Chapter 8

Quadrilateral

Exercise No. 8.1

Multiple Choice Questions:

Write the correct answer in each of the following:

1. Three angles of a quadrilateral are 75º, 90º and 75º. The fourth angle is
(A) 90º
(B) 95º
(C) 105º
(D) 120º

Solution:

Fourth angle of the quadrilateral = $360° - (75° + 90° + 75°)$

$= 360° - 240°$

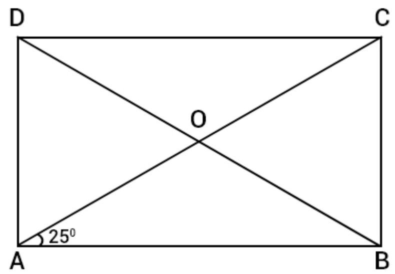
$= 120°$

Hence, the correct option is (D).

2. A diagonal of a rectangle is inclined to one side of the rectangle at 25º. The acute angle between the diagonals is
(A) 55º
(B) 50º
(C) 40º
(D) 25º

Solution:

Let ABCD is a rectangle in which diagonal AC is inclined to one side AB of the rectangle at an angle of $25°$.



Now, AC = BD [Diagonal of a rectangle are equal]

$\frac{1}{2}AC = \frac{1}{2}BD$

OA = OD

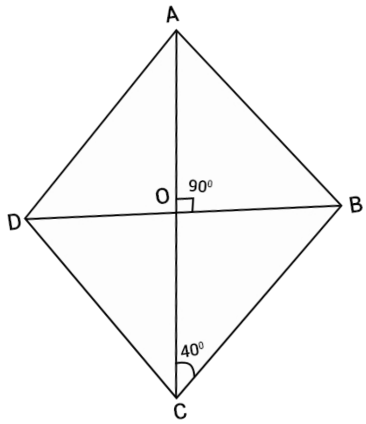
In triangle AOB,

OA = OD

Now, $\angle OBA = \angle OAB = 25°$
And, $\angle AOD = 180° - 130° = 50°$
Hence, the acute angle between the diagonal is $50°$.
Therefore, the correct option is (B).

3. ABCD is a rhombus such that $\angle ACB = 40°$. Then $\angle ADB$ is
(A) 40°
(B) 45°
(C) 50°
(D) 60°

Solution:
Given:
ABCD is a rhombus such that $\angle ACB = 40°$.



The diagonal of a rhombus bisect each other at right angles.
In right triangle BOC,

$$\angle OBC = 180° - (\angle BOC + \angle BCO)$$
$$= 180° - (90° + 40°)$$
$$= 50°$$

So, $\angle DBC = \angle OBC = 50°$
Now, $\angle ADB = \angle DBC$ [Alt. int. $\angle s$]
So, $\angle ADB = 50°$ [$\angle DBC = 50°$]
Hence, the correct option is (C).

4. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rectangle, if
(A) PQRS is a rectangle
(B) PQRS is a parallelogram
(C) Diagonals of PQRS are perpendicular
(D) Diagonals of PQRS are equal.

Solution:

If diagonals of PQRS are perpendicular.
Hence, the correct option is (C).

5. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral PQRS, taken in order, is a rhombus, if
(A) PQRS is a rhombus
(B) PQRS is a parallelogram
(C) Diagonals of PQRS are perpendicular
(D) Diagonals of PQRS are equal.

Solution:
If diagonal of PQRS are equal.
Hence, the correct option is (D).

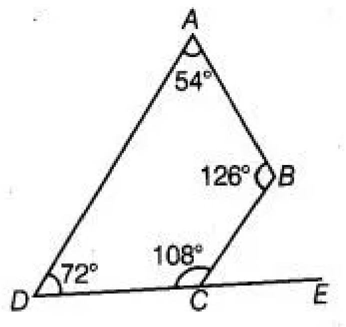
6. If angles A, B, C and D of the quadrilateral ABCD, taken in order, are in the ratio 3:7:6:4, then ABCD is a
(A) rhombus
(B) Parallelogram
(C) Trapezium
(D) Kite

Solution:
Given in the question, ratio of angles of quadrilateral ABCD is 3: 7: 6: 4.
Let the angles of quadrilateral ABCD be 3x, 7x, 6x and 4x respectively. So,
$3x+7x+6x+4x=360°$ [Sum of the all angles of a quadrilateral is $360°$.
$20x=360°$

$$x=\frac{360°}{20}$$

$x=18°$

So, angles of the quadrilateral are:
$\angle A=3\times18°=54°$
$\angle B=7\times18°=126°$
$\angle C=6\times18°=108°$
$\angle D=4\times18°=72°$

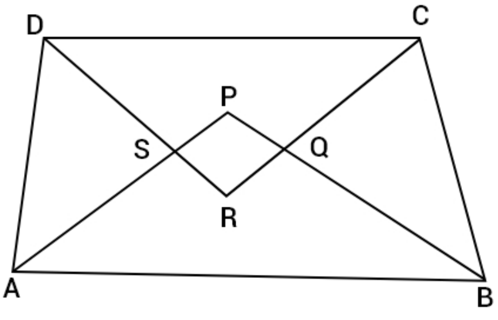


See the figure, $\angle BCE = 180° - \angle BCD$ [Linear pair axiom]
$\angle BCE = 180° - 108° = 72°$
$\angle BCE = \angle ADC = 72°$

Now, BC||AD [The corresponding angles are equal.]
The sum of co interior angles is:
$\angle A + \angle B = 126° + 54° = 180°$
And $\angle C + \angle D = 108° + 72° = 180°$
Hence, ABCD is a trapezium.

7. If bisectors of $\angle$A and $\angle$B of a quadrilateral ABCD intersect each other at P, of ∠$\angle$B and $\angle$C at Q, of $\angle$C and $\angle$D at R and of $\angle$D and $\angle$A at S, then PQRS is a
(A) rectangle
(B) rhombus
(C) parallelogram
(D) quadrilateral whose opposite angles are supplementary

Solution:
PQRS is a quadrilateral whose opposite angles are supplementary.

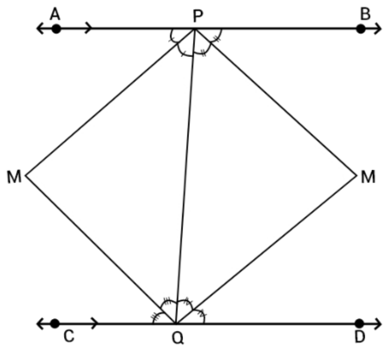


Hence, the correct option is (D).

8. If APB and CQD are two parallel lines, then the bisectors of the angles APQ, BPQ, CQP and PQD form
(A) a square
(B) a rhombus
(C) a rectangle

(D) any other parallelogram

Solution:
PNQM is a rectangle.



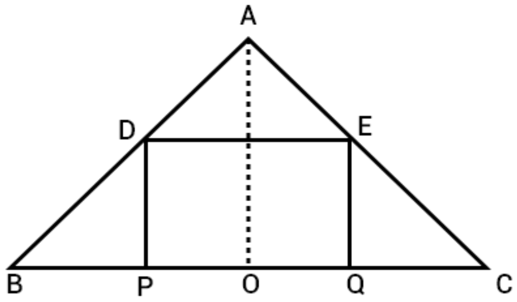
Hence, the correct option is (C).

9. The figure obtained by joining the mid-points of the sides of a rhombus, taken in order, is
(A) a rhombus
(B) a rectangle
(C) a square
(D) any parallelogram

Solution:
The figure will be a rectangle.
Hence, the correct option is (B).

10. D and E are the mid-points of the sides AB and AC of ΔABC **and O is any point on side BC. O is joined to A. If P and Q are the mid-points of OB and OC respectively, then DEQP is**
(A) a square
(B) a rectangle
(C) a rhombus
(D) a parallelogram

Solution:
According to the question, the line segment joining the mid-points of any two sides of a triangle of a triangle is parallel to the third side and is half of it. So,



Now,

$DE = \frac{1}{2}BC$ and DE||BC

Similarly, $DP = \frac{1}{2}AO$ and DP||AO

And, $EQ = \frac{1}{2}AO$ and EQ||AO

DP = EQ [Each = $\frac{1}{2}AO$]

And DP || EQ [Since, DP||AO and EQ||AO]

Now, DEQP is quadrilateral in which one pair of its opposite sides is equal and parallel.

Hence, quadrilateral DEQP is a parallelogram. The correct option is (D).

11. The figure formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order, is a square only if,

(A) ABCD is a rhombus

(B) diagonals of ABCD are equal

(C) diagonals of ABCD are equal and perpendicular

(D) diagonals of ABCD are perpendicular.

Solution:

If diagonals of ABCD are equal and perpendicular.

Hence, the correct option is (C).

12. The diagonals AC and BD of a parallelogram ABCD intersect each other at the point O. If $\angle DAC = 32°$ and $\angle AOB = 70°$, then $\angle DBC$ is equal to

(A) 24º

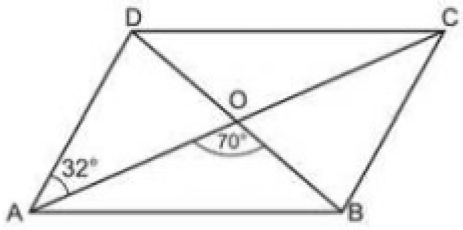
(B) 86º

(C) 38º

(D) 32º

Solution:

According to the question,



AD is parallel to BC and AC cuts it. So,
$\angle DAC = \angle ACB$ [Alt. int. $\angle s$]
$\angle DAC = 32°$ [Given]
So, $\angle ACB = 32°$

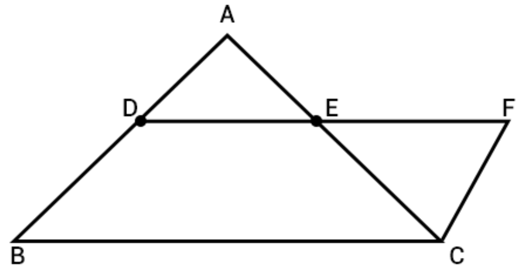
Produce Co to A in triangle AOB. So,
Ext. $\angle BOA = \angle OCB + \angle OBC$ [By exterior angle theorem]
$70° = 32° + \angle OBC$
$\angle OBC = 70° - 32° = 38°$
Hence, $\angle DBC = 38°$. The correct option is (C).

13. Which of the following is not true for a parallelogram?
(A) opposite sides are equal
(B) opposite angles are equal
(C) opposite angles are bisected by the diagonals
(D) diagonals bisect each other.

Solution:
Opposite angles are bisected by the diagonals. That not true for the parallelogram.
Hence, the correct option is (C).

14. D and E are the mid-points of the sides AB and AC respectively of ΔABC. DE is produced to F. To prove that CF is equal and parallel to DA, we need an additional information which is
(A) ∠ $\angle DAE = \angle EFC$
(B) AE = EF
(C) DE = EF
(D) $\angle ADE = \angle ECF$

Solution:
According to the question, we need DE = EF



Hence, the correct option is (C).

Exercise No. 8.2

Short Answer Questions with Reasoning:

1. Diagonals AC and BD of a parallelogram ABCD intersect each other at O. If OA = 3 cm and OD = 2 cm, determine the lengths of AC and BD.

Solution:
As we know that the diagonal of a parallelogram bisect each other. So,
AC = $2\times$OA=2×3 cm = 6 cm
And, BD = 2OD = 2×2 cm = 4 cm
Therefore, lengths of AC and BD are 6 cm and 4 cm respectively.

2. Diagonals of a parallelogram are perpendicular to each other. Is this statement true? Give reason for your answer.

Solution:
The given statement is not true because diagonal of a parallelogram bisect each other.

3. Can the angles 110º, 80º, 70º and 95º be the angles of a quadrilateral? Why or why not?

Solution:
We know that, sum of the angles of a quadrilateral is always $360°$.
Sum of these angles = $110°+80°+70°+95°=355°$ that is not equal to $360°$.
Hence, $110°,80°,70°$ and $95°$ can't be the angle of a quadrilateral.

4. In quadrilateral ABCD, $\angle A+\angle D=180°$. What special name can be given to this quadrilateral?

Solution:
Given:
In quadrilateral ABCD, $\angle A+\angle D=180°$.
We know that the sum of the two consecutive angle is $180°$. So, pair of opposite side AB and CD are parallel.
Since, the quadrilateral ABCD is trapezium.
Hence, special name can be given to this quadrilateral is trapezium.

5. All the angles of a quadrilateral are equal. What special name is given to this quadrilateral?

Solution:
Given:

All the angles of a quadrilateral are equal.

We know that, the sum of angles of a quadrilateral is $360°$. Since, each angle is $\frac{360°}{4} = 90°$.

Hence, special name is given to this quadrilateral is rectangle.

6. Diagonals of a rectangle are equal and perpendicular. Is this statement true? Give reason for your answer.

Solution:

We know that diagonal of a rectangle need not to be perpendicular.
Hence, the given statement is false.

7. Can all the four angles of a quadrilateral be obtuse angles? Give reason for your answer.

Solution:

We know that sum of four angles of a quadrilateral is always equal to $360°$.
Now, if all the four angles of a quadrilateral be obtuse angles then sum of four angle will be more than $360°$.
Hence, the given statement is false.

8. In ΔABC, AB = 5 cm, BC = 8 cm and CA = 7 cm. If D and E are respectively the mid-points of AB and BC, determine the length of DE.

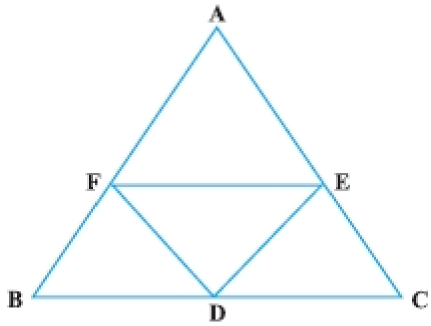
Solution:

Given:

In ΔABC,
AB = 5 cm, BC = 8 cm and CA = 7 cm
According to the question, D and E are respectively the mid-points of AB and BC. So, using mid-point theorem,

$$DE = \frac{1}{2} AC$$
$$= \frac{1}{2} \times 7\text{cm}$$
$$= 3.5cm$$

9. In Fig., it is given that BDEF and FDCE are parallelograms. Can you say that BD = CD? Why or why not?



Solution:

BDEF is a parallelogram. [Given]

So, BD = EF … (I) [Opposite side of a parallelogram]

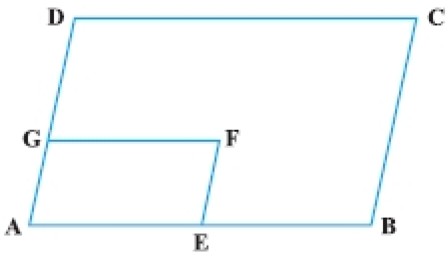
FDCE is a parallelogram. [Given]

So, CD = EF … (II)

Now, from equation (I) and (II), get:

BD = CD

10. In Fig., ABCD and AEFG are two parallelograms. If $\angle C = 55°$, determine $\angle F$.



Solution:

We know that opposite angle of parallelogram are equal.

ABCD is a parallelogram. So,

$\angle A = \angle C$

Now, $\angle C = 55°$ [Given]

In parallelogram AEFG,

$\angle F = \angle A = 55°$

Hence, $\angle F = 55°$.

11. Can all the angles of a quadrilateral be acute angles? Give reason for your answer.

Solution:

We know that an acute angle is less than $90°$ and the sum of angles of quadrilateral is always $360°$.

Hence, all the angle of a quadrilateral can't be acute angle because sum of four angles of a quadrilateral will be less than $360°$.

12. Can all the angles of a quadrilateral be right angles? Give reason for your answer.

Solution:

We know that sum of angles of quadrilateral is always $360°$. Since, all the angles of a quadrilateral can be right angle, which is true because $90° \times 4 = 360°$.

13. Diagonals of a quadrilateral ABCD bisect each other. If ∠A = 35º, determine ∠B.

Solution:

Given:

Diagonals of a quadrilateral ABCD bisect each other.

So, ABCD is a parallelogram.

Now, $\angle A + \angle B = 180°$ [Adjacent angles of a parallelogram are supplementary]

Since,

$35° + \angle B = 180°$

$\angle B = 180° - 35°$

$\angle B = 145°$

14. Opposite angles of a quadrilateral ABCD are equal. If AB = 4 cm, determine CD.

Solution:

Given:

Opposite angles of a quadrilateral ABCD are equal.

So, that is a parallelogram.

Now, ABCD is a parallelogram.

So, AB = CD. [Opposite of a parallelogram are equal]

AB = 4 cm [Given]

Therefore, CD = 4 cm.

Exercise No. 8.3

Short Answer Questions:

1. One angle of a quadrilateral is of 108° and the remaining three angles are equal. Find each of the three equal angles.

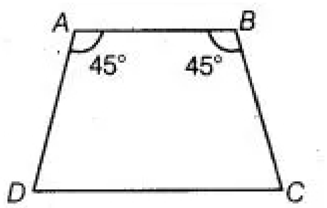
Solution:
We know that the sum of all the angles in a quadrilateral is 360°.
According to the question, the remaining three angles are equal. So, let it is x.
Now,

$$108° + x + x + x = 360°$$
$$3x = 360° - 108°$$
$$3x = 252°$$
$$x = 84°$$

Hence, each of the three equal angles is $84°$.

2. ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 45°$. Find angles C and D of the trapezium.

Solution:
Given:
ABCD is a trapezium in which AB || DC and $\angle A = \angle B = 45°$.



Now, AB||DC and AD is transversal.
So, $\angle A + \angle D = 180°$ [sum of interior angles on the side of the transversal is $180°$]

$$45° + \angle D = 180°$$
$$\angle D = 180° - 45°$$
$$\angle D = 135°$$

Similarly, $\angle B + \angle C = 180°$

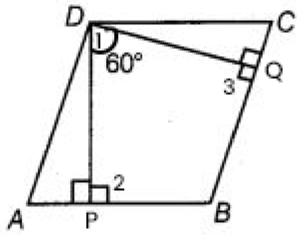
$$45° + \angle C = 180°$$
$$\angle C = 180° - 45°$$
$$\angle C = 135°$$

Therefore, $\angle A = \angle B = 45°$ and $\angle C = \angle D = 135°$.

3. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60º. Find the angles of the parallelogram.

Solution:

In quadrilateral DPBQ:



$\angle 1+\angle 2+\angle B+\angle 3=360°$ [Angle sum property of quadrilateral]

$60°+90°+\angle B+90°=360°$

$\angle B+240°=360°$

$\angle B=360°-240°$

$\angle B=120°$

Since, $\angle ADC=\angle B=120°$ [Opposite angles of a parallelogram are equal]

$\angle A+\angle B=180°$ [Sum of consecutive interior angle is $180°$]

$\angle A+120°=180°$

$\angle A=180°-120°$

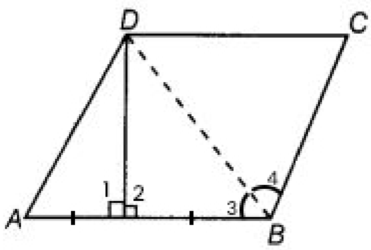
$\angle A=60°$

So, $\angle C=\angle A=60°$ [Opposite angle of a parallelogram are equal]

4. ABCD is a rhombus in which altitude from D to side AB bisects AB. Find the angles of the rhombus.

Solution:

See the below figure, in triangle APD and triangle BPD,



AP = BP [Given]

$\angle 1=\angle 2$ [Each equal to $90°$]

PD = PD [Common side]

So, by SAS criterion of congruence,

$\Delta APD \cong \Delta BPD$

$\angle A = \angle 3$ [CPCT]

$\angle 3 = \angle 4$ [Diagonal bisect opposite angles of a rhombus]

$\angle A = \angle 3 = \angle 4$ … (I)

Now, AD||BC

So, $\angle A + \angle ABC = 180°$ [Sum of consecutive interior angles is $180°$]

$\angle A + \angle 3 + \angle 4 = 180°$

$\angle A + \angle A + \angle A = 180°$

$3\angle A = 180°$

$\angle A = \frac{180°}{3}$

$\angle A = 60°$

Now,

$\angle ABC = \angle 3 + \angle 4$

$= 60° + 60°$

$\angle ABC = 120°$ [Opposite angles of a rhombus are equal]

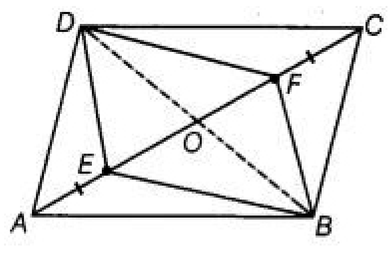
$\angle ADC = \angle ABC = 120°$ [Opposite angles of a rhombus are equal]

5. E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF. Show that BFDE is a parallelogram.

Solution:

Given:

E and F are points on diagonal AC of a parallelogram ABCD such that AE = CF.



To prove that BFDE is parallelogram,

Proof: ABCD is a parallelogram.

OD = OB … (I) [Diagonals of parallelogram bisect each other]

OA = OC … (II) [Diagonals of parallelogram bisect each other]

AE = EF … (III)[Given]

Subtracting equation (III) from equation (II), get:

OA-AE = OC – CF

OE = OF … (IV)

Now, BFDE is parallelogram. [Since, OD = OB and OE = OF]

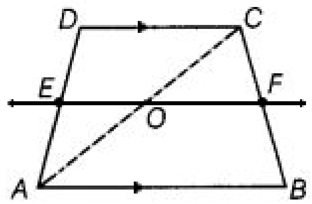
Hence, proved.

6. E is the mid-point of the side AD of the trapezium ABCD with AB || DC. A line through E drawn parallel to AB intersect BC at F. Show that F is the mid-point of BC. [Hint: Join AC]

Solution:

Given

E is the mid-point of the side AD of the trapezium ABCD with AB || DC. Also, EF||AB.



To prove that F is the mid-point of BC.

Construction: Join AC which intersect EF at O.

Proof: In triangle ADC, E is the midpoint of AD and EF||DC. [Since, EF||AB and DC||AB. So, AB||EF||DC]

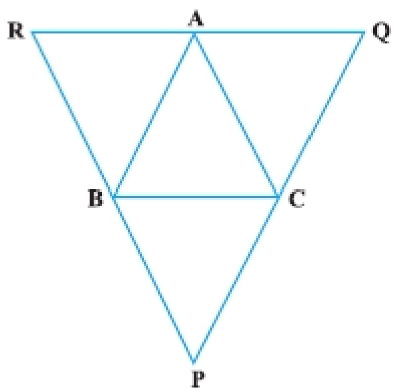
O is the mid-point of AC and OF||AB.

Now, OF bisect BC. [Converse of mid-point theorem]

Or F is the mid-point of BC.

Hence, proved.

7. Through A, B and C, lines RQ, PR and QP have been drawn, respectively parallel to sides BC, CA and AB of a ΔABC as shown in Fig. Show that $BC = \frac{1}{2}QR$.



Solution:

Given in the question, Triangle ABC and PQR in which AB||QP, BC||RQ and CA||PR.

To prove that $BC = \frac{1}{2}QR$

Proof: In quadrilateral BCAR, BR || CA and BC|| RA

So, quadrilateral, BCAR is a parallelogram.
BC = AR … (I)
Now, in quadrilateral BCQA, BC || AQ and AB||QC
So, quadrilateral BCQA is a parallelogram,
BC = AQ … (II)

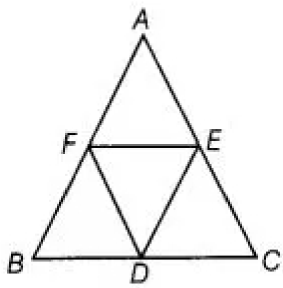
Now, adding equation (I) and (II), get:
2 BC = AR+ AQ
2 BC = RQ
BC = $\frac{1}{2}$ QR

Now, BEDF is a quadrilateral, in which ∠BED = ∠BFD = 90°
∠FSE = 360° – (∠FDE + ∠BED + ∠BFD) = 360° – (60° + 90° + 90°)
= 360°-240°
=120°

8. D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral triangle ABC. Show that ΔDEF is also an equilateral triangle.

Solution:

Given in the question, D, E and F are the mid-points of the sides BC, CA and AB, respectively of an equilateral ΔABC.
To proof that ΔDEF is an equilateral triangle.



Proof: In ΔABC, E and F are the mid-points of AC and AB respectively, then EF || BC. So,

$EF = \frac{1}{2}BC$ …(I)

DF || AC, DE || AB

DE = $\frac{1}{2}$ AB and FD = $\frac{1}{2}$ AC [By mid-point theorem]… **(II)**

Now, ΔABC is an equilateral triangle.
AB = BC = CA

$\frac{1}{2}$ AB = $\frac{1}{2}$ BC = $\frac{1}{2}$ CA [Dividing by 2 in the above equation]

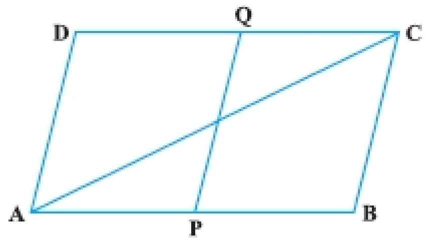
So, DE = EF = FD [**From Equation. (I) and (II)**]

Since, all sides of ADEF are equal.

Hence, ΔDEF is an equilateral triangle.

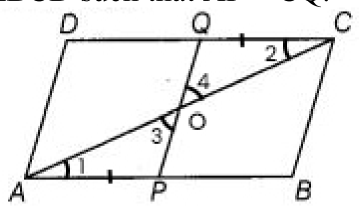
Hence proved.

9. Points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ. Show that AC and PQ bisect each other.



Solution:

Given in the question, points P and Q have been taken on opposite sides AB and CD, respectively of a parallelogram ABCD such that AP = CQ.



In triangle AOP and triangle COQ:

AP = CQ [Given]

$\angle 1 = \angle 2$ [Alternate interior angles]

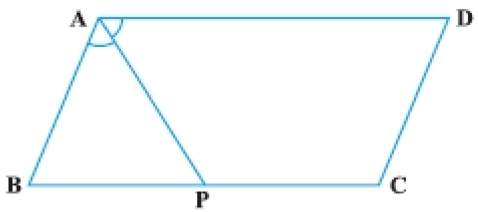
$\angle 3 = \angle 4$ [Vertically opposite angles]

$\Delta AOP \cong \Delta COQ$ [By AAS Congruence rule]

So, OA = OC and OP = OQ [CPCT]

Hence, AC and PQ bisect each other.

10. In Fig., P is the mid-point of side BC of a parallelogram ABCD such that ∠BAP = ∠DAP. Prove that AD = 2CD.



Solution:

Given in the question, in a parallelogram ABCD, P is a mid-point of BC such that $\angle BAP = \angle DAP$.
To prove that AD = 2CD

Proof: ABCD is a parallelogram.
So, AD||BC and AB is transversal, then:
$\angle A + \angle B = 180°$ [Sum of cointerior angles is $180°$]
$\angle B = 180° - \angle A$ … (I)

Now, in triangle ABP,
$\angle PAB + \angle B + \angle BPA = 180°$ [By angle sum property of a triangle]
$\frac{1}{2}\angle A + 180° - \angle A + \angle BPA = 180°$ [From equation (I)]
$\angle BPA - \frac{\angle A}{2} = 0$
$\angle BPA = \frac{\angle A}{2}$ … (II)
$\angle BPA = \angle BPA$
AB = BP [Opposite sides of equal angles are equal]

In above equation multiplying both side by 2, get:
2AB = 2BP
2AB = BC [P is the mid-point of BC]
2CD = AD [ABCD is a parallelogram, then AB = CD and BC = AD]