	COMPLEX NUMBERS Class XI
Q.1)	Convert the given complex number into polar form $z = \frac{1+7i}{(2-i)^2}$
Sol.1)	$z = \dfrac{1+7i}{(2-i)^2}$ First, we have to convert in to standard form $z = \dfrac{1+7i}{4+i^2-4i}$ $z = \frac{1+7i}{4-1-4i}$ ………… $\{i^2 = -1\}$ $z = \frac{1-7i}{3-4i}$ then rationalize $z = \dfrac{1+7i}{3-4i} \times \dfrac{3+4i}{3+4i}$ $z = \dfrac{3+4i+21i-28}{9+16}$ $z = \dfrac{-25+25i}{25}$ $z = -1 + i\ (standard\ form)$ Here, $a = -1$ and $b = 1$ Now, $r\|z\| = \sqrt{\mathrm{a}^2+\mathrm{b}^2}$ $r = \sqrt{1+1} = \sqrt{2}$ $\tan\alpha = \left\|\dfrac{b}{a}\right\| = \left\|\dfrac{1}{-1}\right\| = \|-1\| = 1$ $\Rightarrow \tan\alpha = 1$ $\Rightarrow \alpha = \frac{\pi}{4}$ Now, z is in 2nd quadrant $\therefore \theta = \pi - \alpha$ $\theta = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$ Polar form: $z = r(\cos\theta + i\sin\theta)$ $z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ ans.
Q.2)	Convert the given complex number in to polar form $z = \frac{-16}{1+i\sqrt{3}}$
Sol.2)	We have, $z = \frac{-16}{1+i\sqrt{3}}$ Rationalize $z = \frac{-16}{1+i\sqrt{3}} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$ $z = \dfrac{-16+i16\sqrt{3}}{1-3i^2}$ $z = \dfrac{-16+i16\sqrt{3}}{4}$ $z = -4 + i4\sqrt{3}$ Here, $a = -4$ and $b = 4\sqrt{3}$ Now, $r = \sqrt{\mathrm{a}^2+\mathrm{b}^2} = \sqrt{16+48} = \sqrt{64} = 8$ $\therefore r = 8$ $\tan\alpha = \left\|\dfrac{b}{a}\right\| = \left\|\dfrac{4\sqrt{3}}{-4}\right\| = \left\|-\sqrt{3}\right\| = \sqrt{3}$

	$\Rightarrow \tan\alpha = \sqrt{3}$ $\Rightarrow \alpha = \frac{\pi}{3}$ Now, z is in 2^{nd} quadrant $\therefore \theta = \pi - \alpha$ $\theta = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$ Polar form: $z = r(\cos\theta + i\sin\theta)$ $z = 8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$ ans.	
Q.3)	Convert the given complex number in to polar form $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$	
Sol.3)	We have, $z = \frac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ $\Rightarrow z = \frac{-1+i}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$ $\Rightarrow z = \frac{-2+2i}{1+i\sqrt{3}}$ Rationalize $z = \frac{(2+2i)}{(1+i\sqrt{3})} \times \frac{(1-i\sqrt{3})}{(1-i\sqrt{3})}$ $z = \dfrac{-2+2\sqrt{3}i+2i-2\sqrt{3}i^2}{1-3i^2}$ $z = \dfrac{-2+2\sqrt{3}i+2i-2\sqrt{3}}{1+3}$ $z = \dfrac{(2\sqrt{3}-2)}{4} + \dfrac{i(2\sqrt{3}+2)}{4}$ $z = \left(\dfrac{\sqrt{3}+1}{1}\right) + i\left(\dfrac{\sqrt{3}+1}{2}\right)$ Here, $a = \frac{\sqrt{3}-1}{2}$ and $b = \frac{\sqrt{3}+1}{2}$ Now, $r = \sqrt{\mathrm{a}^2 + \mathrm{b}^2}$ $= \sqrt{\left(\dfrac{\sqrt{3}-1}{2}\right)^2 + \left(\dfrac{\sqrt{3}+1}{2}\right)^2}$ $\Rightarrow r = \sqrt{\frac{3+1-2\sqrt{3}}{4} + \frac{3+1+2\sqrt{3}}{4}}$ $\Rightarrow r = \sqrt{\frac{4+4}{4}} = \sqrt{\frac{8}{4}} = \sqrt{2}$ $\Rightarrow r = \sqrt{2}$ $\tan\alpha = \left\lvert\dfrac{b}{a}\right\rvert = \left\lvert\dfrac{\dfrac{\sqrt{3}+1}{2}}{\dfrac{\sqrt{3}-1}{2}}\right\rvert$ $\Rightarrow \tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1}$ Divide by $\sqrt{3}$ $\Rightarrow \tan\alpha = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$ $\Rightarrow \tan\alpha = \frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{6}\right)}{1-\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\pi}{6}\right)}$……………… $\{= \tan(A+B)\}$	

	$\Rightarrow \tan\alpha = \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$ $\Rightarrow \tan\alpha = \tan\left(\frac{5\pi}{12}\right)$ $\Rightarrow \alpha = \frac{5\pi}{12}$ since, z is in 1st quadrant $\therefore \theta = \alpha$ $\theta = \dfrac{5\pi}{12}$ Polar form: $z = r(\cos\theta + i\sin\theta)$ $z = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ ans.	
	Finding the value of a POYNOMIAL:-	
Q.4)	If $x = -5 + 2\sqrt{-4}$. Find the value of $x^4 + 9x^3 + 35x^2 - x + 4$	
Sol.4)	We have, $x = -5 + 2\sqrt{-4}$ $\Rightarrow x = -5 + 4i$ ………….. $\{\sqrt{-4} = 2i\}$ $\Rightarrow x + 5 = 4i$ Squaring $\Rightarrow x^2 + 25 + 10x = 16i^2$ $\Rightarrow x^2 + 25 + 10x = 16$ $\Rightarrow x^2 + 10x + 41 = 0$ $x^2 + 10x + 41 \overline{) x^4 + 9x^3 + 35x^2 - x + 4}$ $-(x^4 + 10x^3 + 41x^2)$ $-x^3 - 6x^2 - x + 4$ $-(-x^3 - 10x^2 - 41x)$ $4x^2 + 40x + 4$ $-(4x^2 + 40x + 164)$ -160 Now $x^4 + 9x^3 + 35x^2 - x + 4 = (x^2 + 10x + 41)(x^2 - x + 4) - 16$ …….$\{dividend = (division)(quotient) + remainder\}$ $\Rightarrow x^4 + 9x^3 + 35x^2 - x + 4 = 0(x^2 - x + 4) - 160$ ……….. from eq.(i) $= -160$ ans.	
Q.5)	Find the value of $x^3 + 7x^2 - x + 16$ when $x = 1 + 2i$	
Sol.5)	We have, $x = 1 + 2i$ $\Rightarrow x - 1 = 2i$ Squaring $\Rightarrow x^2 + 1 - 2x = 4i^2$ $\Rightarrow x^2 + 1 - 2x = -4$ $\Rightarrow x^2 - 2x + 5 = 0$ ………… eq.(i) Now, $\quad x + 9$ $x^2 - 2x + 5 \overline{) x^3 + 7x^2 - x + 16}$ $-(x^3 + 2x^2 + 5x)$ $9x^2 - 6x + 16$ $-\left(-9x^2 - 18x + 35\right)$ $12x - 29$ $\{dividend = (division)(quotient) + remainder\}$ $\therefore x^3 + 7x^2 - x + 16 = (x^2 - 2x + 5)(x + 9) + 12x - 29$ $= 0(x + 9) + 12x - 29$ ………..$\because x^2 - 2x + 5 = 0\ from\ eq.(i)$ $= 12x - 29$ $= 12(1 + 2i) - 29$	

	$= 12 + 24i - 29$ $= -17 + 24i$ ans.	
	Square Root Of A Complex Numbers:-	
Q.6)	Find $\sqrt{-15 - 8i}$	
Sol.6)	Let $x + iy = \sqrt{-15 - 8i}$ Squaring $x^2 + i^2y^2 + 2ixy = 15 - 8i$ $\Rightarrow (x^2 + y^2) + 2ixy = -15 - 8i$ Equating real & imaginary parts $\Rightarrow x^2 - y^2 = -15$ ……….. (i) And $2xy = -8$ Now $(x^2 + y^2)^2 = (x^2 + y^2)^2 + (2xy)^2$ …………… $(to\ be\ remembered)$ $(x^2 + y^2)^2 = (-15)^2 + (-8)^2$ $(x^2 + y^2)^2 = 289$ $\Rightarrow x^2 + y^2 = 17$ ……….. (ii) Adding eq. (i) & (ii) $\Rightarrow (x^2 - y^2) + (x^2 + y^2) = -15 + 17$ $\Rightarrow 2x^2 = 2$ $\Rightarrow x^2 = 1$ $\Rightarrow x = \pm 1$ Put $x^2 = 1$ in eq. (ii) $\Rightarrow 1 + y^2 = 17$ $\Rightarrow y^2 = 16$ $\Rightarrow y = \pm y$ Now, $\sqrt{-15 - 8i} = \pm(1 - 4i)$ ans.	
Q.7)	Find the value of square of $1 - i$	
Sol.7)	Let $x + iy = \sqrt{1 - i}$ Squaring $x^2 + i^2y^2 + 2ixy = 1 - i$ $\Rightarrow (x^2 + y^2) + 2ixy = 1 - i$ Equating real & imaginary parts $\Rightarrow x^2 - y^2 = 1$ ……….. (i) And $2xy = -1$ Now $(x^2 + y^2)^2 = (x^2 + y^2)^2 + (2xy)^2$ …………… $(to\ be\ remembered)$ $\Rightarrow (x^2 + y^2)^2 = 1 + 1$ $\Rightarrow (x^2 + y^2)^2 = 2$ $\Rightarrow x^2 + y^2 = \sqrt{2}$ ……….. (ii) Adding eq. (i) & (ii) $\Rightarrow (x^2 - y^2) + (x^2 + y^2) = \sqrt{2} + 1$ $\Rightarrow 2x^2 = \sqrt{2} + 1$ $\Rightarrow x^2 = \frac{\sqrt{2}+1}{2}$ $\Rightarrow x = \pm\sqrt{\frac{\sqrt{2}+1}{2}}$ Put $x^2 = \frac{\sqrt{2}+1}{2}$ in eq. (ii) $\Rightarrow \frac{\sqrt{2}+1}{2} + y^2 = \sqrt{2}$	

	$\Rightarrow y^2 = \sqrt{2} - \left(\frac{\sqrt{2}+1}{2}\right)$ $\Rightarrow y^2 = \frac{2\sqrt{2}-\sqrt{2}-1}{2}$ $\Rightarrow y = \sqrt{\frac{\sqrt{2}-1}{2}}$ Now, $1 - i = \pm\left(\sqrt{\frac{\sqrt{2}+1}{2}} - \sqrt{\frac{\sqrt{2}-1}{2}}i\right)$ ans.	
	On Equality:-	
Q.8)	Find real value of $x+y$ for which the complex numbers $-3+ix^2y$ and x^2+y+4i and are conjugate of each other.	
Sol.8)	$-3+ix^2y$ and x^2+y+4i are conjugate of each other $\Rightarrow -3+ix^2y = \overline{(x^2+y)+4\imath}$ $\Rightarrow -3+ix^2y = (x^2+y)-4i$ Equating real & imaginary parts $\Rightarrow x^2+y=-3$ And $x^2y=-4$ $\therefore (-3-y)y=-4$ $\Rightarrow -3+y^2=-4$ $\Rightarrow y^2+3y-4=0$ $\Rightarrow (y+4)(y-1)=0$ $\Rightarrow y=-4$ and $y=1$ Put $x^2y=-4$ when, $y=-4$ when $y=1$ $x^2(-4)=-4$ $x^2(1)=-4$ $x^2=1$ $x^2=-4$ $x=\pm 1$ $x=\pm 2i$ $\therefore x=\pm 1$ and $y=-4$ ans.	
Q.9)	If $(x+iy)^{\frac{1}{3}} = a+ib$, show that $\frac{x}{a}+\frac{y}{b} = 4(a^2-b^2)$	
Sol.9)	We have, $(x+iy)^{\frac{1}{3}} = a+ib$ cubing both sides $x+iy=(a+ib)^3$ $\Rightarrow x+iy = a^3+i^3b^3+3a^2(ib)+3a(i^2b^2)$ $\Rightarrow x+iy = a^3-i^3b^3+3a^2b-3ab^2$ $\Rightarrow x+iy = (a^3-3ab^2)+i(-b^3+3a^2b)$ Equating real & imaginary parts $x=a^3-3ab^2$ and $y=-b^3+3a^2b$ Taking L.H.S. $\frac{x}{a}+\frac{y}{b}$ Putting the values of x and y $= \frac{(a^3-3ab^2)}{a} + \frac{(-b^3+3a^2b)}{b}$ $= \frac{a(a^2-3b^2)}{a} + \frac{b(-b^3+3a^2)}{b}$ $= a^2-3b^2-b^3+3a^2$ $= 4a^2-3b^2$ $= 4(a^2-b^2)$ RHS (proved)	
Q.10)	If $a+ib = \frac{c+i}{c-i}$ show that $\frac{b}{a} = \frac{2c}{c^2-1}$ and hence show $a^2+b^2=1$	

Sol.10)	We have, $a+ib=\frac{c+i}{c-i}$ $a+ib=\dfrac{c+i}{c-i}\times\dfrac{c+i}{c+i}$ $\Rightarrow a+ib=\frac{(c+i)^2}{c^2-i^2}$ $\Rightarrow a+ib=\frac{c^2+i^2+2ic}{c^2+1}$ $\Rightarrow a+ib=\frac{(c^2-1)}{c^2+1}+\frac{2ic}{c^2+1}$ Equating real & imaginary parts $a=\frac{c^2-1}{c^2+1}$ and $b=\frac{2c}{c^2+1}$ Now $\frac{b}{a}=\frac{\frac{2c}{c^2-1}}{\frac{c^2-1}{c^2+1}}$ $\Rightarrow \frac{b}{a}=\frac{2c}{c^2-1}$ (proved) Now $a^2+b^2=\left(\frac{c^2-1}{c^2+1}\right)^2+\left(\frac{2c}{c^2+1}\right)^2$ $=\frac{c^4+1-2c^2}{(c^2+1)^2}+\frac{4c^2}{(c^2+1)^2}$ $=\frac{c^4+2c^2+1}{(c^2+1)^2}$ $=\frac{(c^2+1)^2}{(c^2+1)^2}$ $a^2+b^2=1$ (proved)	