

Mensuration-II Volumes and Surface Areas of a Cuboid and Cube

Ex 21.1

21. MENSURATION-II

Volumes and Surface Areas of a cuboid and a cube

Exercise 21.1

1.)

We have,

i) length = 12 cm, breadth = 8 cm, height = 6 cm.

$$\therefore \text{Volume of the cuboid} = (\text{Length} \times \text{Breadth} \times \text{Height}) \\ = (12 \times 8 \times 6) \text{ cm}^3 = 576 \text{ cm}^3$$

ii) length = 1.2 m = $1.2 \times 100 \text{ cm} = 120 \text{ cm}$,
breadth = 30 cm and height = 15 cm

$$\therefore \text{Volume of the cuboid} = (\text{length} \times \text{Breadth} \times \text{Height}) \\ = (120 \times 30 \times 15) \text{ cm}^3 \\ = 54000 \text{ cm}^3.$$

iii) length = 15 cm, breadth = 1.5 dm = $1.5 \times 10 \text{ cm} = 15 \text{ cm}$,
Height = 8 cm.

$$\therefore \text{Volume of the cuboid} = (\text{length} \times \text{Breadth} \times \text{Height}) \\ = (15 \times 15 \times 8) \text{ cm}^3 \\ = 3000 \text{ cm}^3$$

2.)

We have,

i) side of the cube = 4 cm.

$$\therefore \text{Volume of the cube} = (\text{side})^3 \\ = (4)^3 \text{ cm}^3 = 64 \text{ cm}^3.$$

ii) Side of the cube = 8 cm

$$\therefore \text{Volume of the cube} = (\text{side})^3 = (8)^3 \text{ cm}^3 \\ = 512 \text{ cm}^3.$$

iii) Side of the cube = 1.5 dm = $1.5 \times 10 \text{ cm} = 15 \text{ cm}$.

$$\therefore \text{Volume of the cube} = (\text{side})^3 = (15)^3 \text{ cm}^3 \\ = 3375 \text{ cm}^3$$

iv) Side of the cube = 1.2 m

$$\therefore \text{Volume of the cube} = (\text{side})^3 = (1.2)^3 \text{ m}^3 \\ = 1.728 \text{ m}^3.$$

v) Side of the cube = 25 mm = $\frac{25}{10} \text{ cm} = 2.5 \text{ cm}$.

$$\therefore \text{Volume of the cube} = (\text{side})^3 = (2.5)^3 \text{ cm}^3 \\ = 15.625 \text{ cm}^3.$$

3.) We have,

$$\text{Volume of the cuboid} = 100 \text{ cm}^3$$

length of cuboid = 5 cm, Breadth of cuboid = 4 cm.

$$\therefore \text{Height of cuboid} = \frac{\text{Volume}}{\text{length} \times \text{Breadth}} = \frac{100}{5 \times 4} \text{ cm} \\ = \frac{100}{20} = \underline{\underline{5 \text{ cm}}}$$

4.)

We have,

$$\text{Volume of the cuboidal vessel} = 300 \text{ cm}^3$$

$$\text{length of vessel} = 10 \text{ cm}, \quad \text{Width of the vessel} = 5 \text{ cm}.$$

$$\begin{aligned} \therefore \text{Height of the vessel} &= \frac{\text{Volume}}{\text{length} \times \text{Width}} = \frac{300}{10 \times 5} \text{ cm} \\ &= \frac{300}{50} \text{ cm} = \underline{\underline{6 \text{ cm}}} \end{aligned}$$

5.)

We have,

$$\begin{aligned} \text{Volume of the container} &= 4 \text{ litres} = 4 \times 1000 \text{ cm}^3 \\ &= 4000 \text{ cm}^3. \end{aligned}$$

$$\text{length} = 8 \text{ cm}, \quad \text{Width} = 50 \text{ cm}.$$

$$\begin{aligned} \therefore \text{Height of the container} &= \frac{\text{Volume}}{\text{length} \times \text{Width}} = \frac{4000}{8 \times 50} \text{ cm} \\ &= \frac{4000}{400} = \frac{400}{40} = 10 \text{ cm}. \end{aligned}$$

6.)

We have,

$$\text{Volume of cuboidal block} = 36 \text{ cm}^3$$

$$\text{length of wooden block} = 4 \text{ cm}$$

$$\text{Width of wooden block} = 3 \text{ cm}.$$

$$\begin{aligned} \therefore \text{Height of wooden block} &= \frac{\text{Volume}}{\text{length} \times \text{Width}} = \frac{36}{4 \times 3} \text{ cm} \\ &= \frac{36}{12} = \underline{\underline{3 \text{ cm}}} \end{aligned}$$

7.) Let the edge of the cube be l cm.

Then, its volume ' V ' is given by

$$V = l^3 \text{ cm}^3$$

i) Let V_1 be the volume of the cube when its edge is halved. Then,

$$\Rightarrow V_1 = \left(\frac{l}{2}\right)^3 = \frac{1}{8} l^3 \text{ cm}^3 \quad \left[\because \text{length of the edge of new cube} = \frac{l}{2} \text{ cm} \right]$$

$$\Rightarrow V_1 = \frac{1}{8} V$$

Hence, when the edge is halved then the volume becomes $\frac{1}{8}$ times the original volume.

ii) Let V_2 be the volume of the cube when its edge is trebled. Then,

$$\Rightarrow V_2 = (3l)^3 = 27 l^3 \text{ cm}^3 \quad \left[\because \text{length of the edge of new cube} = 3l \text{ cm} \right]$$

$$\Rightarrow V_2 = 27 V$$

$\therefore$ Hence the volume becomes 27 times the original volume when the edge of cube is trebled.

8.) Let ' V ' be the volume of the cuboid of length ' l ', breadth ' b ' and height ' h ' in cm.

$$\text{Then } V = (l \times b \times h) \text{ cm}^3.$$

i) Let V_1 be the volume of the cuboid when the length is doubled, height is same and breadth is halved.

$\therefore$ length ' l_1 ' of new cuboid is $l_1 = 2l$ cm.

breadth ' b_1 ' of new cuboid is $b_1 = \frac{b}{2}$ cm.

Height is same i.e. $h_1 = h$ cm

$$\text{Then } V_1 = (2l \times \frac{b}{2} \times h) \text{ cm}^3 = (l \times b \times h) \text{ cm}^3 = V.$$

$\therefore$ The volume is same.

ii) Let V_2 be the volume of the cuboid when the length is doubled, height is doubled and breadth is same.

$\therefore$ length of new cuboid is $l_2 = 2l$ cm.

Height of new cuboid is $h_2 = 2h$ cm.

Breadth is same, $b_2 = b$ cm.

$$\text{Then } V_2 = (2l \times 2h \times b) \text{ cm}^3 = 4(l \times b \times h)$$

$$\therefore V_2 = 4V$$

$\therefore$ The volume becomes 4 times the volume of original cuboid.

9) We have,

$$\text{Volume of first cuboid } (V_1) = (5 \times 6 \times 7) \text{ cm}^3 = 210 \text{ cm}^3.$$

$$\text{Volume of second cuboid } (V_2) = (4 \times 7 \times 8) \text{ cm}^3 = 224 \text{ cm}^3$$

$$\text{Volume of third cuboid } (V_3) = (2 \times 3 \times 13) \text{ cm}^3 = 78 \text{ cm}^3.$$

$\therefore$ Total volume of three cuboids is $V = V_1 + V_2 + V_3$

$$\Rightarrow V = (210 + 224 + 78) \text{ cm}^3 = 512 \text{ cm}^3$$

Let side of the new cube be ' x ' cm, then

$$\text{Volume of new cube} = 512 \text{ cm}^3 \quad [\because \text{Volume} = x^3 \text{ cm}^3]$$

$$\Rightarrow x^3 = 512 \text{ cm}^3 \Rightarrow x^3 = 8^3 \text{ cm}^3 \Rightarrow \underline{x = 8 \text{ cm}}$$

10) We have,

$$\begin{aligned} \text{Volume of the given iron piece} &= (50 \times 40 \times 10) \text{ cm}^3 \\ &= 20000 \text{ cm}^3. \end{aligned}$$

$$\text{Since, } 1 \text{ cm}^3 = 8 \text{ gm (given)}$$

$$\begin{aligned} \text{Weight of given iron piece} &= 20000 \text{ cm}^3 \\ &= 20000 \times 8 \text{ gm} \\ &= 1,60,000 \text{ gm} \end{aligned}$$

$$\text{Weight} = \underline{160 \text{ kg}} \quad [\because 1 \text{ kg} = 1000 \text{ gm}]$$

11.) We have,

log of wood size = $3\text{m} \times 75\text{cm} \times 50\text{cm}$.

$$\begin{aligned}\text{So, Volume} &= 3 \times 100\text{cm} \times 75\text{cm} \times 50\text{cm} \\ &= (300 \times 75 \times 50)\text{cm}^3 \\ &= 1125000\text{cm}^3.\end{aligned}$$

Side of wooden cubical block = 25cm .

$$\begin{aligned}\text{Volume of each cubical block} &= (25)^3\text{cm}^3 \quad [\because \text{Volume} = l^3\text{cm}^3] \\ &= 15625\text{cm}^3.\end{aligned}$$

Then we have

Volume of log of wood = $n \times$ Volume of each cubical block
[$\because$ 'n' is no. of cubical blocks].

$$\Rightarrow 1125000 = n \times 15625$$

$$\Rightarrow n = \frac{1125000\text{cm}^3}{15625\text{cm}^3} = \frac{225000}{3125}$$

$$\Rightarrow n = \underline{\underline{72}}.$$

$\therefore$ 72 wooden cubical blocks of side 25cm can be cut

12.) Volume of given silver cuboid is

$$V = 9\text{cm} \times 4\text{cm} \times 3.5\text{cm}$$

$$V = (9 \times 4 \times 3.5)\text{cm}^3 = 126\text{cm}^3.$$

We have,

Volume of each cubical bead = 1.5cm^3 .

Let 'n' be the no. of cubical beads that can be made from the block.

Then,

Volume of silver cuboid = $n \times$ volume of each bead.

$$\Rightarrow 126 \text{ cm}^3 = n \times 1.5 \text{ cm}^3$$

$$\Rightarrow n = \frac{126 \text{ cm}^3}{1.5 \text{ cm}^3} = 84$$

$\therefore$ 84 beads can be made.

(3.) We have,

$$\begin{aligned} \text{Volume of each cuboidal box} &= (2 \times 3 \times 10) \text{ cm}^3 \\ &= 60 \text{ cm}^3 \end{aligned}$$

$$\begin{aligned} \text{Volume of carton} &= (40 \times 36 \times 24) \text{ cm}^3 \\ &= \underline{34560 \text{ cm}^3} \end{aligned}$$

Let ' n ' be the number of cuboidal boxes which can be stored in a carton. Then,

$$\text{Volume of carton} = n \times \text{volume of each cuboidal box}$$

$$\Rightarrow 34560 \text{ cm}^3 = n \times 60 \text{ cm}^3$$

$$\Rightarrow n = \frac{34560 \text{ cm}^3}{60 \text{ cm}^3} = 576$$

$\therefore$ $n = 576$ cuboidal boxes can be stored in the given carton.

14) We have,

$$\begin{aligned}\text{Volume of solid iron, cuboidal block} &= (50 \times 45 \times 34) \text{ cm}^3 \\ &= 76500 \text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of each smaller cuboid cutted} &= 5 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} \\ &= (5 \times 3 \times 2) \text{ cm}^3 \\ &= 30 \text{ cm}^3.\end{aligned}$$

Let 'n' be the number of smaller cuboids obtained from the solid iron cuboidal box.

If there is no wastage of cutting, then

$$\text{Volume of solid iron cuboid} = n \times \text{Volume of each smaller cuboid}$$

$$\Rightarrow 76500 \text{ cm}^3 = n \times 30 \text{ cm}^3$$

$$\Rightarrow n = \frac{76500 \text{ cm}^3}{30 \text{ cm}^3} = 2550$$

$\therefore n = 2550$ cuboids can be obtained.

15) We have,

$$\text{Side of cube A} = 3 \times \text{side of cube B}.$$

Let 'l' be the side of cube 'B'. So,

$$V_1 = \text{Volume of cube B} = l^3 \text{ cm}^3$$

$$V_2 = \text{Volume of cube A} = (3l)^3 \text{ cm}^3 = 27l^3 \text{ cm}^3.$$

$\therefore$ Ratio of volume of cube 'A' to that of cube B is

$$V_2 : V_1 = \frac{V_2}{V_1} = \frac{27l^3}{l^3} = \underline{\underline{27:1}}$$

16) We have,

Volume of deep fridge of inner dimensions $100\text{cm} \times 50\text{cm} \times 120\text{cm}$.

$$\therefore \text{Volume of deep fridge} = (100 \times 50 \times 120) \text{ cm}^3 \\ = 210000 \text{ cm}^3.$$

$$\text{Volume of each ice cream brick} = (20 \times 10 \times 7) \text{ cm}^3 \\ = 1400 \text{ cm}^3$$

Let 'n' be the number of ice cream bricks that can be stored in deep fridge.

$$\therefore \text{Volume of deep fridge} = n \times \text{Volume of each ice cream brick.}$$

$$\Rightarrow 210000 \text{ cm}^3 = n \times 1400 \text{ cm}^3$$

$$\Rightarrow n = \frac{210000 \text{ cm}^3}{1400 \text{ cm}^3} = \frac{2100}{14} = 150$$

$\therefore \underline{n = 150}$ ice cream bricks can be stored.

17) We have,

Volume of first cube of side = 2 cm is

$$V_1 = (2)^3 \text{ cm}^3 = 8 \text{ cm}^3 \quad [\because \text{Volume} = l^3 \text{ cm}^3]$$

Volume of second cube of side = 4 cm is

$$V_2 = (4)^3 \text{ cm}^3 = 64 \text{ cm}^3$$

Then, for comparison take $\frac{V_2}{V_1}$

$$\Rightarrow \frac{V_2}{V_1} = \frac{64 \text{ cm}^3}{8 \text{ cm}^3} = 8 \Rightarrow \underline{V_2 = 8V_1}$$

18) We have,

$$\begin{aligned}\text{Volume of card board box} &= (50 \times 30 \times 0.2 \times 100) \text{ cm}^3 \\ &= (50 \times 30 \times 20) \text{ cm}^3 \\ &= 30000 \text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of each tea packet} &= 10 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm} \\ &= (10 \times 6 \times 4) \text{ cm}^3 \\ &= 240 \text{ cm}^3.\end{aligned}$$

Let 'n' be the number of tea packets that can be placed in a card board. Then,

$$\text{Volume of card board} = n \times \text{Volume of each tea packet}$$

$$\Rightarrow 30000 \text{ cm}^3 = n \times 240 \text{ cm}^3.$$

$$\Rightarrow n = \frac{30000 \cancel{\text{cm}^3}}{240 \cancel{\text{cm}^3}} = \frac{3000}{24} = 125$$

$\therefore n = 125$ tea packets can be placed.

19) We have,

Weight of metal block of size $5 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$ is 1 kg .

$$\text{i.e., Volume} = (5 \times 4 \times 3) \text{ cm}^3 = 60 \text{ cm}^3 = 1 \text{ kg}.$$

Weight of other block of same metal of size

$15 \text{ cm} \times 8 \text{ cm} \times 3 \text{ cm}$ is - - -

$$\therefore \text{Volume of other block} = (15 \times 8 \times 3) \text{ cm}^3 = 360 \text{ cm}^3.$$

$$\text{Weight of the block} = \frac{360 \text{ cm}^3}{60 \text{ cm}^3} \times 1 \text{ kg} = 6 \text{ kg}$$

20) We have,

$$\begin{aligned}\text{Volume of the box} &= 56 \text{ cm} \times 0.4 \text{ m} \times 0.25 \text{ m} \\ &= 56 \text{ cm} \times 0.4 \times 100 \text{ cm} \times 0.25 \times 100 \text{ cm} \\ &= 56 \text{ cm} \times 40 \text{ cm} \times 25 \text{ cm} \\ &= (56 \times 40 \times 25) \text{ cm}^3 \\ &= 56000 \text{ cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of each soap cake} &= 7 \text{ cm} \times 5 \text{ cm} \times 2.5 \text{ cm} \\ &= (7 \times 5 \times 2.5) \text{ cm}^3 \\ &= 87.5 \text{ cm}^3\end{aligned}$$

Let 'n' be the no. of soap cakes that can be placed in the box. Then,

$$\text{Volume of the box} = n \times \text{volume of each soap cake.}$$

$$\Rightarrow 56000 \text{ cm}^3 = n \times 87.5 \text{ cm}^3$$

$$\Rightarrow n = \frac{56000 \text{ cm}^3}{87.5 \text{ cm}^3} = 640$$

$\Rightarrow n = 640$ soap cakes can be placed in the box.

21) We have,

$$\text{Volume of cuboidal box} = 48 \text{ cm}^3.$$

$$\text{Height of cuboidal box} = 3 \text{ cm, length of cuboidal box} = 4 \text{ cm}$$

$$\begin{aligned}\therefore \text{Breadth of cuboidal box} &= \frac{\text{Volume}}{\text{length} \times \text{Height}} = \frac{48}{4 \times 3} \text{ cm} \\ \text{Breadth} &= \frac{48}{12} \text{ cm} = \underline{4 \text{ cm}} \quad \left[\begin{array}{l} \text{Volume} \\ = l \times b \times h \end{array} \right]\end{aligned}$$

Mensuration-II Volumes and Surface Areas of a Cuboid and Cube Ex 21.2

Exercise - 21.2

1.) We have,

i) length = 12 m, breadth = 10 m, height = 4.5 m

$$\therefore \text{Volume} = (12 \times 10 \times 4.5) \text{ m}^3 = 540 \text{ m}^3$$

ii) length = 4 m, breadth = 2.5 m,

$$\text{height} = 50 \text{ cm} = \frac{50}{100} \text{ m} = 0.5 \text{ m}.$$

$$\therefore \text{Volume} = (4 \times 2.5 \times 0.5) \text{ m}^3 = 5 \text{ m}^3$$

iii) length = 10 m, breadth = 25 dm, = $25 \times 10 = 250 \text{ cm}$

$$\text{i.e. breadth} = \frac{25 \times 10}{100} \text{ m} = 2.5 \text{ m, height} = 25 \text{ cm}$$

$$\text{i.e. height} = \frac{25}{100} \text{ m} = 0.25 \text{ m}.$$

$$\left[\begin{array}{l} 1 \text{ m} = 100 \text{ cm} \\ 1 \text{ dm} = 0.1 \text{ m} \end{array} \right]$$

$$\therefore \text{Volume} = (10 \times 2.5 \times 0.25) \text{ m}^3 = 6.25 \text{ m}^3$$

2.) We have,

i) side of cube = 1.5 m = $\frac{100 \times 1.5}{10} \text{ dm} = 15 \text{ dm}$

$$\therefore \text{Volume} = (15)^3 \text{ dm}^3 = 3375 \text{ dm}^3 \left[\begin{array}{l} 1 \text{ dm} = 0.1 \text{ m} \\ 1 \text{ m} = 10 \text{ dm} \end{array} \right]$$

ii) side of cube = 7.5 cm = $\frac{7.5}{10} \text{ dm} = 0.75 \text{ dm}$

$$\left[\begin{array}{l} 1 \text{ dm} = 0.1 \text{ m} \\ 1 \text{ m} = 10 \text{ dm} \end{array} \right]$$

$$\therefore \text{Volume} = (0.75)^3 \text{ dm}^3 = 0.421875 \text{ dm}^3$$

i) Side of cube = $2\text{ dm } 5\text{ cm}$

$$= 2\text{ dm} + \frac{5}{10}\text{ dm}$$

$$= 2.5\text{ dm}$$

$$\therefore \text{Volume} = (2.5)^3\text{ dm}^3 = 15.625\text{ dm}^3$$

3.) We have,

$$\text{Volume of well} = (3 \times 2 \times 5)\text{ m}^3 = 30\text{ m}^3$$

$$\therefore \text{Volume of clay dug out} = 30\text{ m}^3$$

4.) We have,

$$\text{Volume of cuboid} = l \times b \times h = 168\text{ m}^3$$

$$\text{Area of its base} = l \times b = 28\text{ m}^2$$

$$\therefore \text{Height of cuboid } h = \frac{\text{Volume of cuboid}}{\text{Area of base}} = \frac{l \times b \times h}{l \times b}$$

$$h = \frac{168}{28}\text{ m} = 6\text{ m}$$

5.) We have,

$$\text{Volume of the tank} = 8\text{ m} \times 6\text{ m} \times 2\text{ m}$$

$$= (8 \times 6 \times 2)\text{ m}^3 = 96\text{ m}^3$$

$$\therefore \text{Quantity of water it can contain} = 96\text{ m}^3$$

$$= 96 \times (100 \times 100 \times 100)\text{ cm}^3$$

$$= 96000000\text{ cm}^3$$

$$= \frac{96000000}{1000}\text{ litres}$$

$$= 96000\text{ litres} \left[\begin{array}{l} 1\text{ m}^3 \\ = 1000\text{ litres} \end{array} \right]$$

6.) We have,

Capacity of cuboidal tank = 50000 litres

$$= \frac{50000}{1000} \text{ m}^3$$
$$\text{Volume} = 50 \text{ m}^3 \quad \left[\begin{array}{l} \text{1 litre} = 1000 \text{ cm}^3 \\ = \frac{1}{1000} \text{ m}^3 \end{array} \right]$$

Height of tank = 10m, length of tank = 2.5m.

$$\therefore \text{Breadth of tank} = \frac{\text{Volume}}{\text{length} \times \text{height}} = \frac{50}{2.5 \times 10} \text{ m} = \underline{\underline{2 \text{ m}}}$$

7.) We have,

Volume of diesel tanker = $2 \text{ m} \times 2 \text{ m} \times 40 \text{ cm}$

$$= 2 \text{ m} \times 2 \text{ m} \times \frac{40}{100} \text{ m}$$

$$= 2 \text{ m} \times 2 \text{ m} \times 0.4 \text{ m}$$

$$= (2 \times 2 \times 0.4) \text{ m}^3$$

$$= 1.6 \text{ m}^3$$

No. of litres it can hold = $1.6 \text{ m}^3 = 1.6 \times 1000 \text{ litres}$

$\therefore$ No. of litres of diesel it can hold = 1600 litres.

8.) We have,

Volume of the room = $5 \text{ m} \times 4.5 \text{ m} \times 3 \text{ m}$

$$= (5 \times 4.5 \times 3) \text{ m}^3$$

$$= 67.5 \text{ m}^3$$

Volume of air in room = volume of the room
 $= 67.5 \text{ m}^3$

9) We have,

$$\begin{aligned}\text{Volume of water tank} &= 3\text{m} \times 2\text{m} \times 1\text{m} \\ &= (3 \times 2 \times 1)\text{m}^3 = 6\text{m}^3\end{aligned}$$

$$\begin{aligned}\therefore \text{No. of litres of water it can hold} &= 6\text{m}^3 \left[\begin{smallmatrix} 1\text{m}^3 \\ = 1000\text{litre} \end{smallmatrix} \right] \\ &= 6 \times 1000 \text{ litres} \\ &= \underline{\underline{6000 \text{ litres}}}\end{aligned}$$

10) We have,

$$\begin{aligned}\text{Volume of wooden block} &= 6\text{m} \times 75\text{cm} \times 45\text{cm} \\ &= 6 \times 100\text{cm} \times 75\text{cm} \times 45\text{cm} \\ &= (600 \times 75 \times 45)\text{cm}^3 \\ &= 2025000\text{cm}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of each plank} &= 3\text{m} \times 15\text{cm} \times 5\text{cm} \\ &= 3 \times 100\text{cm} \times 15\text{cm} \times 5\text{cm} \\ &= (300 \times 15 \times 5)\text{cm}^3 \\ &= 22500\text{cm}^3.\end{aligned}$$

Let 'n' be the number of planks that can be prepared from wooden block. Then,

$$\text{Volume of wooden block} = n \times \text{Volume of each plank}$$

$$\Rightarrow 2025000\text{cm}^3 = n \times 22500\text{cm}^3$$

$$\Rightarrow n = \frac{2025000\text{cm}^3}{22500\text{cm}^3} = \frac{20250}{225} = \underline{\underline{90}}$$

$\therefore$ n = 90 planks can be prepared.

11) We have,

$$\begin{aligned}\text{Volume of the wall} &= 5\text{m} \times 3\text{m} \times 16\text{cm} \\ &= 5\text{m} \times 3\text{m} \times \frac{16}{100}\text{m} \\ &= (5 \times 3 \times 0.16)\text{m}^3 \\ &= 2.4\text{m}^3 = 2400000\text{cm}^3 \\ \text{Volume of each brick} &= (25 \times 10 \times 8)\text{cm}^3 \quad [1\text{m}^3 = 1000000\text{cm}^3] \\ &= 2000\text{cm}^3.\end{aligned}$$

Let n be number of bricks required to build the wall. If the sand and cement volumes are negligible then,

$$\text{Volume of the wall} = n \times \text{volume of each brick}$$

$$\Rightarrow 2400000\text{cm}^3 = n \times 2000\text{cm}^3$$

$$\Rightarrow n = \frac{2400000\text{cm}^3}{2000\text{cm}^3} = 1200$$

$\therefore n = 1200$ bricks are required

12) We have,

$$\text{Total population of village} = 4000.$$

$$\text{Volume of water required per head per day} = 150\text{ litres.}$$

$$\begin{aligned}\text{Volume of the tank} &= 20\text{m} \times 15\text{m} \times 6\text{m} \\ &= 1800\text{m}^3.\end{aligned}$$

$$\begin{aligned}\text{Volume of water required for total village} \\ \text{per day} &= 4000 \times 150\text{ litres} = 600000\text{ litres}\end{aligned}$$

Let 'n' be the number of days that water will last in tank. Then,

Volume of water tank = n x Volume of water required for total village per day

$$\Rightarrow 1800 \text{ m}^3 = n \times 600000 \text{ litres}$$
$$= n \times \frac{600000}{1000} \text{ m}^3 \quad \left[\begin{array}{l} \because 1 \text{ m}^3 = 1000 \text{ litres} \\ 1 \text{ litre} = \frac{1}{1000} \text{ m}^3 \end{array} \right]$$

$$\Rightarrow 1800 \text{ m}^3 = n \times 600 \text{ m}^3$$

$$\Rightarrow n = \frac{1800 \text{ m}^3}{600 \text{ m}^3} = \underline{\underline{3}}$$

$\therefore n = 3$ days for which water last in the tank.

13) We have,

Volume of the well = Volume of the mud dug-out from earth.

$$= 12 \text{ m} \times 8 \text{ m} \times 6 \text{ m}$$
$$= 672 \text{ m}^3.$$

$$\text{Area of rectangular field} = 12 \times 6$$
$$= 70 \times 60 \text{ m}^2$$
$$= 4200 \text{ m}^2.$$

If the earth dug out is evenly spread on the rectangular field, then the earth level rises by height 'h'. Then,

$$\text{Area of rectangular field} \times h = \text{Volume of well}$$

$$\Rightarrow \text{Volume of rectangular field} = \text{Volume of the well}$$

$$\Rightarrow (4200 \times h) \text{ m}^3 = 672 \text{ m}^3$$

$$\Rightarrow h = \frac{672 \text{ m}^3}{4200 \text{ m}^2} = \frac{4}{25} \text{ m} = \underline{0.16 \text{ m}}$$

$\therefore$ The rise of earth level is $h = 0.16 \text{ m} = 16 \text{ cm}$

14.) We have,

$$\text{Quantity of water pumped} = 3250 \text{ m}^3$$

If 'h' is the rise in the level of water in swimming pool. then,

Volume of swimming pool with height 'h',
length 250 m, width 130 m = Quantity of water.

$$\Rightarrow (250 \times 130 \times h) \text{ m}^3 = 3250 \text{ m}^3$$

$$\Rightarrow h = \frac{3250 \text{ m}^3}{250 \times 130 \text{ m}^2} = \frac{25}{250} \text{ m} = 0.1 \text{ m}$$

$\Rightarrow h = 0.1 \text{ m}$ is the rise in water level,

15.) We have,

$$\text{length of beam} = 5 \text{ m}, \text{ width} = 40 \text{ cm} = 0.4 \text{ m}$$

let 'h' be the thickness of beam. then

$$\text{volume of wood} = 0.6 \text{ m}^3 = \text{volume of beam}$$

$$\Rightarrow 0.6 \text{ m}^3 = 5 \text{ m} \times 0.4 \text{ m} \times h$$

$$\Rightarrow h = \frac{0.6 \text{ m}^3}{5 \times 0.4 \text{ m}^2} = \frac{0.6}{2} \text{ m} = \underline{0.3 \text{ m}}$$

$\therefore h = 0.3 \text{ m}$ is the thickness of beam.

16) We have,

Area of the field = 3 hectares

$$= 3 \times 10000 \text{ m}^2 \quad \left[\because 1 \text{ hectare} = 10000 \text{ m}^2 \right]$$

$$= 30000 \text{ m}^2$$

$$\text{Depth of the water on the field} = \frac{60 \text{ cm}}{100} \text{ m} = 0.06 \text{ m}$$

$$\begin{aligned} \therefore \text{Volume of water} &= \text{Area of field} \times \text{Depth of water} \\ &= (30000 \times 0.06) \text{ m}^3 \\ &= 1800 \text{ m}^3 \\ &= 1800 \times 1000 \text{ litres} \\ \text{Volume of water} &= \underline{1800000 \text{ litres}} \quad \left[\because 1 \text{ m}^3 = 1000 \text{ litres} \right] \\ &= \underline{18 \times 10^5 \text{ litres}} \end{aligned}$$

17) We have,

length of cuboidal beam of wood = 8 m.

if one edge of beam = 0.5 m.

Let third edge be 'h'.

number of cubes of side '1 cm' produced
be 'n' = 4000 (given).

As there is no wastage while slicing the beam,

Volume of beam = $n \times$ Volume of each cube

$$\text{Volume of each cube} = (1\text{cm})^3 = \left(\frac{1}{100}\text{m}\right)^3 = (0.01)^3 \text{m}^3.$$

$$\Rightarrow 8\text{m} \times 0.5 \times h = 4000 \times (0.01)^3 \text{m}^3.$$

$$\Rightarrow h = \frac{4000 \times (0.01)^3 \text{m}^3}{8 \times 0.5 \text{m}^2} = \frac{40000 \times 0.01 \times 0.01 \times 0.01}{8 \times 5}$$

$$\Rightarrow h = \frac{4 \times 0.01}{40} = \underline{0.001 \text{m}}.$$

$\therefore$ length of third edge = 0.001m.

(18) We have,

Dimensions of metal block = 2.25m by 1.5m by 27cm

$$\therefore \text{Volume of metal block} = 2.25\text{m} \times 1.5\text{m} \times \frac{27}{100}\text{m}$$

$$= (2.25 \times 1.5 \times 0.27) \text{m}^3$$

$$= 0.91125 \text{m}^3.$$

Volume of each cube of side 45 cm is

$$V = \left(\frac{45}{100}\text{m}\right)^3 = (0.45)^3 \text{m}^3.$$

Let 'n' be the number of cubes formed.

Then due to melting and recasting

Volume of metal block = $n \times$ Volume of each cube

$$\Rightarrow 0.91125 \text{m}^3 = n \times (0.45)^3 \text{m}^3$$

$$\Rightarrow n = \frac{0.91125 \text{ m}^3}{(0.45)^3 \text{ m}^3} = \frac{0.91125}{0.45 \times 0.45 \times 0.45} =$$

$$\Rightarrow n = \frac{0.91125}{0.091125} = 10$$

$\therefore n = 10$ cubes are formed.

19) We have,

Volume of solid rectangular piece of iron

$$= 6 \text{ m} \times \frac{6}{100} \text{ m} \times \frac{2}{100} \text{ m}$$

$$= 6 \times 100 \text{ cm} \times 6 \text{ cm} \times 2 \text{ cm}$$

$$= (600 \times 6 \times 2) \text{ cm}^3$$

$$= 7200 \text{ cm}^3$$

But, $1 \text{ cm}^3 = 8 \text{ gm}$

$$\therefore \text{Weight of piece} = 7200 \times 8 \text{ gm} = 57600 \text{ gm}$$

$$= \frac{57600}{1000} \text{ kg} = \underline{57.6 \text{ kg}}$$

20) i) $1 \text{ m}^3 = 1 \times (100 \times 100 \times 100) \text{ cm}^3$
 $= \underline{10^6 \text{ cm}^3}$ [$\because 1 \text{ m} = 100 \text{ cm}$]

ii) $1 \text{ litre} = 1000 \text{ cm}^3$
 $= 1000 \times (0.1 \times 0.1 \times 0.1) \text{ dm}^3$ [$\because 1 \text{ dm} = 10 \text{ cm}$
 $1 \text{ cm} = 0.1 \text{ dm}$]

$$1 \text{ litre} = \underline{1 \text{ dm}^3}$$

$$\text{iii)} \quad 1 \text{ kL} = 1000 \text{ litres} = \underline{1 \text{ m}^3} \quad [\because 1 \text{ m}^3 = 1000 \text{ litres}]$$

$$\text{iv)} \quad \text{Side of cube} = 8 \text{ cm}$$

$$\text{Volume} = 8^3 \text{ cm}^3 = \underline{512 \text{ cm}^3}$$

v) We have,

$$\text{Volume of cuboid} = 4000 \text{ cm}^3$$

$$\text{length} = 10 \text{ cm}, \quad \text{breadth} = 8 \text{ cm}, \quad \text{Then}$$

$$\text{height} = \frac{\text{Volume}}{\text{length} \times \text{breadth}} = \frac{4000}{10 \times 8} \text{ cm} = \underline{50 \text{ cm}}$$

$$\begin{aligned} \text{vi)} \quad 1 \text{ cu. dm} &= 1 \text{ dm}^3 = 1 \times (10 \times 10 \times 10) \text{ cm}^3 \\ &= 10^3 (10 \times 10 \times 10) \text{ mm}^3 \\ 1 \text{ dm}^3 &= \underline{10^6 \text{ mm}^3} \quad \left[\begin{array}{l} \because 1 \text{ dm} = 10 \text{ cm} \\ 1 \text{ cm} = 10 \text{ mm} \end{array} \right] \end{aligned}$$

$$\text{vii)} \quad 1 \text{ km}^3 = (1000 \times 1000 \times 1000) \text{ m}^3 = \underline{10^9 \text{ m}^3} \quad [\because 1 \text{ km} = 1000 \text{ m}]$$

$$\text{viii)} \quad 1 \text{ litre} = \underline{1000 \text{ cm}^3} = \underline{10^3 \text{ cm}^3}$$

$$\begin{aligned} \text{ix)} \quad 1 \text{ mL} &= \frac{1}{1000} \times \text{litre} = \frac{1}{1000} \times 1000 \text{ cm}^3 = \underline{1 \text{ cm}^3} \\ \therefore 1 \text{ mL} &= \underline{1 \text{ cm}^3} \quad \left[\begin{array}{l} \because 1 \text{ litre} = 1000 \text{ cm}^3 \\ 1 \text{ mL} = \frac{1}{1000} \text{ litre} \end{array} \right] \end{aligned}$$

$$\begin{aligned} \text{x)} \quad 1 \text{ kL} &= 1 \times 1000 \text{ litre} = 1 \text{ m}^3 = 1 \times (10 \times 10 \times 10) \text{ dm}^3 \\ &= \underline{10^3 \text{ dm}^3} \quad [\because 1 \text{ m} = 10 \text{ dm}] \\ &= 1000 \times 1 \text{ litre} = 1000 \times 1000 \text{ cm}^3 = 10^6 \text{ cm}^3 \\ 1 \text{ kL} &= \underline{10^6 \text{ cm}^3} \end{aligned}$$

Mensuration-II Volumes and Surface Areas of a Cuboid and Cube Ex 21.3

Exercise - 21.3

1. > We have

i) length = 10 cm, breadth = 12 cm, height = 14 cm.

$$\begin{aligned}\text{Surface area of cuboid} &= 2(lb + bh + hl) \\ &= 2(10 \times 12 + 12 \times 14 + 14 \times 10) \text{ cm}^2 \\ &= 2(120 + 168 + 140) \text{ cm}^2 \\ &= 2(428) \text{ cm}^2 \\ &= 856 \text{ cm}^2\end{aligned}$$

ii) length = 6 dm, breadth = 8 dm, height = 10 dm.

$$\begin{aligned}\text{Surface area} &= 2(lb + bh + hl) = 2(6 \times 8 + 8 \times 10 + 10 \times 6) \text{ dm}^2 \\ &= 2(48 + 80 + 60) = 2(188) \text{ dm}^2 \\ &= \underline{376 \text{ dm}^2}\end{aligned}$$

iii) length = 2 m, breadth = 4 m, height = 5 m.

$$\begin{aligned}\text{Surface area} &= 2(lb + bh + hl) = 2(2 \times 4 + 4 \times 5 + 5 \times 2) \text{ m}^2 \\ &= 2(8 + 20 + 10) \text{ m}^2 = 2(38) \text{ m}^2 \\ &= \underline{76 \text{ m}^2}\end{aligned}$$

iv) length = $\frac{3.2 \text{ m}}{10}$, breadth = 30 dm, height = 250 cm.

$$\Rightarrow \text{breadth} = \frac{30}{10} \text{ m} = 3 \text{ m}, \text{ height} = \frac{250}{100} \text{ m} = 2.5 \text{ m}$$

$$\begin{aligned}\text{Surface area} &= 2(lb + bh + hl) = 2\left(\frac{3.2}{10} \times 3 + 3 \times 2.5 + 2.5 \times \frac{3.2}{10}\right) \text{ m}^2 \\ &= 2(9.6 + 7.5 + 8) \text{ m}^2 = 2(25.1) \text{ m}^2 \\ &= 50.2 \text{ m}^2 = \underline{5020 \text{ dm}^2} \quad [\because 1 \text{ m} = 10 \text{ dm}]\end{aligned}$$

2.) We have,

i) Side of cube $(l) = 1.2 \text{ m}$.

$$\begin{aligned}\text{Surface area of cube} &= 6l^2 = 6(1.2)^2 = 6 \times 1.44 \text{ m}^2 \\ &= \underline{8.64 \text{ m}^2}\end{aligned}$$

ii) Edge of cube $(l) = 27 \text{ cm}$.

$$\text{Surface area} = 6l^2 = 6 \times (27)^2 = 6 \times 729 \text{ cm}^2 = \underline{4374 \text{ cm}^2}$$

iii) Edge of cube $(l) = 3 \text{ cm}$.

$$\text{Surface area} = 6l^2 = 6 \times (3)^2 = 6 \times 9 \text{ cm}^2 = \underline{54 \text{ cm}^2}$$

iv) Edge of cube $(l) = 6 \text{ m}$.

$$\text{Surface area} = 6l^2 = 6 \times (6)^2 = 6 \times 36 \text{ m}^2 = \underline{216 \text{ m}^2}$$

v) Edge of cube $(l) = 2.1 \text{ m}$.

$$\text{Surface area} = 6l^2 = 6 \times (2.1)^2 = 6 \times 4.41 \text{ m}^2 = \underline{26.46 \text{ m}^2}$$

3.) We have,

rectangular box of 5 cm by 5 cm by 4 cm .

$$\begin{aligned}\text{Surface area of cuboid} &= 2(lb + bh + hl) \\ &= 2(5 \times 5 + 5 \times 4 + 4 \times 5) \text{ cm}^2 \\ &= 2(25 + 20 + 20) \text{ cm}^2 \\ &= 2(65) \text{ cm}^2 = \underline{130 \text{ cm}^2}\end{aligned}$$

4.) We have

i) Volume of cube $= l^3 = 343 \text{ m}^3$

$$\Rightarrow l^3 = 7^3 \text{ m}^3 \Rightarrow l = 7 \text{ m}$$

$$\therefore \text{Surface area of cube} = 6l^2 = 6(7)^2 \\ = 6 \times (49) \text{ m}^2 = \underline{294 \text{ m}^2}$$

ii) We have,

$$\text{Volume of cube} = l^3 = 216 \text{ dm}^3$$

$$\Rightarrow l^3 = 216 \text{ dm}^3 \Rightarrow l = 6 \text{ dm.}$$

$$\therefore \text{Surface area} = 6l^2 = 6 \times (6)^2 = 6 \times 36 \text{ dm}^2 = \underline{216 \text{ dm}^2}$$

5.) We have

$$\text{i) Surface area of cube} = 96 \text{ cm}^2$$

$$\Rightarrow 6l^2 = 96 \text{ cm}^2 \Rightarrow l^2 = \frac{96}{6} = 16 \text{ cm}^2$$

$$\Rightarrow l = 4 \text{ cm.}$$

$$\therefore \text{Volume of cube} = l^3 = (4)^3 \text{ cm}^3 = \underline{64 \text{ cm}^3.}$$

$$\text{ii) Surface area of cube} = 150 \text{ m}^2$$

$$\Rightarrow 6l^2 = 150 \text{ m}^2 \Rightarrow l^2 = \frac{150}{6} = 25 \text{ m}^2$$

$$\Rightarrow l = 5 \text{ m.}$$

$$\therefore \text{Volume of cube} = l^3 = (5)^3 \text{ m}^3 = \underline{125 \text{ m}^3}$$

6.) We have,

$$\text{Ratio of dimensions} = l : b : h = 5 : 3 : 1$$

$$\Rightarrow \frac{b}{h} = \frac{3}{1} \text{ and } \frac{l}{h} = \frac{5}{1}$$

$$\Rightarrow b = 3h \text{ and } l = 5h.$$

$$\text{Total surface area} = 2(l \times b + b \times h + h \times l) = 414 \text{ m}^2$$

$$\Rightarrow 2(5h \times 3h + 3h \times h + h \times 5h) = 414 \text{ m}^2$$

$$\Rightarrow 2(15h^2 + 3h^2 + 5h^2) = 414 \text{ m}^2$$

$$\Rightarrow 2(23h^2) = 414 \text{ m}^2 \Rightarrow 46h^2 = 414 \text{ m}^2$$

$$\Rightarrow h^2 = \frac{414}{46} \text{ m}^2 \Rightarrow h^2 = 9 \text{ m}^2$$

$$\Rightarrow h = \underline{3 \text{ m.}}$$

$$\therefore \text{length } (l) = 5h = 5 \times 3 = \underline{15 \text{ m.}}$$

$$\text{Breadth } (b) = 3h = 3 \times 3 = \underline{9 \text{ m}}$$

$$\text{height } (h) = h = \underline{3 \text{ m}} \quad \text{are dimensions of cuboid.}$$

7) We have,

$$\text{length} = 25 \text{ cm, breadth} = 0.5 \text{ m} = 0.5 \times 100 \text{ cm} = 50 \text{ cm.}$$

$$\text{height} = 15 \text{ cm of the box (closed).}$$

Then,

$$\text{Area of card board required} = \text{Total surface area of closed box}$$

$$\begin{aligned} \Rightarrow \text{Area of card board required} &= 2(lb + bh + hl) \\ &= 2(25 \times 50 + 50 \times 15 + 15 \times 25) \\ &= 2(1250 + 750 + 375) \text{ cm}^2 \\ &= 2(2375) \end{aligned}$$

$$\text{Area of card board} = \underline{4750 \text{ cm}^2.}$$

8) We have,

$$\text{Edge of a cubic wooden box} = 12 \text{ cm.}$$

$$\begin{aligned} \text{Surface area of cubic wooden box} &= 6l^2 = 6 \times (12)^2 \text{ cm}^2 \\ &= 6(144) \text{ cm}^2 \\ &= \underline{864 \text{ cm}^2} \end{aligned}$$

9.) We have,

dimensions of an oil tin are $26\text{cm} \times 26\text{cm} \times 45\text{cm}$.

let, $l = 26\text{cm}$, $b = 26\text{cm}$, $h = 45\text{cm}$.

Then

Area of tin sheet required for making only

one oil tin = Total surface area of oil tin

$$= 2(lb + bh + hl)$$

$$= 2(26 \times 26 + 26 \times 45 + 45 \times 26) \text{ cm}^2$$

$$= 2(676 + 1170 + 1170) = 2(3016) \text{ cm}^2$$

$$\text{Area for 1 tin} = 6032 \text{ cm}^2$$

Then Area of tin sheet required for making

20 tins = $20 \times$ Area for 1 tin

$$= 20 \times 6032 \text{ cm}^2 = 120640 \text{ cm}^2$$

$$= \frac{120640}{100 \times 100} \text{ m}^2 = 12.064 \text{ m}^2 \left[\because 1\text{cm} = \frac{1}{100}\text{m} \right]$$

But 1m^2 of tin sheet cost Rs. 10.

Then cost of tin sheet for 20 tins

$$= 10 \times \text{Area of tin sheet for 20 tins in m}^2$$

$$= 10 \times 12.064 \text{ m}^2$$

$$\text{Cost} = 120.64$$

$$\therefore \text{Total cost} = \text{Rs. } \underline{120.64}$$

10) We have,

Dimensions of class room as $11\text{m} \times 8\text{m} \times 5\text{m}$

where $l = 11\text{m}$, $b = 8\text{m}$, $h = 5\text{m}$.

Then

$$\text{Area of the floor} = l \times b = 11 \times 8 \text{ m}^2 = 88 \text{ m}^2$$

Area of the four walls (including doors, windows etc.)

$$= 2(lh + bh)$$

$$= 2(11 \times 5 + 8 \times 5) \text{ m}^2 = 2(55 + 40) \text{ m}^2$$

$$= 2(95) = 190 \text{ m}^2$$

Then sum of areas of floor and four walls

is = Area of floor + Area of four walls

$$= 88 + 190 = \underline{278 \text{ m}^2}$$

11) We have,

Dimensions of swimming pool are $20\text{m} \times 15\text{m} \times 3\text{m}$

where $l = 20\text{m}$, $b = 15\text{m}$, $h = 3\text{m}$.

Then area of floor and walls of swimming

$$\text{Pool} = \underbrace{l \times b}_{\text{Floor area}} + \underbrace{2(lh + bh)}_{\text{Area of walls}}$$

$$= (20 \times 15) + 2(20 \times 3 + 15 \times 3)$$

$$= 300 + 2(60 + 45) = 300 + 2(105) = (300 + 210) \text{ m}^2$$

$$\text{Area} = 510 \text{ m}^2$$

Cost of repairing 1m^2 is Rs. 25, Then

$$\text{Cost of repairing floor and walls} = 510 \times 25$$

$$= \text{Rs. } \underline{12750}$$

12) We have,

$$\text{perimeter of a floor} = 30\text{ m} = 2(l+b)$$

$$\Rightarrow l+b = \frac{30}{2} = 15\text{ m and height } (h) = 3\text{ m (given)}$$

$$\text{Area of four walls of room} = 2(lh + bh)$$

$$= 2h(l+b)$$

$$= 2 \times 3 \times 15 \quad \left[\begin{array}{l} \because l+b = 15\text{ m} \\ h = 3\text{ m} \end{array} \right]$$

$$\text{Area of four walls} = 90\text{ m}^2$$

13) We take,

length = l cm, breadth = b cm and height = h cm of a cuboid.

Then

$$\text{Area of floor} = l \times b = lb\text{ cm}^2$$

Product of areas of two adjacent walls

$$= (l \times h) \times (b \times h)$$

$$= lbh^2\text{ cm}^4$$

Product of areas of floor and two adjacent walls is

$$= lb \times lbh^2\text{ cm}^6$$

$$= l^2 b^2 h^2\text{ cm}^6$$

$$= (lbh)^2\text{ cm}^6$$

$$= (\text{Volume of cuboid})^2 \quad (\text{Hence proved})$$

$$\left[\because \text{volume of cuboid} = lbh \right]$$

14) We have,

$$\text{length (l)} = 4.5 \text{ m, breadth (b)} = 3 \text{ m and} \\ \text{height (h)} = 350 \text{ cm} = \frac{350}{100} \text{ m} = 3.5 \text{ m of a room}$$

Then

Area of ceiling + Area of walls

$$\begin{aligned} &= l \times b + 2(l \times h + b \times h) \left[\begin{array}{l} \therefore \text{floor is not} \\ \text{considered} \end{array} \right] \\ &= 4.5 \times 3 + 2(4.5 \times 3.5 + 3 \times 3.5) \\ &= 13.5 + 2(15.75 + 10.5) \\ &= 13.5 + 2(26.25) \\ &= 13.5 + 52.5 = 66 \text{ m}^2 \end{aligned}$$

Cost of plastering 1 m^2 area is Rs. 8

Then cost of plastering the walls and ceiling

$$\begin{aligned} \text{of a room} &= 8 \times [\text{Area of ceiling} + \text{Area of walls}] \\ &= 8 \times 66 = \underline{\underline{\text{Rs. } 528}} \end{aligned}$$

$\therefore$ cost of plastering = Rs. 528

15) We have,

$$\text{Total surface area of cuboid} = 2(lb + bh + hl) = 50 \text{ m}^2$$

$$\begin{aligned} \text{lateral surface area} &= 2(lh + bh) = 30 \text{ m}^2 \\ &= 2h(l + b) = 30 \text{ m}^2 \end{aligned}$$

But we have

$$2 \times (lb) + 2 \times (lh + bh) = 50 \text{ m}^2$$

$$\Rightarrow 2 \times (lb) + 2h(l + b) = 50 \text{ m}^2$$

$$\Rightarrow 2 \times (l \times b) + 30 \text{ m}^2 = 50 \text{ m}^2$$

$$\Rightarrow 2 \times (l \times b) = 50 - 30 = 20 \text{ m}^2$$

$$\Rightarrow l \times b = \frac{20}{2} \text{ m}^2 = \underline{10 \text{ m}^2}$$

$$\therefore \text{Area of its base} = l \times b = \underline{10 \text{ m}^2}$$

16) We have,

Dimensions of class room as $7 \text{ m} \times 6 \text{ m} \times 3.5 \text{ m}$

where $l = 7 \text{ m}$, $b = 6 \text{ m}$, $h = 3.5 \text{ m}$.

Area of four walls including doors and windows

$$= 2(lh + bh)$$

$$= 2(7 \times 3.5 + 6 \times 3.5) \text{ m}^2$$

$$= 2 \times 3.5 \times 13 = \underline{91 \text{ m}^2}$$

Then

Area of walls without doors and windows

$$= (\text{Area including doors and windows}) -$$

(Area occupied by doors and windows)

$$= 91 \text{ m}^2 - 17 \text{ m}^2 \left[\because \text{Area occupied by doors and windows is } 17 \text{ m}^2 \text{ given} \right]$$

$$\text{Area of only walls} = \underline{74 \text{ m}^2}$$

Cost of white washing 1 m^2 area of wall is Rs. 1.50

Then, Total cost of white washing total area of

$$\text{only walls} = 74 \times 1.50 = \underline{\underline{\text{Rs. 111}}}$$

17.) We have,

Dimensions of central hall of a school, i.e.,
length (l) = 80m and height (h) = 8m.

$$\begin{aligned}\text{Area of each door} &= 3\text{m} \times 1.5\text{m} \quad (\text{given}) \\ &= 4.5\text{m}^2\end{aligned}$$

$$\text{Area of each window} = 1.5\text{m} \times 1\text{m} = \underline{1.5\text{m}^2}$$

$$\text{Area of 10 doors} = 10 \times 4.5\text{m}^2 = 45\text{m}^2$$

$$\text{Area of 10 windows} = 10 \times 1.5\text{m}^2 = 15\text{m}^2$$

$$\begin{aligned}\text{Area occupied by windows and doors} &= 45 + 15 \\ &= \underline{60\text{m}^2}\end{aligned}$$

Area of the walls of the hall including doors and windows.

$$= 2(lh + bh)$$

$$= 2(80 \times 8 + b \times 8)\text{m}^2$$

$$= 2(640 + 8b)\text{m}^2$$

Then

Area of only walls i.e., without doors and windows

$$= (\text{Area including doors and walls}) -$$

(Area occupied by doors and walls only)

$$\begin{aligned}\text{Area of only walls} &= [2(640 + 8b)] - 60\text{m}^2\end{aligned}$$

$$= (1280 + 16b - 60)\text{m}^2 = (1220 + 16b)\text{m}^2$$

Cost of white washing walls is Rs. 1.20 per m^2

so, For $1m^2$ cost is Rs 1.20

It is given that the total cost of white washing the walls is Rs 2385.60.

so, cost of white washing walls

$$= (\text{Area of only walls}) \times \text{Rs } 1.20$$

$$= (1220 + 16b) \times 1.20$$

so,

$$\text{Rs. } 2385.60 = (1220 + 16b) \times 1.20$$

Since, cost of white washing only walls area is Rs 2385.60 and only wall area = $(1220 + 16b)m^2$

so,

$$(1220 + 16b) 1.20 = 2385.60$$

$$\Rightarrow (1220 + 16b) = \frac{2385.60}{1.20} = 1988$$

$$\Rightarrow 1220 + 16b = 1988$$

$$\Rightarrow 16b = 1988 - 1220 = 768m.$$

$$\Rightarrow 16b = 768m$$

$$\Rightarrow b = \frac{768}{16}m = \underline{48m}$$

$$\Rightarrow b = \underline{48m}.$$

$\therefore$ Breadth of the hall = 48m

Mensuration-II Volumes and Surface Areas of a Cuboid and Cube Ex 21.4

Exercise - 21.4

1.) We have,
length = 12 m, breadth = 9 m and height = 8 m of a room.

The longest rod that can be placed in a room which is given is nothing but the diagonal length.

$$\begin{aligned}\therefore \text{diagonal} &= \sqrt{l^2 + b^2 + h^2} = \sqrt{12^2 + 9^2 + 8^2} \\ &= \sqrt{144 + 81 + 64} = \sqrt{289} = \underline{17 \text{ m.}}\end{aligned}$$

2.) We have,
dimensions of a cuboid as a, b, c .
If 'V' and 'S' are volume and surface area then

$$V = abc \quad \text{and} \quad S = 2(ab + bc + ca)$$

$$\begin{aligned}\text{Then take } \frac{S}{V} &= \frac{2(ab + bc + ca)}{abc} \\ &= 2 \left(\frac{ab}{abc} + \frac{bc}{abc} + \frac{ca}{abc} \right) \\ &= 2 \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)\end{aligned}$$

$$\Rightarrow \underline{\frac{1}{V} = \frac{2}{S} \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)} \quad (\text{Hence proved})$$

3.) We have,

Areas of three adjacent faces of cuboid as x , y , and z .

let length = l , breadth = b , and height = h of the cuboid. Then

$x = l \times b$, $y = b \times h$, $z = l \times h$. are areas given.

If ' V ' is the volume of the cuboid then

$$V = l b h$$

$$\Rightarrow V^2 = l^2 b^2 h^2 = (l \times b)(b \times h)(l \times h) \quad \left[\begin{array}{l} \therefore \text{rewritten} \\ \text{like this} \end{array} \right]$$

$$\Rightarrow \underline{V^2 = x y z} \quad (\text{Hence proved})$$

4.) We have,

Quantity of water in reservoir = Volume of water = 105 m^3 .

length of base = 12 m and width of base = 3.5 m

then,

$$\text{depth of the water in reservoir} = \frac{\text{volume of water}}{\text{length} \times \text{width}}$$

$$= \frac{105 \text{ m}^3}{12 \times 3.5 \text{ m}^2}$$

$$= \frac{105}{42} \text{ m}$$

$$\text{Depth of water} = 2.5 \text{ m}$$

- 5) Edge length of cube 'A' is 18 cm.
Edge length of cube 'B' is 24 cm.
Edge length of cube 'C' is 30 cm.

Then,

Volume of cube 'A' is $V_1 = (18)^3 \text{ cm}^3$ [∵ Volume of cube = s^3]
 $= 5832 \text{ cm}^3$

Volume of cube 'B' is $V_2 = (24)^3 \text{ cm}^3$
 $= 13824 \text{ cm}^3$

Volume of cube 'C' is $V_3 = (30)^3 \text{ cm}^3$
 $= 27000 \text{ cm}^3$

Then

Total volume of three cubes is

$$V = V_1 + V_2 + V_3 = 5832 + 13824 + 27000$$
$$V = 46656 \text{ cm}^3$$

Let 'x' be the new length of cube 'D' formed after moulding and melting A, B & C.

∴ Volume of cube D = Total volumes of A, B and C

$$\Rightarrow x^3 = 46656 \text{ cm}^3$$

$$\Rightarrow x = \sqrt[3]{46656} \text{ cm}$$

$$\Rightarrow x = 36 \text{ cm.}$$

∴ Edge of bigger cube 'D' is 36 cm.

6.) We have,

Breadth (b) of a room is twice its height (h)

$$\Rightarrow b = 2h \Rightarrow h = \frac{b}{2}$$

Breadth (b) is one half of its length (l)

$$\Rightarrow b = \frac{1}{2}l \Rightarrow l = 2b.$$

Volume of the room = $lbh = 512 \text{ dm}^3$ (given)

$$\Rightarrow (2b) b \left(\frac{b}{2}\right) = 512 \text{ dm}^3 \left[\begin{array}{l} \because h = \frac{b}{2} \\ l = 2b \end{array} \right]$$

$$\Rightarrow b^3 = 512 \text{ dm}^3$$

$$\Rightarrow b^3 = 8^3 \text{ dm}^3$$

$$\Rightarrow \underline{b = 8 \text{ dm}}$$

$$\text{Then length } (l) = 2b = 2 \times 8 \text{ dm} = \underline{16 \text{ dm}}.$$

$$\text{height } (h) = \frac{b}{2} = \frac{8}{2} \text{ dm} = \underline{4 \text{ dm}}.$$

Breadth (b) = $b = \underline{8 \text{ dm}}$ are dimensions.

7.) We have,

length of tank = 12 m , width of tank = 9 m and

depth of tank = 4 m .

Area of iron sheet required = Total surface area of the cuboid

$$= 2(lb + bh + hl)$$

$$= 2(12 \times 9 + 9 \times 4 + 4 \times 12) \text{ m}^2$$

$$= 2(108 + 36 + 48) \text{ m}^2$$

$$\begin{aligned}\text{Area of sheet required} &= 2(192) \text{ m}^2 \\ &= 384 \text{ m}^2.\end{aligned}$$

Let l' be length of sheet and b' is width of sheet.

$$\text{we have } b' = 2 \text{ m (given)}$$

$$\text{Then } l' \times b' = 384 \text{ m}^2 = (192 \times 2) \text{ m}^2$$

$$\Rightarrow l' = 192 \text{ m}, \quad b' = 2 \text{ m}.$$

Then cost of iron sheet is Rs. 5 per metre.

Cost for total length of iron sheet is

$$= 192 \times 5$$

$$= \underline{\underline{\text{Rs. } 960}}$$

87) We have,

Dimensions of tank as $12 \text{ m} \times 8 \text{ m} \times 6 \text{ m}$.

Then, length = l , breadth = b and height = h

$$\Rightarrow l = 12 \text{ m}, \quad b = 8 \text{ m}, \quad h = 6 \text{ m}.$$

Then area of sheet (iron) required for making

the tank = Total surface area of tank with one top open

$$= l \times b + 2(l \times h + b \times h) \quad \left[\because \text{top of the tank is open} \right]$$

$$= 12 \times 8 + 2(12 \times 6 + 8 \times 6) \text{ m}^2$$

$$\begin{aligned}
 \Rightarrow \text{Area of iron sheet} &= 96 + 2(72 + 48) \text{ m}^2 \\
 &= 96 + 2(120) \text{ m}^2 \\
 &= 96 + 240 \\
 &= 336 \text{ m}^2
 \end{aligned}$$

Let ' l ' be the length of iron sheet

' b ' be the width of iron sheet = 4m (given)

$$\text{Then, Area of iron sheet} = l \times b = 336 \text{ m}^2$$

$$\Rightarrow l \times 4 \text{ m} = 336 \text{ m}^2$$

$$\Rightarrow l = \frac{336}{4} = 84 \text{ m.}$$

Cost of iron sheet is Rs 17.50 per metre

Then For $l = 84 \text{ m}$ cost of iron sheet is

$$\Rightarrow \text{Cost} = l \times 17.50 = 84 \times 17.50 = \underline{\text{Rs. 1470}}$$

9.) We have,

Three equal cubes.

Let ' l ' be the edge length of each cube.

Then,

$$\text{Surface area of each cube} = 6l^2$$

sum of surface areas of three cubes

$$= 6l^2 + 6l^2 + 6l^2 = \underline{18l^2}$$

When these three cubes are placed adjacently in a row, they form a cuboid.

$$\therefore \text{length of new cuboid} = l + l + l = 3l$$

$$\text{Breadth of new cuboid} = l$$

$$\text{Height of new cuboid} = l$$

Total surface area of new cuboid

$$= 2(lb + bh + hl)$$

$$= 2(3l \times l + l \times l + l \times 3l)$$

$$= 2(3l^2 + l^2 + 3l^2) = 2(7l^2) = 14l^2$$

$$= 2(7l^2) = 14l^2$$

$\therefore$ Ratio of total surface area of new cuboid to that of sum of the surface areas of three cubes

$$\text{Ratio} = \frac{14l^2}{18l^2} = \frac{14}{18} = \frac{7}{9} = \underline{7:9}$$

10) We have,

Dimensions of a room as 12.5m by 9m by 7m

Dimensions of each door = 2.5m by 1.2m

Dimensions of each window = 1.5m by 1m.

Area of four walls including doors and windows = $2(12.5 \times 7 + 9 \times 7)$

$$= 2(87.5 + 63)$$

$$= 2(150.5)$$

$$\text{Area including doors and windows} = 301 \text{ m}^2$$

Area of 2 doors and 4 windows

$$= 2 \times (2.5 \times 1.2) + 4 \times (1.5 \times 1)$$

$$= 2 \times (3) + 4 \times (1.5)$$

$$= (6 + 6) \text{ m}^2$$

$$= 12 \text{ m}^2$$

Area of only walls (removing areas of 2 doors and 4 windows)

$$= (\text{Area including doors and walls}) - (\text{Area of 2 doors, 4 windows})$$

$$= 301 - 12 = 289 \text{ m}^2$$

Cost of painting walls is Rs. 3.50 per m^2 .

Then for 289 m^2 , the cost is $= 289 \times 3.50$

$$\Rightarrow \text{cost} = \underline{\underline{\text{Rs. } 1011.50}}$$

11) We have,

$$\begin{aligned} \text{Area of the field} &= 150 \text{ m} \times 100 \text{ m} \\ &= 15000 \text{ m}^2 \end{aligned}$$

Amount of the earth dug out in the plot

= Volume of plot with depth 8m

$$= 50\text{m} \times 30\text{m} \times 8\text{m}$$

$$= 12000 \text{ m}^3$$

When the mud dug out is spread evenly in

the field then, the field level is raised.

let 'h' be the field level raised after spreading

Then

Volume of field with 'h' = Amount of earth dug out
in the plot

$$\Rightarrow (\text{Area of field}) \times h = 12000 \text{ m}^3$$

$$\Rightarrow 15000 \text{ m}^2 \times h = 12000 \text{ m}^3$$

$$\Rightarrow h = \frac{12000}{15000} \text{ m} = \frac{12}{15} = \frac{4}{5} = \underline{0.8 \text{ m}}$$

$\therefore$ the level of field raised is 0.8m

12.) We have,

Volume of each cube = 512 cm^3 .

There are 2 cubes joined end to end.

Let 'x' be the edge length of each cube

then

$$l^3 = 512 \text{ cm}^3 = 8^3 \text{ cm}^3$$

$$\Rightarrow l = 8 \text{ cm.}$$

When the two cubes are joined, then

$$\text{length of resulting cuboid} = l + l = 8 + 8 = \underline{16 \text{ cm.}}$$

$$\text{Breadth of resulting cuboid} = l = \underline{8 \text{ cm}}$$

$$\text{Height of resulting cuboid} = l = \underline{8 \text{ cm}}$$

$$\text{Surface area of resulting cuboid} = 2(lb + bh + hl)$$

$$= 2(16 \times 8 + 8 \times 8 + 8 \times 16)$$

$$= 2(128 + 64 + 128)$$

$$= 2(320) \text{ cm}^2$$

$$\text{surface area of cuboid} = \underline{\underline{640 \text{ cm}^2}}$$

13)

We have,

length of room (l) = 12 m, let breadth = b
height = h

cost of preparing the walls at ₹ 1.35 per m^2 .

is ₹. 340.20.

cost of matting the floor at 85 paise per m^2

is ₹. 91.80.

$$\begin{aligned} \text{Area of the floor} &= l \times b = 12 \times b \\ &= 12b \text{ m}^2. \end{aligned}$$

cost of matting floor of area 126 m^2 is Rs. 91.80.

Cost = Rs. 91.80 for 126 m^2

per 1 m^2 cost is 85 paise = 0.85 Rs.

$$\Rightarrow 126 \times 0.85 = 91.80$$

$$\Rightarrow b \times 10.2 = 91.80$$

$$\Rightarrow b = \frac{91.80}{10.2} = \underline{\underline{9 \text{ m}}}$$

$\therefore$ Breadth of room is $b = 9 \text{ m}$.

Then

$$\begin{aligned} \text{Area of 4 walls} &= 2(l \times h + b \times h) \\ &= 2(12h + 9h) \text{ m}^2 \\ &= 2(21h) \text{ m}^2 = \underline{\underline{42h \text{ m}^2}} = 42 \cdot h \text{ m}^2 \end{aligned}$$

Cost of preparing $42h \text{ m}^2$ wall is Rs. 340.20.

per 1 m^2 , cost of preparing the wall is Rs 1.35

$$\Rightarrow 42h \times 1.35 = 340.20$$

$$\Rightarrow 56.7 \times h = 340.20$$

$$\Rightarrow h = \frac{340.20}{56.7} = 6 \text{ m.}$$

$$\Rightarrow \text{Height of the room} = \underline{\underline{6 \text{ m.}}}$$

15) length of hall(l) = 18m

width of hall(b) = 12m.

let ' h ' be the height of the wall.

then

Sum of areas of floor and flat roof

$$= l \times b + l \times b$$

$$= 12 \times 18 + 12 \times 18$$

$$= \underline{432 \text{ m}^2}$$

sum of the areas of 4 walls = $2 \times (l \times h) + 2 \times (b \times h)$

$$= 2[(l \times h) + (b \times h)]$$

$$= 2[18h + 12h]$$

$$= 2(30h) \text{ m}^2$$

$$= 60 \times h \times \text{m}^2$$

Given that, sum of the areas of floor and flat roof is equal to sum of the areas of 4 walls

$$\Rightarrow 60 \times h \times \cancel{\text{m}^2} = 432 \cancel{\text{m}^2}$$

$$\Rightarrow h = \frac{432}{60} \text{ m} = \underline{7.2 \text{ m}}$$

$\therefore$ height of the wall is $h = \underline{7.2 \text{ m}}$

16) We have,

Edge of the metal cube (L) = 12 cm.

$$\text{Volume of the metal cube} = L^3 = (12)^3 \text{ cm}^3 \\ = 1728 \text{ cm}^3$$

When metal cube is melted and formed into three cubes,

Edge of 1st smaller cube (l_1) = 6 cm

Edge of 2nd smaller cube (l_2) = 8 cm.

let ' l_3 ' be the edge of 3rd smaller cube.

$\therefore$ Sum of the volumes of three smaller cubes is equal to volume of the metal cube

$$\Rightarrow l_1^3 + l_2^3 + l_3^3 = L^3 \quad \left[\begin{array}{l} \therefore \text{Volume of cube} \\ = l^3 \end{array} \right]$$

$$\Rightarrow (6)^3 + (8)^3 + l_3^3 = 1728 \text{ cm}^3$$

$$\Rightarrow 216 + 512 + l_3^3 = 1728 \text{ cm}^3$$

$$\Rightarrow 728 \text{ cm}^3 + l_3^3 = 1728 \text{ cm}^3$$

$$\Rightarrow l_3^3 = 1728 - 728 = 1000 \text{ cm}^3$$

$$\Rightarrow l_3^3 = 10^3 \text{ cm}^3$$

$$\Rightarrow l_3 = 10 \text{ cm.}$$

$\therefore$ Edge of 3rd smaller cube is 10 cm

17.) We have,
Dimensions of cinema hall as 100m, 50m, 18m.
Then
Volume of cinema hall = $(100 \times 50 \times 18) \text{ m}^3$
 $= 90000 \text{ m}^3$.

$\therefore$ Volume of air = volume of cinema hall = 90000 m^3 .

Let 'n' be the number of persons that can sit in the hall.

If 150 m^3 of air is required for each person

then for 'n' persons, the air required is
 $= n \times 150 \text{ m}^3$

Then, volume of air in cinema hall is equal to the air required for 'n' persons.

So, $n \times 150 \text{ m}^3 = 90000 \text{ m}^3$

$$\Rightarrow n = \frac{90000 \cancel{\text{m}^3}}{150 \cancel{\text{m}^3}} = \frac{9000}{15} = \underline{600}$$

$\therefore$ $n = 600$ persons can sit in the hall.

18.) We have,
External dimensions of closed wooden box as
48cm, 36cm, 30cm.
Thickness of wood = 1.5 cm

$$\therefore \text{Internal length} = 48 - (2 \times 1.5) = 48 - 3 \\ = 45 \text{ cm.}$$

$$\text{Internal breadth} = 36 - (2 \times 1.5) = 36 - 3 \\ = 33 \text{ cm.}$$

$$\text{Internal height} = 30 - (2 \times 1.5) = 30 - 3 \\ = 27 \text{ cm.}$$

$$\therefore \text{Internal volume} = \frac{\text{Internal length} \times \text{Internal breadth} \times \text{Internal height}}{\text{Internal height}}$$

$$= (45 \times 33 \times 27) \text{ cm}^3$$

$$= 40095 \text{ cm}^3.$$

We have,

bricks of size $6 \text{ cm} \times 3 \text{ cm} \times 0.75 \text{ cm}$

$$\therefore \text{Volume of each brick} = (6 \times 3 \times 0.75) \text{ cm}^3 \\ = 13.5 \text{ cm}^3.$$

Let 'n' be number of bricks that can be put in the box.

$$\therefore \text{Volume of each brick} \times n = \text{Internal volume}$$

$$\Rightarrow n \times 13.5 \text{ cm}^3 = 40095 \text{ cm}^3$$

$$\Rightarrow n = \frac{40095 \text{ cm}^3}{13.5 \text{ cm}^3} = \underline{2970}$$

$\therefore n = \underline{2970}$ bricks can be put into the box.

19) We have,

Ratio of dimensions of rectangular box

i.e. (that is) $l : b : h = 2 : 3 : 4$

$$\Rightarrow \frac{l}{b} = \frac{2}{3} \text{ and } \frac{b}{h} = \frac{3}{4}$$

$$\Rightarrow l = \frac{2b}{3} \text{ and } h = \frac{4}{3}b.$$

Then the total surface area of cuboid is equal to area of sheet of paper required for covering it.

$$\begin{aligned} \text{So, Area of sheet of paper} &= 2(lb + bh + hl) \\ &= 2\left(\frac{2b}{3} \times b + b \times \frac{4}{3}b + \frac{2b}{3} \times \frac{4}{3}b\right) \\ &= 2\left(\frac{2}{3}b^2 + \frac{4}{3}b^2 + \frac{8}{9}b^2\right) \text{ m}^2 \\ &= 2b^2\left[\frac{2}{3} + \frac{4}{3} + \frac{8}{9}\right] \\ &= 2b^2\left[\frac{6+12+8}{9}\right] \\ &= 2b^2\left(\frac{26}{9}\right) \text{ m}^2 \end{aligned}$$

$$\text{Area of sheet of paper} = \frac{52}{9} b^2 \text{ m}^2.$$

Cost of covering with sheet of paper at

$$\begin{aligned} \text{Rs. 8 per m}^2 \text{ is} &= \frac{52}{9} \times b^2 \times 8 \\ &= \text{Rs. } \frac{416}{9} b^2 \end{aligned}$$

Cost of covering it with sheet of paper

$$\text{at Rs. } 9.50 \text{ is } = \frac{52}{9} b^2 \times 9.50 \\ = \text{Rs. } \frac{494}{9} b^2$$

$\therefore$ The difference between the cost of covering it with sheet of paper at Rs 9.50 and Rs. 8 per m^2 is Rs. 1248 given.

$$\Rightarrow \frac{494}{9} b^2 - \frac{416}{9} b^2 = 1248$$

$$\Rightarrow \frac{b^2}{9} (494 - 416) = 1248$$

$$\Rightarrow b^2 (494 - 416) = 1248 \times 9$$

$$\Rightarrow b^2 (78) = 11232$$

$$\Rightarrow b^2 = \frac{11232}{78} = 144$$

$$\Rightarrow b^2 = 12^2 \text{ m}^2$$

$$\Rightarrow \underline{\underline{b = 12 \text{ m}}}$$

$$\therefore \text{ length of box } l = \frac{2b}{3} = \frac{2 \times 12}{3} = \underline{\underline{8 \text{ m}}}$$

$$\text{Breadth of box } b = \underline{\underline{12 \text{ m}}}$$

$$\text{height of box } h = \frac{4b}{3} = \frac{4 \times 12}{3} = \underline{\underline{16 \text{ m}}}$$

are the dimensions of the box.