16. Understanding Shapes-II Quadrilaterals

16. Understanding Shapes-2 (Quadrilaterals).

Solution-01:-

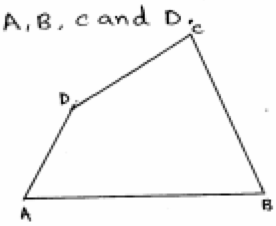
(i) Quadrilateral:-

Let A, B, C and D be four points in a plane such that:

(i) no three of them are collinear, and,

(ii) the line segments AB, BC, CD and DA do not intersect except at their and points.

Then, the figure formed by made up of the four line segments is called the quadrilateral with vertices A, B, C and D.



Convex Quadrilateral:-

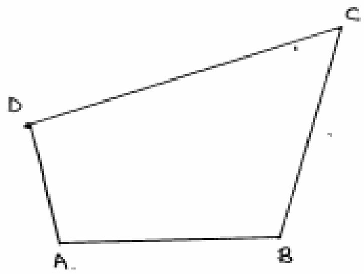
A quadrilateral is called convex quadrilateral, if the line containing any side of the quadrilateral has the remaining vertices on the same side of it. In fig below quadrilateral ABCD is a convex quadrilateral, because.

Vertices A, B lie on the same side of line CD,

vertices B, C lie on the same side of line DA,

vertices C, D lie on the same side of line AB,

and, vertices D, A lie on the same side of line BC



Solution-02 :-

(i) Sides :-

In a quadrilateral ABCD, the four Line segments AB, BC, CD and DA are called its sides.

(ii) vertices :-

In a Quadrilaterals ABCD The Points A, B, C and D are called as Vertices.

(iii) Angles :-

In the quadrilateral ABCD, the angles, $\angle DAB$, $\angle ABC$, $\angle BCD$ and $\angle D$ (or) $\angle CDA$ are called its angles These angles can be denoted $\angle A$, $\angle B$, $\angle C$ and $\angle D$ respectevely.

(iv) Adjacent angles :-

In the Quadrilateral ABCD, the angles $\angle A$, $\angle B$ and $\angle C$, $\angle D$ are called Adjacent angles.

(Common side)

(v) Diagonals:-

In the quadrilaterals ABCD, the line segments AC and BD are called. its diagonals.

(vi) Adjacent sides:-

Two sides of a quadrilateral are called its adjacent sides, if they have a common end-point.

(vii) Opposite sides:-

Two sides of a quadrilateral are called its opposite sides, if they donot have a common end-point.

(viii) Opposite Angles:-

Two angles of a Quadrilateral are called its opposite angles. if they donot have a common end-point. side (or) Arm.

(ix) Interior :-

The part of the plane made up by all such points as are enclosed by Quadrilateral ABCD. This part of the plane is called the interior of the Quadrilateral ABCD and any point of part is called an interior point of the quadrilateral.

Exterior :-

The part of the plane made up by all points as are not enclosed by the quadrilateral ABCD. This part of the plane is called the exterior of the Quadrilateral ABCD and any point of this part is called an exterior point of the Quadrilateral.

Solution - 3 :-

(i) 4

(ii) 4

(iii) 4, Collinear.

(iv) 2.

(v) 4

(vi) 2

(vii) 360°

(viii) opposite

(ix) four

(x) less than

(xi) the interior

(xii) interiors

(xiii) vertices.

Solution - 4 :-

(i) AB, BC or BC, CD or CD, DA or AD, AB.

(ii) AB, CD or BC, DA

(iii) Four

(iv) Two.

(v) $\angle A, \angle B$ or $\angle B, \angle C$ or $\angle C, \angle D$ or $\angle D, \angle A$.

(vi) $\angle A, \angle C$ or $\angle B, \angle D$

(vii) 4.

(viii) 2.

Solution - 05 :-

The angles of a quadrilaterals are $110°, 72°, 55°$ and $x°$.

we know that,

sum of angles of a quadrilaterals

$$\Rightarrow 110° + 72° + 55° + x° = 360°$$

$$\Rightarrow x° = 360° - 237°$$

$$\Rightarrow x = 123°.$$

The opposite angles of a Quadrilateral $= 123°$.

required angle is $123°$.

6. The given three Angles of a Quadrilaterals are respectevely equal to 110°, 50° and 40°.

$\therefore$ Sum of the angles in a Quadrilateral $= 360^\circ$

Let the fourth angle be 'x'

$$\therefore 110^\circ + 50^\circ + 40^\circ + x = 360^\circ$$

$$x = 360^\circ - 200^\circ$$

$$x = 160^\circ.$$

7. Given that.

Three acute angles each measures 80°.

Let the fourth angle be x.

Three angles of Quadrilateral are 80°, 80° and 80°.

We know that,

Sum of the angles in a Quadrilateral $= 360^\circ$.

$$80^\circ + 80^\circ + 80^\circ + x = 360^\circ$$

$$x = 360^\circ - 240^\circ$$

$$x = 120^\circ$$

$\therefore$ The fourth angle be $x = 120^\circ$.

8. Given that,

A quadrilateral has all its four angles of the same measure.

Let it be 'x'

We know that,

Sum of the angles of a Quadrilateral $= 360^\circ$.

$$x + x + x + x = 360^\circ$$

$$\Rightarrow 4x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{4}$$

$$\Rightarrow x = 90^\circ.$$

10. Given that,

Three angles of a quadrilateral are equal.

Let it be x.

Fourth angle is of measure 150°.

WKT, sum of the angles of a quadrilateral =

$$\Rightarrow x + x + x + 150^\circ = 360^\circ$$

$$\Rightarrow 3x + 150^\circ = 360^\circ$$

$$\Rightarrow 3x = 210^\circ$$

$$\Rightarrow x = \frac{210^\circ}{3} = 70^\circ.$$

9. Two angles of a Quadrilateral are of measure 65° and the other two angles are equal.

Let it be 'x'

Then WKT,

sum of the angles of a Quadrilateral $= 360°$.

$\Rightarrow 65° + 65° + x + x = 360°$

$\Rightarrow 2x = 360° - 130°$

$\Rightarrow 2x = 230°$

$\Rightarrow x = \frac{230°}{2}$

$\Rightarrow x = 115°$

11. The four angles of a quadrilaterals are as 3:5:7:9.

we have, $\angle A : \angle B : \angle C : \angle D = 3:5:7:9$.

So, Let $\angle A = 3x°$, $\angle B = 5x°$, $\angle C = 7x°$, $\angle D = 9x°$

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 3x + 5x + 7x + 9x = 360°$

$\Rightarrow 24x = 360°$

$\Rightarrow x = \frac{\cancel{360°}^{30}}{\cancel{24}_{2}}$

$\Rightarrow x = 15°$

$\therefore \angle A = 3(15°) = 45°$, $\angle B = 75°$, $\angle C = 105°$, $\angle D = 135°$.

Solution -12:-

Given that,

sum of the two angles of a Quadrilateral $= 180°$

WKT,

sum of the 4 angles in a quadrilateral $= 360°$

$\Rightarrow$ sum of 2 angles + remaining 2 angles $= 360°$

$\Rightarrow$ $180°$ + Remaining 2 angles $= 360°$

$\Rightarrow$ sum of Remaining 2 angles $= 360° - 180°$

$= 180°$.

Solution -13:-

Given that,

$\angle NOM = 45°$, $\angle OMP = 90°$, $\angle ONP = 90°$ and $\angle MPN = ?$

we know that,

$\angle NOM + \angle OMP + \angle ONP + \angle MPN = 360°$

$\Rightarrow$ $45° + 90° + 90° + \angle MPN = 360°$

$\Rightarrow$ $\angle MPN = 360° - 225°$

$\Rightarrow$ $\angle MPN = 135°$.

$\therefore$ Required angle is $135°$.

14. The sides of a quadrilateral are produced in order.

sum of the four exterior angles = 360°

[$\because$ Let $\theta_1, \theta_2, \theta_3, \theta_4$ be the angles of a quadrilateral Then $180-\theta_1, 180-\theta_2$, $180-\theta_3$ and $180-\theta_4$ respectevely.

$\therefore \quad 180-\theta_1 + 180-\theta_2 + 180-\theta_3 + 180-\theta_4 = 360^\circ$

$\Rightarrow \quad 720^\circ - (\theta_1+\theta_2+\theta_3+\theta_4) = 360^\circ$

$\Rightarrow \quad \theta_1+\theta_2+\theta_3+\theta_4 = 360^\circ$].

15:- Given that,

$\angle C = 100^\circ$ and $\angle D = 50^\circ$ and

$\angle A$ and $\angle B$ meet at a point P.

sum of the 4 angles in a Quadrilateral = 360°.

$\angle A + \angle B + \angle C + \angle D = 360^\circ$

$\angle A + \angle B = 360^\circ - 50^\circ - 100^\circ$

$\angle A + \angle B = 210^\circ$

Let $\angle A = \theta^\circ$

$\angle B = 210^\circ - \theta^\circ$

$\therefore \quad \angle PAB = \frac{\theta}{2}, \ \angle PBA = \frac{210^\circ - \theta^\circ}{2} = 105 - \frac{\theta}{2}$.

We know that,

Sum of the angles in a ~~Quadrilateral~~ triangle $= 180^\circ$

$$\angle PAB + \angle PBA + \angle APB = 180^\circ$$

$$\frac{\theta}{2} + 105 - \frac{\theta}{2} + \angle APB = 180^\circ$$

$$\angle P = 180^\circ - 105^\circ$$

$$\angle P = 75^\circ.$$

$\therefore$ The required angle is 75°.

16. The angles in a Quadrilateral ABCD are $\angle A, B, \angle C, \angle D$ are in the ratio $1:2:4:5$.

$\angle A = x^\circ$

$\angle B = 2x^\circ$

$\angle C = 4x^\circ$

$\angle D = 5x^\circ$.

$$x^\circ + 2x + 4x + 5x = 360^\circ$$

$$12x = 360^\circ$$

$$\Rightarrow x = \frac{360^\circ}{12}$$

$$\Rightarrow x = 30^\circ$$

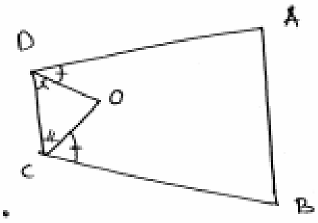
$\therefore$ $\angle A = 30^\circ$, $\angle B = 2(30^\circ) = 60^\circ$

$\angle C = 4(30^\circ) = 120^\circ$, $\angle D = 5(30^\circ) = 150^\circ$.

17.

Consider a quadrilateral ABCD.

let OD & OC be the angular bisectors of D and C respectively.



Let the angle $\angle D$ be $\theta°$.

We know that sum of all angles in quadrilateral = 360°

$$\angle A + \angle B + \angle C + \angle D = 360°$$

$$\angle A + \angle B + \angle C + \theta = 360$$

$$\boxed{\angle C = 360 - (\angle A + B + \theta)°}$$

Here $\angle ODC = \frac{1}{2} \angle D$ (because they are angular bisectors.

$\angle OCD = \frac{1}{2} \angle C$

$$\Rightarrow \boxed{\begin{array}{l} \angle ODC = \frac{\theta°}{2} \\ \angle OCD = 180 - \frac{1}{2}(\angle A + B) - \frac{\theta°}{2} \end{array}}$$

In Δ^{le} ODC,

Sum of all angles = 180°

$\angle ODC + \angle DCO + \angle COD = 180^\circ$

$\cancel{\frac{\theta}{2}} + \angle COD + \cancel{180} - \frac{1}{2}(\angle A + \angle B) - \cancel{\frac{\theta}{2}} = \cancel{180}$

$\Rightarrow \boxed{\angle COD = \frac{1}{2}(\angle A + \angle B)}$

→ Hence proved.

18.

(i) Given Exterior angle $= 180^\circ -$ Given interior angle

Given that,
interior angle $= 160^\circ$

$\Rightarrow$ Exterior angle $= 180^\circ - 160^\circ$
$= 20^\circ$

No. of sides $= \dfrac{360}{\text{Exterior angle}}$

$= \dfrac{360}{20}$

$\boxed{n = 18}$

(ii) Given,
interior angle $= 135^\circ$
Exterior angle $= 180^\circ -$ Given interior angle
$= 180^\circ - 135^\circ = 45^\circ$.

$\therefore$ Number of sides $= \frac{360°}{45°}$

$= 8.$

$\therefore$ No.of sides $= 8.$

(iii) Given

Interior angle $= 175°$

Exterior angle $= 180° -$ interior angle

$= 180° - 175°$

$= 5°$

$\therefore$ No.of sides $= \frac{360°}{5°}$

$= 72°$

(iv) Given,

Interior angle $= 162°.$

Exterior angle $= 180° -$ Interior angle

$= 180° - 162°$

$= 18°.$

$\therefore$ No. of Sides $= \frac{360°}{18}$

$= 20.$

(v) Given,

Interior angle $= 150^\circ$.

Exterior angle $= 180^\circ - 150^\circ$

$= 30^\circ$

No. of sides $= \dfrac{360^\circ}{30^\circ}$

$= 12$.

$\therefore$ No. of sides $= 12$.

19\. Exterior angle of a regular Pentagon $= \dfrac{360^\circ}{5}$

$= 72^\circ$.

$\therefore$ required no. of degrees $= 72^\circ$.

20\. Sum of the exterior angles in a hexagon $= 360^\circ$

Given interior angles are x, $x-5$, $x-5$, $2x-5$, $2x-5$, and $(2x+20)^\circ$

Then Exterior angles are

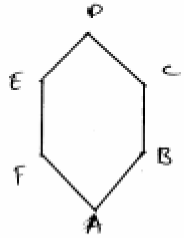
$180(6) - x - x + 5 - x + 5 - 2x + 5 - 2x + 5 - 2x - 20^\circ = 360^\circ$

$\Rightarrow \quad 9x = 1080^\circ - 360^\circ$

$\Rightarrow \quad x = 720/8 \quad \Rightarrow x = 80^\circ$

Solution -21:-

Let ABCDEF be the given hexagon



We know that,

Sum of Exterior angles $= 360°$

$$\angle A_e + \angle B_e + \angle C_e + \angle D_e + \angle E_e + \angle F_e = 180° + 180° \quad \text{——} \; (1)$$

Now,

Sum of interior angles

$= \angle A + \angle B + \angle C + \angle D + \angle E + \angle F$

$= (180 - \angle A_e) + (180 - \angle B_e) + (180 - \angle C_e) + (180 - \angle D_e) + (180 - \angle E_e) + (180 - \angle F_e)$

$= (180 \times 6) - (\angle A_e + \angle B_e + \angle C_e + \angle D_e + \angle E_e + \angle F_e)$

$= 180 \times 6 - 360$

$= 720°$

$= 2(360°)$

$= 2(\angle A_e + \angle B_e + \angle C_e + \angle D_e + \angle E_e + \angle F_e)$

Hence proved

Solution - 22:-

Let the given polygon contain 'n' sides

let the vertices be 1, 2, 3 - - - - - n

Sum of exterior angles = 360°

$$\boxed{\angle 1_e + \angle 2_e + \angle 3_e - - - + \angle n_e = 360°}$$

Sum of interior angles

$= \angle 1 + \angle 2 + \angle 3 - - - + \angle n$

$= (180 - \angle 1_e) + (180 - \angle 2_e) + (180 - \angle 3_e) - - - - - + (180 - \angle n_e)$

$= (180 \times n) - (\angle 1_e + \angle 2_e + - - - \angle n_e)$

$= (180 \times n) - 360$ ———

But given that

Sum of interior angles = $3 \times$ sum of exterior angles

$\Rightarrow (180 \times n) - 360 = 3 \times 360$

$\Rightarrow n = \frac{(3 \times 360 + 360)}{180}$

$\Rightarrow \boxed{n = 8}$

$\Rightarrow$ polygon is octagon.

(23) let the polygon has 'n' sides

Sum of exterior angles = 360 — ①

Sum of interior angles

$$= (180 - \angle 1e) + (180 - \angle 2e) + (180 - \angle 3e) \ldots + (180 - \angle ne)$$

$$= (180 \times n) - 360 \quad — ②$$

$\frac{①}{②} \Rightarrow$

$$\frac{\text{Sum of exterior angles}}{\text{Sum of interior angles}} = \frac{360}{(180 \times n) - 360}$$

But given

$$\frac{\text{Sum of exterior angles}}{\text{Sum of interior angles}} = \frac{1}{5}$$

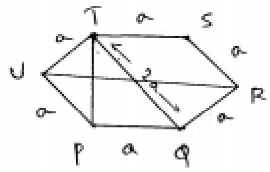
$$\Rightarrow \frac{360}{(180 \times n) - 360} = \frac{1}{5}$$

$$\Rightarrow \frac{2}{n-2} = \frac{1}{5}$$

$$\Rightarrow \boxed{n = 12}$$

No of sides in polygon = 12

24) Let PQRSTU is regular Hexagon.



We know

$\angle TUP = 120^\circ$.

$\angle UTP = \angle TPU$ (because $UT = UP$)

$\Rightarrow \angle TUP + \angle UPT + \angle PTU = 180^\circ$ (sum of angles $= 180^\circ$)

$120^\circ + 2(\angle UTP) = 180^\circ$

$\boxed{\angle UTP = \angle TPU = 30^\circ}$

we know,

$\angle TPQ = \angle UPQ - \angle TPQ$

$\Rightarrow \angle TPQ = 120 - 30$

$\Rightarrow \boxed{\angle TPQ = 90^\circ}$

$\angle PQT = \cos^{-1}\left(\frac{PQ}{QT}\right)$

$= \cos^{-1}\left(\frac{a}{2a}\right)$

$\boxed{\angle PQT = 60^\circ}$

$\boxed{\angle PTQ = 30^\circ}$ (because sum of all angles $= 180^\circ$)