

Chapter 3 Matrices

EXERCISE 3.1

Question 1:

$$A = \begin{pmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{pmatrix},$$

In the matrix, write:

- (i) The order of the matrix
- (ii) The number of elements
- (iii) Write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$

Solution:

- (i) Since, in the given matrix, the number of rows is 3 and the number of columns is 4, the order of the matrix is 3×4 .
- (ii) Since the order of the matrix is 3×4 , there are $3 \times 4 = 12$ elements.
- (iii) Here,

$$a_{13} = 19$$

$$a_{21} = 35$$

$$a_{33} = -5$$

$$a_{24} = 12$$

$$a_{23} = \frac{5}{2}$$

Question 2:

If a matrix has 24 elements, what are the possible order it can have? What, if it has 13 elements?

Solution:

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.

The ordered pairs are: $(1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6)$ and $(6, 4)$.

Hence, the possible orders of a matrix having 24 elements are:

$$(1 \times 24), (24 \times 1), (2 \times 12), (12 \times 2), (3 \times 8), (8 \times 3), (4 \times 6) \text{ and } (6 \times 4).$$

$(1, 13)$ and $(13, 1)$ are the ordered pairs of natural numbers whose product is 13.

Hence, the possible orders of a matrix having 13 elements are (1×13) and (13×1) .

Question 3:

If a matrix has 18 elements, what are the possible order it can have? What, if it has 5 elements?

Solution:

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 18 elements, we have to find all the ordered pairs of natural numbers whose product is 18.

The ordered pairs are: $(1, 18), (18, 1), (2, 9), (9, 2), (3, 6)$ and $(6, 3)$.

Hence, the possible orders of a matrix having 18 elements are:

$(1 \times 18), (18 \times 1), (2 \times 9), (9 \times 2), (3 \times 6)$ and (6×3) .

(1×5) and (5×1) are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are (1×5) and (5×1) .

Question 4:

Construct a 2×2 matrix, $A = [a_{ij}]$, whose elements are given by:

(i) $a_{ij} = \frac{(i+j)^2}{2}$

(ii) $a_{ij} = \frac{i}{j}$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Solution:

In general, a 2×2 matrix is given by $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

(i) $a_{ij} = \frac{(i+j)^2}{2}; i, j = 1, 2$
Therefore,

$$a_{11} = \frac{(1+1)^2}{2} = \frac{4}{2} = 2$$

$$a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$$

$$a_{22} = \frac{(2+2)^2}{2} = \frac{16}{2} = 8$$

$$A = \begin{pmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{pmatrix}$$

Thus, the required matrix is

(ii) $a_{ij} = \frac{i}{j}; \quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{1}{1} = 1$$

$$a_{12} = \frac{1}{2}$$

$$a_{21} = \frac{2}{1} = 2$$

$$a_{22} = \frac{2}{2} = 1$$

$$A = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{pmatrix}$$

Thus, the required matrix is

(iii) $a_{ij} = \frac{(i+2j)^2}{2}; \quad i, j = 1, 2$

Therefore,

$$a_{11} = \frac{(1+2)^2}{2} = \frac{9}{2}$$

$$a_{12} = \frac{(1+4)^2}{2} = \frac{25}{2}$$

$$a_{21} = \frac{(2+2)^2}{2} = 8$$

$$a_{22} = \frac{(2+4)^2}{2} = 18$$

$$A = \begin{pmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{pmatrix}$$

Thus, the required matrix is

Question 5:

In general, a 3×4 matrix whose elements are given by

$$(i) \quad a_{ij} = \frac{1}{2}|-3i + j|$$

$$(ii) \quad a_{ij} = 2i - j$$

Solution:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$

In general, a 3×4 matrix is given by

$$(i) \quad \text{Given } a_{ij} = \frac{1}{2}|-3i + j|; \quad i = 1, 2, 3 \quad j = 1, 2, 3, 4$$

$$a_{11} = \frac{1}{2}|-3(1) + 1| = \frac{1}{2}|-3 + 1| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{21} = \frac{1}{2}|-3(2) + 1| = \frac{1}{2}|-6 + 1| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$a_{31} = \frac{1}{2}|-3(3) + 1| = \frac{1}{2}|-9 + 1| = \frac{1}{2}|-8| = \frac{8}{2} = 4$$

$$a_{12} = \frac{1}{2}|-3(1) + 2| = \frac{1}{2}|-3 + 2| = \frac{1}{2}|-1| = \frac{1}{2}$$

$$a_{22} = \frac{1}{2}|-3(2) + 2| = \frac{1}{2}|-6 + 2| = \frac{1}{2}|-4| = \frac{4}{2} = 2$$

$$a_{32} = \frac{1}{2}|-3(3) + 2| = \frac{1}{2}|-9 + 2| = \frac{1}{2}|-7| = \frac{7}{2}$$

$$a_{13} = \frac{1}{2}|-3(1) + 3| = \frac{1}{2}|-3 + 3| = 0$$

$$a_{23} = \frac{1}{2}|-3(2) + 3| = \frac{1}{2}|-6 + 3| = \frac{1}{2}|-3| = \frac{3}{2}$$

$$a_{33} = \frac{1}{2}|-3(3) + 3| = \frac{1}{2}|-9 + 3| = \frac{1}{2}|-6| = \frac{6}{2} = 3$$

$$a_{14} = \frac{1}{2}|-3(1) + 4| = \frac{1}{2}|-3 + 4| = \frac{1}{2}|1| = \frac{1}{2}$$

$$a_{24} = \frac{1}{2}|-3(2) + 4| = \frac{1}{2}|-6 + 4| = \frac{1}{2}|-2| = \frac{2}{2} = 1$$

$$a_{34} = \frac{1}{2}|-3(3) + 4| = \frac{1}{2}|-9 + 4| = \frac{1}{2}|-5| = \frac{5}{2}$$

$$A = \begin{pmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{pmatrix}$$

Thus, the required matrix is

(ii) $a_{ij} = 2i - j; i = 1, 2, 3 \quad j = 1, 2, 3, 4$

$$a_{11} = 2(1) - 1 = 2 - 1 = 1$$

$$a_{21} = 2(2) - 1 = 4 - 1 = 3$$

$$a_{31} = 2(3) - 1 = 6 - 1 = 5$$

$$a_{12} = 2(1) - 2 = 2 - 2 = 0$$

$$a_{22} = 2(2) - 2 = 4 - 2 = 2$$

$$a_{32} = 2(3) - 2 = 6 - 2 = 4$$

$$a_{13} = 2(1) - 3 = 2 - 3 = -1$$

$$a_{23} = 2(2) - 3 = 4 - 3 = 1$$

$$a_{33} = 2(3) - 3 = 6 - 3 = 3$$

$$a_{14} = 2(1) - 4 = 2 - 4 = -2$$

$$a_{24} = 2(2) - 4 = 4 - 4 = 0$$

$$a_{34} = 2(3) - 4 = 6 - 4 = 2$$

$$A = \begin{pmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{pmatrix}$$

Thus, the required matrix is

Question 6:

Find the value of x , y and z from the following equation:

(i) $\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$

(ii) $\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$

(iii) $\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$

Solution:

(i)
$$\begin{pmatrix} 4 & 3 \\ x & 5 \end{pmatrix} = \begin{pmatrix} y & z \\ 1 & 5 \end{pmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$x = 1, y = 4 \text{ and } z = 3$$

(ii)
$$\begin{pmatrix} x+y & 2 \\ 5+z & xy \end{pmatrix} = \begin{pmatrix} 6 & 2 \\ 5 & 8 \end{pmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$x + y = 6$$

$$xy = 8$$

$$5 + z = 5$$

Hence,

$$\Rightarrow 5 + z = 5$$

$$\Rightarrow z = 0$$

We know that $(a-b)^2 = (a+b)^2 - 4ab$

$$\Rightarrow (x-y)^2 = (6)^2 - 8 \times 4$$

$$\Rightarrow (x-y)^2 = 36 - 32$$

$$\Rightarrow (x-y)^2 = 4$$

$$\Rightarrow (x-y) = \pm 2$$

Equating $x - y = 2$ and $x + y = 6$, we get $x = 4, y = 2$

Similarly, Equating $x - y = -2$ and $x + y = 6$, we get $x = 2, y = 4$

Thus, $x = 4, y = 2, z = 0$ or $x = 2, y = 4, z = 0$

(iii)
$$\begin{pmatrix} x+y+z \\ x+z \\ y+z \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ 7 \end{pmatrix}$$

As the given matrices are equal, their corresponding elements are also equal.
Comparing the corresponding elements, we get:

$$x + y + z = 9 \quad \dots(1)$$

$$x + z = 5 \quad \dots(2)$$

$$y + z = 7 \quad \dots(3)$$

From (1) and (2), we have

$$\Rightarrow y + 5 = 9$$

$$\Rightarrow y = 4$$

From (3), we have

$$\Rightarrow 4 + z = 7$$

$$\Rightarrow z = 3$$

Therefore,

$$\Rightarrow x + z = 5$$

$$\Rightarrow x + 3 = 5$$

$$\Rightarrow x = 2$$

Thus, $x = 2, y = 4, z = 3$

Question 7:

Find the value of a, b, c and d from the equation:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

Solution:

$$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

As the two matrices are equal, their corresponding elements are also equal.

Comparing the corresponding elements, we get:

$$a - b = -1 \quad \dots(1)$$

$$2a - b = 0 \quad \dots(2)$$

$$2a + c = 5 \quad \dots(3)$$

$$3c + d = 13 \quad \dots(4)$$

From (2),

$$b = 2a$$

Putting this value in (1),

$$\Rightarrow a - 2a = -1$$

$$\Rightarrow a = 1$$

Hence,

$$\Rightarrow b = 2$$

Putting $a = 1$ in (3),
 $\Rightarrow 2(1) + c = 5$
 $\Rightarrow c = 3$

Putting $c = 3$ in (4),
 $\Rightarrow 3(3) + d = 13$
 $\Rightarrow d = 4$

Thus, $a = 1, b = 2, c = 3$ and $d = 4$.

Question 8:

$A = [a_{ij}]_{m \times n}$ is a square matrix, if

- (A) $m < n$ (B) $m > n$ (C) $m = n$ (D) None of these

Solution:

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns.

Therefore, $A = [a_{ij}]_{m \times n}$ is a square matrix, if $m = n$.

Thus, the correct option is C.

Question 9:

Which of the given values of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

- (A) $x = \frac{-1}{3}, y = 7$ (B) Not possible to find (C) $y = 7, x = \frac{-2}{3}$ (D) $x = \frac{-1}{3}, y = \frac{-2}{3}$

Solution:

The given matrices are $\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

Equating the corresponding elements, we get:

$$3x + 7 = 0 \Rightarrow x = \frac{-7}{3}$$

$$y - 2 = 5 \Rightarrow y = 7$$

$$y + 1 = 8 \Rightarrow y = 7$$

$$2 - 3x = 4 \Rightarrow x = \frac{-2}{3}$$

We find that on comparing the corresponding elements of the two matrices, we get two different values of x , which is not possible.

Hence, it is not possible to find the values of x and y for which the given matrices are equal.

Thus, the correct option is B.

Question 10:

The number of all possible matrices of order 3×3 with each entry 0 or 1 is:

- (A) 27 (B) 18 (C) 81 (D) 512

Solution:

The given matrix of the order 3×3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Hence, by the multiplication principle, the required number of possible matrices is $2^9 = 512$.

Thus, the correct option is D.

EXERCISE 3.2

Question 1:

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$. Find each of the following:

- (i) $A + B$
- (ii) $A - B$
- (iii) $3A - C$
- (iv) AB
- (v) BA

Solution:

- (i) $A + B$

$$\begin{aligned} &\Rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 3 & 7 \\ 1 & 7 \end{pmatrix} \end{aligned}$$

- (ii) $A - B$

$$\begin{aligned} &\Rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2-1 & 4-3 \\ 3+2 & 2-5 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 1 & 1 \\ 5 & -3 \end{pmatrix} \end{aligned}$$

- (iii) $3A - C$

$$\begin{aligned} &\Rightarrow 3 \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} - \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 3 \times 2 & 3 \times 4 \\ 3 \times 3 & 3 \times 2 \end{pmatrix} - \begin{pmatrix} -2 & 5 \\ 3 & 4 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 8 & 7 \\ 6 & 2 \end{pmatrix} \end{aligned}$$

(iv) AB

$$\begin{aligned} &\Rightarrow \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2(1)+4(-2) & 2(3)+4(5) \\ 3(1)+2(-2) & 3(3)+2(5) \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2-8 & 6+20 \\ 3-4 & 9+10 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} -6 & 26 \\ -1 & 19 \end{pmatrix} \end{aligned}$$

(v) BA

$$\begin{aligned} &\Rightarrow \begin{pmatrix} 1 & 3 \\ -2 & 5 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 3 & 2 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 1(2)+3(3) & 1(4)+3(2) \\ -2(2)+5(3) & -2(4)+5(2) \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2+9 & 4+6 \\ -4+15 & -8+10 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 11 & 10 \\ 11 & 2 \end{pmatrix} \end{aligned}$$

Question 2:

Compute the following:

(i) $\begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix}$

(ii) $\begin{pmatrix} a^2 + b^2 & b^2 + c^2 \\ a^2 + c^2 & a^2 + b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix}$

(iv) $\begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix}$

Solution:

$$\begin{aligned} \text{(i)} \quad & \begin{pmatrix} a & b \\ -b & a \end{pmatrix} + \begin{pmatrix} a & b \\ b & a \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} a+a & b+b \\ -b+b & a+a \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} 2a & 2b \\ 0 & 2a \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad & \begin{pmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{pmatrix} + \begin{pmatrix} 2ab & 2bc \\ -2ac & -2ab \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad & \begin{pmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{pmatrix} + \begin{pmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} -1+12 & 4+7 & -6+6 \\ 8+8 & 5+0 & 16+5 \\ 2+3 & 8+2 & 5+4 \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{pmatrix} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad & \begin{pmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{pmatrix} + \begin{pmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{pmatrix} \\ & \Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{aligned}$$

Question 3:

Compute the indicated products:

$$\text{(i)} \quad \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$$

$$(ii) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$$

$$(iii) \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$$

$$(iv) \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{pmatrix}$$

$$(v) \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix}$$

$$(vi) \begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix}$$

Solution:

$$(i) \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \\ \Rightarrow \begin{pmatrix} a(a)+b(b) & a(-b)+b(a) \\ -b(a)+a(b) & -b(-b)+a(a) \end{pmatrix} \\ \Rightarrow \begin{pmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{pmatrix} \\ \Rightarrow \begin{pmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{pmatrix}$$

$$(ii) \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \\ \Rightarrow \begin{pmatrix} 1(2) & 1(3) & 1(4) \\ 2(2) & 2(3) & 2(4) \\ 3(2) & 3(3) & 3(4) \end{pmatrix} \\ \Rightarrow \begin{pmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{pmatrix}$$

$$\begin{aligned}
 \text{(iii)} \quad & \begin{pmatrix} 1 & -2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 1(1)-2(2) & 1(2)-2(3) & 1(3)-2(1) \\ 2(1)+3(2) & 2(2)+3(3) & 2(3)+3(1) \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv)} \quad & \begin{pmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 2(1)+3(0)+4(3) & 2(-3)+3(2)+4(0) & 2(5)+3(4)+4(5) \\ 3(1)+4(0)+5(3) & 3(-3)+4(2)+5(0) & 3(5)+4(4)+5(5) \\ 4(1)+5(0)+6(3) & 4(-3)+5(2)+6(0) & 4(5)+5(4)+6(5) \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(v)} \quad & \begin{pmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 2(1)+1(-1) & 2(0)+1(2) & 2(1)+1(1) \\ 3(1)+2(-1) & 3(0)+2(2) & 3(1)+2(1) \\ -1(1)+1(-1) & -1(0)+1(2) & -1(1)+1(1) \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & \begin{pmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 3(2)-1(1)+3(3) & 3(-3)-1(0)+3(1) \\ -1(2)+0(1)+2(3) & -1(-3)+0(0)+2(1) \end{pmatrix} \\
 & \Rightarrow \begin{pmatrix} 14 & -6 \\ 4 & 5 \end{pmatrix}
 \end{aligned}$$

Question 4:

If $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$, $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$, then compute $(A+B)$ and $(B-C)$. Also, verify that $A+(B-C) = (A+B)-C$.

Solution:

$$\begin{aligned} (A+B) &= \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{pmatrix} \\ (B-C) &= \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix} - \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{pmatrix} \end{aligned}$$

Now,

$$\begin{aligned} A+(B-C) &= \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix} + \begin{pmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{pmatrix} \\ (A+B)-C &= \begin{pmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{pmatrix} - \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{pmatrix} \end{aligned}$$

Hence, $A+(B-C) = (A+B)-C$.

Question 5:

$$A = \begin{pmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{pmatrix} \quad B = \begin{pmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{pmatrix}$$

If $A = \begin{pmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{pmatrix}$ and $B = \begin{pmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{pmatrix}$, then compute $3A - 5B$.

Solution:

$$\begin{aligned} 3A - 5B &= 3 \begin{pmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{pmatrix} - 5 \begin{pmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{pmatrix} \\ &= \begin{pmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Question 6:

Simplify $\cos \theta \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} + \sin \theta \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix}$.

Solution:

$$\begin{aligned} &\cos \theta \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} + \sin \theta \begin{pmatrix} \sin \theta & -\cos \theta \\ \cos \theta & \sin \theta \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{pmatrix} + \begin{pmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} \cos^2 \theta + \sin^2 \theta & \sin \theta \cos \theta - \sin \theta \cos \theta \\ -\sin \theta \cos \theta + \sin \theta \cos \theta & \cos^2 \theta + \sin^2 \theta \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \end{aligned}$$

Question 7:

Find X and Y , if

$$(i) \quad X + Y = \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} \text{ and } X - Y = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$

$$(ii) \quad 2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \text{ and } 3X + 2Y = \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$$

Solution:

$$(i) \quad \begin{aligned} X + Y &= \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} & \dots(1) \\ X - Y &= \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} & \dots(2) \end{aligned}$$

Adding equations (1) and (2),

$$\begin{aligned} 2X &= \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} + \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 10 & 0 \\ 2 & 8 \end{pmatrix} \\ X &= \frac{1}{2} \begin{pmatrix} 10 & 0 \\ 2 & 8 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix} \end{aligned}$$

Now,

$$\begin{aligned} \Rightarrow X + Y &= \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} \\ \Rightarrow Y &= \begin{pmatrix} 7 & 0 \\ 2 & 5 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 1 & 4 \end{pmatrix} \\ \Rightarrow Y &= \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} \end{aligned}$$

$$(ii) \quad \begin{aligned} 2X + 3Y &= \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} & \dots(1) \\ 3X + 2Y &= \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix} & \dots(2) \end{aligned}$$

Multiplying equation (1) by 2,

$$\begin{aligned} 2(2X + 3Y) &= 2 \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \\ 4X + 6Y &= \begin{pmatrix} 4 & 6 \\ 8 & 0 \end{pmatrix} & \dots(3) \end{aligned}$$

Multiplying equation (2) by 3,

$$3(3X + 2Y) = 3 \begin{pmatrix} 2 & -2 \\ -1 & 5 \end{pmatrix}$$
$$9X + 6Y = \begin{pmatrix} 6 & -6 \\ -3 & 15 \end{pmatrix} \quad \dots(4)$$

From (3) and (4),

$$(4X + 6Y) - (9X + 6Y) = \begin{pmatrix} 4 & 6 \\ 8 & 0 \end{pmatrix} - \begin{pmatrix} 6 & -6 \\ -3 & 15 \end{pmatrix}$$
$$-5X = \begin{pmatrix} 4-6 & 6+6 \\ 8+3 & 0-15 \end{pmatrix}$$
$$-5X = \begin{pmatrix} -2 & 12 \\ 11 & -15 \end{pmatrix}$$
$$X = \frac{-1}{5} \begin{pmatrix} -2 & 12 \\ 11 & -15 \end{pmatrix}$$
$$= \begin{pmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{pmatrix}$$

Now

$$\begin{aligned}
&\Rightarrow 2X + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \\
&\Rightarrow 2 \begin{pmatrix} \frac{2}{5} & \frac{-12}{5} \\ \frac{-11}{5} & 3 \end{pmatrix} + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{pmatrix} + 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} \\
&\Rightarrow 3Y = \begin{pmatrix} 2 & 3 \\ 4 & 0 \end{pmatrix} - \begin{pmatrix} \frac{4}{5} & \frac{-24}{5} \\ \frac{-22}{5} & 6 \end{pmatrix} \\
&\Rightarrow 3Y = \begin{pmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{pmatrix} \\
&\Rightarrow Y = \frac{1}{3} \begin{pmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{pmatrix} \\
&\Rightarrow Y = \begin{pmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{pmatrix}
\end{aligned}$$

Question 8:

Find X , if $Y = \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$.

Solution:

$$2X + Y = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\Rightarrow 2X + \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix}$$

$$\Rightarrow 2X = \begin{pmatrix} 1 & 0 \\ -3 & 2 \end{pmatrix} - \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$$

$$\Rightarrow 2X = \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$$

$$\Rightarrow X = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$

Question 9:

Find x and y , if $2 \begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$.

Solution:

$$\Rightarrow 2 \begin{pmatrix} 1 & 3 \\ 0 & x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2 & 6 \\ 0 & 2x \end{pmatrix} + \begin{pmatrix} y & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2+y & 6 \\ 1 & 2x+2 \end{pmatrix} = \begin{pmatrix} 5 & 6 \\ 1 & 8 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2 + y = 5$$

$$\Rightarrow y = 3$$

$$2x + 2 = 8$$

$$\Rightarrow x = 3$$

Therefore, $x = 3$ and $y = 3$.

Question 10:

Solve the equation for x, y, z and t if $2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$.

Solution:

$$\Rightarrow 2\begin{pmatrix} x & z \\ y & t \end{pmatrix} + 3\begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = 3\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x & 2z \\ 2y & 2t \end{pmatrix} + \begin{pmatrix} 3 & -3 \\ 0 & 6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{pmatrix} = \begin{pmatrix} 9 & 15 \\ 12 & 18 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x + 3 = 9$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$

$$2y = 12$$

$$\Rightarrow y = 6$$

$$2z - 3 = 15$$

$$\Rightarrow 2z = 18$$

$$\Rightarrow z = 9$$

$$2t + 6 = 18$$

$$\Rightarrow 2t = 12$$

$$\Rightarrow t = 6$$

Therefore, $x = 3, y = 6, z = 9$ and $t = 6$.

Question 11:

If $x\begin{pmatrix} 2 \\ 3 \end{pmatrix} + y\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, find values of x and y .

Solution:

$$\Rightarrow x \begin{pmatrix} 2 \\ 3 \end{pmatrix} + y \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x \\ 3x \end{pmatrix} + \begin{pmatrix} -y \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 2x - y \\ 3x + y \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$2x - y = 10 \quad \dots(1)$$

$$3x + y = 5 \quad \dots(2)$$

By adding these two equations, we get

$$5x = 15$$

$$\Rightarrow x = 3$$

Now, putting this value in (2)

$$\Rightarrow 3x + y = 5$$

$$\Rightarrow y = 5 - 3x$$

$$\Rightarrow y = 5 - 3(3)$$

$$\Rightarrow y = 5 - 9$$

$$\Rightarrow y = -4$$

Therefore, $x = 3$ and $y = -4$.

Question 12:

Given $3 \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}$, find values of w, x, y and z .

Solution:

$$\Rightarrow 3 \begin{pmatrix} x & y \\ z & w \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2w \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+w & 3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3x & 3y \\ 3z & 3w \end{pmatrix} = \begin{pmatrix} x+4 & 6+x+y \\ -1+z+w & 2w+3 \end{pmatrix}$$

Comparing the corresponding elements of these two matrices,

$$\Rightarrow 3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$\Rightarrow 3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x$$

$$\Rightarrow 2y = 6 + 2$$

$$\Rightarrow 2y = 8$$

$$\Rightarrow y = 4$$

$$\Rightarrow 3w = 2w + 3$$

$$\Rightarrow w = 3$$

$$\Rightarrow 3z = -1 + z + w$$

$$\Rightarrow 2z = w - 1$$

$$\Rightarrow 2z = 3 - 1$$

$$\Rightarrow 2z = 2$$

$$\Rightarrow z = 1$$

Therefore, $x = 2, y = 4, z = 1$ and $w = 3$

Question 13:

If $F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, show that $F(x)F(y) = F(x+y)$.

Solution:

$$F(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

It is given that

$$F(y) = \begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then,

Now,

$$F(x+y) = \begin{pmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{aligned}
F(x)F(y) &= \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} \cos x \cos y - \sin x \sin y + 0 & -\cos x \sin y - \sin x \cos y + 0 & 0 \\ \sin x \cos y + \cos x \sin y + 0 & -\sin x \sin y + \cos x \cos y + 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
&= \begin{pmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
&= F(x+y)
\end{aligned}$$

Therefore, $F(x)F(y) = F(x+y)$

Question 14:

Show that

$$(i) \quad \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$$

$$(ii) \quad \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Solution:

$$(i) \quad \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$$

$$\begin{aligned}
\begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} &= \begin{pmatrix} 5(2) - 1(3) & 5(1) - 1(4) \\ 6(2) + 7(3) & 6(1) + 7(4) \end{pmatrix} \\
&= \begin{pmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{pmatrix} \\
&= \begin{pmatrix} 7 & 1 \\ 33 & 34 \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
 \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} &= \begin{pmatrix} 2(5)+1(6) & 2(-1)+1(7) \\ 3(5)+4(6) & 3(-1)+4(7) \end{pmatrix} \\
 &= \begin{pmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{pmatrix} \\
 &= \begin{pmatrix} 16 & 5 \\ 39 & 25 \end{pmatrix}
 \end{aligned}$$

$$\text{Thus, } \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \neq \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & 7 \end{pmatrix}$$

$$(ii) \quad \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

$$\begin{aligned}
 \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} &= \begin{pmatrix} 1(-1)+2(0)+3(2) & 1(1)+2(-1)+3(3) & 1(0)+2(1)+3(4) \\ 0(-1)+1(0)+0(2) & 0(1)+1(-1)+0(3) & 0(0)+1(1)+0(4) \\ 1(-1)+1(0)+0(2) & 1(1)+1(-1)+0(3) & 1(0)+1(1)+0(4) \end{pmatrix} \\
 &= \begin{pmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{pmatrix}
 \end{aligned}$$

$$\begin{aligned}
 \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} &= \begin{pmatrix} -1(1)+1(0)+0(1) & -1(2)+1(1)+0(1) & -1(3)+1(0)+0(0) \\ 0(1)+(-1)(0)+1(1) & 0(2)+(-1)(1)+1(1) & 0(3)+(-1)(0)+1(0) \\ 2(1)+3(0)+4(1) & 2(2)+3(1)+4(1) & 2(3)+3(0)+4(0) \end{pmatrix} \\
 &= \begin{pmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{pmatrix}
 \end{aligned}$$

$$\text{Thus, } \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \neq \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

Question 15:

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$$

Find $A^2 - 5A + 6I$, if

Solution:

$$A^2 = AA$$

$$\begin{aligned} &= \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 2(2)+0(2)+1(1) & 2(0)+0(1)+1(-1) & 2(1)+0(3)+1(0) \\ 2(2)+1(2)+3(1) & 2(0)+1(1)+1(1) & 2(1)+1(3)+3(0) \\ 1(2)+(-1)(2)+0(1) & 1(0)+(-1)(1)+0(-1) & 1(1)+(-1)(3)+0(0) \end{pmatrix} \\ &= \begin{pmatrix} 4+0+1 & 0+0-1 & 2+0+0 \\ 4+2+3 & 0+1-3 & 2+3+0 \\ 2-2+0 & 0-1+0 & 1-3+0 \end{pmatrix} \\ &= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} A^2 - 5A + 6I &= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - 5 \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} + 6 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 5-10 & -1-0 & 2-5 \\ 9-10 & -2-5 & 5-15 \\ 0-5 & -1+5 & -2-0 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \\ &= \begin{pmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{pmatrix} + \begin{pmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{pmatrix} \end{aligned}$$

Question 16:

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$.

Solution:

$$A^2 = A.A$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{pmatrix} \\ &= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} \end{aligned}$$

Now,

$$A^3 = A^2.A$$

$$\begin{aligned} &= \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{pmatrix} \\ &= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} \end{aligned}$$

Therefore,

$$\begin{aligned} A^3 - 6A^2 + 7A + 2I &= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - 6 \begin{pmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix} + 2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} + \begin{pmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{pmatrix} + \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 21+7+2 & 0+0+0 & 34+14+0 \\ 12+0+0 & 8+14+2 & 23+7+0 \\ 34+14+0 & 0+0+0 & 55+21+2 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
&= \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} - \begin{pmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{pmatrix} \\
&= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = 0
\end{aligned}$$

Hence, $A^3 - 6A^2 + 7A + 2I = 0$.

Question 17:

If $A = \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, find k so that $A^2 = kA - 2I$.

Solution:

$$\begin{aligned}
A^2 &= A.A \\
&= \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} \\
&= \begin{pmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{pmatrix} \\
&= \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix}
\end{aligned}$$

Now,

$$\begin{aligned}
&\Rightarrow A^2 = kA - 2I \\
&\Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix} = k \begin{pmatrix} 3 & -2 \\ 4 & -2 \end{pmatrix} - 2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 3k & -2k \\ 4k & -2k \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} 1 & -2 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{pmatrix}
\end{aligned}$$

Comparing the corresponding elements, we have:

$$3k - 2 = 1$$

$$\Rightarrow 3k = 3$$

$$\Rightarrow k = 1$$

Therefore, the value of $k = 1$.

Question 18:

If $A = \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix}$ and I is the identity matrix of order 2, show that

$$I + A = (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$
Solution:

$$LHS = I + A$$

$$\begin{aligned} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{pmatrix} \quad \dots(1) \end{aligned}$$

$$\begin{aligned}
RHS &= (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \\
&= \begin{pmatrix} 1 & \tan \frac{\alpha}{2} \\ -\tan \frac{\alpha}{2} & 1 \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \\
&= \begin{pmatrix} \cos \alpha + \sin \alpha \tan \frac{\alpha}{2} & -\sin \alpha + \cos \alpha \tan \frac{\alpha}{2} \\ -\cos \alpha \tan \frac{\alpha}{2} + \sin \alpha & \sin \alpha \tan \frac{\alpha}{2} + \cos \alpha \end{pmatrix} \\
&= \begin{pmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} \\ -\left(2\cos^2 \frac{\alpha}{2} - 1\right) \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} \tan \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{pmatrix} \\
&= \begin{pmatrix} 1 - 2\sin^2 \frac{\alpha}{2} + 2\sin^2 \frac{\alpha}{2} & -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} - \tan \frac{\alpha}{2} \\ -2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \tan \frac{\alpha}{2} + 2\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} & 2\sin^2 \frac{\alpha}{2} + 1 - 2\sin^2 \frac{\alpha}{2} \end{pmatrix} \\
&= \begin{pmatrix} 1 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 1 \end{pmatrix} \quad \dots (2)
\end{aligned}$$

Thus, from (1) and (2), we get

$$I + A = (I - A) \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

Question 19:

A trust fund has ₹30000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide ₹30000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

- (i) ₹ 1800
- (ii) ₹ 2000

Solution:

- (i) Let ₹ x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹ $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of ₹1800, we have:

$$\begin{aligned} & \left[\begin{matrix} x & (30000 - x) \end{matrix} \right] \left[\begin{matrix} \frac{5}{100} \\ \frac{7}{100} \end{matrix} \right] = 1800 & \left[\text{S.I for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right] \\ \Rightarrow & \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800 \\ \Rightarrow & 5x + 210000 - 7x = 180000 \\ \Rightarrow & 210000 - 2x = 180000 \\ \Rightarrow & 2x = 210000 - 180000 \\ \Rightarrow & 2x = 30000 \\ \Rightarrow & x = 15000 \end{aligned}$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹15000 in the first bond and the remaining ₹15000 in the second bond.

- (ii) Let ₹ x be invested in the first bond. Then, the sum of money invested in the second bond will be ₹ $(30000 - x)$.

Therefore, in order to obtain an annual total interest of ₹2000, we have:

$$\begin{aligned} & \left[\begin{matrix} x & (30000 - x) \end{matrix} \right] \left[\begin{matrix} \frac{5}{100} \\ \frac{7}{100} \end{matrix} \right] = 2000 \\ \Rightarrow & \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000 \\ \Rightarrow & 5x + 210000 - 7x = 200000 \\ \Rightarrow & 210000 - 2x = 200000 \\ \Rightarrow & 2x = 210000 - 200000 \\ \Rightarrow & 2x = 10000 \\ \Rightarrow & x = 5000 \end{aligned}$$

Thus, in order to obtain an annual total interest of ₹1800, the trust fund should invest ₹5000 in the first bond and the remaining ₹25000 in the second bond.

Question 20:

The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are ₹ 80, ₹ 60 and ₹ 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Solution:

The total amount of money that will be received from the sale of all these books can be represented in the form of a matrix as:

$$\begin{aligned} 12[10 \quad 8 \quad 10] \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix} &= 12[10(80) + 8(60) + 10(40)] \\ &= 12(800 + 480 + 400) \\ &= 12(1680) \\ &= 20160 \end{aligned}$$

Thus, the bookshop will receive ₹ 20160 from the sale of all these books.

Question 21:

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively.

The restriction on n, k and p so that $PY + WY$ will be defined are:

- (A) $k = 3, p = n$ (B) k is arbitrary, $p = 2$
 (C) p is arbitrary, $k = 3$ (D) $k = 2, p = 3$

Solution:

Matrices P and Y are of the orders $p \times k$ and $3 \times k$ respectively.

Therefore, matrix PY will be defined if $k = 3$.

Consequently, PY will be of the order $p \times k$.

Matrices W and Y are of the orders $n \times 3$ and $3 \times k$ respectively.

Since the number of columns in W is equal to the number of rows in Y , matrix WY is well-defined and is of the order $n \times k$.

Matrices PY and WY can be added only when their orders are the same.

However, PY is of the order $p \times k$ and WY is of the order $n \times k$.

Therefore, we must have $p = n$.

Thus, $k = 3$ and $p = n$ are the restrictions on n, k and p so that $PY + WY$ will be defined.

The correct option is A.

Question 22:

Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$ respectively.

If $n = p$, then the order of the matrix $7X - 5Z$ is:

- (A) $p \times 2$ (B) $2 \times n$
(C) $n \times 3$ (D) $p \times n$

Solution:

Matrix X is of the order $2 \times n$.

Therefore, matrix $7X$ is also of the same order.

Matrix Z is of the order $2 \times p$, i.e., $2 \times n$ [Since $n = p$]

Therefore, matrix $5Z$ is also of the same order.

Now, both the matrices $7X$ and $5Z$ are of the order $2 \times n$.

Thus, matrix $7X - 5Z$ is well-defined and is of the order $2 \times n$.

The correct option is B.

EXERCISE 3.3

Question 1:

Find the transpose of each of the following matrices:

(i) $\begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

Solution:

(i) Let $A = \begin{pmatrix} 5 \\ \frac{1}{2} \\ -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 5 & \frac{1}{2} & -1 \end{pmatrix}$

(ii) Let $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$

Then $A^T = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$

(iii) Let $A = \begin{pmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{pmatrix}$

Then $A^T = \begin{pmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{pmatrix}$

Question 2:

If $A = \begin{pmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$, then verify that

(i) $(A+B)' = A' + B'$

(ii) $(A-B)' = A' - B'$

Solution:

It is given that $A = \begin{pmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix}$

Hence, we have $A' = \begin{pmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{pmatrix}$ and $B' = \begin{pmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{pmatrix}$

(i) $(A+B) = \begin{pmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{pmatrix} + \begin{pmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{pmatrix}$

Hence,

$$(A+B)' = \begin{pmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{pmatrix}$$

Now,

$$\begin{aligned} A' + B' &= \begin{pmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{pmatrix} + \begin{pmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{pmatrix} \end{aligned}$$

Thus, $(A+B)' = A' + B'$.

$$(ii) \quad (A-B) = \begin{pmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{pmatrix} - \begin{pmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{pmatrix}$$

Hence,

$$(A-B)' = \begin{pmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{pmatrix}$$

Now,

$$\begin{aligned} A' - B' &= \begin{pmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{pmatrix} - \begin{pmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{pmatrix} \end{aligned}$$

Thus, $(A-B)' = A' - B'$.

Question 3:

If $A' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, then verify that

$$(i) \quad (A+B)' = A' + B'$$

$$(ii) \quad (A-B)' = A' - B'$$

Solution:

It is known that $A = (A')'$

Hence,

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} \text{ and } B' = \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix}$$

$$(i) \quad A+B = \begin{pmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{pmatrix}$$

Therefore,

$$(A+B)' = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

Now,

$$A'+B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{pmatrix}$$

$$\text{Hence, } (A+B)' = A'+B'.$$

$$(ii) \quad A-B = \begin{pmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{pmatrix}$$

Therefore,

$$(A-B)' = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

Now,

$$A'-B' = \begin{pmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{pmatrix}$$

$$\text{Hence, } (A-B)' = A'-B'.$$

Question 4:

If $A' = \begin{pmatrix} -2 & 3 \\ 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix}$, then find $(A+2B)'$.

Solution:

It is known that $A = (A')'$.

Therefore,

$$A = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$$

Now,

$$\begin{aligned}
 A+2B &= \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} + 2 \begin{pmatrix} -1 & 0 \\ 1 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 2 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} -4 & 1 \\ 5 & 6 \end{pmatrix}
 \end{aligned}$$

Question 5:

For the matrices A and B , verify that $(AB)' = B'A'$ where

(i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}, B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Solution:

(i) It is given that $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$
Hence,

$$\begin{aligned}
 AB &= \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}
 \end{aligned}$$

Therefore,

$$(AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}$$

Now,

$$A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} \text{ and } B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Hence,

$$\begin{aligned}
 B'A' &= \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix}
 \end{aligned}$$

Thus, $(AB)' = B'A'$

(ii) It is given that $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$
Hence,

$$\begin{aligned}
 AB &= \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}
 \end{aligned}$$

Therefore,

$$(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$$

Now,

$$A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \text{ and } B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

Therefore,

$$\begin{aligned}
 B'A' &= \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}
 \end{aligned}$$

Thus, $(AB)' = B'A'$.

Question 6:

If

(i) $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$, then verify $A'A = I$

(ii) $A = \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix}$, then verify $A'A = I$

Solution:

(i) It is given that $A = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$
Therefore,

$$A' = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$$

Now,

$$\begin{aligned} A'A &= \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \\ &= \begin{pmatrix} \cos \alpha \cos \alpha + (-\sin \alpha)(-\sin \alpha) & \sin \alpha \cos \alpha + (-\sin \alpha) \cos \alpha \\ \sin \alpha \cos \alpha + \cos \alpha (-\sin \alpha) & \sin \alpha \sin \alpha + \cos \alpha \cos \alpha \end{pmatrix} \\ &= \begin{pmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ &= I \end{aligned}$$

Thus, $A'A = I$

(ii) It is given that $A = \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix}$
Therefore,

$$A' = \begin{pmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix}$$

Now,

$$\begin{aligned}
A'A &= \begin{pmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{pmatrix} \begin{pmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{pmatrix} \\
&= \begin{pmatrix} \sin \alpha \sin \alpha + (-\cos \alpha)(-\cos \alpha) & \sin \alpha \cos \alpha + (-\cos \alpha) \sin \alpha \\ \sin \alpha \cos \alpha + \sin \alpha (-\cos \alpha) & \sin \alpha \sin \alpha + \cos \alpha \cos \alpha \end{pmatrix} \\
&= \begin{pmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= I
\end{aligned}$$

Thus, $A'A = I$

Question 7:

(i) Show that the matrix $A = \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$ is a symmetric matrix.

(ii) Show that the matrix $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$ is a skew symmetric matrix.

Solution:

(i) $A = \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$

Now,

$$\begin{aligned}
A' &= \begin{pmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix} \\
&= A
\end{aligned}$$

Hence, A is a symmetric matrix.

(ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

$$\begin{aligned}
 A' &= \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \\
 &= -\begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \\
 &= -A
 \end{aligned}$$

Hence, A is a skew symmetric matrix.

Question 8:

For the matrix $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$, verify that

- (i) $(A + A')$ is a symmetric matrix.
- (ii) $(A - A')$ is a skew symmetric matrix.

Solution:

It is given that $A = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix}$

Hence, $A' = \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix}$

$$(i) \quad (A + A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} + \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix}$$

Therefore,

$$\begin{aligned}
 (A + A')' &= \begin{pmatrix} 2 & 11 \\ 11 & 14 \end{pmatrix} \\
 &= (A + A')
 \end{aligned}$$

Thus, $(A + A')$ is a symmetric matrix.

$$(ii) \quad (A - A') = \begin{pmatrix} 1 & 5 \\ 6 & 7 \end{pmatrix} - \begin{pmatrix} 1 & 6 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

Therefore,

$$\begin{aligned}
 (A - A')' &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\
 &= -\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \\
 &= -(A - A')
 \end{aligned}$$

Thus, $(A - A')$ is a skew symmetric matrix.

Question 9:

Find $\frac{1}{2}(A + A')$ and $\frac{1}{2}(A - A')$, when $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$.

Solution:

$$A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$$

It is given that

Hence,

$$A' = \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix}$$

Now,

$$\begin{aligned} (A + A') &= \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} + \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned}$$

Therefore,

$$\frac{1}{2}(A + A') = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Now,

$$\begin{aligned} (A - A') &= \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix} - \begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{pmatrix} \end{aligned}$$

Thus,

$$\begin{aligned}\frac{1}{2}(A - A') &= \begin{pmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}\end{aligned}$$

Question 10:

Express the following as the sum of a symmetric and skew symmetric matrix:

(i) $\begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$

(ii) $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$

(iii) $\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$

(iv) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$

Solution:

(i) Let $A = \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix}$

Hence,

$$A' = \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix}$$

Now,

$$\begin{aligned}(A + A') &= \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix}\end{aligned}$$

Let

$$\begin{aligned}
 P &= \frac{1}{2}(A + A') \\
 &= \frac{1}{2} \begin{pmatrix} 6 & 6 \\ 6 & -2 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 P' &= \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix} \\
 &= P
 \end{aligned}$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,

$$\begin{aligned}
 (A - A') &= \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} - \begin{pmatrix} 3 & 1 \\ 5 & -1 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix}
 \end{aligned}$$

Let

$$\begin{aligned}
 Q &= \frac{1}{2}(A - A') \\
 &= \frac{1}{2} \begin{pmatrix} 0 & 4 \\ -4 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 Q' &= \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \\
 &= -Q
 \end{aligned}$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q :

$$\begin{aligned}
 P+Q &= \begin{pmatrix} 3 & 3 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 5 \\ 1 & -1 \end{pmatrix} \\
 &= A
 \end{aligned}$$

(ii) Let
$$A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

Hence,

$$A' = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

Now,

$$(A+A') = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{pmatrix}$$

Let

$$\begin{aligned}
 P &= \frac{1}{2}(A+A') \\
 &= \frac{1}{2} \begin{pmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{pmatrix} \\
 &= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 P' &= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} \\
 &= P
 \end{aligned}$$

Thus, $P = \frac{1}{2}(A+A')$ is a symmetric matrix.

Now,

$$\begin{aligned}
 (A - A') &= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} - \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
 \end{aligned}$$

Let

$$\begin{aligned}
 Q &= \frac{1}{2}(A - A') \\
 &= \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 Q' &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 &= -Q
 \end{aligned}$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q :

$$\begin{aligned}
 P + Q &= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix} \\
 &= A
 \end{aligned}$$

$$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

(iii) Let

Hence,

$$A' = \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix}$$

Now,

$$\begin{aligned} (A + A') &= \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} + \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} \end{aligned}$$

Let

$$\begin{aligned} P &= \frac{1}{2}(A + A') \\ &= \frac{1}{2} \begin{pmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{pmatrix} \\ &= \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix} \end{aligned}$$

Now,

$$\begin{aligned} P' &= \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix} \\ &= P \end{aligned}$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,

$$\begin{aligned}
 (A - A') &= \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} - \begin{pmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix}
 \end{aligned}$$

Let

$$\begin{aligned}
 Q &= \frac{1}{2}(A - A') \\
 &= \frac{1}{2} \begin{pmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 Q' &= \begin{pmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{pmatrix} \\
 &= -Q
 \end{aligned}$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q :

$$\begin{aligned}
 P+Q &= \begin{pmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{pmatrix} \\
 &= \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix} \\
 &= A
 \end{aligned}$$

(iv) Let $A = \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}$
Hence,

$$A' = \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix}$$

Now,

$$\begin{aligned}
 (A + A') &= \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix} \\
 &= \begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix}
 \end{aligned}$$

Let

$$\begin{aligned}
 P &= \frac{1}{2}(A + A') \\
 &= \frac{1}{2} \begin{pmatrix} 2 & 4 \\ 4 & 4 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}
 \end{aligned}$$

Now,

$$\begin{aligned}
 P' &= \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \\
 &= P
 \end{aligned}$$

Thus, $P = \frac{1}{2}(A + A')$ is a symmetric matrix.

Now,

$$\begin{aligned}(A - A') &= \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ 5 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix}\end{aligned}$$

Let

$$\begin{aligned}Q &= \frac{1}{2}(A - A') \\ &= \frac{1}{2} \begin{pmatrix} 0 & 6 \\ -6 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix}\end{aligned}$$

Now,

$$\begin{aligned}Q' &= \begin{pmatrix} 0 & -3 \\ 3 & 0 \end{pmatrix} \\ &= -Q\end{aligned}$$

Thus, $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

Representing A as the sum of P and Q :

$$\begin{aligned}P + Q &= \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -3 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix} \\ &= A\end{aligned}$$

Question 11:

If A, B are symmetric matrices of the same order, then $AB - BA$ is a

- | | |
|---------------------------|----------------------|
| (A) Skew symmetric matrix | (B) Symmetric matrix |
| (C) Zero matrix | (D) Identity matrix |

Solution:

If A and B are symmetric matrices of the same order, then

$$A' = A \text{ and } B' = B \quad \dots(1)$$

Now consider,

$$\begin{aligned}
 (AB - BA)' &= (AB)' - (BA)' && \left[\because (A - B)' = A' - B' \right] \\
 &= B'A' - A'B && \left[\because (AB)' = B'A' \right] \\
 &= BA - AB && [from (1)] \\
 &= -(AB - BA)
 \end{aligned}$$

Therefore,

$$(AB - BA)' = -(AB - BA)$$

Thus, $AB - BA$ is a skew symmetric matrix.

The Correct option is A.

Question 12:

If $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, then $A + A' = I$, if the value of α is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) π

(D) $\frac{3\pi}{2}$

Solution:

It is given that $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$

Hence,

$$A' = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$$

Now,

$$A + A' = I$$

Therefore,

$$\begin{aligned}
 \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} + \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
 \begin{pmatrix} 2\cos \alpha & 0 \\ 0 & 2\cos \alpha \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
 \end{aligned}$$

Comparing the corresponding elements of the two matrices, we have:

$$\Rightarrow 2 \cos \alpha = 1$$

$$\Rightarrow \cos \alpha = \frac{1}{2}$$

$$\Rightarrow \alpha = \cos^{-1} \frac{1}{2}$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

Thus, the correct option is B.

EXERCISE 3.4

Question 1:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$, if exists.

Solution:

$$\text{Let } A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}$$

We know that $A = IA$

Therefore,

$$\begin{aligned} \Rightarrow \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\ \Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 5 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A & (R_2 \rightarrow R_2 - 2R_1) \\ \Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ -2 & \frac{1}{5} \end{pmatrix} A & (R_2 \rightarrow \frac{1}{5}R_2) \\ \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -2 & \frac{1}{5} \end{pmatrix} A & (R_1 \rightarrow R_1 + R_2) \\ \Rightarrow A^{-1} &= \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -2 & \frac{1}{5} \end{pmatrix} \end{aligned}$$

Question 2:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$, if exists.

Solution:

$$\text{Let } A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - R_2) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} A & (R_2 \rightarrow R_2 - R_1) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}
\end{aligned}$$

Question 3:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} A & (R_2 \rightarrow R_2 - 2R_1) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - 3R_2) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 7 & -3 \\ -2 & 1 \end{pmatrix}
\end{aligned}$$

Question 4:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned} \Rightarrow \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\ \Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 5 & 7 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A & \left(R_1 \rightarrow \frac{1}{2} R_1 \right) \\ \Rightarrow \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & \frac{-1}{2} \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A & (R_2 \rightarrow R_2 - 5R_1) \\ \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{-1}{2} \end{pmatrix} &= \begin{pmatrix} -7 & 3 \\ 0 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 + 3R_2) \\ \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} A & (R_2 \rightarrow -2R_1) \\ \Rightarrow A^{-1} &= \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix} \end{aligned}$$

Question 5:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix}$

We know that $AA^{-1} = I$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & 1 \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 7 & 4 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A & \left(R_1 \rightarrow \frac{1}{2} R_1 \right) \\
&\Rightarrow \begin{pmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ -7 & 1 \end{pmatrix} A & (R_2 \rightarrow R_2 - 7R_1) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - R_2) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix} A & (R_2 \rightarrow 2R_1) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 4 & -1 \\ -7 & 2 \end{pmatrix}
\end{aligned}$$

Question 6:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$

We know that $AA^{-1} = I$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{pmatrix} A & \left(R_1 \rightarrow \frac{1}{2} R_1 \right) \\
&\Rightarrow \begin{pmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{pmatrix} A & (R_2 \rightarrow R_2 - R_1) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -\frac{1}{2} & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - 5R_2) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix} A & (R_2 \rightarrow 2R_2) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 3 & -5 \\ -1 & 2 \end{pmatrix}
\end{aligned}$$

Question 7:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$

We know that $AA^{-1} = I$

Therefore,

$$\begin{aligned}
 &\Rightarrow \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
 &\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} & (C_1 \rightarrow C_1 - 2C_2) \\
 &\Rightarrow \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = A \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} & (C_2 \rightarrow C_2 - C_1) \\
 &\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix} & (C_1 \rightarrow C_1 - C_2) \\
 &\Rightarrow A^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}
 \end{aligned}$$

Question 8:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
 &\Rightarrow \begin{pmatrix} 4 & 5 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
 &\Rightarrow \begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - R_2) \\
 &\Rightarrow \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -3 & 4 \end{pmatrix} A & (R_2 \rightarrow R_2 - 3R_1) \\
 &\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix} A & (R_1 \rightarrow R_1 - R_2) \\
 &\Rightarrow A^{-1} = \begin{pmatrix} 4 & -5 \\ -3 & 4 \end{pmatrix}
 \end{aligned}$$

Question 9:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned} \Rightarrow \begin{pmatrix} 3 & 10 \\ 2 & 7 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\ \Rightarrow \begin{pmatrix} 1 & 3 \\ 2 & 7 \end{pmatrix} &= \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 - R_2) \\ \Rightarrow \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix} A & (R_2 \rightarrow R_2 - 2R_1) \\ \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix} A & (R_1 \rightarrow R_1 - 3R_2) \\ \Rightarrow A^{-1} &= \begin{pmatrix} 7 & -10 \\ -2 & 3 \end{pmatrix} \end{aligned}$$

Question 10:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 3 & -1 \\ -4 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} & (C_1 \rightarrow C_1 + 2C_2) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} = A \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} & (C_2 \rightarrow C_2 + C_1) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix} & \left(C_2 \rightarrow \frac{1}{2}C_2 \right) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{pmatrix}
\end{aligned}$$

Question 11:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix} = A \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&\Rightarrow \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix} = A \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} & (C_2 \rightarrow C_2 + 3C_1) \\
&\Rightarrow \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = A \begin{pmatrix} -2 & 3 \\ -1 & 1 \end{pmatrix} & (C_1 \rightarrow C_1 - C_2) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = A \begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix} & \left(C_1 \rightarrow \frac{1}{2}C_1 \right) \\
&\Rightarrow A^{-1} = \begin{pmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{pmatrix}
\end{aligned}$$

Question 12:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned} \Rightarrow \begin{pmatrix} 6 & -3 \\ -2 & 1 \end{pmatrix} &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\ \Rightarrow \begin{pmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{pmatrix} &= \begin{pmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{pmatrix} A & \quad \left(R_1 \rightarrow \frac{1}{6} R_1 \right) \\ \Rightarrow \begin{pmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{pmatrix} &= \begin{pmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{pmatrix} A & \quad (R_2 \rightarrow R_2 + 2R_1) \end{aligned}$$

In the above equation, we can see all the zeros in the second row of the matrix on the L.H.S.

Thus, A^{-1} does not exist.

Question 13:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} A & (R_1 \rightarrow R_1 + R_2) \\
&\Rightarrow \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} A & (R_2 \rightarrow R_2 + R_1) \\
&\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} A & (R_1 \rightarrow R_1 + R_2) \\
&\Rightarrow A^{-1} = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix}
\end{aligned}$$

Question 14:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$, if exists.

Solution:

Let $A = \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix}$

We know that $A = IA$

Therefore,

$$\begin{aligned}
&\Rightarrow \begin{pmatrix} 2 & 1 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} A \\
&\Rightarrow \begin{pmatrix} 0 & 0 \\ 4 & 2 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-1}{2} \\ 0 & 1 \end{pmatrix} A & \left(R_1 \rightarrow R_1 - \frac{1}{2} R_2 \right)
\end{aligned}$$

In the above equation, we can see all the zeros in the first row of the matrix on the L.H.S.

Thus, A^{-1} does not exist.

Question 15:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$, if exists.

Solution:

$$A = \begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix}$$

Let

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad (R_2 \rightarrow R_2 - R_1)$$

$$\Rightarrow \begin{pmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad \left(R_2 \rightarrow \frac{1}{5} R_2 \right)$$

$$\Rightarrow \begin{pmatrix} -1 & -1 & 1 \\ 0 & 1 & 0 \\ 3 & -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 2 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -1 & 1 & 0 \\ -\frac{2}{5} & \frac{2}{5} & 1 \end{pmatrix} A \quad (R_1 \rightarrow R_1 + R_2 \text{ and } R_3 \rightarrow R_3 + 2R_2)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 5 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -1 & 1 & 0 \\ 2 & 1 & -2 \end{pmatrix} A \quad (R_3 \rightarrow R_3 + 3R_1)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} & \frac{1}{5} & -1 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \quad \left(R_3 \rightarrow \frac{1}{5}R_3 \right)$$

$$\Rightarrow \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{2}{5} & 0 & \frac{-3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \quad (R_1 \rightarrow R_1 - R_3)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix} A \quad (R_1 \rightarrow (-1)R_1)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} \frac{-2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & \frac{-2}{5} \end{pmatrix}$$

Question 16:

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$, if exists.

Solution:

$$A = \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix}$$

Let

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} A$$

$$(R_2 \rightarrow R_2 + 3R_1 \text{ and } R_3 \rightarrow R_3 - 2R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{pmatrix} A$$

$$(R_1 \rightarrow R_1 + 3R_3 \text{ and } R_2 \rightarrow R_2 + 8R_3)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{pmatrix} A$$

$$(R_3 \rightarrow R_3 + R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 10 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -5 & 0 & 3 \\ -13 & 1 & 8 \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A$$

$$\left(R_3 \rightarrow \frac{1}{25} R_3 \right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix} A$$

$$(R_1 \rightarrow R_1 - 10R_3 \text{ and } R_2 \rightarrow R_2 - 21R_3)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 1 & \frac{-2}{5} & \frac{-3}{5} \\ \frac{-2}{5} & \frac{4}{25} & \frac{11}{25} \\ \frac{-3}{5} & \frac{1}{25} & \frac{9}{25} \end{pmatrix}$$

Question 17:

$$\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

Using elementary transformation, Find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$, if exists.

Solution:

$$A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$

Let

We know that $A = IA$

Therefore,

$$\Rightarrow \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad \left(R_1 \rightarrow \frac{1}{2}R_1 \right)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A \quad (R_2 \rightarrow R_2 - 5R_1)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{pmatrix} A \quad (R_3 \rightarrow R_3 - R_2)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{pmatrix} A \quad (R_3 \rightarrow 2R_3)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} A \quad \left(R_1 \rightarrow R_1 + \frac{1}{2}R_3 \text{ and } R_2 \rightarrow R_2 - \frac{5}{2}R_3 \right)$$

$$\Rightarrow A^{-1} = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

Question 18:

Matrices A and B will be the inverse of each other only if:

- (A) $AB = BA$ (B) $AB = BA = 0$
(C) $AB = 0, BA = I$ (D) $AB = BA = I$

Solution:

We know that if A is a square matrix of order m , and if there exists another square matrix B of the same order m , such that $AB = BA = I$, then B is said to be the inverse of A .

In this case, it is clear that A is the inverse of B .

Thus, matrices A and B will be inverses of each other only if $AB = BA = I$.

The correct option is D.

MISCELLANEOUS EXERCISE

Question 1:

Let $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, show that $(aI + bA)^n = a^n I + na^{n-1}bA$, where I is the identity matrix of order 2 and $n \in \mathbb{N}$.

Solution:

It is given that $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): (aI + bA) = aI + ba^0 A = aI + bA$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$

That is, $P(k): (aI + bA)^k = a^k I + ka^{k-1}bA$

Now, we have to prove that the result is true for $n = k + 1$.

Consider,

$$\begin{aligned} (aI + bA)^{k+1} &= (aI + bA)^k (aI + bA) \\ &= (a^k I + ka^{k-1}bA)(aI + bA) \\ &= a^{k+1} I + ka^k bAI + a^k bIA + ka^{k-1}b^2 A^2 \\ &= a^{k+1} I + (k+1)a^k bA + ka^{k-1}b^2 A^2 \quad \dots(1) \end{aligned}$$

Now,

$$A^2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = 0$$

From (1), we have

$$\begin{aligned} (aI + bA)^{k+1} &= a^{k+1} I + (k+1)a^k bA + 0 \\ &= a^{k+1} I + (k+1)a^k bA \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$(aI + bA)^n = a^n I + na^{n-1}bA \text{ where } A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, n \in N$$

Question 2:

If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, prove that $A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$.

Solution:

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

It is given that

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$P(1): \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix} = \begin{pmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} = A$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): A^k = \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix}$$

Now, we have to prove that the result is true for $n = k + 1$.

Since,

$$\begin{aligned}
A^{k+1} &= A.A^k \\
&= \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{pmatrix} \\
&= \begin{pmatrix} 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \\ 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} & 3 \cdot 3^{k-1} \end{pmatrix} \\
&= \begin{pmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{pmatrix}
\end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{pmatrix}, n \in N$$

Question 3:

If $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$, prove that $A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}$, where n is any positive integer.

Solution:

It is given that $A = \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix}$

We shall prove the result by using the principle of mathematical induction.

For $n = 1$, we have:

$$\begin{aligned}
P(1): A^1 &= \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix} \\
&= \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \\
&= A
\end{aligned}$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k): A^k = \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix}, n \in N$$

Now, we have to prove that the result is true for $n = k + 1$.

Since,

$$\begin{aligned} A^{k+1} &= A.A^k \\ &= \begin{pmatrix} 1+2k & -4k \\ k & 1-2k \end{pmatrix} \begin{pmatrix} 3 & -4 \\ 1 & -1 \end{pmatrix} \\ &= \begin{pmatrix} 3(1+2k) - 4k & -4(1+2k) + 4k \\ 3k + 1 - 2k & -4k - 1(1-2k) \end{pmatrix} \\ &= \begin{pmatrix} 3+6k-4k & -4-8k+4k \\ 3k+1-2k & -4k-1+2k \end{pmatrix} \\ &= \begin{pmatrix} 3+2k & -4-4k \\ 1+k & -1-2k \end{pmatrix} \\ &= \begin{pmatrix} 1+2(k+1) & -4(k+1) \\ 1+k & 1-2(k+1) \end{pmatrix} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have:

$$A^n = \begin{pmatrix} 1+2n & -4n \\ n & 1-2n \end{pmatrix}; n \in N$$

Question 4:

If A and B are symmetric matrices, prove that $AB - BA$ is a skew symmetric matrix.

Solution:

It is given that A and B are symmetric matrices.

Therefore, we have:

$$A' = A \quad \text{and} \quad B' = B \quad \dots(1)$$

Now,

$$\begin{aligned}
(AB - BA)' &= (AB)' - (BA)' & \left[(A - B)' = A' - B' \right] \\
&= B'A' - A'B' & \left[(AB)' = B'A' \right] \\
&= BA - AB & \left[\text{Using (1)} \right] \\
&= -(AB - BA)
\end{aligned}$$

Hence,

$$(AB - BA)' = -(AB - BA)$$

Thus, $AB - BA$ is a skew symmetric matrix.

Question 5:

Show that the matrix $B'AB$ is symmetric or skew symmetric according as A is symmetric or skew symmetric.

Solution:

We suppose that A is a symmetric matrix, then

$$A' = A \quad \dots(1)$$

Consider,

$$\begin{aligned}
(B'AB)' &= \{B'(AB)\}' \\
&= (AB)'(B')' & \left[\because (AB)' = B'A' \right] \\
&= B'A'(B) & \left[\because (B')' = B \right] \\
&= B'(A'B) \\
&= B'(AB) & \left[\text{Using (1)} \right]
\end{aligned}$$

Therefore,

$$(B'AB)' = B'AB$$

Thus, if A is symmetric matrix, then $B'AB$ is a symmetric matrix.

Now, we suppose that A is a skew symmetric matrix, then

$$A' = -A \quad \dots(2)$$

Consider,

$$\begin{aligned}
(B'AB)' &= \{B'(AB)\}' \\
&= (AB)'(B')' \\
&= (B'A')B \\
&= B'(-A)B \quad [\text{Using (2)}] \\
&= -B'AB
\end{aligned}$$

Therefore,

$$(B'AB)' = -B'AB$$

Thus, if A is a skew symmetric matrix, then $B'AB$ is a skew symmetric matrix.

Hence, if A is symmetric or skew symmetric matrix, then $B'AB$ is symmetric or skew symmetric accordingly.

Question 6:

$$A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$$

Find the values of x, y, z if the matrix satisfy the equation $A'A = I$.

Solution:

$$A = \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix}$$

It is given that

Therefore,

$$A' = \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix}$$

Now, $A'A = I$

Hence,

$$\begin{aligned} &\Rightarrow \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix} \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 0+x^2+x^2 & 0+xy-xy & 0-xz+xz \\ 0+xy-xy & 4y^2+y^2+y^2 & 2yz-yz-yz \\ 0-xz+xz & 2yz-yz-yz & z^2+z^2+z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &\Rightarrow \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{aligned}$$

On comparing the corresponding elements, we have:

$$2x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

$$6y^2 = 1$$

$$\Rightarrow y = \pm \frac{1}{\sqrt{6}}$$

$$3z^2 = 1$$

$$\Rightarrow z = \pm \frac{1}{\sqrt{3}}$$

Thus, $x = \pm \frac{1}{\sqrt{2}}$, $y = \pm \frac{1}{\sqrt{6}}$ and $z = \pm \frac{1}{\sqrt{3}}$

Question 7:

$$x: \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

For what values of x ?

Solution:

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

Hence,

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = 0$$

$$\Rightarrow [4 + 4x] = 0$$

$$\Rightarrow 4x = -4$$

$$\Rightarrow x = -1$$

Thus, the required value of $x = -1$.

Question 8:

If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$, show that $A^2 - 5A + 7I = 0$

Solution:

It is given that $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$

Therefore,

$$\begin{aligned} A^2 &= A.A \\ &= \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{pmatrix} \\ &= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} \end{aligned}$$

Now,

$$\begin{aligned}
LHS &= A^2 - 5A + 7I \\
&= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - 5 \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix} + 7 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 8 & 5 \\ -5 & 3 \end{pmatrix} - \begin{pmatrix} 15 & 5 \\ -5 & 10 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\
&= \begin{pmatrix} -7 & 0 \\ 0 & -7 \end{pmatrix} + \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix} \\
&= 0 \\
&= RHS
\end{aligned}$$

Thus, $A^2 - 5A + 7I = 0$

Question 9:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Find x , if

Solution:

We have

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

Hence,

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [x(x-2) - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 2x - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 48] = 0$$

$$\Rightarrow x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Thus, $x = \pm 4\sqrt{3}$.

Question 10:

A manufacturer produces three products x, y, z which he sells in two markets. Annual sales are indicated below:

Market	Products		
I	10000	2000	18000
II	6000	20000	8000

- (a) If unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00, respectively, find the total revenue in each market with the help of matrix algebra.
- (b) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively. Find the gross profit.

Solution:

- (a) The unit sale prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively.

Consequently, the total revenue in market I can be represented in the form of a matrix as:

$$\begin{aligned}
 \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} &= 10000 \times 2.50 + 2000 \times 1.50 + 18000 \times 1.00 \\
 &= 25000 + 3000 + 18000 \\
 &= 46000
 \end{aligned}$$

The total revenue in market II can be represented in the form of a matrix as:

$$\begin{aligned}
 \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.50 \\ 1.50 \\ 1.00 \end{bmatrix} &= 6000 \times 2.50 + 20000 \times 1.50 + 8000 \times 1.00 \\
 &= 15000 + 30000 + 8000 \\
 &= 53000
 \end{aligned}$$

Thus, the total revenue in market I is ₹46000 and the total revenue in market II is ₹53000.

- (b) The unit costs of x, y and z are ₹2.00, ₹1.00 and 50 paise respectively.

Consequently, the total cost prices of all the products in market I can be represented in the form of a matrix as:

$$\begin{aligned}
 [10000 \quad 2000 \quad 18000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} &= 10000 \times 2.00 + 2000 \times 1.00 + 18000 \times 0.50 \\
 &= 20000 + 2000 + 9000 \\
 &= 31000
 \end{aligned}$$

Since the total revenue in market I is ₹ 46000 , the gross profit in this market in ₹ is

$$46000 - 31000 = 15000$$

The total cost prices of all the products in market II can be represented in the form of a matrix as:

$$\begin{aligned}
 [6000 \quad 20000 \quad 8000] \begin{bmatrix} 2.00 \\ 1.00 \\ 0.50 \end{bmatrix} &= 6000 \times 2.00 + 20000 \times 1.00 + 8000 \times 0.50 \\
 &= 12000 + 20000 + 4000 \\
 &= 36000
 \end{aligned}$$

Since the total revenue in market I is ₹ 53000, the gross profit in this market in ₹ is

$$53000 - 36000 = 17000$$

Thus, the gross profit in market I is ₹ 15000 and in market II is ₹ 17000.

Question 11:

Find the matrix X so that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution:

It is given that $X \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix.

Therefore, X has to be a 2×2 matrix.

Now, let $X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$
Therefore,

$$\begin{aligned} \Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} &= \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix} \end{aligned}$$

Equating the corresponding elements of the two matrices, we have:

$$\begin{aligned} a+4c &= -7 & 2a+5c &= -8 & 3a+6c &= -9 \\ b+4d &= 2 & 2b+5d &= 4 & 3b+6d &= 6 \end{aligned}$$

Now,

$$\begin{aligned} a+4c &= -7 \\ \Rightarrow a &= -7-4c \end{aligned}$$

Therefore,

$$\begin{aligned} 2a+5c &= -8 \\ \Rightarrow 2(-7-4c)+5c &= -8 \\ \Rightarrow -14-8c+5c &= -8 \\ \Rightarrow -3c &= 6 \\ \Rightarrow c &= -2 \end{aligned}$$

Hence,

$$\begin{aligned} \Rightarrow a &= -7-4(-2) \\ \Rightarrow a &= -7+8 \\ \Rightarrow a &= 1 \end{aligned}$$

Now,

$$\begin{aligned} b+4d &= 2 \\ \Rightarrow b &= 2-4d \end{aligned}$$

Therefore,

$$\begin{aligned} 2b+5d &= 4 \\ \Rightarrow 2(2-4d)+5d &= 4 \\ \Rightarrow 4-8d+5d &= 4 \\ \Rightarrow -3d &= 0 \\ \Rightarrow d &= 0 \end{aligned}$$

Hence,

$$\begin{aligned} b &= 2-4d \\ \Rightarrow b &= 2 \end{aligned}$$

Thus, $a = 1, b = 2, c = -2$ and $d = 0$

Hence, the required matrix $X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Question 12:

If A and B are square matrices of the same order such that $AB = BA$, then prove by induction that $AB^n = B^n A$. Further, prove that $(AB)^n = A^n B^n$ for all $n \in N$.

Solution:

Given: A and B are square matrices of the same order such that $AB = BA$.

To prove: $P(n): AB^n = B^n A, n \in N$

For $n = 1$, we have:

$$\begin{aligned} P(1): AB &= BA && [\text{Given}] \\ \Rightarrow AB^1 &= B^1 A \end{aligned}$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$P(k) = AB^k = B^k A \quad \dots(1)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= AB^k \cdot B \\ &= (B^k A) B && [\text{By (1)}] \\ &= B^k (AB) && [\text{Associative law}] \\ &= B^k (BA) && [AB = BA \text{ (Given)}] \\ &= (B^k B) A && [\text{Associative law}] \\ &= B^{k+1} A \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $AB^n = B^n A, n \in N$

Now, we have to prove that $(AB)^n = A^n B^n$ for all $n \in N$

For $n = 1$, we have:

$$(AB)^1 = A^1 B^1 = AB$$

Therefore, the result is true for $n = 1$.

Let the result be true for $n = k$.

$$(AB)^k = A^k B^k \quad \dots (2)$$

Now, we prove that the result is true for $n = k + 1$.

$$\begin{aligned} AB^{k+1} &= (AB)^k \cdot (AB) \\ &= (A^k B^k) \cdot (AB) && [\text{By (2)}] \\ &= A^k (B^k A) B && [\text{Associative law}] \\ &= A^k (AB^k) B && [AB^n = B^n A, n \in N] \\ &= (A^k A) \cdot (B^k B) && [\text{Associative law}] \\ &= A^{k+1} B^{k+1} \end{aligned}$$

Therefore, the result is true for $n = k + 1$.

Thus, by the principle of mathematical induction, we have $(AB)^n = A^n B^n, n \in N$

Question 13:

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$ then,

(A) $1 + \alpha^2 + \beta\gamma = 0$

(B) $1 - \alpha^2 + \beta\gamma = 0$

(C) $1 - \alpha^2 - \beta\gamma = 0$

(D) $1 + \alpha^2 - \beta\gamma = 0$

Solution:

It is given that $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$

Therefore,

$$\begin{aligned} A^2 &= A.A \\ &= \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \\ &= \begin{pmatrix} \alpha^2 + \beta\gamma & \alpha\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{pmatrix} \\ &= \begin{pmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{pmatrix} \end{aligned}$$

Now, $A^2 = I$

Hence,

$$\begin{pmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \beta\gamma + \alpha^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

On comparing the corresponding elements, we have:

$$\begin{aligned} \alpha^2 + \beta\gamma &= 1 \\ \Rightarrow \alpha^2 + \beta\gamma - 1 &= 0 \\ \Rightarrow 1 - \alpha^2 - \beta\gamma &= 0 \end{aligned}$$

Thus, the correct option is C.

Question 14:

If the matrix A is both symmetric and skew symmetric, then

- (A) A is a diagonal matrix (B) A is a zero matrix
(C) A is a square matrix (D) None of these

Solution:

If the matrix A is both symmetric and skew symmetric, then

$$A' = A \text{ and } A' = -A$$

Hence,

$$\begin{aligned} \Rightarrow A &= -A \\ \Rightarrow A + A &= 0 \\ \Rightarrow 2A &= 0 \\ \Rightarrow A &= 0 \end{aligned}$$

Therefore, A is a zero matrix.

Thus, the correct option is B.

Question 15:

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
(A) A (B) $I - A$ (C) I (D) $3A$

Solution:

It is given that A is a square matrix such that $A^2 = A$.
Now,

$$\begin{aligned}(I + A)^3 - 7A &= I^3 + A^3 + 3I^2A + 3A^2I - 7A \\&= I + A^2.A + 3A + 3A^2 - 7A \\&= I + A.A + 3A + 3A - 7A \quad [\because A^2 = A] \\&= I + A^2 - A \\&= I + A - A \quad [\because A^2 = A] \\&= I\end{aligned}$$

Hence,

$$(I + A)^3 - 7A = I$$

Thus, the correct option is C.