Appendices

A.1 Mathematical Induction

Many formulas, like

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2},$$

can be shown to hold for every positive integer n by applying an axiom called the *mathematical induction principle.* A proof that uses this axiom is called a *proof by mathematical induction* or a *proof by induction.*

The steps in proving a formula by induction are the following.

Step 1: Check that the formula holds for $n = 1$.

Step 2: Prove that if the formula holds for any positive integer $n = k$, then it also holds for the next integer, $n = k + 1$.

Once these steps are completed (the axiom says), we know that the formula holds for all positive integers n. By step 1 it holds for $n = 1$. By step 2 it holds for $n = 2$, and therefore by step 2 also for $n = 3$, and by step 2 again for $n = 4$, and so on. If the first domino falls, and the kth domino always knocks over the $(k + 1)$st when it falls, all the dominoes fall.

From another point of view, suppose we have a sequence of statements S_1, $S_2, \ldots, S_n, \ldots$, one for each positive integer. Suppose we can show that assuming any one of the statements to be true implies that the next statement in line is true. Suppose that we can also show that S_1 is true. Then we may conclude that the statements are true from S_1 on.

EXAMPLE 1 Show that for every positive integer n,

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

Solution We accomplish the proof by carrying out the two steps above.

Step 1: The formula holds for $n = 1$ because

$$1 = \frac{1(1+1)}{2}.$$

Step 2: If the formula holds for $n = k$, does it also hold for $n = k + 1$? The answer is yes, and here's why: If

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2},$$

then

$$1+2+\cdots+k+(k+1) = \frac{k(k+1)}{2}+(k+1) = \frac{k^2+k+2k+2}{2}$$
$$= \frac{(k+1)(k+2)}{2} = \frac{(k+1)((k+1)+1)}{2}.$$

The last expression in this string of equalities is the expression $n(n+1)/2$ for $n=(k+1)$.

The mathematical induction principle now guarantees the original formula for all positive integers n. Notice that all *we* have to do is carry out steps 1 and 2. The mathematical induction principle does the rest. ❑

EXAMPLE 2 Show that for all positive integers n,

$$\frac{1}{2^1}+\frac{1}{2^2}+\cdots+\frac{1}{2^n} = 1-\frac{1}{2^n}.$$

Solution We accomplish the proof by carrying out the two steps of mathematical induction.

Step 1: The formula holds for $n=1$ because

$$\frac{1}{2^1} = 1-\frac{1}{2^1}.$$

Step 2: If

$$\frac{1}{2^1}+\frac{1}{2^2}+\cdots+\frac{1}{2^k} = 1-\frac{1}{2^k},$$

then

$$\frac{1}{2^1}+\frac{1}{2^2}+\cdots+\frac{1}{2^k}+\frac{1}{2^{k+1}} = 1-\frac{1}{2^k}+\frac{1}{2^{k+1}} = 1-\frac{1\cdot 2}{2^k\cdot 2}+\frac{1}{2^{k+1}}$$
$$= 1-\frac{2}{2^{k+1}}+\frac{1}{2^{k+1}} = 1-\frac{1}{2^{k+1}}.$$

Thus, the original formula holds for $n=(k+1)$ whenever it holds for $n=k$.

With these steps verified, the mathematical induction principle now guarantees the formula for every positive integer n. ❑

Other Starting Integers

Instead of starting at $n=1$, some induction arguments start at another integer. The steps for such an argument are as follows.

Step 1: Check that the formula holds for $n=n_1$ (the first appropriate integer).

Step 2: Prove that if the formula holds for any integer $n=k\geq n_1$, then it also holds for $n=(k+1)$.

Once these steps are completed, the mathematical induction principle guarantees the formula for all $n\geq n_1$.

EXAMPLE 3 Show that $n! > 3^n$ if n is large enough.

Solution How large is large enough? We experiment:

n	1	2	3	4	5	6	7
$n!$	1	2	6	24	120	720	5040
3^n	3	9	27	81	243	729	2187

It looks as if $n! > 3^n$ for $n \geq 7$. To be sure, we apply mathematical induction. We take $n_1 = 7$ in step 1 and try for step 2.

Suppose $k! > 3^k$ for some $k \geq 7$. Then

$$(k+1)! = (k+1)(k!) > (k+1)3^k > 7 \cdot 3^k > 3^{k+1}.$$

Thus, for $k \geq 7$,

$$k! > 3^k \quad \Rightarrow \quad (k+1)! > 3^{k+1}.$$

The mathematical induction principle now guarantees $n! \geq 3^n$ for all $n \geq 7$. ❑

Exercises A.1

1. Assuming that the triangle inequality $|a+b| \leq |a| + |b|$ holds for any two numbers a and b, show that

$$|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$$

for any n numbers.

2. Show that if $r \neq 1$, then

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

for every positive integer n.

3. Use the Product Rule, $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$,

and the fact that $\frac{d}{dx}(x) = 1$

to show that $\frac{d}{dx}(x^n) = nx^{n-1}$

for every positive integer n.

4. Suppose that a function $f(x)$ has the property that $f(x_1x_2) = f(x_1) + f(x_2)$ for any two positive numbers x_1 and x_2. Show that

$$f(x_1x_2\cdots x_n) = f(x_1) + f(x_2) + \cdots + f(x_n)$$

for the product of any n positive numbers $x_1, x_2 \ldots, x_n$.

5. Show that

$$\frac{2}{3^1} + \frac{2}{3^2} + \cdots + \frac{2}{3^n} = 1 - \frac{1}{3^n}$$

for all positive integers n.

6. Show that $n! > n^3$ if n is large enough.

7. Show that $2^n > n^2$ if n is large enough.

8. Show that $2^n \geq 1/8$ for $n \geq -3$.

9. *Sums of squares.* Show that the sum of the squares of the first n positive integers is

$$\frac{n\left(n + \frac{1}{2}\right)(n+1)}{3}.$$

10. *Sums of cubes.* Show that the sum of the cubes of the first n positive integers is $(n(n+1)/2)^2$.

11. *Rules for finite sums.* Show that the following finite sum rules hold for every positive integer n.

    a) $\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$

    b) $\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$

    c) $\sum_{k=1}^{n} ca_k = c \cdot \sum_{k=1}^{n} a_k$ (Any number c)

    d) $\sum_{k=1}^{n} a_k = n \cdot c$ (if a_k has the constant value c)

12. Show that $|x^n| = |x|^n$ for every positive integer n and every real number x.

A.2

Proofs of Limit Theorems in Section 1.2

This appendix proves Theorem 1, Parts 2–5, and Theorem 4 from Section 1.2.

Theorem 1
Properties of Limits

The following rules hold if $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = M$ (L and M real numbers).

1. *Sum Rule:* $\lim_{x\to c} [f(x) + g(x)] = L + M$
2. *Difference Rule:* $\lim_{x\to c} [f(x) - g(x)] = L - M$
3. *Product Rule:* $\lim_{x\to c} f(x) \cdot g(x) = L \cdot M$
4. *Constant Multiple Rule:* $\lim_{x\to c} kf(x) = kL$ (any number k)
5. *Quotient Rule:* $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$, if $M \neq 0$
6. *Power Rule:* If m and n are integers, then

$$\lim_{x\to c} [f(x)]^{m/n} = L^{m/n}$$

provided $L^{m/n}$ is a real number.

We proved the Sum Rule in Section 1.3 and the Power Rule is proved in more advanced texts. We obtain the Difference Rule by replacing $g(x)$ by $-g(x)$ and M by $-M$ in the Sum Rule. The Constant Multiple Rule is the special case $g(x) = k$ of the Product Rule. This leaves only the Product and Quotient Rules.

Proof of the Limit Product Rule We show that for any $\epsilon > 0$ there exists a $\delta > 0$ such that for all x in the intersection D of the domains of f and g,

$$0 < |x - c| < \delta \quad \Rightarrow \quad |f(x)\,g(x) - LM| < \epsilon.$$

Suppose then that ϵ is a positive number, and write $f(x)$ and $g(x)$ as

$$f(x) = L + (f(x) - L), \qquad g(x) = M + (g(x) - M).$$

Multiply these expressions together and subtract LM:

$$\begin{aligned} f(x)\cdot g(x) - LM &= (L + (f(x) - L))(M + (g(x) - M)) - LM \\ &= LM + L(g(x) - M) + M(f(x) - L) \\ &\quad + (f(x) - L)(g(x) - M) - LM \\ &= L(g(x) - M) + M(f(x) - L) + (f(x) - L)(g(x) - M). \end{aligned} \tag{1}$$

Since f and g have limits L and M as $x \to c$, there exist positive numbers δ_1, δ_2, δ_3, and δ_4 such that for all x in D

$$
\begin{aligned}
0 < |x - c| < \delta_1 &\Rightarrow |f(x) - L| < \sqrt{\epsilon/3} \\
0 < |x - c| < \delta_2 &\Rightarrow |g(x) - M| < \sqrt{\epsilon/3} \\
0 < |x - c| < \delta_3 &\Rightarrow |f(x) - L| < \epsilon/(3(1 + |M|)) \\
0 < |x - c| < \delta_4 &\Rightarrow |g(x) - M| < \epsilon/(3(1 + |L|))
\end{aligned} \tag{2}
$$

If we take δ to be the smallest numbers δ_1 through δ_4, the inequalities on the right-hand side of (2) will hold simultaneously for $0 < |x - c| < \delta$. Therefore, for all x in D, $0 < |x - c| < \delta$ implies

$$
\begin{aligned}
&|f(x) \cdot g(x) - LM| \\
&\quad \le |L||g(x) - M| + |M||f(x) - L| + |f(x) - L||g(x) - M| && \text{Triangle inequality applied to Eq. (1)} \\
&\quad \le (1 + |L|)|g(x) - M| + (1 + |M|)|f(x) - L| + |f(x) - L||g(x) - M| \\
&\quad \le \frac{\epsilon}{3} + \frac{\epsilon}{3} + \sqrt{\frac{\epsilon}{3}}\sqrt{\frac{\epsilon}{3}} = \epsilon. && \text{Values from (2)}
\end{aligned}
$$

This completes the proof of the Limit Product Rule. ❑

Proof of the Limit Quotient Rule We show that $\lim_{x \to c}(1/g(x)) = 1/M$. We can then conclude that

$$
\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \left(f(x) \cdot \frac{1}{g(x)} \right) = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = L \cdot \frac{1}{M} = \frac{L}{M}
$$

by the Limit Product Rule.

Let $\epsilon > 0$ be given. To show that $\lim_{x \to c}(1/g(x)) = 1/M$, we need to show that there exists a $\delta > 0$ such that for all x

$$
0 < |x - c| < \delta \quad \Rightarrow \quad \left| \frac{1}{g(x)} - \frac{1}{M} \right| < \epsilon.
$$

Since $|M| > 0$, there exists a positive number δ_1 such that for all x

$$
0 < |x - c| < \delta_1 \quad \Rightarrow \quad |g(x) - M| < \frac{M}{2}. \tag{3}
$$

For any numbers A and B it can be shown that $|A| - |B| \le |A - B|$ and $|B| - |A| \le |A - B|$, from which it follows that $||A| - |B|| \le |A - B|$. With $A = g(x)$ and $B = M$, this becomes

$$
||g(x)| - |M|| \le |g(x) - M|,
$$

which can be combined with the inequality on the right in (3) to get, in turn,

$$
\begin{aligned}
||g(x)| - |M|| &< \frac{|M|}{2} \\
-\frac{|M|}{2} < |g(x)| - |M| &< \frac{|M|}{2} \\
\frac{|M|}{2} < |g(x)| &< \frac{3|M|}{2} \\
|M| < 2|g(x)| &< 3|M| \\
\frac{1}{|g(x)|} < \frac{2}{|M|} &< \frac{3}{|g(x)|}
\end{aligned} \tag{4}
$$

Therefore, $0 < |x - c| < \delta_1$ implies that

$$\left|\frac{1}{g(x)} - \frac{1}{M}\right| = \left|\frac{M - g(x)}{Mg(x)}\right| \le \frac{1}{|M|} \cdot \frac{1}{|g(x)|} \cdot |M - g(x)|$$
$$< \frac{1}{|M|} \cdot \frac{2}{|M|} \cdot |M - g(x)|. \qquad \text{Inequality (4)} \tag{5}$$

Since $(1/2)|M|^2 \epsilon > 0$, there exists a number $\delta_2 > 0$ such that for all x

$$0 < |x - c| < \delta_2 \quad \Rightarrow \quad |M - g(x)| < \frac{\epsilon}{2}|M|^2. \tag{6}$$

If we take δ to be the smaller of δ_1 and δ_2, the conclusions in (5) and (6) both hold for all x such that $0 < |x - c| < \delta$. Combining these conclusions gives

$$0 < |x - c| < \delta \quad \Rightarrow \quad \left|\frac{1}{g(x)} - \frac{1}{M}\right| < \epsilon.$$

This concludes the proof of the Limit Quotient Rule. ❑

Theorem 4
The Sandwich Theorem
Suppose that $g(x) \le f(x) \le h(x)$ for all x in some open interval containing c, except possibly at $x = c$ itself. Suppose also that $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$. Then $\lim_{x\to c} f(x) = L$.

Proof for Right-hand Limits Suppose $\lim_{x\to c^+} g(x) = \lim_{x\to c^+} h(x) = L$. Then for any $\epsilon > 0$ there exists a $\delta > 0$ such that for all x the inequality $c < x < c + \delta$ implies

$$L - \epsilon < g(x) < L + \epsilon \quad \text{and} \quad L - \epsilon < h(x) < L + \epsilon. \tag{7}$$

These inequalities combine with the inequality $g(x) \le f(x) \le h(x)$ to give

$$L - \epsilon < g(x) \le f(x) \le h(x) < L + \epsilon,$$
$$L - \epsilon < f(x) < L + \epsilon, \tag{8}$$
$$-\epsilon < f(x) - L < \epsilon.$$

Therefore, for all x, the inequality $c < x < c + \delta$ implies $|f(x) - L| < \epsilon$. ❑

Proof for Left-hand Limits Suppose $\lim_{x\to c^-} g(x) = \lim_{x\to c^-} h(x) = L$. Then for any $\epsilon > 0$ there exists a $\delta > 0$ such that for all x the inequality $c - \delta < x < c$ implies

$$L - \epsilon < g(x) < L + \epsilon \quad \text{and} \quad L - \epsilon < h(x) < L + \epsilon. \tag{9}$$

We conclude as before that for all x, $c - \delta < x < c$ implies $|f(x) - L| < \epsilon$. ❑

Proof for Two-sided Limits If $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, then $g(x)$ and $h(x)$ both approach L as $x \to c^+$ and as $x \to c^-$; so $\lim_{x\to c^+} f(x) = L$ and $\lim_{x\to c^-} f(x) = L$. Hence $\lim_{x\to c} f(x)$ exists and equals L. ❑

Exercises A.2

1. Suppose that functions $f_1(x)$, $f_2(x)$, and $f_3(x)$ have limits L_1, L_2, and L_3, respectively, as $x \to c$. Show that their sum has limit $L_1 + L_2 + L_3$. Use mathematical induction (Appendix 1) to generalize this result to the sum of any finite number of functions.

2. Use mathematical induction and the Limit Product Rule in Theorem 1 to show that if functions $f_1(x), f_2(x), \ldots, f_n(x)$ have limits $L_1, L_2, \ldots, L_n$ as $x \to c$, then

$$\lim_{x\to c} f_1(x) f_2(x) \cdot \cdots \cdot f_n(x) = L_1 \cdot L_2 \cdot \cdots \cdot L_n.$$

3. Use the fact that $\lim_{x\to c} x = c$ and the result of Exercise 2 to show that $\lim_{x\to c} x^n = c^n$ for any integer $n > 1$.

4. *Limits of polynomials.* Use the fact that $\lim_{x\to c}(k) = k$ for any number k together with the results of Exercises 1 and 3 to show that $\lim_{x\to c} f(x) = f(c)$ for any polynomial function

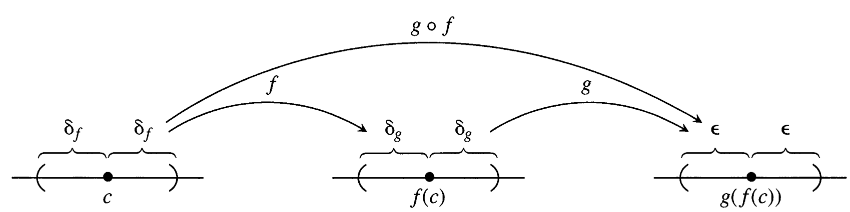
$$f(x) = a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1}x + a_n.$$

5. *Limits of rational functions.* Use Theorem 1 and the result of Exercise 4 to show that if $f(x)$ and $g(x)$ are polynomial functions and $g(c) \neq 0$, then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{f(c)}{g(c)}.$$

6. *Composites of continuous functions.* Figure A.1 gives the diagram for a proof that the composite of two continuous functions is continuous. Reconstruct the proof from the diagram. The statement to be proved is this: If f is continuous at $x = c$ and g is continuous at $f(c)$, then $g \circ f$ is continuous at c.

   Assume that c is an interior point of the domain of f and that $f(c)$ is an interior point of the domain of g. This will make the limits involved two-sided. (The arguments for the cases that involve one-sided limits are similar.)



A.1 The diagram for a proof that the composite of two continuous functions is continuous. The continuity of composites holds for any finite number of functions. The only requirement is that each function be continuous where it is applied. In the figure, f is to be continuous at c and g at $f(c)$.

A.3 Complex Numbers

Complex numbers are expressions of the form $a + ib$, where a and b are real numbers and i is a symbol for $\sqrt{-1}$. Unfortunately, the words "real" and "imaginary" have connotations that somehow place $\sqrt{-1}$ in a less favorable position in our minds than $\sqrt{2}$. As a matter of fact, a good deal of imagination, in the sense of *inventiveness,* has been required to construct the *real* number system, which forms the basis of the calculus. In this appendix we review the various stages of this invention. The further invention of a complex number system will then not seem so strange.

The Development of the Real Numbers

The earliest stage of number development was the recognition of the **counting numbers** 1, 2, 3, . . . , which we now call the **natural numbers** or the **positive integers.** Certain simple arithmetical operations can be performed with these numbers without getting outside the system. That is, the system of positive integers is **closed** under the operations of addition and multiplication. By this we mean that if m and n are any positive integers, then

$$m + n = p \qquad \text{and} \qquad mn = q \tag{1}$$

are also positive integers. Given the two positive integers on the left-hand side of either equation in (1), we can find the corresponding positive integer on the right. More than this, we can sometimes specify the positive integers m and p and find a positive integer n such that $m + n = p$. For instance, $3 + n = 7$ can be solved when the only numbers we know are the positive integers. But the equation $7 + n = 3$ cannot be solved unless the number system is enlarged.

The number zero and the negative integers were invented to solve equations like $7 + n = 3$. In a civilization that recognizes all the **integers**

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots, \tag{2}$$

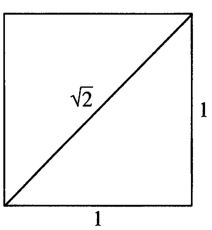
an educated person can always find the missing integer that solves the equation $m + n = p$ when given the other two integers in the equation.

Suppose our educated people also know how to multiply any two of the integers in (2). If, in Eqs. (1), they are given m and q, they discover that sometimes they can find n and sometimes they cannot. If their imagination is still in good working order, they may be inspired to invent still more numbers and introduce fractions, which are just ordered pairs m/n of integers m and n. The number zero has special properties that may bother them for a while, but they ultimately discover that it is handy to have all ratios of integers m/n, excluding only those having zero in the denominator. This system, called the set of **rational numbers,** is now rich enough for them to perform the so-called **rational operations** of arithmetic:

1. a) addition
b) subtraction

2. a) multiplication
b) division

on any two numbers in the system, *except that they cannot divide by zero.*



A.2 With a straightedge and compass, it is possible to construct a segment of irrational length.

The geometry of the unit square (Fig. A.2) and the Pythagorean theorem showed that they could construct a geometric line segment that, in terms of some basic unit of length, has length equal to $\sqrt{2}$. Thus they could solve the equation

$$x^2 = 2$$

by a geometric construction. But then they discovered that the line segment representing $\sqrt{2}$ and the line segment representing the unit of length 1 were incommensurable quantities. This means that the ratio $\sqrt{2}/1$ cannot be expressed as the ratio of two *integer* multiples of some other, presumably more fundamental, unit of length. That is, our educated people could not find a rational number solution of the equation $x^2 = 2$.

There *is* no rational number whose square is 2. To see why, suppose that there were such a rational number. Then we could find integers p and q with no common factor other than 1, and such that

$$p^2 = 2q^2. \tag{3}$$

Since p and q are integers, p must be even; otherwise its product with itself would be odd. In symbols, $p = 2p_1$, where p_1 is an integer. This leads to $2{p_1}^2 = q^2$, which says q must be even, say $q = 2q_1$, where q_1 is an integer. This makes 2 a factor of both p and q, contrary to our choice of p and q as integers with no common factor other than 1. Hence there is no rational number whose square is 2.

Although our educated people could not find a rational solution of the equation $x^2 = 2$, they could get a sequence of rational numbers

$$\frac{1}{1}, \frac{7}{5}, \frac{41}{29}, \frac{239}{169}, \ldots, \tag{4}$$

whose squares form a sequence

$$\frac{1}{1}, \quad \frac{49}{25}, \quad \frac{1681}{841}, \quad \frac{57{,}121}{28{,}561}, \quad \ldots, \tag{5}$$

that converges to 2 as its limit. This time their imagination suggested that they needed the concept of a limit of a sequence of rational numbers. If we accept the fact that an increasing sequence that is bounded from above always approaches a limit and observe that the sequence in (4) has these properties, then we want it to have a limit L. This would also mean, from (5), that $L^2 = 2$, and hence L is *not* one of our rational numbers. If to the rational numbers we further add the limits of all bounded increasing sequences of rational numbers, we arrive at the system of all "real" numbers. The word *real* is placed in quotes because there is nothing that is either "more real" or "less real" about this system than there is about any other mathematical system.

The Complex Numbers

Imagination was called upon at many stages during the development of the real number system. In fact, the art of invention was needed at least three times in constructing the systems we have discussed so far:

1. The *first invented* system: the set of *all integers* as constructed from the counting numbers.
2. The *second invented* system: the set of *rational numbers* m/n as constructed from the integers.
3. The *third invented* system: the set of all *real numbers* x as constructed from the rational numbers.

These invented systems form a hierarchy in which each system contains the previous system. Each system is also richer than its predecessor in that it permits additional operations to be performed without going outside the system:

1. In the system of all integers, we can solve all equations of the form

   $$x + a = 0, \tag{6}$$

   where a can be any integer.
2. In the system of all rational numbers, we can solve all equations of the form

   $$ax + b = 0, \tag{7}$$

   provided a and b are rational numbers and $a \neq 0$.
3. In the system of all real numbers, we can solve all of the equations in (6) and (7) and, in addition, all quadratic equations

   $$ax^2 + bx + c = 0 \quad \text{having} \quad a \neq 0 \quad \text{and} \quad b^2 - 4ac \geq 0. \tag{8}$$

You are probably familiar with the formula that gives the solutions of (8), namely,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \tag{9}$$

and are familiar with the further fact that when the discriminant, $d = b^2 - 4ac$, is negative, the solutions in (9) do *not* belong to any of the systems discussed above. In fact, the very simple quadratic equation

$$x^2 + 1 = 0$$

is impossible to solve if the only number systems that can be used are the three invented systems mentioned so far.

Thus we come to the *fourth invented* system, the set of all complex numbers $a + ib$. We could dispense entirely with the symbol i and use a notation like (a, b). We would then speak simply of a pair of real numbers a and b. Since, under algebraic operations, the numbers a and b are treated somewhat differently, it is essential to keep the *order* straight. We therefore might say that the **complex number system** consists of the set of all ordered pairs of real numbers (a, b), together with the rules by which they are to be equated, added, multiplied, and so on, listed below. We will use both the (a, b) notation and the notation $a + ib$ in the discussion that follows. We call a the **real part** and b the **imaginary part** of the complex number (a, b).

We make the following definitions.

Equality	
$a + ib = c + id$ if and only if $a = c$ and $b = d$	Two complex numbers (a, b) and (c, d) are *equal* if and only if $a = c$ and $b = d$.
Addition	
$(a + ib) + (c + id) = (a + c) + i(b + d)$	The sum of the two complex numbers (a, b) and (c, d) is the complex number $(a + c, b + d)$.
Multiplication	
$(a + ib)(c + id) = (ac - bd) + i(ad + bc)$	The product of two complex numbers (a, b) and (c, d) is the complex number $(ac - bd, ad + bc)$.
$c(a + ib) = ac + i(bc)$	The product of a real number c and the complex number (a, b) is the complex number (ac, bc).

The set of all complex numbers (a, b) in which the second number b is zero has all the properties of the set of real numbers a. For example, addition and multiplication of $(a, 0)$ and $(c, 0)$ give

$$(a, 0) + (c, 0) = (a + c, 0),$$

$$(a, 0) \cdot (c, 0) = (ac, 0),$$

which are numbers of the same type with imaginary part equal to zero. Also, if we multiply a "real number" $(a, 0)$ and the complex number (c, d), we get

$$(a, 0) \cdot (c, d) = (ac, ad) = a(c, d).$$

In particular, the complex number (0, 0) plays the role of zero in the complex number system, and the complex number (1, 0) plays the role of unity.

The number pair (0, 1), which has real part equal to zero and imaginary part equal to one, has the property that its square,

$$(0, 1)(0, 1) = (-1, 0),$$

has real part equal to minus one and imaginary part equal to zero. Therefore, in the system of complex numbers (a, b), there is a number $x = (0, 1)$ whose square can be added to unity $= (1, 0)$ to produce zero $= (0, 0)$; that is,

$$(0, 1)^2 + (1, 0) = (0, 0).$$

The equation

$$x^2 + 1 = 0$$

therefore has a solution $x = (0, 1)$ in this new number system.

You are probably more familiar with the $a + ib$ notation than you are with the notation (a, b). And since the laws of algebra for the ordered pairs enable us to write

$$(a, b) = (a, 0) + (0, b) = a(1, 0) + b(0, 1),$$

while (1, 0) behaves like unity and (0, 1) behaves like a square root of minus one, we need not hesitate to write $a + ib$ in place of (a, b). The i associated with b is like a tracer element that tags the imaginary part of $a + ib$. We can pass at will from the realm of ordered pairs (a, b) to the realm of expressions $a + ib$, and conversely. But there is nothing less "real" about the symbol $(0, 1) = i$ than there is about the symbol $(1, 0) = 1$, once we have learned the laws of algebra in the complex number system (a, b).

To reduce any rational combination of complex numbers to a single complex number, we apply the laws of elementary algebra, replacing i^2 wherever it appears by -1. Of course, we cannot divide by the complex number $(0, 0) = 0 + i\,0$. But if $a + ib \neq 0$, then we may carry out a division as follows:

$$\frac{c + id}{a + ib} = \frac{(c + id)(a - ib)}{(a + ib)(a - ib)} = \frac{(ac + bd) + i(ad - bc)}{a^2 + b^2}.$$

The result is a complex number $x + iy$ with

$$x = \frac{ac + bd}{a^2 + b^2}, \qquad y = \frac{ad - bc}{a^2 + b^2},$$

and $a^2 + b^2 \neq 0$, since $a + ib = (a, b) \neq (0, 0)$.

The number $a - ib$ that is used as multiplier to clear the i from the denominator is called the **complex conjugate** of $a + ib$. It is customary to use $\bar{z}$ (read "z bar") to denote the complex conjugate of z; thus

$$z = a + ib, \qquad \bar{z} = a - ib.$$

Multiplying the numerator and denominator of the fraction $(c + id)/(a + ib)$ by the complex conjugate of the denominator will always replace the denominator by a real number.

EXAMPLE 1

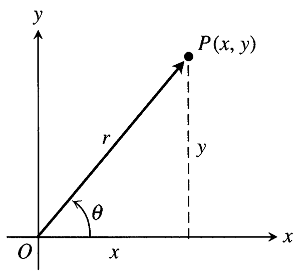
a) $(2 + 3i) + (6 - 2i) = (2 + 6) + (3 - 2)i = 8 + i$

b) $(2 + 3i) - (6 - 2i) = (2 - 6) + (3 - (-2))i = -4 + 5i$

c)
$$\begin{aligned}(2 + 3i)(6 - 2i) &= (2)(6) + (2)(-2i) + (3i)(6) + (3i)(-2i)\\ &= 12 - 4i + 18i - 6i^2 = 12 + 14i + 6 = 18 + 14i\end{aligned}$$

d)
$$\begin{aligned}\frac{2 + 3i}{6 - 2i} &= \frac{2 + 3i}{6 - 2i}\,\frac{6 + 2i}{6 + 2i}\\ &= \frac{12 + 4i + 18i + 6i^2}{36 + 12i - 12i - 4i^2}\\ &= \frac{6 + 22i}{40} = \frac{3}{20} + \frac{11}{20}i\end{aligned}$$

❑



A.3 This Argand diagram represents $z = x + iy$ both as a point $P(x, y)$ and as a vector $\overrightarrow{OP}$.

Argand Diagrams

There are two geometric representations of the complex number $z = x + iy$:

a) as the point $P(x, y)$ in the xy-plane and

b) as the vector $\overrightarrow{OP}$ from the origin to P.

In each representation, the x-axis is called the **real axis** and the y-axis is the **imaginary axis.** Both representations are **Argand diagrams** for $x + iy$ (Fig. A.3).

In terms of the polar coordinates of x and y, we have

$$x = r\cos\theta, \qquad y = r\sin\theta,$$

and

$$z = x + iy = r(\cos\theta + i\sin\theta). \tag{10}$$

We define the **absolute value** of a complex number $x + iy$ to be the length r of a vector $\overrightarrow{OP}$ from the origin to $P(x, y)$. We denote the absolute value by vertical bars, thus:

$$|x + iy| = \sqrt{x^2 + y^2}.$$

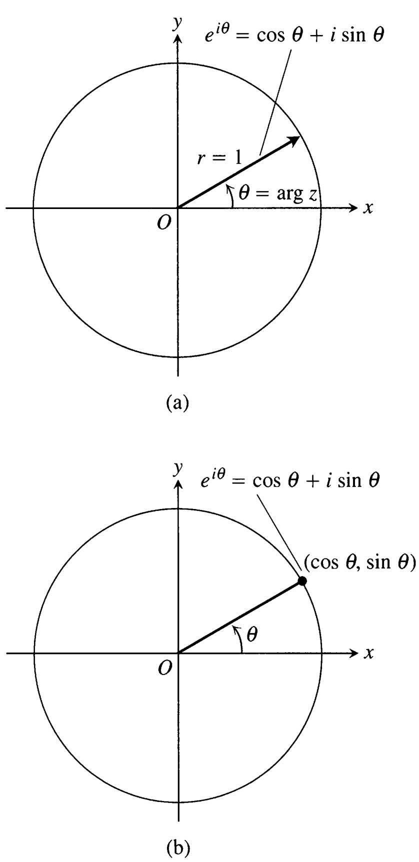
If we always choose the polar coordinates r and θ so that r is nonnegative, then

$$r = |x + iy|.$$

The polar angle θ is called the **argument** of z and is written $\theta = \arg z$. Of course, any integer multiple of 2π may be added to θ to produce another appropriate angle.

The following equation gives a useful formula connecting a complex number z, its conjugate $\bar{z}$, and its absolute value $|z|$, namely,

$$z \cdot \bar{z} = |z|^2.$$



A.4 Argand diagrams for $e^{i\theta} = \cos\theta + i\sin\theta$ (a) as a vector, (b) as a point.

Euler's Formula, Products, and Quotients

The identity

$$e^{i\theta} = \cos\theta + i\sin\theta,$$

called **Euler's formula,** enables us to rewrite Eq. (10) as

$$z = re^{i\theta}.$$

This, in turn, leads to the following rules for calculating products, quotients, powers, and roots of complex numbers. It also leads to Argand diagrams for $e^{i\theta}$. Since $\cos\theta + i\sin\theta$ is what we get from Eq. (10) by taking $r = 1$, we can say that $e^{i\theta}$ is represented by a unit vector that makes an angle θ with the positive x-axis, as shown in Fig. A.4.

Products To multiply two complex numbers, we multiply their absolute values and add their angles. Let

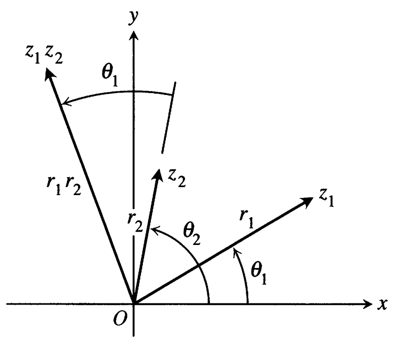
$$z_1 = r_1e^{i\theta_1}, \qquad z_2 = r_2e^{i\theta_2}, \tag{11}$$

so that

$$|z_1| = r_1, \quad \arg z_1 = \theta_1; \qquad |z_2| = r_2, \quad \arg z_2 = \theta_2.$$

Then

$$z_1z_2 = r_1e^{i\theta_1} \cdot r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1+\theta_2)}$$



A.5 When z_1 and z_2 are multiplied, $|z_1z_2| = r_1 \cdot r_2$ and $\arg(z_1z_2) = \theta_1 + \theta_2$.

exp (A) stands for e^A.

and hence

$$\begin{aligned} |z_1z_2| &= r_1r_2 = |z_1| \cdot |z_2|, \\ \arg(z_1z_2) &= \theta_1 + \theta_2 = \arg z_1 + \arg z_2. \end{aligned} \tag{12}$$

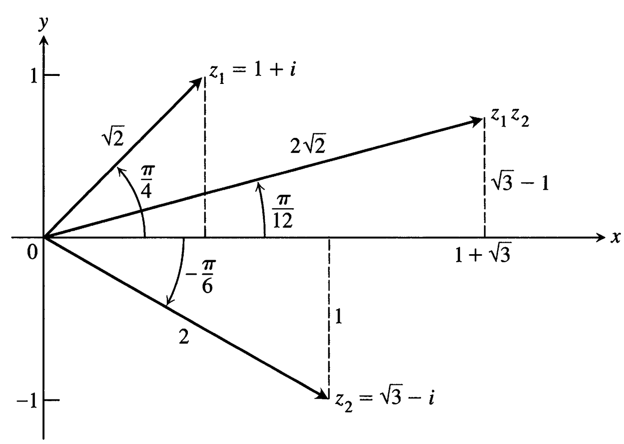
Thus the product of two complex numbers is represented by a vector whose length is the product of the lengths of the two factors and whose argument is the sum of their arguments (Fig. A.5). In particular, a vector may be rotated counterclockwise through an angle θ by multiplying it by $e^{i\theta}$. Multiplication by i rotates 90°, by -1 rotates 180°, by $-i$ rotates 270°, etc.

EXAMPLE 2 Let $z_1 = 1 + i$, $z_2 = \sqrt{3} - i$. We plot these complex numbers in an Argand diagram (Fig. A.6) from which we read off the polar representations

$$z_1 = \sqrt{2}e^{i\pi/4}, \qquad z_2 = 2e^{-i\pi/6}.$$

Then

$$\begin{aligned} z_1z_2 &= 2\sqrt{2}\exp\left(\frac{i\pi}{4} - \frac{i\pi}{6}\right) = 2\sqrt{2}\exp\left(\frac{i\pi}{12}\right) \\ &= 2\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right) \approx 2.73 + 0.73i. \end{aligned}$$



A.6 To multiply two complex numbers, multiply their absolute values and add their arguments. ❑

Quotients

Suppose $r_2 \neq 0$ in Eq. (11). Then

$$\frac{z_1}{z_2} = \frac{r_1e^{i\theta_1}}{r_2e^{i\theta_2}} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}.$$

Hence

$$\left|\frac{z_1}{z_2}\right| = \frac{r_1}{r_2} = \frac{|z_1|}{|z_2|} \quad \text{and} \quad \arg\left(\frac{z_1}{z_2}\right) = \theta_1 - \theta_2 = \arg z_1 - \arg z_2.$$

That is, we divide lengths and subtract angles.

EXAMPLE 3 Let $z_1 = 1 + i$ and $z_2 = \sqrt{3} - i$, as in Example 2. Then

$$\frac{1+i}{\sqrt{3}-i} = \frac{\sqrt{2}e^{i\pi/4}}{2e^{-i\pi/6}} = \frac{\sqrt{2}}{2}e^{5\pi i/12} \approx 0.707\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$$
$$\approx 0.183 + 0.683i. \qquad \square$$

Powers

If n is a positive integer, we may apply the product formulas in (12) to find

$$z^n = z \cdot z \cdot \cdots \cdot z. \qquad n \text{ factors}$$

With $z = re^{i\theta}$, we obtain

$$\begin{aligned} z^n = (re^{i\theta})^n &= r^n e^{i(\theta+\theta+\cdots+\theta)} \qquad n \text{ summands} \\ &= r^n e^{in\theta}. \end{aligned} \tag{13}$$

The length $r = |z|$ is raised to the nth power and the angle $\theta = \arg z$ is multiplied by n.

If we take $r = 1$ in Eq. (13), we obtain De Moivre's theorem.

De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta. \tag{14}$$

If we expand the left-hand side of De Moivre's equation (Eq. 14) by the binomial theorem and reduce it to the form $a + ib$, we obtain formulas for $\cos n\theta$ and $\sin n\theta$ as polynomials of degree n in $\cos\theta$ and $\sin\theta$.

EXAMPLE 4 If $n = 3$ in Eq. (14), we have

$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta.$$

The left-hand side of this equation is

$$\cos^3\theta + 3i\cos^2\theta\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3\theta.$$

The real part of this must equal $\cos 3\theta$ and the imaginary part must equal $\sin 3\theta$. Therefore,

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta,$$
$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta. \qquad \square$$

Roots If $z = re^{i\theta}$ is a complex number different from zero and n is a positive integer, then there are precisely n different complex numbers $w_0, w_1, \ldots, w_{n-1}$, that are nth roots of z. To see why, let $w = \rho e^{i\alpha}$ be an nth root of $z = re^{i\theta}$, so that

$$w^n = z$$

or

$$\rho^n e^{in\alpha} = re^{i\theta}.$$

Then

$$\rho = \sqrt[n]{r}$$

is the real, positive nth root of r. As regards the angle, although we cannot say that $n\alpha$ and θ must be equal, we can say that they may differ only by an integer multiple of 2π. That is,

$$n\alpha = \theta + 2k\pi, \qquad k = 0, \pm 1, \pm 2, \ldots.$$

Therefore,

$$\alpha = \frac{\theta}{n} + k\frac{2\pi}{n}.$$

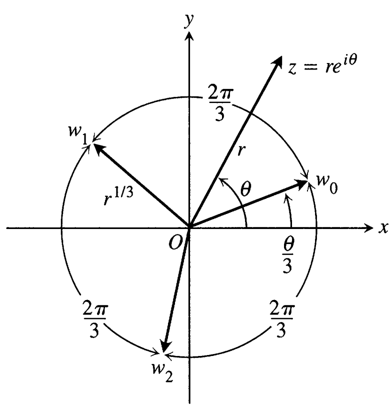
Hence all nth roots of $z = re^{i\theta}$ are given by

$$\sqrt[n]{re^{i\theta}} = \sqrt[n]{r}\ \exp i\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right), \qquad k = 0, \pm 1, \pm 2, \ldots. \tag{15}$$

There might appear to be infinitely many different answers corresponding to the infinitely many possible values of k. But $k = n + m$ gives the same answer as $k = m$ in Eq. (15). Thus we need only take n consecutive values for k to obtain all the different nth roots of z. For convenience, we take

$$k = 0, 1, 2, \ldots, n - 1.$$

All the nth roots of $re^{i\theta}$ lie on a circle centered at the origin O and having radius equal to the real, positive nth root of r. One of them has argument $\alpha = \theta/n$. The others are uniformly spaced around the circle, each being separated from its neighbors by an angle equal to $2\pi/n$. Figure A.7 illustrates the placement of the three cube roots, w_0, w_1, w_2, of the complex number $z = re^{i\theta}$.



A.7 The three cube roots of $z = re^{i\theta}$.

EXAMPLE 5 Find the four fourth roots of -16.

Solution As our first step, we plot the number -16 in an Argand diagram (Fig. A.8) and determine its polar representation $re^{i\theta}$. Here, $z = -16$, $r = +16$, and $\theta = \pi$. One of the fourth roots of $16e^{i\pi}$ is $2e^{i\pi/4}$. We obtain others by successive additions of $2\pi/4 = \pi/2$ to the argument of this first one. Hence

$$\sqrt[4]{16 \exp i\pi} = 2\ \exp i\left(\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right),$$

and the four roots are

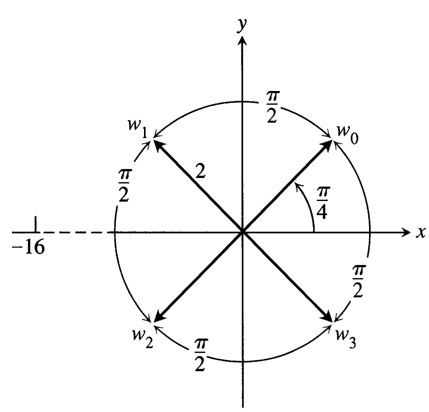
$$w_0 = 2\left[\cos\frac{\pi}{4} + i\ \sin\frac{\pi}{4}\right] = \sqrt{2}(1 + i),$$

$$w_1 = 2\left[\cos\frac{3\pi}{4} + i\ \sin\frac{3\pi}{4}\right] = \sqrt{2}(-1 + i),$$

$$w_2 = 2\left[\cos\frac{5\pi}{4} + i\ \sin\frac{5\pi}{4}\right] = \sqrt{2}(-1 - i),$$

$$w_3 = 2\left[\cos\frac{7\pi}{4} + i\ \sin\frac{7\pi}{4}\right] = \sqrt{2}(1 - i).$$

❑



A.8 The four fourth roots of -16.

The Fundamental Theorem of Algebra One may well say that the invention of $\sqrt{-1}$ is all well and good and leads to a number system that is richer than the real number system alone; but where will this process end? Are we also going to invent still more systems so as to obtain $\sqrt[4]{-1}$, $\sqrt[6]{-1}$, and so on? By now it should be clear that this is not necessary. These numbers are already expressible in terms of the complex number system $a + ib$. In fact, the Fundamental Theorem of Algebra says that with the introduction of the complex numbers we now have enough numbers to factor every polynomial into a product of linear factors and hence enough numbers to solve every possible polynomial equation.

The Fundamental Theorem of Algebra

Every polynomial equation of the form

$$a_0z^n + a_1z^{n-1} + a_2z^{n-2} + \cdots + a_{n-1}z + a_n = 0,$$

in which the coefficients $a_0, a_1, \ldots, a_n$ are any complex numbers, whose degree n is greater than or equal to one, and whose leading coefficient a_0 is not zero, has exactly n roots in the complex number system, provided each multiple root of multiplicity m is counted as m roots.

A proof of this theorem can be found in almost any text on the theory of functions of a complex variable.

Exercises A.3

Operations with Complex Numbers

1. *How computers multiply complex numbers*
   Find $(a, b) \cdot (c, d) = (ac - bd, ad + bc)$.
   a) $(2, 3) \cdot (4, -2)$ **b)** $(2, -1) \cdot (-2, 3)$
   c) $(-1, -2) \cdot (2, 1)$
   (This is how complex numbers are multiplied by computers.)
2. Solve the following equations for the real numbers, x and y.
   a) $(3 + 4i)^2 - 2(x - iy) = x + iy$
   b) $\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{x+iy} = 1 + i$
   c) $(3 - 2i)(x + iy) = 2(x - 2iy) + 2i - 1$

Graphing and Geometry

3. How may the following complex numbers be obtained from $z = x + iy$ geometrically? Sketch.
   a) $\overline{z}$ **b)** $\overline{(-z)}$
   c) $-z$ **d)** $1/z$
4. Show that the distance between the two points z_1 and z_2 in an Argand diagram is equal to $|z_1 - z_2|$.

In Exercises 5–10, graph the points $z = x + iy$ that satisfy the given conditions.

5. **a)** $|z| = 2$ **b)** $|z| < 2$ **c)** $|z| > 2$
6. $|z - 1| = 2$
7. $|z + 1| = 1$
8. $|z + 1| = |z - 1|$
9. $|z + i| = |z - 1|$
10. $|z + 1| \geq |z|$

Express the complex numbers in Exercises 11–14 in the form $re^{i\theta}$, with $r \geq 0$ and $-\pi < \theta \leq \pi$. Draw an Argand diagram for each calculation.

11. $(1 + \sqrt{-3})^2$
12. $\dfrac{1+i}{1-i}$
13. $\dfrac{1+i\sqrt{3}}{1-i\sqrt{3}}$
14. $(2 + 3i)(1 - 2i)$

Theory and Examples

15. Show with an Argand diagram that the law for adding complex numbers is the same as the parallelogram law for adding vectors.
16. Show that the conjugate of the sum (product, or quotient) of two complex numbers z_1 and z_2 is the same as the sum (product, or quotient) of their conjugates.
17. *Complex roots of polynomials with real coefficients come in complex-conjugate pairs.*
    a) Extend the results of Exercise 16 to show that $f(\overline{z}) = \overline{f(z)}$

if

$$f(z) = a_0 z^n + a_1 z^{n-1} + \cdots + a_{n-1} z + a_n$$

is a polynomial with real coefficients $a_0, \ldots, a_n$.

b) If z is a root of the equation $f(z) = 0$, where $f(z)$ is a polynomial with real coefficients as in part (a), show that the conjugate $\bar{z}$ is also a root of the equation. (*Hint:* Let $f(z) = u + iv = 0$; then both u and v are zero. Now use the fact that $f(\bar{z}) = \overline{f(z)} = u - iv$.)

18. Show that $|\bar{z}| = |z|$.

19. If z and $\bar{z}$ are equal, what can you say about the location of the point z in the complex plane?

20. Let $Re(z)$ denote the real part of z and $Im(z)$ the imaginary part. Show that the following relations hold for any complex numbers z, z_1, and z_2.

a) $z + \bar{z} = 2Re(z)$ **b)** $z - \bar{z} = 2iIm(z)$

c) $|Re(z)| \le |z|$

d) $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2 + 2Re(z_1\bar{z}_2)$

e) $|z_1 + z_2| \le |z_1| + |z_2|$

Use De Moivre's theorem to express the trigonometric functions in Exercises 21 and 22 in terms of cos θ and sin θ.

21. cos 4θ **22.** sin 4θ

Roots

23. Find the three cube roots of 1.

24. Find the two square roots of i.

25. Find the three cube roots of $-8i$.

26. Find the six sixth roots of 64.

27. Find the four solutions of the equation $z^4 - 2z^2 + 4 = 0$.

28. Find the six solutions of the equation $z^6 + 2z^3 + 2 = 0$.

29. Find all solutions of the equation $x^4 + 4x^2 + 16 = 0$.

30. Solve the equation $x^4 + 1 = 0$.

A.4 Simpson's One-Third Rule

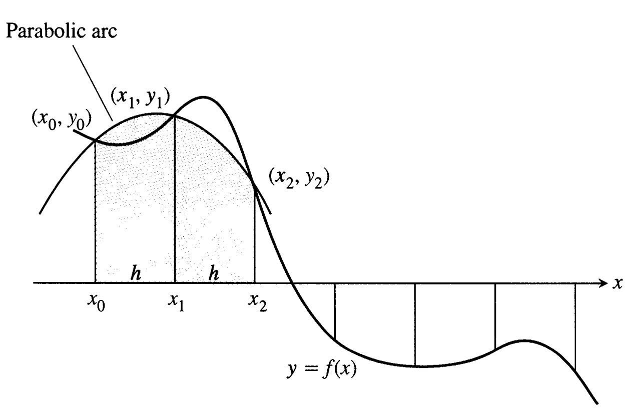
Simpson's rule for approximating $\int_a^b f(x)\,dx$ is based on approximating the graph of f with parabolic arcs.

The area of the shaded region under the parabola in Fig. A.9 is

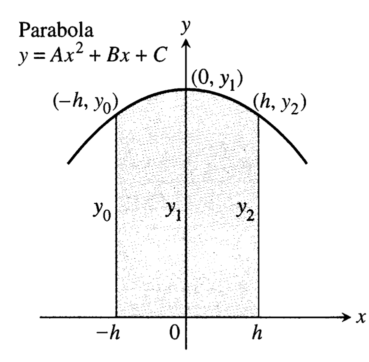
$$\text{Area} = \frac{h}{3}(y_0 + 4y_1 + y_2).$$

This formula is known as Simpson's one-third rule.

We can derive the formula as follows. To simplify the algebra, we use the coordinate system in Fig. A.10. The area under the parabola is the same no matter where the y-axis is, as long as we preserve the vertical scale. The parabola has an equation of the form $y = Ax^2 + Bx + C$, so the area under it from $x = -h$ to



A.9 Simpson's rule approximates short stretches of curve with parabolic arcs.



A.10 By integrating from $-h$ to h, the shaded area is found to be

$$\frac{h}{3}(y_0 + 4y_1 + y_2).$$

$x = h$ is

$$\text{Area} = \int_{-h}^{h} (Ax^2 + Bx + C)\,dx = \left[\frac{Ax^3}{3} + \frac{Bx^2}{2} + Cx\right]_{-h}^{h}$$
$$= \frac{2Ah^3}{3} + 2Ch$$
$$= \frac{h}{3}(2Ah^2 + 6C).$$

Since the curve passes through $(-h, y_0)$, $(0, y_1)$, and (h, y_2), we also have

$$y_0 = Ah^2 - Bh + C, \qquad y_1 = C, \qquad y_2 = Ah^2 + Bh + C.$$

From these equations we obtain

$$C = y_1,$$
$$Ah^2 - Bh = y_0 - y_1,$$
$$Ah^2 + Bh = y_2 - y_1,$$
$$2Ah^2 = y_0 + y_2 - 2y_1.$$

These substitutions for C and $2Ah^2$ give

$$\text{Area} = \frac{h}{3}(2Ah^2 + 6C) = \frac{h}{3}((y_0 + y_2 - 2y_1) + 6y_1) = \frac{h}{3}(y_0 + 4y_1 + y_2).$$

A.5 Cauchy's Mean Value Theorem and the Stronger Form of l'Hôpital's Rule

This appendix proves the finite-limit case of the stronger form of l'Hôpital's Rule (Section 6.6, Theorem 3).

L'Hôpital's Rule (Stronger Form)

Suppose that

$$f(x_0) = g(x_0) = 0$$

and that the functions f and g are both differentiable on an open interval (a, b) that contains the point x_0. Suppose also that $g' \neq 0$ at every point in (a, b) except possibly x_0. Then

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \lim_{x \to x_0} \frac{f'(x)}{g'(x)}, \tag{1}$$

provided the limit on the right exists.

The proof of the stronger form of l'Hôpital's rule is based on Cauchy's Mean Value Theorem, a mean value theorem that involves two functions instead of one. We prove Cauchy's theorem first and then show how it leads to l'Hôpital's rule.

Cauchy's Mean Value Theorem

Suppose that functions f and g are continuous on $[a, b]$ and differentiable throughout (a, b) and suppose also that $g' \neq 0$ throughout (a, b). Then there exists a number c in (a, b) at which

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}. \tag{2}$$

The ordinary Mean Value Theorem (Section 3.2, Theorem 4) is the case $g(x) = x$.

Proof of Cauchy's Mean Value Theorem We apply the Mean Value Theorem of Section 3.2 twice. First we use it to show that $g(a) \neq g(b)$. For if $g(b)$ did equal $g(a)$, then the Mean Value Theorem would give

$$g'(c) = \frac{g(b) - g(a)}{b - a} = 0$$

for some c between a and b. This cannot happen because $g'(x) \neq 0$ in (a, b).

We next apply the Mean Value Theorem to the function

$$F(x) = f(x) - f(a) - \frac{f(b) - f(a)}{g(b) - g(a)}[g(x) - g(a)].$$

This function is continuous and differentiable where f and g are, and $F(b) = F(a) = 0$. Therefore there is a number c between a and b for which $F'(c) = 0$. In terms of f and g this says

$$F'(c) = f'(c) - \frac{f(b) - f(a)}{g(b) - g(a)}[g'(c)] = 0,$$

or

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)},$$

which is Eq. (2). ❑

Proof of the Stronger Form of l'Hôpital's Rule We first establish Eq. (1) for the case $x \to x_0^+$. The method needs almost no change to apply to $x \to x_0^-$, and the combination of these two cases establishes the result.

Suppose that x lies to the right of x_0. Then $g'(x) \neq 0$ and we can apply Cauchy's Mean Value Theorem to the closed interval from x_0 to x. This produces a number c between x_0 and x such that

$$\frac{f'(c)}{g'(c)} = \frac{f(x) - f(x_0)}{g(x) - g(x_0)}.$$

But $f(x_0) = g(x_0) = 0$, so

$$\frac{f'(c)}{g'(c)} = \frac{f(x)}{g(x)}.$$

As x approaches x_0, c approaches x_0 because it lies between x and x_0. Therefore,

$$\lim_{x \to x_0^+} \frac{f(x)}{g(x)} = \lim_{c \to x_0^+} \frac{f'(c)}{g'(c)} = \lim_{x \to x_0^+} \frac{f'(x)}{g'(x)}.$$

This establishes l'Hôpital's rule for the case where x approaches x_0 from above. The case where x approaches x_0 from below is proved by applying Cauchy's Mean Value Theorem to the closed interval $[x, x_0]$, $x < x_0$. ❑

A.6 Limits That Arise Frequently

This appendix verifies limits (4)–(6) in Section 8.2, Table 1.

Limit 4: If $|x| < 1$, $\lim_{n\to\infty} x^n = 0$ We need to show that to each $\epsilon > 0$ there corresponds an integer N so large that $|x^n| < \epsilon$ for all n greater than N. Since $\epsilon^{1/n} \to 1$, while $|x| < 1$, there exists an integer N for which $\epsilon^{1/N} > |x|$. In other words,

$$|x^N| = |x|^N < \epsilon. \tag{1}$$

This is the integer we seek because, if $|x| < 1$, then

$$|x^n| < |x^N| \text{ for all } n > N. \tag{2}$$

Combining (1) and (2) produces $|x^n| < \epsilon$ for all $n > N$, concluding the proof.

Limit 5: For any number x, $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$ Let

$$a_n = \left(1+\frac{x}{n}\right)^n.$$

Then

$$\ln a_n = \ln\left(1+\frac{x}{n}\right)^n = n\ln\left(1+\frac{x}{n}\right) \to x,$$

as we can see by the following application of l'Hôpital's rule, in which we differentiate with respect to n:

$$\lim_{n\to\infty} n\ln\left(1+\frac{x}{n}\right) = \lim_{n\to\infty}\frac{\ln(1+x/n)}{1/n}$$

$$= \lim_{n\to\infty}\frac{\left(\dfrac{1}{1+x/n}\right)\cdot\left(-\dfrac{x}{n^2}\right)}{-1/n^2} = \lim_{n\to\infty}\frac{x}{1+x/n} = x.$$

Apply Theorem 4, Section 8.2, with $f(x) = e^x$ to conclude that

$$\left(1+\frac{x}{n}\right)^n = a_n = e^{\ln a_n} \to e^x.$$

Limit 6: For any number x, $\lim_{n\to\infty}\frac{x^n}{n!} = 0$ Since

$$-\frac{|x|^n}{n!} \le \frac{x^n}{n!} \le \frac{|x|^n}{n!},$$

all we need to show is that $|x|^n/n! \to 0$. We can then apply the Sandwich Theorem for Sequences (Section 8.2, Theorem 3) to conclude that $x^n/n! \to 0$.

The first step in showing that $|x|^n/n! \to 0$ is to choose an integer $M > |x|$, so that $(|x|/M) < 1$. By Limit 4, just proved, we then have $(|x|/M)^n \to 0$. We then restrict our attention to values of $n > M$. For these values of n, we can write

$$\frac{|x|^n}{n!} = \frac{|x|^n}{1 \cdot 2 \cdot \cdots \cdot M \cdot \underbrace{(M+1)(M+2) \cdot \cdots \cdot n}_{(n-M)\text{ factors}}}$$

$$\leq \frac{|x|^n}{M!M^{n-M}} = \frac{|x|^n M^M}{M!M^n} = \frac{M^M}{M!}\left(\frac{|x|}{M}\right)^n.$$

Thus,

$$0 \leq \frac{|x|^n}{n!} \leq \frac{M^M}{M!}\left(\frac{|x|}{M}\right)^n.$$

Now, the constant $M^M/M!$ does not change as n increases. Thus the Sandwich Theorem tell us that $|x|^n/n! \to 0$ because $(|x|/M)^n \to 0$.

A.7 The Distributive Law for Vector Cross Products

In this appendix we prove the distributive law

$$\mathbf{A} \times (\mathbf{B} + \mathbf{C}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{C} \tag{1}$$

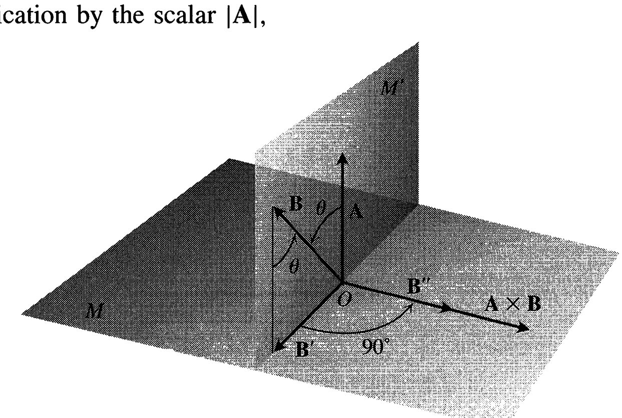
from Eq. (6) in Section 10.4.

Proof To derive Eq. (1), we construct $\mathbf{A} \times \mathbf{B}$ a new way. We draw $\mathbf{A}$ and $\mathbf{B}$ from the common point O and construct a plane M perpendicular to $\mathbf{A}$ at O (Fig. A.11). We then project $\mathbf{B}$ orthogonally onto M, yielding a vector $\mathbf{B}'$ with length $|\mathbf{B}| \sin \theta$. We rotate $\mathbf{B}'$ 90°about $\mathbf{A}$ in the positive sense to produce a vector $\mathbf{B}''$. Finally, we multiply $\mathbf{B}''$ by the length of $\mathbf{A}$. The resulting vector $|\mathbf{A}|\mathbf{B}''$ is equal to $\mathbf{A} \times \mathbf{B}$ since $\mathbf{B}''$ has the same direction as $\mathbf{A} \times \mathbf{B}$ by its construction (Fig. A.11) and

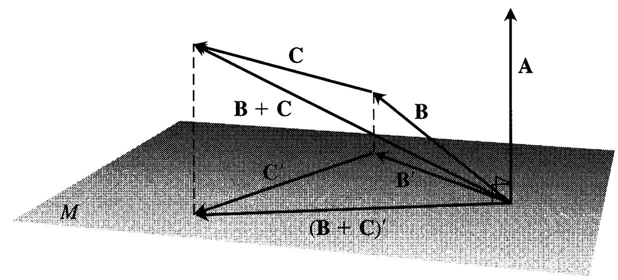
$$|\mathbf{A}||\mathbf{B}''| = |\mathbf{A}||\mathbf{B}'| = |\mathbf{A}||\mathbf{B}| \sin \theta = |\mathbf{A} \times \mathbf{B}|.$$

Now each of these three operations, namely,

1. projection onto M,
2. rotation about $\mathbf{A}$ through 90°,
3. multiplication by the scalar $|\mathbf{A}|$,



A.11 As explained in the text, $\mathbf{A} \times \mathbf{B} = |\mathbf{A}|\mathbf{B}''$.



A.12 The vectors, **B, C, B + C,** and their projections onto a plane perpendicular to **A.**

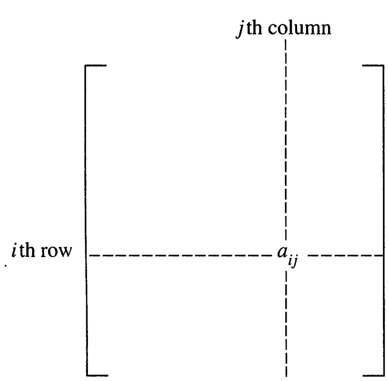
when applied to a triangle whose plane is not parallel to **A,** will produce another triangle. If we start with the triangle whose sides are **B, C,** and **B** + **C** (Fig. A.12) and apply these three steps, we successively obtain

1. a triangle whose sides are $\mathbf{B}'$, $\mathbf{C}'$, and $(\mathbf{B}+\mathbf{C})'$ satisfying the vector equation
$$\mathbf{B}' + \mathbf{C}' = (\mathbf{B}+\mathbf{C})';$$
2. a triangle whose sides are $\mathbf{B}''$, $\mathbf{C}''$, and $(\mathbf{B}+\mathbf{C})''$ satisfying the vector equation
$$\mathbf{B}'' + \mathbf{C}'' = (\mathbf{B}+\mathbf{C})''$$
(the double prime on each vector has the same meaning as in Fig. A.11); and, finally,
3. a triangle whose sides are $|\mathbf{A}|\mathbf{B}''$, $|\mathbf{A}|\mathbf{C}''$, and $|\mathbf{A}|(\mathbf{B}+\mathbf{C})''$ satisfying the vector equation
$$|\mathbf{A}|\mathbf{B}'' + |\mathbf{A}|\mathbf{C}'' = |\mathbf{A}|(\mathbf{B}+\mathbf{C})''. \quad (2)$$

Substituting $|\mathbf{A}|\mathbf{B}'' = \mathbf{A}\times\mathbf{B}$, $|\mathbf{A}|\mathbf{C}'' = \mathbf{A}\times\mathbf{C}$, and $|\mathbf{A}|(\mathbf{B}+\mathbf{C})'' = \mathbf{A}\times(\mathbf{B}+\mathbf{C})$ from our discussion above into Eq. (2) gives
$$\mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C} = \mathbf{A}\times(\mathbf{B}+\mathbf{C}),$$
which is the law we wanted to establish. ❑

A.8 Determinants and Cramer's Rule

A rectangular array of numbers like
$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix}$$
is called a **matrix.** We call A a 2 by 3 matrix because it has two rows and three columns. An m by n matrix has m rows and n columns, and the **entry** or **element** (number) in the ith row and jth column is denoted by a_{ij}. The matrix



$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & -2 \end{bmatrix}$$
has
$$a_{11} = 2, \quad a_{12} = 1, \quad a_{13} = 3,$$
$$a_{21} = 1, \quad a_{22} = 0, \quad a_{23} = -2.$$

A matrix with the same number of rows as columns is a **square matrix.** It is a **matrix of order *n*** if the number of rows and columns is n.

With each square matrix A we associate a number det A or $|a_{ij}|$, called the **determinant** of A, calculated from the entries of A in the following way. For $n = 1$ and $n = 2$, we define

The vertical bars in the notation $|a_{ij}|$ do not mean absolute value.

$$\det [a] = a, \tag{1}$$

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11}a_{22} - a_{21}a_{12}. \tag{2}$$

For a matrix of order 3, we define

$$\det A = \det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{array}{l} \text{Sum of all signed products} \\ \text{of the form } \pm a_{1i}a_{2j}a_{3k}, \end{array} \tag{3}$$

where i, j, k is a permutation of 1, 2, 3 in some order. There are $3! = 6$ such permutations, so there are six terms in the sum. The sign is positive when the index of the permutation is even and negative when the index is odd.

Definition
Index of a Permutation

Given any permutation of the numbers $1, 2, 3, \ldots, n$, denote the permutation by $i_1, i_2, i_3, \ldots, i_n$. In this arrangement, some of the numbers following i_1 may be less than i_1, and the number of these is called the **number of inversions** in the arrangement pertaining to i_1. Likewise, there are a number of inversions pertaining to each of the other i's; it is the number of indices that come after that particular i in the arrangement and are less than it. The **index** of the permutation is the sum of all of the numbers of inversions pertaining to the separate indices.

EXAMPLE 1 For $n = 5$, the permutation

$$5 \quad 3 \quad 1 \quad 2 \quad 4$$

has 4 inversions pertaining to the first element, 5, 2 inversions pertaining to the second element, 3, and no further inversions, so the index is $4 + 2 = 6$. ❑

The following table shows the permutations of 1, 2, 3, the index of each permutation, and the signed product in the determinant of Eq. (3).

Permutation	Index	Signed product
1 2 3	0	$+a_{11}a_{22}a_{33}$
1 3 2	1	$-a_{11}a_{23}a_{32}$
2 1 3	1	$-a_{12}a_{21}a_{33}$
2 3 1	2	$+a_{12}a_{23}a_{31}$
3 1 2	2	$+a_{13}a_{21}a_{32}$
3 2 1	3	$-a_{13}a_{22}a_{31}$

(4)

The sum of the six signed products is

$$a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

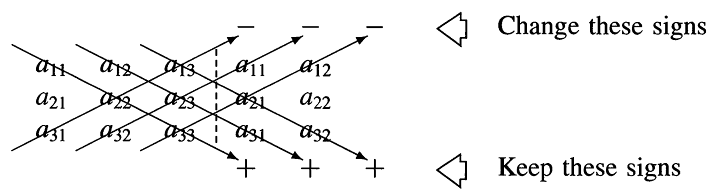
$$= a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \quad (5)$$

The formula

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \quad (6)$$

reduces the calculation of a 3 by 3 determinant to the calculation of three 2 by 2 determinants.

Many people prefer to remember the following scheme for calculating the six signed products in the determinant of a 3 by 3 matrix:



(7)

Minors and Cofactors

The second order determinants on the right-hand side of Eq. (6) are called the **minors** (short for "minor determinants") of the entries they multiply. Thus,

$$\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} \text{ is the minor of } a_{11}, \quad \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} \text{ is the minor of } a_{12},$$

and so on. The minor of the element a_{ij} in a matrix A is the determinant of the matrix that remains after we delete the row and column containing a_{ij}:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \quad \text{The minor of } a_{22} \text{ is } \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}.$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}. \quad \text{The minor of } a_{23} \text{ is } \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}.$$

The **cofactor** A_{ij} of a_{ij} is $(-1)^{i+j}$ times the minor of a_{ij}. Thus,

$$A_{22} = (-1)^{2+2}\begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix},$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}.$$

The factor $(-1)^{i+j}$ changes the sign of the minor when $i + j$ is odd. There is a checkerboard pattern for remembering these changes:

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

In the upper left corner, $i = 1$, $j = 1$ and $(-1)^{1+1} = +1$. In going from any cell to an adjacent cell in the same row or column, we change i by 1 or j by 1, but not both, so we change the exponent from even to odd or from odd to even, which changes the sign from $+$ to $-$ or from $-$ to $+$.

When we rewrite Eq. (6) in terms of cofactors we get

$$\det A = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}. \tag{8}$$

EXAMPLE 2 Find the determinant of

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}.$$

Solution 1 The cofactors are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix}, \qquad A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix},$$

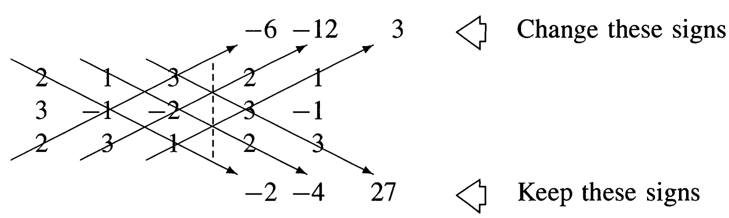
$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}.$$

To find det A, we multiply each element of the first row of A by its cofactor and add:

$$\det A = 2\begin{vmatrix} -1 & -2 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -2 \\ 2 & 1 \end{vmatrix} + 3\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= 2(-1+6) - 1(3+4) + 3(9+2) = 10 - 7 + 33 = 36.$$

Solution 2 From (7) we find



$$\det A = -(-6) - (-12) - 3 + (-2) + (-4) + 27 = 36$$ ❑

Expanding by Columns or by Other Rows

The determinant of a square matrix can be calculated from the cofactors of any row or any column.

If we were to expand the determinant in Example 2 by cofactors according to elements of its third column, say, we would get

$$+3\begin{vmatrix} 3 & -1 \\ 2 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix}$$

$$= 3(9+2) + 2(6-2) + 1(-2-3) = 33 + 8 - 5 = 36.$$

Useful Facts About Determinants

Fact 1: If two rows (or columns) are identical, the determinant is zero.

Fact 2: Interchanging two rows (or columns) changes the sign of the determinant.

Fact 3: The determinant is the sum of the products of the elements of the ith row (or column) by their cofactors, for any i.

Fact 4: The determinant of the transpose of a matrix is the same as the determinant of the original matrix. (The **transpose** of a matrix is obtained by writing the rows as columns.)

Fact 5: Multiplying each element of some row (or column) by a constant c multiplies the determinant by c.

Fact 6: If all elements above the main diagonal (or all below it) are zero, the determinant is the product of the elements on the main diagonal. (The **main diagonal** is the diagonal from upper left to lower right.)

EXAMPLE 3

$$\begin{vmatrix} 3 & 4 & 7 \\ 0 & -2 & 5 \\ 0 & 0 & 5 \end{vmatrix} = (3)(-2)(5) = -30$$

❑

Fact 7: If the elements of any row are multiplied by the cofactors of the corresponding elements of a different row and these products are summed, the sum is zero.

EXAMPLE 4 If A_{11}, A_{12}, A_{13} are the cofactors of the elements of the first row of $A = (a_{ij})$, then the sums

$$a_{21}A_{11} + a_{22}A_{12} + a_{23}A_{13}$$

(elements of second row times cofactors of elements of first row) and

$$a_{31}A_{11} + a_{32}A_{12} + a_{33}A_{13}$$

are both zero. ❑

Fact 8: If the elements of any column are multiplied by the cofactors of the corresponding elements of a different column and these products are summed, the sum is zero.

Fact 9: If each element of a row is multiplied by a constant c and the results added to a different row, the determinant is not changed. A similar result holds for columns.

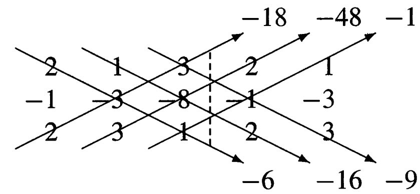
EXAMPLE 5 If we start with

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$$

and add -2 times row 1 to row 2 (subtract 2 times row 1 from row 2), we get

$$B = \begin{bmatrix} 2 & 1 & 3 \\ -1 & -3 & -8 \\ 2 & 3 & 1 \end{bmatrix}.$$

Since det $A = 36$ (Example 2), we should find that det $B = 36$ as well. Indeed we do, as the following calculation shows:



$$\det B = -(-18) - (-48) - (-1) + (-6) + (-16) + (-9)$$
$$= 18 + 48 + 1 - 6 - 16 - 9 = 67 - 31 = 36.$$ ❑

EXAMPLE 6 Evaluate the fourth order determinant

$$D = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & 0 & 2 \\ -1 & 2 & 1 & -2 \\ 0 & 1 & 2 & 1 \end{vmatrix}.$$

Solution We subtract 2 times row 1 from row 2 and add row 1 to row 3 to get

$$D = \begin{vmatrix} 1 & -2 & 3 & 1 \\ 0 & 5 & -6 & 0 \\ 0 & 0 & 4 & -1 \\ 0 & 1 & 2 & 1 \end{vmatrix}.$$

We then multiply the elements of the first column by their cofactors to get

$$D = \begin{vmatrix} 5 & -6 & 0 \\ 0 & 4 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 5(4+2) - (-6)(0+1) + 0 = 36.$$ ❑

Cramer's Rule

If the determinant $D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0$, the system

$$\begin{aligned} a_{11}x + a_{12}y &= b_1, \\ a_{21}x + a_{22}y &= b_2 \end{aligned} \tag{9}$$

has either infinitely many solutions or no solution at all. The system

$$\begin{aligned} x + y &= 0, \\ 2x + 2y &= 0 \end{aligned}$$

whose determinant is

$$D = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$$

has infinitely many solutions. We can find an x to match any given y. The system

$$\begin{aligned} x + y &= 0, \\ 2x + 2y &= 2 \end{aligned}$$

has no solution. If $x + y = 0$, then $2x + 2y = 2(x + y)$ cannot be 2.

If $D \neq 0$, the system (9) has a unique solution, and Cramer's rule states that it may be found from the formulas

$$x = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{D}, \qquad y = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{D}. \tag{10}$$

The numerator in the formula for x comes from replacing the first column in A (the x-column) by the column of constants b_1 and b_2 (the b-column). Replacing the y-column by the b-column gives the numerator of the y-solution.

EXAMPLE 7 Solve the system

$$\begin{aligned} 3x - \ \ y &= \ \ 9, \\ x + 2y &= -4. \end{aligned}$$

Solution We use Eqs. (10). The determinant of the coefficient matrix is

$$D = \begin{vmatrix} 3 & -1 \\ 1 & 2 \end{vmatrix} = 6 + 1 = 7.$$

Hence,

$$x = \frac{\begin{vmatrix} 9 & -1 \\ -4 & 2 \end{vmatrix}}{D} = \frac{18 - 4}{7} = \frac{14}{7} = 2,$$

$$y = \frac{\begin{vmatrix} 3 & 9 \\ 1 & -4 \end{vmatrix}}{D} = \frac{-12 - 9}{7} = \frac{-21}{7} = -3.$$

❑

Systems of three equations in three unknowns work the same way. If

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0,$$

the system

$$\begin{aligned} a_{11}x + a_{12}y + a_{13}z &= b_1, \\ a_{21}x + a_{22}y + a_{23}z &= b_2, \\ a_{31}x + a_{32}y + a_{33}z &= b_3 \end{aligned} \tag{11}$$

has either infinitely many solutions or no solution at all. If $D \neq 0$, the system has a unique solution, given by Cramer's rule:

$$x = \frac{1}{D}\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \qquad y = \frac{1}{D}\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}.$$

$$z = \frac{1}{D}\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

The pattern continues in higher dimensions.

Exercises A.8

Evaluating Determinants

Evaluate the following determinants.

1. $\begin{vmatrix} 2 & 3 & 1 \\ 4 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

2. $\begin{vmatrix} 2 & -1 & -2 \\ -1 & 2 & 1 \\ 3 & 0 & -3 \end{vmatrix}$

3. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 3 & 2 \end{vmatrix}$

4. $\begin{vmatrix} 1 & -1 & 2 & 3 \\ 2 & 1 & 2 & 6 \\ 1 & 0 & 2 & 3 \\ -2 & 2 & 0 & -5 \end{vmatrix}$

Evaluate the following determinants by expanding according to the cofactors of (a) the third row and (b) the second column.

5. $\begin{vmatrix} 2 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 2 & 1 \end{vmatrix}$

6. $\begin{vmatrix} 1 & 0 & -1 \\ 0 & 2 & -2 \\ 2 & 0 & 1 \end{vmatrix}$

7. $\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & -1 & 0 & 7 \\ 3 & 0 & 2 & 1 \end{vmatrix}$

8. $\begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{vmatrix}$

Systems of Equations

Solve the following systems of equations by Cramer's rule.

9. $\begin{aligned} x + 8y &= 4 \\ 3x - y &= -13 \end{aligned}$

10. $\begin{aligned} 2x + 3y &= 5 \\ 3x - y &= 2 \end{aligned}$

11. $\begin{aligned} 4x - 3y &= 6 \\ 3x - 2y &= 5 \end{aligned}$

12. $\begin{aligned} x + y + z &= 2 \\ 2x - y + z &= 0 \\ x + 2y - z &= 4 \end{aligned}$

13. $\begin{aligned} 2x + y - z &= 2 \\ x - y + z &= 7 \\ 2x + 2y + z &= 4 \end{aligned}$

14. $\begin{aligned} 2x - 4y \qquad &= 6 \\ x + y + z &= 1 \\ 5y + 7z &= 10 \end{aligned}$

15. $\begin{aligned} x \qquad - z &= 3 \\ 2y - 2z &= 2 \\ 2x \qquad + z &= 3 \end{aligned}$

16. $\begin{aligned} x_1 + x_2 - x_3 + x_4 &= 2 \\ x_1 - x_2 + x_3 + x_4 &= -1 \\ x_1 + x_2 + x_3 - x_4 &= 2 \\ x_1 \qquad + x_3 + x_4 &= -1 \end{aligned}$

Theory and Examples

17. Find values of h and k for which the system

$$2x + hy = 8,$$
$$x + 3y = k$$

has (a) infinitely many solutions, (b) no solution at all.

18. For what value of x will

$$\begin{vmatrix} x & x & 1 \\ 2 & 0 & 5 \\ 6 & 7 & 1 \end{vmatrix} = 0?$$

19. Suppose u, v, and w are twice-differentiable functions of x that satisfy the relation $au + bv + cw = 0$, where a, b, and c are constants, not all zero. Show that

$$\begin{vmatrix} u & v & w \\ u' & v' & w' \\ u'' & v'' & w'' \end{vmatrix} = 0.$$

20. *Partial fractions.* Expanding the quotient

$$\frac{ax + b}{(x - r_1)(x - r_2)}$$

by partial fractions calls for finding the values of C and D that make the equation

$$\frac{ax + b}{(x - r_1)(x - r_2)} = \frac{C}{x - r_1} + \frac{D}{x - r_2}$$

hold for all x.

a) Find a system of linear equations that determines C and D.

b) Under what circumstances does the system of equations in part (a) have a unique solution? That is, when is the determinant of the coefficient matrix of the system different from zero?

A.9 Euler's Theorem and the Increment Theorem

This appendix derives Euler's Theorem (Theorem 2, Section 12.3) and the Increment Theorem for Functions of Two Variables (Theorem 3, Section 12.4). Euler first published his theorem in 1734, in a series of papers he wrote on hydrodynamics.

Euler's Theorem

If $f(x, y)$ and its partial derivatives f_x, f_y, f_{xy}, and f_{yx} are defined throughout an open region containing a point (a, b) and are all continuous at (a, b), then $f_{xy}(a, b) = f_{yx}(a, b)$.

Proof The equality of $f_{xy}(a,b)$ and $f_{yx}(a,b)$ can be established by four applications of the Mean Value Theorem (Theorem 4, Section 3.2). By hypothesis, the point (a,b) lies in the interior of a rectangle R in the xy-plane on which f, f_x, f_y, f_{xy}, and f_{yx} are all defined. We let h and k be numbers such that the point $(a+h, b+k)$ also lies in the rectangle R, and we consider the difference

$$\Delta = F(a+h) - F(a), \tag{1}$$

where

$$F(x) = f(x, b+k) - f(x, b). \tag{2}$$

We apply the Mean Value Theorem to F (which is continuous because it is differentiable), and Eq. (1) becomes

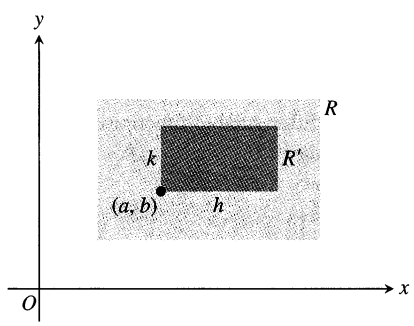
$$\Delta = hF'(c_1), \tag{3}$$

where c_1 lies between a and $a+h$. From Eq. (2),

$$F'(x) = f_x(x, b+k) - f_x(x, b),$$

so Eq. (3) becomes

$$\Delta = h[f_x(c_1, b+k) - f_x(c_1, b)]. \tag{4}$$



A.13 The key to proving $f_{xy}(a,b) = f_{yx}(a,b)$ is the fact that no matter how small R' is, f_{xy} and f_{yx} take on equal values somewhere inside R' (although not necessarily at the same point).

Now we apply the Mean Value Theorem to the function $g(y) = f_x(c_1, y)$ and have

$$g(b+k) - g(b) = kg'(d_1),$$

or

$$f_x(c_1, b+k) - f_x(c_1, b) = kf_{xy}(c_1, d_1),$$

for some d_1 between b and $b+k$. By substituting this into Eq. (4), we get

$$\Delta = hkf_{xy}(c_1, d_1), \tag{5}$$

for some point (c_1, d_1) in the rectangle R' whose vertices are the four points (a,b), $(a+h,b)$, $(a+h,b+k)$, and $(a,b+k)$. (See Fig. A.13.)

By substituting from Eq. (2) into Eq. (1), we may also write

$$\begin{aligned}\Delta &= f(a+h, b+k) - f(a+h, b) - f(a, b+k) + f(a, b)\\ &= [f(a+h, b+k) - f(a, b+k)] - [f(a+h, b) - f(a, b)] \qquad (6)\\ &= \phi(b+k) - \phi(b),\end{aligned}$$

where $$\phi(y) = f(a+h, y) - f(a, y). \tag{7}$$

The Mean Value Theorem applied to Eq. (6) now gives

$$\Delta = k\phi'(d_2), \tag{8}$$

for some d_2 between b and $b+k$. By Eq. (7),

$$\phi'(y) = f_y(a+h, y) - f_y(a, y). \tag{9}$$

Substituting from Eq. (9) into Eq. (8) gives

$$\Delta = k[f_y(a+h, d_2) - f_y(a, d_2)].$$

Finally, we apply the Mean Value Theorem to the expression in brackets and get

$$\Delta = khf_{yx}(c_2, d_2), \tag{10}$$

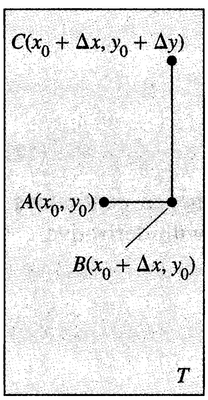
for some c_2 between a and $a+h$.

Together, Eqs. (5) and (10) show that

$$f_{xy}(c_1, d_1) = f_{yx}(c_2, d_2), \tag{11}$$

where (c_1, d_1) and (c_2, d_2) both lie in the rectangle R' (Fig. A.13). Equation (11) is not quite the result we want, since it says only that f_{xy} has the same value at (c_1, d_1) that f_{yx} has at (c_2, d_2). But the numbers h and k in our discussion may be made as small as we wish. The hypothesis that f_{xy} and f_{yx} are both continuous at (a, b) means that $f_{xy}(c_1, d_1) = f_{xy}(a, b) + \epsilon_1$ and $f_{yx}(c_2, d_2) = f_{yx}(a, b) + \epsilon_2$, where $\epsilon_1, \epsilon_2 \to 0$ as $h, k \to 0$. Hence, if we let h and $k \to 0$, we have $f_{xy}(a, b) = f_{yx}(a, b)$. ❑

The equality of $f_{xy}(a, b)$ and $f_{yx}(a, b)$ can be proved with hypotheses weaker than the ones we assumed. For example, it is enough for f, f_x, and f_y to exist in R and for f_{xy} to be continuous at (a, b). Then f_{yx} will exist at (a, b) and will equal f_{xy} at that point.



A.14 The rectangular region T in the proof of the Increment Theorem. The figure is drawn for Δx and Δy positive, but either increment might be zero or negative.

The Increment Theorem for Functions of Two Variables

Suppose that the first partial derivatives of $z = f(x, y)$ are defined throughout an open region R containing the point (x_0, y_0) and that f_x and f_y are continuous at (x_0, y_0). Then the change $\Delta z = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0, y_0)$ in the value of f that results from moving from (x_0, y_0) to another point $(x_0 + \Delta x, y_0 + \Delta y)$ in R satisfies an equation of the form

$$\Delta z = f_x(x_0, y_0)\Delta x + f_y(x_0, y_0)\Delta y + \epsilon_1 \Delta x + \epsilon_2 \Delta y,$$

in which $\epsilon_1, \epsilon_2 \to 0$ as $\Delta x, \Delta y \to 0$.

Proof We work within a rectangle T centered at $A(x_0, y_0)$ and lying within R, and we assume that Δx and Δy are already so small that the line segment joining A to $B(x_0 + \Delta x, y_0)$ and the line segment joining B to $C(x_0 + \Delta x, y_0 + \Delta y)$ lie in the interior of T (Fig. A.14).

We may think of Δz as the sum $\Delta z = \Delta z_1 + \Delta z_2$ of two increments, where

$$\Delta z_1 = f(x_0 + \Delta x, y_0) - f(x_0, y_0)$$

is the change in the value of f from A to B and

$$\Delta z_2 = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0 + \Delta x, y_0)$$

is the change in the value of f from B to C (Fig. A.15, on the following page).

On the closed interval of x-values joining x_0 to $x_0 + \Delta x$, the function $F(x) = f(x, x_0)$ is a differentiable (and hence continuous) function of x, with derivative

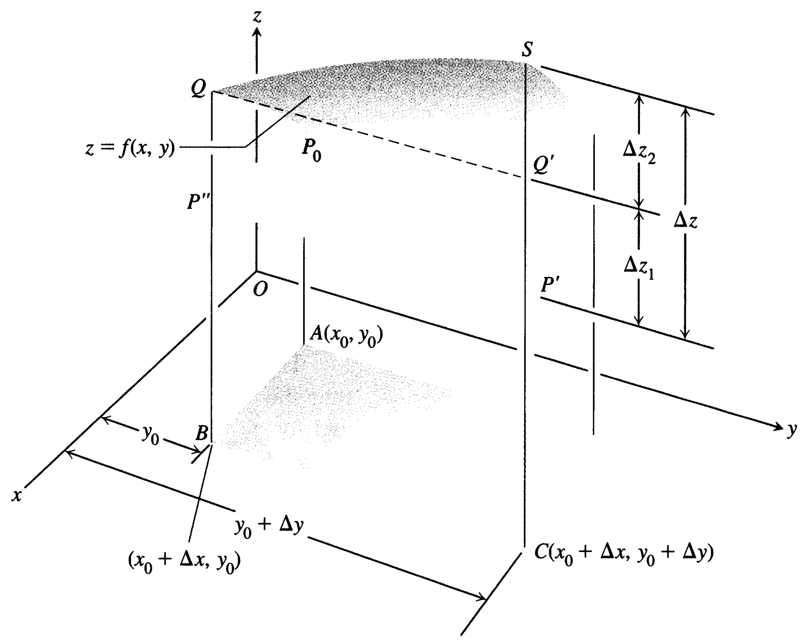
$$F'(x) = f_x(x, y_0).$$

By the Mean Value Theorem (Theorem 4, Section 3.2), there is an x-value c between x_0 and $x_0 + \Delta x$ at which

$$F(x_0 + \Delta x) - F(x_0) = F'(c)\Delta x$$

or

$$f(x_0 + \Delta x, y_0) - f(x_0, y_0) = f_x(c, y_0)\Delta x$$



A.15 Part of the surface $z = f(x, y)$ near $P_0(x_0, y_0, f(x_0, y_0))$. The points P_0, P', and P'' have the same height $z_0 = f(x_0, y_0)$ above the xy-plane. The change in z is $\Delta z = P'S$. The change

$$\Delta z_1 = f(x_0 + \Delta x, y_0) - f(x_0, y_0),$$

shown as $P''Q = P'Q'$, is caused by changing x from x_0 to $x_0 + \Delta x$ while holding y equal to y_0. Then, with x held equal to $x_0 + \Delta x$,

$$\Delta z_2 = f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0 + \Delta x, y_0)$$

is the change in z caused by changing y from y_0 to $y_0 + \Delta y$. This is represented by $Q'S$. The total change in z is the sum of Δz_1 and Δz_2.

or

$$\Delta z_1 = f_x(c, y_0)\Delta x. \tag{12}$$

Similarly, $G(y) = f(x_0 + \Delta x, y)$ is a differentiable (and hence continuous) function of y on the closed y-interval joining y_0 and $y_0 + \Delta y$, with derivative

$$G'(y) = f_y(x_0 + \Delta x, y).$$

Hence there is a y-value d between y_0 and $y_0 + \Delta y$ at which

$$G(y_0 + \Delta y) - G(y_0) = G'(d)\Delta y$$

or

$$f(x_0 + \Delta x, y_0 + \Delta y) - f(x_0 + \Delta x, y) = f_y(x_0 + \Delta x, d)\Delta y$$

or

$$\Delta z_2 = f_y(x_0 + \Delta x, d)\Delta y. \tag{13}$$

Now, as Δx and $\Delta y \to 0$, we know $c \to x_0$ and $d \to y_0$. Therefore, since f_x and f_y are continuous at (x_0, y_0), the quantities

$$\begin{aligned} \epsilon_1 &= f_x(c, y_0) - f_x(x_0, y_0), \\ \epsilon_2 &= f_y(x_0 + \Delta x, d) - f_y(x_0, y_0) \end{aligned} \tag{14}$$

both approach zero as Δx and $\Delta y \to 0$.

Finally,

$$\begin{aligned} \Delta z &= \Delta z_1 + \Delta z_2 \\ &= f_x(c, y_0)\Delta x + f_y(x_0 + \Delta x, d)\,\Delta y && \text{From (12) and (13)} \\ &= [f_x(x_0, y_0) + \epsilon_1]\,\Delta x + [f_y(x_0, y_0) + \epsilon_2]\,\Delta y && \text{From (14)} \\ &= f_x(x_0, y_0)\,\Delta x + f_y(x_0, y_0)\,\Delta y + \epsilon_1\,\Delta x + \epsilon_2\,\Delta y, \end{aligned}$$

where ϵ_1 and $\epsilon_2 \to 0$ as Δx and $\Delta y \to 0$. This is what we set out to prove. ❑

Analogous results hold for functions of any finite number of independent variables. Suppose that the first partial derivatives of

$$w = f(x, y, z)$$

are defined throughout an open region containing the point (x_0, y_0, z_0) and that f_x, f_y, and f_z are continuous at (x_0, y_0). Then

$$\begin{aligned} \Delta w &= f(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z) - f(x_0, y_0, z_0) \\ &= f_x \Delta x + f_y \Delta y + f_z \Delta z + \epsilon_1 \Delta x + \epsilon_2 \Delta y + \epsilon_3 \Delta z, \end{aligned} \tag{15}$$

where

$$\epsilon_1, \epsilon_2, \epsilon_3 \to 0 \quad \text{when} \quad \Delta x, \Delta y, \text{ and } \Delta z \to 0.$$

The partial derivatives f_x, f_y, f_z in this formula are to be evaluated at the point (x_0, y_0, z_0).

The result (15) can be proved by treating Δw as the sum of three increments,

$$\Delta w_1 = f(x_0 + \Delta x, y_0, z_0) - f(x_0, y_0, z_0), \tag{16}$$

$$\Delta w_2 = f(x_0 + \Delta x, y_0 + \Delta y, z_0) - f(x_0 + \Delta x, y_0, z_0), \tag{17}$$

$$\Delta w_3 = f(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z) - f(x_0 + \Delta x, y_0 + \Delta y, z_0), \tag{18}$$

and applying the Mean Value Theorem to each of these separately. Two coordinates remain constant and only one varies in each of these partial increments $\Delta w_1, \Delta w_2, \Delta w_3$. In (17), for example, only y varies, since x is held equal to $x_0 + \Delta x$ and z is held equal to z_0. Since $f(x_0 + \Delta x, y, z_0)$ is a continuous function of y with a derivative f_y, it is subject to the Mean Value Theorem, and we have

$$\Delta w_2 = f_y(x_0 + \Delta x, y_1, z_0)\Delta y$$

for some y_1 between y_0 and $y_0 + \Delta y$.

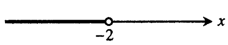
Answers

PRELIMINARY CHAPTER

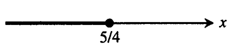
Section 1, pp. 7–8

1. $0.\overline{1}, 0.\overline{2}, 0.\overline{3}, 0.\overline{8}$ **3.** a) Not necessarily true b) True c) True d) True e) True f) True g) True h) True

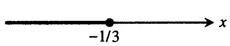
5. $x < -2$



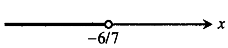
7. $x \le \frac{5}{4}$



9. $x \le -\frac{1}{3}$



11. $x < -\frac{6}{7}$



13. ± 3 **15.** $-\frac{1}{2}, -\frac{9}{2}$ **17.** $\frac{7}{6}, \frac{25}{6}$

19. $-2 < x < 2$

21. $-2 \le t \le 4$

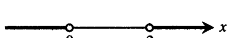
23. $1 < y < \frac{11}{3}$

25. $0 \le z \le 10$

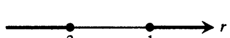
27. $\frac{2}{7} < x < \frac{2}{5}$ or $\frac{10}{35} < x < \frac{14}{35}$

29. $(-\infty, -2] \cup [2, \infty)$

31. $(-\infty, 0) \cup (2, \infty)$



33. $(-\infty, -3] \cup [1, \infty)$

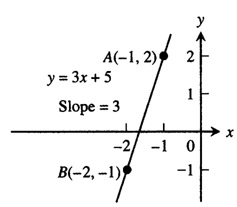


35. $(-\sqrt{2}, \sqrt{2})$ **37.** $(-3, -2) \cup (2, 3)$ **39.** $(-1, 3)$
41. $(0, 1)$ **43.** $a \ge 0$; any negative real number
47. $-\frac{1}{2} < x \le 3$ **49.** a) $(-2, 0) \cup (4, \infty)$

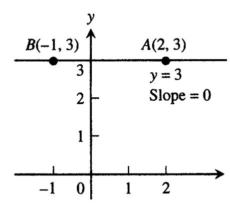
Section 2, pp. 15–17

1. $2, -4; 2\sqrt{5}$ **3.** $-4.9, 0; 4.9$ **5.** Unit circle
7. The circle centered at the origin with points less than a radius of $\sqrt{3}$ and its interior

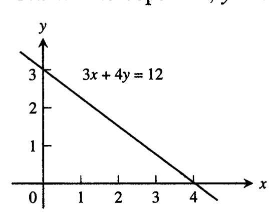
9. $m_\perp = -\frac{1}{3}$



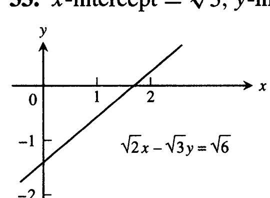
11. $m_\perp$ is undefined.



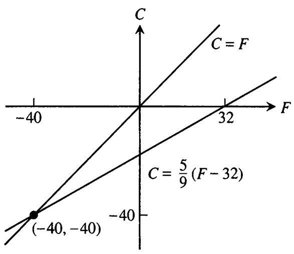
13. a) $x = -1$ b) $y = 4/3$ **15.** a) $x = 0$ b) $y = -\sqrt{2}$
17. $y = -x$ **19.** $y = -\frac{x}{5} + \frac{23}{5}$ **21.** $y = -\frac{5}{4}x + 6$
23. $y = -9$ **25.** $y = 4x + 4$ **27.** $y = -\frac{2}{5}x + 1$
29. $y = -\frac{x}{2} + 12$
31. x-intercept $= 4$, y-intercept $= 3$



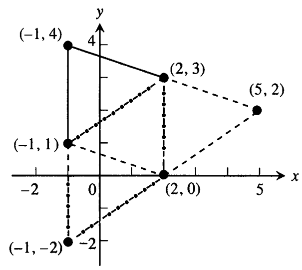
33. x-intercept $= \sqrt{3}$, y-intercept $= -\sqrt{2}$



35. Yes. The lines are perpendicular because their slopes, $-A/B$ and B/A, are negative reciprocals of one another.
37. $(3, -3)$ **39.** $(-2, -9)$ **41.** a) ≈ -2.5 degrees/inch b) ≈ -16.1 degrees/inch c) ≈ -8.3 degrees/inch
43. 5.97 atm
45. Yes: $C = F = -40°$



51.



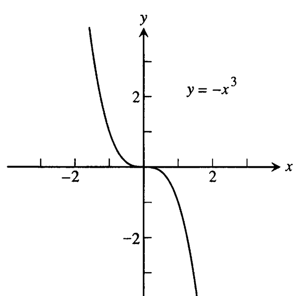
53. $k = -8, \quad k = 1/2$

Section 3, pp. 25–27

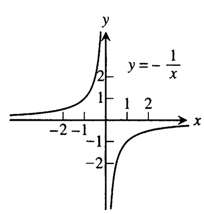
1. $D : (-\infty, \infty), \quad R : [1, \infty)$ **3.** $D : (0, \infty), \quad R : (0, \infty)$
5. $D : [-2, 2], \quad R : [0, 2]$
7. a) Not a function of x because some values of x have two values of y
b) A function of x because for every x there is only one possible y

9. $A = \frac{\sqrt{3}}{4}x^2, \quad p = 3x$ **11.** $x = \frac{d}{\sqrt{3}}, \quad A = 2d^2, \quad V = \frac{d^3}{3\sqrt{3}}$

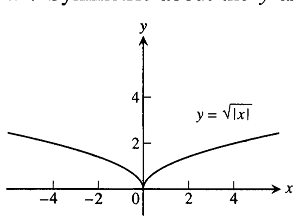
13. Symmetric about the origin



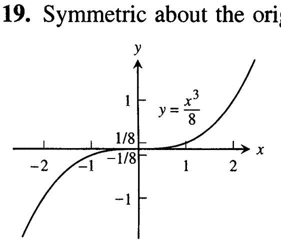
15. Symmetric about the origin



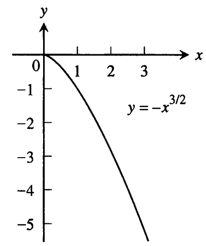
17. Symmetric about the y-axis



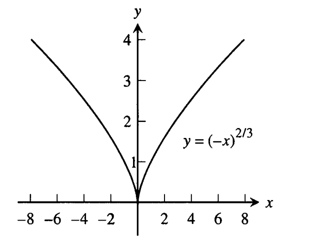
19. Symmetric about the origin



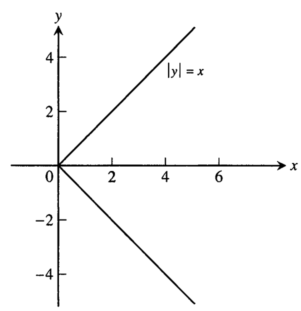
21. No symmetry



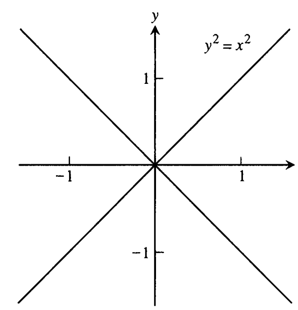
23. Symmetric about the y-axis



25. a) For each positive value of x, there are two values of y.



b) For each value of $x \neq 0$, there are two values of y.



27. Even **29.** Even **31.** Odd **33.** Even **35.** Neither
37. Neither
39. $D_f : -\infty < x < \infty, \quad D_g : x \geq 1, \quad R_f : -\infty < y < \infty,$ $R_g : y \geq 0, \quad D_{f+g} = D_{f \cdot g} = D_g, \quad R_{f+g} : y \geq 1, \quad R_{f \cdot g} : y \geq 0$
41. $D_f : -\infty < x < \infty, \quad D_g : -\infty < x < \infty, \quad R_f : y = 2,$ $R_g : y \geq 1, \quad D_{f/g} : -\infty < x < \infty, \quad R_{f/g} : 0 < y \leq 2,$ $D_{g/f} : -\infty < x < \infty, \quad R_{g/f} : y \geq 1/2$
43. a) 2 b) 22 c) $x^2 + 2$ d) $x^2 + 10x + 22$ e) 5 f) -2 g) $x + 10$ h) $x^4 - 6x^2 + 6$

45. a) $\frac{4}{x^2} - 5$ b) $\frac{4}{x^2} - 5$ c) $\left(\frac{4}{x} - 5\right)^2$ d) $\left(\frac{1}{4x - 5}\right)^2$

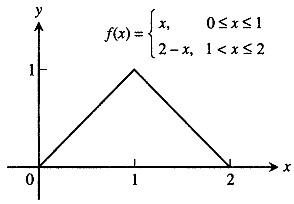
e) $\frac{1}{4x^2 - 5}$ f) $\frac{1}{(4x - 5)^2}$

47. a) $f(g(x))$ b) $j(g(x))$ c) $g(g(x))$ d) $j(j(x))$ e) $g(h(f(x)))$ f) $h(j(f(x)))$

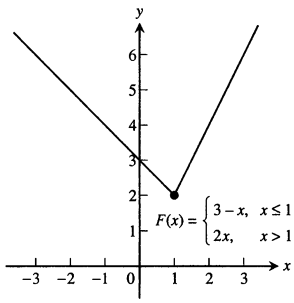
49.

	$g(x)$	$f(x)$	$f \circ g(x)$
a)	$x - 7$	$\sqrt{x}$	$\sqrt{x - 7}$
b)	$x + 2$	$3x$	$3x + 6$
c)	x^2	$\sqrt{x - 5}$	$\sqrt{x^2 - 5}$
d)	$\frac{x}{x - 1}$	$\frac{x}{x - 1}$	x
e)	$\frac{1}{x - 1}$	$1 + \frac{1}{x}$	x
f)	$\frac{1}{x}$	$\frac{1}{x}$	x

51.



53.



55. a) $y = \begin{cases} x, & 0 \le x \le 1 \\ 2-x, & 1 < x \le 2 \end{cases}$

b) $y = \begin{cases} 2, & 0 \le x < 1 \text{ or } 2 \le x < 3 \\ 0, & 1 \le x < 2 \text{ or } 3 \le x \le 4 \end{cases}$

57. a) $0 \le x < 1$ b) $-1 < x \le 0$ **59.** Yes

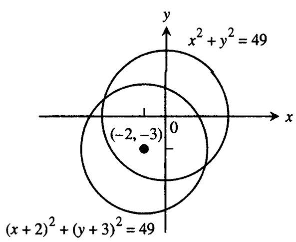
61. a) Odd b) Odd c) Odd d) Even e) Even f) Even g) Even h) Even i) Odd

Section 4, pp. 32–35

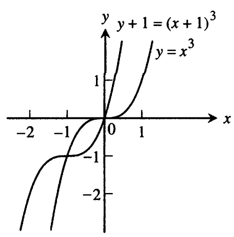
1. a) $y = -(x+7)^2$ b) $y = -(x-4)^2$

3. a) Position 4 b) Position 1 c) Position 2 d) Position 3

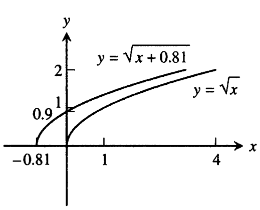
5. $(x+2)^2 + (y+3)^2 = 49$



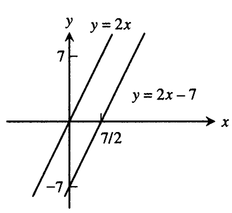
7. $y + 1 = (x+1)^3$



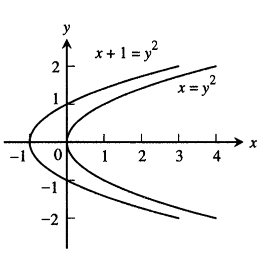
9. $y = \sqrt{x + 0.81}$



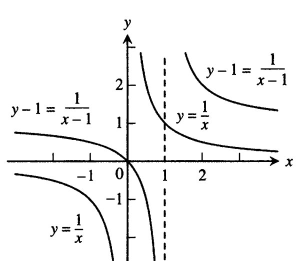
11. $y = 2x$



13. $x + 1 = y^2$



15. $y - 1 = \dfrac{1}{x-1}$



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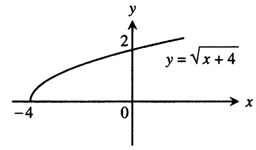

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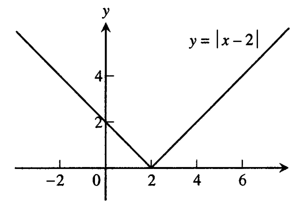

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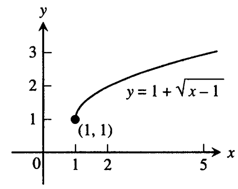

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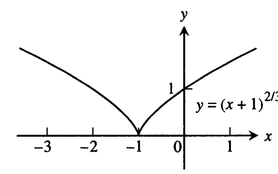

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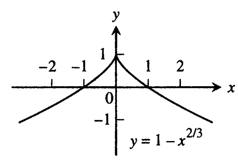

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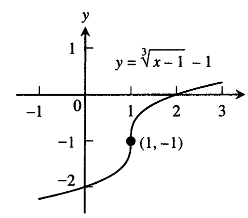

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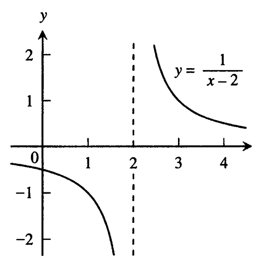
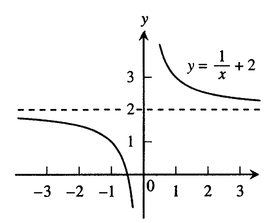
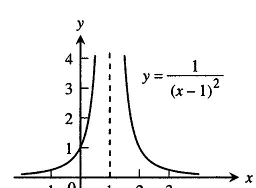


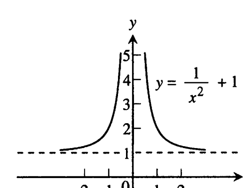
31.



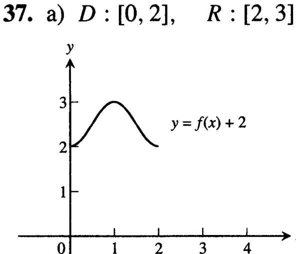
33.



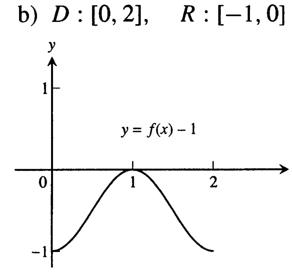
35.



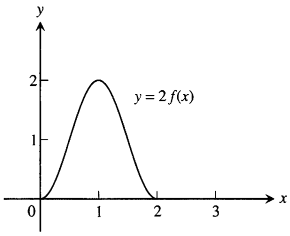
37. a) $D : [0, 2], \quad R : [2, 3]$



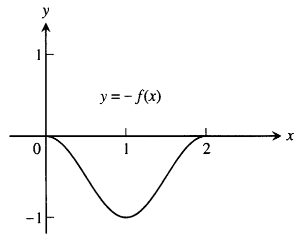
b) $D : [0, 2], \quad R : [-1, 0]$



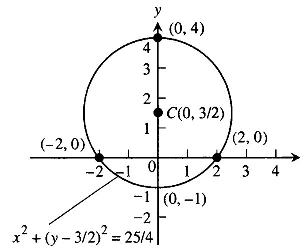
c) $D : [0, 2]$, $R : [0, 2]$



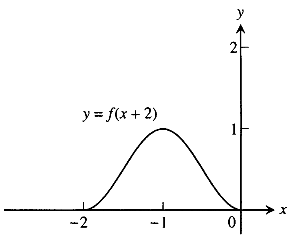
d) $D : [0, 2]$, $R : [-1, 0]$



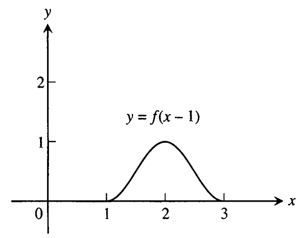
47. $x^2 + (y - 3/2)^2 = 25/4$



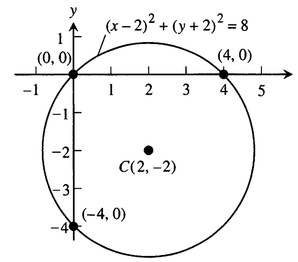
e) $D : [-2, 0]$, $R : [0, 1]$



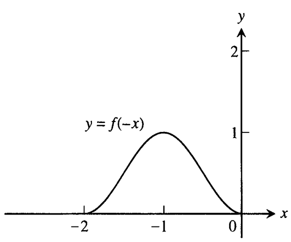
f) $D : [1, 3]$, $R : [0, 1]$



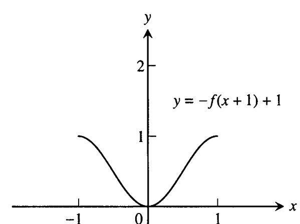
49. $(x - 2)^2 + (y + 2)^2 = 8$



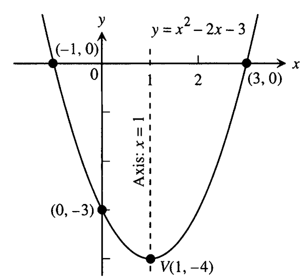
g) $D : [-2, 0]$, $R : [0, 1]$



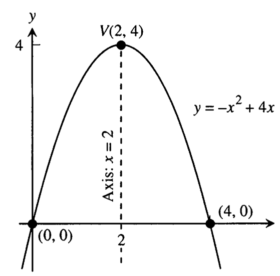
h) $D : [-1, 1]$, $R : [0, 1]$



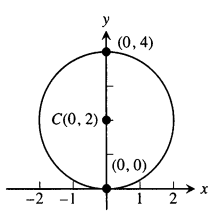
51.



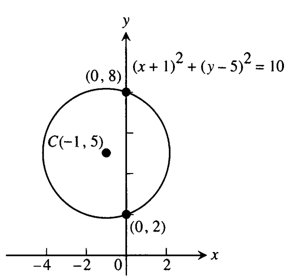
53.



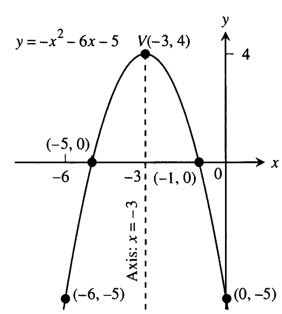
39. $x^2 + (y - 2)^2 = 4$



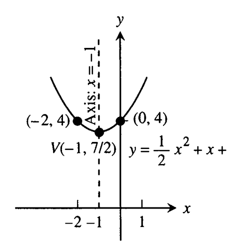
41. $(x + 1)^2 + (y - 5)^2 = 10$



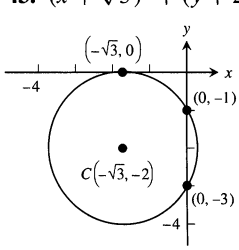
55.



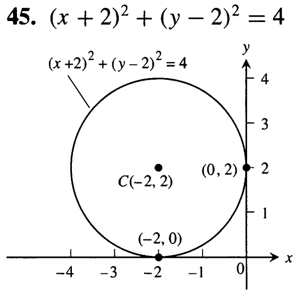
57.



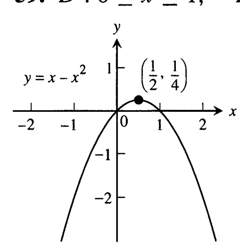
43. $(x + \sqrt{3})^2 + (y + 2)^2 = 4$



45. $(x + 2)^2 + (y - 2)^2 = 4$



59. $D : 0 \le x \le 1$, $R : 0 \le y \le 1/2$



61. Exterior points of a circle of radius $\sqrt{7}$, centered at the origin
63. A circle of radius 2, centered at (1, 0), together with its interior
65. The washer between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ (points with distance from the origin between 1 and 2)
67. The interior points of a circle centered at (0, −3) with a radius of 3 that lie above the line $y = -3$
69. $(x + 2)^2 + (y - 1)^2 < 6$ **71.** $x^2 + y^2 \le 2, \quad x \ge 1$
73. $y = y_0 + m(x - x_0)$ **75.** $\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \quad \left(-\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}\right)$
77. $\left(\frac{1-\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right), \quad \left(\frac{1+\sqrt{5}}{2}, \frac{3+\sqrt{5}}{2}\right)$
79. $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{3}\right), \quad \left(\frac{1}{\sqrt{3}}, -\frac{1}{3}\right)$
81. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), \quad \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

Section 5, pp. 43–47

1. a) 8π m b) $\frac{35\pi}{9}$ m **3.** 8.4 in.

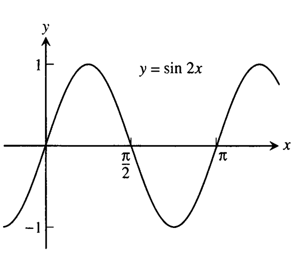
5.

θ	$-\pi$	$-2\pi/3$	0	$\pi/2$	$3\pi/4$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	−1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	UND	−1
$\cot\theta$	UND	$\frac{1}{\sqrt{3}}$	UND	0	−1
$\sec\theta$	−1	−2	1	UND	$-\sqrt{2}$
$\csc\theta$	UND	$-\frac{2}{\sqrt{3}}$	UND	1	$\sqrt{2}$

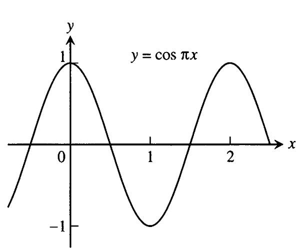
7. $\cos x = -4/5, \tan x = -3/4$ **9.** $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

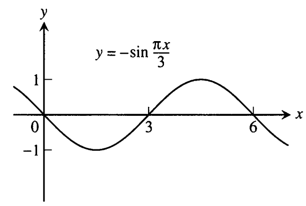
13. Period π



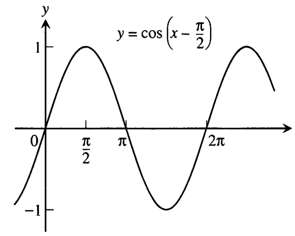
15. Period 2



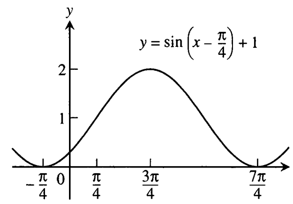
17. Period 6



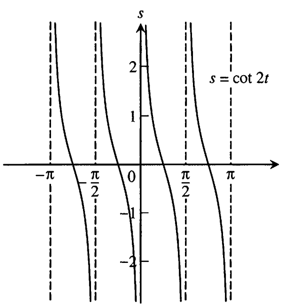
19. Period 2π



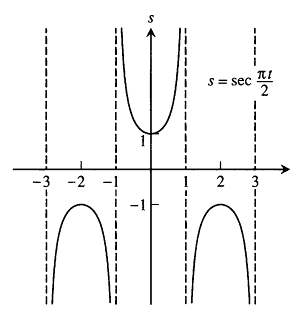
21. Period 2π



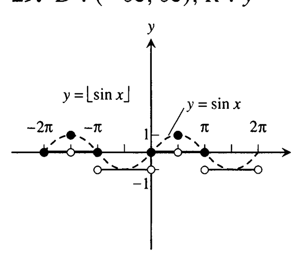
23. Period $\pi/2$, symmetric about the origin



25. Period 4, symmetric about the y-axis



29. $D : (-\infty, \infty), R : y = -1, 0, 1$

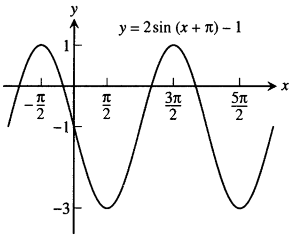


39. $-\cos x$ **41.** $-\cos x$ **43.** $\frac{\sqrt{6}+\sqrt{2}}{4}$ **45.** $\frac{1+\sqrt{3}}{2\sqrt{2}}$

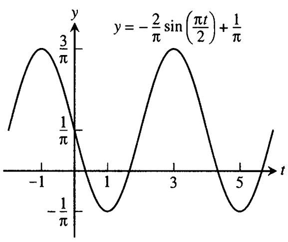
47. $\frac{2+\sqrt{2}}{4}$ **49.** $\frac{2-\sqrt{3}}{4}$ **55.** $c = \sqrt{7} \approx 2.646$

59. $a = 1.464$

61. $A = 2, B = 2\pi, C = -\pi, D = -1$



63. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



65. a) 37 b) 365 c) Right 101 d) Up 25

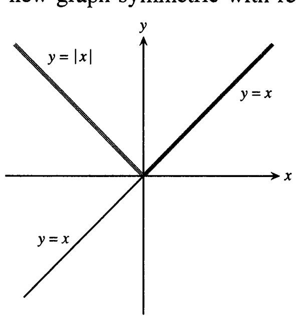
Practice Exercises, pp. 48–49

1. (0, 11)

3. No, no two sides have the same length; no, no two sides are perpendicular

5. $A = \pi r^2, C = 2\pi r, A = \frac{C^2}{4\pi}$ **7.** $x = \tan\theta, y = \tan^2\theta$

9. Replaces the portion for $x < 0$ with mirror image of portion for $x > 0$, to make the new graph symmetric with respect to the y-axis.



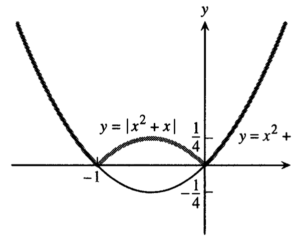
11. It does not change it.

13. Adds the mirror image of the portion for $x > 0$ to make the new graph symmetric with respect to the y-axis.

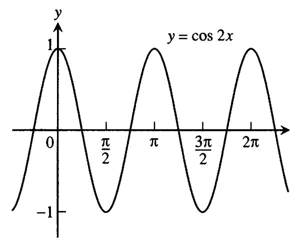
15. Reflects the portion for $y < 0$ across the x-axis.

17. Reflects the portion for $y < 0$ across the x-axis.

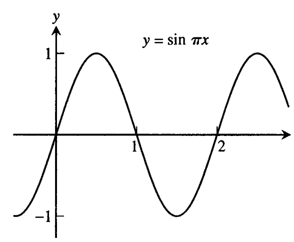
19. Reflects the portion for $y < 0$ across the x-axis.



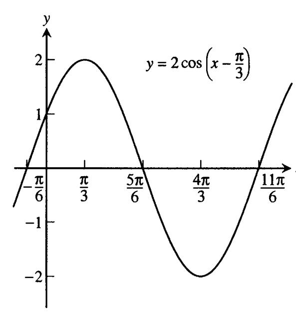
21. Period π



23. Period 2



25.



27. a) $a = 1, \quad b = \sqrt{3}$ b) $a = 2\sqrt{3}/3, \quad c = 4\sqrt{3}/3$

29. a) $a = \frac{b}{\tan B}$ b) $c = \frac{a}{\sin A}$ **31.** 16.98 m

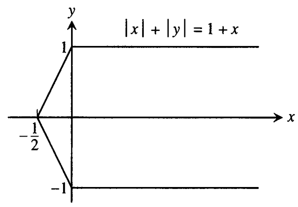
33. $3\sin x\cos^2 x - \sin^3 x$ **35.** b) 4π

Additional Exercises, pp. 49–50

3. Yes. For instance: $f(x) = 1/x$ and $g(x) = 1/x$, or $f(x) = 2x$ and $g(x) = x/2$, or $f(x) = e^x$ and $g(x) = \ln x$.

5. If $f(x)$ is odd, then $g(x) = f(x) - 2$ is not odd. Nor is $g(x)$ even, unless $f(x) = 0$ for all x. If f is even, then $g(x) = f(x) - 2$ is also even.

7.



9. $\sqrt{2}$ **11.** 3/4 **13.** $3\sqrt{15}/16$ **27.** $-4 < m < 0$

CHAPTER 1

Section 1.1, pp. 57–60

1. a) Does not exist. As x approaches 1 from the right, $g(x)$ approaches 0. As x approaches 1 from the left, $g(x)$ approaches 1. There is no single number L that all the values $g(x)$ get arbitrarily close to as $x \to 1$. b) 1 c) 0

3. a) True b) True c) False d) False e) False f) True

5. As x approaches 0 from the left, $x/|x|$ approaches -1. As x approaches 0 from the right, $x/|x|$ approaches 1. There is no single number L that the function values all get arbitrarily close to as $x \to 0$.

11. a) $f(x) = (x^2 - 9)/(x + 3)$

x	−3.1	−3.01	−3.001	−3.0001	−3.00001	−3.000001
$f(x)$	−6.1	−6.01	−6.001	−6.0001	−6.00001	−6.000001

x	−2.9	−2.99	−2.999	−2.9999	−2.99999	−2.999999
$f(x)$	−5.9	−5.99	−5.999	−5.9999	−5.99999	−5.999999

c) $\lim_{x \to -3} f(x) = -6$

13. a) $G(x) = (x + 6)/(x^2 + 4x - 12)$

x	−5.9	−5.99	−5.999	−5.9999	−5.99999	−5.999999
$G(x)$	−.126582	−.1251564	−.1250156	−.1250015	−.1250001	−.1250000

x	−6.1	−6.01	−6.001	−6.0001	−6.00001	−6.000001
$G(x)$	−.123456	−.124843	−.124984	−.124998	−.124999	−.124999

c) $\lim_{x \to -6} G(x) = -1/8 = -0.125$

15. a) $f(x) = (x^2 - 1)/(|x| - 1)$

x	−1.1	−1.01	−1.001	−1.0001	−1.00001	−1.000001
$f(x)$	2.1	2.01	2.001	2.0001	2.00001	2.000001

x	−.9	−.99	−.999	−.9999	−.99999	−.999999
$f(x)$	1.9	1.99	1.999	1.9999	1.99999	1.999999

c) $\lim_{x \to -1} f(x) = 2$

17. a) $g(\theta) = (\sin \theta)/\theta$

θ	.1	.01	.001	.0001	.00001	.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

θ	−.1	−.01	−.001	−.0001	−.00001	−.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

$\lim_{\theta \to 0} g(\theta) = 1$

19. a) $f(x) = x^{1/(1-x)}$

x	.9	.99	.999	.9999	.99999	.999999
$f(x)$	.348678	.366032	.367695	.367861	.367877	.367879

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$f(x)$	.385543	.369711	.368063	.367897	.367881	.367878

$\lim_{x \to 1} f(x) \approx 0.36788$

21. 4 **23.** 0 **25.** 9 **27.** $\pi/2$ **29.** a) 19 b) 1

31. a) $-\frac{4}{\pi}$ b) $-\frac{3\sqrt{3}}{\pi}$ **33.** 1

35. Graphs can shift during a press run, so your estimates may not completely agree with these.

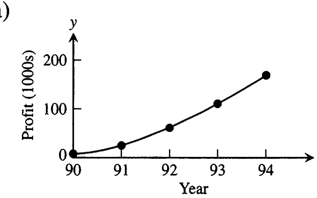
a)

PQ_1	PQ_2	PQ_3	PQ_4
43	46	49	50

The appropriate units are m/sec.

b) $\approx$ 50 m/sec or 180 km/h

37. a)



b) $\approx$ \$56,000/year
c) $\approx$ \$42,000/year

39. a) 0.414213, 0.449489, $(\sqrt{1+h}-1)/h$ b) $g(x)=\sqrt{x}$

$1+h$	1.1	1.01	1.001	1.0001	1.00001	1.000001
$\sqrt{1+h}$	1.04880	1.004987	1.0004998	1.0000499	1.000005	1.0000005
$(\sqrt{1+h}-1)/h$	0.4880	0.4987	0.4998	0.499	0.5	0.5

c) 0.5 d) 0.5

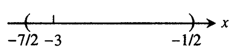
Section 1.2, pp. 65–66

1. -9 **3.** 4 **5.** -8 **7.** 5/8 **9.** 5/2 **11.** 27 **13.** 16 **15.** 3/2 **17.** 1/10 **19.** -7 **21.** 3/2 **23.** $-1/2$ **25.** 4/3 **27.** 1/6 **29.** 4
31. a) Quotient rule b) Difference and Power rules c) Sum and Constant Multiple rules
33. a) -10 b) -20 c) -1 d) 5/7
35. a) 4 b) -21 c) -12 d) $-7/3$
37. 2 **39.** 3 **41.** $1/(2\sqrt{7})$ **43.** $\sqrt{5}$ **45.** a) The limit is 1.
49. 7 **51.** a) 5 b) 5

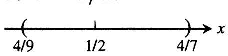
Section 1.3, pp. 74–77

1. $\delta=2$

3. $\delta=1/2$



5. $\delta=1/18$



7. $\delta=0.1$ **9.** $\delta=7/16$ **11.** $\delta=\sqrt{5}-2$ **13.** $\delta=0.36$
15. $(3.99, 4.01)$, $\delta=0.01$ **17.** $(-0.19, 0.21)$, $\delta=0.19$
19. $(3, 15)$, $\delta=5$ **21.** $(10/3, 5)$, $\delta=2/3$
23. $(-\sqrt{4.5}, -\sqrt{3.5})$, $\delta=\sqrt{4.5}-2\approx 0.12$
25. $(\sqrt{15}, \sqrt{17})$, $\delta=\sqrt{17}-4\approx 0.12$

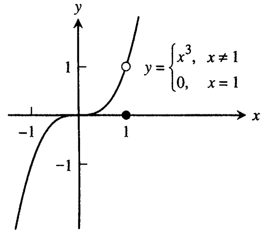
27. $\left(2-\dfrac{0.03}{m}, 2+\dfrac{0.03}{m}\right)$, $\delta=\dfrac{0.03}{m}$

29. $\left(\dfrac{1}{2}-\dfrac{c}{m}, \dfrac{c}{m}+\dfrac{1}{2}\right)$, $\delta=\dfrac{c}{m}$

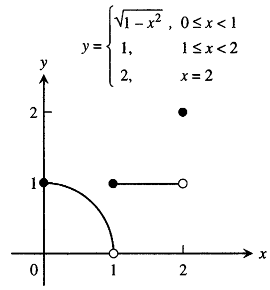
31. $L=-3$, $\delta=0.01$ **33.** $L=4$, $\delta=0.05$
35. $L=4$, $\delta=0.75$
55. [3.384, 3.387]. To be safe, the left endpoint was rounded up and the right endpoint rounded down.

Section 1.4, pp. 83–86

1. a) True b) True c) False d) True e) True f) True g) False h) False i) False j) False k) True l) False
3. a) 2, 1 b) No, $\lim_{x\to 2^+} f(x)\neq \lim_{x\to 2^-} f(x)$ c) 3, 3 d) Yes, 3
5. a) No b) Yes, 0 c) No
7. a)



b) 1, 1 c) Yes, 1
9. a) $D: 0\le x\le 2, R: 0<y\le 1$ and $y=2$ b) $(0,1)\cup(1,2)$ c) $x=2$ d) $x=0$



11. $\sqrt{3}$ **13.** 1 **15.** $2/\sqrt{5}$ **17.** a) 1 b) -1 **19.** a) 1 b) 2/3 **21.** ∞ **23.** $-\infty$ **25.** $-\infty$ **27.** ∞ **29.** a) ∞ b) $-\infty$ **31.** ∞ **33.** ∞ **35.** $-\infty$ **37.** a) ∞ b) $-\infty$ c) $-\infty$ d) ∞ **39.** a) $-\infty$ b) ∞ c) 0 d) 3/2
41. a) $-\infty$ b) 1/4 c) 1/4 d) 1/4 e) It will be $-\infty$.
43. a) $-\infty$ b) ∞ **45.** a) ∞ b) ∞ c) ∞ d) ∞
51. $\delta=\epsilon^2$, $\lim_{x\to 5^+}\sqrt{x-5}=0$ **55.** a) 400 b) 399 c) The limit does not exist.
61. a) For every positive real number B there exists a corresponding number $\delta>0$ such that for all x

$$x_0-\delta<x<x_0 \Rightarrow f(x)>B.$$

b) For every negative real number $-B$ there exists a corresponding number $\delta>0$ such that for all x

$$x_0<x<x_0+\delta \Rightarrow f(x)<-B.$$

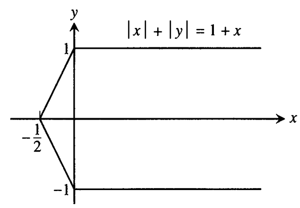
c) For every negative real number $-B$ there exists a corresponding number $\delta>0$ such that for all x

$$x_0-\delta<x<x_0 \Rightarrow f(x)<-B.$$

Section 1.5, pp. 95–97

1. No; discontinuous at $x=2$; not defined at $x=2$
3. Continuous **5.** a) Yes b) Yes c) Yes d) Yes
7. a) No b) No **9.** 0 **11.** 1, nonremovable; 0, removable
13. All x except $x=2$ **15.** All x except $x=3, x=1$
17. All x **19.** All x except $x=0$
21. All x except $x=n\pi/2$, n any integer
23. All x except $n\pi/2$, n an odd integer
25. All $x>-3/2$ **27.** All x **29.** 0 **31.** 1 **33.** $\sqrt{2}/2$
35. $g(3)=6$ **37.** $f(1)=3/2$ **39.** $a=4/3$
63. $x\approx 1.8794, -1.5321, -0.3473$ **65.** $x\approx 1.7549$
67. $x\approx 3.5156$ **69.** $x\approx 0.7391$

7.



9. $\sqrt{2}$ **11.** 3/4 **13.** $3\sqrt{15}/16$ **27.** $-4 < m < 0$

CHAPTER 1

Section 1.1, pp. 57–60

1. a) Does not exist. As x approaches 1 from the right, $g(x)$ approaches 0. As x approaches 1 from the left, $g(x)$ approaches 1. There is no single number L that all the values $g(x)$ get arbitrarily close to as $x \to 1$. b) 1 c) 0

3. a) True b) True c) False d) False e) False f) True

5. As x approaches 0 from the left, $x/|x|$ approaches -1. As x approaches 0 from the right, $x/|x|$ approaches 1. There is no single number L that the function values all get arbitrarily close to as $x \to 0$.

11. a) $f(x) = (x^2 - 9)/(x + 3)$

x	−3.1	−3.01	−3.001	−3.0001	−3.00001	−3.000001
$f(x)$	−6.1	−6.01	−6.001	−6.0001	−6.00001	−6.000001

x	−2.9	−2.99	−2.999	−2.9999	−2.99999	−2.999999
$f(x)$	−5.9	−5.99	−5.999	−5.9999	−5.99999	−5.999999

c) $\lim_{x \to -3} f(x) = -6$

13. a) $G(x) = (x + 6)/(x^2 + 4x - 12)$

x	−5.9	−5.99	−5.999	−5.9999	−5.99999	−5.999999
$G(x)$	−.126582	−.1251564	−.1250156	−.1250015	−.1250001	−.1250000

x	−6.1	−6.01	−6.001	−6.0001	−6.00001	−6.000001
$G(x)$	−.123456	−.124843	−.124984	−.124998	−.124999	−.124999

c) $\lim_{x \to -6} G(x) = -1/8 = -0.125$

15. a) $f(x) = (x^2 - 1)/(|x| - 1)$

x	−1.1	−1.01	−1.001	−1.0001	−1.00001	−1.000001
$f(x)$	2.1	2.01	2.001	2.0001	2.00001	2.000001

x	−.9	−.99	−.999	−.9999	−.99999	−.999999
$f(x)$	1.9	1.99	1.999	1.9999	1.99999	1.999999

c) $\lim_{x \to -1} f(x) = 2$

17. a) $g(\theta) = (\sin \theta)/\theta$

θ	.1	.01	.001	.0001	.00001	.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

θ	−.1	−.01	−.001	−.0001	−.00001	−.000001
$g(\theta)$	.998334	.999983	.999999	.999999	.999999	.999999

$\lim_{\theta \to 0} g(\theta) = 1$

19. a) $f(x) = x^{1/(1-x)}$

x	.9	.99	.999	.9999	.99999	.999999
$f(x)$	.348678	.366032	.367695	.367861	.367877	.367879

x	1.1	1.01	1.001	1.0001	1.00001	1.000001
$f(x)$	.385543	.369711	.368063	.367897	.367881	.367878

$\lim_{x \to 1} f(x) \approx 0.36788$

21. 4 **23.** 0 **25.** 9 **27.** $\pi/2$ **29.** a) 19 b) 1

31. a) $-\dfrac{4}{\pi}$ b) $-\dfrac{3\sqrt{3}}{\pi}$ **33.** 1

35. Graphs can shift during a press run, so your estimates may not completely agree with these.

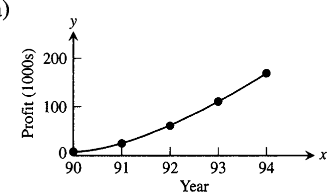
a)

PQ_1	PQ_2	PQ_3	PQ_4
43	46	49	50

The appropriate units are m/sec.

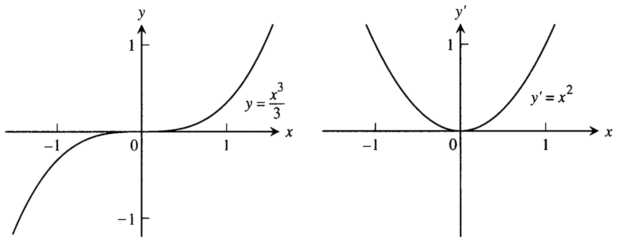
b) ≈ 50 m/sec or 180 km/h

37. a)



b) ≈ \$56,000/year
c) ≈ \$42,000/year

47. a) $y' = x^2$
b)



c) $x \neq 0, x = 0$, none d) $-\infty < x < \infty$, none
49. $y' = 3x^2$ is never negative
51. Yes, $y + 16 = -(x - 3)$ is tangent at $(3, -16)$
53. No, the function $y = \lfloor x \rfloor$ does not satisfy the intermediate value property of derivatives.
55. Yes, $(-f)'(x) = -(f'(x))$
57. For $g(t) = mt$ and $h(t) = t$, $\lim_{t\to 0} \frac{g(t)}{h(t)} = m$, which need not be zero.

Section 2.2, pp. 129–131

1. $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$

3. $\frac{ds}{dt} = 15t^2 - 15t^4$, $\frac{d^2s}{dt^2} = 30t - 60t^3$

5. $\frac{dy}{dx} = 4x^2 - 1$, $\frac{d^2y}{dx^2} = 8x$

7. $\frac{dw}{dz} = -6z^{-3} + \frac{1}{z^2}$, $\frac{d^2w}{dz^2} = 18z^{-4} - \frac{2}{z^3}$

9. $\frac{dy}{dx} = 12x - 10 + 10x^{-3}$, $\frac{d^2y}{dx^2} = 12 - 30x^{-4}$

11. $\frac{dr}{ds} = \frac{-2}{3s^3} + \frac{5}{2s^2}$, $\frac{d^2r}{ds^2} = \frac{2}{s^4} - \frac{5}{s^3}$

13. $y' = -5x^4 + 12x^2 - 2x - 3$ **15.** $y' = 3x^2 + 10x + 2 - \frac{1}{x^2}$

17. $y' = \frac{-19}{(3x-2)^2}$ **19.** $g'(x) = \frac{x^2 + x + 4}{(x + 0.5)^2}$

21. $\frac{dv}{dt} = \frac{t^2 - 2t - 1}{(1 + t^2)^2}$ **23.** $f'(s) = \frac{1}{\sqrt{s}(\sqrt{s} + 1)^2}$

25. $v' = -\frac{1}{x^2} + 2x^{-3/2}$ **27.** $y' = \frac{-4x^3 - 3x^2 + 1}{(x^2 - 1)^2(x^2 + x + 1)^2}$

29. $y' = 2x^3 - 3x - 1$, $y'' = 6x^2 - 3$, $y''' = 12x$, $y^{(4)} = 12$, $y^{(n)} = 0$ for $n \geq 5$

31. $y' = 2x - 7x^{-2}$, $y'' = 2 + 14x^{-3}$

33. $\frac{dr}{d\theta} = 3\theta^{-4}$, $\frac{d^2r}{d\theta^2} = -12\theta^{-5}$

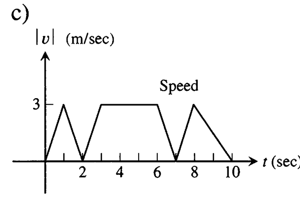
35. $\frac{dw}{dz} = -z^{-2} - 1$, $\frac{d^2w}{dz^2} = 2z^{-3}$

37. $\frac{dp}{dq} = \frac{1}{6}q + \frac{1}{6}q^{-3} + q^{-5}$, $\frac{d^2p}{dq^2} = \frac{1}{6} - \frac{1}{2}q^{-4} - 5q^{-6}$

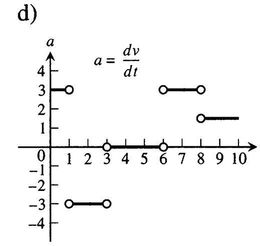
39. a) 13 b) -7 c) 7/25 d) 20 **41.** a) $y = -\frac{x}{8} + \frac{5}{4}$
b) $m = -4$ at $(0, 1)$ c) $y = 8x - 15$, $y = 8x + 17$
43. $y = 4x$, $y = 2$ **45.** $a = 1, b = 1, c = 0$ **47.** a) $y = 2x + 2$,
c) $(2, 6)$ **49.** $\frac{dP}{dV} = -\frac{nRT}{(V - nb)^2} + \frac{2an^2}{V^3}$
51. The Product Rule is then the Constant Multiple Rule, so the latter is a special case of the Product Rule.
55. a) $\frac{3}{2}x^{1/2}$, b) $\frac{5}{2}x^{3/2}$, c) $\frac{7}{2}x^{5/2}$, d) $\frac{d}{dx}\left(x^{n/2}\right) = \frac{n}{2}x^{(n/2)-1}$

Section 2.3, pp. 139–143

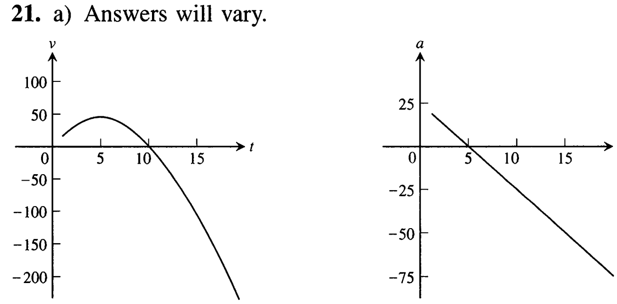
1. a) 80 m, 8 m/sec b) 0 m/sec, 16 m/sec; 1.6 m/sec^2, 1.6 m/sec^2
c) no change in direction
3. a) -9 m, -3 m/sec b) 3 m/sec, 12 m/sec; 6 m/sec^2, -12 m/sec^2
c) no change in direction
5. a) -20 m, -5 m/sec b) 45 m/sec, (1/5) m/sec; 140 m/sec^2, (4/25) m/sec^2 c) no change in direction
7. a) $a(1) = -6$ m/sec^2, $a(3) = 6$ m/sec^2 b) $v(2) = 3$m/sec
c) 6 m
9. Mars: ≈ 7.5 sec, Jupiter: ≈ 1.2 sec
11. a) $24 - 9.8t$ m/sec, -9.8 m/sec^2 b) 2.4 sec c) 29.4 m
d) 0.7 sec going up, 4.2 sec going down e) 4.9 sec
13. 320 sec on the moon, 52 sec on Earth; $\approx 66{,}560$ ft on the moon, $\approx 10{,}816$ ft on Earth
15. a) $9.8t$ m/sec, b) 9.8 m/sec^2
17. a) $t = 2, t = 7$ b) $3 \leq t \leq 6$
c)



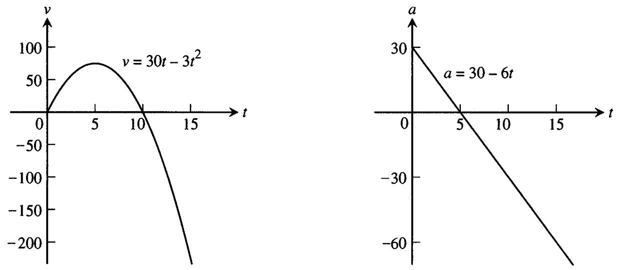
d)



19. a) 190 ft/sec b) 2 sec c) 8 sec, 0 ft/sec, d) 10.8 sec, 90 ft/sec e) 2.8 sec f) greatest acceleration happens 2 sec after launch g) constant acceleration between 2 and 10.8 sec, -32 ft/sec^2
21. a) Answers will vary.



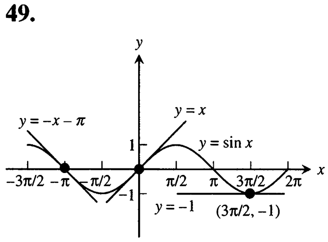
b)



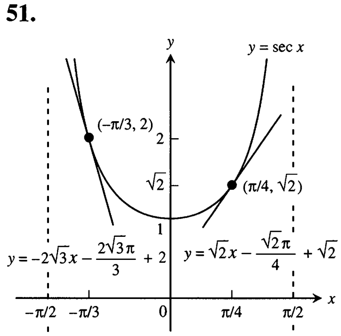
23. C = position, A = velocity, B = acceleration
25. a) \$110/machine b) \$80 c) \$79.90
27. a) 10^4 bacteria/h b) 0 bacteria/h c) -10^4 bacteria/h
29. a) $\frac{t}{12} - 1$ b) Fastest ($dy/dt = -1$ m/h) when $t = 0$, Slowest ($dy/dt = 0$ m/h) when $t = 12$
31. $t = 25$ sec, $D = 6250/9$ m
33. a) $t = 6.25$ sec b) Up on $[0, 6.25)$, down on $(6.25, 12.5]$ c) $t = 6.25$ sec d) Speeds up on $(6.25, 12.5]$, slows down on $[0, 6.25)$ e) Fastest at $t = 0, 12.5$, slowest at $t = 6.25$ f) $t = 6.25$ sec
35. a) $t = (6 \pm \sqrt{15})/3$ b) left on $((6 - \sqrt{15})/3, (6 + \sqrt{15})/3)$; right on $[0, (6 - \sqrt{15})/3) \cup ((6 + \sqrt{15})/3, 4]$ c) $t = (6 \pm \sqrt{15})/3$ d) Speeds up on $((6 - \sqrt{15})/3, 2) \cup ((6 + \sqrt{15})/3, 4]$, slows down on $[0, (6 - \sqrt{15})/3) \cup (2, (6 + \sqrt{15})/3)$ e) Fastest at $t = 0, 4$; slowest at $t = (6 \pm \sqrt{15})/3$ f) $t = (6 + \sqrt{15})/3$

Section 2.4, pp. 152–154

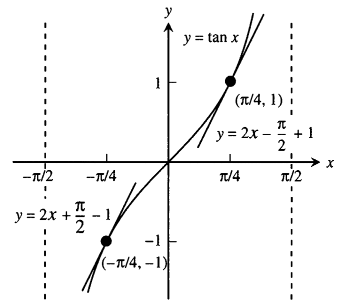
1. $-10 - 3 \sin x$ **3.** $-\csc x \cot x - \frac{2}{\sqrt{x}}$ **5.** 0
7. $\frac{-\csc^2 x}{(1 + \cot x)^2}$ **9.** $4 \tan x \sec x - \csc^2 x$ **11.** $x^2 \cos x$
13. $\sec^2 t - 1$ **15.** $\frac{-2 \csc t \cot t}{(1 - \csc t)^2}$ **17.** $-\theta(\theta \cos \theta + 2 \sin \theta)$
19. $\sec \theta \csc \theta (\tan \theta - \cot \theta) = \sec^2 \theta - \csc^2 \theta$ **21.** $\sec^2 q$
23. $\sec^2 q$ **25.** a) $2 \csc^3 x - \csc x$ b) $2 \sec^3 x - \sec x$
27. 0 **29.** −1 **31.** 0 **33.** 1 **35.** 3/4 **37.** 2 **39.** 1/2
41. 2 **43.** 1 **45.** 1/2 **47.** 3/8
49.



51.



53. Yes, at $x = \pi$ **55.** No
57. $\left(-\frac{\pi}{4}, -1\right)$; $\left(\frac{\pi}{4}, 1\right)$



59. a) $y = -x + \pi/2 + 2$ b) $y = 4 - \sqrt{3}$
61. $-\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec, $\sqrt{2}$ m/sec^2, $\sqrt{2}$ m/sec^3 **63.** $c = 9$
65. $\sin x$

Section 2.5, pp. 160–163

1. $12x^3$ **3.** $3 \cos(3x + 1)$ **5.** $-\sin(\sin x) \cos x$
7. $10 \sec^2(10x - 5)$
9. With $u = (2x + 1)$, $y = u^5$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 5u^4 \cdot 2 = 10(2x + 1)^4$
11. With $u = (1 - (x/7))$, $y = u^{-7}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -7u^{-8} \cdot \left(-\frac{1}{7}\right) = \left(1 - \frac{x}{7}\right)^{-8}$
13. With $u = ((x^2/8) + x - (1/x))$, $y = u^4$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 4u^3 \cdot \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right) = 4\left(\frac{x^2}{8} + x - \frac{1}{x}\right)^3 \left(\frac{x}{4} + 1 + \frac{1}{x^2}\right)$
15. With $u = \tan x$, $y = \sec u$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (\sec u \tan u)(\sec^2 x) = \sec(\tan x) \tan(\tan x) \sec^2 x$
17. With $u = \sin x$, $y = u^3$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2 \cos x = 3 \sin^2 x (\cos x)$
19. $-\frac{1}{2\sqrt{3 - t}}$ **21.** $\frac{4}{\pi}(\cos 3t - \sin 5t)$ **23.** $\frac{\csc \theta}{\cot \theta + \csc \theta}$
25. $2x \sin^4 x + 4x^2 \sin^3 x \cos x + \cos^{-2} x + 2x \cos^{-3} x \sin x$
27. $(3x - 2)^6 - \frac{1}{x^3\left(4 - \frac{1}{2x^2}\right)^2}$ **29.** $\frac{(4x + 3)^3(4x + 7)}{(x + 1)^4}$
31. $\sqrt{x} \sec^2(2\sqrt{x}) + \tan(2\sqrt{x})$ **33.** $\frac{2 \sin \theta}{(1 + \cos \theta)^2}$
35. $\frac{dr}{d\theta} = -2 \sin(\theta^2) \sin 2\theta + 2\theta \cos(2\theta) \cos(\theta^2)$
37. $\frac{dq}{dt} = \left(\frac{t + 2}{2(t + 1)^{3/2}}\right) \cos\left(\frac{t}{\sqrt{t + 1}}\right)$

39. $2\pi \sin(\pi t - 2)\cos(\pi t - 2)$ **41.** $\dfrac{8\sin(2t)}{(1+\cos 2t)^5}$

43. $-2\cos(\cos(2t-5))(\sin(2t-5))$

45. $\left(1+\tan^4\left(\frac{t}{12}\right)\right)^2\left(\tan^3\left(\frac{t}{12}\right)\sec^2\left(\frac{t}{12}\right)\right)$

47. $-\dfrac{t\sin(t^2)}{\sqrt{1+\cos(t^2)}}$ **49.** $\dfrac{6}{x^3}\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)$

51. $2\csc^2(3x-1)\cot(3x-1)$ **53.** $5/2$ **55.** $-\pi/4$ **57.** 0

59. a) $2/3$ b) $2\pi+5$ c) $15-8\pi$ d) $37/6$ e) -1 f) $\sqrt{2}/24$ g) $5/32$ h) $-5/(3\sqrt{17})$

61. 5 **63.** a) 1 b) 1 **65.** a) $y=\pi x+2-\pi$ b) $\pi/2$

67. It multiplies the velocity, acceleration, and jerk by 2, 4, and 8 respectively.

69. $v=\frac{2}{5}$ m/sec, $a=-\frac{4}{125}$ m/sec^2

Section 2.6, pp. 170–172

1. $\frac{9}{4}x^{5/4}$ **3.** $\dfrac{2^{1/3}}{3x^{2/3}}$ **5.** $\dfrac{7}{2(x+6)^{1/2}}$ **7.** $-(2x+5)^{-3/2}$

9. $\dfrac{2x^2+1}{(x^2+1)^{1/2}}$ **11.** $\dfrac{ds}{dt}=\frac{2}{7}t^{-5/7}$

13. $\dfrac{dy}{dt}=-\frac{4}{3}(2t+5)^{-5/3}\cos\left[(2t+5)^{-2/3}\right]$

15. $f'(x)=\dfrac{-1}{4\sqrt{x}(1-\sqrt{x})}$

17. $h'(\theta)=-\frac{2}{3}(\sin 2\theta)(1+\cos 2\theta)^{-2/3}$ **19.** $\dfrac{-2xy-y^2}{x^2+2xy}$

21. $\dfrac{1-2y}{2x+2y-1}$ **23.** $\dfrac{-2x^3+3x^2y-xy^2+x}{x^2y-x^3+y}$ **25.** $\dfrac{1}{y(x+1)^2}$

27. $\cos^2 y$ **29.** $\dfrac{-\cos^2(xy)-y}{x}$

31. $\dfrac{-y^2}{y\sin\left(\frac{1}{y}\right)-\cos\left(\frac{1}{y}\right)+xy}$ **33.** $-\dfrac{\sqrt{r}}{\sqrt{\theta}}$ **35.** $\dfrac{-r}{\theta}$

37. $y'=-\dfrac{x}{y}$, $y''=\dfrac{-y^2-x^2}{y^3}$

39. $y'=\dfrac{x+1}{y}$, $y''=\dfrac{y^2-(x+1)^2}{y^3}$

41. $y'=\dfrac{\sqrt{y}}{\sqrt{y}+1}$, $y''=\dfrac{1}{2(\sqrt{y}+1)^3}$ **43.** -2

45. $(-2,1):m=-1$, $(-2,-1):m=1$ **47.** a) $y=\frac{7}{4}x-\frac{1}{2}$, b) $y=-\frac{4}{7}x+\frac{29}{7}$

49. a) $y=3x+6$, b) $y=-\frac{1}{3}x+\frac{8}{3}$

51. a) $y=\frac{6}{7}x+\frac{6}{7}$, b) $y=-\frac{7}{6}x-\frac{7}{6}$

53. a) $y=-\frac{\pi}{2}x+\pi$, b) $y=\frac{2}{\pi}x-\frac{2}{\pi}+\frac{\pi}{2}$

55. a) $y=2\pi x-2\pi$, b) $y=-\dfrac{x}{2\pi}+\dfrac{1}{2\pi}$

57. Points: $(-\sqrt{7},0)$ and $(\sqrt{7},0)$, Slope: -2

59. $m=-1$ at $\left(\frac{\sqrt{3}}{4},\frac{\sqrt{3}}{2}\right)$, $m=\sqrt{3}$ at $\left(\frac{\sqrt{3}}{4},\frac{1}{2}\right)$

61. $(-3,2):m=-\frac{27}{8}$; $(-3,-2):m=\frac{27}{8}$; $(3,2):m=\frac{27}{8}$; $(3,-2):m=-\frac{27}{8}$

63. a) False b) True c) True d) True **65.** $(3,-1)$

69. $\dfrac{dy}{dx}=-\dfrac{y^3+2xy}{x^2+3xy^2}$, $\dfrac{dx}{dy}=-\dfrac{x^2+3xy^2}{y^3+2xy}$, $\dfrac{dx}{dy}=\dfrac{1}{dy/dx}$

Section 2.7, pp. 176–180

1. $\dfrac{dA}{dt}=2\pi r\dfrac{dr}{dt}$ **3.** a) $\dfrac{dV}{dt}=\pi r^2\dfrac{dh}{dt}$ b) $\dfrac{dV}{dt}=2\pi hr\dfrac{dr}{dt}$ c) $\dfrac{dV}{dt}=\pi r^2\dfrac{dh}{dt}+2\pi hr\dfrac{dr}{dt}$

5. a) 1 volt/sec b) $-\frac{1}{3}$ amp/sec c) $\dfrac{dR}{dt}=\dfrac{1}{I}\left(\dfrac{dV}{dt}-\dfrac{V}{I}\dfrac{dI}{dt}\right)$ d) 3/2 ohms/sec, R is increasing.

7. a) $\dfrac{dS}{dt}=\dfrac{x}{\sqrt{x^2+y^2}}\dfrac{dx}{dt}$ b) $\dfrac{dS}{dt}=\dfrac{x}{\sqrt{x^2+y^2}}\dfrac{dx}{dt}+\dfrac{y}{\sqrt{x^2+y^2}}\dfrac{dy}{dt}$ c) $\dfrac{dx}{dt}=-\dfrac{y}{x}\dfrac{dy}{dt}$

9. a) $\dfrac{dA}{dt}=\frac{1}{2}ab\cos\theta\dfrac{d\theta}{dt}$ b) $\dfrac{dA}{dt}=\frac{1}{2}ab\cos\theta\dfrac{d\theta}{dt}+\frac{1}{2}b\sin\theta\dfrac{da}{dt}$ c) $\dfrac{dA}{dt}=\frac{1}{2}ab\cos\theta\dfrac{d\theta}{dt}+\frac{1}{2}b\sin\theta\dfrac{da}{dt}+\frac{1}{2}a\sin\theta\dfrac{db}{dt}$

11. a) 14 cm^2/sec, increasing b) 0 cm/sec, constant c) $-14/13$ cm/sec, decreasing

13. a) -12 ft/sec b) -59.5 ft^2/sec c) -1 rad/sec

15. 20 ft/sec **17.** a) $\dfrac{dh}{dt}=11.19$ cm/min b) $\dfrac{dr}{dt}=14.92$ cm/min **19.** a) $\dfrac{-1}{24\pi}$ m/min b) $r=\sqrt{26y-y^2}$ m, c) $\dfrac{dr}{dt}=-\dfrac{5}{288\pi}$ m/min

21. 1 ft/min, 40π ft^2/min **23.** 11 ft/sec

25. Increasing at 466/1681 L/min **27.** 1 rad/sec **29.** -5 m/sec

31. -1500 ft/sec **33.** $\dfrac{5}{72\pi}$ in/min, $\dfrac{10}{3}$ in^2/min **35.** 7.1 in/min

37. a) $-32\sqrt{13}\approx -8.875$ ft/sec, b) $d\theta_1/dt=-8/65$ rad/sec, $d\theta_2/dt=8/65$ rad/sec

c) $d\theta_1/dt = -1/6$ rad/sec, $d\theta_2/dt = 1/6$ rad/sec
39. 29.5 knots

Chapter 2 Practice Exercises, pp. 181–185

1. $5x^4 - .25x + .25$ **3.** $3x(x-2)$ **5.** $2(x+1)(2x^2+4x+1)$

7. $3(\theta^2 + \sec\theta + 1)^2(2\theta + \sec\theta\tan\theta)$ **9.** $\dfrac{1}{2\sqrt{t}(1+\sqrt{t})^2}$

11. $2\sec^2 x \tan x$ **13.** $8\cos^3(1-2t)\sin(1-2t)$

15. $5(\sec t)(\sec t + \tan t)^5$ **17.** $\dfrac{\theta\cos\theta + \sin\theta}{\sqrt{2\theta}\sin\theta}$ **19.** $\dfrac{\cos\sqrt{2\theta}}{\sqrt{2\theta}}$

21. $x\csc\left(\dfrac{2}{x}\right) + \csc\left(\dfrac{2}{x}\right)\cot\left(\dfrac{2}{x}\right)$

23. $\dfrac{1}{2}x^{1/2}\sec(2x)^2\left[16\tan(2x)^2 - x^{-2}\right]$ **25.** $-10x\csc^2(x^2)$

27. $8x^3\sin(2x^2)\cos(2x^2) + 2x\sin^2(2x^2)$ **29.** $\dfrac{-(t+1)}{8t^3}$

31. $\dfrac{1-x}{(x+1)^3}$ **33.** $-\dfrac{1}{2x^2\left(1+\dfrac{1}{x}\right)^{1/2}}$ **35.** $\dfrac{-2\sin\theta}{(\cos\theta-1)^2}$

37. $3\sqrt{2x+1}$ **39.** $-9\left(\dfrac{5x+\cos 2x}{(5x^2+\sin 2x)^{5/2}}\right)$ **41.** $-\dfrac{y+2}{x+3}$

43. $\dfrac{-3x^2-4y+2}{4x-4y^{1/3}}$ **45.** $-y/x$ **47.** $\dfrac{1}{2y(x+1)^2}$

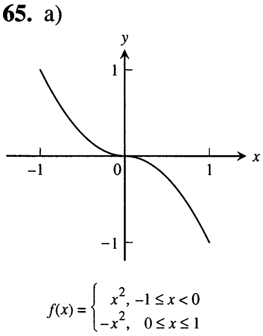
49. $\dfrac{dp}{dq} = \dfrac{6q-4p}{3p^2+4q}$ **51.** $\dfrac{dr}{ds} = (2r-1)(\tan 2s)$

53. a) $\dfrac{d^2y}{dx^2} = \dfrac{-2xy^3-2x^4}{y^5}$ b) $\dfrac{d^2y}{dx^2} = \dfrac{-2xy^2-1}{x^4y^3}$

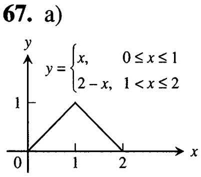
55. a) 1 b) 6 c) 1 d) $-1/9$ e) $-40/3$ f) 2 g) $-4/9$

57. 0 **59.** $\sqrt{3}$ **61.** $-1/2$ **63.** $\dfrac{-2}{(2t+1)^2}$

65. a) b) Yes c) Yes



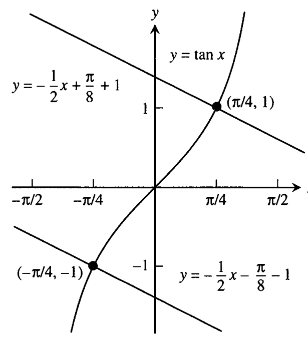
67. a) b) Yes c) No



69. $\left(\dfrac{5}{2}, \dfrac{9}{4}\right)$ and $\left(\dfrac{3}{2}, -\dfrac{1}{4}\right)$ **71.** $(-1, 27)$ and $(2, 0)$

73. a) $(-2, 16), (3, 11)$ b) $(0, 20), (1, 7)$

75.



77. 1/4 **79.** 4

81. Tangent: $y = -\dfrac{1}{4}x + \dfrac{9}{4}$; Normal: $y = 4x - 2$

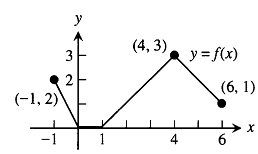
83. Tangent: $y = 2x - 4$; Normal: $y = -\dfrac{1}{2}x + \dfrac{7}{2}$

85. Tangent: $y = -\dfrac{5}{4}x + 6$; Normal: $y = \dfrac{4}{5}x - \dfrac{11}{5}$

87. $(1, 1) : m = -1/2$, $(1, -1)$; m not defined

89. B = graph of f, A = graph of f'

91.



93. a) 0, 0 b) 1700 rabbits, ≈1400 rabbits **95.** 3/2 **97.** −1

99. 1/2 **101.** 4 **103.** 1 **107.** Yes, $k = 1/2$

109. a) $\dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt}$ b) $\dfrac{dS}{dt} = 2\pi r\dfrac{dh}{dt}$

c) $\dfrac{dS}{dt} = (4\pi r + 2\pi h)\dfrac{dr}{dt} + 2\pi r\dfrac{dh}{dt}$ d) $\dfrac{dr}{dt} = -\dfrac{r}{2r+h}\dfrac{dh}{dt}$

111. -40 m^2/sec **113.** 0.02 ohm/sec **115.** 5 m/sec^2

117. a) $r = \dfrac{2}{5}$ h b) $-\dfrac{125}{144\pi}$ ft/min

119. a) $\dfrac{3}{5}$ km/sec or 600 m/sec b) $\dfrac{18}{\pi}$ RPM

Chapter 2 Additional Exercises, pp. 185–187

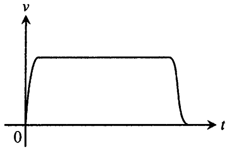
1. a) $\sin 2\theta = 2\sin\theta\cos\theta$; $2\cos 2\theta$
$= 2\sin\theta(-\sin\theta) + \cos\theta(2\cos\theta)$; $2\cos 2\theta$
$= -2\sin^2\theta + 2\cos^2\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$
b) $\cos 2\theta = \cos^2\theta - \sin^2\theta$; $-2\sin 2\theta$
$= 2\cos\theta(-\sin\theta) - 2\sin\theta(\cos\theta)$; $\sin 2\theta$
$= \cos\theta\sin\theta + \sin\theta\cos\theta$; $\sin 2\theta = 2\sin\theta\cos\theta$

3. a) $a = 1, b = 0, c = -1/2$ b) $b = \cos a, c = \sin a$

5. $h = -4, k = 9/2, a = \dfrac{5\sqrt{5}}{2}$

7. a) $0.09y$ b) increasing at 1% per year

9. Answers will vary. Here is one possibility.



11. a) 2 sec, 64 ft/sec b) 12.31 sec, 393.85 ft.

15. a) $m = -\dfrac{b}{\pi}$, b) $m = -1,\ b = \pi$ **17.** a) $a = 3/4,\ b = 9/4$

23. h' is defined but not continuous at $x = 0$; k' is defined *and* continuous at $x = 0$.

CHAPTER 3

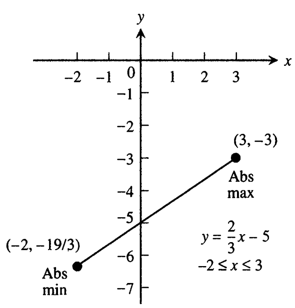
Section 3.1, pp. 195–196

1. Absolute minimum at $x = c_2$, absolute maximum at $x = b$

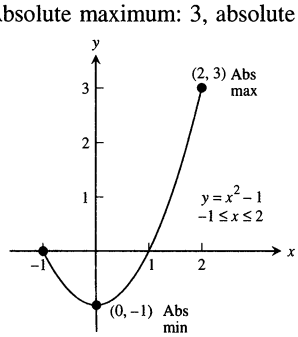
3. Absolute maximum at $x = c$, no absolute minimum

5. Absolute minimum at $x = a$, absolute maximum at $x = c$

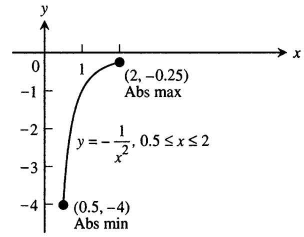
7. Absolute maximum: -3, absolute minimum: $-19/3$



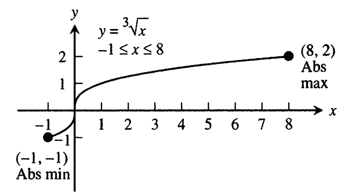
9. Absolute maximum: 3, absolute minimum: -1



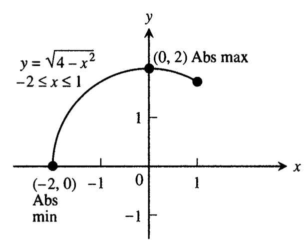
11. Absolute maximum: -0.25, absolute minimum: -4



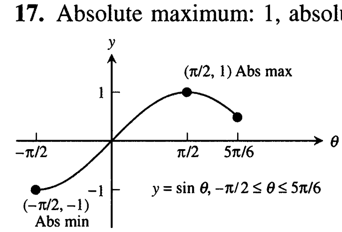
13. Absolute maximum: 2, absolute minimum: -1



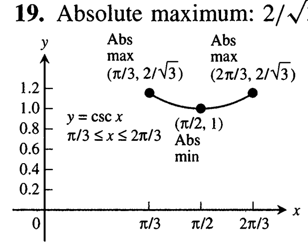
15. Absolute maximum: 2, absolute minimum: 0



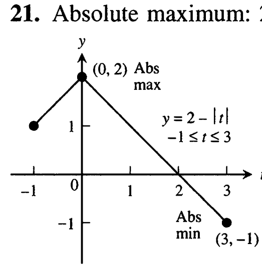
17. Absolute maximum: 1, absolute minimum: -1



19. Absolute maximum: $2/\sqrt{3}$, absolute minimum: 1



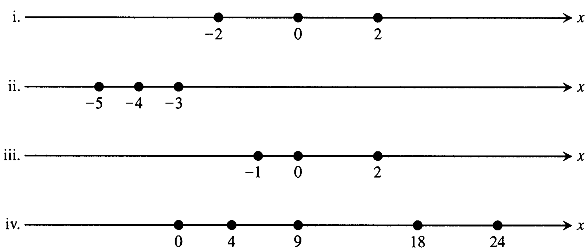
21. Absolute maximum: 2, absolute minimum: -1



23. Increasing on $(0, 8)$, decreasing on $(-1, 0)$, absolute maximum: 16 at $x = 8$, absolute minimum: 0 at $x = 0$
25. Increasing on $(-32, 1)$, absolute maximum: 1 at $\theta = 1$, absolute minimum: -8 at $\theta = -32$
27. a) Local maximum: 0 at $x = \pm 2$, local minimum: -4 at $x = 0$, absolute maximum: 0, absolute minimum: -4
b) Local maximum: 0 at $x = -2$, local minimum: -4 at $x = 0$, absolute maximum: 0, absolute minimum: -4
c) No local maximum, local minimum: -4 at $x = 0$, absolute minimum: -4
d) Local maximum: 0 at $x = -2$, local minimum: -4 at $x = 0$, absolute minimum: -4
e) No local extrema, no absolute extrema **29.** Yes

Section 3.2, pp. 203–205

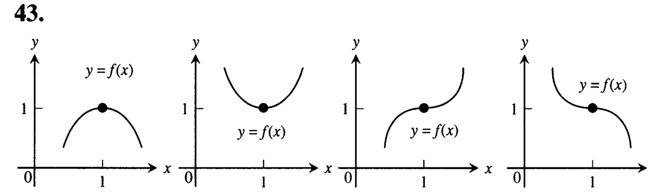
1. $1/2$ **3.** 1
5. Does not; f is not differentiable at the interior domain point $x = 0$.
7. Does
11. a)



27. $1.09999 \le f(0.1) \le 1.1$ **31.** Yes **33.** a) 4 b) 3 c) 3
35. a) $\dfrac{x^2}{2} + C$ b) $\dfrac{x^3}{3} + C$ c) $\dfrac{x^4}{4} + C$
37. a) $\dfrac{1}{x} + C$ b) $x + \dfrac{1}{x} + C$ c) $5x - \dfrac{1}{x} + C$
39. a) $-\dfrac{1}{2}\cos 2t + C$ b) $2\sin\dfrac{t}{2} + C$
c) $-\dfrac{1}{2}\cos 2t + 2\sin\dfrac{t}{2} + C$
41. $f(x) = x^2 - x$ **43.** $r(\theta) = 8\theta + \cot\theta - 2\pi - 1$

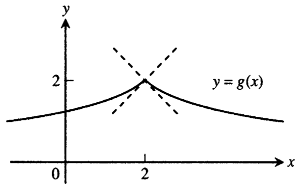
Section 3.3, pp. 208–209

1. a) 0, 1 b) increasing on $(-\infty, 0)$ and $(1, \infty)$, decreasing on $(0, 1)$ c) local maximum at $x = 0$, local minimum at $x = 1$
3. a) -2, 1 b) increasing on $(-2, 1)$ and $(1, \infty)$, decreasing on $(-\infty, -2)$ c) No local maximum, local minimum at $x = -2$
5. a) -2, 1, 3 b) increasing on $(-2, 1)$ and $(3, \infty)$, decreasing on $(-\infty, -2)$ and $(1, 3)$ c) local maximum at $x = 1$, local minima at $x = -2, 3$
7. a) -2, 0 b) increasing on $(-\infty, -2)$ and $(0, \infty)$, decreasing on $(-2, 0)$ c) Local maximum at $x = -2$, local minimum at $x = 0$
9. a) increasing on $(-\infty, -1.5)$, decreasing on $(-1.5, \infty)$
b) local maximum: 5.25 at $t = -1.5$
c) Absolute maximum: 5.25 at $t = -1.5$
11. a) Decreasing on $(-\infty, 0)$, increasing on $(0, 4/3)$, decreasing on $(4/3, \infty)$ b) local minimum at $x = 0$ (0, 0), local maximum at $x = 4/3$ (4/3, 32/27) c) no absolute extrema
13. a) Decreasing on $(-\infty, 0)$, increasing on $(0, 1/2)$, decreasing on $(1/2, \infty)$ b) local minimum at $\theta = 0$ (0, 0), local maximum at $\theta = 1/2$ (1/2, 1/4) c) no absolute extrema
15. a) Increasing on $(-\infty, \infty)$, never decreasing b) no local extrema c) no absolute extrema
17. a) Increasing on $(-2, 0)$ and $(2, \infty)$, decreasing on $(-\infty, -2)$ and $(0, 2)$ b) local maximum: 16 at $x = 0$, local minimum: 0 at $x = \pm 2$ c) no absolute maximum, absolute minimum: 0 at $x = \pm 2$
19. a) Increasing on $(-\infty, -1)$, decreasing on $(-1, 0)$, increasing on $(0, 1)$, decreasing on $(1, \infty)$
b) local maximum at $x = \pm 1$ $(1, 0.5), (-1, 0.5)$, local minimum at $x = 0$ $(0, 0)$ c) absolute maximum: 1/2 at $x = \pm 1$, no absolute minimum
21. a) Decreasing on $(-2\sqrt{2}, -2)$, increasing on $(-2, 2)$, decreasing on $(2, 2\sqrt{2})$ b) local minima: $g(-2) = -4$, $g(2\sqrt{2}) = 0$; local maxima: $g(-2\sqrt{2}) = 0$, $g(2) = 4$ c) absolute maximum: 4 at $x = 2$, absolute minimum: -4 at $x = -2$
23. a) Increasing on $(-\infty, 1)$, decreasing when $1 < x < 2$, decreasing when $2 < x < 3$, discontinuous at $x = 2$, increasing on $(3, \infty)$, b) local minimum at $x = 3$ (3, 6), local maximum at $x = 1$ (1, 2) c) no absolute extrema
25. a) Increasing on $(-2, 0)$ and $(0, \infty)$, decreasing on $(-\infty, -2)$ b) local minimum: $-6\sqrt[3]{2}$ at $x = -2$ c) no absolute maximum, absolute minimum: $-6\sqrt[3]{2}$ at $x = -2$
27. a) Increasing on $(-\infty, -2/\sqrt{7})$ and $(2/\sqrt{7}, \infty)$, decreasing on $(-2/\sqrt{7}, 2/\sqrt{7})$ b) local maximum: $24\sqrt[3]{2}/7^{7/6} \approx 3.12$ at $x = -2/\sqrt{7}$, local minimum: $-24\sqrt[3]{2}/7^{7/6} \approx -3.12$ at $x = 2/\sqrt{7}$
c) no absolute extrema
29. a) Local maximum: 1 at $x = 1$, local minimum: 0 at $x = 2$
b) absolute maximum: 1 at $x = 1$; no absolute minimum
31. a) Local maximum: 1 at $x = 1$, local minimum: 0 at $x = 2$
b) no absolute maximum, absolute minimum: 0 at $x = 2$
33. a) Local maxima: -9 at $t = -3$ and 16 at $t = 2$, local minimum: -16 at $t = -2$ b) absolute maximum: 16 at $t = 2$, no absolute minimum
35. a) Local minimum: 0 at $x = 0$ b) no absolute maximum, absolute minimum: 0 at $x = 0$
37. a) Local minimum: $(\pi/3) - \sqrt{3}$ at $x = 2\pi/3$, local maximum: 0 at $x = 0$, local maximum: π at $x = 2\pi$
39. a) Local minimum: 0 at $x = \pi/4$
41. Local maximum: 3 at $\theta = 0$, Local minimum: -3 at $\theta = 2\pi$
43.

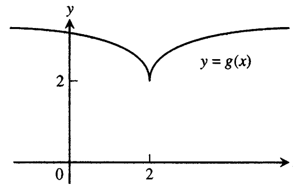


(a) (b) (c) (d)

45. a)



b)



47. Rising

Section 3.4, pp. 217–220

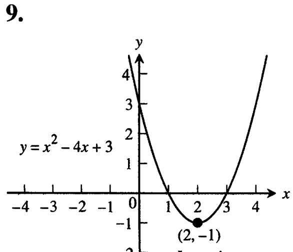
1. Local maximum: $3/2$ at $x = -1$, local minimum: -3 at $x = 2$, point of inflection at $(1/2, -3/4)$, rising on $(-\infty, -1)$ and $(2, \infty)$, falling on $(-1, 2)$, concave up on $(1/2, \infty)$, concave down on $(-\infty, 1/2)$

3. Local maximum: $3/4$ at $x = 0$, local minimum: 0 at $x = \pm 1$, points of inflection at $\left(-\sqrt{3}, \dfrac{3\sqrt[3]{4}}{4}\right)$ and $\left(\sqrt{3}, \dfrac{3\sqrt[3]{4}}{4}\right)$, rising on $(-1, 0)$ and $(1, \infty)$, falling on $(-\infty, -1)$ and $(0, 1)$, concave up on $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$, concave down on $(-\sqrt{3}, \sqrt{3})$

5. Local maxima: $-2\pi/3 + \sqrt{3}/2$ at $x = -2\pi/3$; $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{2}$ at $x = \dfrac{\pi}{3}$, local minima: $-\dfrac{\pi}{3} - \dfrac{\sqrt{3}}{2}$ at $x = -\dfrac{\pi}{3}$; $2\pi/3 - \sqrt{3}/2$ at $x = \dfrac{2\pi}{3}$, points of inflection at $(-\pi/2, -\pi/2)$, $(0, 0)$, and $(\pi/2, \pi/2)$, rising on $(-\pi/3, \pi/3)$, falling on $(-2\pi/3, -\pi/3)$ and $(\pi/3, 2\pi/3)$, concave up on $(-\pi/2, 0)$ and $(\pi/2, 2\pi/3)$, concave down on $(-2\pi/3, -\pi/2)$ and $(0, \pi/2)$

7. Local maxima: 1 at $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$; 0 at $x = -2\pi$ and $x = 2\pi$; local minima: -1 at $x = -\dfrac{3\pi}{2}$ and $x = \dfrac{3\pi}{2}$, 0 at $x = 0$, points of inflection at $(-\pi, 0)$ and $(\pi, 0)$, rising on $(-3\pi/2, -\pi/2)$, $(0, \pi/2)$ and $(3\pi/2, 2\pi)$, falling on $(-2\pi, -3\pi/2)$, $(-\pi/2, 0)$ and $(\pi/2, 3\pi/2)$, concave up on $(-2\pi, -\pi)$ and $(\pi, 2\pi)$, concave down on $(-\pi, \pi)$

9.



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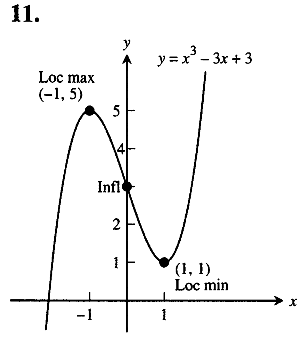

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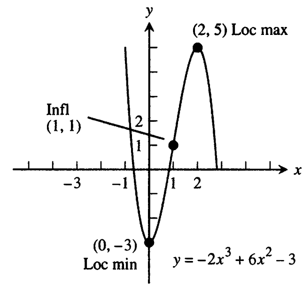

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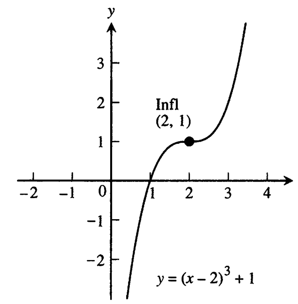

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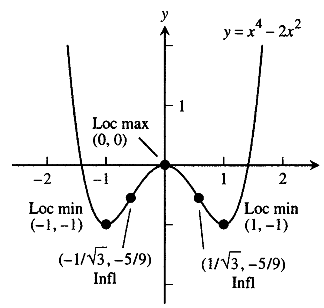

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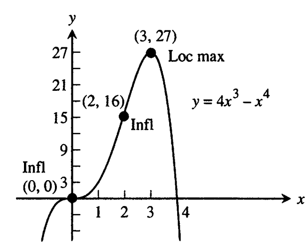

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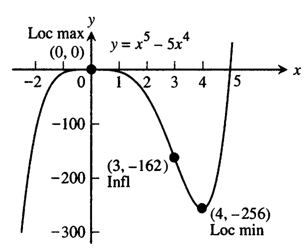

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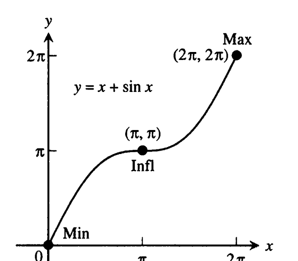

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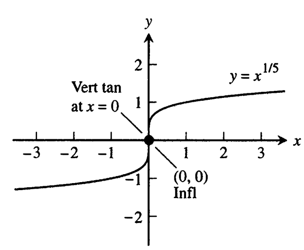

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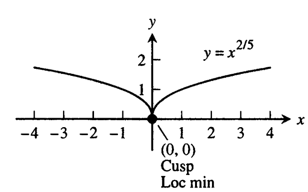

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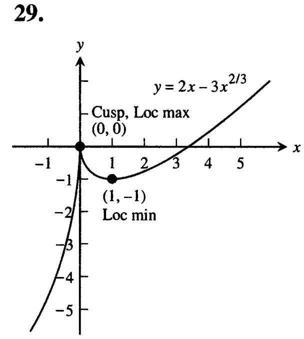
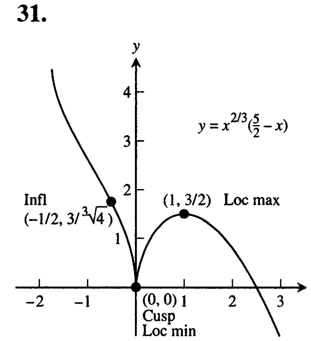


31.



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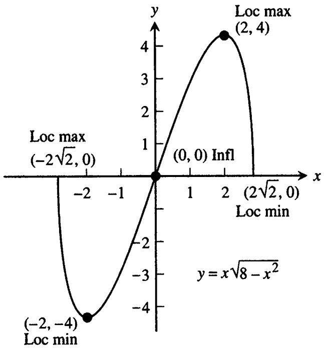

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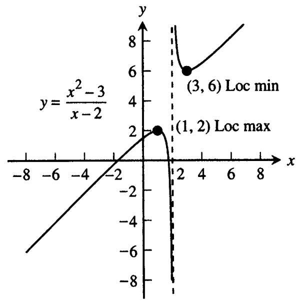

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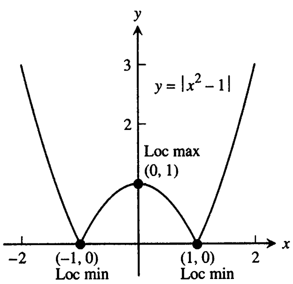
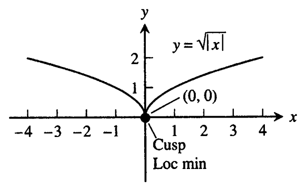
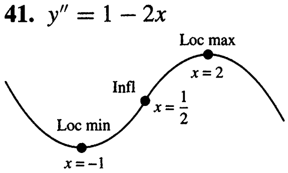


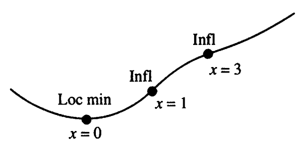
39.



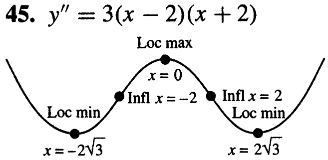
41. $y'' = 1 - 2x$



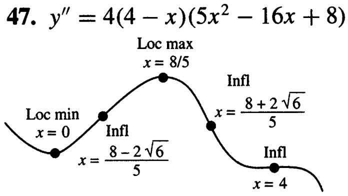
43. $y'' = 3(x - 3)(x - 1)$



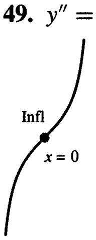
45. $y'' = 3(x - 2)(x + 2)$



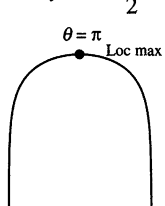
47. $y'' = 4(4 - x)(5x^2 - 16x + 8)$



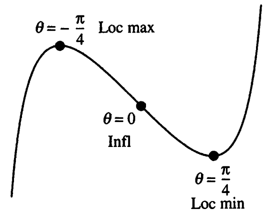
49. $y'' = 2\sec^2 x \tan x$



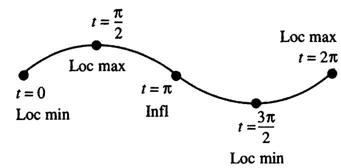
51. $y'' = -\frac{1}{2}\csc^2\frac{\theta}{2}, 0 < \theta < 2\pi$



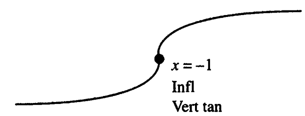
53. $y'' = 2\tan\theta\sec^2\theta, -\frac{\pi}{2} < \theta < \frac{\pi}{2}$



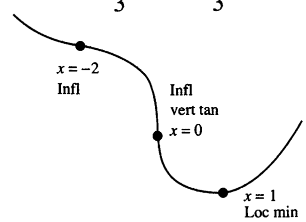
55. $y'' = -\sin t, 0 \le t \le 2\pi$



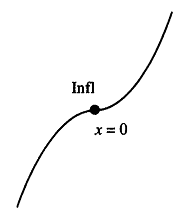
57. $y'' = -\frac{2}{3}(x + 1)^{-5/3}$



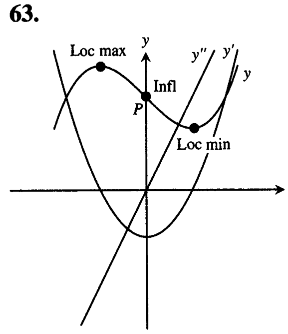
59. $y'' = \frac{1}{3}x^{-2/3} + \frac{2}{3}x^{-5/3}$



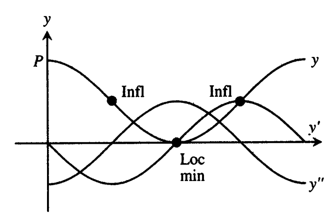
61. $y'' = \begin{cases} -2, & x < 0 \\ 2, & x > 0 \end{cases}$



63.



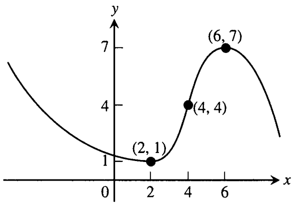
65.



67.

Point	y'	y''
P	−	+
Q	+	0
R	+	−
S	0	−
T	−	−

69.



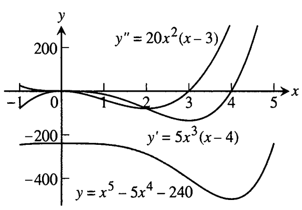
73. ≈ 60 thousand units

75. Local minimum at $x = 2$, inflection points at $x = 1$ and $x = 5/3$

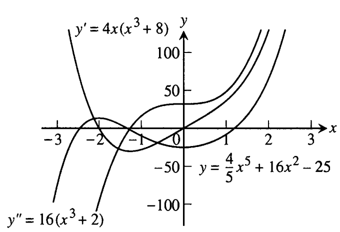
79. $b = -3$

81. a) $\left(-\frac{b}{2a}, \frac{4ac - b^2}{4a}\right)$ b) concave up if $a > 0$, concave down if $a < 0$

85. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively.



87. The zeros of $y' = 0$ and $y'' = 0$ are extrema and points of inflection, respectively. Inflection at $x = -\sqrt[3]{2}$, local maximum at $x = -2$, local minimum at $x = 0$.



91. b) $f'(x) = 3x^2 + k$; $-12k$; positive if $k < 0$, negative if $k > 0$, 0 if $k = 0$; f' has two zeros if $k < 0$, one zero if $k = 0$, no zeros if $k > 0$

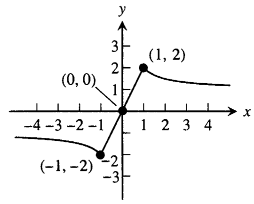
93. b) A cusp since $\lim_{x\to 0^-} y' = \infty$ and $\lim_{x\to 0^+} y' = -\infty$

95. Yes, the graph of y' crosses through zero near -3, so y has a horizontal tangent near -3.

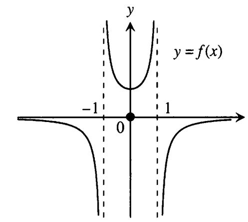
Section 3.5, pp. 230–233

1. a) -3 b) -3 **3.** a) $1/2$ b) $1/2$ **5.** a) $-5/3$ b) $-5/3$
7. 0 **9.** -1 **11.** a) $2/5$ b) $2/5$ **13.** a) 0 b) 0
15. a) $-\infty$ b) ∞ **17.** a) 7 b) 7 **19.** a) $-\infty$ b) ∞
21. a) ∞ b) $-\infty$ **23.** a) $-2/3$ b) $-2/3$ **25.** 0 **27.** 1
29. ∞

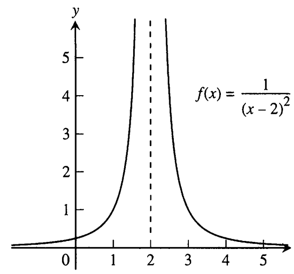
31. Here is one possibility.



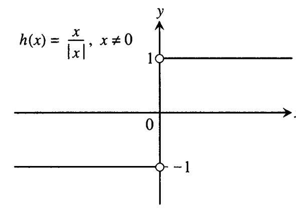
33. Here is one possibility.



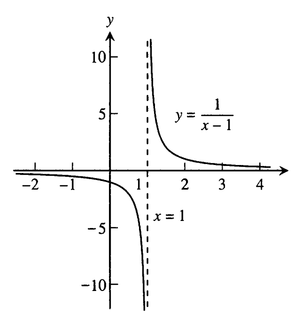
35. Here is one possibility.



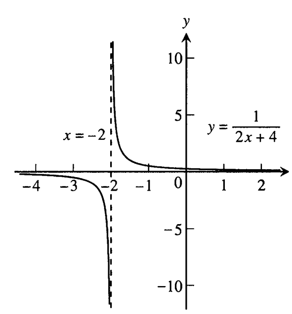
37. Here is one possibility.



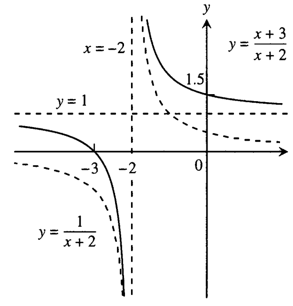
39.



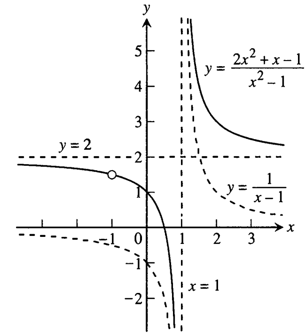
41.



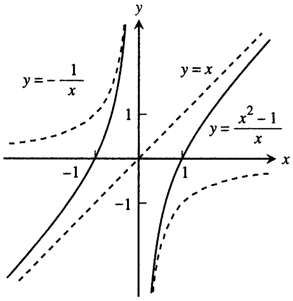
43.



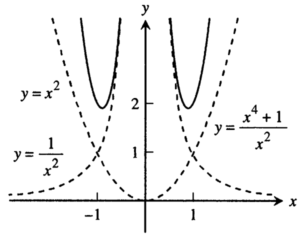
45.



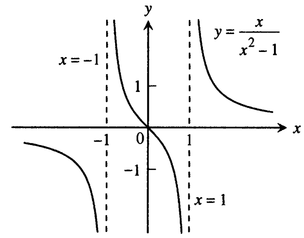
47.



49.



63.



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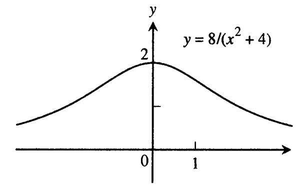
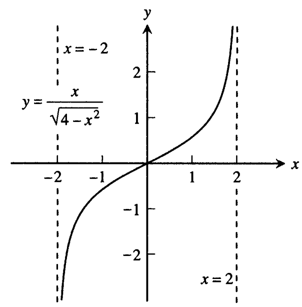
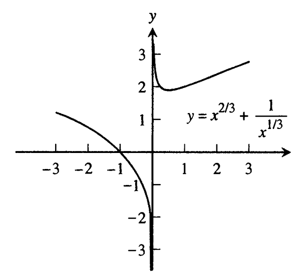


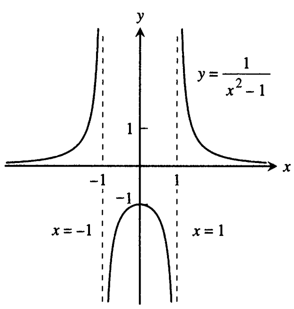
67.



69.



51.



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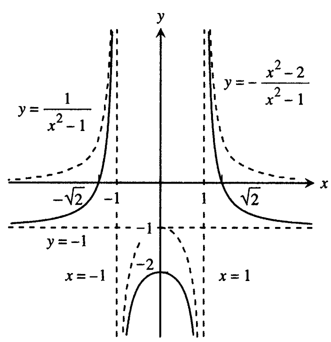
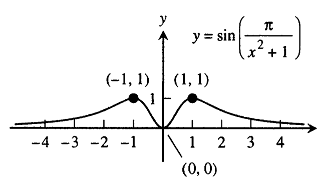
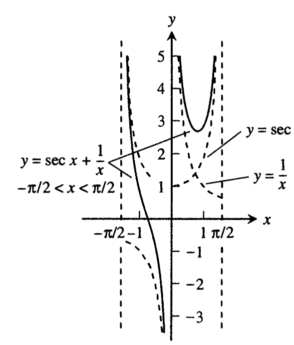


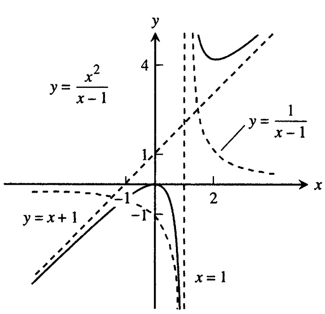
71.



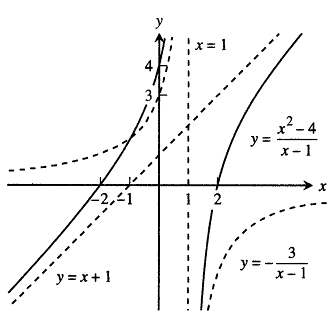
73.



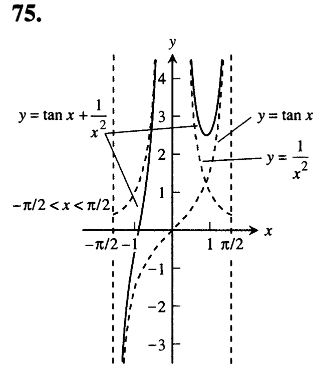
55.



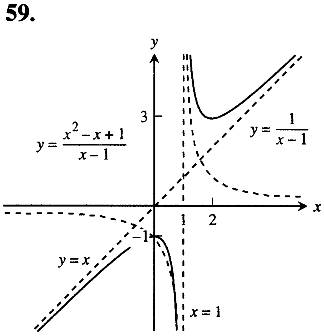
57.



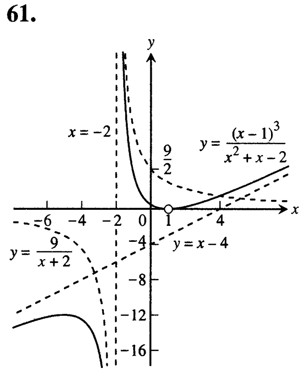
75.



59.



61.



79. Increasing **83.** 2

85. b) One possibility is $f(x) = 2 + (1/x)\sin(x^2)$.

89. $x = -1,\ y = 1 - x$ **91.** $x = 1,\ x = -1,\ y = x - 1$

99. a) $y \to \infty$, b) $y \to \infty$, c) cusp at $x = \pm 1$

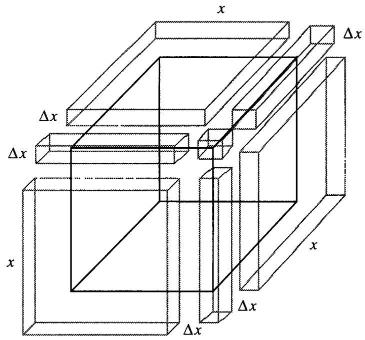
101. The distance in part (c) is so great that small movements are not visible.

103. 1 **105.** 3/2 **107.** 3

Section 3.6, pp. 242–247

1. $r = 25$ m, $s = 50$ m **3.** 16 in. **5.** a) $(x, 1-x)$ b) $A(x) = 2x(1-x)$ c) 1/2 square units **7.** $\frac{14}{3} \times \frac{35}{3} \times \frac{5}{3}$ in. **9.** 80,000 m^2 **11.** base: 10 ft, height: 5 ft **13.** 9×18 in. **15.** $\pi/2$ **17.** $r = h = \frac{10}{\sqrt[3]{\pi}}$ cm **19.** a) $18 \times 18 \times 36$ in.
21. a) 12 cm, 6 cm b) 12 cm, 6 cm
23. a) The circumference of the circle is 4 m.
25. If r is the radius of the semicircle, $2r$ is the base of the rectangle, and h is the height of the rectangle, then $(2r)/h = 8/(4+\pi)$.
27. $\pi/6$ **29.** $\frac{v_0^2}{2g} + s_0$ **31.** a) $4\sqrt{3} \times 4\sqrt{6}$ in. **33.** $2\sqrt{2}$ amps
35. a) When t is an integer multiple of π b) $t = \frac{2\pi}{3}, t = \frac{4\pi}{3}, 3\frac{\sqrt{3}}{2}$ **37.** a) $t = \frac{8}{5}, t = 4$ b) $\frac{8}{5} < t < 4$ c) $\frac{2187}{125}$ units/time
41. No. The function has an absolute minimum of 3/4.
43. a) $\left(c - \frac{1}{2}, \sqrt{c - \frac{1}{2}}\right)$ b) (0, 0) **45.** a) $a = -3, b = -9$ b) $a = -3, b = -24$ **47.** a) $y = -1$ **49.** $(7/2)\sqrt{17}$
53. $M = c/2$ **55.** $\frac{c}{2} + 50$ **57.** $\sqrt{\frac{2km}{h}}$

57. Volume $= (x + \Delta x)^3 = x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$



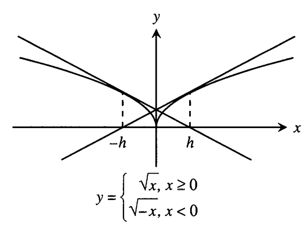
59. $\lim_{x\to 0} \frac{\sqrt{1+x}}{1+\left(\frac{x}{2}\right)} = \frac{\sqrt{1+0}}{1+\left(\frac{0}{2}\right)} = \frac{1}{1} = 1$

Section 3.7, pp. 257–260

1. $4x - 3$ **3.** $2x - 2$ **5.** $\frac{1}{4}x + 1$ **7.** $2x$ **9.** -5
11. $\frac{1}{12}x + \frac{4}{3}$ **13.** a) $L(x) = x$ b) $L(x) = \pi - x$
15. a) $L(x) = 1$ b) $L(x) = 2 - 2\sqrt{3}\left(x + \frac{\pi}{3}\right)$ **17.** a) $1 + 2x$ b) $1 - 5x$ c) $2 + 2x$ d) $1 - 6x$ e) $3 + x$ f) $1 - \frac{x}{2}$
19. $\frac{3}{2}x + 1$. It is equal to their sum. **21.** $\left(3x^2 - \frac{3}{2\sqrt{x}}\right) dx$
23. $\frac{2 - 2x^2}{(1+x^2)^2} dx$ **25.** $\frac{1-y}{3\sqrt{y} + x} dx$ **27.** $\frac{5}{2\sqrt{x}} \cos(5\sqrt{x})\, dx$
29. $(4x^2) \sec^2\left(\frac{x^3}{3}\right) dx$
31. $\frac{3}{\sqrt{x}}(\csc(1 - 2\sqrt{x}) \cot(1 - 2\sqrt{x}))\, dx$ **33.** a) .21 b) .2 c) .01 **35.** a) .231 b) .2 c) .031 **37.** a) $-1/3$ b) $-2/5$ c) 1/15 **39.** $dV = 4\pi r_0^2\, dr$ **41.** $dS = 12x_0\, dx$
43. $dV = 2\pi r_0 h\, dr$ **45.** a) $.08\pi$ m^2 b) 2% **47.** 3%
49. 3% **51.** 1/3% **53.** .05%

Section 3.8, pp. 266–268

1. $x_2 = 13/21, -5/3$ **3.** $x_2 = 5763/4945, -51/31$
5. $x_2 = 2387/2000$ **7.** $x \approx 0.45$ **9.** The root is 1.17951
13.



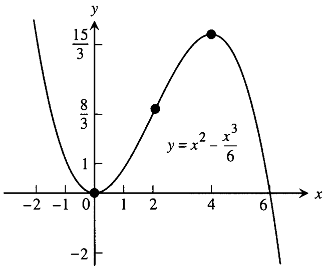
15. a) The points of intersection of $y = x^3$ and $y = 3x + 1$ or $y = x^3 - 3x$ and $y = 1$ have the same x-values as the roots of part (i) or the solutions of part (iv). b) $-1.53209, -0.34730$
17. 2.45, 0.000245 **19.** 1.1655 61185 **21.** a) Two b) 0.3500 35015 05249 and -1.0261 73161 5301
23. ± 1.3065 62964 8764, ± 0.5411 96100 14619
25. Answers will vary with machine used.
27.

x_0	Approximation of corresponding root
−1.0	−0.976823589
0.1	0.100363332
0.6	0.642746671
2.0	1.98371387

Chapter 3 Practice Exercises, pp. 269–272

1. No
3. No minimum, absolute maximum: $f(1) = 16$, critical points: $x = 1$ and 11/3
7. No **11.** b) one **13.** b) 0.8555 99677 2

19.



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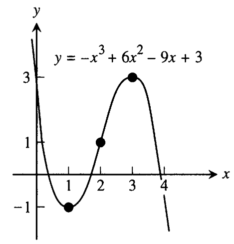

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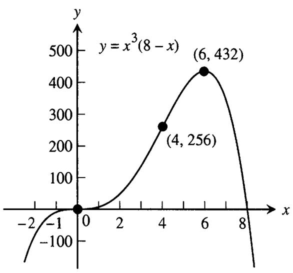

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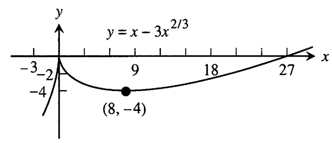
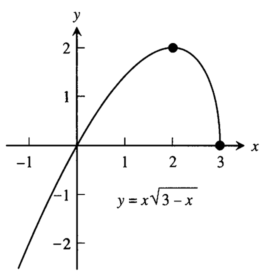
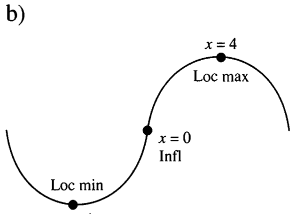


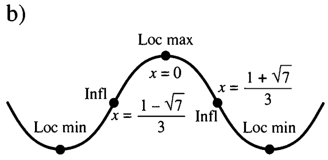
27.



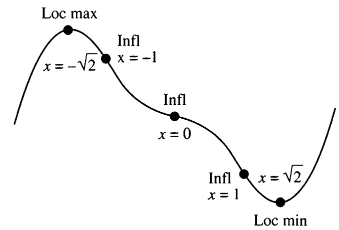
29. a) Local maximum at $x = 4$, local minimum at $x = -4$, inflection point at $x = 0$
b)



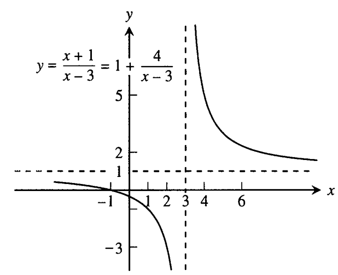
31. a) Local maximum at $x = 0$, local minima at $x = -1$ and $x = 2$, inflection points at $x = (1 \pm \sqrt{7})/3$
b)



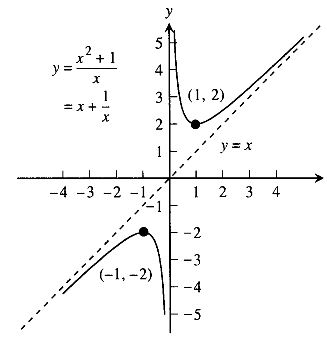
33. a) Local maximum at $x = -\sqrt{2}$, local minimum at $x = \sqrt{2}$, inflection points at $x = \pm 1$ and 0
b)



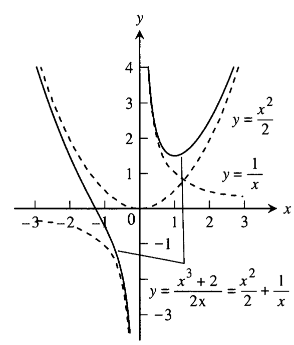
39.



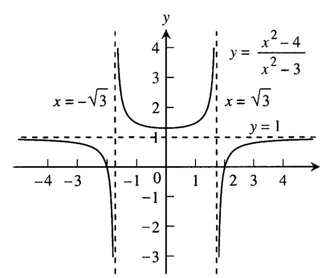
41.



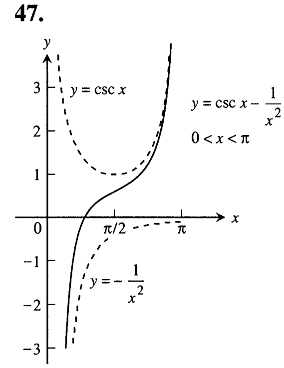
43.



45.

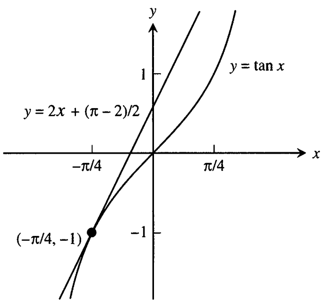


47.

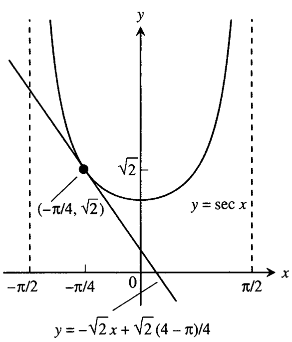


49. a) $t = 0,\ 6,\ 12$ b) $t = 3,\ 9$ c) $6 < t < 12$ d) $0 < t < 6$, $12 < t < 14$
51. 2/5 **53.** 0 **55.** $-\infty$ **57.** 0 **59.** 1 **61.** a) 0, 36 b) 18, 18 **63.** 54 square units **65.** height = 2, radius = $\sqrt{2}$

67. $x = 15$ mi, $y = 9$ mi **69.** $x = 5 - \sqrt{5}$ hundred ≈ 276 tires, $y = 2(5 - \sqrt{5})$ hundred ≈ 553 tires
71. a) $L(x) = 2x + (\pi - 2)/2$



b) $L(x) = -\sqrt{2}x + \sqrt{2}(4 - \pi)/4$



73. $L(x) = 1.5x + 0.5$ **75.** $dV = \frac{2}{3}\pi r_0 h\, dr$
77. a) error $< 1\%$ b) 3% **79.** $dh \approx \pm 2.3271$ ft
81. $x_5 = 2.1958\ 23345$

Chapter 3 Additional Exercises, pp. 272–274

3. The extreme points will not be at the end of an open interval.
5. a) A local minimum at $x = -1$, points of inflection at $x = 0$ and $x = 2$ b) A local maximum at $x = 0$ and local minima at $x = -1$ and $x = 2$, points of inflection at $x = \frac{1 \pm \sqrt{7}}{3}$
11. $a = 1, b = 0, c = 1$ **13.** Yes **15.** Drill the hole at $y = h/2$.
17. $r = \frac{RH}{2(H - R)}$ for $H < 2R, r = R$ if $H \le 2R$
21. a) 0.8156 ft b) 0.00613 sec c) It will lose about 8.83 min/day.

CHAPTER 4

Section 4.1, pp. 280–282

1. a) x^2 b) $\frac{x^3}{3}$ c) $\frac{x^3}{3} - x^2 + x$ **3.** a) x^{-3} b) $-\frac{1}{3}x^{-3}$ c) $-\frac{1}{3}x^{-3} + x^2 + 3x$ **5.** a) $-\frac{1}{x}$ b) $-\frac{5}{x}$ c) $2x + \frac{5}{x}$
7. a) $\sqrt{x^3}$ b) $\sqrt{x}$ c) $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x}$ **9.** a) $x^{2/3}$ b) $x^{1/3}$ c) $x^{-1/3}$ **11.** a) $\cos(\pi x)$ b) $-3\cos x$ c) $-\frac{1}{\pi}\cos(\pi x) + \cos(3x)$ **13.** a) $\tan x$ b) $2\tan\left(\frac{x}{3}\right)$ c) $-\frac{2}{3}\tan\left(\frac{3x}{2}\right)$ **15.** a) $-\csc x$ b) $\frac{1}{5}\csc(5x)$ c) $2\csc\left(\frac{\pi x}{2}\right)$
17. $x + \frac{\cos(2x)}{2}$ **19.** $\frac{x^2}{2} + x + C$ **21.** $t^3 + \frac{t^2}{4} + C$
23. $\frac{x^4}{2} - \frac{5x^2}{2} + 7x + C$ **25.** $-\frac{1}{x} - \frac{x^3}{3} - \frac{x}{3} + C$
27. $\frac{3}{2}x^{2/3} + C$ **29.** $\frac{2}{3}x^{3/2} + \frac{3}{4}x^{4/3} + C$ **31.** $4y^2 - \frac{8}{3}y^{3/4} + C$
33. $x^2 + \frac{2}{x} + C$ **35.** $2\sqrt{t} - \frac{2}{\sqrt{t}} + C$ **37.** $-2\sin t + C$
39. $-21\cos\frac{\theta}{3} + C$ **41.** $3\cot x + C$ **43.** $-\frac{1}{2}\csc\theta + C$
45. $4\sec x - 2\tan x + C$ **47.** $-\frac{1}{2}\cos 2x + \cot x + C$
49. $2y - \sin 2y + C$ **51.** $\frac{t}{2} + \frac{\sin 4t}{8} + C$ **53.** $\tan\theta + C$
55. $-\cot x - x + C$ **57.** $-\cos\theta + \theta + C$
65. a) Wrong: $\frac{d}{dx}\left(\frac{x^2}{2}\sin x + C\right) = \frac{2x}{2}\sin x + \frac{x^2}{2}\cos x = x\sin x + \frac{x^2}{2}\cos x$
b) Wrong: $\frac{d}{dx}(-x\cos x + C) = -\cos x + x\sin x$
c) Right: $\frac{d}{dx}(-x\cos x + \sin x + C) = -\cos x + x\sin x + \cos x = x\sin x$
67. a) Wrong: $\frac{d}{dx}\left(\frac{(2x+1)^3}{3} + C\right) = \frac{3(2x+1)^2(2)}{3} = 2(2x+1)^2$
b) Wrong: $\frac{d}{dx}((2x+1)^3 + C) = 3(2x+1)^2(2) = 6(2x+1)^2$
c) Right: $\frac{d}{dx}((2x+1)^3 + C) = 6(2x+1)^2$
69. a) $-\sqrt{x} + C$ b) $x + C$ c) $\sqrt{x} + C$ d) $-x + C$ e) $x - \sqrt{x} + C$ f) $-x - \sqrt{x} + C$ g) $\frac{x^2}{2} - \sqrt{x} + C$ h) $-3x + C$

Section 4.2, pp. 288–290

1. b **3.** $y = x^2 - 7x + 10$ **5.** $y = -\frac{1}{x} + \frac{x^2}{2} - \frac{1}{2}$
7. $y = 9x^{1/3} + 4$ **9.** $s = t + \sin t + 4$ **11.** $r = \cos(\pi\theta) - 1$

13. $v = \frac{1}{2}\sec t + \frac{1}{2}$ **15.** $y = x^2 - x^3 + 4x + 1$

17. $r = \frac{1}{t} + 2t - 2$ **19.** $y = x^3 - 4x^2 + 5$

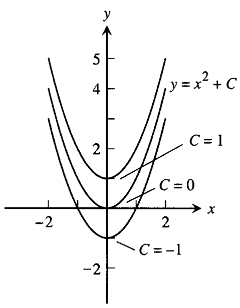
21. $y = -\sin t + \cos t + t^3 - 1$ **23.** $s = 4.9t^2 + 5t + 10$

25. $s = \frac{1 - \cos(\pi t)}{\pi}$ **27.** $s = 16t^2 + 20t + 5$

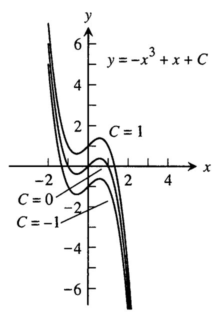
29. $s = \sin(2t) - 3$ **31.** $y = 2x^{3/2} - 50$ **33.** $y = x - x^{4/3} + \frac{1}{2}$

35. $y = -\sin x - \cos x - 2$

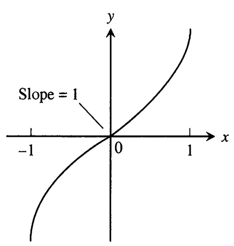
37.



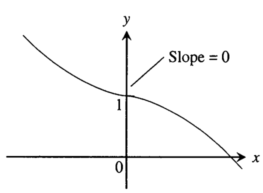
39.



41.



43.



45. 48 m/sec **47.** 14 m/sec **49.** $t = 88/k, k = 16$

51. a) $v = 10t^{3/2} - 6t^{1/2}$ b) $s = 4t^{5/2} - 4t^{3/2}$

55. a) 1: 33.2 units, 2: 33.2 units, 3: 33.2 units b) True

Section 4.3, p. 296

1. $-\frac{1}{3}\cos 3x + C$ **3.** $\frac{1}{2}\sec 2t + C$ **5.** $-(7x - 2)^{-4} + C$

7. $-6(1 - r^3)^{1/2} + C$ **9.** $\frac{1}{3}(x^{3/2} - 1) - \frac{1}{6}\sin(2x^{3/2} - 2) + C$

11. a) $-\frac{1}{4}(\cot^2 2\theta) + C$ b) $-\frac{1}{4}(\csc^2 2\theta) + C$

13. $-\frac{1}{3}(3 - 2s)^{3/2} + C$ **15.** $\frac{2}{5}(5s + 4)^{1/2} + C$

17. $-\frac{2}{5}(1 - \theta^2)^{5/4} + C$ **19.** $-\frac{1}{3}(7 - 3y^2)^{3/2} + C$

21. $(-2/(1 + \sqrt{x})) + C$ **23.** $\frac{1}{3}\sin(3z + 4) + C$

25. $\frac{1}{3}\tan(3x + 2) + C$ **27.** $\frac{1}{2}\sin^6\left(\frac{x}{3}\right) + C$

29. $\left(\frac{r^3}{18} - 1\right)^6 + C$ **31.** $-\frac{2}{3}\cos(x^{3/2} + 1) + C$

33. $\sec\left(v + \frac{\pi}{2}\right) + C$ **35.** $\frac{1}{2\cos(2t + 1)} + C$

37. $-\frac{2}{3}(\cot^3 y)^{1/2} + C$ **39.** $-\sin\left(\frac{1}{t} - 1\right) + C$

41. $-\frac{\sin^2(1/\theta)}{2} + C$ **43.** $\frac{(s^3 + 2s^2 - 5s + 5)^2}{2} + C$

45. $\frac{1}{16}(1 + t^4)^4 + C$ **47.** a) $-\frac{6}{2 + \tan^3 x} + C$
b) $-\frac{6}{2 + \tan^3 x} + C$ c) $-\frac{6}{2 + \tan^3 x} + C$

49. $\frac{1}{6}\sin\sqrt{3(2r - 1)^2 + 6} + C$ **51.** $s = \frac{1}{2}(3t^2 - 1)^4 - 5$

53. $s = 4t - 2\sin\left(2t + \frac{\pi}{6}\right) + 9$

55. $s = \sin\left(2t - \frac{\pi}{2}\right) + 100t + 1$ **57.** 6 m

Section 4.4, pp. 305–309

1. ≈ 44.8, 6.7 L/min **3.** a) 87 in. b) 87 in. **5.** a) 3,490 ft b) 3,840 ft **7.** a) 112 b) 9% **9.** a) 80π b) 6% **11.** a) $93\pi/2$, overestimate b) 9% **13.** a) 40 b) 25% c) 36, 12.5% **15.** a) 118.5π or ≈ 372.28 m^3 b) error $\approx$ 11% **17.** a) 10π, underestimate b) 20% **19.** 31/16 **21.** 1 **23.** a) 74.65 ft/sec b) 45.28 ft/sec c) 146.59 ft **25.** a) upper = 758 gal, lower = 543 gal b) upper = 2363 gal, lower = 1693 gal c) $\approx$ 31.4 hours, $\approx$ 32.4 hours

Section 4.5, pp. 320–323

1. $\frac{6(1)}{1 + 1} + \frac{6(2)}{2 + 1} = 7$

3. $\cos(1)\pi + \cos(2)\pi + \cos(3)\pi + \cos(4)\pi = 0$

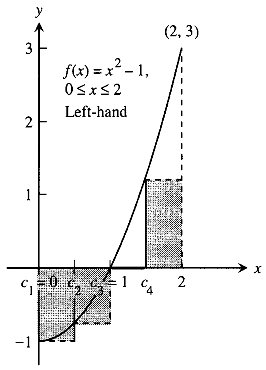
5. $\sin\pi - \sin\frac{\pi}{2} + \sin\frac{\pi}{3} = \frac{\sqrt{3} - 2}{2}$

7. All of them **9.** b **11.** $\sum_{k=1}^{6} k$ **13.** $\sum_{k=1}^{4} \frac{1}{2^k}$

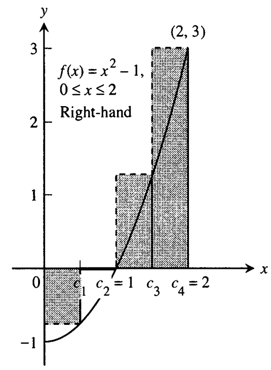
15. $\sum_{k=1}^{5} (-1)^{k+1}\frac{1}{k}$ **17.** a) -15 b) 1 c) 1 d) -11 e) 16

19. a) 55 b) 385 c) 3025 **21.** -56 **23.** -73 **25.** 240 **27.** 3376

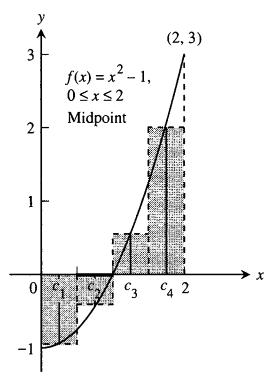
29. a)



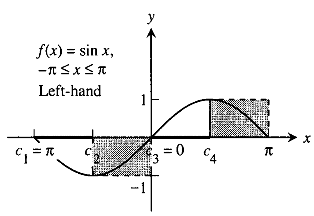
b)



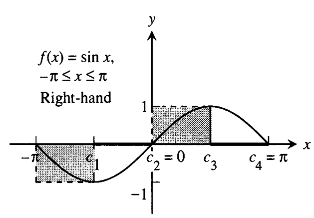
c)



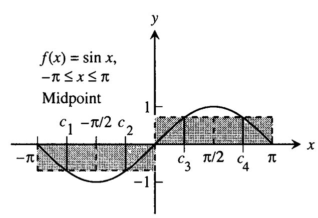
31. a)



b)



c)



33. 1.2 **35.** $\int_0^2 x^2\,dx$ **37.** $\int_{-7}^{5} (x^2 - 3x)\,dx$

39. $\int_2^3 \frac{1}{1-x}\,dx$ **41.** $\int_{-\pi/4}^{0} \sec x\,dx$ **43.** 15 **45.** -480

47. 2.75 **49.** Area $= 21$ square units

51. Area $= 9\pi/2$ square units **53.** Area $= 2.5$ square units

55. Area $= 3$ square units **57.** $b^2/2$ **59.** $b^2 - a^2$ **61.** $1/2$

63. $3\pi^2/2$ **65.** 7/3 **67.** 1/24 **69.** $3a^2/2$ **71.** $b/3$

73. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 3x^2\,dx = b^3$$

75. Using n subintervals of length $\Delta x = b/n$ and right-endpoint values:

$$\text{Area} = \int_0^b 2x\,dx = b^2$$

77. $a = 0$ and $b = 1$ maximize the integral. **81.** $b^3/3$

Section 4.6, pp. 330–332

1. a) 0 b) -8 c) -12 d) 10 e) -2 f) 16 **3.** a) 5 b) $5\sqrt{3}$ c) -5 d) -5 **5.** a) 4 b) -4 **7.** -14 **9.** 10

11. -2 **13.** $-7/4$ **15.** 7 **17.** 0 **19.** 5 1/3 **21.** 19/3

23. a) 6 b) $7\frac{1}{3}$ **25.** a) 0 b) 8/3

27. $\text{av}(f) = 0$, assumed at $x = 1$

29. $\text{av}(f) = -2$, assumed at $x = \sqrt{3}/3$

31. $\text{av}(f) = 1$, assumed at $t = 0$ and $t = 2$

33. a) $\text{av}(g) = -1/2$, assumed at $x = \pm 1/2$ b) $\text{av}(g) = 1$, assumed at $x = 2$ c) $\text{av}(g) = 1/4$, assumed at $x = 5/4$

35. 3/2 **37.** 0 **39.** Upper bound $= 1$, lower bound $= 1/2$

47. Upper bound $= 1/2$ **51.** 37.5 mi/hr

Section 4.7, pp. 338–342

1. 6 **3.** 8 **5.** 1 **7.** 5/2 **9.** 2 **11.** $2\sqrt{3}$ **13.** 0

15. $-\pi/4$ **17.** $\frac{2\pi^3}{3}$ **19.** $-8/3$ **21.** $-3/4$

23. $\sqrt{2} - \sqrt[4]{8} + 1$ **25.** 16 **27.** 0 **29.** $\frac{1}{3}(2\sqrt{2} - 1)$

31. $\frac{\pi}{2} + \sin 2$ **33.** $\sqrt{2}/3$ **35.** 28/3 **37.** 1/2 **39.** 51/4

41. π **43.** $\frac{\sqrt{2}\,\pi}{2}$ **45.** $(\cos\sqrt{x})\left(\frac{1}{2\sqrt{x}}\right)$ **47.** $4t^5$

49. $\sqrt{1 + x^2}$ **51.** $\frac{1}{2}x^{-1/2}\sin x$ **53.** 1

55. d, since $y' = \frac{1}{x}$ and $y(\pi) = \int_\pi^\pi \frac{1}{t}\,dt - 3 = -3$

57. b, since $y' = \sec x$ and $y(0) = \int_0^0 \sec t\,dt + 4 = 4$

59. $y = \int_2^x \sec t\,dt + 3$ **61.** $s = \int_{t_0}^{t} f(x)\,dx + s_0$

63. a) 125/6 b) $h = 25/4$ d) $\frac{2}{3}bh$ **65.** a) \$9.00 b) \$10.00

67. a) $v = \frac{ds}{dt} = \frac{d}{dt}\int_0^t f(x)\,dx = f(t) \Rightarrow v(5) = f(5) = 2$ m/sec

b) $a = df/dt$ is negative since the slope of the tangent line at $t = 5$ is negative.

c) $s = \int_0^3 f(x)\,dx = \frac{1}{2}(3)(3) = \frac{9}{2}$ m since the integral is the area of the triangle formed by $y = f(x)$, the x-axis, and $x = 3$.
d) $t = 6$ since after $t = 6$ to $t = 9$, the region lies below the x-axis.
e) At $t = 4$ and $t = 7$, since there are horizontal tangents there.
f) Toward the origin between $t = 6$ and $t = 9$ since the velocity is negative on this interval. Away from the origin between $t = 0$ and $t = 6$ since the velocity is positive there.
g) Right or positive side, because the integral of f from 0 to 9 is positive, there being more area above the x-axis than below.

69. $\int_{-2}^{2} 4(9 - x^2)\,dx = 368/3$ **71.** $\int_4^8 \pi(64 - x^2)\,dx = 320\pi/3$
75. $2x - 2$ **77.** $-3x + 5$
79. a) True. Since f is continuous, g is differentiable by Part 1 of the Fundamental Theorem of Calculus.
b) True: g is continuous because it is differentiable.
c) True, since $g'(1) = f(1) = 0$.
d) False, since $g''(1) = f'(1) > 0$.
e) True, since $g'(1) = 0$ and $g''(1) = f'(1) > 0$.
f) False: $g''(x) = f'(x) > 0$, so g'' never changes sign.
g) True, since $g'(1) = f(1) = 0$ and $g'(x) = f(x)$ is an increasing function of x (because $f'(x) > 0$).

Section 4.8, pp. 344–345

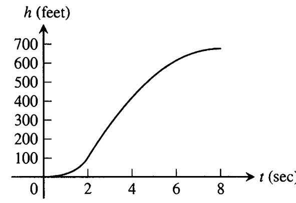
1. a) 14/3 b) 2/3 **3.** a) 1/2 b) −1/2 **5.** a) 15/16 b) 0 **7.** a) 0 b) 1/8 **9.** a) 4 b) 0 **11.** a) 1/6 b) 1/2 **13.** a) 0 b) 0 **15.** $2\sqrt{3}$ **17.** 3/4 **19.** $9^{5/4} - 1$ **21.** 3 **23.** $\pi/3$ **25.** 16/3 **27.** $2^{5/2}$ **29.** $F(6) - F(2)$ **31.** a) −3 b) 3 **33.** $I = a/2$

Section 4.9, pp. 353–356

1. I: a) 1.5, 0 b) 1.5, 0 c) 0%
II: a) 1.5, 0 b) 1.5, 0 c) 0%
3. I: a) 2.75, 0.08 b) 2.67, 0.08 c) $0.0312 \approx 3\%$
II: a) 2.67, 0 b) 2.67, 0 c) 0%
5. I: a) 6.25, 0.5 b) 6, 0.25 c) $0.0417 \approx 4\%$
II: a) 6, 0 b) 6, 0 c) 0%
7. I: a) 0.509, 0.03125 b) 0.5, 0.009 c) $0.018 \approx 2\%$
II: a) 0.5, 0.002604 b) 0.5, 0.0004 c) 0%
9. I: a) 1.8961, 0.161 b) 2, 0.1039 c) $0.052 \approx 5\%$
II: a) 2.0045, 0.0066 b) 2, 0.00454 c) 0%
11. a) 0.31929 b) 0.32812 c) 1/3, 0.01404, 0.00521
13. a) 1.95643 b) 2.00421 c) 2, 0.04357, −0.00421 **15.** a) 1 b) 2 **17.** a) 116 b) 2 **19.** a) 283 b) 2 **21.** a) 71 b) 10 **23.** a) 76 b) 12 **25.** a) 82 b) 8 **27.** 1013
29. ≈ 466.7 in^2 **31.** 4, 4 **33.** a) 3.11571 b) 0.02588
c) With $M = 3.11$, we get $|E_T| \le (\pi^3/1200)(3.11) < 0.081$
37. 1.08943 **39.** 0.82812

Chapter 4 Practice Exercises, pp. 357–360

1. a) about 680 ft
b)

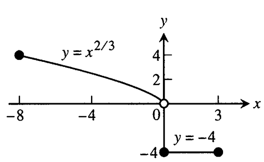


3. a) −1/2 b) 31 c) 13 d) 0
5. $\int_1^5 (2x - 1)^{-1/2}\,dx = 2$
7. $\int_{-\pi}^{0} \cos \frac{x}{2}\,dx = 2$

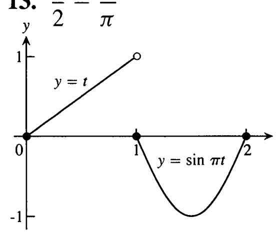
9. a) 4 b) 2 c) −2 d) -2π e) 8/5
11. 8/3 **13.** 62 **15.** $y = x - \frac{1}{x} - 1$
17. $r = 4t^{5/2} + 4t^{3/2} - 8t$ **21.** $y = \int_5^x \left(\frac{\sin t}{t}\right) dt - 3$
23. $\frac{x^4}{4} + \frac{5}{2}x^2 - 7x + C$ **25.** $2t^{3/2} - \frac{4}{t} + C$
27. $-\frac{1}{2(r^2 + 5)} + C$ **29.** $-(2 - \theta^2)^{3/2} + C$
31. $\frac{1}{3}(1 + x^4)^{3/4} + C$ **33.** $10 \tan \frac{s}{10} + C$
35. $-\frac{1}{\sqrt{2}} \csc \sqrt{2}\theta + C$ **37.** $\frac{1}{2}x - \sin \frac{x}{2} + C$
39. $-4(\cos x)^{1/2} + C$ **41.** $\theta^2 + \theta + \sin(2\theta + 1) + C$
43. $\frac{t^3}{3} + \frac{4}{t} + C$ **45.** 16 **47.** 2 **49.** 1 **51.** 8
53. $27\sqrt{3}/160$ **55.** $\pi/2$ **57.** $\sqrt{3}$ **59.** $6\sqrt{3} - 2\pi$ **61.** −1
63. 2 **65.** −2 **67.** 1 **69.** $\sqrt{2} - 1$ **71.** a) b b) b
75. At least 16 **77.** $T = \pi, S = \pi$ **79.** 25°F **81.** Yes
83. $-\sqrt{1 + x^2}$ **85.** cost $\approx$ \$12, 518.10 (trapezoidal rule), no
87. 600, \$18.00 **89.** 300, \$6.00

Chapter 4 Additional Exercises, pp. 360–364

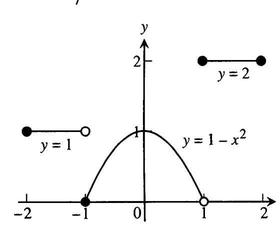
1. a) Yes b) No **5.** a) 1/4 b) $\sqrt[3]{12}$ **7.** $f(x) = \frac{x}{\sqrt{x^2 + 1}}$
9. $y = x^3 + 2x - 4$
11. 36/5



13. $\frac{1}{2} - \frac{2}{\pi}$



15. 13/3



17. 1/2 **19.** $2/x$ **21.** $\frac{\sin 4y}{\sqrt{y}} - \frac{\sin y}{2\sqrt{y}}$ **23.** 1/6

25. $\int_0^1 f(x)\,dx$

CHAPTER 5

Section 5.1, pp. 371–373

1. $\pi/2$ **3.** 1/12 **5.** 128/15 **7.** 5/6 **9.** 38/3 **11.** 49/6 **13.** 32/3 **15.** 48/5 **17.** 8/3 **19.** 8 **21.** 5/3 (There are three intersection points.) **23.** 18 **25.** 243/8 **27.** 8/3 **29.** 2 **31.** 104/15 **33.** 56/15 **35.** 4 **37.** $\frac{4}{3} - \frac{4}{\pi}$ **39.** $\pi/2$ **41.** 2 **43.** 1/2 **45.** 1 **47.** a) $(\pm\sqrt{c}, c)$ b) $c = 4^{2/3}$ c) $c = 4^{2/3}$ **49.** 11/3 **51.** 3/4 **53.** Neither

Section 5.2, pp. 377–378

1. a) $A(x) = \pi(1 - x^2)$ b) $A(x) = 4(1 - x^2)$ c) $A(x) = 2(1 - x^2)$ d) $A(x) = \sqrt{3}(1 - x^2)$ **3.** 16 **5.** 16/3 **7.** a) $2\sqrt{3}$ b) 8 **9.** 8π **11.** a) s^2h b) s^2h

Section 5.3, pp. 385–387

1. $2\pi/3$ **3.** $4 - \pi$ **5.** $32\pi/5$ **7.** 36π **9.** π **11.** $\pi\left(\frac{\pi}{2} + 2\sqrt{2} - \frac{11}{3}\right)$ **13.** 2π **15.** 2π **17.** 3π **19.** $\pi^2 - 2\pi$ **21.** $\frac{2\pi}{3}$ **23.** 2π **25.** $117\pi/5$ **27.** $\pi(\pi - 2)$ **29.** $4\pi/3$ **31.** 8π **33.** $\pi(\sqrt{3})$ **35.** $7\pi/6$ **37.** a) 8π b) $32\pi/5$ c) $8\pi/3$ d) $224\pi/15$ **39.** a) $16\pi/15$ b) $56\pi/15$ c) $64\pi/15$ **41.** $V = 1053\pi$ cm^3 **43.** a) $c = 2/\pi$ b) $c = 0$ **45.** $V = 2a^2b\pi^2$ **47.** b) $V = \frac{\pi r^2 h}{3}$

Section 5.4, pp. 392–393

1. 6π **3.** 2π **5.** $14\pi/3$ **7.** 8π **9.** $5\pi/6$ **11.** $128\pi/5$ **13.** 3π **15.** $\frac{16\pi}{15}(3\sqrt{2} + 5)$ **17.** $8\pi/3$ **19.** $4\pi/3$ **21.** $16\pi/3$ **23.** a) $6\pi/5$ b) $4\pi/5$ c) 2π d) 2π **25.** a) $5\pi/3$ b) $4\pi/3$ c) 2π d) $2\pi/3$ **27.** a) $11\pi/15$ b) $97\pi/105$ c) $121\pi/210$ d) $23\pi/30$ **29.** a) $512\pi/21$ b) $832\pi/21$ **31.** a) $\pi/6$ b) $\pi/6$ **33.** $9\pi/16$ **35.** b) 4π **37.** Disk: 2 integrals; washer: 2 integrals; shell: 1 integral **39.** $3x$

Section 5.5, pp. 398–400

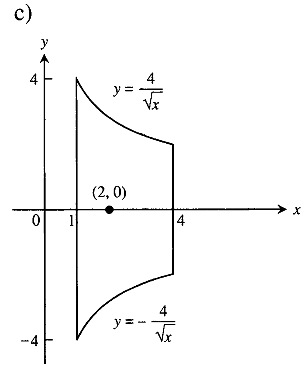
1. a) $\int_{-1}^{2} \sqrt{1 + 4x^2}\,dx$ c) ≈ 6.13 **3.** a) $\int_0^{\pi} \sqrt{1 + \cos^2 y}\,dy$ c) ≈ 3.82 **5.** a) $\int_{-1}^{3} \sqrt{1 + (y+1)^2}\,dy$ c) ≈ 9.29 **7.** a) $\int_0^{\pi/6} \sec x\,dx$ c) ≈ 0.55 **9.** 12 **11.** 53/6 **13.** 123/32 **15.** 99/8 **17.** 2 **19.** a) $y = \sqrt{x}$ or $y = -\sqrt{x} + 2$ b) Two **21.** 1 **23.** 21.07 in.

Section 5.6, pp. 405–407

1. a) $2\pi \int_0^{\pi/4} \tan x \sqrt{1 + \sec^4 x}\,dx$ c) ≈ 3.84 **3.** a) $2\pi \int_1^2 \frac{1}{y}\sqrt{1 + y^{-4}}\,dy$ c) ≈ 5.02 **5.** a) $2\pi \int_1^4 (3 - \sqrt{x})^2 \sqrt{1 + (1 - 3x^{-1/2})^2}\,dx$ c) ≈ 63.37 **7.** a) $2\pi \int_0^{\pi/3} \left(\int_0^y \tan t\,dt\right) \sec y\,dy$ c) ≈ 2.08 **9.** $4\pi\sqrt{5}$ **11.** $3\pi\sqrt{5}$ **13.** $98\pi/81$ **15.** 2π **17.** $\pi(\sqrt{8} - 1)/9$ **19.** $35\pi\sqrt{5}/3$ **21.** $253\pi/20$ **25.** a) $2\pi \int_{-\pi/2}^{\pi/2} (\cos x)\sqrt{1 + \sin^2 x}\,dx$ b) ≈ 14.4236 **27.** Order 226.2 liters of each color. **31.** $5\sqrt{2}\pi$ **33.** 14.4 **35.** 54.9

Section 5.7, pp. 416–418

1. 4 ft **3.** $(L/4, L/4)$ **5.** $M_0 = 8, M = 8, \bar{x} = 1$ **7.** $M_0 = 15/2, M = 9/2, \bar{x} = 5/3$ **9.** $M_0 = 73/6, M = 5, \bar{x} = 73/30$ **11.** $M_0 = 3, M = 3, \bar{x} = 1$ **13.** $\bar{x} = 0, \bar{y} = 12/5$ **15.** $\bar{x} = 1, \bar{y} = -3/5$ **17.** $\bar{x} = 16/105, \bar{y} = 8/15$ **19.** $\bar{x} = 0, \bar{y} = \pi/8$ **21.** $\bar{x} = 1, \bar{y} = -2/5$ **23.** $\bar{x} = \bar{y} = \frac{2}{4 - \pi}$ **25.** $\bar{x} = 3/2, \bar{y} = 1/2$ **27.** a) $\frac{224\pi}{3}$ b) $\bar{x} = 2, \bar{y} = 0$
c)



31. $\bar{x} = \bar{y} = 1/3$ **33.** $\bar{x} = a/3,\ \bar{y} = b/3$ **35.** $13\delta/6$
37. $\bar{x} = 0,\ \bar{y} = \dfrac{a\pi}{4}$

Section 5.8, pp. 424–427

1. 400 ft • lb **3.** 780 J **5.** 72,900 ft • lb **9.** 400 N/m
11. 4 cm, 0.08 J **13.** a) 7238 lb/in.
b) 905 in • lb, 2714 in • lb
15. a) 1,497,600 ft • lb b) 1 hr, 40 min
d) At 62.26 lb/ft^3: a) 1,494,240 ft • lb b) 1 hr, 40 min
At 62.59 lb/ft^3: a) 1,502,160 ft • lb b) 1 hr, 40 min
17. 38,484,510 J **19.** 7,238,229.48 ft • lb **21.** 91.32 in • oz
23. 21,446,605.9 J **25.** 967,611 ft • lb, at a cost of \$4838.05
27. 5.144×10^{10} J **31.** ≈ 85.1 ft • lb **33.** ≈ 64.6 ft • lb
35. ≈ 110.6 ft • lb

Section 5.9, pp. 432–434

1. 114,511,052 lb, 28,627,763 lb **5.** 2808 lb **7.** a) 1164.8 lb
b) 1194.7 lb **9.** a) 374.4 lb b) 7.5 in. c) No **11.** 1309 lb
13. 4.2 lb **15.** 41.6 lb **17.** a) 93.33 lb b) 3 ft **19.** 1035 ft^3
21. $wb/2$
23. No. The tank will overflow because the movable end will have moved only $3\frac{1}{3}$ ft by the time the tank is full.

Section 5.10, pp. 441–443

1. b) 20 m c) 0 m **3.** b) 6 m c) 2 m **5.** b) 245 m
c) 0 m **7.** b) 6 m c) 4 m **9.** b) $2 < t < 4$ c) 6 m
d) $\dfrac{22}{3}$ m **11.** a) Total distance = 7, displacement = 3
b) Total distance = 19.5, displacement = −4.5 **13.** About 65%
15. $\sqrt{3}\pi$ **17.** a) 210 ft^3 b) 13,440 lb
19. $V = 32\pi, S = 32\sqrt{2}\pi$ **21.** $4\pi^2$ **23.** $\bar{x} = 0, \bar{y} = \dfrac{2a}{\pi}$
25. $\bar{x} = 0, \bar{y} = \dfrac{4b}{3\pi}$ **27.** $\sqrt{2}\pi a^3(4 + 3\pi)/6$ **29.** $\dfrac{2a^3}{3}$

Chapter 5 Practice Exercises, pp. 444–447

1. 1 **3.** 1/6 **5.** 18 **7.** 9/8 **9.** $\dfrac{\pi^2}{32} + \dfrac{\sqrt{2}}{2} - 1$ **11.** 4
13. $\dfrac{8\sqrt{2} - 7}{6}$ **15.** Min: −4, max: 0, area: 27/4 **17.** 6/5
19. $9\pi/280$ **21.** π^2 **23.** $72\pi/35$ **25.** a) 2π b) π
c) $12\pi/5$ d) $26\pi/5$ **27.** a) 8π b) $1088\pi/15$ c) $512\pi/15$
29. $\pi(3\sqrt{3} - \pi)/3$ **31.** a) $16\pi/15$ b) $8\pi/5$ c) $8\pi/3$
d) $32\pi/5$ **33.** $28\pi/3$ **35.** 10/3 **37.** 285/8
39. $28\pi\sqrt{2}/3$ **41.** 4π **43.** $\bar{x} = 0, \bar{y} = 8/5$
45. $\bar{x} = 3/2, \bar{y} = 12/5$ **47.** $\bar{x} = 9/5, \bar{y} = 11/10$ **49.** 4640 J
51. 10 ft • lb, 30 ft • lb **53.** 418,208.81 ft • lb
55. $22,500\pi$ ft • lb, 257 sec **57.** 332.8 lb **59.** 2196.48 lb
61. $216w_1 + 360w_2$ **63.** a) 64/3 m b) 0 m
65. a) 15 m b) −5 m

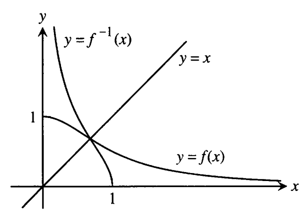
Chapter 5 Additional Exercises, pp. 447–448

1. $f(x) = \sqrt{\dfrac{2x - a}{\pi}}$ **3.** $f(x) = \sqrt{C^2 - 1}\,x + a$, where $C \geq 1$
5. $\bar{x} = 0, \bar{y} = \dfrac{n}{2n + 1}$, (0, 1/2)
9. a) $\bar{x} = \bar{y} = 4(a^2 + ab + b^2)/(3\pi(a + b))$ b) $(2a/\pi, 2a/\pi)$
11. 28/3 **13.** $\dfrac{4h\sqrt{3mh}}{3}$ **15.** $\approx 2{,}329.6$ lb
17. a) $2h/3$ b) $(6a^2 + 8ah + 3h^2)/(6a + 4h)$

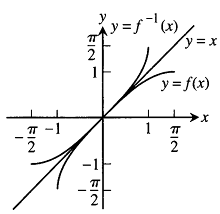
CHAPTER 6

Section 6.1, pp. 454–457

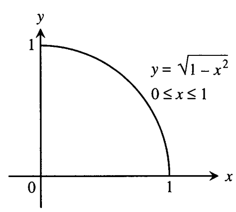
1. One-to-one **3.** Not one-to-one **5.** One-to-one
7. D: (0, 1] R: [0, ∞)



9. D: [−1, 1] R: [−π/2, π/2]

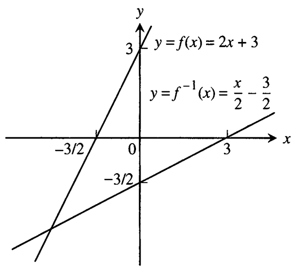


11. a) Symmetric about the line $y = x$



13. $f^{-1}(x) = \sqrt{x - 1}$ **15.** $f^{-1}(x) = \sqrt[3]{x + 1}$
17. $f^{-1}(x) = \sqrt{x} - 1$
19. $f^{-1}(x) = \sqrt[5]{x}$; domain: $-\infty < x < \infty$, range: $-\infty < y < \infty$
21. $f^{-1}(x) = \sqrt[3]{x - 1}$; domain: $-\infty < x < \infty$, range: $-\infty < y < \infty$
23. $f^{-1}(x) = \dfrac{1}{\sqrt{x}}$; domain: $x > 0$, range: $y > 0$
25. a) $f^{-1}(x) = \dfrac{x}{2} - \dfrac{3}{2}$

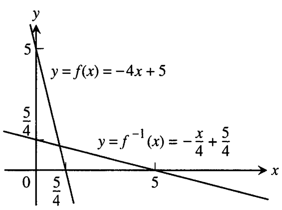
b)



c) 2, 1/2

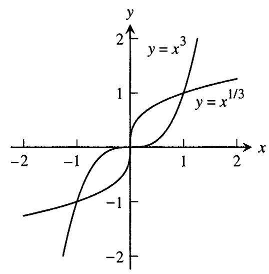
27. a) $f^{-1}(x) = -\frac{x}{4} + \frac{5}{4}$

b)



c) $-4, -1/4$

29. b)



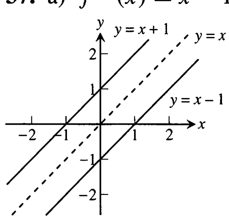
c) Slope of f at (1, 1) : 3, slope of g at (1, 1): 1/3, slope of f at $(-1, -1)$: 3, slope of g at $(-1, -1)$: 1/3

d) $y = 0$ is tangent to $y = x^3$ at $x = 0$; $x = 0$ is tangent to $y = \sqrt[3]{x}$ at $x = 0$

31. 1/9 **33.** 3 **35.** a) $f^{-1}(x) = \frac{1}{m}x$

b) The graph of f^{-1} is the line through the origin with slope $1/m$.

37. a) $f^{-1}(x) = x - 1$



b) $f^{-1}(x) = x - b$. The graph of f^{-1} is a line parallel to the graph of f. The graphs of f and f^{-1} lie on opposite sides of the line $y = x$ and are equidistant from that line.

c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.

41. Increasing, therefore one-to-one; $df^{-1}/dx = \frac{1}{9}x^{-2/3}$

43. Decreasing, therefore one-to-one; $df^{-1}/dx = -\frac{1}{3}x^{-2/3}$

Section 6.2, pp. 465–467

1. a) $\ln 3 - 2\ln 2$ b) $2(\ln 2 - \ln 3)$ c) $-\ln 2$, d) $\frac{2}{3}\ln 3$

e) $\ln 3 + \frac{1}{2}\ln 2$ f) $\frac{1}{2}(3\ln 3 - \ln 2)$ **3.** a) $\ln 5$ b) $\ln(x-3)$

c) $\ln(t^2)$ **5.** $1/x$ **7.** $2/t$ **9.** $-1/x$ **11.** $\frac{1}{\theta + 1}$

13. $3/x$ **15.** $2(\ln t) + (\ln t)^2$ **17.** $x^3 \ln x$ **19.** $\frac{1 - \ln t}{t^2}$

21. $\frac{1}{x(1 + \ln x)^2}$ **23.** $\frac{1}{x \ln x}$ **25.** $2\cos(\ln\theta)$

27. $-\frac{3x+2}{2x(x+1)}$ **29.** $\frac{2}{t(1-\ln t)^2}$ **31.** $\frac{\tan(\ln\theta)}{\theta}$

33. $\frac{10x}{x^2+1} + \frac{1}{2(1-x)}$ **35.** $2x\ln|x| - x\ln\frac{|x|}{\sqrt{2}}$

37. $\left(\frac{1}{2}\right)\sqrt{x(x+1)}\left(\frac{1}{x} + \frac{1}{x+1}\right) = \frac{2x+1}{2\sqrt{x(x+1)}}$

39. $\left(\frac{1}{2}\right)\sqrt{\frac{t}{t+1}}\left(\frac{1}{t} - \frac{1}{t+1}\right) = \frac{1}{2\sqrt{t}(t+1)^{3/2}}$

41. $\sqrt{\theta+3}\,(\sin\theta)\left(\frac{1}{2(\theta+3)} + \cot\theta\right)$

43. $t(t+1)(t+2)\left[\frac{1}{t} + \frac{1}{t+1} + \frac{1}{t+2}\right] = 3t^2 + 6t + 2$

45. $\frac{\theta+5}{\theta\cos\theta}\left[\frac{1}{\theta+5} - \frac{1}{\theta} + \tan\theta\right]$

47. $\frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}\left[\frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3(x+1)}\right]$

49. $\frac{1}{3}\sqrt[3]{\frac{x(x-2)}{x^2+1}}\left(\frac{1}{x} + \frac{1}{x-2} - \frac{2x}{x^2+1}\right)$ **51.** $\ln\left(\frac{2}{3}\right)$

53. $\ln|y^2 - 25| + C$ **55.** $\ln 3$ **57.** $(\ln 2)^2$ **59.** $\frac{1}{\ln 4}$

61. $\ln|6 + 3\tan t| + C$ **63.** $\ln 2$ **65.** $\ln 27$

67. $\ln(1 + \sqrt{x}) + C$

69. a) Max $= 0$ at $x = 0$, min $= -\ln 2$ at $x = \pi/3$

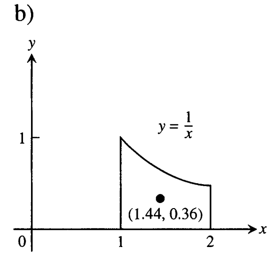
b) Max $= 1$ at $x = 1$, min $= \cos(\ln 2)$ at $x = 1/2$ and $x = 2$

71. $\ln 16$ **73.** $4\pi \ln 4$ **75.** $\pi \ln 16$

77. a) $6 + \ln 2$ b) $8 + \ln 9$

79. a) $\bar{x} \approx 1.44, \bar{y} \approx 0.36$

b)



81. $y = x + \ln|x| + 2$ **83.** b) 0.00469 **85.** 2

Section 6.3, pp. 472–474

1. a) 7.2 b) $\frac{1}{x^2}$ c) $\frac{x}{y}$ **3.** a) 1 b) 1 c) $-x^2 - y^2$

5. e^{2t+4} **7.** $e^{5t}+40$ **9.** $y=2xe^x+1$ **11.** a) $k=\ln 2$ b) $k=(1/10)\ln 2$ c) $k=1000\ln a$ **13.** a) $t=-10\ln 3$ b) $t=-\dfrac{\ln 2}{k}$ c) $t=\dfrac{\ln .4}{\ln .2}$ **15.** $4(\ln x)^2$ **17.** $-5e^{-5x}$
19. $-7e^{(5-7x)}$ **21.** xe^x **23.** x^2e^x **25.** $2e^{\theta}\cos\theta$
27. $2\theta e^{-\theta^2}\sin(e^{-\theta^2})$ **29.** $\dfrac{1-t}{t}$ **31.** $1/(1+e^{\theta})$
33. $e^{\cos t}(1-t\sin t)$ **35.** $(\sin x)/x$ **37.** $\dfrac{ye^y\cos x}{1-ye^y\sin x}$
39. $\dfrac{2e^{2x}-\cos(x+3y)}{3\cos(x+3y)}$ **41.** $\frac{1}{3}e^{3x}-5e^{-x}+C$ **43.** 1
45. $8e^{(x+1)}+C$ **47.** 2 **49.** $2e^{\sqrt{r}}+C$ **51.** $-e^{-t^2}+C$
53. $-e^{1/x}+C$ **55.** e **57.** $\dfrac{1}{\pi}e^{\sec \pi t}+C$ **59.** 1
61. $\ln(1+e^r)+C$ **63.** $y=1-\cos(e^t-2)$
65. $y=2(e^{-x}+x)-1$
67. Maximum: 1 at $x=0$, minimum: $2-2\ln 2$ at $x=\ln 2$
69. Abs max of $1/(2e)$ assumed at $x=1/\sqrt{e}$ **71.** 2
73. $y=e^{x/2}-1$
75. a) $\dfrac{d}{dx}(x\ln x-x+C)=x\cdot\dfrac{1}{x}+\ln x-1+0=\ln x$
b) $\dfrac{1}{e-1}$ **77.** b) $|\text{error}|\approx 0.02140$ **79.** 2.71828183

Section 6.4, pp. 480–482

1. a) 7 b) $\sqrt{2}$ c) 75 d) 2 e) 0.5 f) -1 **3.** a) $\sqrt{x}$ b) x^2 c) $\sin x$ **5.** a) $\dfrac{\ln 3}{\ln 2}$ b) 3 c) 2 **7.** $x=12$
9. $x=3$ or $x=2$ **11.** $2^x\ln x$ **13.** $\left(\dfrac{\ln 5}{2\sqrt{s}}\right)5^{\sqrt{s}}$ **15.** $\pi x^{(\pi-1)}$
17. $-\sqrt{2}\cos\theta^{(\sqrt{2}-1)}\sin\theta$ **19.** $7^{\sec\theta}(\ln 7)^2(\sec\theta\tan\theta)$
21. $(3\cos 3t)(2^{\sin 3t})\ln 2$ **23.** $\dfrac{1}{\theta\ln 2}$ **25.** $\dfrac{3}{x\ln 4}$
27. $\dfrac{2(\ln r)}{r(\ln 2)(\ln 4)}$ **29.** $\dfrac{-2}{(x+1)(x-1)}$
31. $\sin(\log_7\theta)+\dfrac{1}{\ln 7}\cos(\log_7\theta)$ **33.** $\dfrac{1}{\ln 5}$ **35.** $\dfrac{1}{t}(\log_2 3)3^{\log_2 t}$
37. $\dfrac{1}{t}$ **39.** $(x+1)^x\left(\dfrac{x}{x+1}+\ln(x+1)\right)$
41. $(\sqrt{t})^t\left(\dfrac{\ln t}{2}+\dfrac{1}{2}\right)$ **43.** $(\sin x)^x(\ln\sin x+x\cot x)$
45. $\left(x^{\ln x}\right)\left(\dfrac{\ln x^2}{x}\right)$ **47.** $\dfrac{5^x}{\ln 5}+C$ **49.** $\dfrac{1}{2\ln 2}$ **51.** $\dfrac{1}{\ln 2}$
53. $\dfrac{6}{\ln 7}$ **55.** 32760 **57.** $\dfrac{3x^{(\sqrt{3}+1)}}{\sqrt{3}+1}+C$ **59.** $3^{\sqrt{2}+1}$
61. $\dfrac{1}{\ln 10}\left(\dfrac{(\ln x)^2}{2}\right)+C$ **63.** $2(\ln 2)^2$ **65.** $\dfrac{3\ln 2}{2}$
67. $\ln 10$ **69.** $(\ln 10)\ln|\ln x|+C$ **71.** $\ln(\ln x)$, $x>1$
73. $-\ln x$ **75.** $2\ln 5$ **77.** $\left[10^{-7.44}, 10^{-7.37}\right]$ **79.** $k=10$
81. a) 10^{-7} b) 7 c) 1 : 1 **83.** $x\approx -0.76666$
85. a) $L(x)=1+(\ln 2)x\approx 0.69x+1$
87. a) 1.89279 b) -0.35621 c) 0.94575 d) -2.80735 e) 5.29595 f) 0.97041 g) -1.03972 h) -1.61181

Section 6.5, pp. 488–491

1. a) -0.00001 b) 10,536 years c) 82% **3.** 54.88 g
5. 59.8 ft **7.** 2.8147497×10^{14} **9.** a) 8 years b) 32.02 years
11. 15.28 years **13.** a) $A_0e^{0.2}$ b) 17.33 years c) 27.47 years
15. 4.50% **17.** 0.585 days **21.** a) 17.5 min. b) 13.26 min.
23. -3°C **25.** About 6658 years **27.** 41 years old

Section 6.6, pp. 496–498

1. 1/4 **3.** $-23/7$ **5.** 5/7 **7.** 0 **9.** -16 **11.** -2
13. 1/4 **15.** 2 **17.** 3 **19.** -1 **21.** $\ln 3$ **23.** $\dfrac{1}{\ln 2}$
25. $\ln 2$ **27.** 1 **29.** 1/2 **31.** $\ln 2$ **33.** 0 **35.** $-1/2$
37. $\ln 2$ **39.** -1 **41.** 1 **43.** $1/e$ **45.** 1 **47.** $1/e$
49. $e^{1/2}$ **51.** 1 **53.** 3 **55.** 1 **57.** (b) is correct.
59. (d) is correct. **61.** $c=\dfrac{27}{10}$

Section 6.7, pp. 503–504

1. a) Slower b) slower c) slower d) faster e) slower f) slower g) same h) slower
3. a) Same b) faster c) same d) same e) slower f) faster g) slower h) same
5. a) Same b) same c) same d) faster e) faster f) same g) slower h) faster **7.** d, a, c, b
9. a) False b) false c) true d) true e) true f) true g) false h) true
13. When the degree of f is less than or equal to the degree of g.
15. Polynomials of a greater degree grow at a greater rate than polynomials of a lesser degree. Polynomials of the same degree grow at the same rate.
21. b) $\ln\left(e^{17000000}\right)=17{,}000{,}000<\left(e^{17\times 10^6}\right)^{1/10^6}=e^{17}\approx 24{,}154{,}952.75$
c) $x\approx 3.4306311\times 10^{15}$ d) They cross at $x\approx 3.4306311\times 10^{15}$
23. a) The algorithm that takes O ($n\log_2 n$) steps
25. It could take one million for a sequential search; at most 20 steps for a binary search.

Section 6.8, pp. 510–513

1. a) $\pi/4$ b) $-\pi/3$ c) $\pi/6$ **3.** a) $-\pi/6$ b) $\pi/4$ c) $-\pi/3$
5. a) $\pi/3$ b) $3\pi/4$ c) $\pi/6$ **7.** a) $3\pi/4$ b) $\pi/6$ c) $2\pi/3$
9. a) $\pi/4$ b) $-\pi/3$ c) $\pi/6$ **11.** a) $3\pi/4$ b) $\pi/6$ c) $2\pi/3$
13. $\cos\alpha=\dfrac{12}{13}$, $\tan\alpha=\dfrac{5}{12}$, $\sec\alpha=\dfrac{13}{12}$, $\csc\alpha=\dfrac{13}{5}$, $\cot\alpha=\dfrac{12}{5}$

15. $\sin\alpha = \frac{2}{\sqrt{5}}$, $\cos\alpha = -\frac{1}{\sqrt{5}}$, $\tan\alpha = -2$, $\csc\alpha = \frac{\sqrt{5}}{2}$, $\cot\alpha = -\frac{1}{2}$

17. $1/\sqrt{2}$ **19.** $-1/\sqrt{3}$ **21.** $\frac{4+\sqrt{3}}{2\sqrt{3}}$ **23.** 1 **25.** $-\sqrt{2}$

27. $\pi/6$ **29.** $\frac{\sqrt{x^2+4}}{2}$ **31.** $\sqrt{9y^2-1}$ **33.** $\sqrt{1-x^2}$

35. $\frac{\sqrt{x^2-2x}}{x-1}$ **37.** $\frac{\sqrt{9-4y^2}}{3}$ **39.** $\frac{\sqrt{x^2-16}}{x}$ **41.** $\pi/2$

43. $\pi/2$ **45.** $\pi/2$ **47.** 0 **51.** $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 54.7°$

57. a) Defined; there is an angle whose tangent is 2.
b) Not defined; there is no angle whose cosine is 2.
59. a) Not defined; no angle has secant 0.
b) Not defined; no angle has sine $\sqrt{2}$.
61. a) 0.84107 b) −0.72973 c) 0.46365
63. a) Domain; all real numbers except those having the form $\frac{\pi}{2} + k\pi$ where k is an integer; range: $-\pi/2 < y < \pi/2$.
b) Domain: $-\infty < x < \infty$; range: $-\infty < y < \infty$
65. a) Domain: $-\infty < x < \infty$; range: $0 \le y \le \pi$
b) Domain: $-1 \le x \le 1$; range: $-1 \le y \le 1$
67. The graphs are identical.

Section 6.9, pp. 518–520

1. $\frac{-2x}{\sqrt{1-x^4}}$ **3.** $\frac{\sqrt{2}}{\sqrt{1-2t^2}}$ **5.** $\frac{1}{|2s+1|\sqrt{s^2+s}}$

7. $\frac{-2x}{(x^2+1)\sqrt{x^4+2x^2}}$ **9.** $\frac{-1}{\sqrt{1-t^2}}$ **11.** $\frac{-1}{2\sqrt{t}(1+t)}$

13. $\frac{1}{\tan^{-1}(x(1+x^2))}$ **15.** $\frac{-e^t}{|e^t|\sqrt{(e^t)^2-1}} = \frac{-1}{\sqrt{e^{2t}-1}}$

17. $\frac{-2s^2}{\sqrt{1-s^2}}$ **19.** 0 **21.** $\sin^{-1}x$ **23.** $\sin^{-1}\frac{x}{7} + C$

25. $\frac{1}{\sqrt{17}}\tan^{-1}\frac{x}{\sqrt{17}} + C$ **27.** $\frac{1}{\sqrt{2}}\sec^{-1}\left|\frac{5x}{\sqrt{2}}\right| + C$ **29.** $2\pi/3$

31. $\pi/16$ **33.** $-\pi/12$ **35.** $\frac{3}{2}\sin^{-1}2(r-1) + C$

37. $\frac{\sqrt{2}}{2}\tan^{-1}\left(\frac{x-1}{\sqrt{2}}\right) + C$ **39.** $\frac{1}{4}\sec^{-1}\left|\frac{2x-1}{2}\right| + C$

41. π **43.** $\pi/12$ **45.** $\frac{1}{2}\sin^{-1}y^2 + C$ **47.** $\sin^{-1}(x-2) + C$

49. π **51.** $\frac{1}{2}\tan^{-1}\left(\frac{y-1}{2}\right) + C$ **53.** 2π

55. $\sec^{-1}|x+1| + C$ **57.** $e^{\sin^{-1}x} + C$ **59.** $\frac{1}{3}(\sin^{-1}x)^3 + C$

61. $\ln|\tan^{-1}y| + C$ **63.** $\sqrt{3}-1$ **65.** 5 **67.** 2

73. $y = \sin^{-1}(x)$ **75.** $y = \sec^{-1}(x) + \frac{2\pi}{3}, x > 1$ **77.** $3\sqrt{5}$ ft.

79. Yes, $\sin^{-1}(x)$ and $\cos^{-1}(x)$ differ by the constant $\pi/2$.
89. $\pi^2/2$ **91.** a) $\pi^2/2$ b) 2π

Section 6.10, pp. 525–529

1. $\cosh x = 5/4$, $\tanh x = -3/5$, $\coth x = -5/3$, $\operatorname{sech} x = 4/5$, $\operatorname{csch} x = -4/3$
3. $\sinh x = 8/15$, $\tanh x = 8/17$, $\coth x = 17/8$, $\operatorname{sech} x = 15/17$, $\operatorname{csch} x = 15/8$

5. $x + \frac{1}{x}$ **7.** e^{5x} **9.** e^{4x} **13.** $2\cosh\frac{x}{3}$

15. $\operatorname{sech}^2\sqrt{t} + \frac{\tanh\sqrt{t}}{\sqrt{t}}$ **17.** $\coth z$

19. $(\ln \operatorname{sech}\theta)(\operatorname{sech}\theta\tanh\theta)$ **21.** $\tanh^3 v$ **23.** 2

25. $\frac{1}{2\sqrt{x(1+x)}}$ **27.** $\frac{1}{1+\theta} - \tanh^{-1}\theta$ **29.** $\frac{1}{2\sqrt{t}} - \coth^{-1}\sqrt{t}$

31. $-\operatorname{sech}^{-1}x$ **33.** $\frac{\ln 2}{\sqrt{1+\left(\frac{1}{2}\right)^{2\theta}}}$ **35.** $|\sec x|$

41. $\frac{\cosh 2x}{2} + C$ **43.** $12\sinh\left(\frac{x}{2} - \ln 3\right) + C$

45. $7\ln\left|e^{x/7} + e^{-x/7}\right| + C$ **47.** $\tanh\left(x - \frac{1}{2}\right) + C$

49. $-2\operatorname{sech}\sqrt{t} + C$ **51.** $\ln\frac{5}{2}$ **53.** $\frac{3}{32} + \ln 2$ **55.** $e - e^{-1}$

57. 3/4 **59.** $\frac{3}{8} + \ln\sqrt{2}$ **61.** $\ln(2/3)$ **63.** $\frac{-\ln 3}{2}$ **65.** $\ln 3$

67. a) $\sinh^{-1}(\sqrt{3})$ b) $\ln(\sqrt{3}+2)$

69. a) $\coth^{-1}(2) - \coth^{-1}(5/4)$ b) $\left(\frac{1}{2}\right)\ln\left(\frac{1}{3}\right)$

71. a) $-\operatorname{sech}^{-1}\left(\frac{12}{13}\right) + \operatorname{sech}^{-1}\left(\frac{4}{5}\right)$

b) $-\ln\left(\frac{1+\sqrt{1-(12/13)^2}}{(12/13)}\right) + \ln\left(\frac{1+\sqrt{1-(4/5)^2}}{(4/5)}\right)$

$= -\ln\left(\frac{3}{2}\right) + \ln(2) = \ln(4/3)$

73. a) 0 b) 0

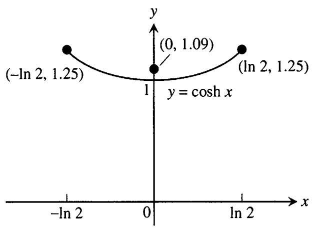
75. b) i) $f(x) = \frac{2f(x)}{2} + 0 = f(x)$, ii) $f(x) = 0 + \frac{2f(x)}{2} = f(x)$

77. b) $\sqrt{\frac{mg}{k}}$ c) $80\sqrt{5} \approx 178.89$ ft/sec

79. $y = \operatorname{sech}^{-1}(x) - \sqrt{1-x^2}$ **81.** 2π **83.** a) $\frac{6}{5}$ b) $\frac{\sinh ab}{a}$

85. a) $\bar{x} = 0$, $\bar{y} = \frac{5}{8} + \frac{\ln 4}{3} \approx 1.09$

b)



89. c) $a \approx 0.0417525$ d) ≈ 47.90 lb

Section 6.11, pp. 537–540

9. $y = \tan(x^2 + C)$ **11.** $\frac{2}{3}y^{3/2} - x^{1/2} = C$ **13.** $e^y - e^x = C$

15. $y = \frac{e^x + C}{x}$ **17.** $y = \frac{C - \cos x}{x^3}, x > 0$

19. $y = \frac{1}{2} - \frac{1}{x} + \frac{C}{x^2}, x > 0$ **21.** $y = \frac{1}{2}xe^{x/2} + Ce^{x/2}$

23. $y = x^2e^{-2x} + Ce^{-2x}$ **25.** $-e^{-y} - e^{\sin x} = C$

27. $s = \frac{t^3}{3(t-1)^4} - \frac{t}{(t-1)^4} + \frac{C}{(t-1)^4}$ **29.** $2\tan\sqrt{x} = t + C$

31. $r = \csc\theta\,(\ln|\sec\theta| + C)$ **33.** $y = -e^{-x}\operatorname{sech} x + C\operatorname{sech} x$

35. $y = \frac{3}{2} - \frac{1}{2}e^{-2t}$ **37.** $y = -\frac{1}{\theta}\cos\theta + \frac{\pi}{2\theta}$

39. $y = 6e^{x^2} - \frac{e^{x^2}}{x+1}$ **41.** $y = y_0e^{kt}$

43. (b) is correct, but (a) is not.

45. a) $c = \frac{G}{100Vk} + \left(c_0 - \frac{G}{100Vk}\right)e^{-kt}$ b) $\frac{G}{100Vk}$

47. 1 hour **49.** a) 550 ft b) $25\ln 22 \approx 77$ sec

51. $t = \frac{L}{R}\ln 2$ seconds

53. a) $i = \frac{V}{R} - \frac{V}{R}e^{-3} = \frac{V}{R}(1 - e^{-3}) \approx 0.95\frac{V}{R}$ amp b) 86%

55. a) 10 lb/min b) $100 + t$ gal c) $4\left(\frac{y}{100+t}\right)$ lb/min

d) $\frac{dy}{dt} = 10 - \frac{4y}{100+t},\ y(0) = 50,\ y = 2(100+t) - \frac{150}{\left(1 + \frac{t}{100}\right)^4}$

e) concentration $= \frac{y(25)}{\text{amt. brine in tank}} = \frac{188.6}{125} \approx 1.5$ lb/gal

57. $y(27.8) \approx 14.8$ lb, $t \approx 27.8$ min

Section 6.12, pp. 545–546

1. y (exact) $= \frac{x}{2} - \frac{4}{x},\ y_1 = -0.25,\ y_2 = 0.3,\ y_3 = 0.75$

3. y (exact) $= 3e^{x(x+2)},\ y_1 = 4.2,\ y_2 = 6.216,\ y_3 = 9.697$

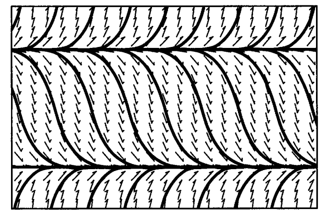
5. y (exact) $= e^{x^2} + 1,\ y_1 = 2.0,\ y_2 = 2.0202,\ y_3 = 2.0618$

7. $y \approx 2.48832$, exact value is e

9. $y \approx -0.2272$, exact value is $1/(1 - 2\sqrt{5}) \approx -0.2880$

11. b **13.** a

15.



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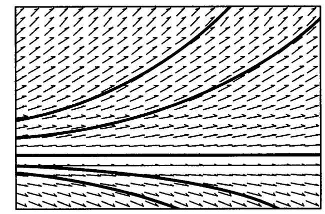

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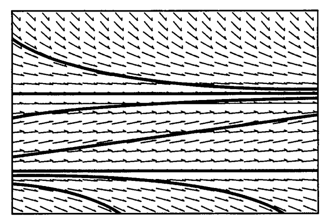


Chapter 6 Practice Exercises, pp. 548–551

1. $-2e^{-x/5}$ **3.** xe^{4x} **5.** $\frac{2\sin\theta\cos\theta}{\sin^2\theta} = 2\cot\theta$ **7.** $\frac{2}{(\ln 2)x}$

9. $-8^{-t}(\ln 8)$ **11.** $18x^{2.6}$ **13.** $(x+2)^{x+2}(\ln(x+2) + 1)$

15. $-\frac{1}{\sqrt{1-u^2}}$ **17.** $\frac{-1}{\sqrt{1-x^2}\cos^{-1}x}$

19. $\tan^{-1}(t) + \frac{t}{1+t^2} - \frac{1}{2t}$ **21.** $\frac{1-z}{\sqrt{z^2-1}} + \sec^{-1}z$ **23.** -1

25. $\frac{2(x^2+1)}{\sqrt{\cos 2x}}\left[\frac{2x}{x^2+1} + \tan 2x\right]$

27. $5\left[\frac{(t+1)(t-1)}{(t-2)(t+3)}\right]^5\left[\frac{1}{t+1} + \frac{1}{t-1} - \frac{1}{t-2} - \frac{1}{t-3}\right]$

29. $\frac{1}{\sqrt{\theta}}(\sin\theta)^{\sqrt{\theta}}(\ln\sqrt{\sin\theta} + \theta\cot\theta)$ **31.** $-\cos e^x + C$

33. $\tan(e^x - 7) + C$ **35.** $e^{\tan x} + C$ **37.** $\frac{-\ln 7}{3}$ **39.** $\ln 8$

41. $\ln(9/25)$ **43.** $-[\ln|\cos(\ln v)|] + C$ **45.** $-\frac{1}{2}(\ln x)^{-2} + C$

47. $-\cot(1 + \ln r) + C$ **49.** $\frac{1}{2\ln 3}\left(3^{x^2}\right) + C$ **51.** $3\ln 7$

53. $15/16 + \ln 2$ **55.** $e - 1$ **57.** $1/6$ **59.** $9/14$

61. $\frac{1}{3}[(\ln 4)^3 - (\ln 2)^3]$ or $\frac{7}{3}(\ln 2)^3$ **63.** $\frac{9\ln 2}{4}$ **65.** π

67. $\pi/\sqrt{3}$ **69.** $\sec^{-1}2y + C$ **71.** $\pi/12$

73. $\sin^{-1}(x+1) + C$ **75.** $\pi/2$ **77.** $\frac{1}{3}\sec^{-1}\left(\frac{t+1}{3}\right) + C$

79. $y = \frac{\ln 2}{\ln(3/2)}$ **81.** $y = \ln x - \ln 3$ **83.** $y = \frac{1}{1 - e^x}$

85. $\ln 10$ **87.** $\ln 2$ **89.** 5 **91.** $-\infty$ **93.** 1 **95.** e^3

97. a) Same rate b) same rate c) faster d) faster e) same rate f) same rate **99.** a) True b) false c) false d) true e) true f) true **101.** 1/3
103. Absolute maximum = 0 at $x = e/2$, absolute minimum = −0.5 at $x = 0.5$
105. 1 **107.** $1/e$ m/sec **109.** $1/\sqrt{2}$ units long by $1/\sqrt{e}$ units high, $A = 1/\sqrt{2e} \approx 0.43$ units^2
111. $\ln 5x - \ln 3x = \ln(5/3)$ **113.** 1/2
115. a) Absolute maximum of $2/e$ at $x = e^2$, inflection point $(e^{8/3}, (8/3)e^{-4/3})$, concave up on $(e^{8/3}, \infty)$, concave down on $(0, e^{8/3})$
b) Absolute maximum of 1 at $x = 0$, inflection points $(\pm 1/\sqrt{2}, 1/\sqrt{e})$, concave up on $(-\infty, -1/\sqrt{2}) \cup (1/\sqrt{2}, \infty)$, concave down on $(-1/\sqrt{2}, 1/\sqrt{2})$
c) Absolute maximum of 1 at $x = 0$, inflection point $(1, 2/e)$, concave up on $(1, \infty)$, concave down on $(-\infty, 1)$
117. 18,935 years **119.** $20(5 - \sqrt{17})$ m
121. $y = \ln(-e^{-x-2} + 2e^{-2})$
123. $y = \dfrac{1}{(x+1)^2} \cdot \left(\dfrac{x^3}{3} + \dfrac{x^2}{2} + 1\right)$
125. y (exact) $= \dfrac{1}{2}x^2 - \dfrac{3}{2}$; $y \approx 0.4$; exact value is 1/2
127. y (exact) $= -e^{(x^2-1)/2}$; $y \approx -3.4192$; exact value is $-e^{3/2} \approx -4.4817$

Chapter 6 Additional Exercises, pp. 551–553

1. $\pi/2$ **3.** $1/\sqrt{e}$ **5.** $\ln 2$ **7.** a) 1 b) $\pi/2$ c) π
9. $a = 2$, $b = -2$ **11.** $\dfrac{1}{\ln 2}$, $\dfrac{1}{2\ln 2}$, 2 : 1 **13.** $x = 2$
15. 2/17 **23.** $\bar{x} = \dfrac{\ln 4}{\pi}$, $\bar{y} = 0$ **27.** b) 61°
29. a) $c - (c - y_0)e^{-(kA/V)t}$ b) c

CHAPTER 7

Section 7.1, pp. 560–561

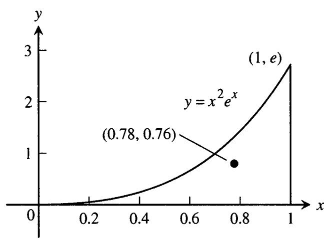
1. $2\sqrt{8x^2+1} + C$ **3.** $2(\sin v)^{3/2} + C$ **5.** $\ln 5$
7. $2\ln(\sqrt{x}+1) + C$ **9.** $-\dfrac{1}{7}\ln|\sin(3-7x)| + C$
11. $-\ln|\csc(e^\theta+1) + \cot(e^\theta+1)| + C$
13. $3\ln\left|\sec\dfrac{t}{3} + \tan\dfrac{t}{3}\right| + C$
15. $-\ln|\csc(s-\pi) + \cot(s-\pi)| + C$ **17.** 1 **19.** $e^{\tan v} + C$
21. $\dfrac{3^{(x+1)}}{\ln 3} + C$ **23.** $\dfrac{2^{\sqrt{w}}}{\ln 2} + C$ **25.** $3\tan^{-1}3u + C$
27. $\pi/18$ **29.** $\sin^{-1}s^2 + C$ **31.** $6\sec^{-1}|5x| + C$
33. $\tan^{-1}e^x + C$ **35.** $\ln(2+\sqrt{3})$ **37.** 2π
39. $\sin^{-1}(t-2) + C$
41. $\sec^{-1}|x+1| + C$, when $|x+1| > 1$
43. $\tan x - 2\ln|\csc x + \cot x| - \cot x - x + C$
45. $x + \sin 2x + C$ **47.** $x - \ln|x+1| + C$ **49.** $7 + \ln 8$
51. $2t^2 - t + 2\tan^{-1}\left(\dfrac{t}{2}\right) + C$ **53.** $\sin^{-1}x + \sqrt{1-x^2} + C$
55. $\sqrt{2}$ **57.** $\tan x - \sec x + C$ **59.** $\ln|1 + \sin\theta| + C$
61. $\cot x + x + \csc x + C$ **63.** 4 **65.** $\sqrt{2}$ **67.** 2
69. $\ln|\sqrt{2}+1| - \ln|\sqrt{2}-1|$ **71.** $4 - \dfrac{\pi}{2}$
73. $-\ln|\csc(\sin\theta) + \cot(\sin\theta)| + C$
75. $\ln|\sin x| + \ln|\cos x| + C$ **77.** $12\tan^{-1}(\sqrt{y}) + C$
79. $\sec^{-1}\left|\dfrac{x-1}{7}\right| + C$ **81.** $\ln|\sec(\tan t)| + C$
83. a) $\sin\theta - \dfrac{1}{3}\sin^3\theta + C$ b) $\sin\theta - \dfrac{2}{3}\sin^3\theta + \dfrac{1}{5}\sin^5\theta + C$
c) $\displaystyle\int \cos^9\theta\, d\theta = \int \cos^8\theta(\cos\theta)\, d\theta = \int (1 - \sin^2\theta)^4(\cos\theta)\, d\theta$
85. a) $\displaystyle\int \tan^3\theta\, d\theta = \frac{1}{2}\tan^2\theta - \int \tan\theta\, d\theta = \frac{1}{2}\tan^2\theta +$ $\ln|\cos\theta| + C$
b) $\displaystyle\int \tan^5\theta\, d\theta = \frac{1}{4}\tan^4\theta - \int \tan^3\theta\, d\theta$
c) $\displaystyle\int \tan^7\theta\, d\theta = \frac{1}{6}\tan^6\theta - \int \tan^5\theta\, d\theta$
d) $\displaystyle\int \tan^{2k+1}\theta\, d\theta = \frac{1}{2k}\tan^{2k}\theta - \int \tan^{2k-1}\theta\, d\theta$
87. $2\sqrt{2} - \ln(3 + 2\sqrt{2})$ **89.** π^2 **91.** $\ln(2+\sqrt{3})$
93. $\bar{x} = 0, \bar{y} = \dfrac{1}{\ln(2\sqrt{2}+3)}$

Section 7.2, pp. 567–569

1. $-2x\cos(x/2) + 4\sin(x/2) + C$
3. $t^2\sin t + 2t\cos t - 2\sin t + C$ **5.** $\ln 4 - \dfrac{3}{4}$
7. $y\tan^{-1}(y) - \ln\sqrt{1+y^2} + C$ **9.** $x\tan x + \ln|\cos x| + C$
11. $(x^3 - 3x^2 + 6x - 6)e^x + C$ **13.** $(x^2 - 7x + 7)e^x + C$
15. $(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)e^x + C$ **17.** $\dfrac{\pi^2 - 4}{8}$
19. $\dfrac{5\pi - 3\sqrt{3}}{9}$ **21.** $\dfrac{1}{2}(-e^\theta\cos\theta + e^\theta\sin\theta) + C$
23. $\dfrac{e^{2x}}{13}(3\sin 3x + 2\cos 3x) + C$
25. $\dfrac{2}{3}\left(\sqrt{3s+9}\,e^{\sqrt{3s+9}} - e^{\sqrt{3s+9}}\right) + C$ **27.** $\dfrac{\pi\sqrt{3}}{3} - \ln(2) - \dfrac{\pi^2}{18}$
29. $\dfrac{1}{2}[-x\cos(\ln x) + x\sin(\ln x)] + C$ **31.** a) π b) 3π c) 5π d) $(2n+1)\pi$ **33.** $2\pi(1 - \ln 2)$ **35.** a) $\pi(\pi - 2)$ b) 2π

37. a) $\bar{x} = \dfrac{6-2e}{e-2} \approx 0.78, \bar{y} = \dfrac{e^2-3}{8(e-2)} \approx 0.76$

b)



39. $\pi^2 + \pi - 4$ **41.** a) $\dfrac{1}{2\pi}\left(1 - e^{-2\pi}\right)$

43. $x \sin^{-1} x + \cos(\sin^{-1} x) + C$

45. $x \sec^{-1} x - \ln\left|x + \sqrt{x^2-1}\right| + C$ **47.** Yes

49. a) $x \sinh^{-1} x - \cosh(\sinh^{-1} x) + C$
b) $x \sinh^{-1} x + (1+x^2)^{1/2} + C$

Section 7.3, pp. 576–578

1. $\dfrac{2}{x-3} + \dfrac{3}{x-2}$ **3.** $\dfrac{1}{x+1} + \dfrac{3}{(x+1)^2}$

5. $\dfrac{-2}{z} + \dfrac{-1}{z^2} + \dfrac{2}{z-1}$ **7.** $1 + \dfrac{17}{t-3} + \dfrac{-12}{t-2}$

9. $\dfrac{1}{2}[\ln|1+x| - \ln|1-x|] + C$

11. $\dfrac{1}{7} \ln|(x+6)^2(x-1)^5| + C$ **13.** $(\ln 15)/2$

15. $-\dfrac{1}{2}\ln|t| + \dfrac{1}{6}\ln|t+2| + \dfrac{1}{3}\ln|t-1| + C$ **17.** $3 \ln 2 - 2$

19. $\dfrac{1}{4}\ln\left|\dfrac{x+1}{x-1}\right| - \dfrac{x}{2(x^2-1)} + C$ **21.** $(\pi + 2\ln 2)/8$

23. $\tan^{-1} y - \dfrac{1}{y^2+1} + C$

25. $-(s-1)^{-2} + (s-1)^{-1} + \tan^{-1} s + C$

27. $\dfrac{-1}{\theta^2+2\theta+2} + \ln|\theta^2+2\theta+2| - \tan^{-1}(\theta+1) + C$

29. $x^2 + \ln\left|\dfrac{x-1}{x}\right| + C$

31. $9x + 2\ln|x| + \dfrac{1}{x} + 7\ln|x-1| + C$

33. $\dfrac{y^2}{2} - \ln|y| + \dfrac{1}{2}\ln(1+y^2) + C$ **35.** $\ln\left|\dfrac{e^t+1}{e^t+2}\right| + C$

37. $\dfrac{1}{5}\ln\left|\dfrac{\sin y - 2}{\sin y + 3}\right| + C$

39. $\dfrac{(\tan^{-1} 2x)^2}{4} - 3\ln|x-2| + \dfrac{6}{x-2} + C$

41. $x = \ln|t-2| - \ln|t-1| + \ln 2$ **43.** $x = \dfrac{6t}{t+2} - 1$

45. $3\pi \ln 25$ **47.** 1.10 **49.** a) $x = \dfrac{1000e^{4t}}{499 + e^{4t}}$ b) 1.55 days

51. a) $\dfrac{22}{7} - \pi$ b) 0.04% c) The area is less than 0.003.

Section 7.4, pp. 582–583

1. $\ln\left|\sqrt{9+y^2} + y\right| + C$ **3.** $\pi/4$ **5.** $\pi/6$

7. $\dfrac{25}{2}\sin^{-1}\left(\dfrac{t}{5}\right) + \dfrac{t\sqrt{25-t^2}}{2} + C$

9. $\dfrac{1}{2}\ln\left|\dfrac{2x}{7} + \dfrac{\sqrt{4x^2-49}}{7}\right| + C$

11. $7\left[\dfrac{\sqrt{y^2-49}}{7} - \sec^{-1}\left(\dfrac{y}{7}\right)\right] + C$ **13.** $\dfrac{\sqrt{x^2-1}}{x} + C$

15. $\dfrac{1}{3}(x^2+4)^{3/2} - 4\sqrt{x^2+4} + C$ **17.** $\dfrac{-2\sqrt{4-w^2}}{w} + C$

19. $4\sqrt{3} - 4\pi/3$ **21.** $-\dfrac{x}{\sqrt{x^2-1}} + C$

23. $-\dfrac{1}{5}\left(\dfrac{\sqrt{1-x^2}}{x}\right)^5 + C$ **25.** $2\tan^{-1} 2x + \dfrac{4x}{(4x^2+1)} + C$

27. $\dfrac{1}{3}\left(\dfrac{v}{\sqrt{1-v^2}}\right)^3 + C$ **29.** $\ln 9 - \ln(1+\sqrt{10})$ **31.** $\pi/6$

33. $\sec^{-1}|x| + C$ **35.** $\sqrt{x^2-1} + C$

37. $y = 2\left[\dfrac{\sqrt{x^2-4}}{2} - \sec^{-1}\left(\dfrac{x}{2}\right)\right]$ **39.** $y = \dfrac{3}{2}\tan^{-1}\left(\dfrac{x}{2}\right) - \dfrac{3\pi}{8}$

41. $3\pi/4$ **43.** $\dfrac{2}{1-\tan(x/2)} + C$ **45.** 1 **47.** $\dfrac{\sqrt{3}\pi}{9}$

49. $\dfrac{1}{\sqrt{2}}\ln\left|\dfrac{\tan(t/2) + 1 - \sqrt{2}}{\tan(t/2) + 1 + \sqrt{2}}\right| + C$ **51.** $\ln\left|\dfrac{1+\tan(\theta/2)}{1-\tan(\theta/2)}\right| + C$

Section 7.5, pp. 591–594

1. $\dfrac{2}{\sqrt{3}}\left(\tan^{-1}\sqrt{\dfrac{x-3}{3}}\right) + C$ **3.** $\sqrt{x-2}\left(\dfrac{2(x-2)}{3} + 4\right) + C$

5. $\dfrac{(2x-3)^{3/2}(x+1)}{5} + C$

7. $\dfrac{-\sqrt{9-4x}}{x} - \dfrac{2}{3}\ln\left|\dfrac{\sqrt{9-4x}-3}{\sqrt{9-4x}+3}\right| + C$

9. $\dfrac{(x+2)(2x-6)\sqrt{4x-x^2}}{6} + 4\sin^{-1}\left(\dfrac{x-2}{2}\right) + C$

11. $-\frac{1}{\sqrt{7}}\ln\left|\frac{\sqrt{7}+\sqrt{7+x^2}}{x}\right|+C$

13. $\sqrt{4-x^2}-2\ln\left|\frac{2+\sqrt{4-x^2}}{x}\right|+C$

15. $\frac{p}{2}\sqrt{25-p^2}+\frac{25}{2}\sin^{-1}\frac{p}{5}+C$

17. $2\sin^{-1}\frac{r}{2}-\frac{1}{2}r\sqrt{4-r^2}+C$

19. $-\frac{1}{3}\tan^{-1}\left[\frac{1}{3}\tan\left(\frac{\pi}{4}-\theta\right)\right]+C$

21. $\frac{e^{2t}}{13}(2\cos 3t+3\sin 3t)+C$

23. $\frac{x^2}{2}\cos^{-1}(x)+\frac{1}{4}\sin^{-1}(x)-\frac{1}{4}x\sqrt{1-x^2}+C$

25. $\frac{s}{18(9-s^2)}+\frac{1}{108}\ln\left|\frac{s+3}{s-3}\right|+C$

27. $-\frac{\sqrt{4x+9}}{x}+\frac{2}{3}\ln\left|\frac{\sqrt{4x+9}-3}{\sqrt{4x+9}+3}\right|+C$

29. $2\sqrt{3t-4}-4\tan^{-1}\sqrt{\frac{3t-4}{4}}+C$

31. $\frac{x^3}{3}\tan^{-1}x-\frac{x^2}{6}+\frac{1}{6}\ln(1+x^2)+C$

33. $-\frac{\cos 5x}{10}-\frac{\cos x}{2}+C$ **35.** $8\left[\frac{\sin(7t/2)}{7}-\frac{\sin(9t/2)}{9}\right]+C$

37. $6\sin(\theta/12)+\frac{6}{7}\sin(7\theta/12)+C$

39. $\frac{1}{2}\ln|x^2+1|+\frac{x}{2(1+x^2)}+\frac{1}{2}\tan^{-1}x+C$

41. $\left(x-\frac{1}{2}\right)\sin^{-1}\sqrt{x}+\frac{1}{2}\sqrt{x-x^2}+C$

43. $\sin^{-1}\sqrt{x}-\sqrt{x-x^2}+C$

45. $\sqrt{1-\sin^2 t}-\ln\left|\frac{1+\sqrt{1-\sin^2 t}}{\sin t}\right|+C$

47. $\ln\left|\ln y+\sqrt{3+(\ln y)^2}\right|+C$ **49.** $\ln\left|3r+\sqrt{9r^2-1}\right|+C$

51. $x\cos^{-1}\sqrt{x}+\frac{1}{2}\sin^{-1}\sqrt{x}-\frac{1}{2}\sqrt{x-x^2}+C$

53. $-\frac{\sin^4 2x\cos 2x}{10}-\frac{2\sin^2 2x\cos 2x}{15}-\frac{4\cos 2x}{15}+C$

55. $\frac{\cos^3 2\pi t\sin 2\pi t}{\pi}+\frac{3}{2}\frac{\cos 2\pi t\sin 2\pi t}{\pi}+3t+C$

57. $\frac{\sin^3 2\theta\cos^2 2\theta}{10}+\frac{\sin^3 2\theta}{15}+C$ **59.** $\frac{2}{3}\tan^3 t+C$

61. $\tan^2 2x-2\ln|\sec 2x|+C$

63. $8\left[-\frac{1}{3}\cot^3 t+\cot t+t\right]+C$

65. $\frac{(\sec\pi x)(\tan\pi x)}{\pi}+\frac{1}{\pi}\ln|\sec\pi x+\tan\pi x|+C$

67. $\frac{\sec^2 3x\tan 3x}{3}+\frac{2}{3}\tan 3x+C$

69. $\frac{-\csc^3 x\cot x}{4}-\frac{3\csc x\cot x}{8}-\frac{3}{8}\ln|\csc x+\cot x|+C$

71. $4x^4(\ln x)^2-2x^4(\ln x)+\frac{x^2}{2}+C$ **73.** $\frac{e^{3x}}{9}(3x-1)+C$

75. $2x^3e^{x/2}-12x^2e^{x/2}+96e^{x/2}\left(\frac{x}{2}-1\right)+C$

77. $\frac{x^2 2^x}{\ln 2}-\frac{2}{\ln 2}\left[\frac{x2^x}{\ln 2}-\frac{2^x}{(\ln 2)^2}\right]+C$ **79.** $\frac{x\pi^x}{\ln\pi}-\frac{\pi^x}{(\ln\pi)^2}+C$

81. $\frac{1}{2}\left[\sec(e^t-1)\tan(e^t-1)+\ln|\sec(e^t-1)+\tan(e^t-1)|\right]+C$

83. $\sqrt{2}+\ln(\sqrt{2}+1)$ **85.** $\pi/3$

87. $\frac{1}{120}\sinh^4 3x\cosh 3x-\frac{1}{90}\sinh^2 3x\cosh 3x+\frac{2}{90}\cosh 3x+C$

89. $\frac{x^2}{3}\sinh 3x-\frac{2x}{9}\cosh 3x+\frac{2}{27}\sinh 3x+C$

91. $-\frac{\operatorname{sech}^7 x}{7}+C$ **101.** $\pi(2\sqrt{3}+\sqrt{2})\ln(\sqrt{2}+\sqrt{3})$

103. $\bar{x}=4/3$, $\bar{y}=\ln\sqrt{2}$ **105.** 7.62 **107.** $\pi/8$ **111.** $\pi/4$

Section 7.6, pp. 603–605

1. $\pi/2$ **3.** 2 **5.** 6 **7.** $\pi/2$ **9.** ln 3 **11.** ln 4 **13.** 0
15. $\sqrt{3}$ **17.** π **19.** $\ln\left(1+\frac{\pi}{2}\right)$ **21.** -1 **23.** 1
25. $-1/4$ **27.** $\pi/2$ **29.** $\pi/3$ **31.** 6 **33.** ln 2
35. Diverges **37.** Converges **39.** Converges **41.** Converges
43. Diverges **45.** Converges **47.** Converges **49.** Diverges
51. Converges **53.** Converges **55.** Diverges **57.** Converges
59. Diverges **61.** Converges **63.** Converges
65. b) ≈ 0.88621 **69.** 1 **71.** 2π **73.** ln 2 **79.** Diverges
81. Converges **83.** Converges **85.** Diverges **91.** b) $\pi/2$

Chapter 7 Practice Exercises, pp. 606–609

1. $\frac{1}{12}(4x^2-9)^{3/2}+C$ **3.** $\frac{(2x+1)^{5/2}}{10}-\frac{(2x+1)^{3/2}}{6}+C$

5. $\frac{\sqrt{8x^2+1}}{8}+C$ **7.** $\frac{1}{2}\ln(25+y^2)+C$

9. $\frac{-\sqrt{9-4t^4}}{8}+C$ **11.** $\frac{9}{25}(z^{5/3}+1)^{5/3}+C$

13. $-\dfrac{1}{2(1-\cos 2\theta)}+C$ **15.** $-\dfrac{1}{4}\ln|3+4\cos t|+C$

17. $-\dfrac{1}{2}e^{\cos 2x}+C$ **19.** $-\dfrac{1}{3}\cos^3(e^\theta)+C$ **21.** $\dfrac{2^{x-1}}{\ln 2}+C$

23. $\ln|\ln v|+C$ **25.** $\ln|2+\tan^{-1}x|+C$ **27.** $\sin^{-1}(2x)+C$

29. $\dfrac{1}{3}\sin^{-1}\left(\dfrac{3t}{4}\right)+C$ **31.** $\dfrac{1}{3}\tan^{-1}\left(\dfrac{t}{3}\right)+C$

33. $\dfrac{1}{5}\sec^{-1}\left|\dfrac{5x}{4}\right|+C$ **35.** $\sin^{-1}\left(\dfrac{x-2}{2}\right)+C$

37. $\dfrac{1}{2}\tan^{-1}\left(\dfrac{y-2}{2}\right)+C$ **39.** $\sec^{-1}|x-1|+C$

41. $\dfrac{x}{2}-\dfrac{\sin 2x}{4}+C$

43. $\dfrac{2}{3}\cos^3\left(\dfrac{\theta}{2}\right)-2\cos\left(\dfrac{\theta}{2}\right)+C$

45. $\dfrac{\tan^2(2t)}{4}-\dfrac{1}{2}\ln|\sec 2t|+C$

47. $-\dfrac{1}{2}\ln|\csc(2x)+\cot(2x)|+C$ **49.** $\ln\sqrt{2}$ **51.** 2

53. $2\sqrt{2}$ **55.** $x-2\tan^{-1}\left(\dfrac{x}{2}\right)+C$

57. $x+x^2+2\ln|2x-1|+C$

59. $\ln(y^2+4)-\dfrac{1}{2}\tan^{-1}\left(\dfrac{y}{2}\right)+C$

61. $-\sqrt{4-t^2}+2\sin^{-1}\left(\dfrac{t}{2}\right)+C$ **63.** $x-\tan x+\sec x+C$

65. $-\dfrac{1}{3}\ln|\sec(5-3x)+\tan(5-3x)|+C$

67. $4\ln\left|\sin\left(\dfrac{x}{4}\right)\right|+C$ **69.** $-2\left(\dfrac{(\sqrt{1-x})^3}{3}-\dfrac{(\sqrt{1-x})^5}{5}\right)+C$

71. $\dfrac{1}{2}\left(z\sqrt{z^2+1}+\ln\left|z+\sqrt{z^2+1}\right|\right)+C$

73. $\ln\left|y+\sqrt{25+y^2}\right|+C$ **75.** $\dfrac{-\sqrt{1-x^2}}{x}+C$

77. $\dfrac{\sin^{-1}x}{2}-\dfrac{x\sqrt{1-x^2}}{2}+C$ **79.** $\ln\left|\dfrac{x}{3}+\dfrac{\sqrt{x^2-9}}{3}\right|+C$

81. $\sqrt{w^2-1}-\sec^{-1}(w)+C$

83. $[(x+1)(\ln(x+1))-(x+1)]+C$

85. $x\tan^{-1}(3x)-\dfrac{1}{6}\ln(1+9x^2)+C$

87. $(x+1)^2e^x-2(x+1)e^x+2e^x+C$

89. $\dfrac{2e^x\sin 2x}{5}+\dfrac{e^x\cos 2x}{5}+C$

91. $2\ln|x-2|-\ln|x-1|+C$

93. $\ln|x|-\ln|x+1|+\dfrac{1}{x+1}+C$ **95.** $-\dfrac{1}{3}\ln\left|\dfrac{\cos\theta-1}{\cos\theta+2}\right|+C$

97. $4\ln|x|-\dfrac{1}{2}\ln(x^2+1)+4\tan^{-1}x+C$

99. $\dfrac{1}{16}\ln\left|\dfrac{(v-2)^5(v+2)}{v^6}\right|+C$

101. $\dfrac{1}{2}\tan^{-1}t-\dfrac{\sqrt{3}}{6}\tan^{-1}\dfrac{t}{\sqrt{3}}+C$

103. $\dfrac{x^2}{2}+\dfrac{4}{3}\ln|x+2|+\dfrac{2}{3}\ln|x-1|+C$

105. $\dfrac{x^2}{2}-\dfrac{9}{2}\ln|x+3|+\dfrac{3}{2}\ln|x+1|+C$

107. $\dfrac{1}{3}\ln\left|\dfrac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C$ **109.** $\ln|1-e^{-x}|+C$ **111.** $\pi/2$

113. 6 **115.** $\ln 3$ **117.** 2 **119.** $\pi/6$ **121.** Diverges

123. Diverges **125.** Converges **127.** $-\sqrt{16-y^2}+C$

129. $-\dfrac{1}{2}\ln|4-x^2|+C$ **131.** $\ln\dfrac{1}{\sqrt{9-x^2}}+C$

133. $\dfrac{1}{6}\ln\left|\dfrac{x+3}{x-3}\right|+C$

135. $\dfrac{2x^{3/2}}{3}-x+2\sqrt{x}-2\ln(\sqrt{x}+1)+C$

137. $\ln\left|\dfrac{x}{\sqrt{x^2+1}}\right|-\dfrac{1}{2}\left(\dfrac{x}{\sqrt{x^2+1}}\right)^2+C$ **139.** $\sin^{-1}(x+1)+C$

141. $\ln|u+\sqrt{1+u^2}|+C$

143. $-2\cot x-\ln|\csc x+\cot x|+\csc x+C$

145. $\dfrac{1}{12}\ln\left|\dfrac{3+v}{3-v}\right|+\dfrac{1}{6}\tan^{-1}\dfrac{v}{3}+C$

147. $\dfrac{\theta\sin(2\theta+1)}{2}+\dfrac{\cos(2\theta+1)}{4}+C$

149. $\dfrac{x^2}{2}+2x+3\ln|x-1|-\dfrac{1}{x-1}+C$ **151.** $-\cos(2\sqrt{x})+C$

153. $-\ln|\csc(2y)+\cot(2y)|+C$ **155.** $\dfrac{1}{2}\tan^2 x+C$

157. $-\sqrt{4-(r+2)^2}+C$ **159.** $\dfrac{1}{4}\sec^2\theta+C$ **161.** $\dfrac{\sqrt{2}}{2}$

163. $2\left(\dfrac{(\sqrt{2-x})^3}{3}-2\sqrt{2-x}\right)+C$ **165.** $\tan^{-1}(y-1)+C$

167. $\dfrac{1}{3}\ln|\sec\theta^3|+C$

169. $\dfrac{1}{4}\ln|z|-\dfrac{1}{4z}-\dfrac{1}{4}\left[\dfrac{1}{2}\ln(z^2+4)+\dfrac{1}{2}\tan^{-1}\left(\dfrac{z}{2}\right)\right]+C$

171. $-\dfrac{1}{4}\sqrt{9-4t^2}+C$ **173.** $\ln|\sin\theta|-\dfrac{1}{2}\ln(1+\sin^2\theta)+C$

175. $\ln|\sec\sqrt{y}|+C$ **177.** $-\theta+\ln\left|\dfrac{\theta+2}{\theta-2}\right|+C$ **179.** $x+C$

181. $-\frac{\cos x}{2}+C$ **183.** $\ln(1+e^t)+C$ **185.** 1/4

187. $\ln|\ln \sin v|+C$ **189.** $\frac{2}{3}x^{3/2}+C$

191. $-\frac{1}{5}\tan^{-1}\cos(5t)+C$ **193.** $\frac{1}{3}\left(\frac{27^{3\theta+1}}{\ln 27}\right)+C$

195. $2\sqrt{r}-2\ln(1+\sqrt{r})+C$ **197.** $\ln\left|\frac{y}{y+2}\right|+\frac{2}{y}-\frac{2}{y^2}+C$

199. $4\sec^{-1}\left(\frac{7m}{2}\right)+C$ **201.** $\frac{\sqrt{8}-1}{6}$ **203.** $\frac{\pi}{2}(3b-a)+2$

Chapter 7 Additional Exercises, pp. 609–612

1. $x(\sin^{-1}x)^2+2(\sin^{-1}x)\sqrt{1-x^2}-2x+C$

3. $\frac{x^2\sin^{-1}x}{2}+\frac{x\sqrt{1-x^2}-\sin^{-1}x}{4}+C$

5. $\frac{\ln|\sec 2\theta+\tan 2\theta|+2\theta}{4}+C$

7. $\frac{1}{2}\left(\ln\left(t-\sqrt{1-t^2}\right)-\sin^{-1}t\right)+C$

9. $\frac{1}{16}\ln\left|\frac{x^2+2x+2}{x^2-2x+2}\right|+\frac{1}{8}\left(\tan^{-1}(x+1)+\tan^{-1}(x-1)\right)+C$

11. 0 **13.** $\ln(4)-1$ **15.** 1 **17.** $32\pi/35$ **19.** 2π

21. a) π b) $\pi(2e-5)$ **23.** b) $\pi\left(\frac{8(\ln 2)^2}{3}-\frac{16(\ln 2)}{9}+\frac{16}{27}\right)$

25. $\left(\frac{e^2+1}{4},\frac{e-2}{2}\right)$

27. $\sqrt{1+e^2}-\ln\left(\frac{\sqrt{1+e^2}}{e}+\frac{1}{e}\right)-\sqrt{2}+\ln(1+\sqrt{2})$ **29.** 6

31. $y=\sqrt{x},\quad 0\le x\le 4$ **33.** b) 1 **37.** $a=\frac{1}{2},-\frac{\ln 2}{4}$

39. $\frac{1}{2}<p\le 1$

41. $\frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+C$

43. $\frac{\cos x\sin 3x-3\sin x\cos 3x}{8}+C$

45. $\frac{e^{ax}}{a^2+b^2}(a\sin bx-b\cos bx)+C$

47. $x\ln(ax)-x+C$

CHAPTER 8

Section 8.1, pp. 619–622

1. $a_1=0,\ a_2=-1/4,\ a_3=-2/9,\ a_4=-3/16$

3. $a_1=1,\ a_2=-1/3,\ a_3=1/5,\ a_4=-1/7$

5. $a_1=1/2,\ a_2=1/2,\ a_3=1/2,\ a_4=1/2$

7. $1,\ \frac{3}{2},\ \frac{7}{4},\ \frac{15}{8},\ \frac{31}{16},\ \frac{63}{32},\ \frac{127}{64},\ \frac{255}{128},\ \frac{511}{256},\ \frac{1023}{512}$

9. $2,\ 1,\ -\frac{1}{2},\ -\frac{1}{4},\ \frac{1}{8},\ \frac{1}{16},\ -\frac{1}{32},\ -\frac{1}{64},\ \frac{1}{128},\ \frac{1}{256}$

11. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 **13.** $a_n=(-1)^{n+1},\ n\ge 1$

15. $a_n=(-1)^{n+1}(n)^2,\ n\ge 1$ **17.** $a_n=n^2-1,\ n\ge 1$

19. $a_n=4n-3,\ n\ge 1$ **21.** $a_n=\frac{1+(-1)^{n+1}}{2},\ n\ge 1$

23. $N=692,\ a_n=\sqrt[n]{0.5},\ L=1$

25. $N=65,\ a_n=(0.9)^n,\ L=0$ **27.** b) $\sqrt{3}$

31. Nondecreasing, bounded **33.** Not nondecreasing, bounded

35. Converges, nondecreasing sequence theorem

37. Converges, nondecreasing sequence theorem

39. Diverges, definition of divergence **43.** Converges

45. Converges

Section 8.2, pp. 628–630

1. Converges, 2 **3.** Converges, −1 **5.** Converges, −5

7. Diverges **9.** Diverges **11.** Converges, 1/2

13. Converges, 0 **15.** Converges, $\sqrt{2}$ **17.** Converges, 1

19. Converges, 0 **21.** Converges, 0 **23.** Converges, 0

25. Converges, 1 **27.** Converges, e^7 **29.** Converges, 1

31. Converges, 1 **33.** Diverges **35.** Converges, 4

37. Converges, 0 **39.** Diverges **41.** Converges, e^{-1}

43. Converges, $e^{2/3}$ **45.** Converges, x $(x>0)$ **47.** Converges, 0

49. Converges, 1 **51.** Converges, 1/2 **53.** Converges, $\pi/2$

55. Converges, 0 **57.** Converges, 0 **59.** Converges, 1/2

61. Converges, 0 **63.** $x_n=2^{n-2}$

65. a) $f(x)=x^2-2,\ 1.414213562\approx\sqrt{2}$

b) $f(x)=\tan(x)-1,\ 0.7853981635\approx\pi/4$

c) $f(x)=e^x$, diverges **67.** b) 1 **75.** 1 **77.** −0.73908456

79. 0.853748068 **83.** −3

Section 8.3, pp. 638–640

1. $s_n=\frac{2(1-(1/3)^n)}{1-(1/3)},\ 3$ **3.** $s_n=\frac{1-(-1/2)^n}{1-(-1/2)},\ 2/3$

5. $s_n=\frac{1}{2}-\frac{1}{n+2},\ \frac{1}{2}$ **7.** $1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\cdots,\ \frac{4}{5}$

9. $\frac{7}{4}+\frac{7}{16}+\frac{7}{64}+\cdots,\ \frac{7}{3}$

11. $(5+1)+\left(\frac{5}{2}+\frac{1}{3}\right)+\left(\frac{5}{4}+\frac{1}{9}\right)+\left(\frac{5}{8}+\frac{1}{27}\right)+\cdots,\ \frac{23}{2}$

13. $(1+1)+\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{4}+\frac{1}{25}\right)+\left(\frac{1}{8}-\frac{1}{125}\right)+\cdots,\ \frac{17}{6}$

15. 1 **17.** 5 **19.** Converges, 1 **21.** Converges, $-\frac{1}{\ln 2}$

23. Converges, $2+\sqrt{2}$ **25.** Converges, 1 **27.** Diverges

29. Converges, $\frac{e^2}{e^2-1}$ **31.** Converges, 2/9 **33.** Converges, 3/2

35. Diverges **37.** Diverges **39.** Converges, $\dfrac{\pi}{\pi - e}$

41. $a = 1, r = -x$; converges to $1/(1+x)$ for $|x| < 1$

43. $a = 3, r = (x-1)/2$; converges to $6/(3-x)$ for x in $(-1, 3)$

45. $|x| < \dfrac{1}{2}, \dfrac{1}{1-2x}$ **47.** $-2 < x < 0, \dfrac{1}{2+x}$

49. $x \neq (2k+1)\dfrac{\pi}{2}$, k an integer; $\dfrac{1}{1 - \sin x}$ **51.** 23/99

53. 7/9 **55.** 1/15 **57.** 41251/33300

59. a) $\displaystyle\sum_{n=-2}^{\infty} \frac{1}{(n+4)(n+5)}$ b) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(n+2)(n+3)}$

c) $\displaystyle\sum_{n=5}^{\infty} \frac{1}{(n-3)(n-2)}$ **61.** a) Answers may vary.

b) Answers may vary. c) Answers may vary. **69.** a) $r = 3/5$

b) $r = -3/10$ **71.** $|r| < 1, \dfrac{1+2r}{1-r^2}$ **73.** 28 m **75.** 8 m^2

77. a) $3\left(\dfrac{4}{3}\right)^{n-1}$

b) $A_n = A + \dfrac{1}{3}A + \dfrac{1}{3}\left(\dfrac{4}{9}\right)A + \cdots + \dfrac{1}{3}\left(\dfrac{4}{9}\right)^{n-2} A$,

$\lim_{n\to\infty} A_n = 2\sqrt{3}/5$

Section 8.4, pp. 643–644

1. Converges; geometric series, $r = \dfrac{1}{10} < 1$

3. Diverges; $\lim_{n\to\infty} \dfrac{n}{n+1} = 1 \neq 0$

5. Diverges; p-series, $p < 1$

7. Converges; geometric series, $r = \dfrac{1}{8} < 1$

9. Diverges; Integral Test

11. Converges; geometric series, $r = 2/3 < 1$

13. Diverges; Integral Test

15. Diverges; $\lim_{n\to\infty} \dfrac{2^n}{n+1} \neq 0$

17. Diverges; $\lim_{n\to\infty} (\sqrt{n}/\ln n) \neq 0$

19. Diverges; geometric series, $r = \dfrac{1}{\ln 2} > 1$

21. Converges; Integral Test

23. Diverges; nth-Term Test

25. Converges; Integral Test

27. Converges; Integral Test

29. Converges; Integral Test **31.** $a = 1$ **33.** b) About 41.55

35. True

Section 8.5, p. 649

1. Diverges; limit comparison with $\sum(1/\sqrt{n})$

3. Converges; compare with $\sum(1/2^n)$ **5.** Diverges; nth-Term Test

7. Converges; $\left(\dfrac{n}{3n+1}\right)^n < \left(\dfrac{n}{3n}\right)^n = \left(\dfrac{1}{3}\right)^n$

9. Diverges; direct comparison with $\sum(1/n)$

11. Converges; limit comparison with $\sum(1/n^2)$

13. Diverges; limit comparison with $\sum(1/n)$

15. Diverges; limit comparison with $\sum(1/n)$

17. Diverges; Integral Test

19. Converges; compare with $\sum(1/n^{3/2})$

21. Converges; $\dfrac{1}{n2^n} \leq \dfrac{1}{2^n}$

23. Converges; $\dfrac{1}{3^{n-1}+1} < \dfrac{1}{3^{n-1}}$

25. Diverges; limit comparison with $\sum(1/n)$

27. Converges; compare with $\sum(1/n^2)$

29. Converges; $\dfrac{\tan^{-1} n}{n^{1.1}} < \dfrac{\pi/2}{n^{1.1}}$

31. Converges; compare with $\sum(1/n^2)$

33. Diverges; $3n > n\sqrt[n]{n} \Rightarrow \dfrac{1}{3n} < \dfrac{1}{n\sqrt[n]{n}} \Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt[n]{n}}$ diverges

35. Converges; limit comparison with $\sum(1/n^2)$

Section 8.6, pp. 654–655

1. Converges; Ratio Test **3.** Diverges; Ratio Test

5. Converges; Ratio Test **7.** Converges; compare with $\sum(3/(1.25)^n)$

9. Diverges; $\lim_{n\to\infty} \left(1 - \dfrac{3}{n}\right)^n = e^{-3} \neq 0$

11. Converges; compare with $\sum(1/n^2)$

13. Diverges; compare with $\sum(1/(2n))$

15. Diverges; compare with $\sum(1/n)$ **17.** Converges; Ratio Test

19. Converges; Ratio Test **21.** Converges; Ratio Test

23. Converges; Root Test **25.** Converges; compare with $\sum(1/n^2)$

27. Converges; Ratio Test **29.** Diverges; Ratio Test

31. Converges; Ratio Test **33.** Converges; Ratio Test

35. Diverges; $a_n = \left(\dfrac{1}{3}\right)^{(1/n!)} \to 1$ **37.** Converges; Ratio Test

39. Diverges; Root Test **41.** Converges; Root Test

43. Converges; Ratio Test **47.** Yes

Section 8.7, pp. 661–663

1. Converges by Theorem 8 **3.** Diverges; $a_n \not\to 0$

5. Converges by Theorem 8 **7.** Diverges; $a_n \to 1/2$

9. Converges by Theorem 8

11. Converges absolutely. Series of absolute values is a convergent geometric series.

13. Converges conditionally. $1/\sqrt{n} \to 0$ but $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ diverges.

15. Converges absolutely. Compare with $\sum_{n=1}^{\infty}(1/n^2)$.

17. Converges conditionally. $1/(n+3) \to 0$ but $\sum_{n=1}^{\infty} \frac{1}{n+3}$ diverges (compare with $\sum_{n=1}^{\infty}(1/n)$).

19. Diverges; $\frac{3+n}{5+n} \to 1$

21. Converges conditionally; $\left(\frac{1}{n^2}+\frac{1}{n}\right) \to 0$ but $(1+n)/n^2 > 1/n$

23. Converges absolutely; Root Test

25. Converges absolutely by Integral Test **27.** Diverges; $a_n \not\to 0$

29. Converges absolutely by the Ratio Test

31. Converges absolutely; $\frac{1}{n^2+2n+1} < \frac{1}{n^2}$

33. Converges absolutely since $\left|\frac{\cos n\pi}{n\sqrt{n}}\right| = \left|\frac{(-1)^{n+1}}{n^{3/2}}\right| = \frac{1}{n^{3/2}}$ (convergent p–series)

35. Converges absolutely by Root Test **37.** Diverges; $a_n \to \infty$

39. Converges conditionally; $\sqrt{n+1}-\sqrt{n} = 1/(\sqrt{n}+\sqrt{n+1}) \to 0$, but series of absolute values diverges (compare with $\sum(1/\sqrt{n})$)

41. Diverges, $a_n \to 1/2 \neq 0$

43. Converges absolutely; $\operatorname{sech} n = \frac{2}{e^n+e^{-n}} = \frac{2e^n}{e^{2n}+1} < \frac{2e^n}{e^{2n}}$ $= \frac{2}{e^n}$, a term from a convergent geometric series.

45. $|\text{Error}| < 0.2$ **47.** $|\text{Error}| < 2 \times 10^{-11}$ **49.** 0.54030

51. a) $a_n \geq a_{n+1}$ b) $-1/2$

Section 8.8, pp. 671–672

1. a) 1, $-1 < x < 1$ b) $-1 < x < 1$ c) none

3. a) 1/4, $-1/2 < x < 0$ b) $-1/2 < x < 0$ c) none

5. a) 10, $-8 < x < 12$ b) $-8 < x < 12$ c) none

7. a) 1, $-1 < x < 1$ b) $-1 < x < 1$ c) none

9. a) 3, $[-3, 3]$ b) $[-3, 3]$ c) none **11.** a) ∞, for all x b) for all x c) none **13.** a) ∞, for all x b) for all x c) none

15. a) 1, $-1 \leq x < 1$ b) $-1 < x < 1$ c) $x = -1$

17. a) 5, $-8 < x < 2$ b) $-8 < x < 2$ c) none

19. a) 3, $-3 < x < 3$ b) $-3 < x < 3$ c) none

21. a) 1, $-1 < x < 1$ b) $-1 < x < 1$ c) none

23. a) 0, $x = 0$ b) $x = 0$ c) none **25.** a) 2, $-4 < x \leq 0$ b) $-4 < x < 0$ c) $x = 0$ **27.** a) 1, $-1 \leq x \leq 1$ b) $-1 \leq x \leq 1$ c) none **29.** a) 1/4, $1 \leq x \leq 3/2$ b) $1 \leq x \leq 3/2$ c) none **31.** a) 1, $(-1-\pi) \leq x < (1-\pi)$ b) $(-1-\pi) < x < (1-\pi)$ c) $x = -1-\pi$

33. $-1 < x < 3$, $4/(3+2x-x^2)$ **35.** $0 < x < 16$, $2/(4-\sqrt{x})$

37. $-\sqrt{2} < x < \sqrt{2}$, $3/(2-x^2)$

39. $1 < x < 5$, $2/(x-1)$, $1 < x < 5$, $-2/(x-1)^2$

41. a) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$; converges for all x

b) and c) $2x - \frac{2^3x^3}{3!} + \frac{2^5x^5}{5!} - \frac{2^7x^7}{7!} + \frac{2^9x^9}{9!} - \frac{2^{11}x^{11}}{11!} + \cdots$

43. a) $\frac{x^2}{2} + \frac{x^4}{12} + \frac{x^6}{45} + \frac{17x^8}{2520} + \frac{31x^{10}}{14175}, -\frac{\pi}{2} < x < \frac{\pi}{2}$

b) $1 + x^2 + \frac{2x^4}{3} + \frac{17x^6}{45} + \frac{62x^8}{315} + \cdots, -\frac{\pi}{2} < x < \frac{\pi}{2}$

Section 8.9, pp. 677–678

1. $P_0(x) = 0,\ P_1(x) = x-1,\ P_2(x) = (x-1) - \frac{1}{2}(x-1)^2,$

$P_3(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3$

3. $P_0(x) = \frac{1}{2},\ P_1(x) = \frac{1}{2} - \frac{1}{4}(x-2),$

$P_2(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2,$

$P_3(x) = \frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$

5. $P_0(x) = \frac{\sqrt{2}}{2},\ P_1(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right),$

$P_2(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2,$

$P_3(x) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x-\frac{\pi}{4}\right)^2 - \frac{\sqrt{2}}{12}\left(x-\frac{\pi}{4}\right)^3$

7. $P_0(x) = 2,\ P_1(x) = 2 + \frac{1}{4}(x-4),$

$P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2,$

$P_3(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2 + \frac{1}{512}(x-4)^3$

9. $\sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$

11. $\sum_{n=0}^{\infty}(-1)^n x^n = 1 - x + x^2 - x^3 + \cdots$

13. $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n+1} x^{2n+1}}{(2n+1)!}$ **15.** $7\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$ **17.** $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

19. $x^4 - 2x^3 - 5x + 4$ **21.** $8 + 10(x-2) + 6(x-2)^2 + (x-2)^3$

23. $21 - 36(x+2) + 25(x+2)^2 - 8(x+2)^3 + (x+2)^4$

25. $\sum_{n=0}^{\infty}(-1)^n(n+1)(x-1)^n$ **27.** $\sum_{n=0}^{\infty} \frac{e^2}{n!}(x-2)^n$

33. $L(x) = 0,\ Q(x) = -x^2/2$ **35.** $L(x) = 1,\ Q(x) = 1 + x^2/2$

37. $L(x) = x,\ Q(x) = x$

Section 8.10, pp. 686–688

1. $\sum_{n=0}^{\infty} \frac{(-5x)^n}{n!} = 1 - 5x + \frac{5^2x^2}{2!} - \frac{5^3x^3}{3!} + \cdots$

3. $\sum_{n=0}^{\infty} \frac{5(-1)^n(-x)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{5(-1)^{n+1}x^{2n+1}}{(2n+1)!}$

$= -5x + \frac{5x^3}{3!} - \frac{5x^5}{5!} + \frac{5x^7}{7!} + \cdots$

5. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(2n)!}$ **7.** $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!} = x + x^2 + \frac{x^3}{2!} + \frac{x^4}{3!} + \frac{x^5}{4!} + \cdots$

9. $\sum_{n=2}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \cdots$

11. $x - \frac{\pi^2 x^3}{2!} + \frac{\pi^4 x^5}{4!} - \frac{\pi^6 x^7}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n} x^{2n+1}}{(2n)!}$

13. $1 + \sum_{n=1}^{\infty} \frac{(-1)^n (2x)^{2n}}{2 \cdot (2n)!} = 1 - \frac{(2x)^2}{2 \cdot 2!} +$

$\frac{(2x)^4}{2 \cdot 4!} - \frac{(2x)^6}{2 \cdot 6!} + \frac{(2x)8}{2 \cdot 8!} - \cdots$

15. $\sum_{n=0}^{\infty} (2x)^{n+2} = 2^2x^2 + 2^3x^3 + 2^4x^4 + \cdots$

17. $\sum_{n=1}^{\infty} nx^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots$

19. $|x| < (0.06)^{1/5} < 0.56968$

21. $|\text{Error}| < (10^{-3})^3/6 < 1.67 \times 10^{-10}, \quad -10^{-3} < x < 0$

23. $|\text{Error}| < (3^{0.1})(0.1)^3/6 < 1.87 \times 10^{-5}$ **25.** 0.000293653

27. $|x| < 0.02$ **31.** $\sin x,\ x = 0.1; \sin(0.1)$

33. $\tan^{-1} x,\ x = \pi/3; \sqrt{3}$

35. $e^x \sin x = x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} \cdots$

43. a) $Q(x) = 1 + kx + \frac{k(k-1)}{2}x^2$ b) for $0 \le x < 100^{-1/3}$

49. a) -1 b) $(1/\sqrt{2})(1+i)$ c) $-i$

53. $x + x^2 + \frac{1}{3}x^3 - \frac{1}{30}x^5 \cdots$; will converge for all x

Section 8.11, pp. 697–699

1. $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$ **3.** $1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \cdots$

5. $1 - x + \frac{3x^2}{4} - \frac{x^3}{2}$ **7.** $1 - \frac{x^3}{2} + \frac{3x^6}{8} - \frac{5x^9}{16}$

9. $1 + \frac{1}{2x} - \frac{1}{8x^2} + \frac{1}{16x^3}$

11. $(1+x)^4 = 1 + 4x + 6x^2 + 4x^3 + x^4$

13. $(1-2x)^3 = 1 - 6x + 12x^2 - 8x^3$

15. $y = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^n = e^{-x}$ **17.** $y = \sum_{n=1}^{\infty} (x^n/n!) = e^x - 1$

19. $y = \sum_{n=2}^{\infty} (x^n/n!) = e^x - x - 1$ **21.** $y = \sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = e^{x^2/2}$

23. $y = \sum_{n=0}^{\infty} 2x^n = \frac{2}{1-x}$ **25.** $y = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = \sinh x$

27. $y = 2 + x - 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n}}{(2n)!}$

29. $y = \sum_{n=0}^{\infty} \frac{-2(x-2)^{2n+1}}{(2n+1)!}$

31. $y = a + bx + \frac{1}{6}x^3 - \frac{ax^4}{3 \cdot 4} - \frac{bx^5}{4 \cdot 5} - \frac{x^7}{6 \cdot 6 \cdot 7} + \frac{ax^8}{3 \cdot 4 \cdot 7 \cdot 8}$

$+ \frac{bx^9}{4 \cdot 5 \cdot 8 \cdot 9} \cdots$. For $n \ge 6$, $a_n = (n-2)(n-3)\,a_{n-4}$.

33. 0.00267 **35.** 0.1 **37.** 0.0999 44461 1 **39.** 0.1000 01

41. $1/(13 \cdot 6!) \approx 0.00011$ **43.** $\frac{t^3}{3} - \frac{t^7}{7 \cdot 3!} + \frac{t^{11}}{11 \cdot 5!}$

45. a) $\frac{x^2}{2} - \frac{x^4}{12}$

b) $\frac{x^2}{2} - \frac{x^4}{3 \cdot 4} + \frac{x^6}{5 \cdot 6} - \frac{x^8}{7 \cdot 8} + \cdots + (-1)^{15} \frac{x^{32}}{31 \cdot 32}$

47. 1/2 **49.** $-1/24$ **51.** 1/3 **53.** -1 **55.** 2

59. 500 terms

61. 3 terms

63. a) $x + \frac{x^3}{6} + \frac{3x^5}{40} + \frac{5x^7}{112}$, radius of convergence = 1

b) $\frac{\pi}{2} - x - \frac{x^3}{6} - \frac{3x^5}{40} - \frac{5x^7}{112}$ **65.** $1 - 2x + 3x^2 - 4x^3 + \cdots$

71. c) $3\pi/4$

Chapter 8 Practice Exercises, pp. 700–702

1. Converges to 1 **3.** Converges to -1 **5.** Diverges

7. Converges to 0 **9.** Converges to 1 **11.** Converges to e^{-5}

13. Converges to 3 **15.** Converges to ln 2 **17.** Diverges

19. 1/6 **21.** 3/2 **23.** $e/(e-1)$ **25.** Diverges

27. Converges conditionally **29.** Converges conditionally

31. Converges absolutely **33.** Converges absolutely

35. Converges absolutely **37.** Converges absolutely

39. Converges absolutely **41.** a) 3, $-7 \le x < -1$

b) $-7 < x < -1$ c) $x = -7$ **43.** a) 1/3, $0 \le x \le 2/3$

b) $0 \le x \le 2/3$ c) none **45.** a) ∞, for all x b) for all x

c) none **47.** a) $\sqrt{3}$, $-\sqrt{3} < x < \sqrt{3}$ b) $-\sqrt{3} < x < \sqrt{3}$

c) none **49.** a) e, $(-e, e)$ b) $(-e, e)$ c) { }

51. $\frac{1}{1+x}, \frac{1}{4}, \frac{4}{5}$ **53.** $\sin x,\ \pi,\ 0$ **55.** e^x, ln 2, 2

57. $\sum_{n=0}^{\infty} 2^n x^n$ **59.** $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1} x^{2n+1}}{(2n+1)!}$ **61.** $\sum_{n=0}^{\infty} \frac{(-1)^n x^{5n}}{(2n)!}$

63. $\sum_{n=0}^{\infty} \frac{((\pi x)/2)^n}{n!}$

65. $2 - \frac{(x+1)}{2 \cdot 1!} + \frac{3(x+1)^2}{2^3 \cdot 2!} + \frac{9(x+1)^3}{2^5 \cdot 3!} + \cdots$

67. $\frac{1}{4} - \frac{1}{4^2}(x-3) + \frac{1}{4^3}(x-3)^2 - \frac{1}{4^4}(x-3)^3$

69. $y = \sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{n!} x^n = -e^{-x}$

71. $y = 3 \sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{n!} x^n = 3e^{-2x}$

73. $y = -1 - x + 2 \sum_{n=2}^{\infty} (x^n/n!) = 2e^x - 3x - 3$

75. $y = 1 + x + 2 \sum_{n=0}^{\infty} (x^n/n!) = 2e^x - 1 - x$ **77.** 0.4849 17143 1

79. $\approx 0.4872\ 22358\ 3$ **81.** $7/2$ **83.** $1/12$ **85.** -2

87. $r = -3,\ s = 9/2$

89. b) $|\text{error}| < |\sin(1/42)| < 0.02381$; an underestimate because the remainder is positive

91. $2/3$ **93.** $\ln\left(\frac{n+1}{2n}\right)$; the series converges to $\ln\left(\frac{1}{2}\right)$.

95. a) ∞ b) $a = 1,\quad b = 0$ **97.** It converges.

Chapter 8 Additional Exercises, pp. 703–707

1. Converges; Comparison Test **3.** Diverges; nth Term Test

5. Converges; Comparison Test **7.** Diverges; nth Term Test

9. With $a = \pi/3$, $\cos x = \frac{1}{2} - \frac{\sqrt{3}}{2}(x - \pi/3) - \frac{1}{4}(x - \pi/3)^2 + \frac{\sqrt{3}}{12}(x - \pi/3)^3 + \cdots$

11. With $a = 0$, $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$

13. With $a = 22\pi$, $\cos x = 1 - \frac{1}{2}(x - 22\pi)^2 + \frac{1}{4!}(x - 22\pi)^4 - \frac{1}{6!}(x - 22\pi)^6 + \cdots$

15. Converges, limit $= b$ **17.** $\pi/2$ **23.** $b = \pm\frac{1}{5}$

25. $a = 2,\ L = -7/6$ **29.** b) Yes

35. a) $\sum_{n=1}^{\infty} nx^{n-1}$ b) 6 c) $1/q$

37. a) $R_n = C_0 e^{-kt_0}\left(1 - e^{-nkt_0}\right) / \left(1 - e^{-kt_0}\right)$, $R = C_0\left(e^{-kt_0}\right) / \left(1 - e^{-kt_0}\right) = C_0 / \left(e^{kt_0} - 1\right)$
b) $R_1 = 1/e \approx 0.368$,
$R_{10} = R(1 - e^{-10}) \approx R(0.9999546) \approx 0.58195$;
$R \approx 0.58198$; $0 < (R - R_{10})/R < 0.0001$ c) 7

CHAPTER 9

Section 9.1, pp. 719–722

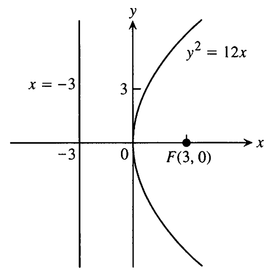
1. $y^2 = 8x$, $F(2, 0)$, directrix: $x = -2$

3. $x^2 = -6y$, $F(0, -3/2)$, directrix: $y = 3/2$

5. $\frac{x^2}{4} - \frac{y^2}{9} = 1$, $F(\pm\sqrt{13}, 0)$, $V(\pm 2, 0)$, asymptotes: $y = \pm\frac{3}{2}x$

7. $\frac{x^2}{2} + y^2 = 1$, $F(\pm 1, 0)$, $V(\pm\sqrt{2}, 0)$

9.



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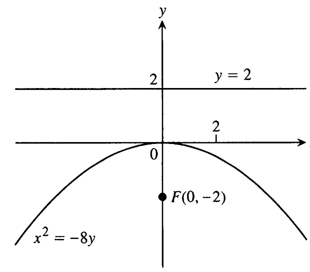

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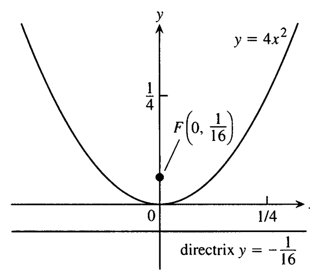

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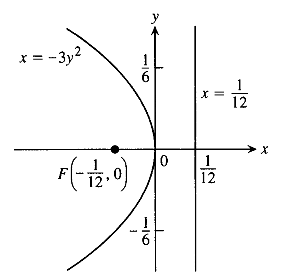

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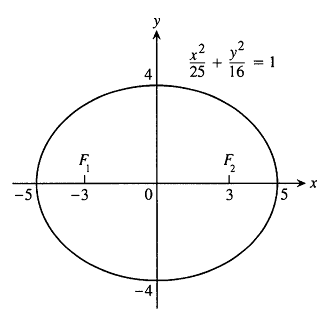

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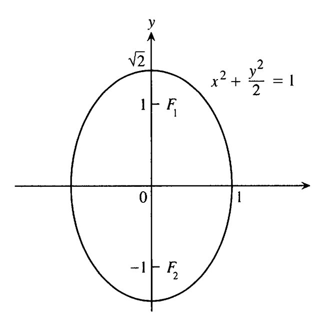

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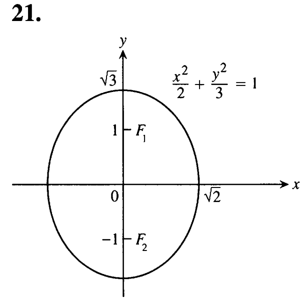

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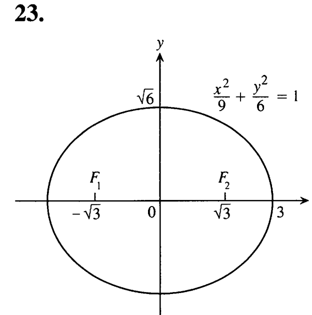
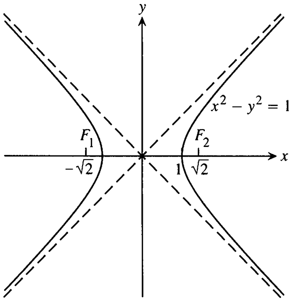
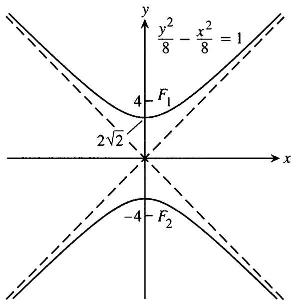


25. $\frac{x^2}{4} + \frac{y^2}{2} = 1$

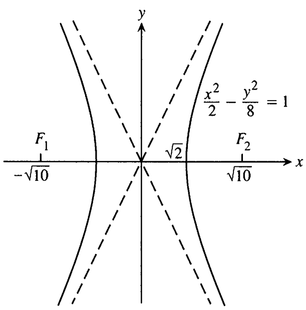
27. Asymptotes: $y = \pm x$



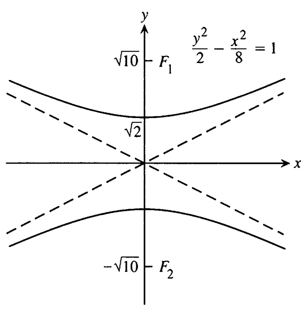
29. Asymptotes: $y = \pm x$



31. Asymptotes: $y = \pm 2x$

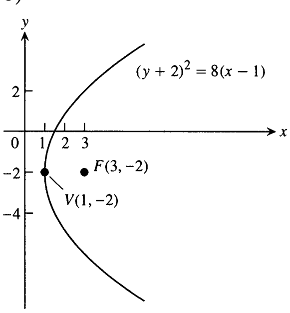


33. Asymptotes: $y = \pm \dfrac{x}{2}$

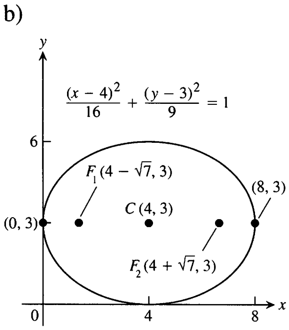


35. $y^2 - x^2 = 1$ **37.** $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$

39. a) Vertex: $(1, -2)$; focus: $(3, -2)$; directrix: $x = -1$
b)

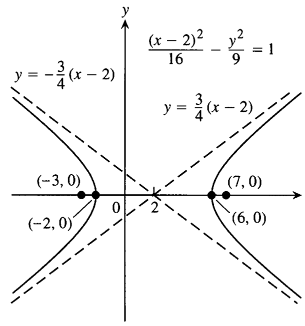


41. a) Foci: $(4 \pm \sqrt{7}, 3)$; vertices: $(8, 3)$ and $(0, 3)$; center: $(4, 3)$
b)



43. a) Center: $(2, 0)$; foci: $(7, 0)$ and $(-3, 0)$; vertices: $(6, 0)$ and $(-2, 0)$; asymptotes: $y = \pm \dfrac{3}{4}(x - 2)$

b)



45. $(y + 3)^2 = 4(x + 2)$, $V(-2, -3)$, $F(-1, -3)$, directrix: $x = -3$

47. $(x - 1)^2 = 8(y + 7)$, $V(1, -7)$, $F(1, -5)$, directrix: $y = -9$

49. $\dfrac{(x + 2)^2}{6} + \dfrac{(y + 1)^2}{9} = 1$, $F(-2, \pm\sqrt{3} - 1)$, $V(-2, \pm 3 - 1)$, $C(-2, -1)$

51. $\dfrac{(x - 2)^2}{3} + \dfrac{(y - 3)^2}{2} = 1$, $F(3, 3)$ and $F(1, 3)$, $V(\pm\sqrt{3} + 2, 3)$, $C(2, 3)$

53. $\dfrac{(x - 2)^2}{4} - \dfrac{(y - 2)^2}{5} = 1$, $C(2, 2)$, $F(5, 2)$ and $F(-1, 2)$, $V(4, 2)$ and $V(0, 2)$; asymptotes: $(y - 2) = \pm\dfrac{\sqrt{5}}{2}(x - 2)$

55. $(y + 1)^2 - (x + 1)^2 = 1$, $C(-1, -1)$, $F(-1, \sqrt{2} - 1)$ and $F(-1, -\sqrt{2} - 1)$, $V(-1, 0)$ and $V(-1, -2)$; asymptotes: $(y + 1) = \pm(x + 1)$

57. $C(-2, 0)$, $a = 4$ **59.** $V(-1, 1)$, $F(-1, 0)$

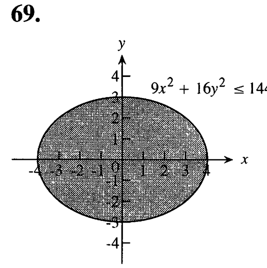
61. Ellipse: $\dfrac{(x + 2)^2}{5} + y^2 = 1$, $C(-2, 0)$, $F(0, 0)$ and $F(-4, 0)$, $V(\sqrt{5} - 2, 0)$ and $V(-\sqrt{5} - 2, 0)$

63. Ellipse: $\dfrac{(x - 1)^2}{2} + (y - 1)^2 = 1$, $C(1, 1)$, $F(2, 1)$ and $F(0, 1)$, $V(\sqrt{2} + 1, 1)$ and $V(-\sqrt{2} + 1, 1)$

65. Hyperbola: $(x - 1)^2 - (y - 2)^2 = 1$, $C(1, 2)$, $F(1 + \sqrt{2}, 2)$ and $F(1 - \sqrt{2}, 2)$, $V(2, 2)$ and $V(0, 2)$; asymptotes: $(y - 2) = \pm(x - 1)$

67. Hyperbola: $\dfrac{(y - 3)^2}{6} - \dfrac{x^2}{3} = 1$, $C(0, 3)$, $F(0, 6)$ and $F(0, 0)$, $V(0, \sqrt{6} + 3)$ and $V(0, -\sqrt{6} + 3)$; asymptotes: $y = \sqrt{2}x + 3$ or $y = -\sqrt{2}x + 3$

69.



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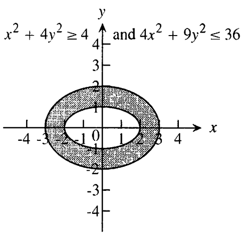

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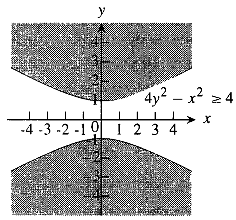


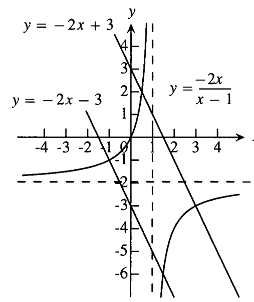
77. $3x^2 + 3y^2 - 7x - 7y + 4 = 0$
79. $(x+2)^2 + (y-1)^2 = 13$. The point is inside the circle.
81. b) $1 : 1$ **83.** Length $= 2\sqrt{2}$, width $= \sqrt{2}$, area $= 4$
85. 24π **87.** $(0, 16/(3\pi))$

Section 9.2, pp. 726–727

1. $e = 3/5,\ F(\pm 3, 0),\ x = \pm 25/3$
3. $e = 1/\sqrt{2},\ F(0, \pm 1),\ y = \pm 2$
5. $e = 1/\sqrt{3},\ F(0, \pm 1),\ y = \pm 3$
7. $e = \sqrt{3}/3,\ F(\pm\sqrt{3}, 0),\ x = \pm 3\sqrt{3}$
9. $\dfrac{x^2}{27} + \dfrac{y^2}{36} = 1$ **11.** $\dfrac{x^2}{4851} + \dfrac{y^2}{4900} = 1$
13. $e = \dfrac{\sqrt{5}}{3},\ \dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ **15.** $e = 1/2,\ \dfrac{x^2}{64} + \dfrac{y^2}{48} = 1$
19. $\dfrac{(x-1)^2}{4} + \dfrac{(y-4)^2}{9} = 1,\ F(1, 4 \pm \sqrt{5}),\ e = \sqrt{5}/3,$ $y = 4 \pm (9\sqrt{5}/5)$
21. $a = 0,\ b = -4,\ c = 0,\ e = \sqrt{3}/2$
23. $e = \sqrt{2},\ F(\pm\sqrt{2}, 0),\ x = \pm 1/\sqrt{2}$
25. $e = \sqrt{2},\ F(0, \pm 4),\ y = \pm 2$
27. $e = \sqrt{5},\ F(\pm\sqrt{10}, 0),\ x = \pm 2/\sqrt{10}$
29. $e = \sqrt{5},\ F(0, \pm\sqrt{10}),\ y = \pm 2/\sqrt{10}$ **31.** $y^2 - \dfrac{x^2}{8} = 1$
33. $x^2 - \dfrac{y^2}{8} = 1$ **35.** $e = \sqrt{2},\ \dfrac{x^2}{8} - \dfrac{y^2}{8} = 1$
37. $e = 2,\ x^2 - \dfrac{y^2}{3} = 1$ **39.** $\dfrac{(y-6)^2}{36} - \dfrac{(x-1)^2}{45} = 1$

Section 9.3, pp. 733–734

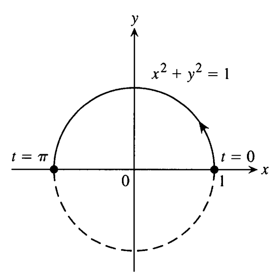
1. Hyperbola **3.** Ellipse **5.** Parabola **7.** Parabola
9. Hyperbola **11.** Hyperbola **13.** Ellipse **15.** Ellipse
17. $x'^2 - y'^2 = 4$, hyperbola **19.** $4x'^2 + 16y' = 0$, parabola
21. $y'^2 = 1$, parallel lines **23.** $2\sqrt{2}x'^2 + 8\sqrt{2}y' = 0$, parabola
25. $4x'^2 + 2y'^2 = 19$, ellipse
27. $\sin\alpha = 1/\sqrt{5},\ \cos\alpha = 2/\sqrt{5}$; or $\sin\alpha = -2/\sqrt{5}$, $\cos\alpha = 1/\sqrt{5}$
29. $A' = 0.88,\ B' = 0.00,\ C' = 3.10,\ D' = 0.74,\ E' = -1.20,$ $F' = -3,\ 0.88x'^2 + 3.10y'^2 + 0.74x' - 1.20y' - 3 = 0$, ellipse
31. $A' = 0.00,\ B' = 0.00,\ C' = 5.00,\ D' = 0,\ E' = 0,\ F' = -5,$ $5.00y'^2 - 5 = 0$ or $y' = \pm 1.00$, parallel lines
33. $A' = 5.05,\ B' = 0.00,\ C' = -0.05,\ D' = -5.07,\ E' = -6.18,$ $F' = -1,\ 5.05x'^2 - 0.05y'^2 - 5.07x' - 6.18y' - 1 = 0$, hyperbola
35. a) $\dfrac{x'^2}{b^2} + \dfrac{y'^2}{a^2} = 1$ b) $\dfrac{y'^2}{a^2} - \dfrac{x'^2}{b^2} = 1$ c) $x'^2 + y'^2 = a^2$
d) $y' = -\dfrac{1}{m}x'$ e) $y' = -\dfrac{1}{m}x' + \dfrac{b}{m}$
37. a) $x'^2 - y'^2 = 2$ b) $x'^2 - y'^2 = 2a$ **43.** a) Parabola
45. a) Hyperbola
b)



c) $y = -2x - 3,\ y = -2x + 3$

Section 9.4, pp. 741–744

1.



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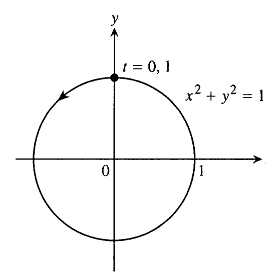

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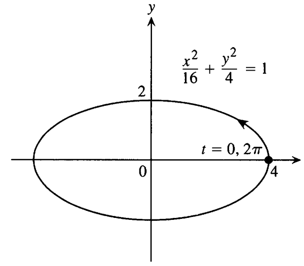

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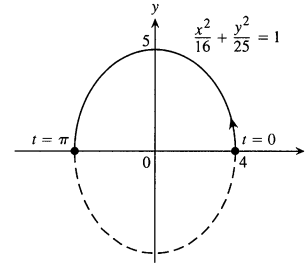

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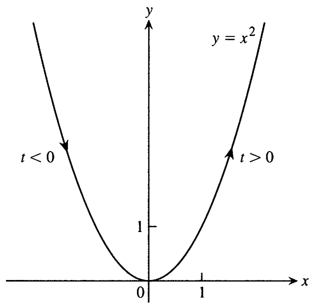

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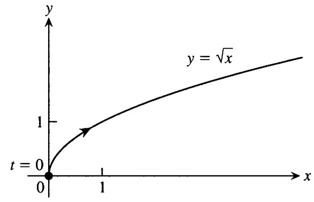

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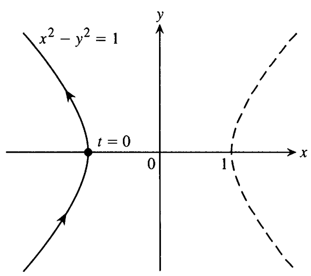

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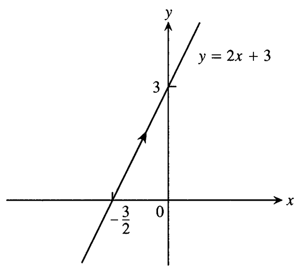

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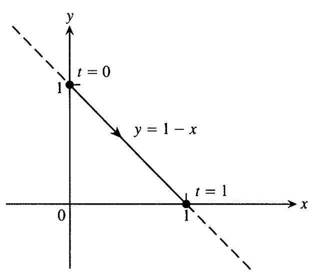

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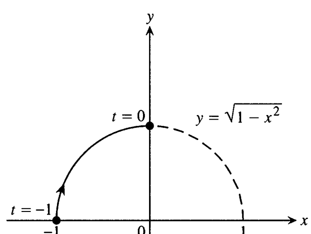

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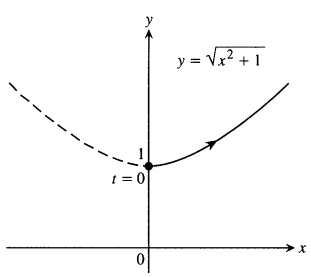

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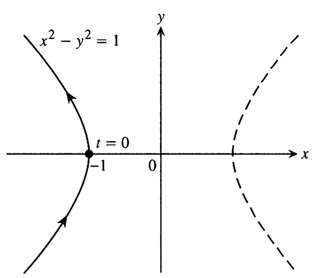


25. a) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 2\pi$ b) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 2\pi$ c) $x = a\cos t,\ y = -a\sin t,\ 0 \le t \le 4\pi$ d) $x = a\cos t,\ y = a\sin t,\ 0 \le t \le 4\pi$

27. $x = \dfrac{-at}{\sqrt{1+t^2}},\ y = \dfrac{a}{\sqrt{1+t^2}},\ -\infty < t < \infty$

29. $x = 2\cot t,\ y = 2\sin^2 t,\ 0 < t < \pi$

31. b) $x = x_1 t,\ y = y_1 t$ (answer not unique) c) $x = -1 + t,\ y = t$ (answer not unique)

33. $x = (a-b)\cos\theta + b\cos\left(\dfrac{a-b}{b}\theta\right),$
$y = (a-b)\sin\theta - b\sin\left(\dfrac{a-b}{b}\theta\right)$

35. $x = a\sin^2 t\tan t,\ y = a\sin^2 t$ **37.** $(1, 1)$

Section 9.5, pp. 749–751

1. $y = -x + 2\sqrt{2},\ \dfrac{d^2y}{dx^2} = -\sqrt{2}$

3. $y = -\dfrac{1}{2}x + 2\sqrt{2},\ \dfrac{d^2y}{dx^2} = -\dfrac{\sqrt{2}}{4}$ **5.** $y = x + \dfrac{1}{4},\ \dfrac{d^2y}{dx^2} = -2$

7. $y = 2x - \sqrt{3},\ \dfrac{d^2y}{dx^2} = -3\sqrt{3}$ **9.** $y = x - 4,\ \dfrac{d^2y}{dx^2} = \dfrac{1}{2}$

11. $y = \sqrt{3}x - \dfrac{\pi\sqrt{3}}{3} + 2,\ \dfrac{d^2y}{dx^2} = -4$ **13.** 0 **15.** -6 **17.** 4

19. 12 **21.** π^2 **23.** $8\pi^2$ **25.** $52\pi/3$ **27.** $3\pi\sqrt{5}$

29. a) $(\bar{x}, \bar{y}) = \left(\dfrac{12}{\pi} - \dfrac{24}{\pi^2}, \dfrac{24}{\pi^2} - 2\right)$ b) Centroid: (1.4, 0.4)

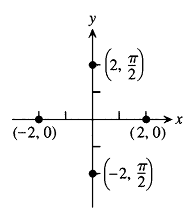
31. a) $(\bar{x}, \bar{y}) = \left(\dfrac{1}{3}, \pi - \dfrac{4}{3}\right)$ **33.** a) π b) π **37.** $3\pi a^2$

39. $64\pi/3$ **41.** $\left(\dfrac{\sqrt{2}}{2}, 1\right)$, $y = 2x$ at $t = 0$, $y = -2x$ at $t = \pi$

Section 9.6, pp. 755–756

1. a) e; b) g; c) h; d) f

3.



a) $\left(2, \dfrac{\pi}{2} + 2n\pi\right)$ and $\left(-2, \dfrac{\pi}{2} + (2n+1)\pi\right)$, n an integer

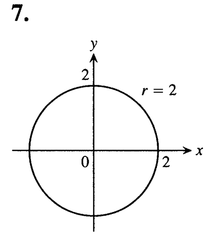
b) $(2, 2n\pi)$ and $(-2, (2n+1)\pi)$, n an integer

c) $\left(2, \dfrac{3\pi}{2} + 2n\pi\right)$ and $\left(-2, \dfrac{3\pi}{2} + (2n+1)\pi\right)$, n an integer

d) $(2, (2n+1)\pi)$ and $(-2, 2n\pi)$, n an integer

5. a) (3, 0) b) (−3, 0) c) $(-1, \sqrt{3})$ d) $(1, \sqrt{3})$, e) (3, 0) f) $(1, \sqrt{3})$ g) (−3, 0) h) $(-1, \sqrt{3})$

7.



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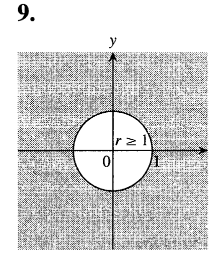

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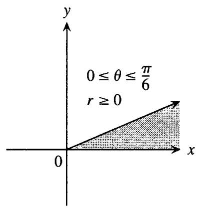

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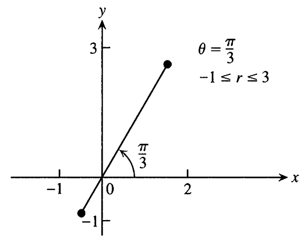

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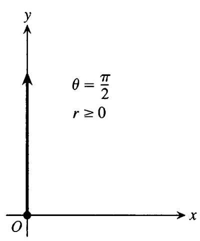

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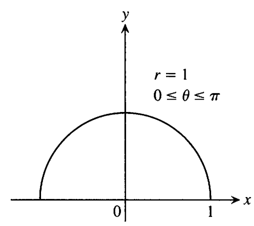

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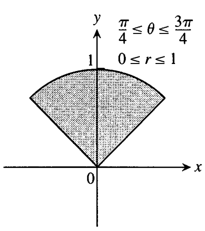

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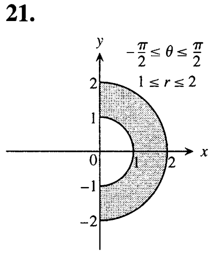
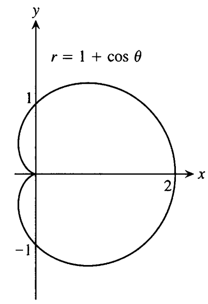


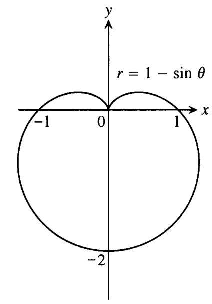
23. $x = 2$, vertical line through $(2, 0)$ **25.** $y = 0$, the x-axis
27. $y = 4$, horizontal line through $(0, 4)$
29. $x + y = 1$, line, $m = -1$, $b = 1$
31. $x^2 + y^2 = 1$, circle, $C(0, 0)$, radius 1
33. $y - 2x = 5$, line, $m = 2$, $b = 5$
35. $y^2 = x$, parabola, vertex $(0, 0)$, opens right
37. $y = e^x$, graph of natural exponential function
39. $x + y = \pm 1$, two straight lines of slope -1, y-intercepts $b = \pm 1$
41. $(x + 2)^2 + y^2 = 4$, circle, $C(-2, 0)$, radius 2
43. $x^2 + (y - 4)^2 = 16$, circle, $C(0, 4)$, radius 4
45. $(x - 1)^2 + (y - 1)^2 = 2$, circle, $C(1, 1)$, radius $\sqrt{2}$
47. $\sqrt{3}y + x = 4$ **49.** $r \cos \theta = 7$ **51.** $\theta = \pi/4$
53. $r = 2$ or $r = -2$ **55.** $4r^2 \cos^2 \theta + 9r^2 \sin^2 \theta = 36$
57. $r \sin^2 \theta = 4 \cos \theta$ **59.** $r = 4 \sin \theta$
61. $r^2 = 6r \cos \theta - 2r \sin \theta - 6$ **63.** $(0, \theta)$, where θ is any angle

Section 9.7, pp. 763–764

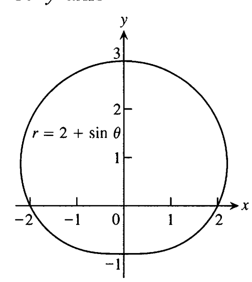
1. x-axis



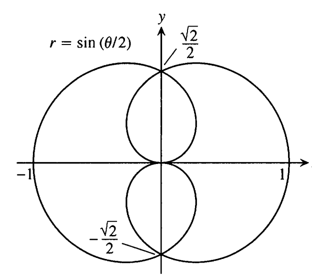
3. y-axis



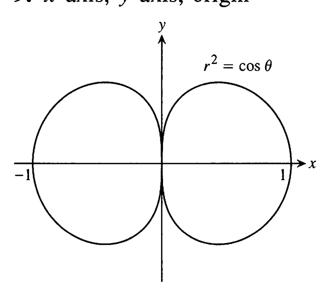
5. y-axis



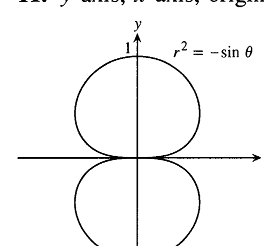
7. x-axis



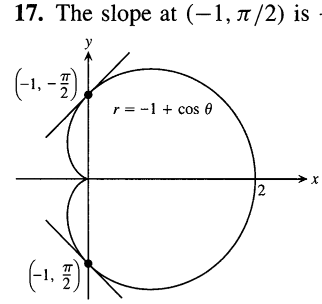
9. x-axis, y-axis, origin



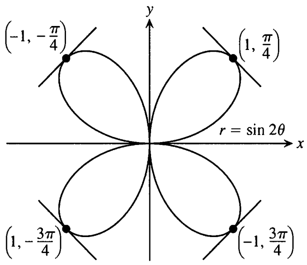
11. y-axis, x-axis, origin



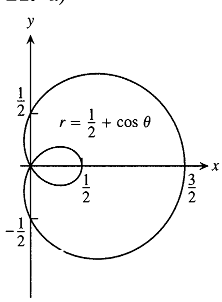
13. x-axis, y-axis, origin **15.** Origin
17. The slope at $(-1, \pi/2)$ is -1, at $(-1, -\pi/2)$ is 1.



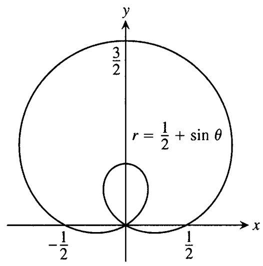
19. The slope at $(1, \pi/4)$ is -1, at $(-1, -\pi/4)$ is 1, at $(-1, 3\pi/4)$ is 1, at $(1, -3\pi/4)$ is -1.



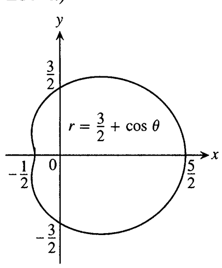
21. a)



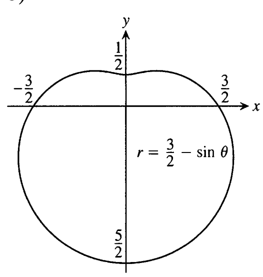
b)



23. a)



b)



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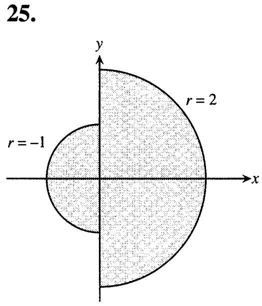

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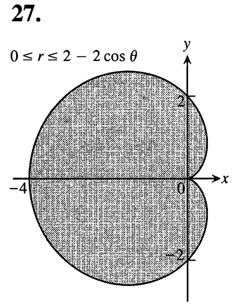


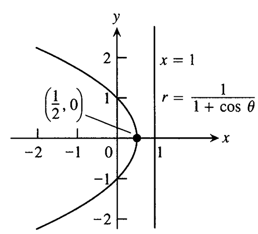
31. $(0, 0)$, $(1, \pi/2)$, $(1, 3\pi/2)$
33. $(0, 0)$, $(\sqrt{3}, \pi/3)$, $(-\sqrt{3}, -\pi/3)$
35. $(\sqrt{2}, \pm\pi/6)$, $(\sqrt{2}, \pm 5\pi/6)$
37. $(1, \pi/12)$, $(1, 5\pi/12)$, $(1, 13\pi/12)$, $(1, 17\pi/12)$ **43.** a

51. $2y = \dfrac{2\sqrt{6}}{9}$

Section 9.8, pp. 768-770

1. $r\cos(\theta - \pi/6) = 5$, $y = -\sqrt{3}x + 10$
3. $r\cos(\theta - 4\pi/3) = 3$, $y = -(\sqrt{3}/3)x - 2\sqrt{3}$ **5.** $y = 2 - x$
7. $y = (\sqrt{3}/3)x + 2\sqrt{3}$ **9.** $r\cos\left(\theta - \dfrac{\pi}{4}\right) = 3$
11. $r\cos\left(\theta + \dfrac{\pi}{2}\right) = 5$ **13.** $r = 8\cos\theta$ **15.** $r = 2\sqrt{2}\sin\theta$
17. $C(2, 0)$, radius $= 2$ **19.** $C(1, \pi)$, radius $= 1$
21. $(x - 6)^2 + y^2 = 36$, $r = 12\cos\theta$
23. $x^2 + (y - 5)^2 = 25$, $r = 10\sin\theta$
25. $(x + 1)^2 + y^2 = 1$, $r = -2\cos\theta$
27. $x^2 + (y + 1/2)^2 = 1/4$, $r = -\sin\theta$ **29.** $r = 2/(1 + \cos\theta)$
31. $r = 30/(1 - 5\sin\theta)$ **33.** $r = 1/(2 + \cos\theta)$
35. $r = 10/(5 - \sin\theta)$

37.



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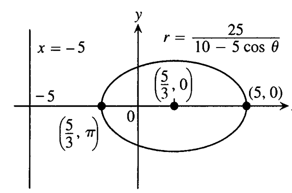

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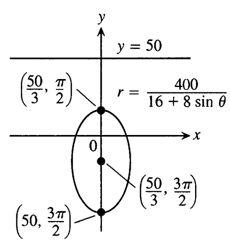

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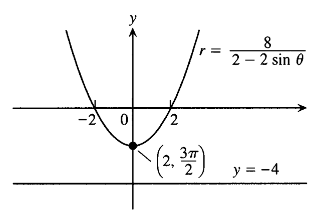
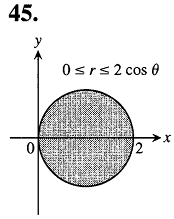


45.

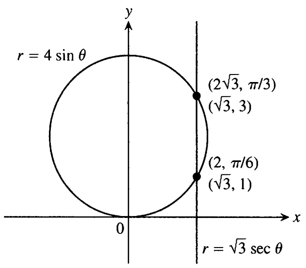


57. b)

Planet	Perihelion	Aphelion
Mercury	0.3075 AU	0.4667 AU
Venus	0.7184 AU	0.7282 AU
Earth	0.9833 AU	1.0167 AU
Mars	1.3817 AU	1.6663 AU
Jupiter	4.9512 AU	5.4548 AU
Saturn	9.0210 AU	10.0570 AU
Uranus	18.2977 AU	20.0623 AU
Neptune	29.8135 AU	30.3065 AU
Pluto	29.6549 AU	49.2251 AU

59. a) $x^2 + (y - 2)^2 = 4$, $x = \sqrt{3}$

b)



61. $r = 4/(1 + \cos\theta)$ **63.** b) The pins should be 2 in. apart.
65. $r = 2a\sin\theta$ (a circle) **67.** $r\cos(\theta - \alpha) = p$ (a line)

Section 9.9, pp. 775–777

1. 18π **3.** $\pi/8$ **5.** 2 **7.** $\frac{\pi}{2} - 1$ **9.** $5\pi - 8$

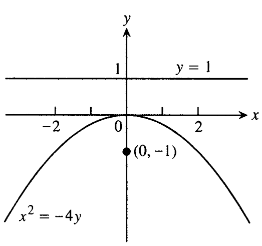
11. $3\sqrt{3} - \pi$ **13.** $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$ **15.** $12\pi - 9\sqrt{3}$ **17.** a) $\frac{3}{2} - \frac{\pi}{4}$

19. $19/3$ **21.** 8 **23.** $3(\sqrt{2} + \ln(1 + \sqrt{2}))$ **25.** $\frac{\pi}{8} + \frac{3}{8}$

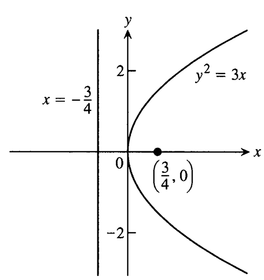
27. 2π **29.** $\pi\sqrt{2}$ **31.** $2\pi(2 - \sqrt{2})$ **37.** $\left(\frac{5}{6}a, 0\right)$

Chapter 9 Practice Exercises, pp. 778–782

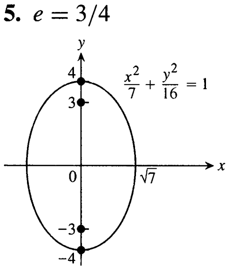
1.



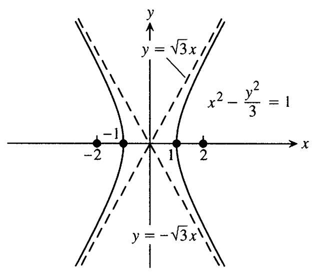
3.



5. $e = 3/4$



7. $e = 2$



9. $(x - 2)^2 = -12(y - 3)$, $V(2, 3)$, $F(2, 0)$; directrix: $y = 6$

11. $\frac{(x + 3)^2}{9} + \frac{(y + 5)^2}{25} = 1$, $C(-3, -5)$, $V(-3, 0)$ and $V(-3, -10)$, $F(-3, -1)$ and $F(-3, -9)$

13. $\frac{(y - 2\sqrt{2})^2}{8} - \frac{(x - 2)^2}{2} = 1$, $C(2, 2\sqrt{2})$, $V(2, 4\sqrt{2})$ and $V(2, 0)$, $F(2, \sqrt{10} + 2\sqrt{2})$ and $F(2, -\sqrt{10} + 2\sqrt{2})$; asymptotes: $y = 2x - 4 + 2\sqrt{2}$ and $y = -2x + 4 + 2\sqrt{2}$

15. Hyperbola: $\frac{(x - 2)^2}{4} - y^2 = 1$, $F(2 \pm \sqrt{5}, 0)$, $V(2 \pm 2, 0)$, $C(2, 0)$; asymptotes: $y = \pm\frac{1}{2}(x - 2)$

17. Parabola: $(y - 1)^2 = -16(x + 3)$, $V(-3, 1)$, $F(-7, 1)$; directrix: $x = 1$

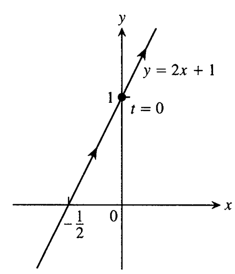
19. Ellipse: $\frac{(x + 3)^2}{16} + \frac{(y - 2)^2}{9} = 1$, $F(\pm\sqrt{7} - 3, 2)$, $V(\pm 4 - 3, 2)$, $C(-3, 2)$

21. Circle: $(x - 1)^2 + (y - 1)^2 = 2$, $C(1, 1)$, radius $= \sqrt{2}$

23. Ellipse **25.** Hyperbola **27.** Line

29. Ellipse, $5x'^2 + 3y'^2 = 30$ **31.** Hyperbola, $x'^2 - y'^2 = 2$

33.



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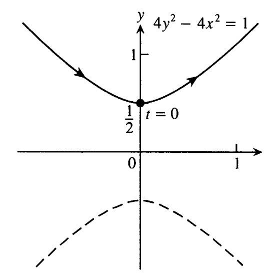
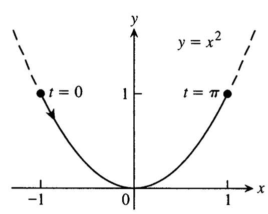


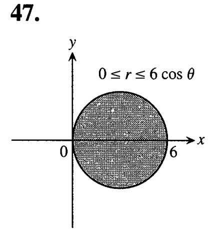
37.



39. $x = 3\cos t$, $y = 4\sin t$, $0 \le t \le 2\pi$ **41.** $y = \frac{\sqrt{3}}{2}x + \frac{1}{4}$, $\frac{1}{4}$

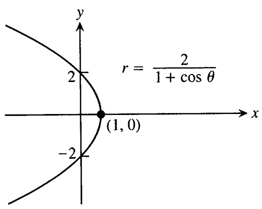
43. $3 + \frac{\ln 2}{8}$ **45.** $76\pi/3$

47.

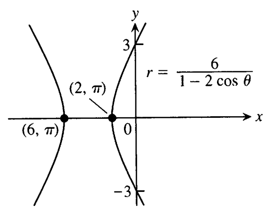


49. d **51.** l **53.** k **55.** i **57.** $(0, 0)$
59. $(0, 0)$, $(1, \pm\pi/2)$ **61.** The graphs coincide. **63.** $(\sqrt{2}, \pi/4)$
65. $y = x$, $y = -x$

67. At $(1, \pi/4) : r \cos(\theta - \pi/4) = 1$, At $(1, 3\pi/4)$: $r \cos(\theta - 3\pi/4) = 1$, At $(1, 5\pi/4) : r \cos(\theta - 5\pi/4) = 1$, At $(1, 7\pi/4) : r \cos(\theta - 7\pi/4) = 1$
69. $y = (\sqrt{3}/3)x - 4$ **71.** $x = 2$ **73.** $y = -3/2$
75. $x^2 + (y+2)^2 = 4$ **77.** $(x - \sqrt{2})^2 + y^2 = 2$
79. $r = -5 \sin\theta$ **81.** $r = 3 \cos\theta$
83.



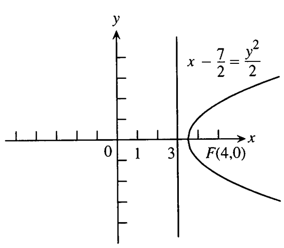
85.



87. $r = \dfrac{4}{1 + 2\cos\theta}$ **89.** $r = \dfrac{2}{2 + \sin\theta}$ **91.** $9\pi/2$
93. $2 + \pi/4$ **95.** 8 **97.** $\pi - 3$ **99.** $(2 - \sqrt{2})\pi$
101. a) 24π b) 16π **111.** $\pi/2$ **115.** $\left(2, \pm\dfrac{\pi}{3}\right)$, $\dfrac{\pi}{2}$
119. $\pi/2$ **121.** $\pi/4$

Chapter 9 Additional Exercises, pp. 783–786

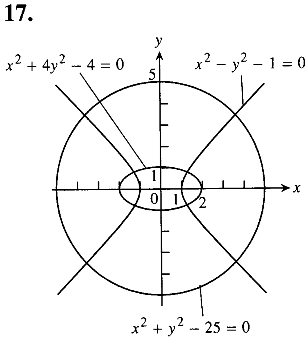
1.



3. $3x^2 + 3y^2 - 8y + 4 = 0$ **5.** $(0, \pm 1)$

7. a) $\dfrac{(y-1)^2}{16} - \dfrac{x^2}{48} = 1$ b) $\dfrac{16\left(y + \dfrac{3}{4}\right)^2}{25} - \dfrac{2x^2}{75} = 1$

17.



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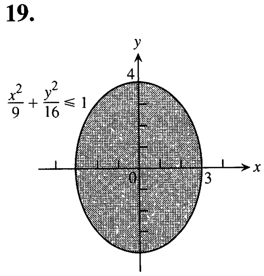
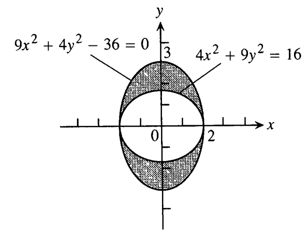
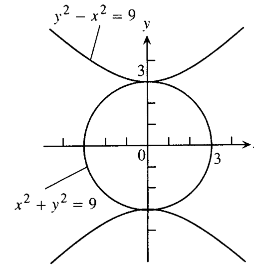


21.



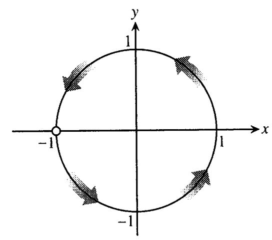
23.



25. $x = (a+b)\cos\theta - b\cos\left(\dfrac{a+b}{b}\theta\right)$,

$y = (a+b)\sin\theta - b\sin\left(\dfrac{a+b}{b}\theta\right)$

27. $(-1, 0)$, $t = -1, 0, 1$



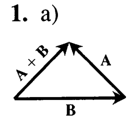
29. a) $r = e^{2\theta}$ b) $\dfrac{\sqrt{5}}{2}(e^{4\pi} - 1)$
31. $\dfrac{32\pi - 4\pi\sqrt{2}}{5}$ **33.** $r = \dfrac{4}{1 + 2\cos\theta}$ **35.** $r = \dfrac{2}{2 + \sin\theta}$
37. a) 120° **39.** 1×10^7 mi. **41.** $e = \sqrt{2/3}$
43. Yes, a parabola **45.** a) $r = \dfrac{2a}{1 + \cos\left(\theta - \dfrac{\pi}{4}\right)}$

b) $r = \dfrac{8}{3 - \cos\theta}$ c) $r = \dfrac{3}{1 + 2\sin\theta}$

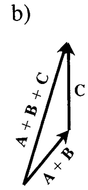
CHAPTER 10

Section 10.1, pp. 794–795

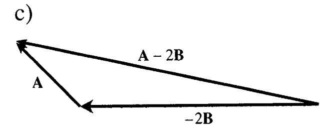
1. a)



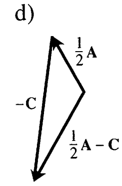
b)



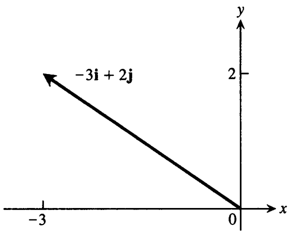
c)



d)



3. $4\mathbf{i}+5\mathbf{j}$ **5.** $(6-(\sqrt{3}/\pi))\,\mathbf{i}-20\,\mathbf{j}$
7. a) $\mathbf{w}=\mathbf{v}+\mathbf{u}$ b) $\mathbf{v}=\mathbf{w}-\mathbf{u}$
9.



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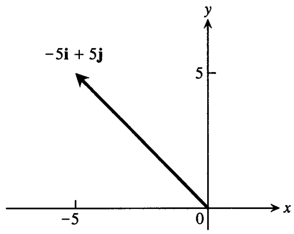

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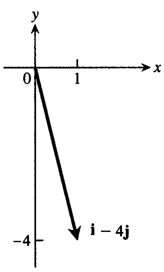

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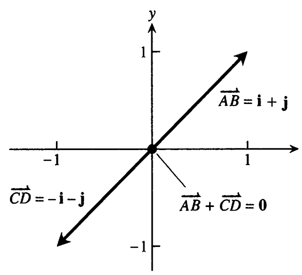
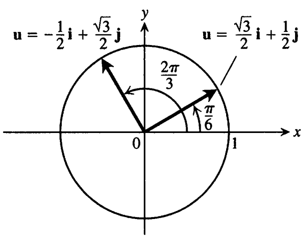
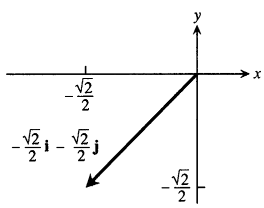


17. (5, 8)
19.



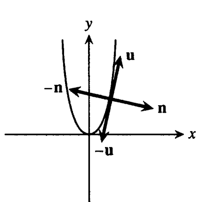
21.



23. $\frac{3}{5}\mathbf{i}-\frac{4}{5}\mathbf{j}$

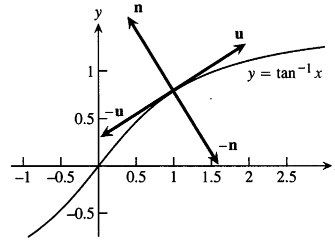
25. $\mathbf{u}=\frac{1}{\sqrt{17}}\mathbf{i}+\frac{4}{\sqrt{17}}\mathbf{j},\ -\mathbf{u}=-\frac{1}{\sqrt{17}}\mathbf{i}-\frac{4}{\sqrt{17}}\mathbf{j},$

$\mathbf{n}=\frac{4}{\sqrt{17}}\mathbf{i}-\frac{1}{\sqrt{17}}\mathbf{j},\ -\mathbf{n}=-\frac{4}{\sqrt{17}}\mathbf{i}+\frac{1}{\sqrt{17}}\mathbf{j}$



27. $\mathbf{u}=\frac{1}{\sqrt{5}}(2\mathbf{i}+\mathbf{j}),\ -\mathbf{u}=\frac{1}{\sqrt{5}}(-2\mathbf{i}-\mathbf{j}),\ \mathbf{n}=\frac{1}{\sqrt{5}}(-\mathbf{i}+2\mathbf{j}),$

$-\mathbf{n}=\frac{1}{\sqrt{5}}(\mathbf{i}-2\mathbf{j})$



29. $\mathbf{u}=\frac{\pm 1}{5}(-4\mathbf{i}+3\mathbf{j}),\ \mathbf{v}=\frac{\pm 1}{5}(3\mathbf{i}+4\mathbf{j})$

31. $\mathbf{u}=\frac{\pm 1}{2}(\mathbf{i}+\sqrt{3}\mathbf{j}),\ \mathbf{v}=\frac{\pm 1}{2}(-\sqrt{3}\mathbf{i}+\mathbf{j})$

33. $13\left(\frac{5}{13}\mathbf{i}+\frac{12}{13}\mathbf{j}\right)$

35. $\frac{3}{5}\mathbf{i}-\frac{4}{5}\mathbf{j}$ and $-\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{j}$ **39.** $5\sqrt{3}\mathbf{i},\ 5\mathbf{j}$

41. $\alpha=3/2,\ \beta=1/2$

43. a) $(5\cos 60°, 5\sin 60°)=\left(\frac{5}{2},\frac{5\sqrt{3}}{2}\right)$

b) $(5\cos 60°+10\cos 315°,\ 5\sin 60°+10\sin 315°)$

$=\left(\frac{5+\sqrt{2}}{2},\frac{5\sqrt{3}-10\sqrt{2}}{2}\right)$

45. The slope of $-\mathbf{v}=-a\,\mathbf{i}-b\mathbf{j}$ is $(-b)/(-a)=b/a$, the same as the slope of $\mathbf{v}$.

Section 10.2, pp. 804–806

1. The line through the point (2, 3, 0) parallel to the z-axis
3. The x-axis **5.** The circle $x^2+y^2=4$ in the xy-plane
7. The circle $x^2+z^2=4$ in the xz-plane
9. The circle $y^2+z^2=1$ in the yz-plane
11. The circle $x^2+y^2=16$ in the xy-plane
13. a) The first quadrant of the xy-plane
b) The fourth quadrant of the xy-plane
15. a) The ball of radius 1 centered at the origin
b) All points greater than 1 unit from the origin
17. a) The upper hemisphere of radius 1 centered at the origin
b) The solid upper hemisphere of radius 1 centered at the origin
19. a) $x=3$ b) $y=-1$ c) $z=-2$ **21.** a) $z=1$ b) $x=3$
c) $y=-1$ **23.** a) $x^2+(y-2)^2=4,\ z=0$
b) $(y-2)^2+z^2=4,\ x=0$ c) $x^2+z^2=4,\ y=2$
25. a) $y=3,\ z=-1$ b) $x=1,\ z=-1$ c) $x=1,\ y=3$
27. $x^2+y^2+z^2=25,\ z=3$ **29.** $0\le z\le 1$ **31.** $z\le 0$
33. a) $(x-1)^2+(y-1)^2+(z-1)^2<1$
b) $(x-1)^2+(y-1)^2+(z-1)^2>1$ **35.** $3\left(\frac{2}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}\right)$

37. $9\left(\frac{1}{9}\mathbf{i}+\frac{4}{9}\mathbf{j}-\frac{8}{9}\mathbf{k}\right)$ **39.** $5(\mathbf{k})$ **41.** $1\left(\frac{3}{5}\mathbf{i}+\frac{4}{5}\mathbf{k}\right)$

43. $\sqrt{\frac{1}{2}}\left(\frac{1}{\sqrt{3}}\mathbf{i}-\frac{1}{\sqrt{3}}\mathbf{j}-\frac{1}{\sqrt{3}}\mathbf{k}\right)$ **45.** a) $2\mathbf{i}$ b) $-\sqrt{3}\mathbf{k}$
c) $\frac{3}{10}\mathbf{j}+\frac{2}{5}\mathbf{k}$ d) $6\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ **47.** $\frac{7}{13}(12\mathbf{i}-5\mathbf{k})$
49. $-\frac{10}{7}\mathbf{i}+\frac{15}{7}\mathbf{j}-\frac{30}{7}\mathbf{k}$ **51.** a) 3 b) $\frac{2}{3}\mathbf{i}+\frac{2}{3}\mathbf{j}-\frac{1}{3}\mathbf{k}$
c) (2, 2, 1/2) **53.** a) 7 b) $\frac{3}{7}\mathbf{i}-\frac{6}{7}\mathbf{j}+\frac{2}{7}\mathbf{k}$ c) (5/2, 1, 6)
55. a) $2\sqrt{3}$ b) $\frac{1}{\sqrt{3}}\mathbf{i}-\frac{1}{\sqrt{3}}\mathbf{j}-\frac{1}{\sqrt{3}}\mathbf{k}$ c) $(1, -1, -1)$
57. $A(4, -3, 5)$ **59.** $C(-2, 0, 2)$, $a = 2\sqrt{2}$
61. $C(\sqrt{2}, \sqrt{2}, -\sqrt{2})$, $a = \sqrt{2}$
63. $(x-1)^2 + (y-2)^2 + (z-3)^2 = 14$
65. $(x+2)^2 + y^2 + z^2 = 3$ **67.** $C(-2, 0, 2)$, $a = \sqrt{8}$
69. $C\left(-\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}\right)$, $a = \frac{5\sqrt{3}}{4}$
71. a) $\sqrt{y^2+z^2}$ b) $\sqrt{x^2+z^2}$ c) $\sqrt{x^2+y^2}$
73. a) $\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}-3\mathbf{k}$ b) $\mathbf{i}+\mathbf{j}-2\mathbf{k}$ c) (2, 2, 1)

Section 10.3, pp. 812–814

1. a) -25, 5, 5 b) -1 c) -5 d) $-2\mathbf{i}+4\mathbf{j}-\sqrt{5}\mathbf{k}$
3. a) 25, 15, 5 b) 1/3 c) 5/3 d) $\frac{1}{9}(10\mathbf{i}+11\mathbf{j}-2\mathbf{k})$
5. a) 0, $\sqrt{53}$, 1 b) 0 c) 0 d) 0 **7.** a) 2, $\sqrt{34}$, $\sqrt{3}$
b) $\frac{2}{\sqrt{3}\sqrt{34}}$ c) $\frac{2}{\sqrt{34}}$ d) $\frac{1}{17}(5\mathbf{j}-3\mathbf{k})$ **9.** a) $\sqrt{3}-\sqrt{2}$, $\sqrt{2}$, 3
b) $\frac{\sqrt{3}-\sqrt{2}}{3\sqrt{2}}$ c) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}}$ d) $\frac{\sqrt{3}-\sqrt{2}}{2}(-\mathbf{i}+\mathbf{j})$
11. $\left(\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}\right)+\left(-\frac{3}{2}\mathbf{i}+\frac{3}{2}\mathbf{j}+4\mathbf{k}\right)$
13. $\left(\frac{14}{3}\mathbf{i}+\frac{28}{3}\mathbf{j}-\frac{14}{3}\mathbf{k}\right)+\left(\frac{10}{3}\mathbf{i}-\frac{16}{3}\mathbf{j}-\frac{22}{3}\mathbf{k}\right)$
15. The sum of two vectors of equal length is *always* orthogonal to their difference, as we can see from the equation

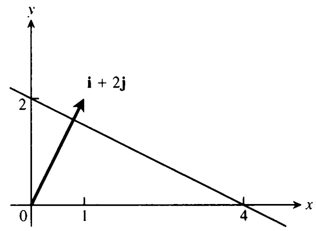
$$(\mathbf{v}_1+\mathbf{v}_2)\cdot(\mathbf{v}_1-\mathbf{v}_2) = \mathbf{v}_1\cdot\mathbf{v}_1+\mathbf{v}_2\cdot\mathbf{v}_1-\mathbf{v}_1\cdot\mathbf{v}_2-\mathbf{v}_2\cdot\mathbf{v}_2 = |\mathbf{v}_1|^2-|\mathbf{v}_2|^2.$$

21. $\tan^{-1}\sqrt{2}$ **23.** 0.75 rad **25.** 1.77 rad
27. $\angle A \approx 1.24$ rad, $\angle B \approx 0.66$ rad, $\angle C \approx 1.24$ rad **29.** 0.62 rad
31. a) Since $|\cos\theta| \le 1$, we have

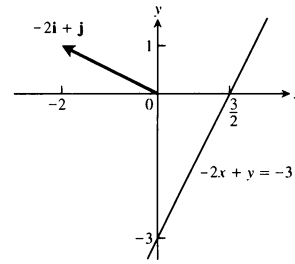
$$|\mathbf{u}\cdot\mathbf{v}| = |\mathbf{u}||\mathbf{v}||\cos\theta| \le |\mathbf{u}||\mathbf{v}|(1) = |\mathbf{u}||\mathbf{v}|.$$

b) We have equality precisely when $|\cos\theta| = 1$ or when one or both of **u** and **v** are **0**. In the case of nonzero vectors, we have equality when $\theta = 0$ or π, i.e., when the vectors are parallel.
33. a **35.** a) $\sqrt{70}$ b) $\sqrt{568}$ **37.** 5 J **39.** 3464.10 J

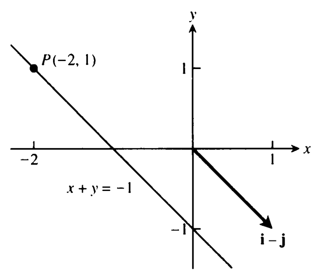
43. $x + 2y = 4$



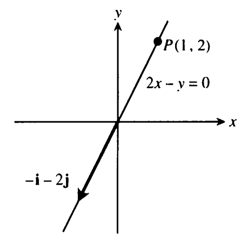
45. $-2x + y = -3$



47. $x + y = -1$



49. $2x - y = 0$

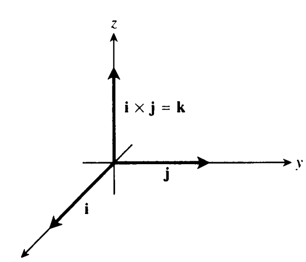


51. $\pi/4$ **53.** $\pi/6$ **55.** 0.14 **57.** $\pi/3$ and $2\pi/3$ at each point
59. At (0, 0): $\pi/2$, at (1, 1): $\pi/4$ and $3\pi/4$

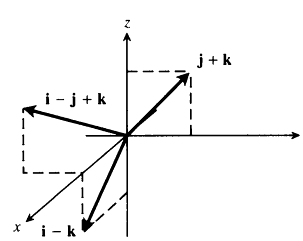
Section 10.4, pp. 820–821

1. $|\mathbf{A}\times\mathbf{B}| = 3$, direction is $\frac{2}{3}\mathbf{i}+\frac{1}{3}\mathbf{j}+\frac{2}{3}\mathbf{k}$; $|\mathbf{B}\times\mathbf{A}| = 3$, direction is $-\frac{2}{3}\mathbf{i}-\frac{1}{3}\mathbf{j}-\frac{2}{3}\mathbf{k}$
3. $|\mathbf{A}\times\mathbf{B}| = 0$, no direction; $|\mathbf{B}\times\mathbf{A}| = 0$, no direction
5. $|\mathbf{A}\times\mathbf{B}| = 6$, direction is $-\mathbf{k}$; $|\mathbf{B}\times\mathbf{A}| = 6$, direction is $\mathbf{k}$
7. $|\mathbf{A}\times\mathbf{B}| = 6\sqrt{5}$, direction is $\frac{1}{\sqrt{5}}\mathbf{i}-\frac{2}{\sqrt{5}}\mathbf{k}$; $|\mathbf{B}\times\mathbf{A}| = 6\sqrt{5}$, direction is $-\frac{1}{\sqrt{5}}\mathbf{i}+\frac{2}{\sqrt{5}}\mathbf{k}$

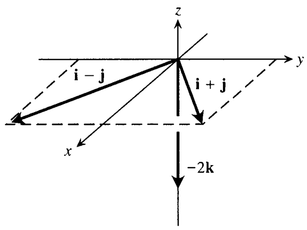
9.



11.



13.



15. a) $2\sqrt{6}$ b) $\pm\frac{1}{\sqrt{6}}(2\mathbf{i}+\mathbf{j}+\mathbf{k})$ **17.** a) $\frac{\sqrt{2}}{2}$ b) $\pm\frac{1}{\sqrt{2}}(\mathbf{i}-\mathbf{j})$
19. a) None b) **A** and **C** **21.** $10\sqrt{3}$ ft • lb **23.** 8 **25.** 7
27. a) True b) Not always true c) True d) True e) Not always true f) True g) True h) True
29. a) $\text{proj}_{\mathbf{B}}\mathbf{A} = \frac{\mathbf{A}\cdot\mathbf{B}}{\mathbf{B}\cdot\mathbf{B}}\mathbf{B}$ b) $\pm\mathbf{A}\times\mathbf{B}$ c) $\pm(\mathbf{A}\times\mathbf{B})\times\mathbf{C}$
d) $|(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}|$ **31.** a) Yes b) No c) Yes d) No
33. No, **B** need not equal **C**. For example, $\mathbf{i}+\mathbf{j} \neq -\mathbf{i}+\mathbf{j}$, but

$$\mathbf{i}\times(\mathbf{i}+\mathbf{j}) = \mathbf{i}\times\mathbf{i}+\mathbf{i}\times\mathbf{j} = \mathbf{0}+\mathbf{k} = \mathbf{k}$$

$$\mathbf{i}\times(-\mathbf{i}+\mathbf{j}) = -\mathbf{i}\times\mathbf{i}+\mathbf{i}\times\mathbf{j} = \mathbf{0}+\mathbf{k} = \mathbf{k}$$

35. 2 **37.** 13 **39.** 11/2 **41.** 25/2
43. If $\mathbf{A} = a_1\mathbf{i}+a_2\mathbf{j}$ and $\mathbf{B} = b_1\mathbf{i}+b_2\mathbf{j}$, then

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & 0 \\ b_1 & b_2 & 0 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\mathbf{k}$$

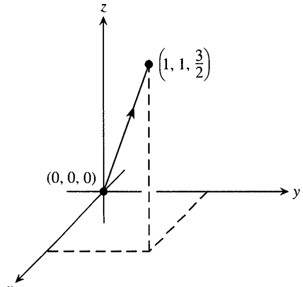
and the triangle's area is

$$\frac{1}{2}|\mathbf{A}\times\mathbf{B}| = \pm\frac{1}{2}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}.$$

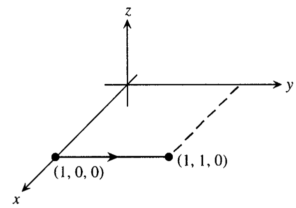
The applicable sign is (+) if the acute angle from **A** to **B** runs counterclockwise in the xy-plane, and (−) if it runs clockwise.

Section 10.5, pp. 827–829

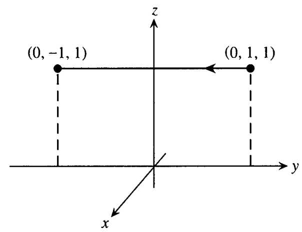
1. $x = 3+t,\ y = -4+t,\ z = -1+t$
3. $x = -2+5t,\ y = 5t,\ z = 3-5t$ **5.** $x = 0,\ y = 2t,\ z = t$
7. $x = 1,\ y = 1,\ z = 1+t$ **9.** $x = t,\ y = -7+2t,\ z = 2t$
11. $x = t,\ y = 0,\ z = 0$
13. $x = t,\ y = t,\ z = \frac{3}{2}t,\ 0 \le t \le 1$



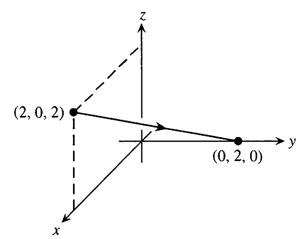
15. $x = 1,\ y = 1+t,\ z = 0,\ -1 \le t \le 0$



17. $x = 0,\ y = 1-2t,\ z = 1,\ 0 \le t \le 1$



19. $x = 2-2t,\ y = 2t,\ z = 2-2t,\ 0 \le t \le 1$

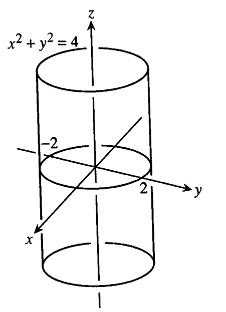


21. $3x-2y-z = -3$ **23.** $7x-5y-4z = 6$
25. $x+3y+4z = 34$ **27.** $(1, 2, 3),\ -20x+12y+z = 7$
29. $y+z = 3$ **31.** $x-y+z = 0$ **33.** $2\sqrt{30}$ **35.** 0
37. $\frac{9\sqrt{42}}{7}$ **39.** 3 **41.** 19/5 **43.** 5/3 **45.** $9/\sqrt{41}$
47. $\pi/4$ **49.** 1.76 rad **51.** 0.82 rad **53.** $\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right)$
55. (1, 1, 0) **57.** $x = 1-t,\ y = 1+t,\ z = -1$
59. $x = 4,\ y = 3+6t,\ z = 1+3t$
61. $L1$ intersects $L2$; $L2$ is parallel to $L3$; $L1$ and $L3$ are skew.
63. $x = 2+2t,\ y = -4-t,\ z = 7+3t;\ x = -2-t,\ y = -2+(1/2)t,\ z = 1-(3/2)t$
65. $\left(0, -\frac{1}{2}, -\frac{3}{2}\right),\ (-1, 0, -3),\ (1, -1, 0)$
69. Many possible answers. One possibility: $x+y = 3$ and $2y+z = 7$
71. $(x/a)+(y/b)+(z/c) = 1$ describes all planes *except* those through the origin or parallel to a coordinate axis.

Section 10.6, pp. 839–841

1. d, ellipsoid **3.** a, cylinder **5.** l, hyperbolic paraboloid
7. b, cylinder **9.** k, hyperbolic paraboloid **11.** h, cone

13.



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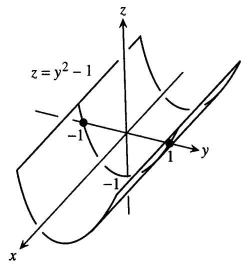

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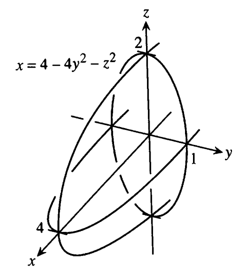

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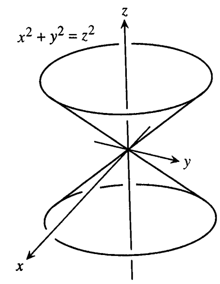

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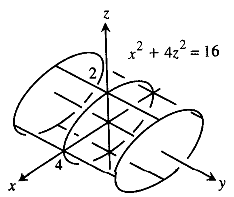

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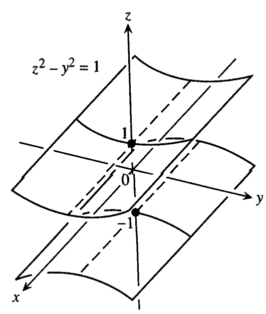

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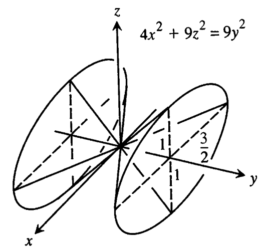

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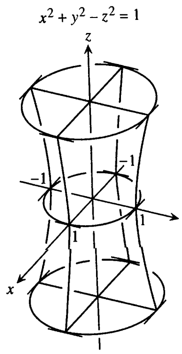

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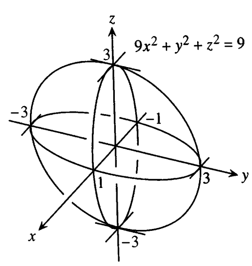

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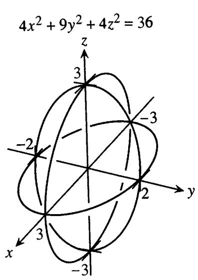

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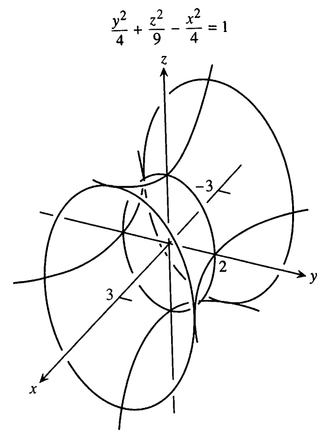
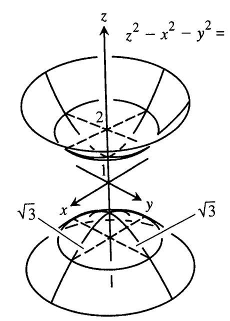


39.



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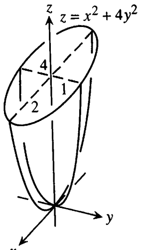

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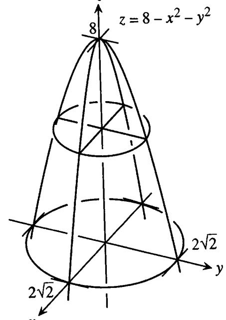

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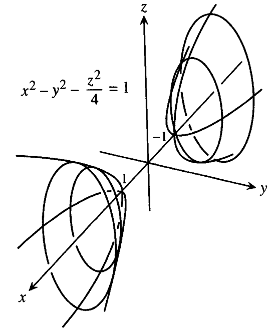

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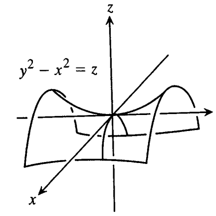

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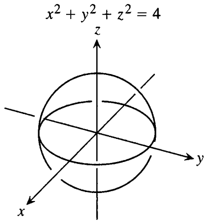

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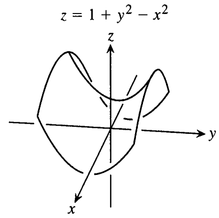

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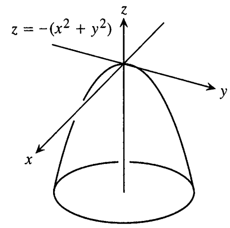

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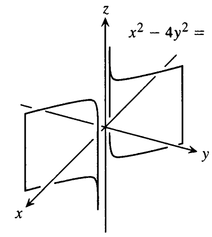

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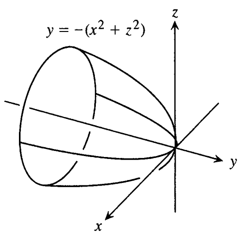

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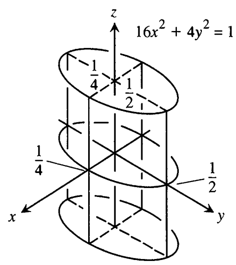

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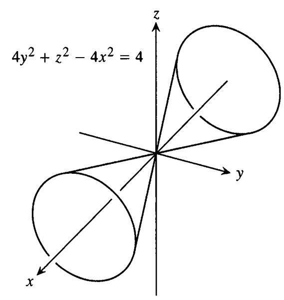

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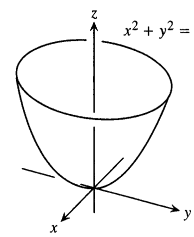

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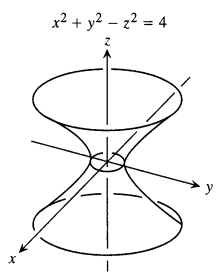

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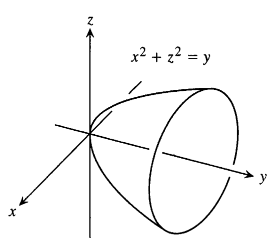

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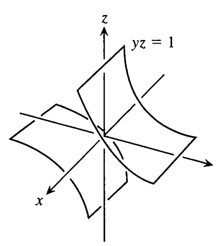

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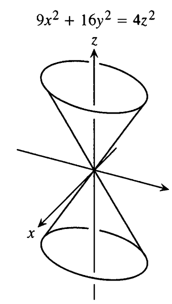

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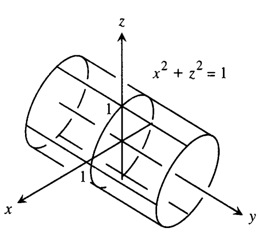

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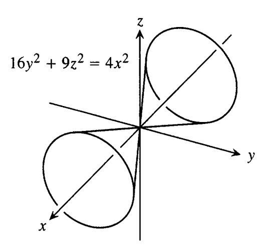

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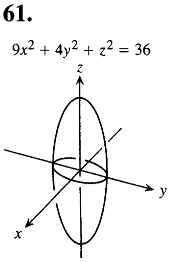
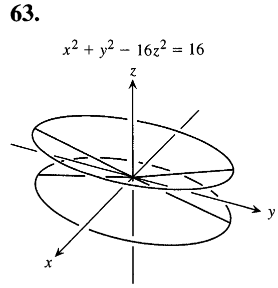


63.



77. a) $\dfrac{2\pi(9 - c^2)}{9}$ b) 8π c) $\dfrac{4\pi abc}{3}$

81. Vertex $(0, y_1, cy_1^2/b^2)$, focus $(0, y_1, c(y_1^2/b^2) - a^2/(4c))$

Section 10.7, pp. 846–847

	Rectangular	Cylindrical	Spherical
1.	$(0, 0, 0)$	$(0, 0, 0)$	$(0, 0, 0)$
3.	$(0, 1, 0)$	$(1, \pi/2, 0)$	$(1, \pi/2, \pi/2)$
5.	$(1, 0, 0)$	$(1, 0, 0)$	$(1, \pi/2, 0)$
7.	$(0, 1, 1)$	$(1, \pi/2, 1)$	$(\sqrt{2}, \pi/4, \pi/2)$
9.	$(0, -2\sqrt{2}, 0)$	$(2\sqrt{2}, 3\pi/2, 0)$	$(2\sqrt{2}, \pi/2, 3\pi/2)$

11. $x^2 + y^2 = 0$, $\theta = 0$ or $\theta = \pi$, the z-axis

13. $z = 0$, $\phi = \pi/2$, the xy-plane

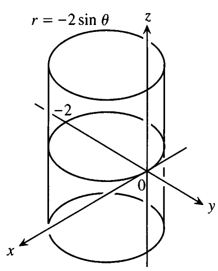
15. $z = r$, $0 \le r \le 1$; $\phi = \pi/4$, $0 \le \rho \le \sqrt{2}$; a (finite) cone

17. $x = 0$, $\theta = \pi/2$, the yz-plane

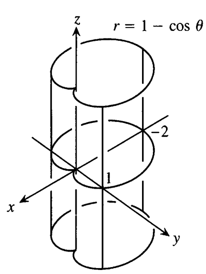
19. $r^2 + z^2 = 4$, $\rho = 2$, sphere of radius 2 centered at the origin

21. $x^2 + y^2 + \left(z - \frac{5}{2}\right)^2 = \frac{25}{4}$, $r^2 + z^2 = 5z$, sphere of radius $5/2$ centered at $(0, 0, 5/2)$ (rectangular)

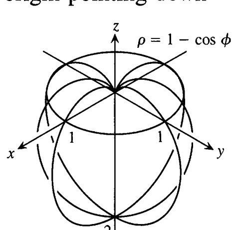
23. $y = 1$, $\rho \sin\phi \sin\theta = 1$, the plane $y = 1$
25. $z = \sqrt{2}$, the plane $z = \sqrt{2}$
27. $r^2 + z^2 = 2z$, $z \le 1$; $\rho = 2\cos\phi$, $\pi/4 \le \phi \le \pi/2$; lower half (hemisphere) of the sphere of radius 1 centered at (0, 0, 1) (rectangular)
29. $x^2 + y^2 + z^2 = 9$, $-3/2 \le z \le 3/2$; $r^2 + z^2 = 9$, $-3/2 \le z \le 3/2$; the portion of the sphere of radius 3 centered at the origin between the planes $z = -3/2$ and $z = 3/2$
31. $z = 4 - 4(x^2 + y^2)$, $0 \le z \le 4$; $\rho\cos\phi = 4 - 4\rho^2\sin^2\phi$, $0 \le \phi \le \pi/2$; the upper portion cut from the paraboloid $z = 4 - 4(x^2 + y^2)$ by the xy-plane
33. $z = -\sqrt{x^2 + y^2}$, $-1 \le z \le 0$; $z = -r$, $0 \le r \le 1$; cone, vertex at origin, base the circle $x^2 + y^2 = 1$ in the plane $z = -1$
35. $z + x^2 - y^2 = 0$ or $z = y^2 - x^2$, $\cos\phi + \rho\sin^2\phi\cos 2\theta = 0$, hyperbolic paraboloid
37. (2, 3, 1)
39. Right circular cylinder parallel to the z-axis generated by the circle $r = -2\sin\theta$ in the $r\theta$-plane



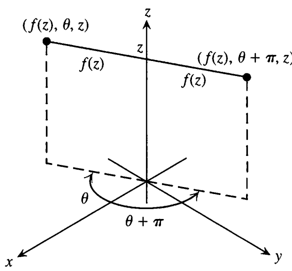
41. Cylinder of lines parallel to the z-axis generated by the cardioid $r = 1 - \cos\theta$ in the $r\theta$-plane



43. Cardioid of revolution symmetric about the y-axis, cusp at the origin pointing down

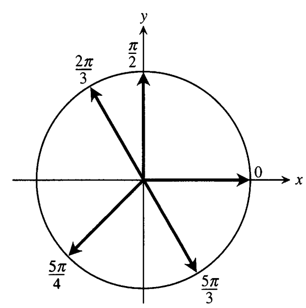


45. b) $\phi = \pi/2$
49. The surface's equation $r = f(z)$ tells us that the point $(r, \theta, z) = (f(z), \theta, z)$ will lie on the surface for all θ. In particular $(f(z), \theta + \pi, z)$ lies on the surface whenever $(f(z), \theta, z)$ lies on the surface, so the surface is symmetric with respect to the z-axis.



Chapter 10 Practice Exercises, pp. 848–851

1.

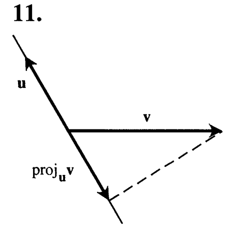


3. $2 \cdot \left(\frac{1}{\sqrt{2}}\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j}\right)$ **5.** $7 \cdot \left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{6}{7}\mathbf{k}\right)$

7. $\frac{8}{\sqrt{33}}\mathbf{i} - \frac{2}{\sqrt{33}}\mathbf{j} + \frac{8z}{\sqrt{33}}\mathbf{k}$
9. a) $\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB}$

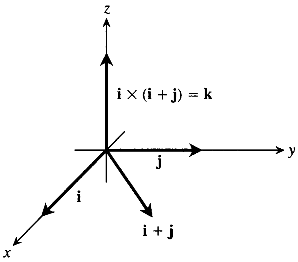
b) $\overrightarrow{AP} = \frac{1}{2}(\overrightarrow{AB} + \overrightarrow{AD})$

11.



13. $|\mathbf{A}| = \sqrt{2}$, $|\mathbf{B}| = 3$, $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A} = 3$, $\mathbf{A} \times \mathbf{B} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\mathbf{B} \times \mathbf{A} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, $|\mathbf{A} \times \mathbf{B}| = 3$, $\theta = \pi/4$, $|\mathbf{B}|\cos\theta = 3/\sqrt{2}$, $\text{proj}_{\mathbf{A}}\mathbf{B} = (3/2)(\mathbf{i} + \mathbf{j})$
15. $\frac{4}{3}(2\mathbf{i} + \mathbf{j} - \mathbf{k}) - \frac{1}{3}(5\mathbf{i} + \mathbf{j} + 11\mathbf{k})$

17.



19. unit tangents $\pm\left(\frac{1}{\sqrt{5}}\mathbf{i}+\frac{2}{\sqrt{5}}\mathbf{j}\right)$, unit normals $\pm\left(-\frac{2}{\sqrt{5}}\mathbf{i}+\frac{1}{\sqrt{5}}\mathbf{j}\right)$

23. $2\sqrt{7}$ **25.** a) $A=\sqrt{14}$ b) $V=1$ **29.** $\sqrt{78}/3$

31. $x=1-3t,\ y=2,\ z=3+7t$ **33.** $\sqrt{2}$

35. $2x+y-z=3$ **37.** $-9x+y+7z=4$

39. $\left(0,-\frac{1}{2},-\frac{3}{2}\right)$, $(-1,0,-3)$, $(1,-1,0)$

41. $\pi/3$ **43.** $x=-5+5t,\ y=3-t,\ z=-3t$

45. b) $x=-12t,\ y=19/12+15t,\ z=1/6+6t$

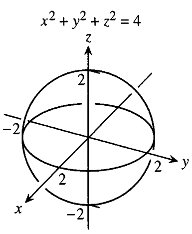
47. Yes; $\mathbf{v}$ is parallel to the plane. **49.** 3 **51.** $-3\mathbf{j}+3\mathbf{k}$

53. $\frac{2}{\sqrt{35}}(5\mathbf{i}-\mathbf{j}-3\mathbf{k})$ **55.** $\left(\frac{11}{9},\frac{26}{9},\frac{7}{9}\right)$

57. $(1,-2,-1)$; $x=1-5t,\ y=-2+3t,\ z=-1+4t$

59. $2x+7y+2z+10=0$ **61.** a) No b) no c) no d) no e) yes **63.** $11/\sqrt{107}$

65.



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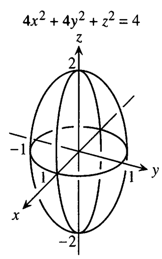

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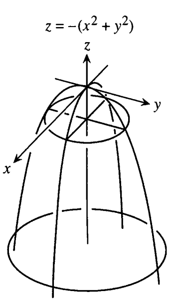

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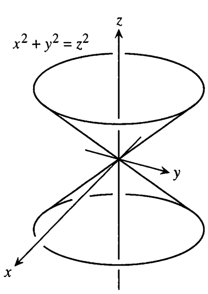

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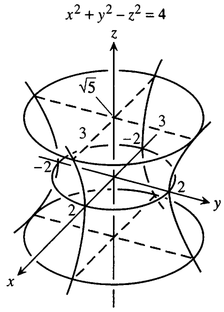

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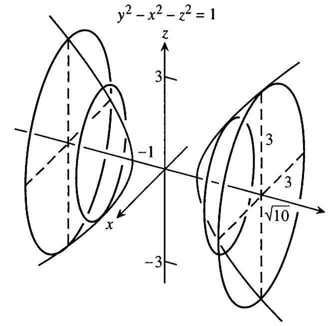


77. The y-axis in the xy-plane; the yz-plane in three dimensional space

79. The circle centered at $(0,0)$ with radius 2 in the xy-plane; the cylinder parallel to the z-axis in three dimensional space with the circle as a generating curve

81. The parabola $x=y^2$ in the xy-plane; the cylinder parallel to the z-axis in three dimensional space with the parabola as a generating curve

83. A cardioid in the $r\theta$-plane; a cylinder parallel with the z-axis in three dimensional space with the cardioid as a generating curve

85. A horizontal lemniscate of length $2\sqrt{2}$ in the $r\theta$-plane; the cylinder parallel to the z-axis in three dimensional space with the lemniscate as a generating curve

87. The sphere of radius 2 centered at the origin

89. The upper nappe of the cone having its vertex at the origin and making a $\pi/6$ angle with the z-axis

91. The upper hemisphere of the sphere of radius 1 centered at the origin

	Rectangular	**Cylindrical**	**Spherical**
93.	$(1, 0, 0)$	$(1, 0, 0)$	$(1, \pi/2, 0)$
95.	$(0, 1, 1)$	$(1, \pi/2, 1)$	$(\sqrt{2}, \pi/4, \pi/2)$
97.	$(-1, 0, -1)$	$(1, \pi, -1)$	$(\sqrt{2}, 3\pi/4, \pi)$

99. Cylindrical: $z=2$, spherical: $\rho\cos\phi=2$, a plane parallel with the xy-plane

101. Cylindrical: $r^2+z^2=-2z$, spherical: $\rho=-2\cos\phi$, sphere of radius 1 centered at $(0,0,-1)$ (rectangular)

103. Rectangular: $z=x^2+y^2$, spherical: $\rho=0$ or $\rho=\frac{\cos\phi}{\sin^2\phi}$ when $0<\phi<\pi/2$, a paraboloid symmetric to the z-axis, opening upward, vertex at the origin

105. Rectangular: $x^2+(y-7/2)^2=49/4$, spherical: $\rho\sin\phi=7\sin\theta$, cylinder parallel to the z-axis generated by the circle

107. Rectangular: $x^2+y^2+z^2=16$, cylindrical: $r^2+z^2=16$, sphere of radius 4 centered at the origin

109. Rectangular: $-\sqrt{x^2+y^2}=z$, cylindrical: $z=-r,\ r\geq 0$, single cone making an angle of $3\pi/4$ with the positive z-axis, vertex at the origin

Chapter 10 Additional Exercises, pp. 851–853

1. $(26, 23, -1/3)$ **3.** $\approx 34{,}641$ J

17. b) $6/\sqrt{14}$ c) $2x - y + 2z = 8$
d) $x - 2y + z = 3 + 5\sqrt{6}$ and $x - 2y + z = 3 - 5\sqrt{6}$

23. $\mathbf{v} - 2\dfrac{\mathbf{v} \cdot \mathbf{z}}{|\mathbf{z}|^2}\mathbf{z}$ **25.** a) $|\mathbf{F}| = \dfrac{GMm}{d^2}\left(1 + \sum_{i=1}^{n} \dfrac{2}{(i^2+1)^{3/2}}\right)$
b) Yes

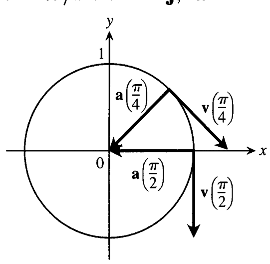
CHAPTER 11

Section 11.1, pp. 865–868

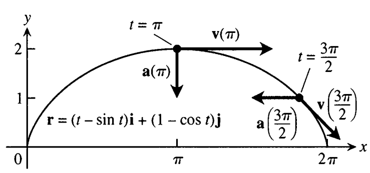
1. $y = x^2 - 2x,\ \mathbf{v} = \mathbf{i} + 2\mathbf{j},\ \mathbf{a} = 2\mathbf{j}$

3. $y = \dfrac{2}{9}x^2,\ \mathbf{v} = 3\mathbf{i} + 4\mathbf{j},\ \mathbf{a} = 3\mathbf{i} + 8\mathbf{j}$

5. $t = \dfrac{\pi}{4} : \mathbf{v} = \dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j},\ \mathbf{a} = \dfrac{-\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j};$
$t = \pi/2 : \mathbf{v} = -\mathbf{j},\ \mathbf{a} = -\mathbf{i}$



7. $t = \pi : \mathbf{v} = 2\mathbf{i},\ \mathbf{a} = -\mathbf{j};\ t = \dfrac{3\pi}{2} : \mathbf{v} = \mathbf{i} - \mathbf{j},\ \mathbf{a} = -\mathbf{i}$



9. $\mathbf{v} = \mathbf{i} + 2t\,\mathbf{j} + 2\mathbf{k};\ \mathbf{a} = 2\mathbf{j}$; speed: 3; direction: $\dfrac{1}{3}\mathbf{i} + \dfrac{2}{3}\mathbf{j} + \dfrac{2}{3}\mathbf{k}$;
$\mathbf{v}(1) = 3\left(\dfrac{1}{3}\mathbf{i} + \dfrac{2}{3}\mathbf{j} + \dfrac{2}{3}\mathbf{k}\right)$

11. $\mathbf{v} = (-2\sin t)\,\mathbf{i} + (3\cos t)\,\mathbf{j} + 4\mathbf{k}$;
$\mathbf{a} = (-2\cos t)\,\mathbf{i} - (3\sin t)\,\mathbf{j}$; speed: $2\sqrt{5}$;
direction: $(-1/\sqrt{5})\,\mathbf{i} + (2/\sqrt{5})\,\mathbf{k}$;
$\mathbf{v}(\pi/2) = 2\sqrt{5}[(-1/\sqrt{5})\,\mathbf{i} + (2/\sqrt{5})\,\mathbf{k}]$

13. $\mathbf{v} = \left(\dfrac{2}{t+1}\right)\mathbf{i} + 2t\,\mathbf{j} + t\,\mathbf{k};\ \mathbf{a} = \left(\dfrac{-2}{(t+1)^2}\right)\mathbf{i} + 2\mathbf{j} + \mathbf{k}$;
speed: $\sqrt{6}$; direction: $\dfrac{1}{\sqrt{6}}\mathbf{i} + \dfrac{2}{\sqrt{6}}\mathbf{j} + \dfrac{1}{\sqrt{6}}\mathbf{k}$;
$\mathbf{v}(1) = \sqrt{6}\left(\dfrac{1}{\sqrt{6}}\mathbf{i} + \dfrac{2}{\sqrt{6}}\mathbf{j} + \dfrac{1}{\sqrt{6}}\mathbf{k}\right)$

15. $\pi/2$ **17.** $\pi/2$ **19.** $t = 0,\ \pi,\ 2\pi$

21. $(1/4)\,\mathbf{i} + 7\mathbf{j} + (3/2)\,\mathbf{k}$ **23.** $\left(\dfrac{\pi + 2\sqrt{2}}{2}\right)\mathbf{j} + 2\mathbf{k}$

25. $(\ln 4)\,\mathbf{i} + (\ln 4)\,\mathbf{j} + (\ln 2)\,\mathbf{k}$

27. $\mathbf{r}(t) = \left(\dfrac{-t^2}{2} + 1\right)\mathbf{i} + \left(\dfrac{-t^2}{2} + 2\right)\mathbf{j} + \left(\dfrac{-t^2}{2} + 3\right)\mathbf{k}$

29. $\mathbf{r}(t) = ((t+1)^{3/2} - 1)\,\mathbf{i} + (-e^{-t} + 1)\,\mathbf{j} + (\ln(t+1) + 1)\,\mathbf{k}$

31. $\mathbf{r}(t) = 8t\,\mathbf{i} + 8t\,\mathbf{j} + (-16t^2 + 100)\,\mathbf{k}$

33. $x = t,\ y = -1,\ z = 1 + t$

35. $x = at,\ y = a,\ z = 2\pi b + bt$

37. a) (i): It has constant speed 1 (ii): Yes (iii): Counterclockwise (iv): Yes b) (i): It has constant speed 2 (ii): Yes (iii): Counterclockwise (iv): Yes c) (i): It has constant speed 1 (ii): Yes (iii): Counterclockwise (iv): It starts at $(0, -1)$ instead of $(1, 0)$ d) (i): It has constant speed 1 (ii): Yes (iii): Clockwise (iv): Yes e) (i): It has variable speed (ii): No (iii): Counterclockwise (iv): Yes

39. $\mathbf{r}(t) = \left(\dfrac{3}{2}t^2 + \dfrac{6}{\sqrt{11}}t + 1\right)\mathbf{i} - \left(\dfrac{1}{2}t^2 + \dfrac{2}{\sqrt{11}}t - 2\right)\mathbf{j} +$
$\left(\dfrac{1}{2}t^2 + \dfrac{2}{\sqrt{11}}t + 3\right)\mathbf{k} = \left(\dfrac{1}{2}t^2 + \dfrac{2t}{\sqrt{11}}\right)(3\mathbf{i} - \mathbf{j} + \mathbf{k}) +$
$(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$

41. $\mathbf{v} = 2\sqrt{5}\,\mathbf{i} + \sqrt{5}\,\mathbf{j}$

43. $\max |\mathbf{v}| = 3,\ \min |\mathbf{v}| = 2,\ \max |\mathbf{a}| = 3,\ \min |\mathbf{a}| = 2$

Section 11.2, pp. 873–876

1. 50 sec **3.** a) 72.2 sec, 25,510 m b) 4020 m c) 6378 m

5. $t \approx 2.135$ sec, $x \approx 66.42$ ft

7. $v_0 = 9.9$ m/sec, $\alpha = 18.4°$ or $71.6°$ **9.** 190 mph

11. The golf ball will clip the leaves at the top. **13.** 46.6 ft/sec

17. 141% **21.** 1.92 sec, 73.7 ft (approx.)

25. $\mathbf{v}(t) = -gt\,\mathbf{k} + \mathbf{v}_0,\ \mathbf{r}(t) = -\dfrac{1}{2}gt^2\,\mathbf{k} + \mathbf{v}_0 t$

Section 11.3, pp. 880–881

1. $\mathbf{T} = \left(-\dfrac{2}{3}\sin t\right)\mathbf{i} + \left(\dfrac{2}{3}\cos t\right)\mathbf{j} + \dfrac{\sqrt{5}}{3}\mathbf{k},\ 3\pi$

3. $\mathbf{T} = \dfrac{1}{\sqrt{1+t}}\mathbf{i} + \dfrac{\sqrt{t}}{\sqrt{1+t}}\mathbf{k},\ \dfrac{52}{3}$ **5.** $\mathbf{T} = -\cos t\,\mathbf{j} + \sin t\,\mathbf{k},\ \dfrac{3}{2}$

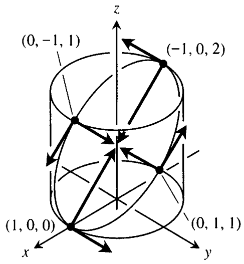
7. $\mathbf{T} = \left(\dfrac{\cos t - t\sin t}{t+1}\right)\mathbf{i} + \left(\dfrac{\sin t + t\cos t}{t+1}\right)\mathbf{j} + \left(\dfrac{\sqrt{2}\,t^{1/2}}{t+1}\right)\mathbf{k},$
$\dfrac{\pi^2}{2} + \pi$

9. $(0, 5, 24\pi)$ **11.** $s(t) = 5t,\ L = \dfrac{5\pi}{2}$

13. $s(t) = \sqrt{3}e^t - \sqrt{3},\ L = \dfrac{3\sqrt{3}}{4}$ **15.** $\sqrt{2} + \ln(1 + \sqrt{2})$

17. a) Cylinder is $x^2 + y^2 = 1$, plane is $x + z = 1$

b) and c)



d) $L = \int_0^{2\pi} \sqrt{1 + \sin^2 t}\, dt$ e) $L \approx 7.64$

Section 11.4, pp. 890–893

1. $\mathbf{T} = (\cos t)\,\mathbf{i} - (\sin t)\,\mathbf{j},\ \mathbf{N} = (-\sin t)\,\mathbf{i} - (\cos t)\,\mathbf{j},\ \kappa = \cos t$

3. $\mathbf{T} = \dfrac{1}{\sqrt{1+t^2}}\mathbf{i} - \dfrac{t}{\sqrt{1+t^2}}\mathbf{j},\ \mathbf{N} = \dfrac{-t}{\sqrt{1+t^2}}\mathbf{i} - \dfrac{1}{\sqrt{1+t^2}}\mathbf{j},$
$\kappa = \dfrac{1}{2\left(\sqrt{1+t^2}\right)^3}$

5. $\mathbf{a} = \dfrac{2t}{\sqrt{1+t^2}}\mathbf{T} + \dfrac{2}{\sqrt{1+t^2}}\mathbf{N}$ **7.** b) $\cos x$

9. b) $\mathbf{N} = \dfrac{-2e^{2t}}{\sqrt{1+4e^{4t}}}\mathbf{i} + \dfrac{1}{\sqrt{1+4e^{4t}}}\mathbf{j}$

c) $\mathbf{N} = -\dfrac{1}{2}\left(\sqrt{4-t^2}\,\mathbf{i} + t\,\mathbf{j}\right)$

11. $\mathbf{T} = \dfrac{3\cos t}{5}\mathbf{i} - \dfrac{3\sin t}{5}\mathbf{j} + \dfrac{4}{5}\mathbf{k},\ \mathbf{N} = (-\sin t)\,\mathbf{i} - (\cos t)\,\mathbf{j},$
$\mathbf{B} = \left(\dfrac{4}{5}\cos t\right)\mathbf{i} - \left(\dfrac{4}{5}\sin t\right)\mathbf{j} - \dfrac{3}{5}\mathbf{k},\ \kappa = \dfrac{3}{25},\ \tau = -\dfrac{4}{25}$

13. $\mathbf{T} = \left(\dfrac{\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\dfrac{\cos t + \sin t}{\sqrt{2}}\right)\mathbf{j},$
$\mathbf{N} = \left(\dfrac{-\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\dfrac{-\sin t + \cos t}{\sqrt{2}}\right)\mathbf{j},$
$\mathbf{B} = \mathbf{k},\ \kappa = \dfrac{1}{e^t\sqrt{2}},\ \tau = 0$

15. $\mathbf{T} = \dfrac{t}{\sqrt{t^2+1}}\mathbf{i} + \dfrac{1}{\sqrt{t^2+1}}\mathbf{j},\ \mathbf{N} = \dfrac{\mathbf{i}}{\sqrt{t^2+1}} - \dfrac{t\,\mathbf{j}}{\sqrt{t^2+1}},$
$\mathbf{B} = -\mathbf{k},\ \kappa = \dfrac{1}{t(t^2+1)^{3/2}},\ \tau = 0$

17. $\mathbf{T} = \left(\operatorname{sech}\dfrac{t}{a}\right)\mathbf{i} + \left(\tanh\dfrac{t}{a}\right)\mathbf{j},$
$\mathbf{N} = \left(-\tanh\dfrac{t}{a}\right)\mathbf{i} + \left(\operatorname{sech}\dfrac{t}{a}\right)\mathbf{j},$
$\mathbf{B} = \mathbf{k},\ \kappa = \dfrac{1}{a}\operatorname{sech}^2\dfrac{t}{a},\ \tau = 0$

19. $\mathbf{a} = |a|\,\mathbf{N}$ **21.** $\mathbf{a}(1) = \dfrac{4}{3}\mathbf{T} + \dfrac{2\sqrt{5}}{3}\mathbf{N}$ **23.** $\mathbf{a}(0) = 2\,\mathbf{N}$

25. $\mathbf{r}\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k},\ \mathbf{T}\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j},$
$\mathbf{N}\left(\dfrac{\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j},\ \mathbf{B}\left(\dfrac{\pi}{4}\right) = \mathbf{k}$; osculating plane: $z = -1$; normal plane: $-x + y = 0$; rectifying plane: $x + y = \sqrt{2}$

27. Yes. If the car is moving on a curved path ($\kappa \neq 0$), then $a_{\mathbf{N}} = \kappa|\mathbf{v}|^2 \neq 0$ and $\mathbf{a} \neq \mathbf{0}$.

31. $|\mathbf{F}| = \kappa\left(m\left(\dfrac{ds}{dt}\right)^2\right)$ **35.** $1/(2b)$ **39.** a) $b - a$ b) π

45. $\kappa(x) = 2/(1+4x^2)^{3/2}$ **47.** $\kappa(x) = |\sin x|/(1+\cos^2 x)^{3/2}$

57. Components of $\mathbf{v}$: −1.8701, 0.7089, 1.0000
Components of $\mathbf{a}$: −1.6960, −2.0307, 0
Speed: 2.2361; Components of $\mathbf{T}$: −0.8364, 0.3170, 0.4472
Components of $\mathbf{N}$: −0.4143, −0.8998, −0.1369
Components of $\mathbf{B}$: 0.3590, −0.2998, 0.8839; Curvature: 0.5060
Torsion: 0.2813; Tangential component of acceleration: 0.7746
Normal component of acceleration: 2.5298

59. Components of $\mathbf{v}$: 2.0000, 0, 0.1629
Components of $\mathbf{a}$: 0, −1.0000, 0.0086; Speed: 2.0066
Components of $\mathbf{T}$: 0.9967, 0, 0.0812
Components of $\mathbf{N}$: −0.0007, −1.0000, 0.0086
Components of $\mathbf{B}$: 0.0812, −0.0086, −0.9967; Curvature: 0.2484
Torsion: −0.0411; Tangential component of acceleration: 0.0007
Normal component of acceleration: 1.0000

Section 11.5, pp. 901–902

1. $T = 93.2$ min **3.** $a = 6763$ km **5.** $T = 1655$ min

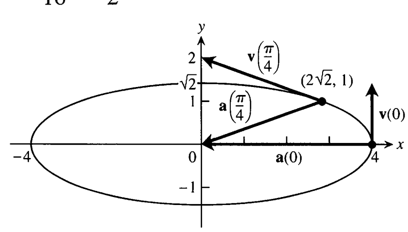
7. $a = 20{,}430$ km **9.** $|v| = 1.9966 \times 10^7 r^{-1/2}$ m/sec

11. Circle: $v_0 = \sqrt{\dfrac{GM}{r_0}}$; ellipse: $\sqrt{\dfrac{GM}{r_0}} < v_0 < \sqrt{\dfrac{2GM}{r_0}}$; parabola: $v_0 = \sqrt{\dfrac{2GM}{r_0}}$; hyperbola: $v_0 > \sqrt{\dfrac{2GM}{r_0}}$

15. a) $x(t) = 2 + (3 - 4\cos(\pi t))\cos(\pi t),\ y(t) = (3 - 4\cos(\pi t))\sin(\pi t)$

Chapter 11 Practice Exercises, pp. 902–905

1. $\dfrac{x^2}{16} + \dfrac{y^2}{2} = 1$



At $t = 0 : a_{\mathbf{T}} = 0,\ a_{\mathbf{N}} = 4,\ \kappa = 2;$

At $t = \dfrac{\pi}{4}$: $a_{\mathbf{T}} = \dfrac{7}{3}$, $a_{\mathbf{N}} = \dfrac{4\sqrt{2}}{3}$, $\kappa = \dfrac{4\sqrt{2}}{27}$

3. $|\mathbf{v}|_{\max} = 1$ **5.** $\kappa = 1/5$ **7.** $dy/dt = -x$; clockwise

11. Shot put is on the ground, about 66 ft, 5 in. from the stopboard.

15. a) 59.19 ft/sec b) 74.58 ft/sec **19.** $\kappa = \pi s$

21. Length $= \dfrac{\pi}{4}\sqrt{1 + \dfrac{\pi^2}{16}} + \ln\left(\dfrac{\pi}{4} + \sqrt{1 + \dfrac{\pi^2}{16}}\right)$

23. $\mathbf{T}(0) = \dfrac{2}{3}\mathbf{i} - \dfrac{2}{3}\mathbf{j} + \dfrac{1}{3}\mathbf{k}$; $\mathbf{N}(0) = \dfrac{1}{\sqrt{2}}\mathbf{i} + \dfrac{1}{\sqrt{2}}\mathbf{j}$;

$\mathbf{B}(0) = -\dfrac{1}{3\sqrt{2}}\mathbf{i} + \dfrac{1}{3\sqrt{2}}\mathbf{j} + \dfrac{4}{3\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{\sqrt{2}}{3}$; $\tau = \dfrac{1}{6}$

25. $\mathbf{T}(\ln 2) = \dfrac{1}{\sqrt{17}}\mathbf{i} + \dfrac{4}{\sqrt{17}}\mathbf{j}$; $\mathbf{N}(\ln 2) = -\dfrac{4}{\sqrt{17}}\mathbf{i} + \dfrac{1}{\sqrt{17}}\mathbf{j}$;

$\mathbf{B}(\ln 2) = \mathbf{k}$; $\kappa = \dfrac{8}{17\sqrt{17}}$; $\tau = 0$

27. $\mathbf{a}(0) = 10\mathbf{T} + 6\mathbf{N}$

29. $\mathbf{T} = \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{i} - (\sin t)\mathbf{j} + \left(\dfrac{1}{\sqrt{2}}\cos t\right)\mathbf{k}$;

$\mathbf{N} = \left(-\dfrac{1}{\sqrt{2}}\sin t\right)\mathbf{i} - (\cos t)\mathbf{j} - \left(\dfrac{1}{\sqrt{2}}\sin t\right)\mathbf{k}$;

$\mathbf{B} = \dfrac{1}{\sqrt{2}}\mathbf{i} - \dfrac{1}{\sqrt{2}}\mathbf{k}$; $\kappa = \dfrac{1}{\sqrt{2}}$; $\tau = 0$

31. $\pi/3$ **33.** $x = 1 + t$, $y = t$, $z = -t$

35. 5971 km, 1.639×10^7 km^2, 3.21% visible

Chapter 11 Additional Exercises, pp. 905–907

1. a) $\mathbf{r}(t) = \left(-\dfrac{8}{15}t^3 + 4t^2\right)\mathbf{i} + (-20t + 100)\mathbf{j}$; b) $\dfrac{100}{3}$ m

3. $\mathbf{v} = \dfrac{\sqrt{3}}{2}\mathbf{i} + \dfrac{\sqrt{3}}{2}\mathbf{j} - \sqrt{3}\mathbf{k}$ **5.** a) $\left.\dfrac{d\theta}{dt}\right|_{\theta=2\pi} = 2\sqrt{\dfrac{\pi gb}{a^2 + b^2}}$

b) $\theta = \dfrac{gbt^2}{2(a^2 + b^2)}$, $z = \dfrac{gb^2t^2}{2(a^2 + b^2)}$

c) $\mathbf{v}(t) = \dfrac{gbt}{\sqrt{a^2 + b^2}}\mathbf{T}$; $\dfrac{d^2\mathbf{r}}{dt^2} = \dfrac{bg}{\sqrt{a^2 + b^2}}\mathbf{T} + a\left(\dfrac{bgt}{a^2 + b^2}\right)^2\mathbf{N}$

There is no component in the direction of **B**.

9. a) $\dfrac{dx}{dt} = \dot{r}\cos\theta - r\dot{\theta}\sin\theta$, $\dfrac{dy}{dt} = \dot{r}\sin\theta + r\dot{\theta}\cos\theta$

b) $\dfrac{dr}{dt} = \dot{x}\cos\theta + \dot{y}\sin\theta$, $r\dfrac{d\theta}{dt} = -\dot{x}\sin\theta + \dot{y}\cos\theta$

11. a) $\mathbf{a}(1) = -9\mathbf{u}_r - 6\mathbf{u}_\theta$, $\mathbf{v}(1) = -\mathbf{u}_r + 3\mathbf{u}_\theta$ b) 6.5 in.

13. c) $\mathbf{v} = \dot{r}\,\mathbf{u}_r + r\dot{\theta}\,\mathbf{u}_\theta + \dot{z}\,\mathbf{k}$, $\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\,\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\mathbf{u}_\theta + \ddot{z}\,\mathbf{k}$

15. a) $\mathbf{u}_\rho = \sin\phi\cos\theta\,\mathbf{i} + \sin\phi\sin\theta\,\mathbf{j} + \cos\phi\,\mathbf{k}$,
$\mathbf{u}_\phi = \cos\phi\cos\theta\,\mathbf{i} + \cos\phi\sin\theta\,\mathbf{j} - \sin\phi\,\mathbf{k}$,
$\mathbf{u}_\theta = -\sin\theta\,\mathbf{i} + \cos\theta\,\mathbf{j}$

CHAPTER 12

Section 12.1, pp. 914–917

1. a) All points in the xy-plane b) All reals c) The lines $y - x = c$ d) No boundary points e) Both open and closed f) Unbounded

3. a) All points in the xy-plane b) $z \geq 0$ c) For $f(x, y) = 0$, the origin; for $f(x, y) \neq 0$, ellipses with the center $(0, 0)$, and major and minor axes, along the x- and y-axes, respectively d) No boundary points e) Both open and closed f) Unbounded

5. a) All points in the xy-plane b) All reals c) For $f(x, y) = 0$, the x- and y-axes; for $f(x, y) \neq 0$, hyperbolas with the x- and y-axes as asymptotes d) No boundary points e) Both open and closed f) Unbounded

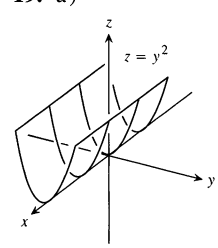
7. a) All (x, y) satisfying $x^2 + y^2 < 16$ b) $z \geq 1/4$ c) Circles centered at the origin with radii $r < 4$ d) Boundary is the circle $x^2 + y^2 = 16$ e) Open f) Bounded

9. a) $(x, y) \neq (0, 0)$ b) All reals c) The circles with center $(0, 0)$ and radii $r > 0$ d) Boundary is the single point $(0, 0)$ e) Open f) Unbounded

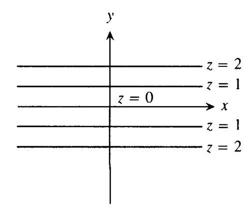
11. a) All (x, y) satisfying $-1 \leq y - x \leq 1$ b) $-\pi/2 \leq z \leq \pi/2$ c) Straight lines of the form $y - x = c$ where $-1 \leq c \leq 1$ d) Boundary is two straight lines $y = 1 + x$ and $y = -1 + x$ e) Closed f) Unbounded

13. f **15.** a **17.** d

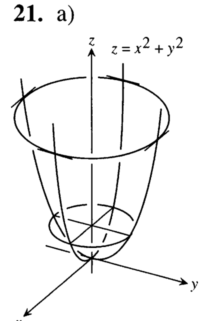
19. a)



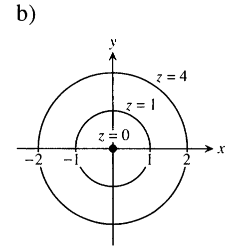
b)



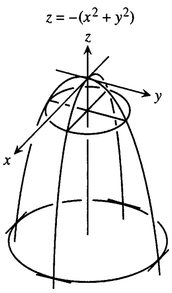
21. a)



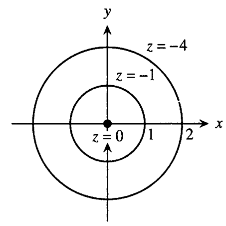
b)



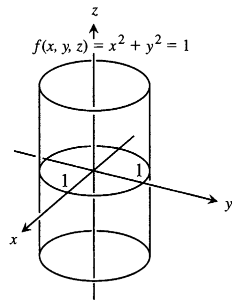
23. a)



b)



33.



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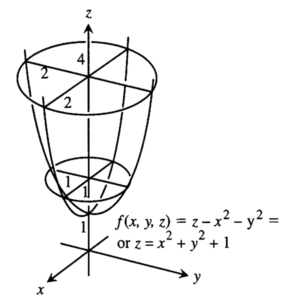
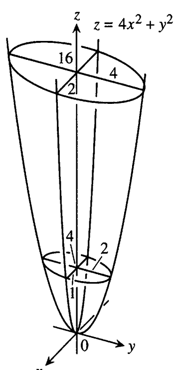


37. $x^2 + y^2 = 10$ **39.** $\tan^{-1} y - \tan^{-1} x = 2 \tan^{-1} \sqrt{2}$

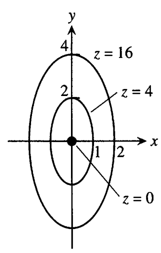
41. $\sqrt{x - y} - \ln z = 2$ **43.** $\dfrac{x + y}{z} = \ln 2$ **45.** Yes, 2000

47. 63 km

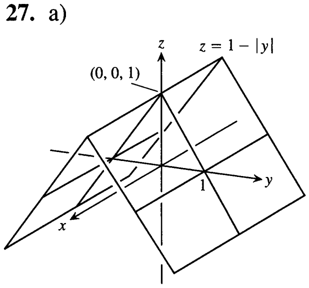
25. a)



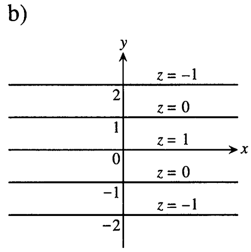
b)



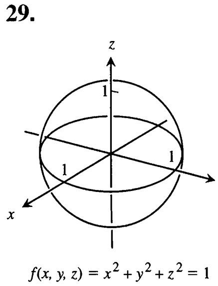
27. a)



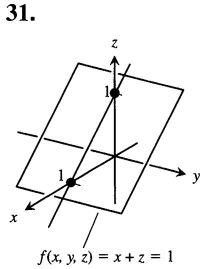
b)



29.



31.



Section 12.2, pp. 921–923

1. 5/2 **3.** $2\sqrt{6}$ **5.** 1 **7.** 1/2 **9.** 1 **11.** 0 **13.** 0
15. −1 **17.** 2 **19.** 1/4 **21.** 19/12 **23.** 2 **25.** 3
27. a) All (x, y) b) All (x, y) except $(0, 0)$
29. a) All (x, y) except where $x = 0$ or $y = 0$ b) All (x, y)
31. a) All (x, y, z) b) All (x, y, z) except the interior of the cylinder $x^2 + y^2 = 1$
33. a) All (x, y, z) with $z \neq 0$ b) All (x, y, z) with $x^2 + z^2 \neq 1$
35. Consider paths along $y = x, x > 0$, and along $y = x, x < 0$
37. Consider the paths $y = kx^2$, k a constant
39. Consider the paths $y = mx$, m a constant, $m \neq -1$
41. Consider the paths $y = kx^2$, k a constant, $k \neq 0$ **43.** No
45. The limit is 1 **47.** The limit is 0
49. a) $f(x, y)|_{y=mx} = \sin 2\theta$ where $\tan \theta = m$ **51.** 0
53. Does not exist **55.** $\pi/2$ **57.** $f(0, 0) = \ln 3$ **61.** $\delta = 0.1$
63. $\delta = 0.005$ **65.** $\delta = \sqrt{0.015}$ **67.** $\delta = 0.005$

Section 12.3, pp. 931–933

1. $\dfrac{\partial f}{\partial x} = 4x,\ \dfrac{\partial f}{\partial y} = -3$ **3.** $\dfrac{\partial f}{\partial x} = 2x(y + 2),\ \dfrac{\partial f}{\partial y} = x^2 - 1$

5. $\dfrac{\partial f}{\partial x} = 2y(xy - 1),\ \dfrac{\partial f}{\partial y} = 2x(xy - 1)$

7. $\dfrac{\partial f}{\partial x} = \dfrac{x}{\sqrt{x^2 + y^2}},\ \dfrac{\partial f}{\partial y} = \dfrac{y}{\sqrt{x^2 + y^2}}$

9. $\dfrac{\partial f}{\partial x} = \dfrac{-1}{(x + y)^2},\ \dfrac{\partial f}{\partial y} = \dfrac{-1}{(x + y)^2}$

11. $\dfrac{\partial f}{\partial x} = \dfrac{-y^2 - 1}{(xy - 1)^2},\ \dfrac{\partial f}{\partial y} = \dfrac{-x^2 - 1}{(xy - 1)^2}$

13. $\dfrac{\partial f}{\partial x} = e^{x+y+1},\ \dfrac{\partial f}{\partial y} = e^{x+y+1}$ **15.** $\dfrac{\partial f}{\partial x} = \dfrac{1}{x + y},\ \dfrac{\partial f}{\partial y} = \dfrac{1}{x + y}$

17. $\dfrac{\partial f}{\partial x} = 2 \sin (x - 3y) \cos (x - 3y),$

$\dfrac{\partial f}{\partial y} = -6 \sin (x - 3y) \cos (x - 3y)$

19. $\frac{\partial f}{\partial x} = yx^{y-1},\ \frac{\partial f}{\partial y} = x^y \ln x$ 21. $\frac{\partial f}{\partial x} = -g(x),\ \frac{\partial f}{\partial y} = g(y)$

23. $f_x = y^2,\ f_y = 2xy,\ f_z = -4z$

25. $f_x = 1,\ f_y = -y(y^2+z^2)^{-1/2},\ f_z = -z(y^2+z^2)^{-1/2}$

27. $f_x = \frac{yz}{\sqrt{1-x^2y^2z^2}},\ f_y = \frac{xz}{\sqrt{1-x^2y^2z^2}},\ f_z = \frac{xy}{\sqrt{1-x^2y^2z^2}}$

29. $f_x = \frac{1}{x+2y+3z},\ f_y = \frac{2}{x+2y+3z},\ f_z = \frac{3}{x+2y+3z}$

31. $f_x = -2xe^{-(x^2+y^2+z^2)},\ f_y = -2ye^{-(x^2+y^2+z^2)},$
$f_z = -2ze^{-(x^2+y^2+z^2)}$

33. $f_x = \operatorname{sech}^2(x+2y+3z),\ f_y = 2\operatorname{sech}^2(x+2y+3z),$
$f_z = 3\operatorname{sech}^2(x+2y+3z)$

35. $\frac{\partial f}{\partial t} = -2\pi \sin(2\pi t - \alpha),\ \frac{\partial f}{\partial \alpha} = \sin(2\pi t - \alpha)$

37. $\frac{\partial h}{\partial \rho} = \sin\phi\cos\theta,\ \frac{\partial h}{\partial \phi} = \rho\cos\phi\cos\theta,\ \frac{\partial h}{\partial \theta} = -\rho\sin\phi\sin\theta$

39. $W_P(P,V,\delta,v,g) = V,\ W_V(P,V,\delta,v,g) = P + \frac{\delta v^2}{2g},$
$W_P(P,V,\delta,v,g) = \frac{Vv^2}{2g},\ W_v(P,V,\delta,v,g) = \frac{V\delta v}{g},$
$W_g(P,V,\delta,v,g) = -\frac{V\delta v^2}{2g^2}$

41. $\frac{\partial f}{\partial x} = 1+y,\ \frac{\partial f}{\partial y} = 1+x,\ \frac{\partial^2 f}{\partial x^2} = 0,\ \frac{\partial^2 f}{\partial y^2} = 0,$
$\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} = 1$

43. $\frac{\partial g}{\partial x} = 2xy + y\cos x,\ \frac{\partial g}{\partial y} = x^2 - \sin y + \sin x,$
$\frac{\partial^2 g}{\partial x^2} = 2y - y\sin x,\ \frac{\partial^2 g}{\partial y^2} = -\cos y,\ \frac{\partial^2 g}{\partial y\,\partial x} = \frac{\partial^2 g}{\partial x\,\partial y} = 2x + \cos x$

45. $\frac{\partial r}{\partial x} = \frac{1}{x+y},\ \frac{\partial r}{\partial y} = \frac{1}{x+y},\ \frac{\partial^2 r}{\partial x^2} = \frac{-1}{(x+y)^2},$
$\frac{\partial^2 r}{\partial y^2} = \frac{-1}{(x+y)^2},\ \frac{\partial^2 r}{\partial y\,\partial x} = \frac{\partial^2 r}{\partial x\,\partial y} = \frac{-1}{(x+y)^2}$

47. $\frac{\partial w}{\partial x} = \frac{2}{2x+3y},\ \frac{\partial w}{\partial y} = \frac{3}{2x+3y},\ \frac{\partial^2 w}{\partial y\,\partial x} = \frac{-6}{(2x+3y)^2},$
$\frac{\partial^2 w}{\partial x\,\partial y} = \frac{-6}{(2x+3y)^2}$

49. $\frac{\partial w}{\partial x} = y^2 + 2xy^3 + 3x^2y^4,\ \frac{\partial w}{\partial y} = 2xy + 3x^2y^2 + 4x^3y^3,$
$\frac{\partial^2 w}{\partial y\,\partial x} = 2y + 6xy^2 + 12x^2y^3,\ \frac{\partial^2 w}{\partial x\,\partial y} = 2y + 6xy^2 + 12x^2y^3$

51. a) x first b) y first c) x first d) x first e) y first f) y first 53. $f_x(1,2) = -13,\ f_y(1,2) = -2$ 55. 12

57. -2 59. $\frac{\partial A}{\partial a} = \frac{a}{bc\sin A},\ \frac{\partial A}{\partial b} = \frac{c\cos A - b}{bc\sin A}$

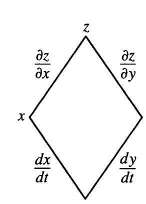
61. $v_x = \frac{\ln v}{(\ln u)(\ln v) - 1}$

Section 12.4, pp. 942–944

1. a) $L(x,y) = 1$ b) $L(x,y) = 2x + 2y - 1$
3. a) $L(x,y) = 3x - 4y + 5$ b) $L(x,y) = 3x - 4y + 5$
5. a) $L(x,y) = 1 + x$ b) $L(x,y) = -y + (\pi/2)$
7. $L(x,y) = 7 + x - 6y$; 0.06 9. $L(x,y) = x + y + 1$; 0.08
11. $L(x,y) = 1 + x$; 0.0222
13. Pay more attention to the smaller of the two dimensions. It will generate the larger partial derivative.
15. Maximum error (estimate) ≤ 0.31 in magnitude
17. Maximum percentage error $= \pm 4.83\%$
19. Let $|x-1| \le 0.014,\ |y-1| \le 0.014$ 21. $\approx 0.1\%$
23. a) $L(x,y,z) = 2x + 2y + 2z - 3$ b) $L(x,y,z) = y + z$ c) $L(x,y,z) = 0$ 25. a) $L(x,y,z) = x$ b) $L(x,y,z) = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y$ c) $L(x,y,z) = \frac{1}{3}x + \frac{2}{3}y + \frac{2}{3}z$
27. a) $L(x,y,z) = 2 + x$ b) $L(x,y,z) = x - y - z + \frac{\pi}{2} + 1$ c) $L(x,y,z) = x - y - z + \frac{\pi}{2} + 1$
29. $L(x,y,z) = 2x - 6y - 2z + 6,\ 0.0024$
31. $L(x,y,z) = x + y - z - 1, 0.00135$
33. a) $S_0\left(\frac{1}{100}\,dp + dx - 5\,dw - 30\,dh\right)$ b) More sensitive to a change in height
35. f is most sensitive to a change in d. 37. (47/24) ft^3
39. Magnitude of possible error ≤ 4.8 41. Yes

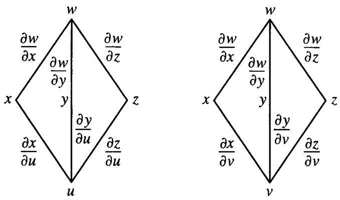
Section 12.5, pp. 950–952

1. $\frac{dw}{dt} = 0,\ \frac{dw}{dt}(\pi) = 0$ 3. $\frac{dw}{dt} = 1,\ \frac{dw}{dt}(3) = 1$
5. $\frac{dw}{dt} = 4t\tan^{-1}t + 1,\ \frac{dw}{dt}(1) = \pi + 1$
7. a) $\frac{\partial z}{\partial r} = 4\cos\theta\ln(r\sin\theta) + 4\cos\theta,$
$\frac{\partial z}{\partial \theta} = -4r\sin\theta\ln(r\sin\theta) + \frac{4r\cos^2\theta}{\sin\theta}$
b) $\frac{\partial z}{\partial r} = \sqrt{2}(\ln 2 + 2),\ \frac{\partial z}{\partial \theta} = -2\sqrt{2}\ln 2 + 4\sqrt{2}$
9. a) $\frac{\partial w}{\partial u} = 2u + 4uv,\ \frac{\partial w}{\partial v} = -2v + 2u^2$ b) $\frac{\partial w}{\partial u} = 3,\ \frac{\partial w}{\partial v} = -\frac{3}{2}$
11. a) $\frac{\partial u}{\partial x} = 0,\ \frac{\partial u}{\partial y} = \frac{z}{(z-y)^2},\ \frac{\partial u}{\partial z} = \frac{-y}{(z-y)^2}$
b) $\frac{\partial u}{\partial x} = 0,\ \frac{\partial u}{\partial y} = 1,\ \frac{\partial u}{\partial z} = -2$
13. $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$

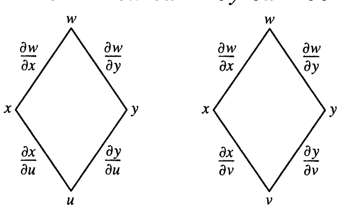


15. $\dfrac{\partial w}{\partial u}=\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u}+\dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}+\dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial u}$,

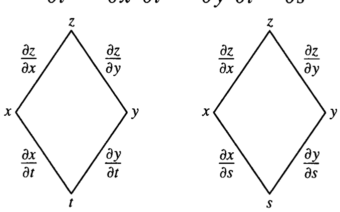
$\dfrac{\partial w}{\partial v}=\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v}+\dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}+\dfrac{\partial w}{\partial z}\dfrac{\partial z}{\partial v}$



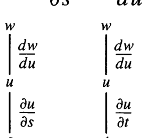
17. $\dfrac{\partial w}{\partial u}=\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial u}+\dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial u}$, $\dfrac{\partial w}{\partial v}=\dfrac{\partial w}{\partial x}\dfrac{\partial x}{\partial v}+\dfrac{\partial w}{\partial y}\dfrac{\partial y}{\partial v}$



19. $\dfrac{\partial z}{\partial t}=\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t}+\dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}$, $\dfrac{\partial z}{\partial s}=\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial s}+\dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial s}$

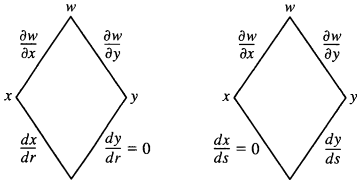


21. $\dfrac{\partial w}{\partial s}=\dfrac{dw}{du}\dfrac{\partial u}{\partial s}$, $\dfrac{\partial w}{\partial t}=\dfrac{dw}{du}\dfrac{\partial u}{\partial t}$



23. $\dfrac{\partial w}{\partial r}=\dfrac{\partial w}{\partial x}\dfrac{dx}{dr}+\dfrac{\partial w}{\partial y}\dfrac{dy}{dr}=\dfrac{\partial w}{\partial x}\dfrac{dx}{dr}$ since $\dfrac{dy}{dr}=0$,

$\dfrac{\partial w}{\partial s}=\dfrac{\partial w}{\partial x}\dfrac{dx}{ds}+\dfrac{\partial w}{\partial y}\dfrac{dy}{ds}=\dfrac{\partial w}{\partial y}\dfrac{dy}{ds}$ since $\dfrac{dx}{ds}=0$



25. $4/3$ **27.** $-4/5$ **29.** $\dfrac{\partial z}{\partial x}=\dfrac{1}{4}$, $\dfrac{\partial z}{\partial y}=-\dfrac{3}{4}$

31. $\dfrac{\partial z}{\partial x}=-1$, $\dfrac{\partial z}{\partial y}=-1$ **33.** 12 **35.** -7

37. $\dfrac{\partial z}{\partial u}=2$, $\dfrac{\partial z}{\partial v}=1$ **39.** -0.00005 amps/sec

45. $(\cos 1, \sin 1, 1)$ and $(\cos(-2), \sin(-2), -2)$

47. a) Maximum at $\left(-\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ and $\left(\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$, minimum at $\left(\dfrac{\sqrt{2}}{2}, \dfrac{\sqrt{2}}{2}\right)$ and $\left(-\dfrac{\sqrt{2}}{2}, -\dfrac{\sqrt{2}}{2}\right)$

b) max $=6$, min $=2$ **49.** $2x\sqrt{x^8+x^3}+\displaystyle\int_0^{x^2}\frac{3x^2}{2\sqrt{t^4+x^3}}\,dt$

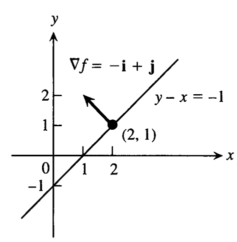
Section 12.6, pp. 956–957

1. a) 0 b) $1+2z$ c) $1+2z$ **3.** a) $\dfrac{\partial U}{\partial P}+\dfrac{\partial U}{\partial T}\left(\dfrac{V}{nR}\right)$

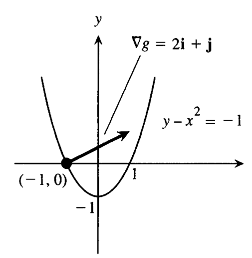
b) $\dfrac{\partial U}{\partial P}\left(\dfrac{nR}{V}\right)+\dfrac{\partial U}{\partial T}$ **5.** a) 5 b) 5 **7.** $\dfrac{x}{\sqrt{x^2+y^2}}$

Section 12.7, pp. 967–969

1.



3.



5. $\nabla f = 3\mathbf{i}+2\mathbf{j}-4\mathbf{k}$ **7.** $\nabla f = -\dfrac{26}{27}\mathbf{i}+\dfrac{23}{54}\mathbf{j}-\dfrac{23}{54}\mathbf{k}$ **9.** -4

11. $31/13$ **13.** 3 **15.** 2

17. $\mathbf{u}=-\dfrac{1}{\sqrt{2}}\mathbf{i}+\dfrac{1}{\sqrt{2}}\mathbf{j}$, $(D_{\mathbf{u}}f)_{P_0}=\sqrt{2}$; $-\mathbf{u}=\dfrac{1}{\sqrt{2}}\mathbf{i}-\dfrac{1}{\sqrt{2}}\mathbf{j}$, $(D_{-\mathbf{u}}f)_{P_0}=-\sqrt{2}$

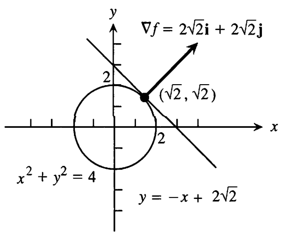
19. $\mathbf{u}=\dfrac{1}{3\sqrt{3}}\mathbf{i}-\dfrac{5}{3\sqrt{3}}\mathbf{j}-\dfrac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{\mathbf{u}}f)_{P_0}=3\sqrt{3}$;

$-\mathbf{u}=-\dfrac{1}{3\sqrt{3}}\mathbf{i}+\dfrac{5}{3\sqrt{3}}\mathbf{j}+\dfrac{1}{3\sqrt{3}}\mathbf{k}$, $(D_{-\mathbf{u}}f)_{P_0}=-3\sqrt{3}$

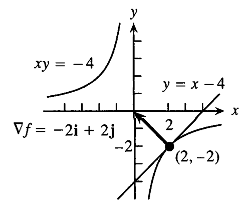
21. $\mathbf{u}=\dfrac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(D_{\mathbf{u}}f)_{P_0}=2\sqrt{3}$; $-\mathbf{u}=-\dfrac{1}{\sqrt{3}}(\mathbf{i}+\mathbf{j}+\mathbf{k})$, $(D_{-\mathbf{u}}f)_{P_0}=-2\sqrt{3}$

23. $df=\dfrac{9}{910}\approx 0.01$ **25.** $dg=0$

27. Tangent: $x+y+z=3$, normal line: $x=1+2t,\ y=1+2t,\ z=1+2t$
29. Tangent: $2x-z-2=0$, normal line: $x=2-4t,\ y=0,\ z=2+2t$
31. Tangent: $2x+2y+z-4=0$, normal line: $x=2t,\ y=1+2t,\ z=2+t$
33. Tangent: $x+y+z-1=0$, normal line: $x=t,\ y=1+t,\ z=t$
35. $2x-z-2=0$ **37.** $x-y+2z-1=0$
39.



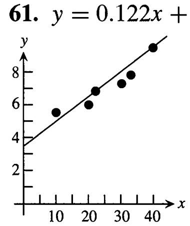
41.



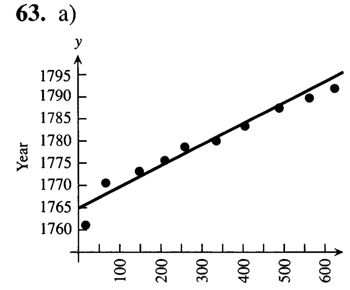
43. $x=1,\ y=1+2t,\ z=1-2t$
45. $x=1-2t,\ y=1,\ z=\frac{1}{2}+2t$
47. $x=1+90t,\ y=1-90t,\ z=3$
49. $\mathbf{u}=\frac{7}{\sqrt{53}}\mathbf{i}-\frac{2}{\sqrt{53}}\mathbf{j},\ -\mathbf{u}=-\frac{7}{\sqrt{53}}\mathbf{i}+\frac{2}{\sqrt{53}}\mathbf{j}$
51. No, the maximum rate of change is $\sqrt{185}<14$. **53.** $-\frac{7}{\sqrt{5}}$
55. a) $\frac{\sqrt{3}}{2}\sin\sqrt{3}-\frac{1}{2}\cos\sqrt{3}\approx 0.935°\text{C/ft}$
b) $\sqrt{3}\sin\sqrt{3}-\cos\sqrt{3}\approx 1.87°\text{C/sec}$
57. At $-\frac{\pi}{4},\ -\frac{\pi}{2\sqrt{2}}$; at 0, 0; at $\frac{\pi}{4},\ \frac{\pi}{2\sqrt{2}}$

Section 12.8, pp. 975–979

1. $f(-3,3)=-5$, local minimum
3. $f\left(\frac{2}{3},\frac{4}{3}\right)=0$, local maximum **5.** $f(-2,1)$, saddle point
7. $f\left(\frac{6}{5},\frac{69}{25}\right)$, saddle point **9.** $f(2,1)$, saddle point
11. $f(2,-1)=-6$, local minimum **13.** $f(1,2)$, saddle point
15. $f(0,0)$, saddle point
17. $f(0,0)$, saddle point; $f\left(-\frac{2}{3},\frac{2}{3}\right)=\frac{170}{27}$, local maximum
19. $f(0,0)=0$, local minimum; $f(1,-1)$, saddle point
21. $f(0,0)$, saddle point; $f\left(\frac{4}{9},\frac{4}{3}\right)=-\frac{64}{81}$, local minimum
23. $f(0,0)$, saddle point; $f(0,2)=-12$, local minimum; $f(-2,0)=-4$, local maximum; $f(-2,2)$, saddle point
25. $f(0,0)$, saddle point; $f(1,1)=2,\ f(-1,-1)=2$, local maxima
27. $f(0,0)=-1$, local maximum
29. $f(n\pi,0)$, saddle point; $f(n\pi,0)=0$ for every n
31. Absolute maximum: 1 at (0, 0); absolute minimum: −5 at (1, 2)
33. Absolute maximum: 4 at (0, 2); absolute minimum: 0 at (0, 0)
35. Absolute maximum: 11 at (0, −3); absolute minimum: −10 at (4, −2)
37. Absolute maximum: 4 at (2, 0); absolute minimum: $\frac{3\sqrt{2}}{2}$ at $\left(3,-\frac{\pi}{4}\right),\ \left(3,\frac{\pi}{4}\right),\ \left(1,-\frac{\pi}{4}\right)$, and $\left(1,\frac{\pi}{4}\right)$ **39.** $a=-3, b=2$
41. Hottest: $2\frac{1}{4}^{\circ}$ at $\left(-\frac{1}{2},\frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{1}{2},-\frac{\sqrt{3}}{2}\right)$; coldest: $-\frac{1}{4}^{\circ}$ at $\left(\frac{1}{2},0\right)$
43. a) $f(0,0)$, saddle point b) $f(1,2)$, local minimum
c) $f(1,-2)$, local minimum; $f(-1,-2)$, saddle point
49. $\left(\frac{1}{6},\frac{1}{3},\frac{355}{36}\right)$
53. a) On the semicircle, max $f=2\sqrt{2}$ at $t=\pi/4$, min $f=-2$ at $t=\pi$. On the quarter circle, max $f=2\sqrt{2}$ at $t=\pi/4$, min $f=2$ at $t=0,\pi/2$.
b) On the semicircle, max $g=2$ at $t=\pi/4$, min $g=-2$ at $t=3\pi/4$. On the quarter circle, max $g=2$ at $t=\pi/4$, min $g=0$ at $t=0,\pi/2$.
c) On the semicircle, max $h=8$ at $t=0,\pi$; min $h=4$ at $t=\pi/2$. On the quarter circle, max $h=8$ at $t=0$, min $h=4$ at $t=\pi/2$.
55. i) min $f=-1/2$ at $t=-1/2$; no max ii) max $f=0$ at $t=-1,\ 0$; min $f=-1/2$ at $t=-1/2$ iii) max $f=4$ at $t=1$; min $f=0$ at $t=0$
57. $y=-\frac{20}{13}x+\frac{9}{13},\ y|_{x=4}=-\frac{71}{13}$
59. $y=\frac{3}{2}x+\frac{1}{6},\ y|_{x=4}=\frac{37}{6}$
61. $y=0.122x+3.58$



63. a)



b) $y=0.0427\text{K}+1764.8$ c) 1780

Section 12.9, pp. 987–989

1. $\left(\pm\frac{1}{\sqrt{2}},\frac{1}{2}\right),\ \left(\pm\frac{1}{\sqrt{2}},-\frac{1}{2}\right)$ **3.** 39 **5.** $(3,\pm3\sqrt{2})$ **7.** a) 8
b) 64 **9.** $r=2$ cm, $h=4$ cm **11.** $l=4\sqrt{2},\ w=3\sqrt{2}$
13. $f(0,0)=0$ is minimum, $f(2,4)=20$ is maximum

15. Minimum $= 0°$, maximum $= 125°$ **17.** $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$ **19.** 1

21. $(0, 0, 2)$, $(0, 0, -2)$

23. $f(1, -2, 5) = 30$ is maximum, $f(-1, 2, -5) = -30$ is minimum

25. 3, 3, 3 **27.** $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ by $\frac{2}{\sqrt{3}}$ units **29.** $\left(\pm\frac{4}{3}, -\frac{4}{3}, -\frac{4}{3}\right)$

31. $U(8, 14) = \$128$ **33.** $f\left(\frac{2}{3}, \frac{4}{3}, -\frac{4}{3}\right) = \frac{4}{3}$ **35.** $(2, 4, 4)$

37. Maximum is $1 + 6\sqrt{3}$ at $(\pm\sqrt{6}, \sqrt{3}, 1)$, minimum is $1 - 6\sqrt{3}$ at $(\pm\sqrt{6}, -\sqrt{3}, 1)$

39. Maximum is 4 at $(0, 0, \pm 2)$, minimum is 2 at $(\pm\sqrt{2}, \pm\sqrt{2}, 0)$

Section 12.10, p. 993

1. Quadratic: $x + xy$; cubic: $x + xy + \frac{1}{2}xy^2$

3. Quadratic: xy; cubic: xy

5. Quadratic: $y + \frac{1}{2}(2xy - y^2)$;

cubic: $y + \frac{1}{2}(2xy - y^2) + \frac{1}{6}(3x^2y - 3xy^2 + 2y^3)$

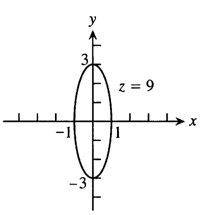
7. Quadratic: $\frac{1}{2}(2x^2 + 2y^2) = x^2 + y^2$; cubic: $x^2 + y^2$

9. Quadratic: $1 + (x + y) + (x + y)^2$;
cubic: $1 + (x + y) + (x + y)^2 + (x + y)^3$

11. Quadratic: $1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$, $E(x, y) \le 0.00134$

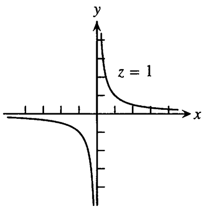
Chapter 12 Practice Exercises, pp. 994–998

1.



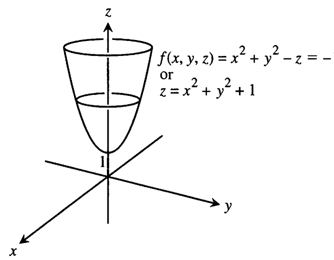
Domain: all points in the xy-plane; range: $z \ge 0$. Level curves are ellipses with major axis along the y-axis and minor axis along the x-axis.

3.



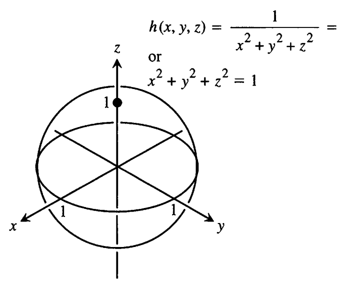
Domain: all (x, y) such that $x \ne 0$ and $y \ne 0$; range: $z \ne 0$. Level curves are hyperbolas with the x- and y-axes as asymptotes.

5.



Domain: all (x, y, z) such that $(x, y, z) \ne (0, 0, 0)$; range: all real numbers. Level surfaces are paraboloids of revolution with the z-axis as axis.

7.



Domain: all (x, y, z) such that $(x, y, z) \ne (0, 0, 0)$; range: positive real numbers. Level surfaces are spheres with center $(0, 0, 0)$ and radius $r > 0$.

9. -2 **11.** $1/2$ **13.** 1 **15.** Let $y = kx^2$, $k \ne 1$

17. a) Does not exist b) Not continuous at $(0, 0)$

19. $\frac{\partial g}{\partial r} = \cos\theta + \sin\theta$, $\frac{\partial g}{\partial \theta} = -r\sin\theta + r\cos\theta$

21. $\frac{\partial f}{\partial R_1} = -\frac{1}{R_1^2}$, $\frac{\partial f}{\partial R_2} = -\frac{1}{R_2^2}$, $\frac{\partial f}{\partial R_3} = -\frac{1}{R_3^2}$

23. $\frac{\partial P}{\partial n} = \frac{RT}{V}$, $\frac{\partial P}{\partial R} = \frac{nT}{V}$, $\frac{\partial P}{\partial T} = \frac{nR}{V}$, $\frac{\partial P}{\partial V} = -\frac{nRT}{V^2}$

25. $\frac{\partial^2 g}{\partial x^2} = 0$, $\frac{\partial^2 g}{\partial y^2} = \frac{2x}{y^3}$, $\frac{\partial^2 g}{\partial y\,\partial x} = \frac{\partial^2 g}{\partial x\,\partial y} = -\frac{1}{y^2}$

27. $\frac{\partial^2 f}{\partial x^2} = -30x + \frac{2 - 2x^2}{(x^2 + 1)^2}$, $\frac{\partial^2 f}{\partial y^2} = 0$, $\frac{\partial^2 f}{\partial y\,\partial x} = \frac{\partial^2 f}{\partial x\,\partial y} = 1$

29. Answers will depend on the upper bound used for $|f_{xx}|$, $|f_{xy}|$, $|f_{yy}|$. With $M = \sqrt{2}/2$, $|E| \le 0.0142$. With $M = 1$, $|E| \le 0.02$.

31. $L(x, y, z) = y - 3z$, $L(x, y, z) = x + y - z - 1$

33. Be more careful with the diameter.

35. $dl = 0.038$, % change in $V = -4.17\%$, % change in $R = -20\%$, % change in $l = 15.83\%$

37. a) 5% **39.** $\left.\frac{dw}{dt}\right|_{t=0} = -1$

41. $\left.\frac{\partial w}{\partial r}\right|_{(r,s)=(\pi,0)} = 2$, $\left.\frac{\partial w}{\partial s}\right|_{(r,s)=(\pi,0)} = 2 - \pi$

43. $\left.\frac{df}{dt}\right|_{t=1} = -(\sin 1 + \cos 2)\sin 1 + (\cos 1 + \cos 2)\cos 1 - 2(\sin 1 + \cos 1)\sin 2$

45. $\left.\dfrac{dy}{dx}\right|_{(x,y)=(0,1)} = -1$

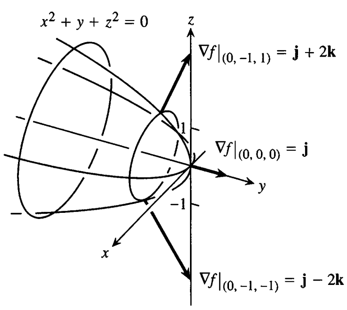
47. a) $(2y + x^2 z)e^{yz}$ b) $x^2 e^{yz}\left(y - \dfrac{z}{2y}\right)$ c) $(1 + x^2 y)e^{yz}$

49. Increases most rapidly in the direction $\mathbf{u} = -\dfrac{\sqrt{2}}{2}\mathbf{i} - \dfrac{\sqrt{2}}{2}\mathbf{j}$; decreases most rapidly in the direction $-\mathbf{u} = \dfrac{\sqrt{2}}{2}\mathbf{i} + \dfrac{\sqrt{2}}{2}\mathbf{j}$; $D_{\mathbf{u}}f = \dfrac{\sqrt{2}}{2}$, $D_{-\mathbf{u}}f = -\dfrac{\sqrt{2}}{2}$, $D_{\mathbf{u}_1}f = -\dfrac{7}{10}$

51. Increases most rapidly in the direction $\mathbf{u} = \dfrac{2}{7}\mathbf{i} + \dfrac{3}{7}\mathbf{j} + \dfrac{6}{7}\mathbf{k}$; decreases most rapidly in the direction $-\mathbf{u} = -\dfrac{2}{7}\mathbf{i} - \dfrac{3}{7}\mathbf{j} - \dfrac{6}{7}\mathbf{k}$; $D_{\mathbf{u}}f = 7$, $D_{-\mathbf{u}}f = -7$, $D_{\mathbf{u}_1}f = 7$

53. $\pi/\sqrt{2}$ **55.** a) $f_x(1,2) = f_y(1,2) = 2$ b) $14/5$

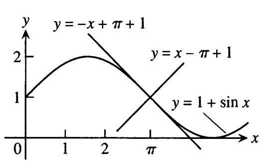
57.



59. Tangent: $4x - y - 5z = 4$, normal line: $x = 2 + 4t$, $y = -1 - t$, $z = 1 - 5t$

61. $2y - z - 2 = 0$

63. Tangent: $x + y = \pi + 1$, normal line: $y = x - \pi + 1$



65. $x = 1 - 2t$, $y = 1$, $z = \dfrac{1}{2} + 2t$

67. Local minimum of -8 at $(-2, -2)$

69. Saddle point at $(0, 0)$, $f(0,0) = 0$; local maximum of $1/4$ at $\left(-\dfrac{1}{2}, -\dfrac{1}{2}\right)$

71. Saddle point at $(0, 0)$, $f(0,0) = 0$; local minimum of -4 at $(0, 2)$; local maximum of 4 at $(-2, 0)$; saddle point at $(-2, 2)$, $f(-2, 2) = 0$

73. Absolute maximum: 28 at $(0, 4)$, absolute minimum: $-9/4$ at $(3/2, 0)$

75. Absolute maximum: 18 at $(2, -2)$, absolute minimum: $-17/4$ at $(-2, 1/2)$

77. Absolute maximum: 8 at $(-2, 0)$, absolute minimum: -1 at $(1, 0)$

79. Absolute maximum: 4 at $(1, 0)$, absolute minimum: -4 at $(0, -1)$

81. Absolute maximum: 1 at $(0, \pm 1)$ and $(1, 0)$, absolute minimum: -1 at $(-1, 0)$

83. Maximum: 5 at $(0, 1)$, minimum: $-1/3$ at $(0, -1/3)$

85. Maximum: $\sqrt{3}$ at $(1/\sqrt{3}, -1/\sqrt{3}, 1/\sqrt{3})$, minimum: $-\sqrt{3}$ at $(-1/\sqrt{3}, 1/\sqrt{3}, -1/\sqrt{3})$

87. Width $= \left(\dfrac{c^2 V}{ab}\right)^{1/3}$, depth $= \left(\dfrac{b^2 V}{ac}\right)^{1/3}$, height $= \left(\dfrac{a^2 V}{bc}\right)^{1/3}$

89. Maximum $= 3/2$ at $(1/\sqrt{2}, 1/\sqrt{2}, \sqrt{2})$ and $(-1/\sqrt{2}, -1/\sqrt{2}, -\sqrt{2})$, minimum $= 1/2$ at $(-1/\sqrt{2}, 1/\sqrt{2}, -\sqrt{2})$ and $(1/\sqrt{2}, -1/\sqrt{2}, \sqrt{2})$

91. $\dfrac{\partial w}{\partial x} = \cos\theta \dfrac{\partial w}{\partial r} - \dfrac{\sin\theta}{r}\dfrac{\partial w}{\partial \theta}$, $\dfrac{\partial w}{\partial y} = \sin\theta \dfrac{\partial w}{\partial r} + \dfrac{\cos\theta}{r}\dfrac{\partial w}{\partial \theta}$

97. $(t, -t \pm 4, t)$, t a real number

Chapter 12 Additional Exercises, pp. 998–1000

1. $f_{xy}(0,0) = -1$, $f_{yx}(0,0) = 1$ **7.** c) $r^2 = \dfrac{1}{2}(x^2 + y^2 + z^2)$

15. $V = \dfrac{\sqrt{3}abc}{2}$ **19.** $f(x, y) = \dfrac{y}{2} + 4$, $g(x, y) = \dfrac{x}{2} + \dfrac{9}{2}$

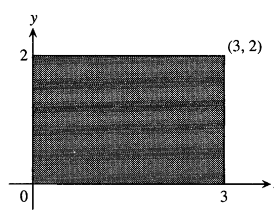
21. $y = 2\ln|\sin x| + \ln 2$ **23.** a) $\dfrac{1}{\sqrt{53}}(2\mathbf{i} + 7\mathbf{j})$ b) $\dfrac{-1}{\sqrt{29097}}(98\mathbf{i} - 127\mathbf{j} + 58\mathbf{k})$ **25.** $w = e^{-c^2\pi^2 t}\sin \pi x$

27. 0.213%

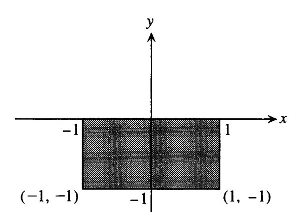
CHAPTER 13

Section 13.1, pp. 1010–1011

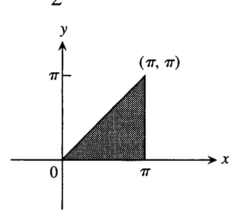
1. 16



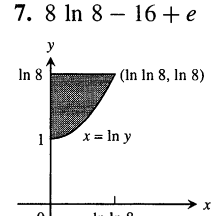
3. 1



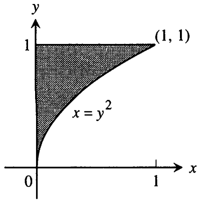
5. $\dfrac{\pi^2}{2} + 2$



7. $8\ln 8 - 16 + e$

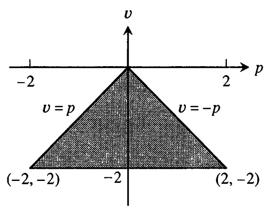


9. $e - 2$

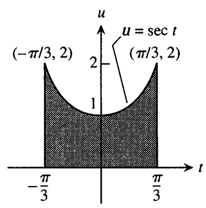


11. $\frac{3}{2} \ln 2$ **13.** $1/6$ **15.** $-1/10$

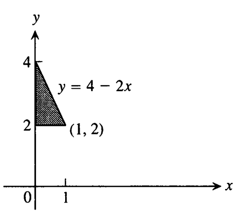
17. 8



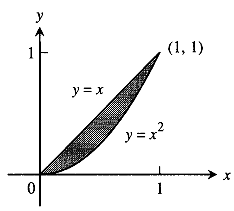
19. 2π



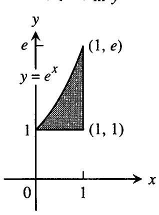
21. $\int_2^4 \int_0^{(4-y)/2} dx\, dy$



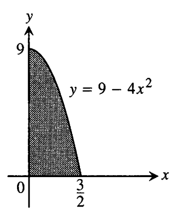
23. $\int_0^1 \int_{x^2}^{x} dy\, dx$



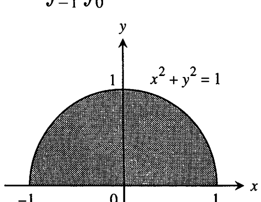
25. $\int_1^e \int_{\ln y}^{1} dx\, dy$



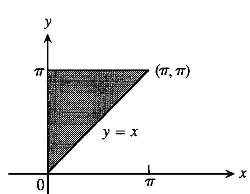
27. $\int_0^9 \int_0^{(\sqrt{9-y})/2} 16x\, dx\, dy$



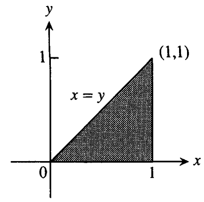
29. $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 3y\, dy\, dx$



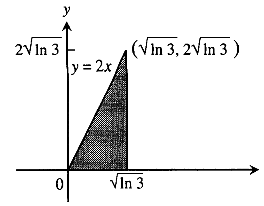
31. 2



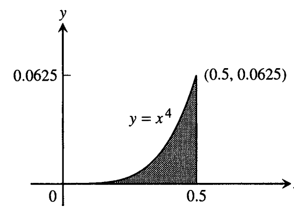
33. $\frac{e-2}{2}$



35. 2

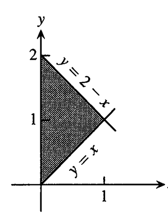


37. $\frac{1}{80\pi}$



39. $-2/3$ **41.** $4/3$ **43.** $625/12$ **45.** 16 **47.** 20
49. $2(1 + \ln 2)$ **51.** 1 **53.** π^2 **55.** $-1/4$ **57.** $20\sqrt{3}/9$

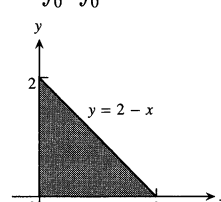
59. $\int_0^1 \int_x^{2-x} (x^2 + y^2)\, dy\, dx = \frac{4}{3}$



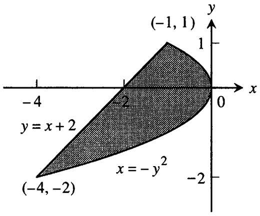
67. 0.603 **69.** 0.233

Section 13.2, pp. 1018–1020

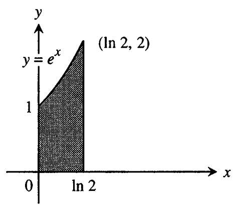
1. $\int_0^2 \int_0^{2-x} dy\, dx = 2$ or $\int_0^2 \int_0^{2-y} dx\, dy = 2$



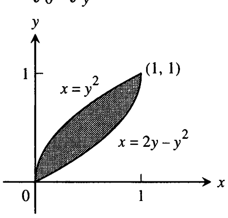
3. $\int_{-2}^{1}\int_{y-2}^{-y^2} dx\,dy = 9/2$



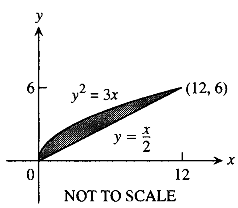
5. $\int_0^{\ln 2}\int_0^{e^x} dy\,dx = 1$



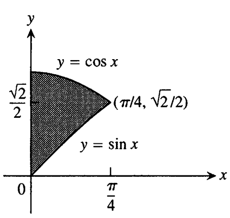
7. $\int_0^1\int_{y^2}^{2y-y^2} dx\,dy = 1/3$



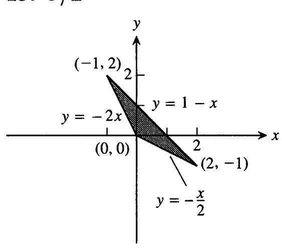
9. 12



11. $\sqrt{2}-1$



13. 3/2



15. a) 0 b) $4/\pi^2$ **17.** 8/3 **19.** $\bar{x}=\frac{5}{14},\ \bar{y}=\frac{38}{35}$

21. $\bar{x}=\frac{64}{35},\ \bar{y}=\frac{5}{7}$ **23.** $\bar{x}=0,\ \bar{y}=\frac{4}{3\pi}$ **25.** $\bar{x}=\bar{y}=\frac{4a}{3\pi}$

27. $\bar{x}=\frac{\pi}{2},\ \bar{y}=\frac{\pi}{8}$ **29.** $\bar{x}=-1,\ \bar{y}=\frac{1}{4}$

31. $I_x=\frac{64}{105},\ R_x=2\sqrt{\frac{2}{7}}$ **33.** $\bar{x}=\frac{3}{8},\ \bar{y}=\frac{17}{16}$

35. $\bar{x}=\frac{11}{3},\ \bar{y}=\frac{14}{27},\ I_y=432,\ R_y=4$

37. $\bar{x}=0,\ \bar{y}=\frac{13}{31},\ I_y=\frac{7}{5},\ R_y=\sqrt{\frac{21}{31}}$

39. $\bar{x}=0,\ \bar{y}=7/10;\ I_x=9/10,\ I_y=3/10,\ I_0=6/5;$

$R_x=\frac{3\sqrt{6}}{10},\ R_y=\frac{3\sqrt{2}}{10},\ R_0=\frac{3\sqrt{2}}{5}$

41. $40{,}000(1-e^{-2})\ln\left(\frac{7}{2}\right)\approx 43{,}329$

43. If $0<a\le 5/2$, then the appliance will have to be tipped more than 45° to fall over.

45. $(\bar{x},\bar{y})=(2/\pi,0)$ **47.** a) 3/2 b) They are the same.

53. a) $\left(\frac{7}{5},\frac{31}{10}\right)$ b) $\left(\frac{19}{7},\frac{18}{7}\right)$ c) $\left(\frac{9}{2},\frac{19}{8}\right)$ d) $\left(\frac{11}{4},\frac{43}{16}\right)$

55. In order for c.m. to be on the common boundary, $h=a\sqrt{2}$. In order for c.m. to be inside T, $h>a\sqrt{2}$.

Section 13.3, pp. 1024–1026

1. $\pi/2$ **3.** $\pi/8$ **5.** πa^2 **7.** 36 **9.** $(1-\ln 2)\pi$

11. $(2\ln 2-1)(\pi/2)$ **13.** $\frac{\pi}{2}+1$ **15.** $\pi(\ln(4)-1)$

17. $2(\pi-1)$ **19.** 12π **21.** $\frac{3\pi}{8}+1$ **23.** 4 **25.** $6\sqrt{3}-2\pi$

27. $\bar{x}=5/6,\ \bar{y}=0$ **29.** $2a/3$ **31.** $2a/3$ **33.** 2π

35. $\frac{4}{3}+\frac{5\pi}{8}$ **37.** a) $\sqrt{\pi}/2$ b) 1 **39.** $\pi\ln 4$, no

41. $\frac{1}{2}(a^2+2h^2)$

Section 13.4, pp. 1031–1034

1. 1

3. $\int_0^1\int_0^{2-2x}\int_0^{3-3x-3y/2} dz\,dy\,dx,\quad \int_0^2\int_0^{1-y/2}\int_0^{3-3x-3y/2} dz\,dx\,dy,$

$\int_0^1\int_0^{3-3x}\int_0^{2-2x-2z/3} dy\,dz\,dx,\quad \int_0^3\int_0^{1-z/3}\int_0^{2-2x-2z/3} dy\,dx\,dz,$

$\int_0^2\int_0^{3-3y/2}\int_0^{1-y/2-z/3} dx\,dz\,dy,\quad \int_0^3\int_0^{2-2z/3}\int_0^{1-y/2-z/3} dx\,dy\,dz.$

The value of all six integrals is 1.

5. $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dy\,dx,$

$\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{x^2+y^2}^{8-x^2-y^2} 1\,dz\,dx\,dy,$

$\int_{-2}^{2}\int_{4}^{8-y^2}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dz\,dy+\int_{-2}^{2}\int_{y^2}^{4}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dz\,dy,$

$\int_{4}^{8}\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-y^2}}^{\sqrt{8-z-y^2}} 1\,dx\,dy\,dz+\int_{0}^{4}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-y^2}}^{\sqrt{z-y^2}} 1\,dx\,dy\,dz,$

$\int_{-2}^{2}\int_{4}^{8-x^2}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dz\,dx+\int_{-2}^{2}\int_{x^2}^{4}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dz\,dx,$

$\int_{4}^{8}\int_{-\sqrt{8-z}}^{\sqrt{8-z}}\int_{-\sqrt{8-z-x^2}}^{\sqrt{8-z-x^2}} 1\,dy\,dx\,dz+\int_{0}^{4}\int_{-\sqrt{z}}^{\sqrt{z}}\int_{-\sqrt{z-x^2}}^{\sqrt{z-x^2}} 1\,dy\,dx\,dz.$

The value of all six integrals is 16π.

7. 1 **9.** 1 **11.** $\frac{\pi^3}{2}(1-\cos 1)$ **13.** 18 **15.** 7/6 **17.** 0

19. $\frac{1}{2}-\frac{\pi}{8}$ **21.** a) $\int_{-1}^{1}\int_0^{1-x^2}\int_{x^2}^{1-z} dy\,dz\,dx$

b) $\int_0^1\int_{-\sqrt{1-z}}^{\sqrt{1-z}}\int_{x^2}^{1-z} dy\,dx\,dz$ c) $\int_0^1\int_0^{1-z}\int_{-\sqrt{y}}^{\sqrt{y}} dy\,dz\,dx$

d) $\int_0^1 \int_0^{1-y} \int_{-\sqrt{y}}^{\sqrt{y}} dx\,dz\,dy$ e) $\int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{1-y} dz\,dx\,dy$ **23.** 2/3

25. 20/3 **27.** 1 **29.** 16/3 **31.** $8\pi - \frac{32}{3}$ **33.** 2 **35.** 4π

37. 31/3 **39.** 1 **41.** $2\sin 4$ **43.** 4 **45.** $a = 3$ or $a = \frac{13}{3}$

Section 13.5, pp. 1036–1039

1. $R_x = \sqrt{\frac{b^2+c^2}{12}},\ R_y = \sqrt{\frac{a^2+c^2}{12}},\ R_z = \sqrt{\frac{a^2+b^2}{12}}$

3. $I_x = \frac{M}{3}(b^2+c^2),\ I_y = \frac{M}{3}(a^2+c^2),\ I_z = \frac{M}{3}(a^2+b^2)$

5. $\bar{x} = \bar{y} = 0,\ \bar{z} = \frac{12}{5},\ I_x = \frac{7904}{105} \approx 75.28,\ I_y = \frac{4832}{63} \approx 76.70,$ $I_z = \frac{256}{45} \approx 5.69$

7. a) $\bar{x} = \bar{y} = 0,\ \bar{z} = \frac{8}{3}$ b) $c = 2\sqrt{2}$

9. $I_L = 1386,\ R_L = \sqrt{\frac{77}{2}}$ **11.** $I_L = \frac{40}{3},\ R_L = \sqrt{\frac{5}{3}}$ **13.** a) $\frac{4}{3}$ b) $\bar{x} = \frac{4}{5},\ \bar{y} = \bar{z} = \frac{2}{5}$ **15.** a) $\frac{5}{2}$ b) $\bar{x} = \bar{y} = \bar{z} = \frac{8}{15}$ c) $I_x = I_y = I_z = \frac{11}{6}$ d) $R_x = R_y = R_z = \sqrt{\frac{11}{15}}$ **17.** 3

19. a) $\frac{4}{3}g$ b) $\frac{4}{3}g$

23. a) $I_{\text{c.m.}} = \frac{abc(a^2+b^2)}{12},\ R_{\text{c.m.}} = \sqrt{\frac{a^2+b^2}{12}}$

b) $I_L = \frac{abc(a^2+7b^2)}{3},\ R_L = \sqrt{\frac{a^2+7b^2}{3}}$

27. a) $h = a\sqrt{3}$ b) $h = a\sqrt{2}$

Section 13.6, pp. 1044–1047

1. $4\pi(\sqrt{2}-1)/3$ **3.** $17\pi/5$ **5.** $\pi\,(6\sqrt{2}-8)$ **7.** $3\pi/10$

9. $\pi/3$ **11.** a) $\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_0^1 r\,dr\,dz\,d\theta + \int_0^{2\pi}\int_{\sqrt{3}}^2\int_0^{\sqrt{4-z^2}} r\,dr\,dz\,d\theta$

c) $\int_0^1\int_0^{\sqrt{4-r^2}}\int_0^{2\pi} r\,d\theta\,dz\,dr$

13. $\int_{-\pi/2}^{\pi/2}\int_0^{\cos\theta}\int_0^{3r^2} f(r,\theta,z)\,r\,dz\,dr\,d\theta$

15. $\int_0^{\pi}\int_0^{2\sin\theta}\int_0^{4-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$

17. $\int_{-\pi/2}^{\pi/2}\int_1^{1+\cos\theta}\int_0^4 f(r,\theta,z)\,dz\,r\,dr\,d\theta$

19. $\int_0^{\pi/4}\int_0^{\sec\theta}\int_0^{2-r\sin\theta} f(r,\theta,z)\,dz\,r\,dr\,d\theta$ **21.** π^2 **23.** $\pi/3$

25. 5π **27.** 2π **29.** $\left(\frac{8-5\sqrt{2}}{2}\right)\pi$

31. a) $\int_0^{2\pi}\int_0^{\pi/6}\int_0^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta +$ $\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\int_0^{\csc\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $\int_0^{2\pi}\int_1^2\int_{\pi/6}^{\sin^{-1}(1/\rho)} \rho^2\sin\phi\,d\phi\,d\rho\,d\theta +$ $\int_0^{2\pi}\int_0^2\int_0^{\pi/6} \rho^2\sin\phi\,d\phi\,d\rho\,d\theta$

33. $\int_0^{2\pi}\int_0^{\pi/2}\int_{\cos\phi}^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{31\pi}{6}$

35. $\int_0^{2\pi}\int_0^{\pi}\int_0^{1-\cos\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{8\pi}{3}$

37. $\int_0^{2\pi}\int_{\pi/4}^{\pi/2}\int_0^{2\cos\phi} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta = \frac{\pi}{3}$

39. a) $8\int_0^{\pi/2}\int_0^{\pi/2}\int_0^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $8\int_0^{\pi/2}\int_0^2\int_0^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

c) $8\int_0^2\int_0^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$

41. a) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^2 \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

b) $\int_0^{2\pi}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-r^2}} r\,dz\,dr\,d\theta$

c) $\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_1^{\sqrt{4-x^2-y^2}} dz\,dy\,dx$ d) $\frac{5\pi}{3}$ **43.** $8\pi/3$

45. 9/4 **47.** $(3\pi-4)/18$ **49.** $\frac{2\pi a^3}{3}$ **51.** $5\pi/3$ **53.** $\pi/2$

55. $\frac{4(2\sqrt{2}-1)\pi}{3}$ **57.** 16π **59.** $5\pi/2$ **61.** $\frac{4\pi(8-3\sqrt{3})}{3}$

63. 2/3 **65.** 3/4 **67.** $\bar{x} = \bar{y} = 0,\ \bar{z} = 3/8$

69. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 3/8)$ **71.** $\bar{x} = \bar{y} = 0,\ \bar{z} = 5/6$

73. $I_z = 30\pi,\ R_z = \sqrt{\frac{5}{2}}$ **75.** $I_x = \pi/4$ **77.** $\frac{a^4 h\pi}{10}$

79. a) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{4}{5}\right),\ I_z = \frac{\pi}{12},\ R_z = \sqrt{\frac{1}{3}}$

b) $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{5}{6}\right),\ I_z = \frac{\pi}{14},\ R_z = \sqrt{\frac{5}{14}}$

83. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{2h^2 + 3h}{3h + 6}\right),\ I_z = \frac{\pi a^4(h^2 + 2h)}{4},\ R_z = \frac{a}{\sqrt{2}}$

85. $\frac{3M}{\pi R^3}$

Section 13.7, pp. 1054–1055

1. a) $x = \frac{u + v}{3},\ y = \frac{v - 2u}{3};\ \frac{1}{3}$

b) Triangular region with boundaries $u = 0,\ v = 0,$ and $u + v = 3$

3. a) $x = \frac{1}{5}(2u - v),\ y = \frac{1}{10}(3v - u);\ \frac{1}{10}$

b) Triangular region with boundaries $3v = u,\ v = 2u,$ and $3u + v = 10$

5. a) $\begin{vmatrix} \cos v & -u \sin v \\ \sin v & u \cos v \end{vmatrix} = u \cos^2 v + u \sin^2 v = u$

b) $\begin{vmatrix} \sin v & u \cos v \\ \cos v & -u \sin v \end{vmatrix} = -u \sin^2 v - u \cos^2 v = -u$

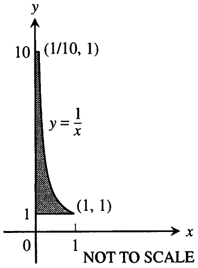
9. $64/5$ **11.** $\int_1^2 \int_1^3 (u + v)\frac{2u}{v}\, du\, dv = 8 + \frac{52}{3} \ln 2$

13. $\frac{\pi ab(a^2 + b^2)}{4}$ **15.** $\frac{1}{3}\left(1 + \frac{3}{e^2}\right) \approx 0.4687$ **19.** $\frac{4\pi\, abc}{3}$

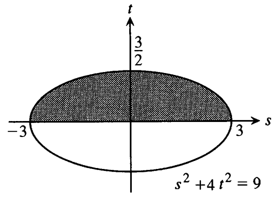
21. $\int_0^3 \int_0^2 \int_1^2 \left(\frac{v}{3} + \frac{vw}{3u}\right) du\, dv\, dw = 2 + \ln 8$

Chapter 13 Practice Exercises, pp. 1056–1058

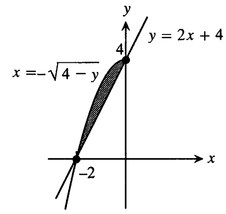
1. $9e - 9$



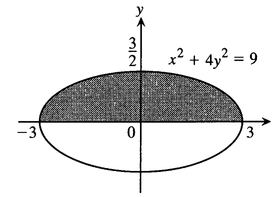
3. $9/2$



5. $\int_{-2}^{0} \int_{2x+4}^{4-x^2} dy\, dx = \frac{4}{3}$



7. $\int_{-3}^{3} \int_{0}^{(1/2)\sqrt{9-x^2}} y\, dy\, dx = \frac{9}{2}$



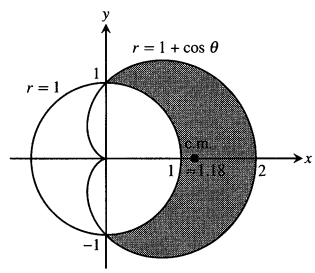
9. $\sin 4$ **11.** $\frac{\ln 17}{4}$ **13.** $4/3$ **15.** $4/3$ **17.** $1/4$

19. $\bar{x} = \bar{y} = \frac{1}{2 - \ln 4}$ **21.** $I_0 = 104$ **23.** $I_x = 2\delta,\ R_x = \sqrt{\frac{2}{3}}$

25. $M = 4,\ M_x = 0,\ M_y = 0$ **27.** π **29.** $\bar{x} = \frac{3\sqrt{3}}{\pi},\ \bar{y} = 0$

31. a) $\bar{x} = \frac{15\pi + 32}{6\pi + 48},\ \bar{y} = 0$

b)



33. $\frac{\pi - 2}{4}$ **35.** 0 **37.** $8/35$ **39.** $\pi/2$ **41.** $\frac{2(31 - 3^{5/2})}{3}$

43. a) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 3\, dz\, dx\, dy$

b) $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2} 3\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$ c) $2\pi(8 - 4\sqrt{2})$

45. $\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sec\phi} \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \frac{\pi}{3}$

47. $\int_0^1 \int_{\sqrt{1-x^2}}^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} z^2 xy\, dz\, dy\, dx +$

$\int_1^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} z^2 xy\, dz\, dy\, dx$

49. a) $\int_{-\sqrt{3}}^{\sqrt{3}} \int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \int_1^{\sqrt{4-x^2-y^2}} dz\, dy\, dx$

b) $\int_0^{2\pi} \int_0^{\sqrt{3}} \int_1^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$

c) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

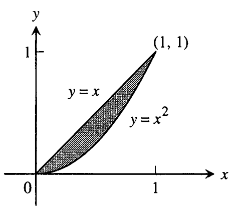
51. a) $\dfrac{8\pi(4\sqrt{2}-5)}{3}$ b) $\dfrac{8\pi(4\sqrt{2}-5)}{3}$ **53.** $I_z = \dfrac{8\pi\,\delta(b^5 - a^5)}{15}$

Chapter 13 Additional Exercises, pp. 1058–1060

1. a) $\int_{-3}^{2}\int_{x}^{6-x^2} x^2 \, dy \, dx$ b) $\int_{-3}^{2}\int_{x}^{6-x^2}\int_{0}^{x^2} dz \, dy \, dx$ c) $\dfrac{125}{4}$

3. 2π **5.** $3\pi/2$ **7.** a) Hole radius = 1, sphere radius = 2 b) $4\sqrt{3}\pi$ **9.** $\pi/4$

11. $\int_0^1\int_y^{\sqrt{y}} f(x, y) \, dx \, dy$



13. $\ln\left(\dfrac{b}{a}\right)$ **17.** $1/\sqrt[4]{3}$ **19.** $\overline{x} = \dfrac{15\pi + 32}{6\pi + 48}$, $\overline{y} = 0$

21. Mass $= a^2 \cos^{-1}\left(\dfrac{b}{a}\right) - b\sqrt{a^2 - b^2}$,

$I_0 = \dfrac{a^4}{2}\cos^{-1}\left(\dfrac{b}{a}\right) - \dfrac{b^3}{2}\sqrt{a^2 - b^2} - \dfrac{b^3}{6}(a^2 - b^2)^{3/2}$

23. a) $\overline{x} = \overline{y} = 0$, $\overline{z} = 1/2$; $I_z = \dfrac{\pi}{8}$, $R_z = \dfrac{\sqrt{3}}{2}$

b) $\overline{x} = \overline{y} = 0$, $\overline{z} = 5/14$; $I_z = \dfrac{2\pi}{7}$, $R_z = \sqrt{\dfrac{5}{7}}$

25. $\overline{x} = \overline{y} = 0$, $\overline{z} = \dfrac{3a}{8}$ **27.** $\dfrac{1}{ab}\left(e^{a^2b^2} - 1\right)$ **29.** b) 1 c) 0

33. $h = \sqrt{20}$ in., $h = \sqrt{60}$ in. **37.** $\dfrac{1}{2}\pi^2$

CHAPTER 14

Section 14.1, pp. 1065–1067

1. c **3.** g **5.** d **7.** f **9.** $\sqrt{2}$ **11.** 13/2 **13.** $3\sqrt{14}$

15. $(1/6)(5\sqrt{5} + 9)$ **17.** $\sqrt{3}\ln(b/a)$ **19.** $\dfrac{10\sqrt{5} - 2}{3}$ **21.** 8

23. $2\sqrt{2} - 1$ **25.** a) $4\sqrt{2} - 2$ b) $\sqrt{2} + \ln(1 + \sqrt{2})$

27. $I_z = 2\pi\delta a^3$, $R_z = a$ **29.** a) $I_z = 2\pi\sqrt{2}\,\delta$, $R_z = 1$

b) $I_z = 4\pi\sqrt{2}\,\delta$, $R_z = 1$ **31.** $I_x = 2\pi - 2$, $R_x = 1$

Section 14.2, pp. 1074–1076

1. $\nabla f = -(x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})(x^2 + y^2 + z^2)^{-3/2}$

3. $\nabla g = -(2x/(x^2 + y^2))\,\mathbf{i} - (2y/(x^2 + y^2))\,\mathbf{j} + e^z\,\mathbf{k}$

5. $\mathbf{F} = -\dfrac{kx}{(x^2 + y^2)^{3/2}}\mathbf{i} - \dfrac{ky}{(x^2 + y^2)^{3/2}}\mathbf{j}$, any $k > 0$

7. a) $\dfrac{9}{2}$ b) $\dfrac{13}{3}$ c) $\dfrac{9}{2}$ **9.** a) $\dfrac{1}{3}$ b) $-\dfrac{1}{5}$ c) 0 **11.** a) 2

b) $\dfrac{3}{2}$ c) $\dfrac{1}{2}$ **13.** 1/2 **15.** $-\pi$ **17.** 207/12 **19.** $-39/2$

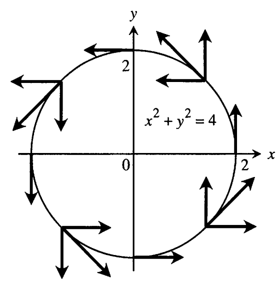
21. 25/6 **23.** a) $\text{Circ}_1 = 0$, $\text{circ}_2 = 2\pi$, $\text{flux}_1 = 2\pi$, $\text{flux}_2 = 0$

b) $\text{Circ}_1 = 0$, $\text{circ}_2 = 8\pi$, $\text{flux}_1 = 8\pi$, $\text{flux}_2 = 0$

25. Circ = 0, flux = $a^2\pi$ **27.** Circ = $a^2\pi$, flux = 0

29. a) $-\pi/2$ b) 0 c) 1

31.



33. a) $\mathbf{G} = -y\,\mathbf{i} + x\,\mathbf{j}$ b) $\mathbf{G} = \sqrt{x^2 + y^2}\,\mathbf{F}$

35. $\mathbf{F} = -(x\,\mathbf{i} + y\,\mathbf{j})/\sqrt{x^2 + y^2}$ **37.** 48 **39.** π **41.** 0

43. 1/2

Section 14.3, pp. 1083–1084

1. Conservative **3.** Not conservative **5.** Not conservative

7. $f(x, y, z) = x^2 + \dfrac{3y^2}{2} + 2z^2 + C$ **9.** $f(x, y, z) = xe^{y+2z} + C$

11. $f(x, y, z) = x\ln x - x + \tan(x + y) + \dfrac{1}{2}\ln(y^2 + z^2) + C$

13. 49 **15.** -16 **17.** 1 **19.** $9\ln 2$ **21.** 0 **23.** -3

27. $\mathbf{F} = \nabla\left(\dfrac{x^2 - 1}{y}\right)$ **29.** a) 1 b) 1 c) 1 **31.** a) 2 b) 2

33. $f(x, y, z) = \dfrac{GmM}{(x^2 + y^2 + z^2)^{1/2}}$ **35.** a) $c = b = 2a$

b) $c = b = 2$

37. It does not matter what path you use. The work will be the same on any path because the field is conservative.

Section 14.4, pp. 1093–1095

1. Flux = 0, circ = $2\pi a^2$ **3.** Flux = $-\pi a^2$, circ = 0

5. Flux = 2, circ = 0 **7.** Flux = -9, circ = 9

9. Flux = 1/2, circ = 1/2 **11.** Flux = 1/5, circ = $-1/12$

13. 0 **15.** 2/33 **17.** 0 **19.** -16π **21.** πa^2 **23.** $\dfrac{3}{8}\pi$

25. a) 0 b) $(h - k)$(area of the region) **35.** a) 0

Section 14.5, pp. 1103–1105

1. $\frac{13}{3}\pi$ **3.** 4 **5.** $6\sqrt{6}-2\sqrt{2}$ **7.** $\pi\sqrt{c^2+1}$

9. $\frac{\pi}{6}(17\sqrt{17}-5\sqrt{5})$ **11.** $3+2\ln 2$ **13.** $9a^3$

15. $\frac{abc}{4}(ab+ac+bc)$ **17.** 2 **19.** 18 **21.** $\pi a^3/6$

23. $\pi a^2/4$ **25.** $\pi a^3/2$ **27.** -32 **29.** -4 **31.** $3a^4$

33. $\left(\frac{a}{2}, \frac{a}{2}, \frac{a}{2}\right)$

35. $(\bar{x}, \bar{y}, \bar{z}) = \left(0, 0, \frac{14}{9}\right)$, $I_z = \frac{15\pi\sqrt{2}}{2}\delta$, $R_z = \frac{\sqrt{10}}{2}$

37. a) $\frac{8\pi}{3}a^4\delta$ b) $\frac{20\pi}{3}a^4\delta$ **39.** $\frac{\pi}{6}(13\sqrt{13}-1)$ **41.** $5\pi\sqrt{2}$

43. $\frac{2}{3}(5\sqrt{5}-1)$

Section 14.6, pp. 1112–1114

1. $\mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + r^2\mathbf{k}$, $0 \le r \le 2$, $0 \le \theta \le 2\pi$
3. $\mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (r/2)\mathbf{k}$, $0 \le r \le 6$, $0 \le \theta \le \pi/2$
5. $\mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + \sqrt{9-r^2}\,\mathbf{k}$, $0 \le r \le 3\sqrt{2}/2$, $0 \le \theta \le 2\pi$; Also: $\mathbf{r}(\phi,\theta) = (3\sin\phi\cos\theta)\mathbf{i} + (3\sin\phi\sin\theta)\mathbf{j} + (3\cos\phi)\mathbf{k}$, $0 \le \phi \le \pi/4$, $0 \le \theta \le 2\pi$
7. $\mathbf{r}(\phi,\theta) = (\sqrt{3}\sin\phi\cos\theta)\mathbf{i} + (\sqrt{3}\sin\phi\sin\theta)\mathbf{j} + (\sqrt{3}\cos\phi)\mathbf{k}$, $\pi/3 \le \phi \le 2\pi/3$, $0 \le \theta \le 2\pi$
9. $\mathbf{r}(x,y) = x\mathbf{i} + y\mathbf{j} + (4-y^2)\mathbf{k}$, $0 \le x \le 2$, $-2 \le y \le 2$
11. $\mathbf{r}(u,v) = u\mathbf{i} + (3\cos v)\mathbf{j} + (3\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$
13. a) $\mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (1 - r\cos\theta - r\sin\theta)\mathbf{k}$, $0 \le r \le 3$, $0 \le \theta \le 2\pi$
b) $\mathbf{r}(u,v) = (1 - u\cos v - u\sin v)\mathbf{i} + (u\cos v)\mathbf{j} + (u\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$
15. $\mathbf{r}(u,v) = (4\cos^2 v)\mathbf{i} + u\mathbf{j} + (4\cos v\sin v)\mathbf{k}$, $0 \le u \le 3$, $-(\pi/2) \le v \le (\pi/2)$; Another way: $\mathbf{r}(u,v) = (2+2\cos v)\mathbf{i} + u\mathbf{j} + (2\sin v)\mathbf{k}$, $0 \le u \le 3$, $0 \le v \le 2\pi$

17. $\int_0^{2\pi}\int_0^1 \frac{\sqrt{5}}{2} r\,dr\,d\theta = \frac{\pi\sqrt{5}}{2}$

19. $\int_0^{2\pi}\int_1^3 r\sqrt{5}\,dr\,d\theta = 8\pi\sqrt{5}$

21. $\int_0^{2\pi}\int_1^4 1\,du\,dv = 6\pi$

23. $\int_0^{2\pi}\int_0^1 u\sqrt{4u^2+1}\,du\,dv = \frac{(5\sqrt{5}-1)}{6}\pi$

25. $\int_0^{2\pi}\int_{\pi/4}^{\pi} 2\sin\phi\,d\phi\,d\theta = (4+2\sqrt{2})\pi$

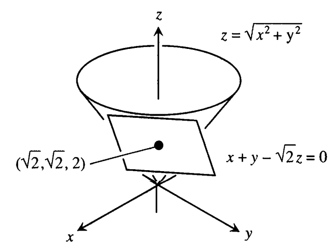
27. $\iint_S x\,d\sigma = \int_0^3\int_0^2 u\sqrt{4u^2+1}\,du\,dv = \frac{17\sqrt{17}-1}{4}$

29. $\iint_S x^2\,d\sigma = \int_0^{2\pi}\int_0^{\pi} \sin^3\phi\cos^2\theta\,d\phi\,d\theta = \frac{4\pi}{3}$

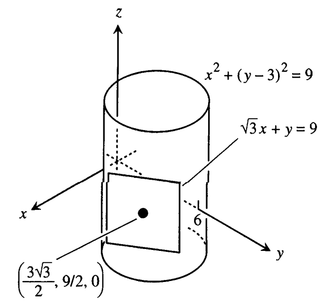
31. $\iint_S z\,d\sigma = \int_0^1\int_0^1 (4-u-v)\sqrt{3}\,dv\,du = 3\sqrt{3}$
(for $x = u$, $y = v$)

33. $\iint_S x^2\sqrt{5-4z}\,d\sigma = \int_0^1\int_0^{2\pi} u^2\cos^2 v \cdot \sqrt{4u^2+1}\cdot u\sqrt{4u^2+1}\,dv\,du = \int_0^1\int_0^{2\pi} u^3(4u^2+1)\cos^2 v\,dv\,du = \frac{11\pi}{12}$

35. -32 **37.** $\pi a^3/6$ **39.** $13a^4/6$ **41.** $2\pi/3$ **43.** $-73\pi/6$
45. $(a/2, a/2, a/2)$ **47.** $8\delta\pi a^4/3$
49.



51.



55. b) $A = \int_0^{2\pi}\int_0^{\pi} [a^2b^2\sin^2\phi\cos^2\phi + b^2c^2\cos^4\phi\cos^2\theta + a^2c^2\cos^4\phi\sin^2\theta]^{1/2}\,d\phi\,d\theta$

57. $x_0x + y_0y = 25$

Section 14.7, pp. 1122–1123

1. 4π **3.** $-5/6$ **5.** 0 **7.** -6π **9.** $2\pi a^2$ **13.** 12π

15. $-\frac{\pi}{4}$ **17.** -15π **25.** $16\,I_y + 16\,I_x$

Section 14.8, pp. 1132–1134

1. 0 **3.** 0 **5.** -16 **7.** -8π **9.** 3π **11.** $-40/3$
13. 12π **15.** $12\pi(4\sqrt{2}-1)$
21. The integral's value never exceeds the surface area of S.

Chapter 14 Practice Exercises, pp. 1134–1137

1. Path 1: $2\sqrt{3}$, Path 2: $1+3\sqrt{2}$ **3.** $4a^2$ **5.** 0 **7.** 0

9. 0 **11.** $\pi\sqrt{3}$ **13.** $2\pi\left(1-\frac{1}{\sqrt{2}}\right)$ **15.** $\frac{abc}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$

17. 50
19. $\mathbf{r}(\phi,\theta) = (6\sin\phi\cos\theta)\mathbf{i} + (6\sin\phi\sin\theta)\mathbf{j} + (6\cos\phi)\mathbf{k}$, $(\pi/6) \le \phi \le 2\pi/3$, $0 \le \theta \le 2\pi$
21. $\mathbf{r}(r,\theta) = (r\cos\theta)\mathbf{i} + (r\sin\theta)\mathbf{j} + (1+r)\mathbf{k}$, $0 \le r \le 2$, $0 \le \theta \le 2\pi$

23. $\mathbf{r}(u, v) = (u \cos v)\,\mathbf{i} + 2u^2\,\mathbf{j} + (u \sin v)\,\mathbf{k}$, $0 \le u \le 1$, $0 \le v \le \pi$
25. $\sqrt{6}$ **27.** $\pi[\sqrt{2} + \ln(1 + \sqrt{2})]$ **29.** Conservative
31. Not conservative **33.** $f(x, y, z) = y^2 + yz + 2x + z$
35. Path 1: 2, Path 2: 8/3 **37.** a) $1 - e^{-2\pi}$ b) $1 - e^{-2\pi}$
39. a) $-\pi/2$ b) 0 c) 1 **41.** 0 **43.** a) $4\sqrt{2} - 2$
b) $\sqrt{2} + \ln(1 + \sqrt{2})$
45. $(\bar{x}, \bar{y}, \bar{z}) = \left(1, \frac{16}{15}, \frac{2}{3}\right)$; $I_x = \frac{232}{45}$, $I_y = \frac{64}{15}$, $I_z = \frac{56}{9}$;
$R_x = \sqrt{\frac{116}{45}}$, $R_y = \sqrt{\frac{32}{15}}$, $R_z = \sqrt{\frac{28}{9}}$
47. $\bar{z} = \frac{3}{2}$, $I_z = \frac{7\sqrt{3}}{3}$, $R_z = \sqrt{\frac{7}{3}}$
49. $(\bar{x}, \bar{y}, \bar{z}) = (0, 0, 49/12)$, $I_z = 640\pi$, $R_z = 2\sqrt{2}$
51. Flux: 3/2, Circ: −1/2 **55.** 3 **57.** $\frac{2\pi}{3}(7 - 8\sqrt{2})$ **59.** 0
61. π

Chapter 14 Additional Exercises, pp. 1137–1139

1. 6π **3.** 2/3 **5.** a) $\mathbf{F}(x, y, z) = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$
b) $\mathbf{F}(x, y, z) = z\,\mathbf{i} + y\,\mathbf{k}$ c) $\mathbf{F}(x, y, z) = z\,\mathbf{i}$ **7.** $\frac{16\pi R^3}{3}$
9. $a = 2, b = 1$. The minimum flux is −4. **11.** b) $\frac{16}{3}g$
c) Work $= \left(\int_C g\,xy\,ds\right)\bar{y} = g\int_C xy^2\,ds$ **13.** c) $\frac{4}{3}\pi w$
19. False if $\mathbf{F} = y\,\mathbf{i} + x\,\mathbf{j}$

APPENDICES

Appendix A.3, pp. A-16–A-17

1. a) (14, 8) b) (−1, 8) c) (0, −5)
3. a) By reflecting z across the real axis b) By reflecting z across the imaginary axis c) By reflecting z in the origin d) By reflecting z in the real axis and then multiplying the length of the vector by $1/|z|^2$
5. a) Points on the circle $x^2 + y^2 = 4$ b) points inside the circle $x^2 + y^2 = 4$ c) points outside the circle $x^2 + y^2 = 4$
7. Points on a circle of radius 1, center (−1, 0)
9. Points on the line $y = -x$ **11.** $4e^{2\pi i/3}$ **13.** $1e^{2\pi i/3}$
21. $\cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$ **23.** 1, $-\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$
25. $2i$, $-\sqrt{3} - i$, $\sqrt{3} - i$ **27.** $\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i$, $-\frac{\sqrt{6}}{2} \pm \frac{\sqrt{2}}{2}i$
29. $1 \pm \sqrt{3}i$, $-1 \pm \sqrt{3}i$

Appendix A.8, p. A-29

1. −5 **3.** 1 **5.** −7 **7.** 38 **9.** $x = -4$, $y = 1$
11. $x = 3$, $y = 2$ **13.** $x = 3$, $y = -2$, $z = 2$
15. $x = 2$, $y = 0$, $z = -1$ **17.** a) $h = 6$, $k = 4$
b) $h = 6$, $k \neq 4$

Index

Note: Numbers in parentheses refer to exercises on the pages indicated.

A Brief Table of Integrals

1. $\int u\,dv = uv - \int v\,du$

2. $\int a^u\,du = \frac{a^u}{\ln a} + C, \quad a \neq 1, \quad a > 0$

3. $\int \cos u\,du = \sin u + C$

4. $\int \sin u\,du = -\cos u + C$

5. $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$

6. $\int (ax+b)^{-1}\,dx = \frac{1}{a} \ln |ax+b| + C$

7. $\int x(ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a^2}\left[\frac{ax+b}{n+2} - \frac{b}{n+1}\right] + C, \quad n \neq -1, -2$

8. $\int x(ax+b)^{-1}\,dx = \frac{x}{a} - \frac{b}{a^2} \ln |ax+b| + C$

9. $\int x(ax+b)^{-2}\,dx = \frac{1}{a^2}\left[\ln |ax+b| + \frac{b}{ax+b}\right] + C$

10. $\int \frac{dx}{x(ax+b)} = \frac{1}{b} \ln \left|\frac{x}{ax+b}\right| + C$

11. $\int (\sqrt{ax+b})^n\,dx = \frac{2}{a}\frac{(\sqrt{ax+b})^{n+2}}{n+2} + C, \quad n \neq -2$

12. $\int \frac{\sqrt{ax+b}}{x}\,dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}$

13. a) $\int \frac{dx}{x\sqrt{ax-b}} = \frac{2}{\sqrt{b}} \tan^{-1}\sqrt{\frac{ax-b}{b}} + C$

b) $\int \frac{dx}{x\sqrt{ax+b}} = \frac{1}{\sqrt{b}} \ln \left|\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}}\right| + C$

14. $\int \frac{\sqrt{ax+b}}{x^2}\,dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}} + C$

15. $\int \frac{dx}{x^2\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}} + C$

16. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\frac{x}{a} + C$

17. $\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3} \tan^{-1}\frac{x}{a} + C$

18. $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left|\frac{x+a}{x-a}\right| + C$

19. $\int \frac{dx}{(a^2-x^2)^2} = \frac{x}{2a^2(a^2-x^2)} + \frac{1}{4a^3} \ln \left|\frac{x+a}{x-a}\right| + C$

20. $\int \frac{dx}{\sqrt{a^2+x^2}} = \sinh^{-1}\frac{x}{a} + C = \ln(x + \sqrt{a^2+x^2}) + C$

21. $\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2+x^2}) + C$

22. $\int x^2\sqrt{a^2+x^2}\,dx = \frac{x}{8}(a^2+2x^2)\sqrt{a^2+x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2+x^2}) + C$

23. $\int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a \ln \left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

24. $\int \frac{\sqrt{a^2+x^2}}{x^2}\,dx = \ln(x + \sqrt{a^2+x^2}) - \frac{\sqrt{a^2+x^2}}{x} + C$

25. $\int \frac{x^2}{\sqrt{a^2+x^2}}\,dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2+x^2}) + \frac{x\sqrt{a^2+x^2}}{2} + C$

26. $\int \frac{dx}{x\sqrt{a^2+x^2}} = -\frac{1}{a} \ln \left|\frac{a+\sqrt{a^2+x^2}}{x}\right| + C$

27. $\int \frac{dx}{x^2\sqrt{a^2+x^2}} = -\frac{\sqrt{a^2+x^2}}{a^2x} + C$

28. $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\frac{x}{a} + C$

29. $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + C$

30. $\int x^2\sqrt{a^2-x^2}\,dx = \frac{a^4}{8}\sin^{-1}\frac{x}{a} - \frac{1}{8}x\sqrt{a^2-x^2}(a^2-2x^2)+C$

31. $\int \frac{\sqrt{a^2-x^2}}{x}\,dx = \sqrt{a^2-x^2} - a\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

32. $\int \frac{\sqrt{a^2-x^2}}{x^2}\,dx = -\sin^{-1}\frac{x}{a} - \frac{\sqrt{a^2-x^2}}{x} + C$

33. $\int \frac{x^2}{\sqrt{a^2-x^2}}\,dx = \frac{a^2}{2}\sin^{-1}\frac{x}{a} - \frac{1}{2}x\sqrt{a^2-x^2}+C$

34. $\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-x^2}}{x}\right| + C$

35. $\int \frac{dx}{x^2\sqrt{a^2-x^2}} = -\frac{\sqrt{a^2-x^2}}{a^2x} + C$

36. $\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\frac{x}{a} + C = \ln\left|x+\sqrt{x^2-a^2}\right| + C$

37. $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

38. $\int (\sqrt{x^2-a^2})^n\,dx = \frac{x(\sqrt{x^2-a^2})^n}{n+1} - \frac{na^2}{n+1}\int(\sqrt{x^2-a^2})^{n-2}dx,\quad n \neq -1$

39. $\int \frac{dx}{(\sqrt{x^2-a^2})^n} = \frac{x(\sqrt{x^2-a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2}\int\frac{dx}{(\sqrt{x^2-a^2})^{n-2}},\quad n \neq 2$

40. $\int x(\sqrt{x^2-a^2})^n\,dx = \frac{(\sqrt{x^2-a^2})^{n+2}}{n+2} + C,\quad n \neq -2$

41. $\int x^2\sqrt{x^2-a^2}\,dx = \frac{x}{8}(2x^2-a^2)\sqrt{x^2-a^2} - \frac{a^4}{8}\ln\left|x+\sqrt{x^2-a^2}\right| + C$

42. $\int \frac{\sqrt{x^2-a^2}}{x}\,dx = \sqrt{x^2-a^2} - a\sec^{-1}\left|\frac{x}{a}\right| + C$

43. $\int \frac{\sqrt{x^2-a^2}}{x^2}\,dx = \ln\left|x+\sqrt{x^2-a^2}\right| - \frac{\sqrt{x^2-a^2}}{x} + C$

44. $\int \frac{x^2}{\sqrt{x^2-a^2}}\,dx = \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + \frac{x}{2}\sqrt{x^2-a^2} + C$

45. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + C = \frac{1}{a}\cos^{-1}\left|\frac{a}{x}\right| + C$

46. $\int \frac{dx}{x^2\sqrt{x^2-a^2}} = \frac{\sqrt{x^2-a^2}}{a^2x} + C$

47. $\int \frac{dx}{\sqrt{2ax-x^2}} = \sin^{-1}\left(\frac{x-a}{a}\right) + C$

48. $\int \sqrt{2ax-x^2}\,dx = \frac{x-a}{2}\sqrt{2ax-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

49. $\int (\sqrt{2ax-x^2})^n dx = \frac{(x-a)(\sqrt{2ax-x^2})^n}{n+1} + \frac{na^2}{n+1}\int(\sqrt{2ax-x^2})^{n-2}\,dx$

50. $\int \frac{dx}{(\sqrt{2ax-x^2})^n} = \frac{(x-a)(\sqrt{2ax-x^2})^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2}\int\frac{dx}{(\sqrt{2ax-x^2})^{n-2}}$

51. $\int x\sqrt{2ax-x^2}\,dx = \frac{(x+a)(2x-3a)\sqrt{2ax-x^2}}{6} + \frac{a^3}{2}\sin^{-1}\left(\frac{x-a}{a}\right) + C$

52. $\int \frac{\sqrt{2ax-x^2}}{x}\,dx = \sqrt{2ax-x^2} + a\sin^{-1}\left(\frac{x-a}{a}\right) + C$

53. $\int \frac{\sqrt{2ax-x^2}}{x^2}\,dx = -2\sqrt{\frac{2a-x}{x}} - \sin^{-1}\left(\frac{x-a}{a}\right) + C$

54. $\int \frac{x\,dx}{\sqrt{2ax-x^2}} = a\sin^{-1}\left(\frac{x-a}{a}\right) - \sqrt{2ax-x^2} + C$

55. $\int \frac{dx}{x\sqrt{2ax-x^2}} = -\frac{1}{a}\sqrt{\frac{2a-x}{x}} + C$

56. $\int \sin ax\,dx = -\frac{1}{a}\cos ax + C$

57. $\int \cos ax\,dx = \frac{1}{a}\sin ax + C$

58. $\int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$

59. $\int \cos^2 ax\,dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$

60. $\int \sin^n ax\,dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n}\int \sin^{n-2} ax\,dx$

61. $\int \cos^n ax\,dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n}\int \cos^{n-2} ax\,dx$

62. a) $\int \sin ax \cos bx\,dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$

b) $\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$

c) $\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$

63. $\int \sin ax \cos ax\,dx = -\frac{\cos 2ax}{4a} + C$

64. $\int \sin^n ax \cos ax\,dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

65. $\int \frac{\cos ax}{\sin ax}\,dx = \frac{1}{a}\ln|\sin ax| + C$

66. $\int \cos^n ax \sin ax\,dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$

67. $\int \frac{\sin ax}{\cos ax}\,dx = -\frac{1}{a}\ln|\cos ax| + C$

68. $\int \sin^n ax \cos^m ax\,dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n}\int \sin^{n-2} ax \cos^m ax\,dx, \quad n \neq -m$ (reduces $\sin^n ax$)

69. $\int \sin^n ax \cos^m ax\,dx = \frac{\sin^{n+1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n}\int \sin^n ax \cos^{m-2} ax\,dx, \quad m \neq -n$ (reduces $\cos^m ax$)

70. $\int \frac{dx}{b + c\sin ax} = \frac{-2}{a\sqrt{b^2-c^2}}\tan^{-1}\left[\sqrt{\frac{b-c}{b+c}}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right)\right] + C, \quad b^2 > c^2$

71. $\int \frac{dx}{b + c\sin ax} = \frac{-1}{a\sqrt{c^2-b^2}}\ln\left|\frac{c + b\sin ax + \sqrt{c^2-b^2}\cos ax}{b + c\sin ax}\right| + C, \quad b^2 < c^2$

72. $\int \frac{dx}{1 + \sin ax} = -\frac{1}{a}\tan\left(\frac{\pi}{4} - \frac{ax}{2}\right) + C$

73. $\int \frac{dx}{1 - \sin ax} = \frac{1}{a}\tan\left(\frac{\pi}{4} + \frac{ax}{2}\right) + C$

74. $\int \frac{dx}{b + c\cos ax} = \frac{2}{a\sqrt{b^2-c^2}}\tan^{-1}\left[\sqrt{\frac{b-c}{b+c}}\tan\frac{ax}{2}\right] + C, \quad b^2 > c^2$

75. $\int \frac{dx}{b + c\cos ax} = \frac{1}{a\sqrt{c^2-b^2}}\ln\left|\frac{c + b\cos ax + \sqrt{c^2-b^2}\sin ax}{b + c\cos ax}\right| + C, \quad b^2 < c^2$

76. $\int \frac{dx}{1 + \cos ax} = \frac{1}{a}\tan\frac{ax}{2} + C$

77. $\int \frac{dx}{1 - \cos ax} = -\frac{1}{a}\cot\frac{ax}{2} + C$

78. $\int x\sin ax\,dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$

79. $\int x\cos ax\,dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$

80. $\int x^n \sin ax\, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax\, dx$

81. $\int x^n \cos ax\, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax\, dx$

82. $\int \tan ax\, dx = \frac{1}{a} \ln |\sec ax| + C$

83. $\int \cot ax\, dx = \frac{1}{a} \ln |\sin ax| + C$

84. $\int \tan^2 ax\, dx = \frac{1}{a} \tan ax - x + C$

85. $\int \cot^2 ax\, dx = -\frac{1}{a} \cot ax - x + C$

86. $\int \tan^n ax\, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax\, dx, \quad n \neq 1$

87. $\int \cot^n ax\, dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax\, dx, \quad n \neq 1$

88. $\int \sec ax\, dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$

89. $\int \csc ax\, dx = -\frac{1}{a} \ln |\csc ax + \cot ax| + C$

90. $\int \sec^2 ax\, dx = \frac{1}{a} \tan ax + C$

91. $\int \csc^2 ax\, dx = -\frac{1}{a} \cot ax + C$

92. $\int \sec^n ax\, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax\, dx, \quad n \neq 1$

93. $\int \csc^n ax\, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax\, dx, \quad n \neq 1$

94. $\int \sec^n ax \tan ax\, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$

95. $\int \csc^n ax \cot ax\, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$

96. $\int \sin^{-1} ax\, dx = x \sin^{-1} ax + \frac{1}{a}\sqrt{1 - a^2x^2} + C$

97. $\int \cos^{-1} ax\, dx = x \cos^{-1} ax - \frac{1}{a}\sqrt{1 - a^2x^2} + C$

98. $\int \tan^{-1} ax\, dx = x \tan^{-1} ax - \frac{1}{2a} \ln (1 + a^2x^2) + C$

99. $\int x^n \sin^{-1} ax\, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1}\, dx}{\sqrt{1 - a^2x^2}}, \quad n \neq -1$

100. $\int x^n \cos^{-1} ax\, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1}\, dx}{\sqrt{1 - a^2x^2}}, \quad n \neq -1$

101. $\int x^n \tan^{-1} ax\, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1}\, dx}{1 + a^2x^2}, \quad n \neq -1$

102. $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$

103. $\int b^{ax} dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0,\ b \neq 1$

104. $\int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1) + C$

105. $\int x^n e^{ax} dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax}\, dx$

106. $\int x^n b^{ax} dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax}\, dx, \quad b > 0, b \neq 1$

107. $\int e^{ax} \sin bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \sin bx - b \cos bx) + C$

108. $\int e^{ax} \cos bx\, dx = \frac{e^{ax}}{a^2 + b^2}(a \cos bx + b \sin bx) + C$

109. $\int \ln ax\, dx = x \ln ax - x + C$

110. $\int x^n (\ln ax)^m\, dx = \frac{x^{n+1}(\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx, \quad n \neq -1$

111. $\int x^{-1}(\ln ax)^m\, dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$

112. $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$

113. $\int \sinh ax\, dx = \frac{1}{a}\cosh ax + C$

114. $\int \cosh ax\, dx = \frac{1}{a}\sinh ax + C$

115. $\int \sinh^2 ax\, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$

116. $\int \cosh^2 ax\, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$

117. $\int \sinh^n ax\, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n}\int \sinh^{n-2} ax\, dx, \quad n \neq 0$

118. $\int \cosh^n ax\, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n}\int \cosh^{n-2} ax\, dx, \quad n \neq 0$

119. $\int x \sinh ax\, dx = \frac{x}{a}\cosh ax - \frac{1}{a^2}\sinh ax + C$

120. $\int x \cosh ax\, dx = \frac{x}{a}\sinh ax - \frac{1}{a^2}\cosh ax + C$

121. $\int x^n \sinh ax\, dx = \frac{x^n}{a}\cosh ax - \frac{n}{a}\int x^{n-1}\cosh ax\, dx$

122. $\int x^n \cosh ax\, dx = \frac{x^n}{a}\sinh ax - \frac{n}{a}\int x^{n-1}\sinh ax\, dx$

123. $\int \tanh ax\, dx = \frac{1}{a}\ln(\cosh ax) + C$

124. $\int \coth ax\, dx = \frac{1}{a}\ln|\sinh ax| + C$

125. $\int \tanh^2 ax\, dx = x - \frac{1}{a}\tanh ax + C$

126. $\int \coth^2 ax\, dx = x - \frac{1}{a}\coth ax + C$

127. $\int \tanh^n ax\, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax\, dx, \quad n \neq 1$

128. $\int \coth^n ax\, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax\, dx, \quad n \neq 1$

129. $\int \operatorname{sech} ax\, dx = \frac{1}{a}\sin^{-1}(\tanh ax) + C$

130. $\int \operatorname{csch} ax\, dx = \frac{1}{a}\ln\left|\tanh \frac{ax}{2}\right| + C$

131. $\int \operatorname{sech}^2 ax\, dx = \frac{1}{a}\tanh ax + C$

132. $\int \operatorname{csch}^2 ax\, dx = -\frac{1}{a}\coth ax + C$

133. $\int \operatorname{sech}^n ax\, dx = \frac{\operatorname{sech}^{n-2} ax \tanh ax}{(n-1)a} + \frac{n-2}{n-1}\int \operatorname{sech}^{n-2} ax\, dx, \quad n \neq 1$

134. $\int \operatorname{csch}^n ax\, dx = -\frac{\operatorname{csch}^{n-2} ax \coth ax}{(n-1)a} - \frac{n-2}{n-1}\int \operatorname{csch}^{n-2} ax\, dx, \quad n \neq 1$

135. $\int \operatorname{sech}^n ax \tanh ax\, dx = -\frac{\operatorname{sech}^n ax}{na} + C, \quad n \neq 0$

136. $\int \operatorname{csch}^n ax \coth ax\, dx = -\frac{\operatorname{csch}^n ax}{na} + C, \quad n \neq 0$

137. $\int e^{ax}\sinh bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} - \frac{e^{-bx}}{a-b}\right] + C, \quad a^2 \neq b^2$

138. $\int e^{ax}\cosh bx\, dx = \frac{e^{ax}}{2}\left[\frac{e^{bx}}{a+b} + \frac{e^{-bx}}{a-b}\right] + C, \quad a^2 \neq b^2$

139. $\int_0^\infty x^{n-1}e^{-x}\, dx = \Gamma(n) = (n-1)!, \quad n > 0$

140. $\int_0^\infty e^{-ax^2}\, dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}, \quad a > 0$

141. $\int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \begin{cases} \frac{1\cdot 3\cdot 5\cdots(n-1)}{2\cdot 4\cdot 6\cdots n}\cdot\frac{\pi}{2}, & \text{if } n \text{ is an even integer} \geq 2 \\ \frac{2\cdot 4\cdot 6\cdots(n-1)}{3\cdot 5\cdot 7\cdots n}, & \text{if } n \text{ is an odd integer} \geq 3 \end{cases}$