Chapter 10 - Direct and Inverse Variations

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Exercise 10.1

Question1.

Solution:

When two variables are connected to each other in such a way that if we increase the value of one variable, the value of other variable also increases and vice — versa. Similarly, if we decrease the value of one variable, the value of other variable also decreases and vice — versa. Therefore, if the ratio between two variables remains constant, it is said to be in direct variation.

Question 2.

Solution:

(i) The number of articles is directly related to the price. Therefore, they will vary directly with each other.

(ii) The number of articles is directly related to the weight of the articles. Therefore, they will vary directly with each other.

(iii) Speed is constant. Therefore, distance and time does not vary directly.

(iv) The number of hours is directly related to the wages. Therefore, it is a direct variation.

(v) Distance is constant. Therefore, speed and time does not vary directly.

(vi) If the area of a land is large, its cost will also be high. Thus, it is a direct variation.

Thus, the respective values in (i), (ii), (iv) and (vi) vary directly with each other.

Question 3.

Solution:

(i)

a	**7**	**9**	**13**	**21**	**25**
b	**21**	**27**	**39**	**63**	**75**

(ii)

a	**10**	**20**	**30**	**40**	**46**

b	**5**	**10**	**15**	**20**	**23**

(iii)

A	**2**	**3**	**4**	**5**	**6**
B	**6**	**9**	**12**	**17**	**20**

(iv)

a	1^2	2^2	3^2	4^2	5^2
b	1^3	2^3	3^3	4^3	5^3

Solution:

If x and y vary directly, the ratio of the corresponding values of x and y remains constant

(i) $\frac{x}{y}=\frac{7}{21}=\frac{1}{3}, \frac{x}{y}=\frac{9}{27}=\frac{1}{3}, \frac{x}{y}=\frac{13}{39}=\frac{1}{3}, \frac{x}{y}=\frac{25}{75}=\frac{1}{3}.$

In all the cases, the ratio is the same. Therefore, x and y vary directly.

(ii) $\frac{x}{y}=\frac{10}{5}=2, \frac{x}{y}=\frac{20}{10}=2, \frac{x}{y}=\frac{30}{15}=2, \frac{x}{y}=\frac{40}{20}=2, \frac{x}{y}=\frac{46}{23}=2$

In all the cases, the ratio is the same. Therefore, x and y vary directly.

(iii) $\frac{x}{y}=\frac{2}{6}=\frac{1}{3}, \frac{x}{y}=\frac{3}{9}=\frac{1}{3}, \frac{x}{y}=\frac{4}{12}=\frac{1}{3}, \frac{x}{y}=\frac{5}{17}=\frac{5}{17}, \frac{x}{y}=\frac{6}{20}=\frac{3}{10}$

In all the cases, the ratio is not the same. Therefore, x and y do not vary directly.

(iv) $\frac{x}{y}=\frac{1^2}{1^3}=1, \frac{x}{y}=\frac{2^2}{2^3}=\frac{1}{2}, \frac{x}{y}=\frac{3^2}{3^3}=\frac{1}{3}, \frac{x}{y}=\frac{4^2}{4^3}=\frac{1}{4}\ \frac{x}{y}=\frac{5^2}{5^3}=\frac{1}{5}$

In all the cases, the ratio is not the same. Therefore, x and y do not vary directly. Thus, in (i) and (ii) x and y vary directly.

Question 4**:**

Solution:

i) Directly

(ii) x and y are said to vary directly with each other if $\frac{x}{y}$ = k, where k is a positive number

(iii) Because u = 3v, u and y vary directly with each other

Question 5.

Solution:

Here, x and y vary directly. ∴ x = ky
(i) x = 2.5 and y = 5 i.e. 2.5 = 5k

=> 2.5/ 5 = 0.5.

For y = 8 and k = 0.5, we have: x = ky

=> x= 8 X 0.5 = 4

For y = 12 and k = 0.5, we have: x = ky

=> x = 12 X 0.5 = 6

For x = 15 and k =0.5, we have: x = ky

=> 15 = 0.5 X y

$$=Y=\frac{15}{0.5}=30$$

(ii) x = 5 and y = 8 i.e. 5 = 8 X k

$$\Rightarrow k=\frac{5}{8}=0.625$$

For y = 12 and k = 0.625, we have; x = ky

=> x = 12 X 0.625 = 7.5

For x = 10 and k = 0.625, we have: x = ky

=> 10 = 0.625 X y

$$\Rightarrow y=\frac{10}{0.625}=16$$

For x = 35 and k = 0.625, we have: x = ky

=> 35 = 0.625 X y

$$\Rightarrow y=\frac{35}{0.625}=56$$

For x = 25 and k = 0.625, we know: x = ky

=> 25 = 0.625 X y

$$\Rightarrow y=\frac{25}{0.625}=40$$

For y = 32 and k = 0.625, we know: x = ky

=> x = 0.625 X 32 = 20

(iii) x = 6 and y = 15 i.e. 6 = 15k

$\Rightarrow k = \frac{6}{15} = 0.4$

For x = 10 and k = 0.4, we have:

$y = \frac{10}{0.4} = 25$

For y = 40 and k = 0.4, we have:

x = 0.4 X 40 = 16

For x = 20 and k = 0.4, we have: $y = \frac{20}{0.4} = 50$

(iv) x = 4 and y = 16 i.e. 4 = 16k

$\Rightarrow k = \frac{4}{16} = \frac{1}{4}$

For x = 9 and k = 14, we have: 9 = ky

$\Rightarrow$ y = 4 X 9 = 36

For y = 48 and k = 14, we have: x = ky

$\Rightarrow \frac{1}{4} \times 48 = 12$

For y = 36 and k = 14, we have: x = ky

$\Rightarrow \frac{1}{4} \times 36 = 9$

For x = 3 and k = 14, we have: x = ky

$\Rightarrow 3 = \frac{1}{4} \times y$

$\Rightarrow$ y = 12

For y = 4 and k = 14, we have: x = ky

$= \frac{1}{4} \times 4 = 1$

(v) x = 5 and y = 20 i.e. 5 = 20k

$\Rightarrow k = \frac{5}{20} = \frac{1}{4}$

For x = 3 and $k = \frac{1}{4}$, we have:

$3 = \frac{1}{4} \times y$

$\Rightarrow y = 12$

For x = 9, k = 14, we have: x = ky

$\Rightarrow 9 = \frac{1}{4} \times y$

$\Rightarrow y = 36$

Question 6.

Solution:

Since it is a direct variation, $\frac{x}{y} = k$.

For x = 3 and y = 12, we have: $k = \frac{3}{12} = \frac{1}{4}$

Thus, in all cases, $k = \frac{1}{4}$

Question 7.

Solution:

Let the cost of y registers be Rs x.

Register	12	7
Cost (in Rs)	156	x

If he buys less number of registers, the cost will also be less. Therefore, it is a direct variation.

We get: 12:7 = 156:x => $\frac{12}{7} = \frac{156}{x}$

Applying cross multiplication, we get: x = $\frac{156 \times 7}{12} = 91$

Thus, the cost of 7 such registers will be Rs 91.

Question 8.

Solution:

Let the distance travelled is more, the time needed to cover it will be more. Therefore, it is direct variation. We get: 125: 135 = 100:x

$$\Rightarrow \frac{125}{315} = \frac{100}{x}$$

Applying cross multiplication, we get:

$$X = \frac{100 \times 315}{125} = 252$$

Thus, Anupama would cover 252 meter in 315 minutes.

Question 9.

Solution:

Length of plastic sheet (in meter)	**93**	**105**
Cost (in Rs)	**1395**	**x**

Let the cost of the plastic sheet per meter be Rs x

If more sheets are brought, the cost will also be more. Therefore, it is a direct variation. We get: 93: 105 = 1395: x

$$\Rightarrow \frac{93}{105} = \frac{1395}{x}$$

Applying cross multiplication, we get: $x = \frac{105 \times 1395}{93} = 1575$.

Thus, the required cost will be Rs 1,575.

Question

Solution:

Number of words	**1080**	**x**
Time (in minute)	**60**	**1**

Let x be her GWAM.

If the time taken is less, GWAM will also be less. Therefore, it is a direct = 60: 1

$$\Rightarrow \frac{1080}{x} = \frac{60}{1}$$

Applying cross multiplication, we get: $x = \frac{1080 \times 1}{60} = GWAM$ will be 18.

Question 11.

Solution:

Distance (in km)	**50**	**x**
Time (in minute)	**60**	**12**

Let the distance be x km.

If the time taken is less, the distance covered will also be less. Therefore, it is a direct variation.

50: x = 60: 12 => $\frac{50}{x} = \frac{60}{12}$

Applying cross multiplication, we get: x = $\frac{50 \times 12}{60} = 10$.

Thus, the required distance will be 10 km.

Question 12.

Solution:

Number of boxes	**68**	**x**
Shelf- length (in m)	**13.6**	**20.4**

Let x be the number of boxes that occupy a shelf- length of 20.4 m

If the length of the shelf increases, the number of boxes will also increase. Therefore, it a case of direct variation

$\frac{68}{x} = \frac{13.6}{20.4}$.

$68 \times 20.4 = x \times 13.6\ x = x = \frac{68 \times 20.4}{13.6} = \frac{1387.2}{13.6} = 102$

Thus, 102 boxes will occupy a shelf- length of 20.4 m.

Question 13.

Solution:

Number of copies	136	x
Shelf- length (in m)	3.4	5.1

Let x be the number of copies that would occupy a shelf- length of 5.1 m

Since the number of copies and the length of the shelf are in direct variation,

we have: $\frac{136}{x} = \frac{3.4}{5.1} => 136 \times 5.1 = x \times 3.4$

$\Rightarrow x = x = \frac{136 \times 5.1}{3.4} = 204$

Thus, 204 copies will occupy a shelf of length 5.1 m

Question 14

Solution:

Let Rs x be the fare for a journey of 139.2 km

Distance (in km)	**240**	**139.2**
Fare (in Rs)	**15**	**x**

Since the distance travelled and the fares are in direct variation,

we have: $\frac{240}{139.2} = \frac{15}{x}$

$\Rightarrow 240 \times x = 15 \times 139.2$

$\Rightarrow x = x = \frac{15 \times 139.2}{240} = \frac{2008}{240} = 8.7$

Thus, the fare for a journey of 139.2 km will be Rs 8.70

Question 15.

Solution:

Let x cm be the thickness of a pile of 294 cardboard

Thickness (in cm)	**3.5**	**x**
Cardboard	**12**	**294**

Since the pile of the cardboard and its thickness are in direct variation, we have: $x =$

$x = \frac{3.5}{x} = \frac{12}{294} \Rightarrow 3.5 \times 294 = x \times 12$

$\Rightarrow x = x = \frac{3.5 \times 294}{12} = \frac{1029}{12} = 85.75cm$

Thus, the thickness of a pile of 294 cardboard will be 85.75 cm (or 857.5 mm).

Question 16.

Solution:

Let x meter be the length of the cloth that can be purchased for Rs. 320.50

Length (in m)	**97**	**x**

Cost (in Rs)	242.50	302.50

Since the length of the cloth and its cost are in direct variation, we have:

$\frac{97}{x}=\frac{242.50}{302.50}$

=> 97×302.50=x×242.50

=> $x=\frac{97\times 302.50}{242.50}=\frac{29342.50}{242.52}=121$

Thus, the required length will be 121 meter.

Question 17.

Solution:

Let x be the number of men required to dig a trench of 27 meter.

Number of men	11	x
Length (in m)	27/ 4	27

Since the length of the trench and the number of men are in direct variation, we have:

=> $11\times 27=x\times\frac{27}{4}$

=> $x=\frac{11\times 27\times 4}{27}=44$

Thus, 44 men will be required to dig a trench of 27 meter

Question 18.

Solution:

Let x be the number of days for which the worker is paid Rs 875.

Income (in Rs)	210	875
Number of days	6	x

Since the income of the worker and the number of working days are in direct variation, we have: $\frac{210}{875}=\frac{6}{x}$

=> 210×x=875×6

=> $x=\frac{875\times 6}{210}=\frac{5250}{210}=25$

Thus, the required number of days is 25.

Question 19.

Solution:

Let Rx x be the income for 20 days work.

Income (in Rs)	200	x
Number of days	8	20

Since the income and the number of working days are in direct variation,

we have: $\frac{200}{x}=\frac{8}{20}$

=> 200×20=8x

=> $x=\frac{200\times 20}{8}=\frac{4000}{8}=500$

Thus, the worker will get Rs 500 for working 20 days.

Question 20.

Solution:

Let x gm be the weight that would produce an extension of 17.4 cm.

Weight (in gm)	150	x
Length (in cm)	2.9	17.4

Since the amount of extension in an elastic string and the weight hung on it are in direct variation, we have: $\frac{150}{x}=\frac{2.9}{17.4}$

=> 17.4×150=2.9×x

=> $x=\frac{17.4\times 150}{2.9}=\frac{2610}{2.9}=900$

Thus, the required weight will be 900 gm.

Question 21

Solution:

Let x cm be the extension produced by the weight of 700 gm

Weight (in gm)	250	700
Length (in cm)	3.5	x

Since the amount of extension in an elastic spring varies and the weight hung on it is in direct variation, we have: $\frac{250}{700}=\frac{3.5}{x}$

=> x×250=3.5×700

$\Rightarrow x = \frac{3.5 \times 700}{250} = \frac{2450}{250} = 9.8$

Thus, the required extension will be 9.8 cm

Question .22:

Solution:

Let the amount of dust picked up by the earth in 45 days be x pounds.

Since the amount of dust picked up by the earth and the number of days are in direct variation, we have: Ratio of the dust picked up by the earth in pounds = ratio of the number of days taken

$\Rightarrow \frac{10}{45} = \frac{2.6 \times 10^8}{x}$

$\Rightarrow x \times 10 = 45 \times 2.6 \times 10^8$

Thus, 11.7×10^8 pounds of dust will be picked up by the earth in 45 days.

Q.23.

Solution:

Let x be the number of days taken by the earth to pick up 4.8×10^8 kg of dust

Since the amount of dust picked up by the earth and the number of days are in direct variation, we get:

$\frac{15}{x} = \frac{1.2 \times 10^8}{4.8 \times 10^8} \Rightarrow x = 15 \times \frac{4.8}{1.2}$

$\Rightarrow$ x = 60

Thus, the required number of days will be 60.

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Exercise 10.2

Question 1.

Solution:

(i) Since x and y vary inversely, we have: $y = kx \Rightarrow xy = k$

$\therefore$ the product of x and y is constant. In all cases, the product xy is constant (i.e. 24)
Thus, in this case x and y vary inversely.

(ii) In all cases, the product xy is constant for any two pairs of values for x and y. Here, xy = 100 for all cases.

Thus, in this cases, x and y do not vary inversely.

(iii) If x and y vary inversely, the product xy should be constant. Here, in one cases, product = $6 \times 8 = 48$ and in the rest, product = 36
Thus, in this case, x and y do not vary inversely.

(iv) If x and y inversely, the product xy should be constant. Here, the product is different for all cases.
Thus in this cases, x and y do not vary inversely.

Question 2.

Solution:

(i) Since x and y vary inversely, we have: $xy = k$

For $x = 16$ and $y = 6$, we have: $16 \times 6 = k$

$\Rightarrow k = 96$

For $x = 12$ and $k = 96$, we have:

$xy = k$

$\Rightarrow 12y = 96$

$\Rightarrow y = 96/12 = 8$

For $y = 4$ and $k = 96$, we have: $xy = k$

$\Rightarrow 4x = 96$

$\Rightarrow x = 24$

For $x = 8$ and $k = 96$, we have:

$xy = k$

=> 8y = 96 => y = 96/ 8 = 12

For y = 0.25 and k = 96, we have: xy = k

=> 0.25x = 96

=> x = 96/ 0.25 = 384

(ii) Since x and y vary inversely, we have: xy = k

For x = 16 and y = 4, we have: 16 X 4 = k

=> k = 64

For x = 32 and k = 64, we have: xy = k

=> 32y = 64

=> y = 64/ 32 = 2

For x = 8 and k = 64

=> xy = k

=> 8y = 64

=> y = 8

(iii) Since x and y vary inversely, we have: xy = k

For x = 9 and y = 27

9 X 27 = k

=> k = 243

For y = 9 and k = 243, we have: xy = k

=> 9x = 243

=> x = 243/ 9 = 27

For x = 81 and k = 243, we have:

xy = k

=> 81y = 243

=> y = 243/ 81 = 3

Question 3.

Solution:

(i) If the number of men is more, the time taken to construct a wall will be less. Therefore, it is in inverse variation.

(ii) If the length of a journey is more, the price of the ticket will also be more. Therefore, it is not in inverse variation.

(iii) If the length of the journey is more, the amount of petrol consumed by the car will also be more.

Therefore, it is not in inverse variation.

Thus, only (i) is in inverse variation.

Question 4.

Solution:

Since the volume and pressure for the given mass vary inversely, we have: vp = k

For v = 60 and p = 30, we have: k = 60 X 32 = 90

For p = 2 and k = 90, we have: 2v = 90

=> v = 45

For v = 48 and k =90, we have: 48p = 90

=> p = 48/ 90

For p = 1 and k = 90, we have: 1v = 90

=> v = 90

For v = 100 and k = 90, we have: 100p = 90

=> v = 9/ 10

For p = 12 and k = 90, we have: 12v = 90

=> v = 90 / 12 = 15/ 2

For v = 200 and k = 90, we have:

200p = 90

=> p = 9/ 20

Question 5

Solution:

Let x be the number of days in which 15 men can do a piece of work.

Number of men	36	15
Number of days	25	x

Since the number of men hired and the number of days taken to do a piece of work are in inverse variation, we have:

$$36 \times 25 = x \times 15 \Rightarrow x = \frac{36 \times 25}{15} = \frac{900}{15} = 60$$

Thus, the required number of days is 60.

Question 6

Solution:

Let x be the number of days required to complete a piece of work by 125 men.

Number of men	50	125
Months	5	x

Since the number of men engaged and the number of days taken to do a piece of work are in inverse variation, we have:

$50\times5=125x$

$\Rightarrow x=\frac{50\times5}{125}=2$

Thus, the required number of months is 2

Question 7.

Solution:

Let x be the extra number of men employed to complete the job in 7 months.

Number of men	**420**	**x**
Months	**9**	**7**

Since the number of men hired and the time required to finish the piece of work are in inverse variation, we have:

$420\times9=7x \;\; x=\frac{420\times9}{7}=540$

Thus, the number of extra men required to complete the job in 7 months

$= 540 – 420 = 120$

Question 8.

Solution:

Number of men	1200	x
Days	35	25

Let x be the number of addition men required to finish the stock in 25 days.

Since the number of men and the time taken to finish a stock are in inverse variation, we have: $1200\times35=25x$

$\Rightarrow x=\frac{1200\times35}{25}=1680$

$\therefore$ Required number of men = 1680 – 1200 = 480.

Thus, an additional 480 men should join the existing 1200 men finish the stock in 25 days.

Question 9.

Solution:

Let x be the number of days with food provisions for 80 (i.e. 50 + 30) girls.

Number of girls	50	80
Number of days	40	x

Soln: Since the number of girls and number of days with food provisions are in inverse variation, we have: $50\times40=80x$

$\Rightarrow x=\frac{50\times40}{80}=\frac{2000}{80}=25$

Thus, the required number of days is 25

Question 10.

Solution:

Let the increased speed be x km/h.

Time (in h)	10	8
Speed (km/h)	48	x + 48

Since speed and time taken are in inverse variation, we get:

$10\times48=8(x+48)$
$\Rightarrow 8x=480–384$
$\Rightarrow 8x=96=12$
Thus, the speed should be increased by 12 km/h.

Question 11

Solution:

It is given that after 4 days, out of 28 days, the fort had enough food for 1200 soldiers for (28 – 4 = 24)

Let x be the number of soldiers who left the fort.

Number of soldiers	1200	1200-x
Number of days for which food lasts	24	32

Since the number of soldiers and the number of days for which the food lasts are in inverse variation, we have:

$1200\times24=1200–x\times32$
$\Rightarrow 1200\times2432=1200–x$
$\Rightarrow 900 = 1200 – x$

$\Rightarrow x = 300$

Thus, 300 soldiers left the fort

Question 12.

Solution:

Let the time taken by 5 spraying machines to finish a painting job be x minutes.

Number of machines	3	5
Time (in minutes)	60	x

Since the number of spraying machines and the time taken by them to finish a painting job are in inverse variation, we have:

$3\times60=5\times x$
$\Rightarrow 180 = 5x$

$\Rightarrow x = 36$

Thus, the required time will be 36 minutes.

Question 13.

Solution:

Let x be the number of new members in group.

Number of members	3	X
Number of days	30	18

Since more members can finish the wheat in less number of days, it is a case of inverse variation. Therefore, we get:

$3\times30=x\times18$
$\Rightarrow 90 = 18x$

$\Rightarrow x = 90/ 18 = 5$

Thus, the number of new members in the group = 5 – 3 = 2

Question 14.

Solution:

Let the number of cows and the number of days taken by them to graze the field is in inverse variation, we have:

$16\times55=10\times x$

$\Rightarrow x=\frac{16\times55}{10}=88$

The required number of cows is 88

Question 15.

Solution:

Let the number of men required to reap the field in 15 days be x.

Number of days	35	15
Number of men	18	X

Since the number of days and the number of men required to reap the field are in inverse variation, we have:

$35\times18=15\times x$

$\Rightarrow x=\frac{35\times18}{15}=42$

Thus, the required number of men is 42.

Question 16.

Solution:

Let x be the number of cycles bought if each cycle costs Rs 125 more

Cost of a cycle (in Rs)	500	625
Number of cycles	25	x

It is in inverse variation. Therefore, we get:

500 **x** 25 = 625 **x** x

$\Rightarrow x=\frac{500\times256}{25}=20$

∴ The required number of cycles is 20.

Question:17
Solution:
Let x be the number of machines he can buy if a discount of Rs. 50 is offered on each machine.

Number of machines	75	x
Price of each machine (in Rs)	200	150

Since Raghu is getting a discount of Rs 50 on each machine, the cost of each machine will get decreased by 50.

If the price of a machine is less, he can buy more number of machines. It is a case of inverse variation. Therefore, we have:

$75\times200=x\times150$

$\Rightarrow x=\frac{75\times200}{150}=\frac{15000}{150}=100$

∴ The number of machines he can buy is 100.

Question 18.

Solution:

(i) Since x and y vary inversely, we have: xy = k

For x =3 and y = 8, we have:

=> 3 **x** 8 = k

=> k = 24

For x = 4, we have:

4y = 24

=> y = 6

∴ y = 6

(ii) Since x and y vary inversely, we have: xy = k

For x = 5 and y = 15, we have:

=> 5 **x** 15 = k

=> k = 75

For y = 12, we have:

12x = 75

=> x = 75/ 12 = 25/ 4

∴ x = 25/ 4

(iii) Given: x = 30 and k = 900

∴ xy = k
=> 30y = 900

=> y = 900/ 30 = 30

=> y = 30

(iv) Given: y = 35 and k = 7 Now, xy = k

=> 35x = 7

=> x = 1/ 5

∴ x = 1/ 5