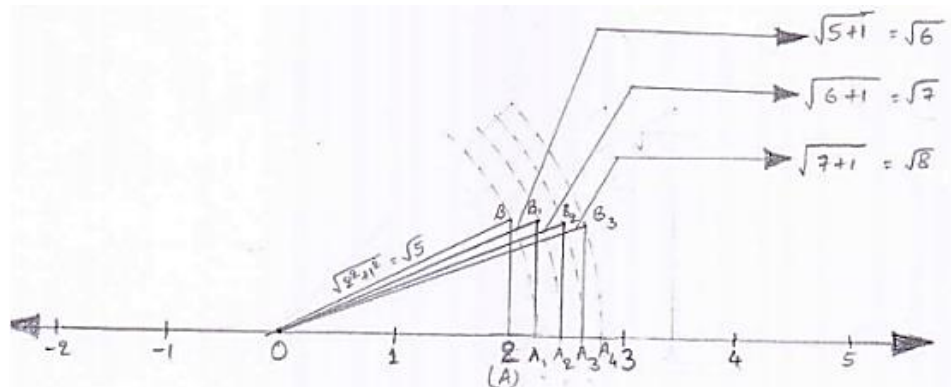
Sol:

- Every point on the number line corresponds to a **Real** number which may be either **rational** or **irrational**.
- The decimal form of an irrational number is neither **terminating** nor **repeating**
- The decimal representation of a rational number is either **terminating, non-terminating** or **recurring**.
- Every real number is either **a rational** number or **an irrational** number.

2. Represent $\sqrt{6}, \sqrt{7}, \sqrt{8}$ on the number line.

Sol:

Draw a number line and mark point O, representing zero, on it



Suppose point A represents 2 as shown in the figure

Then $OA = 2$. Now, draw a right triangle OAB such that $AB = 1$.

By Pythagoras theorem, we have

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow OB^2 = 2^2 + 1^2$$

$$\Rightarrow OB^2 = 4 + 1 = 5 \Rightarrow OB = \sqrt{5}$$

Now, draw a circle with center O and radius OB.

We find that the circle cuts the number line at A

Clearly, $OA_1 = OB = \text{radius of circle} = \sqrt{5}$

Thus, A_1 represents $\sqrt{5}$ on the number line.

But, we have seen that $\sqrt{5}$ is not a rational number. Thus we find that there is a point on the number which is not a rational number.

Now, draw a right triangle OA_1B_1 , Such that $A_1B_1 = AB = 1$

Again, by Pythagoras theorem, we have

$$(OB_1)^2 = (OA_1)^2 + (A_1B_1)^2$$

$$\Rightarrow (OB_1)^2 = (\sqrt{5})^2 + (1)^2$$

$$\Rightarrow (OB_1)^2 = 5 + 1 = 6 \Rightarrow OB_1 = \sqrt{6}$$

Draw a circle with center O and radius $OB_1 = \sqrt{6}$. This circle cuts the number line at A_2 as shown in figure

Clearly $OA_2 = OB_1 = \sqrt{6}$

Thus, A_2 represents $\sqrt{6}$ on the number line.

Also, we know that $\sqrt{6}$ is not a rational number.

Thus, A_2 is a point on the number line not representing a rational number

Continuing in this manner, we can represent $\sqrt{7}$ and $\sqrt{8}$ also on the number lines as shown in the figure

Thus, $OA_3 = OB_2 = \sqrt{7}$ and $OA_4 = OB_3 = \sqrt{8}$

3. Represent $\sqrt{3 \cdot 5}$, $\sqrt{9 \cdot 4}$, $\sqrt{10 \cdot 5}$ on the real number line.

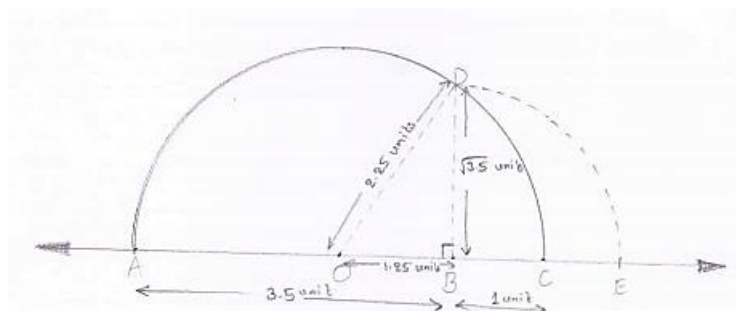
Sol:

Given to represent $\sqrt{3 \cdot 5}$, $\sqrt{9 \cdot 4}$, $\sqrt{10 \cdot 5}$ on the real number line

Representation of $\sqrt{3 \cdot 5}$ on real number line:

Steps involved:

- (i) Draw a line and mark A on it.



- (ii) Mark a point B on the line drawn in step - (i) such that $AB = 3.5$ units
- (iii) Mark a point C on AB produced such that $BC = 1$ unit
- (iv) Find mid-point of AC. Let the midpoint be O
 $\Rightarrow AC = AB + BC = 3.5 + 1 = 4.5$
 $\Rightarrow AO = OC = \frac{AC}{2} = \frac{4.5}{2} = 2.25$
- (v) Taking O as the center and $OC = OA$ as radius drawn a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D. Consider triangle OBD , it is right angled at B.

$$BD^2 = OD^2 - OB^2$$

$$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \quad [\because OC = OD = \text{radius}]$$

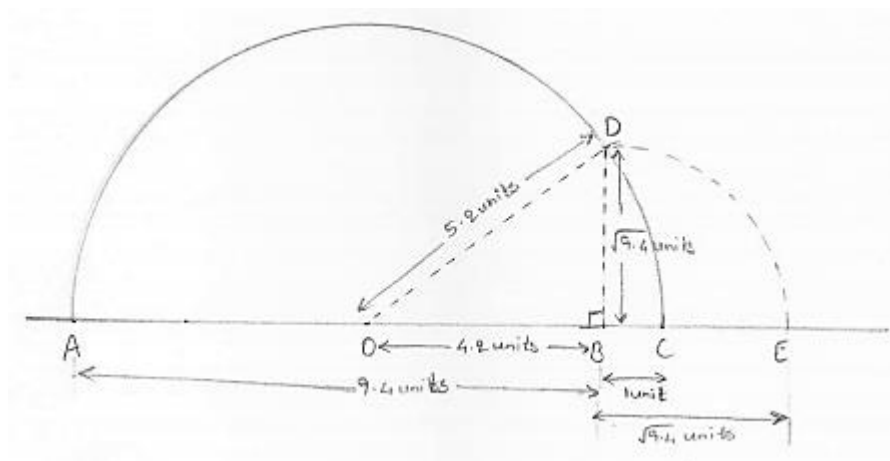
$$\Rightarrow BD^2 = 2OC \cdot BC - (BC)^2$$

$$\Rightarrow BD = \sqrt{2 \times 2.25 \times 1 - (1)^2} \Rightarrow BD = \sqrt{3.5}$$

- (vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents $\sqrt{3.5}$ as $BD = BE = \sqrt{3.5} = \text{radius}$
 Thus, E represents the required point on the real number line.

Representation of $\sqrt{9.4}$ on real number line steps involved:

- (i) Draw a line and mark A on it

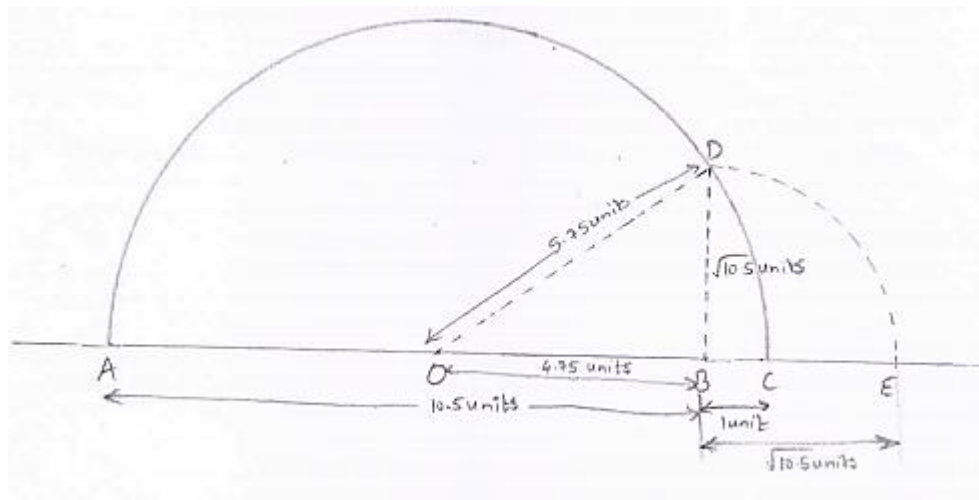


- (ii) Mark a point B on the line drawn in step (i) such that $AB = 9 \cdot 4$ units
- (iii) Mark a point C on AB produced such that $BC = 1$ unit.
- (iv) Find midpoint of AC. Let the midpoint be O.
 $\Rightarrow AC = AB + BC = 9 \cdot 4 + 1 = 10 \cdot 4$ units
 $\Rightarrow AD = OC = \frac{AC}{2} = \frac{10 \cdot 4}{2} = 5 \cdot 2$ units
- (v) Taking O as the center and $OC = OA$ as radius draw a semi-circle. Also draw a line passing through B perpendicular to OB. Suppose it cuts the semi-circle at D. Consider triangle OBD, it is right angled at B.
 $\Rightarrow BD^2 = OD^2 - OB^2$
 $\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \quad [\because OC = OD = \text{radius}]$
 $\Rightarrow BD^2 = OC^2 - (OC^2 - 2OC \cdot BC + (BC)^2)$
 $\Rightarrow BD^2 = 2OC \cdot (BC - (BC)^2)$
 $\Rightarrow BD^2 = \sqrt{2 \times (5 \cdot 2) \times 1 - 1^2} \Rightarrow BD = \sqrt{9 \cdot 4}$ units
- (vi) Taking B as center and BD as radius draw an arc cutting OC produced at E so obtained represents $\sqrt{9 \cdot 4}$ as $BD = BE = \sqrt{9 \cdot 4} = \text{radius}$
 Thus, E represents the required point on the real number line.

Representation of $\sqrt{10 \cdot 5}$ on the real number line:

Steps involved:

- (i) Draw a line and mark A on it



- (ii) Mark a point B on the line drawn in step (i) such that $AB = 10 \cdot 5$ units
- (iii) Mark a point C on AB produced such that $BC = 1$ unit
- (iv) Find midpoint of AC. Let the midpoint be O.
 $\Rightarrow AC = AB + BC = 10 \cdot 5 + 1 = 11 \cdot 5$ units

$$\Rightarrow AO = OC = \frac{AC}{2} = \frac{11 \cdot 5}{2} = 5 \cdot 75 \text{ units}$$

- (v) Taking O as the center and $OC = OA$ as radius, draw a semi-circle. Also draw a line passing through B perpendicular to DB. Suppose it cuts the semi-circle at D. consider triangle OBD, it is right angled at B

$$\Rightarrow BD^2 = OD^2 - OB^2$$

$$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \quad [\because OC = OD = \text{radius}]$$

$$\Rightarrow BD^2 = OC^2 - [OC^2 - 2OC \cdot BC + (BC)^2]$$

$$\Rightarrow BD^2 = 2OC \cdot BC - BC^2$$

$$\Rightarrow BC^2 = OD^2 - OB^2$$

$$\Rightarrow BD^2 = OC^2 - (OC - BC)^2 \quad [\because OC = OD = \text{radius}]$$

$$\Rightarrow BD^2 = OC^2 - [OC^2 - 2OC \cdot BC + (BC)^2]$$

$$\Rightarrow BD^2 = 2OC \cdot BC - BC^2$$

$$\Rightarrow BD = \sqrt{2 \times 575 \times 1 - (1)^2} \Rightarrow BD = \sqrt{10 \cdot 5}$$

- (vi) Taking B as the center and BD as radius draw an arc cutting OC produced at E. point E so obtained represents $\sqrt{10 \cdot 5}$ as $BD = BE = \sqrt{10 \cdot 5} = \text{radius arc}$
Thus, E represents the required point on the real number line

4. Find whether the following statements are true or false.

- (i) Every real number is either rational or irrational.
- (ii) it is an irrational number.
- (iii) Irrational numbers cannot be represented by points on the number line.

Sol:

- (i) True

As we know that rational and irrational numbers taken together form the set of real numbers.

- (ii) True

As, π is ratio of the circumference of a circle to its diameter, it is an irrational number

$$\Rightarrow \pi = \frac{2\pi r}{2r}$$

- (iii) False

Irrational numbers can be represented by points on the number line.

Exercise -1.6

Mark the correct alternative in each of the following:

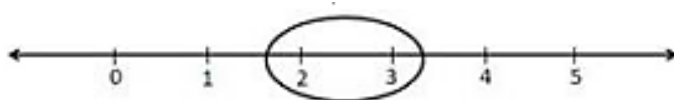
1. Which one of the following is a correct statement?

- (a) Decimal expansion of a rational number is terminating
- (b) Decimal expansion of a rational number is non-terminating
- (c) Decimal expansion of an irrational number is terminating
- (d) Decimal expansion of an irrational number is non-terminating and non-repeating

Sol:

The following steps for successive magnification to visualise 2.665 are:

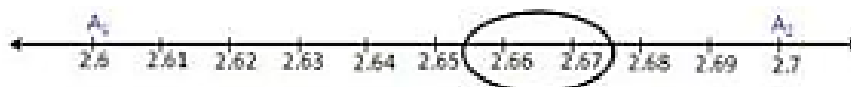
- (1) We observe that 2.665 is located somewhere between 2 and 3 on the number line. So, let us look at the portion of the number line between 2 and 3.



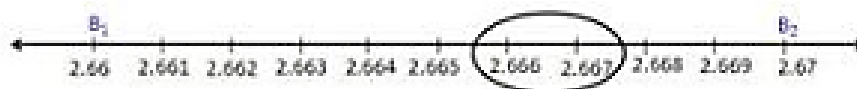
- (2) We divide this portion into 10 equal parts and mark each point of division. The first mark to the right of 2 will represent 2.1, the next 2.2 and soon. Again we observe that 2.665 lies between 2.6 and 2.7.



- (3) We mark these points A_1 and A_2 respectively. The first mark on the right side of A_1 will represent 2.61, the number 2.62, and soon. We observe 2.665 lies between 2.66 and 2.67.



- (4) Let us mark 2.66 as B_1 and 2.67 as B_2 . Again divide the B_1B_2 into ten equal parts. The first mark on the right side of B_1 will represent 2.661. Then next 2.662, and so on. Clearly, fifth point will represent 2.665.



2. Which one of the following statements is true?
- (a) The sum of two irrational numbers is always an irrational number
 - (b) The sum of two irrational numbers is always a rational number
 - (c) The sum of two irrational numbers may be a rational number or an irrational number
 - (d) The sum of two irrational numbers is always an integer

Sol:

Once again we proceed by successive magnification, and successively decrease the lengths of the portions of the number line in which $5.\overline{37}$ is located. First, we see that $5.\overline{37}$ is located between 5 and 6. In the next step, we locate $5.\overline{37}$ between 5.3 and 5.4. To get a more accurate visualization of the representation, we divide this portion of the number line into 10 equal parts and use a magnifying glass to visualize that $5.\overline{37}$ lies between 5.37 and 5.38. To visualize $5.\overline{37}$ more accurately, we again divide the portion between 5.37 and 5.38 into ten equal parts and use a magnifying glass to visualize that $5.\overline{37}$ lies between 5.377 and 5.378. Now to visualize $5.\overline{37}$ still more accurately, we divide the portion between 5.377 and 5.378 into 10 equal parts, and visualize the representation of $5.\overline{37}$ as in fig.,(iv) . Notice that $5.\overline{37}$ is located closer to 5.3778 than to 5.3777(iv)

