8. Division of Algebraic Expressions

Exercise 8.1

1. Question

Write the degree of each of the following polynomials:

(i) $2x^3 + 5x^2 - 7$

(ii) $5x^2 - 3x + 2$

(iii) $2x + x^2 - 8$

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

(v) $3x^3 + 1$

(vi) 5

(vii) $20x^3 + 12x^2y^2 + 20$

Answer

(i) $2x^3 + 5x^2 - 7$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(ii) $5x^2 - 3x + 2$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iii) $2x + x^2 - 8$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 2.

Therefore degree of the polynomial is 2.

(iv) $\frac{1}{2}y^7 - 12y^6 + 48y^5 - 10$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 7.

Therefore degree of the polynomial is 7.

(v) $3x^3 + 1$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 3.

Therefore degree of the polynomial is 3.

(vi) 5

Degre is the highest power of the variable of a polynomial. In the given polynomial there is no variable term.

Therefore degree of the polynomial is 0.

(vii) $20x^3 + 12x^2y^2 + 20$

Degre is the highest power of the variable of a polynomial. In the given polynomial highest power is 4.

Therefore degree of the polynomial is 4.

2. Question

Which of the following expressions are not polynomiasl?

(i) $x^2 + 2x^{-2}$

(ii) $\sqrt{a}x + x^2 - x^3$

(iii) $3y^3 - \sqrt{5}y + 9$

(iv) $ax^{1/2} + ax + 9x^2 + 4$

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

Answer

(i) $x^2 + 2x^{-2}$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

(ii) $\sqrt{a}x + x^2 - x^3$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iii) $3y^3 - \sqrt{5}y + 9$

A polynomial always has positive power.

Therefore the given expression is a polynomial.

(iv) $ax^{1/2} + ax + 9x^2 + 4$

A polynomial never has negative or fractional power. In the given expression x has fractional power.

Therefore it is not a polynomial.

(v) $3x^{-3} + 2x^{-1} + 4x + 5$

A polynomial never has negative or fractional power. In the given expression x has negative power.

Therefore it is not a polynomial.

3. Question

Write each of the following polynomicals in the standard from. Also, write their drgree:

(i) $x^2 + 3 + 6x + 5x^4$

(ii) $a^2 + 4 + 5a^6$

(iii) $(x^3 - 1)(x^3 - 4)$

(iv) $(y^3 - 2)(y^3 + 11)$

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

Answer

(i) $x^2 + 3 + 6x + 5x^4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5x^4 + x^2 + 6x + 3$ or $3 + 6x + x^2 + 5x^4$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 4

(ii) $a^2 + 4 + 5a^6$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $5a^6 + a^2 + 4$ or $4 + a^2 + 5a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iii) $(x^3 - 1)(x^3 - 4)$

$(x^3 - 1)(x^3 - 4) = x^6 - 4x^3 - x^3 + 4$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $x^6 - 5x^3 + 4$ or $4 - 5x^3 + x^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(iv) $(y^3 - 2)(y^3 + 11)$

$(y^3 - 2)(y^3 + 11) = y^6 + 11y^3 - 2y^3 - 22$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $v^6 + 9v^3 - 22$ or $-22 + 9v^3 + v^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(v) $\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right)$

$\left(a^3 - \frac{3}{8}\right)\left(a^3 + \frac{16}{17}\right) = a^6 + \frac{16a^3}{17} - \frac{3a^3}{8} - \frac{6}{17}$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^6 + \frac{11a^3}{\square} - \frac{6}{\square}$ or $-\frac{6}{\square} + \frac{11a^3}{\square} + a^6$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 6

(vi) $\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right)$

$\left(a + \frac{3}{4}\right)\left(a + \frac{4}{3}\right) = a^2 + \frac{4a}{3} + \frac{3a}{4} + 1$

A polynomial in the standard form is written in the decreasing or increasing power of the variable.

Standard form of the polynomial: $a^2 + \frac{25a}{\square} + 1$ or $1 + \frac{25a}{\square} + a^2$

Degree is the highest power of the variable in the given expression.

Therefore degree of the polynomial is: 2

Exercise 8.2

1. Question

Divide:

$6x^3y^3z^2$ by $3x^2yz$

Answer

$\frac{6x^3y^3z^2}{3x^2yz} = (\frac{6}{3}x^{3-2}y^{3-1}z^{2-1}) = 2xy^2z$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$15m^3n^3$ by $5m^2n^2$

Answer

$\frac{15m^3n^3}{5m^2n^2} = (\frac{15}{5}m^{3-2}n^{3-2}) = 3mn$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$24a^3b^3$ by $-8ab$

Answer

$\frac{24a^3b^3}{-8ab} = (\frac{24}{-8}a^{3-1}b^{3-1}) = -3a^2b^2$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$-21abc^2$ by $-7abc$

Answer

$\frac{-21abc^2}{-7abc} = (\frac{-21}{-7}a^{1-1}b^{1-1}c^{2-1}) = 3a^\circ b^\circ c = 3c$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

5. Question

Divide:

xyz^2 by $-9xz$

Answer

$\frac{xyz^2}{-9xz} = (\frac{1}{-9}x^{1-1}yz^{2-1}) = -\frac{1yz}{9} = -\frac{yx}{9}$ [Using $a^n \div a^m = a^{n-m}$] and [$a^\circ = 1$]

6. Question

Divide:

$-72a^4b^5c^8$ by $-9a^2b^2c^3$

Answer

$\frac{-72a^4b^5c^8}{-9a^2b^2c^3}$

$= 8a^2b^3c^5$

7. Question

Simplify:

$\frac{16m^3y^2}{4m^2y}$

Answer

$\frac{16m^3y^2}{4m^2y} = \left(\frac{16}{4}m^{3-2}y^{2-1}\right) = 4my$ [Using $a^n \div a^m = a^{n-m}$]

8. Question

Simplify:

$\frac{32\,m^2n^2p^2}{4\,mnp}$

Answer

$\frac{32m^2n^2p^2}{4mnp} = (\frac{32}{4}m^{2-1}n^{2-1}p^{2-1}) = 8mnp$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.3

1. Question

Divide:

$x + 2x^2 + 3x^4 - x^5$ by $2x$

Answer

$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^4}{2x} - \frac{x^5}{2x} = \frac{1}{2} + x + \frac{3x^3}{2} - \frac{x^4}{5}$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$y^4 - 3y^3 + \frac{1}{2}y^2$ by $3y$

Answer

$\frac{y^4-3y^3+\frac{1}{2}y^2}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{y^2}{6y} = \frac{y^3}{3} - y^2 + \frac{y}{6}$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$-4a^3 + 4a^2 + a$ by $2a$

Answer

$-\frac{4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a} = -2a^2 + 2a + \frac{1}{2} =$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$-x^6 + 2x^4 + 4x^3 + 2x^2$ by $\sqrt{2}x^2$

Answer

$-\frac{x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} = -\frac{x^4}{\sqrt{2}} + \frac{2x^2}{\sqrt{2}} + \frac{4x}{\sqrt{2}} = -\frac{x^4}{\sqrt{2}} + \sqrt{2}x^2 + \sqrt{2}x$ [Using $a^n \div a^m = a^{n-m}$]

5. Question

Divide:

$5z^3 - 6z^2 + 7z$ by $2z$

Answer

$\frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z} = = \frac{5z^2}{2} - 3z + \frac{7}{2}$ [Using $a^n \div a^m = a^{n-m}$]

6. Question

Divide:

$\sqrt{3}a^4 + 2\sqrt{3}a^3 + 3^2 - 6a$ by $3a$

Answer

$\frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{9}{3a} - \frac{6a}{3a} = \frac{\sqrt{3}a^3}{3} + \frac{2\sqrt{3}a^2}{3} + \frac{3}{a} - 2$ [Using $a^n \div a^m = a^{n-m}$]

Exercise 8.4

1. Question

Divide:

$5x^3 - 15x^2 + 25x$ by $5x$

Answer

$\frac{5x^3}{5x} - \frac{15x^2}{5x} + \frac{25x}{5x} = = 5x^2 - 3x + 5$ [Using $a^n \div a^m = a^{n-m}$]

2. Question

Divide:

$4z^3 + 6z^2 - z$ by $-\frac{1}{2}z$

Answer

$\frac{2\times4z^3}{-1z} + \frac{2\times6z^2}{-1z} - \frac{2\times z}{-1z} = = -8z^2 - 12z + 2$ [Using $a^n \div a^m = a^{n-m}$]

3. Question

Divide:

$9x^2y - 6xy + 12xy^2$ by $-\frac{3}{2}xy$

Answer

$\frac{2\times9x^2y}{-3xy} - \frac{2\times6xy}{-3xy} + \frac{2\times12xy^2}{-3xy} = = -6x^2y + 4y - 8y$ [Using $a^n \div a^m = a^{n-m}$]

4. Question

Divide:

$3x^3y^2 + 2x^2y + 15xy$ by $3xy$

Answer

$\frac{3x^3y^2}{3xy} + \frac{2x^2y}{3xy} + \frac{15xy}{3xy} = = x^2y + \frac{2x}{3} + 5$ [Using $a^n \div a^m = a^{n-m}$]

5. Question

Divide:

$x^3 + 7x + 12$ by $x + 4$

Answer

$$
\begin{array}{r|l}
x+4 & x^2+7x+12 \;\big)\; x+3 \\
 & \underline{x^2+4x} \\
 & \quad 3x+12 \\
 & \quad \underline{3x+12} \\
 & \quad\quad 0
\end{array}
$$

Ans: x+3

6. Question

Divide:

$4y^2 + 3y + \frac{1}{2}$ by $2y + 1$

Answer

$$
\begin{array}{r|l}
2y+1 & 4y^2+3y+\frac{1}{2} \;\big)\; 2y+\frac{1}{2} \\
 & \underline{4y^2+2y} \\
 & \quad y+\frac{1}{2} \\
 & \quad \underline{y+\frac{1}{2}} \\
 & \quad\quad 0
\end{array}
$$

7. Question

Divide:

$3x^3 + 4x^2 + 5x + 18$ by $x + 2$

Answer

$$
\begin{array}{r|l}
x+2 & 3x^3+4x^2+5x+18 \;\big)\; 3x^2-2x+9 \\
 & \underline{3x^3+6x^2} \\
 & \quad -2x^2+5x \\
 & \quad \underline{-2x^2-4x} \\
 & \quad\quad 9x+18 \\
 & \quad\quad \underline{9x+18} \\
 & \quad\quad\quad 0
\end{array}
$$

8. Question

Divide:

$14x^2 - 53x + 45$ by $7x - 9$

Answer

$$
\begin{array}{r|l}
7x-9 & 14x^2-53x+45 \;\big)\; 2x-5 \\
 & \underline{14x^2-18x} \\
 & \quad -35x+45 \\
 & \quad \underline{-35x+45} \\
 & \quad\quad 0
\end{array}
$$

9. Question

Divide:

$-21 + 71x - 31x^2 - 24x^3$ by $3 - 8x$

Answer

$$
\begin{array}{l|l}
-8x+3 & -24x^3-31x^2+71x-21 \;\big)\; 3x^2+5x-7 \\
& \quad -24x^3+9x^2 \\
& \quad +\quad - \\
& \quad\quad -40x^2+71x \\
& \quad\quad -40x^2+15x \\
& \quad\quad +\quad - \\
& \quad\quad\quad\quad 56x-21 \\
& \quad\quad\quad\quad 56x-21 \\
& \quad\quad\quad\quad -\quad + \\
& \quad\quad\quad\quad\quad 0
\end{array}
$$

10. Question

Divide:

$3y^4 - 3y^3 - 4y^2 - 4y$ by $y^2 - 2y$

Answer

$$
\begin{array}{l|l}
y^2-2y & 3y^4-3y^3-4y^2-4y \;\big)\; 3y^2+3y+2 \\
& \quad 3y^4-6y^3 \\
& \quad -\quad + \\
& \quad\quad 3y^3-4y^2 \\
& \quad\quad 3y^3-6y^2 \\
& \quad\quad -\quad + \\
& \quad\quad\quad\quad 2y^2-4y \\
& \quad\quad\quad\quad 2y^2-4y \\
& \quad\quad\quad\quad -\quad + \\
& \quad\quad\quad\quad\quad 0
\end{array}
$$

11. Question

Divide:

$2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3$ by $2y^3 + 1$

Answer

$$
\begin{array}{l|l}
2y^3+1 & 2y^5+10y^4+6y^3+y^2+5y+3 \;\big)\; y^2+5y+3 \\
& \quad 2y^5 \quad\quad +y^2 \\
& \quad - \quad\quad\quad -\\
& \quad\quad 10y^4+6y^3+5y+3 \\
& \quad\quad 10y^4 \quad\quad +5y \\
& \quad\quad - \quad\quad\quad -\\
& \quad\quad\quad\quad 6y^3 \quad +3 \\
& \quad\quad\quad\quad 6y^3 \quad +3 \\
& \quad\quad\quad\quad -\quad\quad - \\
& \quad\quad\quad\quad\quad 0
\end{array}
$$

12. Question

Divide:

$x^4 - 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$

Answer

$$
\begin{array}{l|l}
x^2+x+1 & x^4-2x^3+2x^2+x+4 \;\big)\; x^2-3x+4 \\
& \quad x^4+x^3+x^2 \\
& \quad -\quad -\quad - \\
& \quad\quad -3x^3+x^2+x+4 \\
& \quad\quad -3x^3-3x^2-3x \\
& \quad\quad +\quad +\quad + \\
& \quad\quad\quad\quad 4x^2+4x+4 \\
& \quad\quad\quad\quad 4x^2+4x+4 \\
& \quad\quad\quad\quad -\quad -\quad - \\
& \quad\quad\quad\quad\quad 0
\end{array}
$$

13. Question

Divide:

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$

Answer

$$
\begin{array}{l}
m^2 - 12m + 13 \overline{\big)\ m^3 - 14m^2 + 37m - 26\ \big(\ m - 2} \\
\quad\ m^3 - 12m^2 + 13m \\
\quad\ - \quad - \quad - \\
\hline
\quad\quad -2m^2 + 24m - 26 \\
\quad\quad -2m^2 + 24m - 26 \\
\quad\quad + \quad + \quad + \\
\hline
\quad\quad\quad 0
\end{array}
$$

14. Question

Divide:

$x^4 + x^2 + 1$ by $x^2 + x + 1$

Answer

$$
\begin{array}{l}
x^2 + x + 1 \overline{\big)\ x^4 + 0x^3 + x^2 + 1\ \big(\ x^2 - x + 1} \\
\quad\ x^4 + x^3 + x^2 \\
\quad - \quad - \quad - \\
\hline
\quad\quad -x^3 + 1 \\
\quad\quad -x^3 \quad - x^2 - x \\
\quad\quad + \quad\quad + \quad + \\
\hline
\quad\quad\quad\quad x^2 + x + 1 \\
\quad\quad\quad\quad x^2 + x + 1 \\
\quad\quad\quad\quad - \quad - \quad - \\
\hline
\quad\quad\quad\quad\quad 0
\end{array}
$$

15. Question

Divide:

$x^5 + x^4 + x^3 + x^2 + x + 1$ by $x^3 + 1$

Answer

$$
\begin{array}{l}
x^3 + 1 \overline{\big)\ x^5 + x^4 + x^3 + x^2 + x + 1\ \big(\ x^2 + x + 1} \\
\quad\ x^5 \quad\quad + x^2 \\
\quad - \quad\quad - \\
\hline
\quad\quad x^4 + x^3 + x + 1 \\
\quad\quad x^4 \quad + x \\
\quad\quad - \quad\quad - \\
\hline
\quad\quad\quad\quad x^3 + 1 \\
\quad\quad\quad\quad x^3 + 1 \\
\quad\quad\quad\quad - \quad - \\
\hline
\quad\quad\quad\quad\quad 0
\end{array}
$$

16. Question

Divide each of the following and find the quotient and remainder:

$14x^3 - 5x^2 + 9x - 1$ by $2x - 1$

Answer

$$
\begin{array}{l}
2x - 1 \overline{\big)\ 14x^3 - 5x^2 + 9x - 1\ \big(\ 7x^2 + x + 5} \\
\quad\ 14x^3 - 7x^2 \\
\quad - \quad + \\
\hline
\quad\quad 2x^2 + 9x - 1 \\
\quad\quad\ 2x^2 - x \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad\ 10x - 1 \\
\quad\quad\quad\ 10x - 5 \\
\quad\quad\quad - \quad + \\
\hline
\quad\quad\quad\quad 4
\end{array}
$$

Quotient: $7x^2 + x + 5$

Remainder: 4

17. Question

Divide each of the following and find the quotient and remainder:

$3x^3 - x^2 - 10x - 3$ by $x - 3$

Answer

$$
\begin{array}{r|l}
x-3 & 3x^3 - x^2 - 10x - 3 \\
\cline{2-2}
 & 3x^3 - 9x^2 \\
 & \underline{- \quad +} \\
 & 8x^2 - 10x - 3 \\
 & 8x^2 - 24x \\
 & \underline{- \quad +} \\
 & 14x - 3 \\
 & 14x - 42 \\
 & \underline{- \quad +} \\
 & 39
\end{array}
\quad 3x^2 + 8x + 14
$$

Quotient: $3x^2 + 8x + 14$

Remainder: 39

18. Question

Divide each of the following and find the quotient and remainder:

$6x^3 + 11x^2 - 39x - 65$ by $3x^2 + 13x + 13$

Answer

$$
\begin{array}{r|l}
3x^2+13x+13 & 6x^3 + 11x^2 - 39x - 65 \\
\cline{2-2}
 & 6x^3 + 26x^2 + 26x \\
 & \underline{- \quad - \quad -} \\
 & -15x^2 - 65x - 65 \\
 & -15x^2 - 65x - 65 \\
 & \underline{+ \quad + \quad +} \\
 & 0
\end{array}
\quad 2x - 5
$$

Quotient: $2x - 5$

Remainder: 0

19. Question

Divide each of the following and find the quotient and remainder:

$30x^4 + 11x^3 - 82x^2 - 12x + 48$ by $3x^2 + 2x - 4$

Answer

$$
\begin{array}{r|l}
3x^2+2x-4 & 30x^4 + 11x^3 - 82x^2 - 12x + 48 \\
\cline{2-2}
 & 30x^4 + 20x^3 - 40x^2 \\
 & \underline{- \quad - \quad +} \\
 & -9x^3 - 42x^2 - 12x + 48 \\
 & -9x^3 - 6x^2 + 12x \\
 & \underline{+ \quad + \quad -} \\
 & -36x^2 - 24x + 48 \\
 & -36x^2 - 24x + 48 \\
 & \underline{+ \quad + \quad -} \\
 & 0
\end{array}
\quad 10x^2 - 3x - 12
$$

Quotient: $10x^2 - 3x - 12$

Remainder: 0

20. Question

Divide each of the following and find the quotient and remainder:

$9x - 4x^2 + 4$ by $3x^2 - 4x + 2$

Answer

$$
\begin{array}{l}
3x^2-4x+2 \;\overline{)\; 9x^4+0x^3-4x^2+4 \;)\; 3x^2+4x+2} \\
\quad 9x^4-12x^3+6x^2 \\
\quad - \quad + \quad - \\
\hline
\quad\quad 12x^3-10x^2+4 \\
\quad\quad 12x^3-16x^2+8x \\
\quad\quad - \quad + \quad - \\
\hline
\quad\quad\quad 6x^2-8x+4 \\
\quad\quad\quad 6x^2-8x+4 \\
\quad\quad\quad - \quad + \quad - \\
\hline
\quad\quad\quad\quad 0
\end{array}
$$

Quotient: $3x^2 + 4x + 2$

Remainder: 0

21. Question

Verify division algorithm i.e. Dividend=Divisor $\times$ Quotient + Remainder, in each of the following. Also, write the quotient and remainder;

Dividend	Divisor
(i) $14x^2 + 13x - 15$	$7x - 4$
(ii) $15z^3 - 20z^2 + 13z - 12$	$3z - 6$
(iii) $6y^5 - 28y^3 + 3y^2 + 30y - 9$	$2x^2 - 6$
(iv) $34x - 22x^3 - 12x^4 - 10x^2 - 75$	$3x + 7$
(v) $15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6$	$3y - 2$
(vi) $4y^3 + 8y + 8y^2 + 7$	$2y^2 - y + 1$
(vii) $6y^4 + 4y^4 + 4y^3 + 7y^2 + 27y + 6$	$2y^3 + 1$

Answer

(i)

$$
\begin{array}{l}
7x-4 \;\overline{)\; 14x^2+13x-15 \;)\; 2x+3} \\
\quad 14x^2-8x \\
\quad - \quad + \\
\hline
\quad\quad 21x-15 \\
\quad\quad 21x-12 \\
\quad\quad - \quad + \\
\hline
\quad\quad\quad -3
\end{array}
$$

Dividend = Divisor $\times$ Quotient + Remainder

$14x^2 + 13x - 15 = (7x - 4) \times (2x + 3) + (-3)$

$14x^2 + 13x - 15 = 14x^2 + 21x - 8x - 12 - 3$

$14x^2 + 13x - 15 = 14x^2 + 13x - 15$

(ii)

$$
\begin{array}{l|l}
3z-6 & 15z^3-20z^2+13z-12 \\
\cline{2-2}
 & 5z^2+\frac{10z}{3}+11
\end{array}
$$

$$
\begin{array}{r}
15z^3-30z^2 \\
\hline
10z^2+13z-12 \\
10z^2-20z \\
\hline
33z-12 \\
33z-66 \\
\hline
54
\end{array}
$$

Dividend = Divisor $\times$ Quotient + Remainder

$$15z^3-20z^2+13z-12=(3z-6)\times\left(5z^2+\frac{10z}{3}+11\right)+54$$

$$15z^3-20z^2+13z-12=15z^3+10z^2+33z-30z^2-20z+54$$

$$15z^3-20z^2+13z-12=15z^3-20z^2+13z-12$$

(iii)

$$
\begin{array}{l|l}
2y^2-6 & 6y^5-28y^3+3y^2+30y-9 \\
\cline{2-2}
 & 3y^3-5y+\frac{3}{2}
\end{array}
$$

$$
\begin{array}{r}
6y^5-18y^3 \\
\hline
-10y^3+3y^2+30y-9 \\
-10y^3+30y \\
\hline
3y^2-9 \\
3y^2-9 \\
\hline
0
\end{array}
$$

Dividend = Divisor $\times$ Quotient + Remainder

$$6y^5-28y^3+3y^2+30y-9=(2y^2-6)\times\left(3y^3-5y+\frac{3}{2}\right)+0$$

$$6y^5-28y^3+3y^2+30y-9=6y^5-10y^3+3y^2-18y^3+30y-9$$

$$6y^5-28y^3+3y^2+30y-9=6y^5-28y^3+3y^2+30y-9$$

(iv)

$$
\begin{array}{l|l}
3x+7 & -12x^4-22x^3-10x^2+34x-75 \\
\cline{2-2}
 & -4x^3+2x^2-8x+30
\end{array}
$$

$$
\begin{array}{r}
-12x^4-28x^3 \\
\hline
6x^3-10x^2+34x-75 \\
6x^3+14x^2 \\
\hline
-24x^2+34x-75 \\
-24x^2-56x \\
\hline
90x-75 \\
90x+210 \\
\hline
-285
\end{array}
$$

Dividend = Divisor $\times$ Quotient + Remainder

$$-12x^4-22x^3-10x^2+34x-75=(3x+7)\times(-4x^3+2x^2-8x+30)-285$$

$$-12x^4-22x^3-10x^2+34x-75=-12x^4+6x^3-24x^2-28x^3+14x^2+90x-56x+210-285$$

$$-12x^4-22x^3-10x^2+34x-75=-12x^4-22x^3-10x^2+34x-75$$

(v)

$$3y-2\overline{)15y^4-16y^3+9y^2-\frac{10y}{3}+6}\ \big(5y^3-2y^2+\frac{5y}{3}$$

$15y^4-10y^3$ (− +)

$-6y^3+9y^2-\frac{10y}{3}+6$

$-6y^3+4y^2$ (+ −)

$5y^2-\frac{10y}{3}+6$

$5y^2-\frac{10y}{3}$ (+ +)

6

Dividend = Divisor × Quotient + Remainder

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=(3y-2)\times\left(5y^3-2y^2+\frac{5y}{3}\right)+6$$

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=15y^4-6y^3+5y^2-10y^3+4y^2-\frac{10y}{3}+6$$

$$15y^4-16y^3+9y^2-\frac{10y}{3}+6=15y^4-16y^3+9y^2-\frac{10y}{3}+6$$

(vi)

$$2y^2-y+1\overline{)4y^3+8y^2+8y+7}\ \big(2y+5$$

$4y^3-2y^2+2y$ (− + −)

$10y^2+6y+7$

$10y^2-5y+5$ (− + −)

$11y+2$

Dividend = Divisor × Quotient + Remainder

$$4y^3+8y^2+8y+7=(2y^2-y+1)\times(2y+5)+11y+2$$

$$4y^3+8y^2+8y+7=4y^3+10y^2-2y^2-5y+2y+5+11y+2$$

$$4y^3+8y^2+8y+7=4y^3+8y^2+8y+7$$

(vii)

$$2y^3+1\overline{)6y^5+4y^4+4y^3+7y^2+27y+6}\ \big(3y^2+2y+2$$

$6y^5+3y^2$ (− −)

$4y^4+4y^3+4y^2+27y+6$

$4y^4+2y$ (− −)

$4y^3+4y^2+25y+6$

$4y^3+2$ (− −)

$4y^2+25y+4$

Dividend = Divisor × Quotient + Remainder

$$6y^5+4y^4+4y^3+7y^2+27y+6=(2y^3+1)\times(3y^2+2y+2)+4y^2+25y+4$$

$$6y^5+4y^4+4y^3+7y^2+27y+6=6y^5+4y^4+4y^3+3y^2+2y+2\ +\ 4y^2+25y+4$$

$$6y^5+4y^4+4y^3+7y^2+27y+6=6y^5+4y^4+4y^3+7y^2+27y+6$$

22. Question

Divide $15y^4+16y^3+\frac{10}{3}y-9y^2-6$ by $3y-z$ Write down the coeficients of the terms in the quotient.

Answer

$$3y-2\ \overline{)\ 15y^4+16y^3-9y^2+\frac{10y}{3}-6\ }\ 5y^3+\frac{26y^2}{3}+\frac{25y}{9}+\frac{80}{27}$$

$$15y^4-10y^3$$
$$-\quad+$$
$$26y^3-9y^2+\frac{10y}{3}-6$$
$$26y^3-\frac{52y^2}{3}$$
$$-\quad+$$
$$\frac{25y^2}{3}+\frac{10y}{3}-6$$
$$\frac{25y^2}{3}-\frac{50y}{9}$$
$$-\quad+$$
$$\frac{80y}{9}-6$$
$$\frac{80y}{9}-\frac{160}{27}$$
$$-\quad+$$
$$-\frac{2}{27}$$

Quotient: $5y^3+\frac{26y^2}{3}+\frac{25y}{9}+\frac{80}{27}$

Coefficient of $y^3 = 5$; Coefficient of $y^2 = \frac{26}{3}$; Coefficient of $y = \frac{25}{9}$; Constant term = Coefficient of $y^2 = \frac{80}{27}$

23. Question

Using division of polynomials state whether

(i) $x+6$ is a factor of x^2-x-42 3

(ii) 4x-1 is a factor of $4x^2-13x-12$

(iii) 2y-5 is a factor of $4y^4-10y^3-10y^2+30y-15$

(iv) $3y^2+5$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

(v) z^2+3 is a factor of z^5-9z

(vi) $2x^2-x+3$ is a factor of $6x^5-x^4+4x^3-5x^2-x-15$

Answer

(i) $x+6$ is a factor of x^2-x-42

$$x+6\ \overline{)\ x^2-x-42\ }\ x-7$$
$$x^2+6x$$
$$-\quad-$$
$$-7x-42$$
$$-7x-42$$
$$+\quad+$$
$$0$$

Quotient: $x-7$

Remainder: 0

Since remainder is 0 therefore $(x+6)$ is a factor of x^2-x-42

(ii) 4x-1 is a factor of $4x^2-13x-12$

$$4x-1\ \overline{)\ 4x^2-13x-12\ }\ x-3$$
$$4x^2-x$$
$$-\quad+$$
$$-12x-12$$
$$-12x+3$$
$$+\quad-$$
$$15$$

Quotient: $x-3$

Remainder: 15

Since remainder is 15 therefore $(4x-1)$ is **NOT** a factor of $4x^2-13x-12$

(iii) 2y-5 is a factor of $4y^4-10y^3-10y^2+30y-15$

$$
\begin{array}{l}
2y-5 \;\overline{)\; 4y^4-10y^3-10y^2+30y-15 \;\big(\; 2y^3-5y+\frac{5}{2}} \\
\quad\; 4y^4-10y^3 \\
\quad\; -\quad + \\
\hline
\qquad\qquad -10y^2+30y-15 \\
\qquad\qquad -10y^2+25y \\
\qquad\quad +\qquad - \\
\hline
\qquad\qquad\qquad 5y-15 \\
\qquad\qquad\qquad 5y-\frac{25}{2} \\
\qquad\qquad\quad -\quad + \\
\hline
\qquad\qquad\qquad\quad -\frac{5}{2}
\end{array}
$$

Quotient: $2y^3-5y+\frac{5}{2}$

Remainder: $-\frac{5}{2}$

Since remainder is $-\frac{5}{2}$ therefore $(2y-5)$ is **NOT** a factor of $4y^4-10y^3-10y^2+30y-15$

(iv) $3y^2+5$ is a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

$$
\begin{array}{l}
3y^2+5 \;\overline{)\; 6y^5+15y^4+16y^3+4y^2+10y-35 \;\big(\; 2y^3+2y+\frac{4}{3}} \\
\quad\; 6y^5 \qquad +10y^3 \\
-\quad\; \overline{\qquad\qquad\quad -} \\
\qquad\qquad\quad 6y^3+4y^2+10y-35 \\
\qquad\quad\; 6y^3 \qquad +10y \\
\qquad -\quad\; \overline{\qquad\quad\; -} \\
\qquad\qquad\quad 4y^2 \qquad -35 \\
\qquad\qquad\quad 4y^2 \qquad +\frac{20}{3} \\
\qquad\qquad\quad\; -\qquad\quad -\\
\hline
\qquad\qquad\qquad\quad -\frac{125}{3}
\end{array}
$$

Quotient: $2y^3+2y+\frac{4}{3}$

Remainder: $-\frac{125}{3}$

Since remainder is $-\frac{125}{3}$ therefore $(3y^2+5)$ is **NOT** a factor of $6y^5+15y^4+16y^3+4y^2+10y-35$

(v) z^2+3 is a factor of z^5-9z

$$
\begin{array}{l}
z^2+3 \;\overline{)\; z^5-9z \;\big(\; z^3-3z} \\
\quad\; z^5+3z^3 \\
-\quad -\\
\hline
\qquad -3z^3-9z \\
\qquad +\quad + \\
\hline
\qquad\quad 0
\end{array}
$$

Quotient: z^3-3z

Remainder: 0

Since remainder is 0 therefore (z^3-3z) is a factor of z^5-9z

(vi) $2x^2-x+3$ is a factor of $6x^5-x^4+4x^3-5x^2-x-15$

$$
\begin{array}{r|l}
2x^2 - x + 3 & 6x^5 - x^4 + 4x^3 - 5x^2 - x - 15 \\
\cline{2-2}
 & 3x^3 + x^2 - 2x - 3 \\
 & 6x^5 - 3x^4 + 9x^3 \\
 & \text{- + -} \\
 & \cline{1-1} 2x^4 - 5x^3 - 5x^2 - x - 15 \\
 & 2x^4 - x^3 + 3x^2 \\
 & \text{- + -} \\
 & \cline{1-1} -4x^3 - 8x^2 - x - 15 \\
 & -4x^3 - 2x^2 - 6x \\
 & \text{+ + +} \\
 & \cline{1-1} -6x^2 + 5x - 15 \\
 & -6x^2 + 3x - 9 \\
 & \text{+ - +} \\
 & \cline{1-1} 2x - 6
\end{array}
$$

Quotient: $3x^3 + x^2 - 2x - 3$

Remainder: $2x^2 - x + 3$

Since remainder is $2x - 6$ therefore $(3y^2 + 5)$ is **NOT** a factor of $6y^5 + 15y^4 + 16y^3 + 4y^2 + 10y - 35$

24. Question

Find the value of a, if x+2 is a factor of $4x^4 + 2x^3 - 3x^2 + 8x + 5a$.

Answer

$x + 2$ *is a factor of* $4x^4 + 2x^3 - 3x^2 + 8x + 5a$

$x + 2 = 0$

$x = -2$ Therefore substitute x = -2 in the given equation we get,

$4(-2)^4 + 2(-2)^3 - 3(-2)^2 + 8(-2) + 5a = 0$

$64 - 16 - 12 - 16 + 5a = 0$

$20 + 5a = 0$

$5a = -20$

$a = -\frac{20}{5} = -4$

$\boldsymbol{a = -4}$

25. Question

What must be added to $x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polymonial is exactly divible by $x^2 + 2x - 3$.

Answer

$$
\begin{array}{r|l}
x^2 + 2x - 3 & x^4 + 2x^3 - 2x^2 + x - 1 \\
\cline{2-2}
 & x^2 + 1 \\
 & x^4 + 2x^3 - 3x^2 \\
 & \text{- - +} \\
 & \cline{1-1} x^2 + x - 1 \\
 & x^2 + 2x - 3 \\
 & \text{- - +} \\
 & \cline{1-1} -x + 2
\end{array}
$$

Quotient: $x^2 + 1$

Remainder: $-x + 2$

Therefore $x - 2$ to be added.

Exercise 8.5

1. Question

Divide the first polynomial by the second polynomial in each of the following Also write the quotient and remainder:

(i) $3x^2 + 4x + 5, x - 2$

(ii) $10x^2 - 7x + 8, 5x - 3$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

(iv) $x^4 - x^3 + 5x, x - 1$

(v) $y^4 + y^2, y^2 - 2$

Answer

(i) $3x^2 + 4x + 5, x - 2$

$$
\begin{array}{r|l}
x-2 & 3x^2+4x+5 \\
 & 3x^2-6x \\
 & \hline
 & 10x+5 \\
 & 10x-20 \\
 & \hline
 & 25
\end{array}
\qquad \text{Quotient: } 3x+10
$$

Remainder: 25

(ii) $10x^2 - 7x + 8, 5x - 3$

$$
\begin{array}{r|l}
5x-3 & 10x^2-7x+8 \\
 & 10x^2-6x \\
 & \hline
 & -x+8 \\
 & -x+\frac{3}{5} \\
 & \hline
 & 8-\frac{3}{5}=\frac{37}{5}
\end{array}
\qquad \text{Quotient: } 2x-\frac{1}{5}
$$

Remainder: $\frac{37}{5}$

(iii) $5y^3 - 6y^2 + 6y - 1, 5y - 1$

$$
\begin{array}{r|l}
5y-1 & 5y^3-6y^2+6y-1 \\
 & 5y^3-y^2 \\
 & \hline
 & -5y^2+6y-1 \\
 & -5y^2+y \\
 & \hline
 & 5y-1 \\
 & 5y-1 \\
 & \hline
 & 0
\end{array}
\qquad \text{Quotient: } y^2-y+1
$$

Remainder: 0

(iv) $x^4 - x^3 + 5x, x - 1$

$$
\begin{array}{r|l}
x-1 & x^3+5 \\
\cline{2-2}
x^4-x^3 & \\
-\ \ + & \\
\cline{1-1}
5x & \\
5x-5 & \\
-\ \ + & \\
\cline{1-1}
5 &
\end{array}
$$

Quotient: $x^3 + 5$

Remainder: 5

(v) $y^4 + y^2, y^2 - 2$

$$
\begin{array}{r|l}
y^2-2 & y^2+1 \\
\cline{2-2}
y^4-2y^2 & \\
-\ \ + & \\
\cline{1-1}
y^2 & \\
y^2-2 & \\
-\ \ + & \\
\cline{1-1}
2 &
\end{array}
$$

Quotient: $y^2 + 1$

Remainder: 2

2. Question

Find Whether or not the first polynomial is a factor of the second:

(i) $x + 1, 2x^2 + 5x + 4$

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

(iii) $4x^2 - 5, 4x^4 + 7x^2 + 15$

(iv) $4 - z, 3z^2 - 13z + 4$

(v) $2a - 3, 10a^2 - 9a - 5$

(vi) $4y + 1, 8y^2 - 2y + 1$

Answer

(i) $x + 1, 2x^2 + 5x + 4$

$$
\begin{array}{r|l}
x+1 & 2x+3 \\
\cline{2-2}
2x^2+2x & \\
-\ \ - & \\
\cline{1-1}
3x+4 & \\
3x+3 & \\
-\ \ - & \\
\cline{1-1}
1 &
\end{array}
$$

Quotient: $2x + 3$

Remainder: 1

Since remainder is 1 therefore the first polynomial is **NOT** a factor of the second polynomial.

(ii) $y - 2, 3y^3 + 5y^2 + 5y + 2$

$$
\begin{array}{r|l}
y-2 & 3y^3+5y^2+5y+2 \\
 & 3y^3-6y^2 \\
 & \hline
 & 11y^2+5y+2 \\
 & 11y^2-22y \\
 & \hline
 & 27y+2 \\
 & 27y-54 \\
 & \hline
 & 56
\end{array}
\quad \Big) 3y^2+11y+27
$$

Quotient: $3y^2+11y+27$

Remainder: 56

Since remainder is 56 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iii) $4x^2-5, 4x^4+7x^2+15$

$$
\begin{array}{r|l}
4x^2-5 & 4x^4+7x^2+15 \\
 & 4x^4-5x^2 \\
 & \hline
 & 12x^2+15 \\
 & 12x^2-15 \\
 & \hline
 & 30
\end{array}
\quad \Big) x^2+3
$$

Quotient: x^2+3

Remainder: 30

Since remainder is 30 therefore the first polynomial is **NOT** a factor of the second polynomial.

(iv) $4-z, 3z^2-13z+4$

$$
\begin{array}{r|l}
-z+4 & 3z^2-13z+4 \\
 & 3z^2-12z \\
 & \hline
 & -z+4 \\
 & -z+1 \\
 & \hline
 & 0
\end{array}
\quad \Big) -3z+1
$$

Quotient: $-3z+1$

Remainder: 0

Since remainder is 0 therefore the first polynomial is a factor of the second polynomial.

(v) $2a-3, 10a^2-9a-5$

$$
\begin{array}{r|l}
2a-3 & 10a^2-9a-5 \\
 & 10a^2-15a \\
 & \hline
 & 6a-5 \\
 & 6a-9 \\
 & \hline
 & 4
\end{array}
\quad \Big) 5a+3
$$

Quotient: $5a+3$

Remainder: 4

Since remainder is 4 therefore the first polynomial is **NOT** a factor of the second polynomial.

(vi) $4y+1, 8y^2-2y+1$

$$
\begin{array}{r|l}
4y+1 & 8y^2-2y+1 \;\big)\; 2y-1 \\
 & \underline{8y^2+2y} \\
 & \quad -4y+1 \\
 & \quad \underline{-4y-1} \\
 & \quad\quad 2
\end{array}
$$

Quotient: $2y-1$

Remainder: 2

Since remainder is 2 therefore the first polynomial is **NOT** a factor of the second polynomial.

Exercise 8.6

1. Question

Divide:

$x^2 - 5x + 6$ by $x - 3$

Answer

$$
\begin{array}{r|l}
x-3 & x^2-5x+6 \;\big)\; x-2 \\
 & \underline{x^2-3x} \\
 & \quad -2x+6 \\
 & \quad \underline{-2x+6} \\
 & \quad\quad 0
\end{array}
$$

Quotient: $x-2$

Remainder: 0

2. Question

Divide:

$ax^2 - ay^2$ by $ax + ay$

Answer

$$
\begin{array}{r|l}
ax-ay & ax^2-ay^2 \;\big)\; x-y \\
 & \underline{ax^2-axy} \\
 & \quad -ay^2+axy \\
 & \quad \underline{-ay^2+axy} \\
 & \quad\quad 0
\end{array}
$$

Quotient: $x-y$

Remainder: 0

3. Question

Divide:

$x^4 - y^4$ by $x^2 - y^2$

Answer

$$
\begin{array}{r|l}
x^2-y^2 & x^4-y^4 \;\big)\; x^2+y^2 \\
 & \underline{x^4-x^2y^2} \\
 & \quad -y^4+x^2y^2 \\
 & \quad \underline{-y^4+x^2y^2} \\
 & \quad\quad 0
\end{array}
$$

Quotient: $x^2 + y^2$

Remainder: 0

4. Question

Divide:

$acx^2 + (bc + ad)x + bd$ by $(ax + b)$

Answer

$$ax + b \,\overline{\big)\, acx^2 + bcx + adx + bd \,\big(}\, cx + d$$
$$acx^2 + bcx$$
$$- \quad -$$
$$\overline{\qquad adx + bd}$$
$$adx + bd$$
$$+ \quad -$$
$$\overline{\qquad 0 \qquad}$$

Quotient: $cx + d$

Remainder: 0

5. Question

Divide:

$(a^2 + 2ab + b^2) - (a^2 + 2ac + c^2)$ by $2a + b + c$

Answer

$$2a + b + c \,\overline{\big)\, 2ab - 2ac + b^2 - c^2 \,\big(}\, b - c$$
$$2ab \quad + b^2 \quad + bc$$
$$- \quad - \quad -$$
$$\overline{\qquad -2ac \quad - c^2 - bc}$$
$$-2ac \quad - c^2 - bc$$
$$+ \quad + \quad +$$
$$\overline{\qquad 0 \qquad}$$

Quotient: $b - c$

Remainder: 0

6. Question

Divide:

$\frac{1}{4}x^2 - \frac{1}{2}x - 12$ by $\frac{1}{2}x - 4$

Answer

$$\frac{x}{2} - 4 \,\overline{\big)\, \frac{x^2}{4} - \frac{x}{2} - 12 \,\big(}\, \frac{x}{2} + 3$$
$$\frac{x^2}{4} - 2x$$
$$- \quad +$$
$$\overline{\qquad \frac{3x}{2} - 12}$$
$$\frac{3x}{2} - 12$$
$$- \quad +$$
$$\overline{\qquad 0 \qquad}$$

Quotient: $\frac{x}{2} + 3$

Remainder: 0