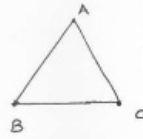


Chapter-15 Properties of Triangles

Exercise-15.1

Solution-01:-



→ By joining of AB, BC and CA figure obtained is Triangle ABC. Where A, B and C are Three non-collinear points.

- (i) the side opposite to $\angle B$ is AC
- (ii) the angle opposite to side AB is $\angle ACB$
- (iii) The vertex opposite to side BC is A
- (iv) The side opposite to vertex B is AC.

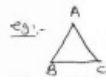
Solution-02:-

No, By definition of a triangle.

Solution-03:-

Triangle:-

A plane figure formed by three non parallel line segments is called a triangle



Triangular Region:-

The interior of $\triangle ABC$ together with the $\triangle ABC$ itself, is called the triangular region of $\triangle ABC$.

Solution-04:-

Triangles observed in the figure are $\triangle ACD$, $\triangle ADB$ and $\triangle ABC$.

Total no. of triangles are '3'

Solution-05:-

Eight triangles observed in the figure are

1. $\triangle ABC$
2. $\triangle ABP$
3. $\triangle ABO$
4. $\triangle BCO$
5. $\triangle POC$
6. $\triangle AOD$
7. $\triangle ACD$
8. $\triangle BOP$

Solution-06:-

→ A plane figure formed by three non parallel line segments is called a triangle where as Triangular Region is the interior of $\triangle ABC$ together with the $\triangle ABC$ itself, is called the triangular region ABC .

Solution-07:-

(i) Triangle:-

A Plane figure formed by three non-parallel line segments is called a triangle.

(ii) Parts or Elements of a triangle:-

The three sides AB, BC, CA and three angles $\angle A, \angle B, \angle C$ of a $\triangle ABC$ are together called the six parts or elements of $\triangle ABC$.

(iii) Scalene Triangle:-

A triangle whose no two sides are equal, is called Scalene Triangle.

(iv) Isosceles triangle:-

A triangle whose two sides are equal, is called isosceles triangle.

(v) Equilateral triangle:-

A triangle whose all sides are equal, is called Equilateral triangle.

(vi) Acute triangle:-

A triangle whose all the angles are acute is called Acute triangle.

vii) Right triangle:-

A triangle whose one of the angles is right angle is called Right triangle.

viii) Obtuse triangle:-

A triangle whose one of the angles is obtuse angle is called Obtuse triangle.

(ix) Interior of a Triangle:-

The interior of a triangle is made up of all such points P of the plane, are enclosed by the triangle.

(x) Exterior of a Triangle:-

The Exterior of a triangle is made up of part of the plane which consists of those points Q, which are neither on the triangle nor in its interior.

Solution - 08:-

(i) $AB \neq BC \neq CA$

Scalene triangle

(ii) $PA = PR$; $AR = 5\text{cm}$.

Isosceles triangle.

(iii) $XY = YZ = ZX$

Equilateral triangle

(iv) $UV \neq VW \neq UW$

Scalene triangle

(v) Two sides are equal

→ Isosceles triangle

Solution - 09:-

(i) Angle given is 90°

∴ Right angle Triangle

(ii) Angle given is 120° [$120^\circ > 90^\circ$]

∴ Obtuse triangle

(iii) All the angles are acute [$< 90^\circ \rightarrow$

Acute]

∴ Acute triangle.

(iv) Right triangle

(v) Obtuse triangle.

Solution-10:-

- (i) Three
- (ii) Three
- (iii) Three
- (iv) Six
- (v) Scalene
- (vi) Isosceles
- (vii) Equilateral
- (viii) Right triangle
- (ix) Acute triangle
- (x). Obtuse triangle.

Exercise-15.2

Exercise -15.2 :-

Solution-01:-

Let $\triangle ABC$ be a triangle such that $\angle B = 105^\circ$ & $\angle C = 30^\circ$
Then, we have find the measure of the third angle A .

$$\text{Now, } \angle B = 105^\circ \text{ and } \angle C = 30^\circ$$

$$\Rightarrow \angle B + \angle C = 105^\circ + 30^\circ = 135^\circ$$

By the angle sum property of a triangle,
we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle A + 135^\circ = 180^\circ$$

$$\Rightarrow \angle A = 180^\circ - 135^\circ$$

$$\Rightarrow \angle A = 45^\circ$$

Solution-02:-

Let $\triangle ABC$ be a triangle such that $\angle A = 130^\circ$ and
the other two angles $\angle B = \angle C$

By the angle sum property of a triangle, we
have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$130^\circ + \angle B + \angle C = 180^\circ \quad [\angle A = 130^\circ \text{ and } \angle C = \angle B]$$

$$130^\circ + 2\angle C = 180^\circ \Rightarrow 2\angle C = 180^\circ - 130^\circ$$

$$\therefore \angle C = 25^\circ = \angle B$$

$$\therefore \angle A = 130^\circ, \angle B = 25^\circ \text{ \& } \angle C = 25^\circ$$

Solution-03:-

Given that,

Three triangles of triangles are equal.

The measured angles be $\angle A = \angle B = \angle C$.

By the angle sum property of a triangle,
we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle A + \angle A + \angle A = 180^\circ$$

$$3\angle A = 180^\circ$$

$$\angle A = \frac{180^\circ}{3} = 60^\circ$$

$$\therefore \angle A = \angle B = \angle C = 60^\circ$$

Solution-04:-

Given that,

three angles of a triangle in the ratio 1:2:3

Let the measured angles be $x, 2x$ & $3x$.

By the angle sum property of a triangle
we have

$$x + 2x + 3x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

$$1^{\text{st}} \text{ angle} \rightarrow 30^\circ$$

$$2^{\text{nd}} \text{ Angle} \rightarrow 2(30^\circ) \rightarrow 60^\circ \quad [2x]$$

$$3^{\text{rd}} \text{ Angle} \rightarrow 3(30^\circ) \rightarrow 90^\circ \quad [3x]$$

Solution-05:-

Given angles of a triangle are

$$(x-40)^\circ, (x-20)^\circ \text{ \& } (\frac{1}{2}x-10)^\circ.$$

By the Angle Sum Property of a triangle,
we have

$$(x-40)^\circ + (x-20)^\circ + (\frac{1}{2}x-10)^\circ = 180^\circ$$

$$2x + \frac{x}{2} - 70^\circ = 180^\circ \Rightarrow \frac{4x + x - 140^\circ}{2} = 180^\circ$$

$$5x - 140^\circ = 360^\circ$$

$$5x = 360^\circ + 140^\circ$$

$$x = \frac{500^\circ}{5}$$

$$\boxed{x = 100^\circ}$$

Required angles are

$$(x-40)^\circ = (100-40)^\circ = 60^\circ$$

$$(x-20)^\circ = (100-20)^\circ = 80^\circ$$

$$(\frac{x}{2}-10)^\circ = (\frac{100}{2}-10)^\circ$$

$$= (50-10)^\circ$$

$$= 40^\circ$$

$\therefore$ Required angles are $40^\circ, 80^\circ$ and 60° .

Solution-06:-

Given that,

Angles are arranged in the ascending
order say ~~$A > B > C$~~ . $A < B < C$. and

Difference between two angles is 10° .

Then the measured angles be say

$$x, x+10 \text{ and } x+20.$$

By Angle Sum property of a triangle, we have

$$x + x + 10 + x + 20 = 180^\circ$$

$$3x + 30^\circ = 180^\circ$$

$$3x = 180^\circ - 30^\circ$$

$$x = \frac{150^\circ}{3}$$

$$x = 50^\circ$$

$\therefore$ Required angles are $x, x+10$ and $x+20$

$$LA = x = 50^\circ$$

$$LB = x + 10 = 60^\circ$$

$$LC = x + 10 + 10 = x + 20^\circ = 50^\circ + 20^\circ = 70^\circ$$

$\therefore$ Three angles are $50^\circ, 60^\circ, 70^\circ$.

Solution-08:-

Given that,

one angle of a triangle is equal to the sum of the other two

Let the measures of angles be x, y, z

By the Angle sum property of a Δ^e , we have

$$x + y + z = 180^\circ$$

$$x + x = 180^\circ \quad [x = y + z]$$

$$2x = 180^\circ$$

$$x = 90^\circ$$

If one angle is 90° then the given triangle is a right angle triangle.

Solution-09:-

Given that,

each angle of a triangle is less than the sum of the other two.

measure of angles be x, y and z

$$x < y + z$$

$$y < x + z$$

$$z < x + y$$

$$\therefore x < 90^\circ, y < 90^\circ, z < 90^\circ \quad [\text{By the Angle sum Property of a } \Delta^e]$$

$\therefore$ The triangle is acute angled.

Solution-10:-

$$(i) \quad 63^\circ + 37^\circ + 80^\circ = 180^\circ$$

[By the angle sum of Properties of a triangle].

Angles form a triangle

$$(ii) \quad 45^\circ + 61^\circ + 73^\circ \neq 180^\circ$$

$$(iii) \quad 59^\circ + 62^\circ + 61^\circ \neq 180^\circ$$

$$(iv) \quad 45^\circ + 45^\circ + 90^\circ = 180^\circ$$

Angles form a triangle.

$$(v) \quad 20^\circ + 20^\circ + 125^\circ \neq 180^\circ$$

Solution-11

Given that,

angles of a triangle in ratio $3:4:5$

measure of Angles be $3x, 4x$ and $5x$

Angle sum Property of a triangle, we have

$$3x + 4x + 5x = 180^\circ$$

$$12x = 180^\circ$$

$$x = \frac{180^\circ}{12} = 15^\circ$$

$$\text{Smallest angle} = 3x$$

$$= 3(15^\circ)$$

$$= 45^\circ$$

Solution-12:-

Given,

Two acute angles of a right angle triangle are equal.

Right triangle:-

Triangle whose one of the angle is right angle.

measured angles be $x, x, 90^\circ$

By Angle sum property of a triangle, we have

$$x + x + 90^\circ = 180^\circ$$

$$2x = 180^\circ - 90^\circ$$

$$x = \frac{90^\circ}{2} \Rightarrow x = 45^\circ$$

The two angles are $45^\circ, 45^\circ$.

Solution-13:-

Angle of a triangle is greater than the sum of the other two.

measure of angles be x, y and z .

$$x > y + z \text{ (or)}$$

$$y > x + z \text{ (or)}$$

$$z > x + y.$$

x (or) y (or) $z > 90^\circ$ which is obtuse

$\therefore$ triangle is a obtuse angle.

Solution-14:-

$$\angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA.$$

We know that the sum of angles of a triangle is 180° .

$\therefore$ In $\triangle ABC$, we have

$$\angle CAB + \angle ABC + \angle BCA = 180^\circ \dots (i)$$

In $\triangle ACD$, we have.

$$\angle PAC + \angle ACD + \angle CPA = 180^\circ \dots (ii)$$

In $\triangle ADE$, we have

$$\angle EAD + \angle ADE + \angle DEA = 180^\circ \dots (iii)$$

In $\triangle AEF$, we have.

$$\angle FAE + \angle AEF + \angle EFA = 180^\circ \dots (iv)$$

Adding (i), (ii), (iii) and (iv) we get

$$\begin{aligned} & (\angle CAB + \angle ABC + \angle BCA) + (\angle PAC + \angle ACD + \angle CPA) \\ & + (\angle EAD + \angle ADE + \angle DEA) + (\angle FAE + \angle AEF + \angle EFA) \\ & = 720^\circ \end{aligned}$$

$$\therefore \angle FAB + \angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFA = 720^\circ$$

$$[\angle FAB = \angle CAB + \angle PAC + \angle EAD + \angle FAE;$$

$$\angle BCD = \angle BCA + \angle ACD;$$

$$\angle CDE = \angle CPA + \angle ADE;$$

$$\angle DEF = \angle AEF + \angle DEA; \angle EFA = \angle EFA]$$

Solution-15:-

(i) By Angle Sum Property of a triangle we have

$$\angle 40^\circ + \angle 30^\circ + \angle Z = 180^\circ$$

$$\angle Z = 180^\circ - 70^\circ$$

$$\angle Z = 110^\circ$$

$\angle X$ and 40° are corresponding angles

$$\angle X = 40^\circ$$

$\angle Y$ and 30° are corresponding angles

$$\angle Y = 30^\circ$$

(ii) By Angle Sum Property of a $\triangle$, we have

In $\triangle ABC$

$$\angle x^\circ + 40^\circ + 90^\circ = 180^\circ$$

$$\angle x = 180^\circ - 130^\circ$$

$$\angle x = 50^\circ$$

In $\triangle ACD$

$$45^\circ + 90^\circ + \angle y^\circ = 180^\circ$$

$$\angle y^\circ = 180^\circ - 135^\circ$$

$$\angle y = 45^\circ$$

(iii) By Angle sum property of a $\triangle ABC$, we have.

In $\triangle ABC$

$$50^\circ + 50^\circ + \angle x^\circ = 180^\circ$$

$$\angle x^\circ = 180^\circ - 100^\circ$$

$$\angle x^\circ = 80^\circ$$

~~Q~~ $\angle x^\circ$ and $\angle y^\circ$ is a linear pair

$$\angle x^\circ + \angle y^\circ = 180^\circ$$

$$80^\circ + \angle y^\circ = 180^\circ$$

$$\angle y^\circ = 100^\circ$$

(iv) By Angle sum property of triangle, we have

In $\triangle ADE$

$$\angle x^\circ + 50^\circ + 40^\circ = 180^\circ$$

$$\angle x^\circ = 180^\circ - 90^\circ$$

$$\angle x^\circ = 90^\circ$$

$$\angle y = 80^\circ \quad [\text{corresponding angles}]$$

$$\angle z = 40^\circ \quad [\text{corresponding angles}]$$

Solution-16:-

Given that angle D = 60°

Let the other angles be x and $2x$

[$\because$ two angles are in the ratio 1:2]

$$60^\circ + 2x + x = 180^\circ$$

$$60^\circ + 3x = 180^\circ$$

$$x = \frac{120^\circ}{3}$$

$$x = 40^\circ$$

two angles are $40^\circ, 80^\circ$

Solution-17:-

Given one angle of a triangle = 100°

the two angles are in the ratio-2:3

measures of angles be $2x$ and $3x$

respectively

$$2x + 3x + 100^\circ = 180^\circ$$

$$5x + 100^\circ = 180^\circ$$

$$5x = 180 - 100$$

$$x = 16^\circ$$

$\therefore$ Two angles are $32^\circ = 2(16^\circ)$

$$48^\circ = 3(16^\circ)$$

Solution-18:-

In a $\triangle ABC$

By Angle sum property of a triangle, we have

$$3\angle A + 4\angle B + 6\angle C = 180^\circ$$

$$3\angle A + 3\angle A + 3\angle A = 180^\circ \quad [\because 3\angle A = 4\angle B = 6\angle C]$$

$$9\angle A = 180^\circ$$

$$\angle A = 20^\circ$$

$$3\angle A = 3(20^\circ) = 60^\circ$$

$$4\angle B = 60^\circ \Rightarrow \angle B = 15^\circ$$

$$6\angle C = 60^\circ \Rightarrow \angle C = \frac{60^\circ}{6} = 10^\circ$$

Solution-20:-

Given $\angle A = 100^\circ$

$$\angle PAB = 50^\circ, \angle P = 90^\circ$$

$$\therefore \angle B + \angle PAB + \angle P = 180^\circ$$

$$\angle B = 180^\circ - 140^\circ = 40^\circ$$



Solution - 21:-

Given that,

$$\angle A = 50^\circ, \angle B = 70^\circ$$

By Angle sum property of a triangle, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$50^\circ + 70^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 120^\circ$$

$$\angle C = 60^\circ$$

Given that CD bisects AB in D .

In $\triangle ACD$

$$\angle ACD + \angle ADC + \angle A = 180^\circ \quad [\text{A.S.P}]$$

$$50^\circ + 30^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 100^\circ$$

In $\triangle DCB$

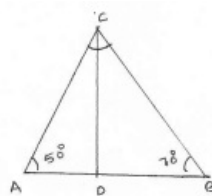
$$\angle B + \angle C + \angle CDB = 180^\circ$$

$$70^\circ + 30^\circ + \angle CDB = 180^\circ$$

$$\angle CDB = 180^\circ - 100^\circ$$

$$\therefore \angle CDB = 80^\circ$$

$$\therefore \angle CDB = 80^\circ, \angle ADC = 100^\circ$$



Solution - 22:-

Given,

$$\text{In } \triangle ABC, \angle A = 60^\circ, \angle B = 80^\circ$$

1) By Angle sum property of a triangle, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$60^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 140^\circ$$

$$\angle C = 40^\circ$$

(ii) In $\triangle BOC$, we have

By Angle sum property of a triangle, we have

$$\angle B = 40^\circ, \angle C = 20^\circ, \angle BOC = ?$$

$$\angle B + \angle C + \angle BOC = 180^\circ \quad [\text{A.S.P}]$$

$$40^\circ + 20^\circ + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - 60^\circ$$

$$\angle BOC = 120^\circ$$

Solution-23:-

Given that,

The bisectors of the acute angles of a right angle meet at O.

$$\angle B = 90^\circ$$

$$\angle A + \angle C = 90^\circ$$

$$\angle A + \angle C = 90^\circ$$

$$\therefore \angle CAO + \angle OCA = \frac{90^\circ}{2} = 45^\circ \quad [\text{bisectors}]$$

In $\triangle OAC$, we have

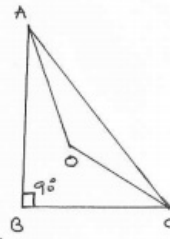
By Angle sum property of a triangle

$$\angle CAO + \angle OCA + \angle COA = 180^\circ$$

$$45^\circ + \angle COA = 180^\circ$$

$$\angle COA = 180^\circ - 45^\circ$$

$$\angle COA = 135^\circ$$



Exercise-15.3

Exercise-15.3

Solution-01:-

(i) The interior adjacent angle of $\angle Bx$ is $\angle ABC$

(ii) $\angle BAC, \angle ACB$;
 $\angle ABC, \angle ACB$.

interior opposite angles to exterior $\angle Bx$.

Solution-02:-

Given,

exterior angle of a vertex $= 95^\circ$

interior opposite angle $= 55^\circ$



Let ABC be a triangle whose side BC produced to D to form an exterior angle $\angle ACD$ such that $\angle ACD = 95^\circ$.

Let $\angle B = 55^\circ$. By exterior angle theorem, we have

$$\angle ACD = \angle B + \angle A$$

$$95^\circ = 55^\circ + \angle A$$

$$\angle A = 95^\circ - 55^\circ$$

$$\angle A = 40^\circ$$

Now, By using Angle sum property of a triangle, we have

$$\angle A + \angle B + \angle C = 180^\circ$$

$$40^\circ + 55^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 95^\circ$$

$$\angle C = 85^\circ$$

The angles of a triangle are $40^\circ, 55^\circ$ and 85° .

Solution-06:-

Given

$$\angle ACD = 105^\circ$$

$$\angle EAF = 45^\circ$$

$\angle ACD$ and $\angle ACB$ is a linear pair

$$\angle ACD + \angle ACB = 180^\circ$$

$$105^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 105^\circ$$

$$\angle ACB = 75^\circ$$

Since $\angle ACB$ and $\angle EAF$ are vertically opposite angles

$$\angle CAB = \angle EAF = 45^\circ$$

By using Angle sum property of a triangle, we have

$$\angle CAB + \angle ACB + \angle ABC = 180^\circ$$

$$45^\circ + 75^\circ + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ$$

$$\angle ABC = 60^\circ$$

Solution-07:-

Given

$$\angle A : \angle B : \angle C = 3 : 2 : 1$$

By Angle sum property of a triangle, we have
measure of angles be $3x$, $2x$ and x

$$x + 2x + 3x = 180^\circ$$

$$6x = 180^\circ$$

$$x = 30^\circ$$

$$\angle A = 30^\circ (3) = 90^\circ$$

$$\angle B = 60^\circ$$

$$\angle C = 30^\circ$$

$$\angle ACB + \angle ACE + \angle ECD = 180^\circ$$

[Linear pair]

$$90^\circ + 30^\circ + \angle ECD = 180^\circ$$

$$\angle ECD = 180^\circ - 120^\circ$$

$$\angle ECD = 60^\circ$$

Solution-08:-

No, since sum of interior angles A and B
 $= 183^\circ > 180^\circ$.

Solution-09:-

Given, $\angle FCD = 50^\circ$.

$\angle BAD = ?$.

In $\triangle ABD$, we have

$\angle ADB = 90^\circ$.

$\angle FCD$ and $\angle BAD$ are vertically opposite angles

$\angle FCD = \angle BAD$

$\therefore \angle BAD = 50^\circ$.

Solution-10:-

Given

$\angle CFB = 60^\circ$,

$\angle FDC = 90^\circ$

$\angle BCF = ?$.

$\angle CFB + \angle FDC + \angle BCF = 180^\circ$ [A.S.P]

$\angle BCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

$\angle BCF + x^\circ = 180^\circ \Rightarrow x^\circ = 180^\circ - 30^\circ = 150^\circ$
[Linear pair] $\therefore x^\circ = 150^\circ$

Here,

$\angle AED + 120^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \angle AED = 180^\circ - 120^\circ = 60^\circ$

We know that the sum of all angles of a triangle is 180° .

Therefore, for $\triangle ADE$, we can say that:

$\angle ADE + \angle AED + \angle DAE = 180^\circ$

$\Rightarrow 60^\circ + \angle ADE + 30^\circ = 180^\circ$

Or,

$\angle ADE = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

From the given figure, we can also say that:

$\angle FDC + 90^\circ = 180^\circ$ (Linear pair)

$\Rightarrow \angle FDC = 180^\circ - 90^\circ = 90^\circ$

Using the above rule for $\triangle CDF$, we can say that:

$\angle CDF + \angle DCF + \angle DFC = 180^\circ$

$\Rightarrow 90^\circ + \angle DCF + 60^\circ = 180^\circ$

$\angle DCF = 180^\circ - 60^\circ - 90^\circ = 30^\circ$

Also,

$\angle DCF + x = 180^\circ$ (Linear pair)

$\Rightarrow 30^\circ + x = 180^\circ$

Or,

$x = 180^\circ - 30^\circ = 150^\circ$

Q11

(i)

Here,

$$\angle BAF + \angle FAD = 180^\circ \text{ (Linear pair)}$$

$$\Rightarrow \angle FAD = 180^\circ - \angle BAF = 180^\circ - 90^\circ = 90^\circ$$

Also,

$$\angle AFE = \angle ADF + \angle FAD \text{ (Exterior angle property)}$$

$$\angle ADF + 90^\circ = 130^\circ$$

$$\angle ADF = 130^\circ - 90^\circ = 40^\circ$$

(ii)

We know that the sum of all the angles of a triangle is 180° .

Therefore, for $\triangle BDE$, we can say that :

$$\angle BDE + \angle BED + \angle DBE = 180^\circ.$$

$$\Rightarrow \angle DBE = 180^\circ - \angle BDE - \angle BED = 180^\circ - 90^\circ - 40^\circ = 50^\circ \dots (i)$$

Also,

$$\angle FAD = \angle ABC + \angle ACB \text{ (Exterior angle property)}$$

$$\Rightarrow 90^\circ = 50^\circ + \angle ACB$$

Or,

$$\angle ACB = 90^\circ - 50^\circ = 40^\circ$$

$$(iii) \angle ABC = \angle DBE = 50^\circ \text{ [From (i)]}$$

Solution -11:-

$$\text{Given } \angle AFE = 130^\circ$$

$$\angle CAB = 90^\circ$$

$$\angle DEB = 90^\circ$$

(i) $\triangle BDE$

$$\angle AFE = 130^\circ$$

$$\angle ACB = 40^\circ$$

$$\therefore \angle BAE + \angle ACB + \angle ABC = 180^\circ$$

$$\angle ABC + \angle BAE = 180^\circ - 40^\circ - 90^\circ$$

$$\angle ABC = 50^\circ$$

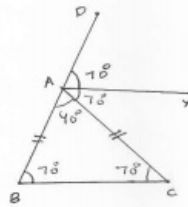
$$(i) \angle BDE = 180^\circ - 50^\circ - 90^\circ$$

$$= 40^\circ$$

$$(ii) \angle BCA = 40^\circ$$

$$(iii) \angle ABC = 50^\circ$$

Solution-12:-



Given,

$$\angle DAX = 70^\circ$$

AX Bisects exterior angle DAC

$$\text{So, } \angle DAX = \angle DAC$$

$$\therefore \angle DAC = 70^\circ$$

$\angle DAX$, $\angle DAC$ and $\angle BAC$ is linear pair

$$\angle DAX + \angle DAC + \angle BAC = 180^\circ$$

$$70^\circ + 70^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 140^\circ$$

$$\angle BAC = 40^\circ$$

$$\text{Let } \angle B = \angle C = x$$

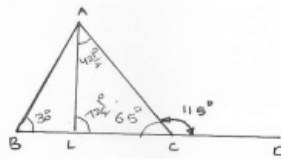
By using Angle sum property of a triangle we have, In $\triangle ABC$

$$40^\circ + x + x = 180^\circ \Rightarrow 2x = 180^\circ - 40^\circ = 140^\circ$$

$$\Rightarrow x = 70^\circ$$

$$\therefore \angle B = \angle C = x = 70^\circ$$

Solution-13:-



Given,

$$\angle ABC = 30^\circ \text{ \& } \angle ACD = 115^\circ$$

we have, In $\triangle ABC$

By using Angle sum property of a triangle

$$30^\circ + 65^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 95^\circ$$

$$\angle BAC = 42\frac{1}{2}^\circ + 42\frac{1}{2}^\circ$$

$$\Rightarrow \angle BAL + \angle CAL = 42\frac{1}{2}^\circ + 42\frac{1}{2}^\circ$$

$$\therefore \angle BAL = 42\frac{1}{2}^\circ = \angle CAL$$

By using Angle sum property of a triangle, we have,

$$65^\circ + 42\frac{1}{2}^\circ + \angle ALC = 180^\circ$$

$$\angle ALC = 180^\circ - 107\frac{1}{2}^\circ$$

$$\angle ALC = 72\frac{1}{2}^\circ$$

$$\angle APD = 9^\circ$$

By using Angle Sum property of a triangle we have

In $\triangle ABC$,

$$60^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 140^\circ$$

$$\angle C = 40^\circ$$

In $\triangle ADC$,

$$40^\circ + 40^\circ + \angle ADC = 180^\circ$$

$$\angle ADC = 180^\circ - 40^\circ - 40^\circ$$

$$\angle ADC = 100^\circ$$

$$\angle ADP + \angle PDC = 100^\circ$$

$$\angle ADP = 100^\circ - 15^\circ$$

$$= 85^\circ$$

By using Angle sum property of a $\triangle$ we have

$$\angle APD + \angle ADP + \angle PAD = 180^\circ$$

$$\angle APD = 180^\circ - 85^\circ - 40^\circ$$

$$\angle APD = 180^\circ - 125^\circ$$

$$\angle APD = 55^\circ$$

$$[\angle A = \angle ADC +$$

$$\angle ADB]$$

$$[\angle ADC = \angle ADP]$$

Solution-15:-

(i) $\angle ACD$ and $\angle ACD$ is a Linear pair

$$75^\circ + \angle ACD = 180^\circ$$

$$\angle ACD = 105^\circ$$

[BC produced is
let D]

By using Angle sum property of a triangle we have

In $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B = 180^\circ - 70^\circ - 40^\circ$$

$$\angle B = 180^\circ - 110^\circ$$

$$\angle B = 70^\circ$$

Solution-15 (ii):-

In $\triangle ABC$ Let AB produced is D.

$\angle BAC$ and $\angle DAC$ is a Linear pair

$$\angle BAC + \angle DAC = 180^\circ$$

$$\angle BAC = 180^\circ - 80^\circ$$

$$\angle BAC = 100^\circ$$

By using Angle ^{sum} property of a $\triangle$ we have

$$\angle BAC + \angle ABC + \angle C = 180^\circ$$

$$\angle ABC = 180^\circ - 100^\circ - 30^\circ$$

$$\angle ABC = 50^\circ$$

Solution-15 (iii):-

By using Angle sum property of a Δ^e we have

In ΔACD ,

$$\angle CAD + \angle ACD + \angle CDA = 180^\circ$$

$$\angle CDA = 180^\circ - 100^\circ - 30^\circ$$

$$\angle CDA = 50^\circ$$

$\angle CDA$ and $\angle ADB$ is a Linear pair

$$\angle ACD + \angle ADB = 180^\circ$$

$$\angle ADB = 180^\circ - 50^\circ - 100^\circ$$

$$\angle ADB = 30^\circ$$

By using Angle sum property of a Δ^e we have

$$\angle BAC + 45^\circ + 80^\circ = 180^\circ$$

$$\angle BAC = 180^\circ - 125^\circ$$

$$\angle BAC = 55^\circ$$

$$x = 55^\circ, y = 50^\circ.$$

Solution-15 (iv):-

By using Angle sum property of a Δ^e , we have

$$\angle PBC + \angle PCA + \angle BPC = 180^\circ$$

$$\angle PBC = 180^\circ - 30^\circ - 50^\circ = 100^\circ$$

$\angle PBC$ and $\angle PBA$ is a Linear pair

$$\angle PBC + \angle PBA = 180^\circ$$

$$\angle PBA = 180^\circ - 100^\circ$$

$$= 80^\circ.$$

$$x = 80^\circ$$

In ΔAEB

$$\angle EAB + \angle AEB + \angle ABE = 180^\circ$$

$$30^\circ + 80^\circ + \angle AEB = 180^\circ$$

$$\angle AEB = 180^\circ - 110^\circ$$

$$\angle AEB = 70^\circ$$

$\angle AEB$ and $\angle AED$ is a Linear pair

$$70^\circ + \angle AED = 180^\circ$$

$$\angle AED = 180^\circ - 70^\circ$$

$$= 110^\circ$$

$$x = 80^\circ, y = 110^\circ$$

Solution-16:-

(i) $\angle BAE$ and $\angle BAC$ is a Linear pair

$$\angle BAE + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 120^\circ$$

$$\angle BAC = 60^\circ$$

$\angle ACB$ and $\angle ACD$ is a Linear pair

$$\angle ACB + \angle ACD = 180^\circ$$

$$\angle ACB = 180^\circ - 112^\circ$$

$$\angle ACB = 68^\circ$$

A.S.P

$$\rightarrow \angle BAC + \angle BCA + \angle ABC = 180^\circ$$

$$\Rightarrow x^\circ = 180^\circ - 60^\circ - 68^\circ = 52^\circ. \therefore x = 52^\circ.$$

Solution-16:-

(ii) $\angle ACD$ and $\angle ACB$ is a Linear pair

$$\angle ACD + \angle ACB = 180^\circ$$

$$110^\circ + \angle ACB = 180^\circ$$

$$\angle ACB = 180^\circ - 110^\circ$$

$$\angle ACB = 70^\circ$$

$\angle EBA$ and $\angle ABC$ is a Linear pair

$$\angle EBA + \angle ABC = 180^\circ$$

$$\angle ABC = 180^\circ - 120^\circ$$

$$\angle ABC = 60^\circ$$

By using Angle sum property of a triangle, we have

$$\angle ACB + \angle ABC + \angle BAC = 180^\circ$$

$$60^\circ + 70^\circ + \angle BAC = 180^\circ$$

$$\angle BAC = 180^\circ - 130^\circ = 50^\circ \therefore x = 50^\circ$$

(iii) $\angle BAD$ and $\angle ADC$ are vertically opposite angles

$$\angle BAD = \angle ADC = 52^\circ$$

By using Angle sum property of a triangle, we have

$$\angle ADC = \angle EDC$$

$$\text{In } \triangle EDC: \angle EDC + \angle CED + \angle ECD = 180^\circ$$

$$52^\circ + 40^\circ + x = 180^\circ$$

$$x = 180^\circ - 52^\circ - 40^\circ$$

$$x = 180^\circ - 92^\circ$$

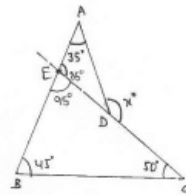
$$x^\circ = 88^\circ$$

Solution 16 :-

(iv)

Given $\angle ABC = 45^\circ$,
 $\angle BCD = 50^\circ$,
 $\angle BAD = 35^\circ$.

Construction:- Extend line DC and it intersects AB at 'E'.



Required to find $\angle ADC$

In $\triangle BCE$,

$$\angle ECB + \angle BCE + \angle CEB = 180^\circ \quad (\text{Sum of angles in a triangle is } 180^\circ)$$

$$\angle ABC + \angle BCD + \angle CEB = 180^\circ \quad \left(\because \angle BCE = \angle ABC \right)$$

$$45^\circ + 50^\circ + \angle CEB = 180^\circ$$

$$\angle CEB = 180^\circ - 95^\circ$$

$$\angle CEB = 85^\circ$$

Consider line AB and line EC,

$\angle BEC$ and $\angle AEC$ form a linear pair.

$$\therefore \angle BEC + \angle AEC = 180^\circ$$

$$85^\circ + \angle AEC = 180^\circ$$

$$\angle AEC = 180^\circ - 85^\circ$$

$$= 95^\circ$$

$$\therefore \angle AEC = 95^\circ$$

In $\triangle AED$, $\angle ADC$ is exterior angle.

$\therefore \angle ADC = \text{Sum of two opposite interior angle}$

$$\angle ADC = \angle DAE + \angle AED$$

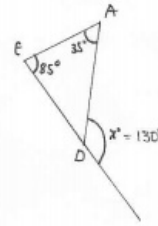
$$x^\circ = 35^\circ + \angle AED$$

$$= 35^\circ + 95^\circ$$

$$= 130^\circ$$

$$\therefore \angle ADC = 130^\circ$$

$$\therefore x^\circ = 130^\circ$$



Exercise-15.4

Exercise-15.4.

1) solution:-

i) we have,

$$5+7 > 4, 5+9 > 7, 9+7 > 5$$

That is, the sum of any two of the given numbers is greater than the third number.

So, 5cm, 7cm & 9cm can be the lengths of the legs sides of a triangle

ii) we have,

$$2+10 \not> 15$$

So, the given numbers cannot be the lengths of the sides of a triangle

(iii) we have,

$$3+4 > 5, 4+5 > 3, 5+3 > 4$$

That is, the sum of any two of the given numbers is greater than the third number

So 3cm, 4cm & 5cm can be the lengths of the sides of a triangle.

(iv) we have

$$2+5 \not> 7$$

So, the given numbers cannot be the lengths of the sides of a triangle.

(v) we have

$$5+8 \not> 20$$

So, the given numbers cannot be the lengths of the sides of a triangle.

Solution-02:-

$$(i) <$$

$$(ii) <$$

$$(iii) <$$

Solution-03:-

$$(i) \text{ False}$$

$$(ii) \text{ True}$$

$$(iii) \text{ False}$$

Solution-04:-

$$OA + OB > AB \quad \text{--- (1)}$$

$$\text{Similarly } OB + OC > BC \quad \text{--- (2)}$$

$$OC + OA > CA \quad \text{--- (3)}$$

$$\text{Add (1), (2) \& (3)}$$

$$(OA + OB) + (OB + OC) + (OC + OA) > AB + BC + CA$$

$$\Rightarrow 2(OA + OB + OC) > AB + BC + CA$$

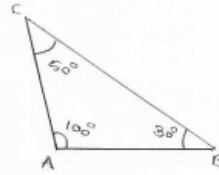
$$\Rightarrow OA + OB + OC > \frac{1}{2}(AB + BC + CA)$$

Solution-05:-

Given,

In $\triangle ABC$

$$\angle A = 100^\circ, \angle B = 30^\circ, \angle C = 50^\circ.$$



→ AC is the smallest side which is opposite to the smallest angle $\angle B$

→ BC is the largest side which is opposite to the largest angle $\angle A$.

Exercise-15.5

Exercise -15.5.

Solution-01:-

Pythagoras theorem:-

In a right triangle, the square of the hypotenuse equal to the sum of the squares of its remaining two sides.

Converse of Pythagoras theorem:-

If the square of one side of triangle is equal to the sum of the squares of the other two sides, then the triangle is a right triangle with the angle opposite the first side as right angle

In $\triangle ABC$, we have

$$AB^2 = AC^2 + BC^2.$$

Solution-2:-

(i) $a = 6\text{cm}$, $b = 8\text{cm}$, $c = ?$.

WKT, by Pythagoras theorem

$$a^2 + b^2 = c^2$$

$$c^2 = 6^2 + 8^2$$

$$c^2 = 36 + 64$$

$$c = \sqrt{100} = 10\text{cm}.$$

Given,

(ii) $a = 8\text{cm}$, $b = 15\text{cm}$, $c = ?$.

$$a^2 + b^2 = c^2 \quad [\text{by Pythagoras theorem}]$$

$$8^2 + 15^2 = c^2$$

$$c^2 = 64 + 225$$

$$c^2 = 289$$

$$c = \sqrt{289} = 17\text{cm}$$

$$c = 17\text{cm}$$

(iii) Given,

$a = 3\text{cm}$, $b = 4\text{cm}$ and $c = ?$

WKT, By Pythagoras theorem

$$a^2 + b^2 = c^2$$

$$3^2 + 4^2 = c^2$$

$$c^2 = 25$$

$$c = \sqrt{25} = 5\text{cm}$$

$$c = 5\text{cm}$$

(iv) Given,

$a = 2\text{cm}$, $b = 1.5\text{cm}$ and $c = ?$.

WKT, By Pythagoras theorem

$$a^2 + b^2 = c^2$$

$$2^2 + 1.5^2 = c^2$$

$$c = \sqrt{4 + 1.5 \times 1.5}$$

$$c = \sqrt{2.5 \times 2.5}$$

$$c = 2.5$$

Solution - 03:-

Given,

$$\text{Hypotenuse} = 2.5 \text{ cm}$$

$$\text{side} = 1.5 \text{ cm}$$

$$\text{side}_2 = ?$$

$$\text{side}^2 + \text{side}^2 = \text{Hyp}^2$$

$$2.5^2 - 1.5^2 = \text{side}^2$$

$$\text{side}^2 = 4$$

$$\text{side} = \sqrt{4} = 2 \text{ cm.}$$

Solution-04:-

$$\text{Given Ladder Length} = 3.7 \text{ m}$$

$$\text{Ladder Base} = 1.2 \text{ m}$$

∴ By using Pythagoras theorem

$$3.7^2 = 1.2^2 + \text{side}^2$$

$$13.69 - 1.44 = \text{side}^2$$

$$\text{side} = \sqrt{12.25}$$

$$\text{side} = 3.5 \text{ m.}$$

$$\therefore \text{required height} = 3.5 \text{ m}$$

Solution-05:-

Given,

$$a = 3 \text{ cm, } b = 4 \text{ cm and } c = 6 \text{ cm}$$

Here the Largest side is $c = 6 \text{ cm}$.

$$\text{we have } a^2 + b^2 = c^2$$

$$\text{clearly, } 3^2 + 4^2 \neq 6^2$$

$$16 \neq 25$$

So, the triangle with the given sides is not a right angle.

Solution-06:-

(i) Given

$$a = 7 \text{ cm, } b = 24 \text{ cm and } c = 25 \text{ cm}$$

Here the Largest side is $c = 25 \text{ cm}$

$$\text{we have: } a^2 + b^2 = c^2$$

$$7^2 + 24^2 = 25^2$$

$$516 + 49 = 625$$

$$625 = 625$$

So, the triangle with the given sides is a right triangle.

(ii) Here the Largest side $c = 18 \text{ cm}$.

$$\text{we have: } a^2 + b^2 = c^2$$

$$18^2 \neq 9^2 + 16^2$$

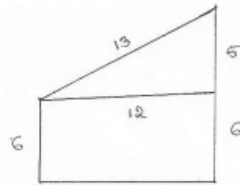
$$81 + 256 \neq 324$$

$$337 \neq 324$$

So, the triangle with the given sides is not a right triangle

Solution-07:-

Two poles of heights 6m and 11m
distance between their tops = 13m.



$$\begin{aligned}\text{Side} &= 11\text{m} - 6\text{m} \\ &= 5\text{m}\end{aligned}$$

∴ By using Pythagoras theorem

$$5^2 + 12^2 = \text{Hyp}^2$$

$$\text{Hyp}^2 = 25 + 144$$

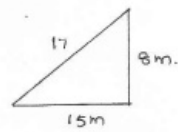
$$\text{Hyp} = \sqrt{169}$$

$$\text{Hyp} = 13\text{m}.$$

Solution-08:-

Given

distances of sides are
15m and 8m.



How far is he from ^{the} starting point's Hypotenuse

By using Pythagoras theorem

$$\text{Hyp}^2 = \text{side}^2 + \text{Side}^2$$

$$\text{Hyp}^2 = 15^2 + 8^2 = 225 + 64 = 289$$

$$\text{Hyp} = \sqrt{289}$$

$$\text{Hyp} = 17.$$

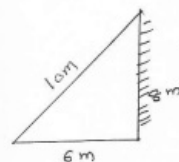
Solution-09:-

Given,

In initial position

Foot of a ladder = 6m

height = 8m.



By Applying Pythagoras theorem, we get

$$6^2 + 8^2 = \text{hyp}^2$$

$$36 + 64 = \text{hyp}^2$$

$$\text{hyp} = \sqrt{100}$$

$$= 10\text{m}$$

In Final position

foot of a Ladder = 8m

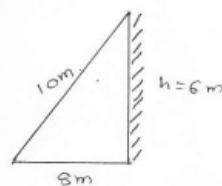
$l = \text{hyp} = 10\text{m}.$

$$10^2 = 8^2 + \text{height}^2$$

$$\text{height}^2 = 100 - 64$$

$$h = \sqrt{36}$$

$$\text{height} = 6\text{m}.$$



Solution-10:-

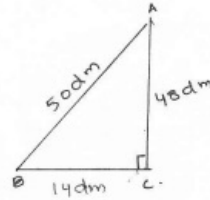
Given

Ladder Length = 50 dm

height = 48 dm

base of the wall = ?

Ladder Length = 50 dm



By Applying Pythagoras theorem we get

$$50^2 = 48^2 + \text{base}^2$$

$$\text{base}^2 = 2500 - 2304$$

$$\text{base}^2 = 196$$

$$\text{base} = \sqrt{196}$$

$$\text{base} = 14 \text{ dm}$$

$\therefore$ base of the wall = 14 dm.

Solution-11:-

The two legs of a right triangle are equal

and $\text{Hyp}^2 = 50$

side₁ = side₂ = side

$$\text{side}_1^2 + \text{side}_2^2 = \text{Hyp}^2 \quad [\because \text{Pythagoras theorem}]$$

$$2 \text{ side}^2 = \text{Hyp}^2$$

$$\text{side}^2 = \frac{50}{2}$$

$$\text{side} = \sqrt{25}$$

$$\text{side} = 5 \text{ units.}$$

$\therefore$ length of each leg = 5 units.

Solution-12:-

(i) Given numbers are 12, 35 & 37

$$12^2 + 35^2 = 37^2$$

$$144 + 1225 = 1369$$

(12, 35, 37) is a triplet

$\rightarrow$ yes.

(ii) Given numbers are 7, 24 and 25

By Pythagoras theorem

$$7^2 + 24^2 = 25^2$$

$$49 + 576 = 625$$

$$625 = 625$$

(7, 24, 25) is a triplet

$\rightarrow$ yes.

(iii) Given numbers are , 27, 36 and 45

By using pythagoras theorem

$$27^2 + 36^2 \neq 45^2$$

$$729 + 1296 = 2025$$

$$2025 = 2025.$$

(27, 36 & 45) is a triplet

→ yes.

(iv) Given numbers are 15, 36 and 39

By using pythagoras theorem

$$15^2 + 36^2 = 39^2$$

$$225 + 1296 = 1521$$

$$1521 = 1521$$

(15, 36, 39) is a triplet

→ yes.

Solution -13:-

Given,

$$\angle A = 105^\circ$$

$$\angle ABC = 105^\circ, \angle BAC = 35^\circ$$

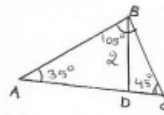
By using Angle sum Property of a triangle, we have

$$100^\circ + 35^\circ + \angle C = 180^\circ$$

$$\angle C = 180^\circ - 135^\circ = 45^\circ$$

$$BD = 8 \text{ cm}$$

$$DC = 8 \text{ cm}.$$



Solution -14:-

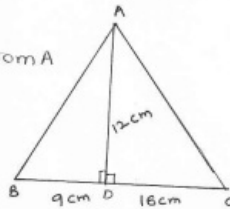
Given,

In $\triangle ABC$, AD is the altitude from A

$$AD = 12 \text{ cm}$$

$$DC = 16 \text{ cm}$$

$$DB = 9 \text{ cm}$$



By using pythagoras theorem $\triangle ABD$, we have

$$9^2 + 12^2 = \text{hyp}^2$$

$$\text{hyp}^2 = 144 + 81$$

$$\text{hyp}^2 = 225$$

$$\text{hyp} = AB = 15 \text{ cm}$$

In $\triangle ADC$, we have

$$12^2 + 16^2 = \text{hyp}^2 = AC^2$$

$$144 + 256 = AC^2$$

$$AC = \sqrt{400}$$

$$AC = 20 \text{ cm}$$

No, $\triangle ABC$ is right angled at A.

Solution-15:-

Given that

$$Ac = 4 \text{ cm}$$

$$Bc = 3 \text{ cm}$$

$$\angle C = 105^\circ$$

By construction

$$AB = 4.5 \text{ cm}$$

$$(AB)^2 = Ac^2 + Bc^2$$

$$(4.5)^2 \neq 4^2 + 3^2$$

$$20.25 \neq 25$$

$$20.25 < 25$$

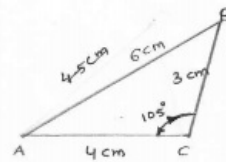
$$(6)^2 \neq 3^2 + 4^2$$

$$36 \neq 3^2 + 4^2$$

$$36 > 25$$

$$\text{No, } (AB)^2 \neq Ac^2 + Bc^2$$

$$(AB)^2 > Ac^2 + Bc^2$$



Solution-16:-

Given

$$Ac = 4 \text{ cm},$$

$$Bc = 3 \text{ cm and } \angle C = 80^\circ$$

By construction, we get

$$AB = 4.6$$

By Pythagoras theorem

$$(AB)^2 = Ac^2 + Bc^2$$

$$(4.6)^2 \neq 4^2 + 3^2$$

$\rightarrow$ No.

$$(4.6)^2 < 4^2 + 3^2 = 25.16 < 25$$

$$\therefore (AB)^2 < Ac^2 + Bc^2$$

