Division Of Algebraic Expressions
Ex-8.1

8. Division of ALGEBRAIC EXPRESSIONS.

1. Degree of a polynomial in one Variable:-

In a polynomial in one variable, the highest power of the variable is called its degree.

(i) $2x^3+5x^2-7$.

highest power of a polynomial $=3$.

degree $=3$.

(ii) $5x^2+3x+2$.

highest power of a polynomial $=2$.

degree $=2$.

(iii) $2x+x^2-8$.

highest power of a polynomial $=2$.

degree $=2$.

(iv) $\frac{1}{2}y^7-12y^6+48y^5-10$.

highest power of a polynomial $=7$.

degree $=7$.

(v) $3x^3+1$

highest power of a polynomial $=3$.

degree $=3$.

(vi) 5

highest power of a polynomial $=0$, degree $=0$.

(vii) $20x^3+12x^2y^2*-10y^2+20$.

highest power of a polynomial $=4$, degree $=4$.

2. Polynomials :-

An algebraic expression in which the variables involved have only non-negative integral powers, is called a polynomial.

In the given expressions

(i), (iv), (v) are not polynomials.

because. The expressions in which the variables involved have only non-negative integral powers.

3. (i) $3+6x+x^2+5x^4$ (or) $5x^4+x^2+6x+3$.

degree = 4.

(ii) $4+a^2+5a^6$ (or) $5a^6+a^2+4$.

degree = 6.

(iii) x^6-5x^3+4 (or) $4-5x^3+x^6$

degree = 6.

(iv) y^6+9y^3-22 (or) $-22+9y^3+y^6$

degree = 6.

(v) $a^6+\frac{27}{136}a^3-\frac{48}{136}$ (or), $-\frac{48}{136}+\frac{27}{136}a^3+a^6$.

degree = 6.

(vi) $a^2+\frac{25}{12}a+1$ (or), $1+\frac{25}{12}a+a^2$

degree = 2.

Division Of Algebraic Expressions
Ex 8.2

Exercise - 8.2.

Divide:

1. $6x^3y^2z^2$ by $3x^2yz$.

We have, $\Rightarrow \dfrac{6x^3y^2z^2}{3x^2yz} = 2xyz.$

$$\frac{6x^3y^2z^2}{3x^2yz} = 2xyz$$

2. $15m^2n^2$ by $5m^2n^2$

$$\frac{15m^2n^3}{5m^2n^2} = \frac{15 \cdot m \cdot m \cdot n \cdot n \cdot n}{5 \cdot m \cdot m \cdot n \cdot n}$$

$$= 3n.$$

3. $24a^3b^3$ by $-8ab$.

$$\frac{24a^3b^3}{-8ab} = \frac{24 \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b}{-8 \cdot a \cdot b}$$

$$= -3a^2b^2.$$

4. $-21abc^2$ by $7abc$.

$$\frac{-21abc^2}{7abc} = \frac{-21 a \cdot b \cdot c \cdot c}{7 \cdot a \cdot b \cdot c}$$

$$= -3c.$$

5. $72xyz^2$ by $-9xz$

$$\frac{72xyz^2}{-9xz} = \frac{8 \times 9 \cdot x \cdot y \cdot z \cdot z}{-9 \cdot x \cdot z} = -8yz.$$

Solution-06:-

$$\frac{-72a^4b^5c^8}{-9a^2b^2c^3} = \frac{+72 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b \cdot b \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c \cdot c}{+9 \cdot a \cdot a \cdot b \cdot b \cdot c \cdot c \cdot c}$$

$$= 8a^2b^3c^5$$

Solution-07:-

$$\frac{16m^3y^2}{4m^2y} = \frac{16 \cdot m \cdot m \cdot m \cdot y \cdot y}{4 \cdot m \cdot m \cdot y}$$

$$= 4m \cdot y.$$

Solution-08:-

$$\frac{32m^2n^3p^2}{4m \cdot n \cdot p} = \frac{32 \, m \cdot m \cdot n \cdot n \cdot n \cdot p \cdot p}{4 \cdot m \cdot n \cdot p}$$

$$= 8mn^2p.$$

Division Of Algebraic Expressions
Ex 8.3

Exercise-8.3

Divide.

1. $x+2x^2+3x^4-x^5$ by $2x$

$$\frac{x+2x^2+3x^4-x^5}{2x} = \frac{x(1+2x+3x^3-x^4)}{2x}$$

$$= \frac{x}{2x} + \frac{2x^2}{2x} + \frac{3x^3 \cdot x}{2 \cdot x} - \frac{x^4 \cdot x}{2x}$$

$$= \frac{1}{2} + x + \frac{3}{2}x^3 - \frac{1}{2}x^4.$$

2. $y^4-3y^3+\frac{1}{2}y^2$ by $3y$.

$$\frac{y^4-3y^3+\frac{1}{2}y^2}{3y} = \frac{y^4}{3y} - \frac{3y^3}{3y} + \frac{1}{2}\cdot\frac{y^2}{3\cdot y}$$

$$= \frac{1}{3}\cdot y^3 - y^2 + \frac{1}{6}\cdot y.$$

3. $-2a^2+2a+\frac{1}{2}$

3. $-4a^3+4a^2+a$ by $2a$.

$$\frac{-4a^3+4a^2+a}{2a} = \frac{-4a^3}{2a} + \frac{4a^2}{2a} + \frac{a}{2a}$$

$$= -2a^2+2a+\frac{1}{2}.$$

4. $-x^6+2x^4+4x^3+2x^2$ by $\sqrt{2}\,x^2$.

$$\frac{-x^6+2x^4+4x^3+2x^2}{\sqrt{2}\,x^2} = \frac{-x^6}{\sqrt{2}x^2} + \frac{2x^4}{\sqrt{2}x^2} + \frac{4x^3}{\sqrt{2}x^2} + \frac{2x^2}{\sqrt{2}x^2}$$

$$= \frac{-1}{\sqrt{2}}x^4 + \sqrt{2}x^2 + 2\sqrt{2}x + \sqrt{2}.$$

5. $5z^3-6z^2+7z$ by $2z$.

$$\frac{5z^3-6z^2+7z}{2z} = \frac{5z^3}{2z} - \frac{6z^2}{2z} + \frac{7z}{2z}$$

$$= \frac{5}{2}z^2 - 3z + \frac{7}{2}.$$

6. $\sqrt{3}a^4+2\sqrt{3}a^3+3a^2-6a$ by $3a$.

$$\frac{\sqrt{3}a^4+2\sqrt{3}a^3+3a^2-6a}{3a} = \frac{\sqrt{3}a^4}{3a} + \frac{2\sqrt{3}a^3}{3a} + \frac{3a^2}{3a} - \frac{6a}{3a}$$

$$= \frac{1}{\sqrt{3}}a^2 + \frac{2}{\sqrt{3}}a^2 + a - 2.$$

Division Of Algebraic Expressions

Ex 8.4

Exercise-8.4.

Divide.

1. $5x^3 - 15x^2 + 25x$ by $5x$.

$$\frac{5x^3 - 15x^2 + 25x}{5x} = \frac{5x^3}{5x} + \left(\frac{-15}{5}\right)\cdot\frac{x^2}{x} + \frac{25}{5}\cdot\frac{x}{x}$$

$$= x^2 - 3x + 5.$$

2. $4z^3 + 6z^2 - z$ by $\frac{-1}{2}z$

$$\frac{4z^3 + 6z^2 - z}{\frac{-1}{2}z} = \frac{4z^3\cdot(2)}{-z} - \frac{6z^2\cdot 2}{z} + \frac{z\cdot 2}{z}$$

$$= -8z^2 - 12z + 2.$$

3. $9x^2y - 6xy + 12xy^2$ by $\frac{-3}{2}xy$.

$$\frac{9x^2y - 6xy + 12xy^2}{\frac{-3}{2}xy} = \frac{9x^2y}{-3x\cdot y}\cdot 2 + \frac{6xy\cdot 2}{3\cdot x\cdot y} + \frac{12x\cdot y^2\cdot 2}{-3xy}$$

$$= -6x + 4 - 8y.$$

4. $3x^3\cdot y^2 + 2x^2\cdot y + 15xy$ by $3xy$.

$$\frac{3x^3\cdot y^2 + 2x^2\cdot y + 15xy}{3xy} = \frac{3\cdot x^3\cdot y^2}{3xy} + \frac{2x^2\cdot y}{3xy} + \frac{15xy}{3xy}$$

$$= x^2y + \frac{2}{3}x + 5.$$

5. $x^2+7x+12$ by $x+4$.

step 1:-

we devide the first term x^2 of the dividend by the first term x of the divisor and obtain $\frac{x^2}{x} = x$ as the first term of the quotient.

$$\begin{array}{r|l} & x+3 \\ \hline x+4 & x^2+7x+12 \\ & x^2+4x \\ \hline & 3x+12 \\ & 3x+12 \\ \hline & 0 \end{array}$$

Step -2 :-

We multiply the divisor $x+4$ by the first term x of the quotient and subtract the result from the dividend $x^2+7x+12$. we obtain $3x+12$ as the remainder

step-3 :-

Now we treat $3x+12$ as the new dividend and divide the first term $3x$ by the first term x of the divisor to obtain $\frac{3x}{x} = 3$ as the third term of the quotient.

step-IV:-

we multiply the divisor $x+4$ and the ~~third~~ second term 3 of the quotient and subtract the result $3x+12$ from the new dividend. we obtain 0 as the remainder.

Thus, we can say that

$$\frac{x^2+7x+12}{x+4} = x+3.$$

Solution-06:-

$4y^2+3y+\frac{1}{2}$ by $2y+1$

$$
\begin{array}{r|l}
2y+1 & 2y+\frac{1}{2} \\
\hline
 & 4y^2+3y+\frac{1}{2} \\
 & \underline{-4y^2+2y} \\
 & \quad y+\frac{1}{2} \\
 & \quad \underline{y+\frac{1}{2}} \\
 & \quad 0
\end{array}
$$

Solution-07.

$3x^3+4x^2+5x+18$ by $x+2$.

$$
\begin{array}{r|l}
x+2 & 3x^2-2x+9 \\
\hline
 & 3x^3+4x^2+5x+18 \\
 & \underline{3x^3+6x^2} \\
 & \quad -2x^2+5x \\
 & \quad \underline{-2x^2+(+4x} \\
 & \quad\quad 9x+18 \\
 & \quad\quad \underline{9x+18} \\
 & \quad\quad 0
\end{array}
$$

Solution-08:-

$$
\begin{array}{r|l}
7x-9 & 2x-5 \\
\hline
 & 14x^2-53x+45 \\
 & \underline{-14x^2+18x} \\
 & \quad -35x+45 \\
 & \quad \underline{-35x+45} \\
 & \quad 0
\end{array}
$$

Solution - 09.

$$\frac{-(-21+71x-31x^2-24x^3)}{-(3-8x)} = \frac{21-71x+31x^2+24x^3}{8x-3}$$

$$\begin{array}{r|l} & 3x^2+5x-7 \\ \hline 8x-3 & 24x^3+31x^2-71x+21 \\ & -24x^3 \mp 9x^2 \\ \hline & \quad 40x^2-71x \\ & \quad -40x^2 \mp 15x \\ \hline & \quad\quad -56x+21 \\ & \quad\quad \mp 56x \pm 21 \\ \hline & \quad\quad\quad (0) \end{array}$$

Solution-10:-

$3y^4-3y^3-4y^2-4y$ by y^2-2y.

$$\begin{array}{r|l} & 3y^2+3y+2 \\ \hline y^2-2y & 3y^4-3y^3-4y^2-4y \\ & 3y^4-6y^3 \\ & - \quad + \\ \hline & \quad 3y^3-4y^2 \\ & \quad 3y^3-6y^2 \\ & \quad + \\ \hline & \quad\quad 2y^2-4y \\ & \quad\quad \frac{2y^2+4y}{- \quad +} \\ & \quad\quad\quad (0) \end{array}$$

$(y^2-2y)(3y^2+3y+2) = 3y^4-3y^3-4y^2-4y$.

Solution-11

$2y^5 + 10y^4 + 6y^3 + y^2 + 5y + y + 3$ by $2y^3 + 1$

$$
\begin{array}{r|l}
 & y^2 + 5y + 3 \\
\hline
2y^3 + 1 & 2y^5 + 10y^4 + 6y^3 + y^2 + 5y + 3 \\
 & \underline{2y^5 + 0 + 0 + y^2} \\
 & \quad 10y^4 + 6y^3 + 0 + 5y \\
 & \quad \underline{10y^4 + 0 + 0 + 5y} \\
 & \qquad 6y^3 + 0 + 0 + 3 \\
 & \qquad \underline{6y^3 + 0 + 0 + 3} \\
 & \qquad\quad (0)
\end{array}
$$

Solution -12:-

$x^4 - 2x^3 + 2x^2 + x + 4$ by $x^2 + x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 3x + 4 \\
\hline
x^2 + x + 1 & x^4 - 2x^3 + 2x^2 + x + 4 \\
 & \underline{x^4 + x^3 + x^2} \\
 & \quad -3x^3 + x^2 + x \\
 & \quad \underline{-3x^3 - 3x^2 - 3x} \\
 & \qquad 4x^2 + 4x + 4 \\
 & \qquad \underline{4x^2 + 4x + 4} \\
 & \qquad\quad (0).
\end{array}
$$

Solution-13:-

$m^3 - 14m^2 + 37m - 26$ by $m^2 - 12m + 13$

$$
\begin{array}{r|l}
 & m - 2 \\
\hline
m^2 - 12m + 13 & m^3 - 14m^2 + 37m - 26 \\
 & m^3 - 12m^2 + 13m \\
 & \quad - \quad + \quad - \\
\hline
 & -2m^2 + 24m - 26 \\
 & -2m^2 + 24m - 26 \\
 & \quad + \quad - \quad + \\
\hline
 & 0
\end{array}
$$

Solution-014 :-

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2 + x + 1 & x^4 + x^2 + 1 + 0 \\
 & x^4 + x^2 + 0 + x^3 \\
\hline
 &
\end{array}
$$

solution-14 :-

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
\hline
x^2 + x + 1 & x^4 + 0 + x^2 + 0 + 1 \\
 & x^4 + x^3 + x^2 \\
\hline
 & x^3 + 0 + 0 + 1 \\
 & x^3 - x^2 - x + 0 \\
 & \quad + \quad + \\
\hline
 & x^2 + x + 1 \\
 & x^2 + x + 1 \\
\hline
 & 0
\end{array}
$$

Solution-15:-

$$
\begin{array}{r|l}
x^3+1 & x^2+x+1 \\
\hline
 & x^5+x^4+x^3+x^2+x+1 \\
 & x^5+0+0+x^2 \\
\hline
 & x^4+x^3+0+x \\
 & x^4+0+0+x \\
\hline
 & x^3+0+0+1 \\
 & x^3+0+0+1 \\
\hline
 & 0
\end{array}
$$

16:- Divide the following and find the quotient and remainder.

Solution-16:-

$14x^3-5x^2+9x-1$ by $2x-1$.

$$
\begin{array}{r|l}
2x-1 & 7x^2+x+5 \\
\hline
 & 14x^3-5x^2+9x-1 \\
 & 14x^3-7x^2 \\
\hline
 & 2x^2+9x \\
 & 2x^2-x \\
\hline
 & 10x-1 \\
 & 10x+5 \\
\hline
 & 4
\end{array}
$$

$\therefore$ $Q = 7x^2+x+5$; $R = 4$.

Solution-17 :-

$$
\begin{array}{r|l}
2x-3 & 3x^2+4x+1 \\
\hline
 & 6x^3-x^2-10x-3 \\
 & -6x^3+9x^2 \\
\hline
 & 8x^2-10x \\
 & 8x^2-8x \\
\hline
 & 2x-3 \\
 & -2x+3 \\
\hline
 & 0
\end{array}
$$

$\therefore$ Quotient $= 3x^2+4x+1$; Remainder $= 0$.

Solution-18 :-

$6x^3+11x^2-39x-65$ by $3x^2+13x+13$

$$
\begin{array}{r|l}
3x^2+13x+13 & 2x-5 \\
\hline
 & 6x^3+11x^2-39x-65 \\
 & 6x^3+26x^2+26x \\
 & - \quad - \quad - \\
\hline
 & -15x^2-65x-65 \\
 & -15x^2-65x-65 \\
 & + \quad + \quad + \\
\hline
 & 0
\end{array}
$$

$\therefore$ Quotient $= 2x-5$; Remainder $= 0$.

Solution-19:-

$30x^4+11x^3-82x^2-12x+48$ by $3x^2+2x-4$.

$$
\begin{array}{r|l}
3x^2+2x-4 & 10x^2-3x-12 \\
\hline
 & 30x^4+11x^3-82x^2-12x+48 \\
 & \underline{30x^4+20x^3-40x^2} \\
 & \quad -9x^3-42x^2-12x \\
 & \quad \underline{-9x^3-6x^2+12x} \\
 & \qquad -36x^2-24x+48 \\
 & \qquad \underline{-36x^2-24x+48} \\
 & \qquad 0
\end{array}
$$

$\therefore$ Quotient $= 10x^2-3x-12$; Remainder $=0$.

Solution-20:-

$9x^4-4x^2+4$ by $3x^2-4x+2$.

$$
\begin{array}{r|l}
3x^2-4x+2 & 3x^2+4x+2 \\
\hline
 & 9x^4-0+(-4x^2)+0+4 \\
 & \underline{9x^4-12x^3+6x^2} \\
 & \quad +12x^3-10x^2+0+4 \\
 & \quad \underline{12x^3-16x^2+8x+0} \\
 & \qquad 6x^2-8x+4 \\
 & \qquad \underline{6x^2-8x+4} \\
 & \qquad 0
\end{array}
$$

Quotient $= 3x^2-4x+2$; Remainder $=0$.

Soution -21.

(i) Dividend = $14x^2+13x-15$.

Divisor = $7x-4$

$$
\begin{array}{r|l}
7x-4 & 2x+3 \\
\hline
 & 14x^2+13x-15 \\
 & 14x^2-8x \quad (+) \\
\hline
 & \quad 21x-15 \\
 & \quad 21x-12 \quad (+) \\
\hline
 & \quad (-3)
\end{array}
$$

$\therefore$ Quotient = $2x+3$; Remainder = -3.

$14x^2+13x-15 = (7x-4)(2x+3)-3$

$= 14x^2+21x-8x-12-3$

$= 14x^2+13x-15$.

$\therefore$ LHS = RHS.

(ii)

$$
\begin{array}{r|l}
3z-6 & 5z^2+\frac{10}{3}z+11 \\
\hline
 & 15z^3-20z^2+13z-12 \\
 & 15z^3-30z^2 \\
\hline
 & \quad 10z^2+13z \\
 & \quad 10z^2-20z \quad (+) \\
\hline
 & \quad 33z-12 \\
 & \quad 33z-66 \quad (+) \\
\hline
 & \quad (54)
\end{array}
$$

$\therefore$ $15z^3-20z^2+13z-12 = (3z-6)(5z^2+\frac{10z}{3}+11)+54$

$= 15z^3-20z^2+13z-66+54$

$= 15z^3-20z^2+13z-12$.

Solution - 21 :-

(iii)

$2y^2-6$ ← divisor

$6y^5-28y^3+3y^2+30y-9$ ← dividend

$$
\begin{array}{r|l}
 & 3y^3-5y+\frac{3}{2} \\
\hline
2y^2-6 & 6y^5+0-28y^3+3y^2+30y-9 \\
 & 6y^5+0-18y^3 \\
 & \quad + \\
\hline
 & \quad -10y^3+3y^2+30y \\
 & \quad -10y^3+0+30y \\
 & \quad + \quad - \\
\hline
 & \quad\quad 3y^2-9 \\
 & \quad\quad 3y^2-9 \\
 & \quad\quad - \quad + \\
\hline
 & \quad\quad (0)
\end{array}
$$

$$6y^5-28y^3+3y^2+30y-9 = (2y^2-6)(3y^3-5y+\tfrac{3}{2})+0$$

$$= 6y^5-10y^3+3y^2-18y^3+30y-9$$

$$= 6y^5-28y^3+3y^2+30y-9$$

(iv)

$$
\begin{array}{r|l}
 & -4x^3+2x^2-8x+30 \\
\hline
3x+7 & -12x^4-22x^3-10x^2+34x-75 \\
 & -12x^4-28x^3 \\
 & \quad + \quad + \\
\hline
 & \quad\quad 6x^3-10x^2+34x \\
 & \quad\quad 6x^3+14x^2+0 \\
 & \quad\quad - \quad - \quad - \\
\hline
 & \quad\quad\quad -24x^2+34x-75 \\
 & \quad\quad\quad -24x^2-56x \\
 & \quad\quad\quad + \quad + \\
\hline
 & \quad\quad\quad\quad 90x-75 \\
 & \quad\quad\quad\quad 90x+210 \\
 & \quad\quad\quad\quad - \quad - \\
\hline
 & \quad\quad\quad\quad (-285)
\end{array}
$$

Quotient = $-4x^3 + 2x^2 - 8x + 30$

$R = -285$

$\therefore \quad 34x - 22x^3 - 12x^4 - 10x^2 - 75 = (3x+7)(-4x^3 + 2x^2 - 8x + 30) - 285$

$= -12x^4 - 22x^3 - 10x^2 + 34x - 75.$

(v)

$3y - 2 \;\overline{)\; 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6}$

$5y^3 - 2y^2 + \frac{5}{3}y$

$15y^4 - 10y^3$

$-6y^3 + 9y^2$

$-6y^3 + 4y^2$

$5y^2 - \frac{10}{3}y + 6$

$5y^2 - \frac{10}{3}y + 0$

6

$\therefore \quad 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6 = (3y-2)(5y^3 - 2y^2 + \frac{5}{3}y) + 6$

$= 15y^4 - 6y^3 + 5y^2 - 10y^3 + 4y^2 - \frac{10y}{3} + 6$

$= 15y^4 - 16y^3 + 9y^2 - \frac{10}{3}y + 6.$

$\therefore$ LHS = RHS.

(vi)

$$
\begin{array}{r|l}
2y^2-y+1 & 2y+5 \\
\hline
 & 4y^3+8y^2+8y+7 \\
 & \underline{4y^3-2y^2+2y} \\
 & \quad 10y^2+6y+7 \\
 & \quad \underline{10y^2-5y+5} \\
 & \quad\quad 11y+2.
\end{array}
$$

$$(2y^2-y+1)(2y+5)+11y+2 = 4y^3+10y^2-2y^2-5y+8y+5+11y+2$$

$$= 4y^3+8y^2+8y+7.$$

(vii)

$$
\begin{array}{r|l}
2y^3+1 & 3y^2+2y+2 \\
\hline
 & 6y^5+4y^4+4y^3+7y^2+2y+6 \\
 & \underline{6y^5+0+0+3y^2} \\
 & \quad 4y^4+4y^3+4y^2+2y+6 \\
 & \quad \underline{4y^4+0+0+2y} \\
 & \quad\quad 4y^3+4y^2+2y+6 \\
 & \quad\quad \underline{4y^3+0+0+2} \\
 & \quad\quad\quad 4y^2+2y+4.
\end{array}
$$

$\therefore$ Quotient $= 4y^2+2y+4$; Divisor $= 2y^3+1$.

Remainder $= 4y^2+2y+4$.

$\Rightarrow 6y^5+4y^4+4y^3+7y^2+2y+6 = (2y^3+1)(4y^2+2y+4)+4y^2+2y+4$

$= 6y^5+4y^4+4y^3+7y^2+2y+6.$

Solution-22 :-

$Q = 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$

$3y-2 \;\big|\; 15y^4 + 16y^3 + \frac{103}{y} - 9y^2 - 6$

can be written as

$15y^4 + 16y^3 - 9y^2 + \frac{10}{3}y - 6$

$15y^4 \mp 10y^3$

$26y^3 - 9y^2$

$26y^3 - \frac{52}{3}y^2$

$\frac{25}{3}y^2 + \frac{10}{3}y$

$\frac{25y^2}{3} - \frac{50}{9}y$

$\frac{180}{9}y - 6$

$\frac{180}{9}y - 6$

(0)

$\therefore$ Quotient $= 5y^3 + \frac{26}{3}y^2 + \frac{25}{9}y + \frac{80}{27}$.

coefficient of $y^3 = 5$

" $y^2 = \frac{26}{3}$

" $y = \frac{25}{9}$.

constant term $= \frac{80}{27}$.

23. (i)

$$x+6 \;\overline{)\; x^2-x+42 \;(\; x-7}$$

$x^2 + 6x$

$-7x + 42$

$-7x - 42$

$+ \quad +$

0

Yes, $x+6$ is a factor of x^2-x-42.

(ii)

$$4x-1 \;\overline{)\; 4x^2-13x-12 \;(\; x-3}$$

$4x^2 - x$

$-12x - 12$

$-12x + 3$

$+ \quad -$

-15

$4x-1$ is not a factor of $4x^2-13x-12$.

23. (iii)

$$2y-5 \;\overline{)\; 4y^4-10y^3-10y^2+30y-15 \;(\; 2y^3+5y-3}$$

$4y^4 - 10y^3$

$-10y^2 + 30y$

$10y^2 - 25y$

$5y - 15$

$2y-5$ is not a factor of $4y^4-10y^3-10y^2+30y-15$

(iv)

$$3y^2+5 \;\overline{)\; 6y^5+15y^4+16y^3+4y^2+10y-35 \;(\; 2y^3+5y^2+2y-7}$$

$6y^5 + 10y^4 + 10y^3$

$15y^4 + 6y^3 + 4y^2$

$15y^4 + 0 + 25y^2$

$6y^3 - 21y^2 + 10y$

$6y^3 + 0 + 10y$

$-21y^2 - 35$

$-21y^2 - 35$

$+ \quad +$

0

$\therefore$ $3y^2+5$ is a factor of given polynamial.

(v)

$$z^2+3 \;\overline{)\; z^5-9z = z^5+0+0+0-9z+0}$$

$$z^3+3z$$

$$-z^5+0\pm 3z^3$$

$$-3z^3-9z$$

$$3z^3+9z$$

$$0$$

z^2+3 is a factor of polynomial z^5-9z.

(vi)

$$2x^2+x+3 \;\overline{)\; 6x^5-x^4+4x^3-5x^2-x-15}$$

$$3x^3+\square x^2-2x+5$$

$$6x^5-3x^4+9x^3$$

$$2x^4-5x^3-5x^2$$

$$2x^4-x^3+3x^2$$

$$-4x^3-8x^2-x$$

$$-4x^3+2x^2-6x$$

$$-10x^2+5x-15$$

$$10x^2-5x+15$$

$$0$$

$2x^2-x+3$ is factor of given polynomial.

25. x^2+2x-3

$$x^4+2x^3-2x^2+x+2$$

$$x^2+1$$

$$x^4+2x^3-3x^2$$

$$x^2+x+1$$

$$x^2+2x-3$$

$$-x+2$$

$x-2$ added to $x^4+2x^3-2x^2+x-1$ so that the resulting Polynomial exactly divisible by x^2+2x-3.

24. $x+2 \;\overline{)\; 4x^4+2x^3-3x^2+8x+5a}$

$$4x^3-6x^2+9x+10$$

$$4x^3+8x^3$$

$$-6x^3-3x^2$$

$$-6x^3+12x^2$$

$$9x^2+8x$$

$$-9x^2+18x$$

$$-10x+5a$$

$$10x+20$$

$$5a+20$$

$-10x+5a$

$5a+20=0$

$$a=\frac{-20}{5}$$

$$\boxed{a=-4}$$

Division Of Algebraic Expressions

Ex 8.5

Exercise - 8.5.

1. Divide the first polynomial by the second polynomial in each of the following. Also, write the quotient and the remainder:

(i) $3x^2+4x+5$, $x-2$.

we have,

$$3x^2+4x+5 = x(3x+10)-2(3x+10)+5+20$$
$$= (x-2)(3x+10)+20+5$$

Quotient $= 3x+10$, Remainder $= 25$.

(ii) $10x^2-7x+8$, $5x-3$

we have

$$10x^2-7x+8 = 5x\left(2x-\frac{1}{5}\right)-3\left(2x-\frac{1}{5}\right)+8-\frac{3}{5}$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{40-3}{5}$$
$$= (5x-3)\left(2x-\frac{1}{5}\right)+\frac{37}{5}$$

Quotient $= \left(2x-\frac{1}{5}\right)$, Remainder $= \frac{37}{5}$.

(iii) $5y^3-6y^2+6y-1$, $5y-1$.

$$5y^3-6y^2+6y-1 = 5y(y^2-y+1)-1(y^2-y+1)-1+1.$$
$$= (5y-1)(y^2-y+1)+0.$$

$\therefore$ Quotient $= y^2-y+1$, Remainder $= 0$.

② (iii) $4x^2-5$) $4x^4+0+7x^2+15$ ($2x^2-3$

$4x^4+0-5x^2$

$12x^2+15$

$-12x^2+15$

30

No, $4x^2-5$ is not a factor of $4x^4+7x^2+15$.

② (v) $2a-3$) $10a^2-9a-5$ ($5a+3$ $\quad$ $2a-5=5a+3$

$10a^2-9a-5$

$10a^2-15a$

$6a-5$

$6a-9$

4

No, $(2a-3)$ is not a factor of $10a^2-9a-5$

1>

(iv) x^4-x^3+5x, $x-1$.

We have,

$x^4-x^3+5x = x(x^3+5)-1(x^3+5)+5$

$= (x-1)(x^3+5)+5$.

∴ Quotient $= (x^3+5)$, Remainder $= 5$

(v) y^4+y^2, y^2-2.

$y^2(y^2-2)-2(y^2-2)+4$.

We have,

$y^4+y^2 = y^2(y^2+3)-2(y^2+3)+6$

$= (y^2-2)(y^2+3)+6$.

∴ Quotient $= y^2+3$, Remainder $= 6$.

②

(i) $x+1$) $2x^2+5x+4$ ($2x+3$

$2x^2+2x$

$3x+4$

$3x+3$

1.

NO, $(x+1)$ is not a factor of the second.

(ii) $y-2$) $3y^3+5y^2+5y+2$ ($3y^2+11y+27$

$3y^3-6y^2$

$11y^2+5y$

$11y^2-22y$

$27y+2$

$27y-54$

-52

No, $(y-2)$ is not a factor of the second.

(5)

(vi)

$4y+1$) $8y^2-2y+2$ ($2y+1$

$8y^2+2y$

$-4y+1$

$4y+1$

(2)

No, $4y+1$ is not a factor of given Polynomial

Division Of Algebraic Expressions
Ex 8.6

Exercise - 8.6.

1. x^2-5x+6 by $x-3$.

$$x^2-5x+6 = x^2-2x-3x+6$$
$$= x(x-2)-3(x-2)$$
$$= (x-3)(x-2)$$
$$\therefore \frac{x^2-5x+6}{(x-3)} = \frac{(x-3)(x-2)}{(x-3)} = x-2.$$

2. ax^2-ay^2 by $ax+ay$.

$$ax^2-ay^2 = a(x^2-y^2)$$
$$= a(x+y)(x-y)$$
$$\therefore \frac{ax^2-ay^2}{ax+ay} = \frac{a(x+y)(x-y)}{a(x+y)} = (x-y)$$

3. x^4-y^4 by x^2-y^2

$$x^4-y^4 = (x^2)^2-(y^2)^2 = (x^2+y^2)(x^2-y^2)$$
$$\therefore \frac{x^4-y^4}{x^2-y^2} = \frac{(x^2+y^2)(x^2-y^2)}{(x^2-y^2)} = (x^2+y^2).$$

4. $[acx^2+(bc+ad)x+bd]$ by $[ax+b]$

$$acx^2+(bc+ad)x+bd = (ax+b)(cx+d)$$

$$\therefore \frac{acx^2+(bc+ad)x+bd}{(ax+b)} = \frac{(ax+b)(cx+d)}{(ax+b)}$$

$$= cx+d.$$

5. $(a^2+2ab+b^2)-(a^2+2ac+c^2)$ by $2a+b+c$.

$$a^2+2ab+b^2-a^2-2ac-c^2 = (b^2-c^2)+2a(b-c)$$

$$= (b-c)(b+c)+2a(b-c)$$

$$= (b-c)(2a+b+c)$$

$$\therefore \frac{(a^2+2ab+b^2)-(a^2+2ac+c^2)}{(2a+b+c)} = \frac{(2a+b+c)(b-c)}{(2a+b+c)}$$

$$= b-c.$$

6. $\frac{1}{4}x^2-\frac{1}{2}x-12$ by $\frac{1}{2}x-4$.

$$\frac{1}{4}x^2-\frac{1}{2}x-12 = \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{1}{2}x+3\right)$$

$$= \frac{1}{2}x\left(\frac{1}{2}x+3\right)-4\left(\frac{x}{2}+3\right)$$

$$= \left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)$$

$$\therefore \frac{\frac{1}{4}x^2-\frac{1}{2}x-12}{\frac{x}{2}-4} = \frac{\left(\frac{x}{2}-4\right)\left(\frac{x}{2}+3\right)}{\frac{x}{2}-4} = \frac{x}{2}+3.$$