

	COMPLEX NUMBERS Class XI	
	Quadratic Equation:-	
Q.1)	Find all non-zero complex numbers z satisfying $ z = iz^2$	
Sol.1)	Let $z = x + iy$ We have $z = iz^2$ $\Rightarrow x - iy = i(x + iy)^2$ $\Rightarrow x - iy = i(x^2 - y^2 + 2ixy)$ $\Rightarrow x - iy = ix^2 - iy^2 - 2xy$ $\Rightarrow x - iy = -2xy + i(x^2 - y^2)$ Equating real & imaginary parts $\Rightarrow x = -2xy \text{ and } -y = x^2 - y^2$ $\Rightarrow x + 2xy = 0 \text{ and } x^2 - y^2 + y = 0$ Consider $x + 2xy = 0$ $\Rightarrow x(1 + 2y) = 0$ $\Rightarrow x = 0 \text{ (or) } y = \frac{-1}{2}$ Case 1, when $x = 0$ $\Rightarrow x^2 - y^2 + y = 0$ $\Rightarrow y(-y + 1) = 0$ $\Rightarrow y = 0 \text{ (or) } y = 1$ $\therefore x = 0 \text{ \& } y = 0 \text{ and } x = 0 \text{ \& } y = 1$ $\Rightarrow z = 0 + 0i \text{ and } z = 0 + 1i$ (rejected) $\therefore z = i$ Case 2, when $y = \frac{-1}{2}$ $\Rightarrow x^2 - y^2 + y = 0$ $\Rightarrow x^2 - \frac{1}{4} - \frac{1}{2} = 0$ $\Rightarrow x^2 = \frac{3}{4}$ $\Rightarrow x = \pm \frac{\sqrt{3}}{2}$ $\therefore x = \frac{\sqrt{3}}{2}, y = \frac{-1}{2} \text{ and } x = \frac{-\sqrt{3}}{2}, y = \frac{-1}{2}$ $\Rightarrow z = \frac{\sqrt{3}}{2} - \frac{1}{2}i \text{ and } z = \frac{-\sqrt{3}}{2} - \frac{1}{2}i$ $\therefore \text{required complex numbers are } i, \frac{\sqrt{3}}{2} - \frac{1}{2}i \text{ and } \frac{-\sqrt{3}}{2} - \frac{1}{2}i \text{ ans.}$	
Q.2)	Solve the equation $z^2 + z = 0$.	
Sol.2)	Let $z = x + iy$ We have $z^2 + z = 0$ $\Rightarrow (x + iy)^2 + \sqrt{x^2 + y^2} = 0$ $\Rightarrow x^2 - y^2 + 2ixy + \sqrt{x^2 + y^2} = 0$ $\Rightarrow (x^2 - y^2 + \sqrt{x^2 + y^2}) + i2xy = 0 + 0i$ $\Rightarrow x^2 - y^2 + \sqrt{x^2 + y^2} = 0 \text{ and } 2xy = 0$ Consider $2xy = 0$	

	$x = 0$ or $y = 0$ Case 1, when $x = 0$ Then $x^2 - y^2 + \sqrt{x^2 + y^2} = 0$ $\Rightarrow -y^2 + \sqrt{y^2} = 0$ $\Rightarrow -y^2 + y = 0$ If $y > 0$ then $ y = y$ $\Rightarrow -y^2 + y = 0$ $\Rightarrow y(-y + 1) = 0$ $y = 0$ or $y = 1$ If $y < 0$ then $ y = -y$ $\Rightarrow -y^2 + y = 0$ $\Rightarrow -y(y + 1) = 0$ $y = 0$ or $y = -1$ $x = 0; y = 0$ or $x = 0; y = -1$ (ii) Case 2, when $y = 0$ Then $x^2 - y^2 + \sqrt{x^2 - y^2} = 0$ $\Rightarrow x^2 + \sqrt{x^2} = 0$ $\Rightarrow x^2 + x = 0$ If $x > 0$ then $ x = x$ $\Rightarrow x^2 + x = 0$ $\Rightarrow x(x + 1) = 0$ $x = 0$ or $x = -1$ (rejected since $x > 0$) $x = 0; y = 0$ (iii) If $x < 0$ then $ x = -x$ $\Rightarrow x^2 - x = 0$ $\Rightarrow x(x - 1) = 0$ $x = 0$ or $x = 1$ (rejected since $x < 0$) $x = 0; y = 0$ (iv) From (i), (ii), (iii) & (iv) $z = 0 + 0i; z = 0 + i; z = 0 - i$ ans.	
Q.3)	If $ z_1 = z_2 = z_3 \dots \dots \dots z_n = 1$ show that $ z_1 + z_2 + z_3 + \dots \dots \dots z_n = \left \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots \dots \dots \frac{1}{z_n} \right $	
Sol.3)	We have, $ z_1 + z_2 + z_3 + \dots \dots \dots z_n $ $= \left \frac{z_1 \bar{z}_1}{\bar{z}_1} + \frac{z_2 \bar{z}_2}{\bar{z}_2} + \frac{z_3 \bar{z}_3}{\bar{z}_3} + \dots \dots \dots \frac{z_n \bar{z}_n}{\bar{z}_n} \right $ (multiply & divide) $= \frac{ z_1 ^2}{\bar{z}_1} + \frac{ z_2 ^2}{\bar{z}_2} + \frac{ z_3 ^2}{\bar{z}_3} + \dots \dots \dots \frac{ z_n ^2}{\bar{z}_n}$ $\{z\bar{z} = z ^2\}$ $= \left \frac{1}{\bar{z}_1} + \frac{1}{\bar{z}_2} + \frac{1}{\bar{z}_3} + \dots \dots \dots \frac{1}{\bar{z}_n} \right $ $\{ z_1 = z_2 = 1\}$ $= \left \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots \dots \dots \frac{1}{z_n} \right $ $\{\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2\}$ $= \left \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \dots \dots \dots \frac{1}{z_n} \right $ $ \bar{z} = z $	
Q.4)	If $ z^2 - 1 = z ^2 + 1$, show that z lies on imaginary axis.	
Sol.4)	Let $z = x + iy$	

	Then $z^2 - 1 = z ^2 + 1$ $= (x + iy)^2 - 1 = \sqrt{x^2 + y^2}^2 + 1$ $= (x^2 - y^2 - 1) + 2ixy = (x^2 + y^2) + 1$ $= \sqrt{(x^2 - y^2 - 1) + 4x^2y^2} = x^2 + y^2 + 1$ Squaring both sides $= (x^2 - y^2 - 1)^2 + 4x^2y^2 = (x^2 + y^2 + 1)^2$ $= x^4 + y^4 + 1 - 2x^2y^2 + 2y^2 - 2x^2 + 4x^2y^2$ $= x^4 + y^4 + 1 + 2x^2y^2 + 2y^2 + 2x^2 + 4x^2y^2$ $= 4x^2 = 0$ $= x^2 = 0 = x = 0$ $z = 0 + iy$ Clearly z lies on y - axis	
Q.5)	If the imaginary part of $\frac{2z+1}{iz+1}$ is -2, then show that the locus of the point representing z in the argand plane is a straight line. LOCUS is a path traced by a moving point (x, y)	
Sol.5)	Let $z = x + iy$ Here, we have prove that equation containing x & y must be linear i.e., straight line Now $\frac{2z+1}{iz+1} = \frac{2(x+iy)+1}{i(x+iy)+1}$ $= \frac{(2x+1)+i2y}{(1-y)+ix}$ Rationalize $= \frac{(2x+1)+i2y}{(1-y)+ix} \times \frac{(1-y)-ix}{(1-y)-ix}$ $= \frac{(2x+1)(1-y) - ix(2x+1) + i2y(1-y) - i^22xy}{(1-y)^2 - i^2x^2}$ $= \frac{2x - 2xy + 1 - y - i(2x^2 + x) + i(2y - 2y^2) + 2xy}{1 + y^2 - 2y + x^2}$ $= \frac{(2x+1-y)}{x^2 + y^2 - 2y + 1} + i \frac{(2y - 2y^2 - 2x^2 - x)}{x^2 + y^2 - 2y + 1}$ Here $Im\left(\frac{2z+1}{iz+1}\right) = \frac{2y-2y^2-2x^2-x}{x^2+y^2-2y+1}$ Given $Im\left(\frac{2z+1}{iz+1}\right) = -2$ $= \frac{2y-2y^2-2x^2-x}{x^2+y^2-2y+1} = -2$ $\Rightarrow 2y - 2y^2 - 2x^2 - x = -2x^2 - 2y^2 + 4y - 2$ $\Rightarrow x + 2y - 2 = 0$ Clearly this represents the equation of straight line	
Q.6)	Let z_1 & z_2 be two complex numbers such that $z_1 + z_2 = z_1 + z_2$ then show that $\arg(z_1) - \arg(z_2) = 0$.	
Sol.6)	Let $z_1 = r_1 (\cos \theta_1 + i \sin \theta_1)$ And $z_2 = r_2 (\cos \theta_2 + i \sin \theta_2)$ Here $z_1 = r_1$ $\arg(z_1) = \theta_1$ $\Rightarrow z_2 = r_2$ $\arg(z_2) = \theta_2$	

	We have, $z_1 + z_2 = z_1 + z_2$ $\Rightarrow r_1(\cos \theta_1 + i \sin \theta_1) + r_2(\cos \theta_2 + i \sin \theta_2) = r_1 + r_2$ $\Rightarrow (r_1 \cos \theta_1 + r_2 \cos \theta_2) + i(r_1 \sin \theta_1 + r_2 \sin \theta_2) = r_1 + r_2$ $\Rightarrow \sqrt{(r_1 \cos \theta_1 + r_2 \cos \theta_2)^2 + (r_1 \sin \theta_1 + r_2 \sin \theta_2)^2} = r_1 + r_2$ Squaring both sides $\Rightarrow r_1^2 \cos^2 \theta_1 + r_2^2 \cos^2 \theta_2 + 2r_1 r_2 \cos \theta_1 \cos \theta_2 + r_1^2 \sin^2 \theta_1 + r_2^2 \sin^2 \theta_2 + 2r_1 r_2 \sin \theta_1 \sin \theta_2 = (r_1 + r_2)^2$ $\Rightarrow r_1^2(\cos^2 \theta_1 + \sin^2 \theta_1) + r_2^2(\cos^2 \theta_2 + \sin^2 \theta_2) + 2r_1 r_2(\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2) = r_1^2 + r_2^2 + 2r_1 r_2$ $\Rightarrow r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2) = r_1^2 + r_2^2 + 2r_1 r_2$ $\Rightarrow \cos(\theta_1 - \theta_2) = 1$ $\Rightarrow \theta_1 - \theta_2 = 0 \dots\dots\dots \{\cos \theta = 1 \text{ then } \theta = 0\}$ $\Rightarrow \arg(z_1) - \arg(z_2) = 0$	
Q.7)	What does this equation $ z + 1 - i = z - 1 + i $ represents.	
Sol.7)	Let $z = x + iy$ Then $z + 1 - i = z - 1 + i$ $\Rightarrow x + iy + 1 - i = x + iy - 1 + i$ $\Rightarrow (x + 1) + i(y - 1) = (x - 1) + i(y + 1)$ $\Rightarrow \sqrt{(x + 1)^2 + (y - 1)^2} = \sqrt{(x - 1)^2 + (y + 1)^2}$ $\Rightarrow \sqrt{x^2 + 1 + 2x + y^2 + 1 - 2y} = \sqrt{x^2 + 1 - 2x + y^2 + 1 + 2y}$ Squaring $\Rightarrow x^2 + y^2 + 2x - 2y + 2 = x^2 + y^2 + 2y - 2x + 2$ $\Rightarrow 4x - 4y = 0$ $\Rightarrow x - y = 0$ Clearly this represents the equation of a straight line.	
Q.8)	Evaluate: $1 + i^2 + i^4 + i^6 + \dots \dots \dots i^{20}$	
SOL.8)	$= 1 - 1 + 1 - 1 + \dots + 1$ $= 1 \text{ ans.}$	
Q.9)	Find the value of $\frac{i^{4n+1} - i^{4n-1}}{2}$	
Sol.9)	$\Rightarrow \frac{i^{4n} \cdot i - i^{4n} \cdot i^{-1}}{2}$ $\Rightarrow \frac{1(i) - 1(\frac{1}{i})}{2} \dots\dots\dots \{i^{4n} = (i^4)^n = (1)^n = 1\}$ $\Rightarrow \frac{i - (i)}{2} \dots\dots\dots \frac{1}{1} = \frac{1}{1} \times \frac{i}{i} = -i$ $\Rightarrow \frac{2i}{2} = i$	
Q.10)	What is the smallest positive integer n for which $(1 + i)^{2n} = (1 - i)^{2n}$.	
Sol.10)	We have, $(1 + i)^{2n} = (1 - i)^{2n}$ $\Rightarrow \frac{(1+i)^{2n}}{(1-i)^{2n}} = 1$ $\Rightarrow \left(\frac{1+i}{1-i}\right)^{2n} = 1$	