

QUADRATIC EQUATIONS

KEY POINTS

1. The general form of a quadratic equation is $ax^2+bx+c=0$, $a \neq 0$. a , b and c are real numbers.
2. A real number α is said to be a root of quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$ if $a\alpha^2 + b\alpha + c = 0$. The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the corresponding quadratic equation $ax^2 + bx + c = 0$ are the same.
3. Discriminant: - The expression b^2-4ac is called discriminant of the equation $ax^2+bx+c=0$ and is usually denoted by D . Thus discriminant $D=b^2-4ac$.
4. Every quadratic equation has two roots which may be real, coincident or imaginary.

5. If α and β are the roots of the equation $ax^2+bx+c=0$ then

$$\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a} \quad \text{And} \quad \beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$$

6. Sum of the roots, $\alpha + \beta = -\frac{b}{a}$ and product of the roots, $\alpha\beta = \frac{c}{a}$

7. Forming quadratic equation, when the roots α and β are given.

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

8. Nature of roots of $ax^2+bx+c=0$

- i. If $D > 0$, then roots are real and unequal.
- ii. $D=0$, then the equation has equal and real roots.
- iii. $D < 0$, then the equation has no real roots
- iv. If $D > 0$ and D is a perfect square, then roots are rational and unequal.
- v. If $D > 0$ and D is not a perfect square then roots are irrational.

9. Irrational roots always occur in conjugate pairs. If $2+\sqrt{3}$ is one of the root of the quadratic equation then other root is $2-\sqrt{3}$.

10. If $a.b > 0$ then $a > 0$ and $b > 0$ or $a < 0$ and $b < 0$
If $a.b < 0$ then $a > 0$ and $b < 0$ or $a < 0$ and $b > 0$.

LEVEL-I

1. If $\frac{1}{2}$ is a root of the equation $x^2+kx-5/4=0$, then find the value of K .
2. If $D > 0$, then write the roots of a quadratic equation $ax^2+bx+c=0$
3. Find the Discriminant of $x^2+5x+5=0$.
4. Find the the sum of roots of a quadratic equation $x^2+4x-32=0$
5. Find the product of roots of a quadratic equation $2x^2+7x-4=0$.
6. Find the values of K for which the equation $9x^2+2kx+1=0$ has real roots.
7. Find the Value of K if the equation $x^2-2(k+1)x+k^2=0$ has equal roots.
8. For what value of k , $x=a$ is a solution of equation $x^2-(a+b)x+k=0$?

9. Represent the situation in the form of Quadratic equation:

The Product of Rahman's age (in years) 5 years ago with his age 9 years later is 15.

10. Find the roots of $x^2 - 3x - 10 = 0$

11. The product of two consecutive odd numbers is 483. Find the numbers.

LEVEL - II

1. If $x = 2$ and $x = 3$ are roots of the equation $3x^2 - 2kx + 2m = 0$ find the value of k and m.

2. Solve the equation:

$$\frac{x}{x+1} + \frac{x+1}{x} = \frac{34}{15}, x \neq 0, x \neq -1$$

3. Solve the equation $2x^2 - 5x + 3 = 0$ by the method of completing square.

4. Using quadratic formula, solve the equation: $p^2x^2 + (p^2 - q^2)x - q^2 = 0$.

5. 300 apples are distributed equally among a certain number of student's .Had there been 10 more students, each would have received one apple less. Find the number of students.

6. Find the roots of Quadratic equation $16x^2 - 24x - 1 = 0$ by using the quadratic formula.

7. Find the discriminant of the Quadratic equation $2x^2 - 4x + 3 = 0$ and hence find the nature of its roots.

LEVEL – III

1. In a class test, the sum of Shefali's marks in math's and English is 30. Had she got 2 marks more in math's and 3 marks less in English, the product of their marks would have been 210. Find her marks in two subjects.

2. A two digit number is such that the product of its digit is 35. When 18 is added to the number, the digits interchange the places. Find the number.

3. Solve $3x^2 - 23x - 110 = 0$

4. Solve the following equation for 'x', $9x^2 - 9(a+b)x + (2a^2 + 5ab + 2b^2) = 0$

5. If the roots of the equation $(a-b)x^2 + (b-c)x + (c-a) = 0$ are equal, prove that $2a = b+c$.

Self-Evaluation

1. Find the value of p so that the equation $3x^2 - 5x + 2p = 0$ has equal roots. Also find the roots.

2. The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.

3. Find the quadratic equation whose roots are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

4. A person on tour has Rs. 360 for his daily expenses. If he exceeds his tour Programme by four days, he must cut down his daily expenses by Rs 3 per day. Find the number of days of his tour Programme.
5. Divide 29 into two parts so that the sum of squares of the parts is 425.
6. Solve for x: $9x^2 - 6ax + (a^2 - b^2) = 0$
7. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots, show that $c^2 = a^2(1 + m^2)$

VALUE Based Questions

Q1. If the price of petrol is increased by Rs. 2 per liter, a person had to buy 1 liter less petrol for Rs. 1740. Find the original price of the petrol at that time.

(a) Why do you think the price of petrol is increasing day by day?

(b) What should we do to save petrol?

2. Ramesh wants to design a rectangular park of perimeter 80 m and area 400 m^2 for jogging and morning walk for the people of his colony. Is it possible to design the park? If so find the length and breadth of the park. Which value of Ramesh is depicted here?

Answer

LEVEL-I

1. 2
2. $\frac{-b \pm \sqrt{D}}{2a}$
3. 5
4. -4
5. -2
6. $k \geq 3$ or $k \leq -3$
7. $-1/2$
8. $K=ab$
9. $x^2+4x-60=0$
10. -2, 5
11. 21, 23

LEVEL-II

1. $K = \frac{15}{2}, m = 9$
2. $x = \frac{3}{2}$ or $x = \frac{-5}{2}$
3. $x = \frac{3}{2}$ or $x = 1$
4. $x = -1$, or $x = \frac{q^2}{p^2}$
5. 50
6. $\frac{3+\sqrt{10}}{4}, \frac{3-\sqrt{10}}{4}$
7. $D = -8 < 0$ it has no real roots.

LEVEL- III

1. (Marks in maths = 12 , marks in English =18) or (marks in maths = 13 , marks in English = 17)
2. 57
3. $-10/3, 11$
4. $\frac{2a+b}{3}, \frac{a+2b}{3}$

SELF EVALUATION

1. 25/24
2. (10, 5) or (5, 10)
3. $x^2 - 4x + 1 = 0$
4. 20 days.
5. (16, 13) or (13, 16)
6. $(a+b)/3, (a-b)/3$

VALUE BASED QUESTIONS

1. Rs 58 per liter
2. Yes, $l=20m$ and $b=20m$.