

Boolean Algebra

TOPIC- 1

Basics of Boolean Algebra

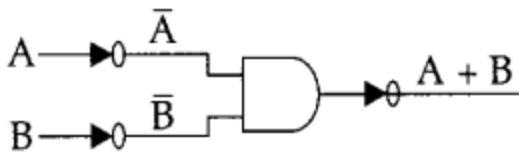
Very Short Answer Type Questions (1 Mark each)

Question 1:

Express the OR operator in terms of AND and NOT operator.

Answer:

$$(A \cdot B)' = \bar{A} + \bar{B} \quad [\text{De Morgan's law}]$$
$$(\bar{A} + \bar{B})' = A \cdot B.$$



Question 2:

Specify which axioms/theorems are being used in the following Boolean reductions:

- (a) $(bc)' + bc = 1$
- (b) $xyz + zx = xz$

Answer:

- (a) $x+x'=1$ -Complementary law
- (b) $xy+x=x$ & absorption law

Question 3:

Which gates are known as universal gates? Why?

Answer:

Universal gates are the ones which can be used for implementing any gate like AND, OR and NOT or any combination of these basic gates. NAND and NOR gates are universal gates.

Question 4:

State and verify Associative law using Truth Table.

Answer:

Associative Law: This law states that:

$$(A+B)+C=A+(B+C)$$

$$(A.B).C=A.(B.C)$$

Proof:

A	B	C	A + B	(A + B) + C	B + C	A + (B + C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	1	1	0	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

From the above tables,

$$(A+B)+C=A+(B+C)$$

Similarly, we can prove,

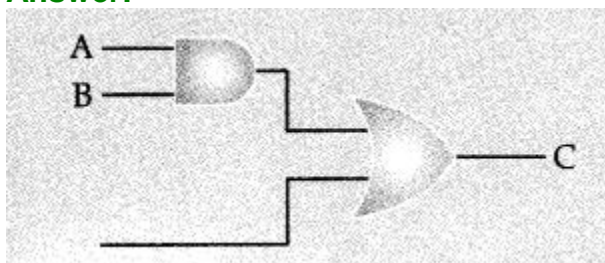
$$A.(B.C)=(A.B).C$$

A	B	C	B.C	A. (B. C)	A.B	(A.B).C
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	1	0	0	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	0	0	1	0
1	1	1	1	1	1	1

Question 5:

Draw the equivalent logic circuit for the following Boolean expression :

Answer:



Short Answer Type Questions

Question 1:

State and define principle of duality. Why is it so important in Boolean Algebra?

Answer:

Principle of duality: Duality principle states that from every boolean relation another boolean relation can be derived by:

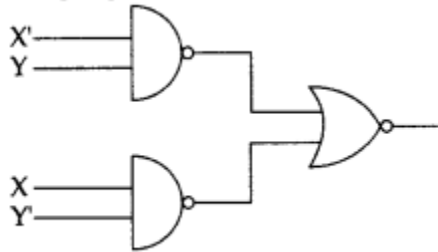
1. Changing each OR sign (+) to an AND sign (\cdot)
2. Changing each AND sign (\cdot) to an OR sign (+)

ex : Dual of $A + A'B = A \cdot (A' + B)$

Importance in Boolean Algebra: The principle of duality is an important concept in Boolean algebra, particularly in proving various theorems. The principle of duality is used extensively in proving Boolean algebra theorem. Once we prove that an expression is valid, by the principle of duality, its dual is also valid. Hence, our effort in proving various theorems is reduced to half.

Question 2:

Write the equivalent boolean expression for the following logic circuit.

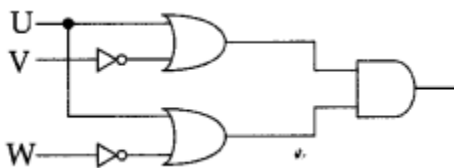


Answer:

$$((X' \cdot Y)' + (X \cdot Y'))'$$

Question 3:

Write the equivalent boolean expression for the following logic circuit.

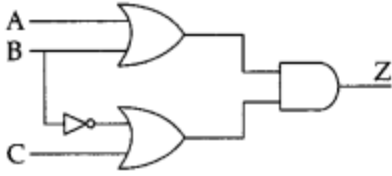


Answer:

$$Y = (U + \bar{V}) \cdot (U + \bar{W})$$

Question 4:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$Z = (A+B).(B' +C)$$

$$= A.B' + AC + B.B' + BC$$

$$=A.B' + AC + BC$$

Question 5:

Name the law shown below & verify it using a truth table.

$$X + \bar{X} \cdot Y = X + Y.$$

Answer:

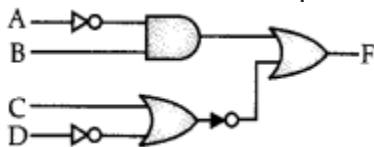
This law is called “Absorption Law” also referred as redundance law.

Prove by Truth table

X	Y	X'	$X+X'.Y$	$X+Y$
0	0	1	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

Question 6:

Obtain the Boolean Expression for the logic circuit shown below.



Answer:

$$F = (\bar{A} \cdot B) + (\overline{C+D})$$

$$= (\bar{A} \cdot B) + \bar{C} \cdot \bar{D}$$

Question 7:

Verify the following using truth table:

(i) $X+0=X$

(ii) $X+x'=1$

Answer:

(i)

X	0	$Y = X + 0$
0	0	0
1	0	1

Hence, $X + 0 = X$

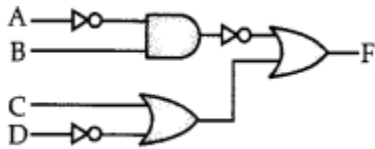
(ii)

X	X'	$Y = X + X'$
0	1	1
1	0	1

Hence, $X + X' = 1$

Question 8:

Obtain the Boolean Expression for the logic circuit shown below.



Answer:

$$F = (\overline{X} \cdot \overline{Y}) + (\overline{Z} + W).$$

$$F = \overline{X} + Y + \overline{Z} + W.$$

Question 9:

State Demorgan's law. Verify one of them using truth table

There are two Demorgan's law :

(i) $\overline{A \cdot B} = \overline{A} + \overline{B}$

(ii) $\overline{A + B} = \overline{A} \cdot \overline{B}$

Answer:

Proof :

(i) $\overline{A \cdot B} = \overline{A} + \overline{B}$

A	B	\overline{A}	\overline{B}	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

Proof :

(i) $\overline{A \cdot B} = \overline{A} + \overline{B}$

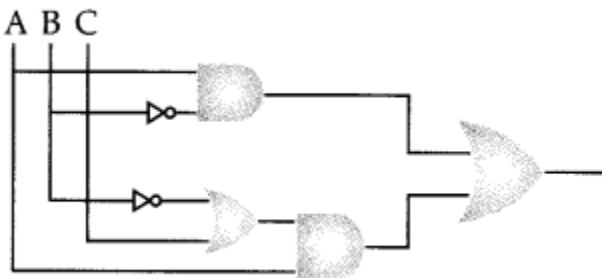
A	B	\overline{A}	\overline{B}	$\overline{A \cdot B}$	$\overline{A} + \overline{B}$
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

Question 10:

Draw a logic circuit for the Boolean expression

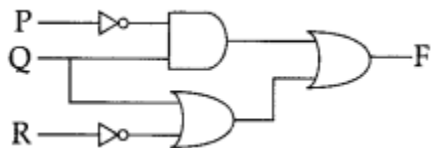
$A \cdot \overline{B} + (C + \overline{B}) \cdot \overline{A}$.

Answer:

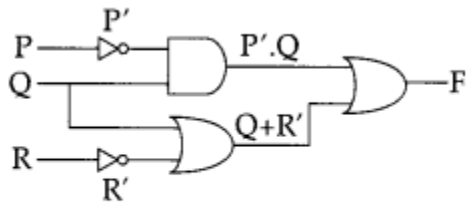


Question 11:

Obtain the boolean expression for the logic circuit shown below:



Answer:



$$\begin{aligned} F &= P' Q + (Q + R') \\ &= Q \cdot (P' + R') \end{aligned} \quad \text{(Distributive Law)}$$

Question 12:

Verify the following using Boolean Lawas:
 $X+Z=X+X'Z+Y.Z$

Answer:

Taking RHS

$$\begin{aligned} &X + X'Z + YZ \\ &= (X + X') \cdot (X + Z) + YZ \quad \text{(Distribution Law)} \\ &= 1 \cdot (X + Z) + YZ \quad \quad \quad (A + A' = 1) \\ &= X + Z + YZ \\ &= X + Z(1 + Y) \\ &= X + Z \quad \quad \quad (1 + A = 1; 1 \cdot A = A) \\ &= \text{Hence Verified} \end{aligned}$$

Question 13:

Verify the following using Boolean Lawas:
 $A+C=A+A'.C+B.C$

Answer:

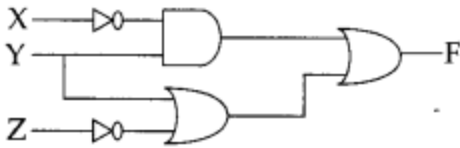
$$A + C = A + A'.C + BC$$

Solve RHS

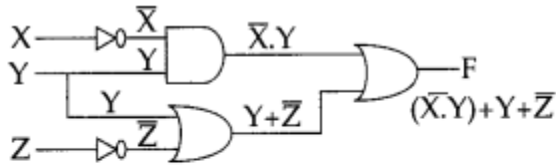
$$\begin{aligned} &A + A'.C + BC \\ &(A + A') \cdot (A + C) + BC \quad \text{[Using distributive law]} \\ &1 \cdot (A + C) + BC \quad \quad \quad [Q A + = 1] \\ &= A + C + BC \\ &= A + C(1 + B) \\ &= A + C \cdot 1 = A + C = \text{L.H.S. (Hence, verified)} \end{aligned}$$

Question 14:

Obtain the Boolean Expression for the logic circuit shown below:



Answer:



Expression at F :

$$(\bar{X} \cdot Y) + (Y + \bar{Z})$$

$$\Rightarrow (\bar{X} + 1) Y + \bar{Z} \quad \text{[Distributive law]}$$

$$\Rightarrow Y + \bar{Z} \quad [\because 1 + \bar{Z} = 1] 2$$

Question 15:

Verify the following using truth table:

(i) $X \cdot X' = 0$

(ii) $X + 1 = 1$

Answer:

(i)

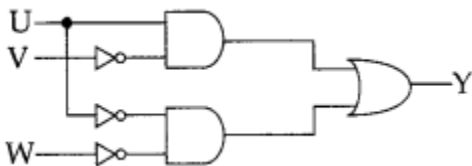
X	X'	Y = X.X'
0	1	0
1	0	0

(ii)

X	1	Y = X + 1
0	1	1
1	1	1

Question 16:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$Y = U\bar{V} + \bar{U}W$$

Question 17:

Answer:

Distribution law: This law states that

(i) $x(y+z)=x.y+x.z$

(ii) $x+yz=(x+y)(x+z)$

Now let us prove using truth table.

(i)

x	y	z	y + z	x(y + z)	xy	x.z	x.y + x.z
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

Hence $x(y + z) = x.y + x.z$.

(ii)

x	y	z	yz	x+yz	x+y	x+z	(x+y)(x+z)
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Hence $x + yz = (x + y).(x + z)$

Question 18:

Name the law shown below & verify it using a truth table.

$A+B.C=(A+B).(A+C)$.

Answer:

This law is called "Distribute Law".

Prove by Truth table

A	B	C	A+B.C	A+B	A+C	(A+B).(A+C)
0	0	0	0	0	0	0
0	0	1	0	0	1	0
0	1	0	0	1	0	0
0	1	1	1	1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Question 19:

Write the POS form of a Boolean Function F, which is represented in a truth table as follows:

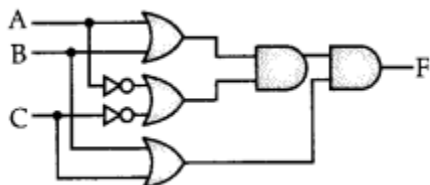
P	Q	R	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Answer:

$$(P+Q+R).(P'+Q+R).(P'+Q'+R)$$

Question 20:

Write the Boolean Expression for the result of the Logic Circuit as shown below:



Answer:

The boolean expression is :

$$F = [(A+b).(A'+C')].(B+C)$$

Question 21:

Derive a Canonical POS expression for a Boolean function G, represented by the following truth table:

X	Y	Z	G(X, Y, Z)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Answer:

$$\begin{aligned} \text{POS} &= x'y'z' + x'yz' + x'yz + xy'z + xyz' \\ &= \Sigma m(0, 2, 3, 5, 6) \end{aligned}$$

Question 22:

Correct the following boolean statements:

- (i) $X + 1 = X$
- (ii) $(A')' = A$
- (iii) $A + A' = 0$
- (iv) $(A + B)' = A'.B'$

Answer:

- (i) $X + 1 = X$ or $X + 0 = X$
- (ii) $((A')) = A$
- (iii) $A + A' = 1$ or $A . A' = 0$
- (iv) $(A + B)' = A' . B'$

Question 23:

Derive a Canonical SOP expression for a Boolean function G, represented by the following truth table:

A	B	C	G (A, B, C)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$G(A, B, C) = A'B'C' + A'B.C' + A.B.C' + A.B.C$$

OR

$$G(A, B, C) = S(0, 2, 6, 7)$$

Question 24:

Derive a Canonical POS expression for a Boolean function F, represented by the following truth table:

P	Q	R	F (P, Q, R)
0	0	0	0

0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

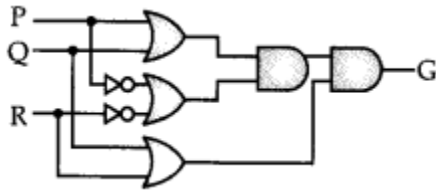
$$F(P, Q, R) = (P' + Q' + R') \cdot (P + Q' R') \cdot (P + Q' R) \cdot (P' + Q + R')$$

OR

$$F(P, Q, R) = \pi(0, 3, 4, 5)$$

Question 25:

Write the Boolean Expression for the result of the Logic Circuit shown below:

**Answer:**

$$C = P \cdot Q' + P \cdot R + Q \cdot R'$$

Question 26:

Verify the following using Boolean Laws

$$P' + Q' \cdot R = P' \cdot Q' \cdot R' + P' \cdot Q \cdot R' + P' \cdot Q \cdot R + P' \cdot Q' \cdot R + P \cdot Q' \cdot R$$

Answer:

$$\text{LHS } P' + Q' R$$

$$= P' (Q + Q') (R + R') + Q' R (P + P')$$

(Simplification)

$$= P'QR + P'QR' + P'Q'R + P'Q'R' + PQR + P'Q'R$$

$$= P'QR + P'QR' + P'Q'R + P'Q'R' + PQR$$

$$= P'Q'R' + P'Q'R' + P'QR + P'Q'R' + PQR$$

$$= \text{R.H.S.}$$

2

Question 27:

Verify the following using Boolean Laws $x' + y'z = x'y'z' + x'y'z + x'yz + x'y'z + xy'z$

Answer:

$$\text{LHS}$$

$$X' + Y' \cdot Z$$

$$= X' \cdot (Y + Y') \cdot (Z + Z') + (X + X') \cdot Y' \cdot Z$$

$$= X' \cdot Y \cdot Z + X' \cdot Y \cdot Z' + X' \cdot Y' \cdot Z + X' \cdot Y' \cdot Z' + X \cdot Y' \cdot Z + X' \cdot Y' \cdot Z$$

$$Y'Z + X' \cdot Y' \cdot Z$$

$$= X' \cdot Y \cdot Z + X' \cdot Y \cdot Z' + X' \cdot Y' \cdot Z + X' \cdot Y' \cdot Z' + X \cdot Y' \cdot Z + X' \cdot Y' \cdot Z$$

$$= X' \cdot Y \cdot Z + X' \cdot Y \cdot Z' + X' \cdot Y' \cdot Z + X' \cdot Y' \cdot Z' + X \cdot Y' \cdot Z$$

$$= \text{RHS}$$

2

OR

RHS

$$\begin{aligned} & X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z + X' \cdot Y \cdot Z + X' \cdot Y' \cdot Z + X \cdot Y' \cdot Z \\ &= X' \cdot Y' \cdot Z + X' \cdot Y' \cdot Z + X' \cdot Y \cdot Z + X' \cdot Y \cdot Z + X \cdot Y' \cdot Z \\ &= X' \cdot Y' \cdot (Z + Z) + X' \cdot Y \cdot (Z + Z) + X \cdot Y' \cdot Z \\ &= X' \cdot (Y' + Y) + X \cdot Y' \cdot Z \\ &= X' + X \cdot Y' \cdot Z \\ &= (X' + X) \cdot (X' + Y' \cdot Z) \\ &= X' + Y' \cdot Z \\ &= \text{LHS} \end{aligned}$$

Question 28:

State and prove De-Morgan's laws in boolean algebra.

DeMorgan's theorem states that:

- (i) $(X+Y)'=X'.Y'$
- (ii) $(X.Y)'=X'+Y'$

Answer:

Now ,let us prove using truth table

(i) $(X + Y)' = X'.Y'$

X	Y	X+Y	(X+Y)'	X'	Y'	X'.Y'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Hence, $(X + Y)' = X'.Y'$

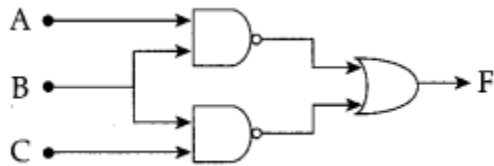
(ii) $(X.Y)' = X' + Y'$

X	Y	X.Y	(X.Y)'	X'	Y'	X'+Y'
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

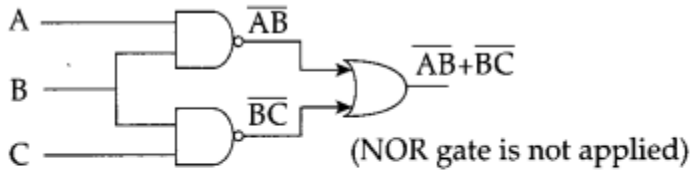
Hence, $(X.Y)' = X' + Y'$

Question 29:

Write the equivalent boolean expression for the following logic circuit:



Answer:



$$\begin{aligned} \overline{\overline{AB} + \overline{BC}} &= \overline{\overline{AB} \cdot \overline{BC}} && \text{(NOR gate is applied)} \\ &= AB \cdot BC \\ &= B(A \cdot C) \end{aligned}$$

Question 30:

Verify the following using truth table:

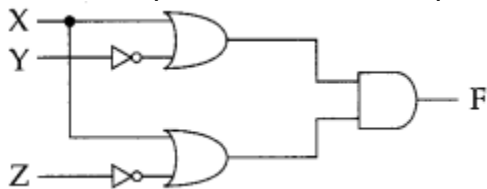
Answer:

$$U \cdot (U' + V) = (U \cdot V)$$

U	V	U'	U'+V	U.(U'+V)	U.V
0	0	1	1	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	1	0	1	1	1

Question 31:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$F = (X+Y') \cdot (X+Z')$$

Question 32:

Verify the following using truth table:

$$X+Y.Z=(X+Y).(X+Z)$$

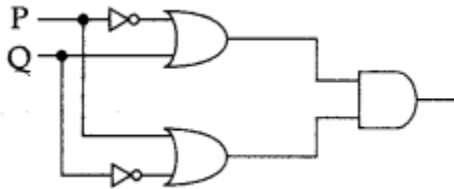
Answer:

This law is called **Distributive Law**.

X	Y	Z	X+Y	X+Y.Z	X+Z	(X+Y)(X+Z)
0	0	1	0	0	1	0
0	1	0	1	0	0	0
0	1	1	1	1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	1	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Question 33:

Write the equivalent boolean expression for the following logic circuit:

**Answer:**

$$PQ'+PR'$$

$$=P(Q'+R')$$

Question 34:

State and verify De-Morgan's laws algebraically

Answer:

$$X + X.Y = X$$

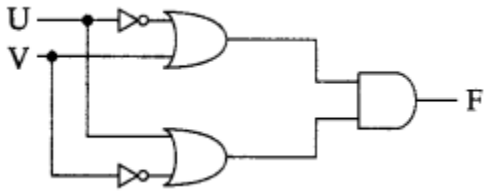
$$\begin{aligned} \text{L.H.S.} &= X+X.Y \\ &= X.1+X.Y \\ &= X.(1+Y) \\ &= X.1 \\ &= X = \text{R.H.S.} \end{aligned}$$

$$X + X'.Y = X+Y$$

$$\begin{aligned} \text{L.H.S.} &= X+X'.Y \\ &= (X+X').(X+Y) \\ &= 1.(X+Y) \\ &= X+Y = \text{R.H.S.} \end{aligned}$$

Question 35:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$F(P,Q)=(P'+Q).(P+Q')$$

Question 36:

State and algebraically verify Absorption law

Answer:

$$X + X.Y = X$$

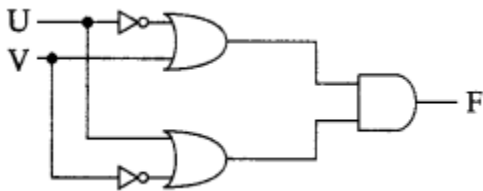
$$\begin{aligned} \text{L.H.S.} &= X+X.Y \\ &= X.1+X.Y \\ &= X.(1+Y) \\ &= X.1 \\ &= X = \text{R.H.S.} \end{aligned}$$

$$X + X'.Y = X+Y$$

$$\begin{aligned} \text{L.H.S.} &= X+X'.Y \\ &= (X+X').(X+Y) \\ &= 1.(X+Y) \\ &= X+Y = \text{R.H.S.} \end{aligned}$$

Question 37:

Write the equivalent boolean expression for the following logic circuit:



Answer:

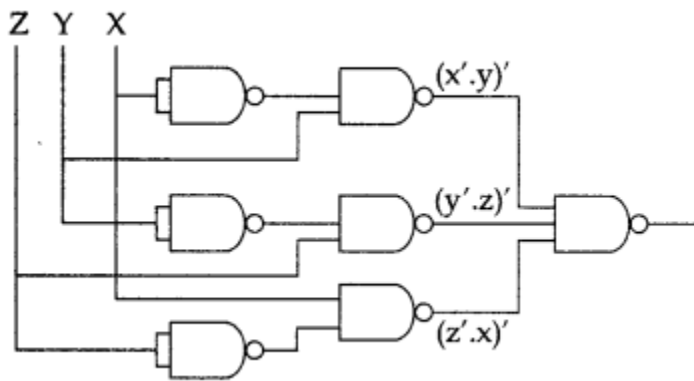
$$F(U,V)=(U'+V).(U+V')$$

Question 38:

Represent the expression $X'.Y+Y'.Z+Z'X$ using NAND gates

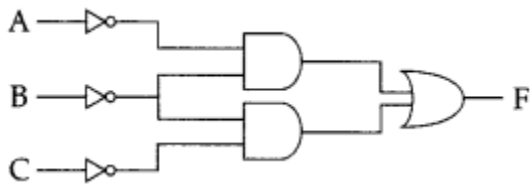
Answer:

$$\begin{aligned}\text{Expression} &= ((X'.Y + Y'.Z + Z'.X)') \\ &= [(x'.y)'.(y'.z)'.(z'.x)']\end{aligned}$$



Question 39:

Write the equivalent expression for the following logical circuit:



Answer:

$$F = A'.B' + B'.C'$$

Question 40:

Verify the following algebraically.

$$(A'+B').(A+B) = A'.B + A.B'$$

Answer:

$$(A'+B').(A+B) = A'B + A.B'$$

L.H.S.

$$(A' + B'). (A + B)$$

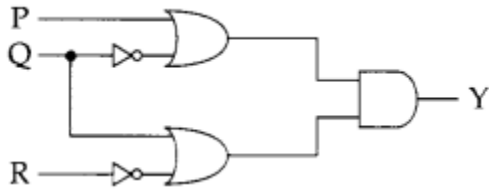
$$A'A + A'B + B'A + B'B$$

$$0 + A'.B + B'.A + 0$$

$$A'.B + A.B' = \text{R.H.S. (Hence, verified)}$$

Question 41:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$Y = (P + Q') \cdot (Q + R')$$

Question 42:

Verify the following algebraically:

$$X' \cdot Y + X \cdot Y' = (X' + Y') \cdot (X + Y)$$

Answer:

$$X' \cdot Y + X \cdot Y' = (X' + Y') \cdot (X + Y)$$

$$\text{R.H.S.} = (X' + Y') \cdot (X + Y)$$

$$= X'X + X'Y + Y'X + Y'Y$$

$$= 0 + X'Y + Y'X + 0 \quad (\text{Q } AA' = 0)$$

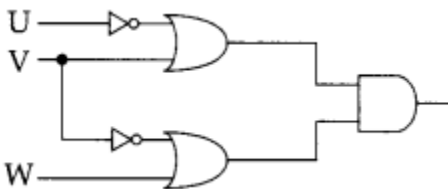
$$= X'Y + Y'X \quad (\text{Q } A + 0 = A)$$

$$= X'Y + XY' \quad (\text{Q } AB' = B'A)$$

$$= \text{L.H.S.} \quad \quad \quad \mathbf{2}$$

Question 43:

Write the equivalent boolean expression for the following logic circuit:



Answer:

$$(U' + V) \cdot (V' + W)$$

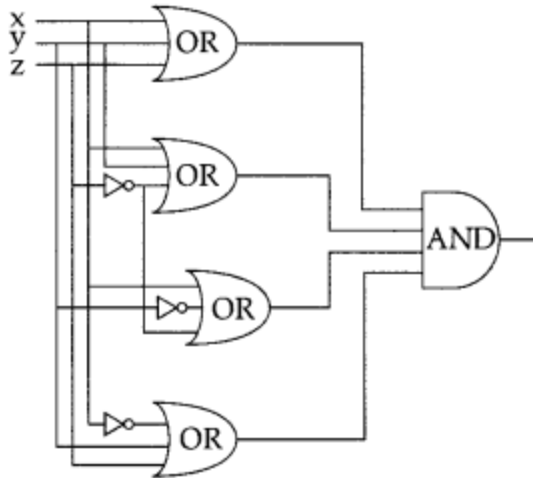
Question 44:

Answer:

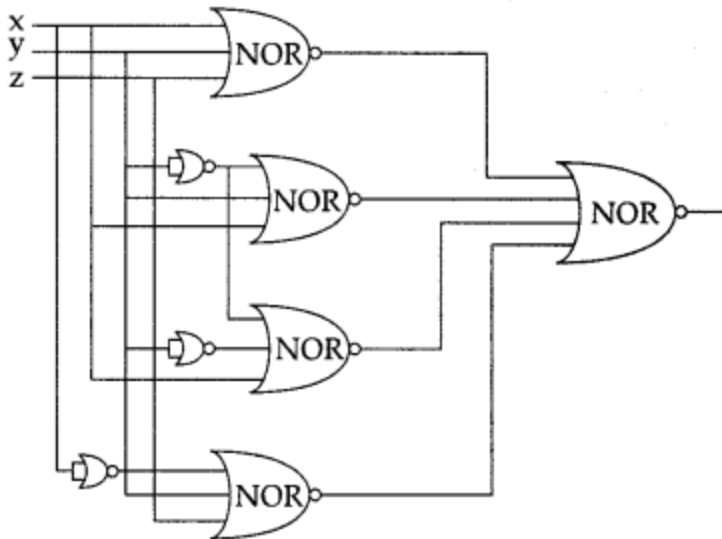
$\pi (0, 1, 3, 4) = (x + y + z) \cdot (x + y + z')$
 $(x + y' + z') \cdot (x' + y + z)$

(i) Converted to POS

(ii) Draw circuit diagram using normal (basic gates).



(iii) Convert all gates to NOR gates.



TOPIC-2
Karnaugh Map Minimization and Applications of Boolean Algebra
Very Short Answer Type Questions

Question 1:

Convert the following Boolean expression into its equivalent Canonical Sum of Products form (SOP):

$$(U + V + W)(U+V + W')(U'+V + W) (U' + V' + W')$$

Answer:

$$\pi (0, 1, 4, 7)$$

$$\Sigma (2, 3, 5, 6)$$

$$010 \ 011 \ 101 \ 110$$

$$= U'VW' + U'VW + UV'W + UVW'$$

Question 2:

Write Product of Sum expression of the function F (a, b, c, d) from the given truth table.

a	b	c	d	f
0	0	0	0	0

0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Answer:

$$F(a, b, c, d) =$$

$$(a + b + c + d).(a + b + c + d') . (a + b' + c + d) .$$

$$(a + b' + c' + d') . (a' + b + c + d) .$$

$$(a' + b + c + d') . (a' + b' + c + d) . (a' + b' + c + d')$$

$$. (a' + b' + c' + d) \quad \mathbf{1}$$

Question 3:

Write the Sum of Product form of the function $F(P, Q, R)$ for the following truth table representation of F :

P	Q	R	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$(\bar{P} \bar{Q} \bar{R} + \bar{P} QR + PQ\bar{R} + PQR).$$

Question 4:

Write the Product of Sum form of the function $F(X, Y, Z)$ for the following truth table representation of F :

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$(X + Y + \bar{Z}).(X + \bar{Y} + Z).(\bar{X} + Y + Z).(\bar{X} + Y + \bar{Z}).$$

Question 5:

Write the Product of Sum form of the function $G(U, V, W)$ for the following truth table representation of G :

U	V	W	G(U, V, W)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Answer:

$$(U + V + W) \cdot (U + \bar{V} + W) \cdot (\bar{U} + \bar{V} + \bar{W})$$

$$(\bar{U} + \bar{V} + \bar{W})$$

1

Question 6:

Write the Product of Sum form of function G(U, V, W) for the following truth table representation of G :

Maxterm

U	V	W	G
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

U	V	W	G	POS
0	0	0	1	
0	0	1	0	$U + V + \bar{W}$
0	1	0	1	
0	1	1	0	$U + \bar{V} + \bar{W}$
1	0	0	1	
1	0	1	0	$+ V + \bar{W}$
1	1	0	0	$+ \bar{V} + W$
1	1	1	1	

$$POS = (U + V + \bar{W}) \cdot (U + \bar{V} + \bar{W}) \cdot (\bar{U} + V + \bar{W}) \cdot (\bar{U} + \bar{V} + W)$$

1

Question 7:

Write the Sum of Product form of the function $F(A, B, C)$ for the following truth table representation of F .

Minterm

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

A	B	C	F	SOP
0	0	0	0	
0	0	1	0	
0	1	0	1	$A'BC'$
0	1	1	1	$A'BC$
1	0	0	1	$AB'C'$
1	0	1	0	
1	1	0	0	
1	1	1	1	ABC

$$SOP = A'BC' + A'BC + AB'C' + ABC \quad \mathbf{1}$$

Question 8:

Write the SOP form of a boolean function F , which is represented in a truth table as follows:

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

$$F(X,Y,Z) = X' \cdot Y' Z' + X' \cdot Y' \cdot Z + X \cdot Y' \cdot Z' + X \cdot Y \cdot Z$$

Question 9:

Write the POS form of boolean function G, which is represented in a truth table as follows :

A	B	C	G
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$G(A,B,C) = (A+B+C) \cdot (A+B'+C') \cdot (A'+B+C) \cdot (A'+B+C')$$

Question 10:

Write the SOP form of a boolean function F, which is represented in a truth table as follows :

X	Y	Z	F(X, Y, Z)
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

SOP form is given by

$$X'Y'Z' + X'Y'Z + X'YZ + XY'Z' + XYZ$$

Question 11:

Write the POS form of a boolean function F, which is represented in a truth table as follows :

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

$$(A + B + C). (A+B' + C'). (A' + B + C'). (A' + B' + C)$$

Question 12:

Write the SOP form of a boolean function F, which is represented in a truth table as follows :

U	V	W	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$F=(0,3,6,7)$$

So, SOP form of F is

$$F = (U'.V'W') + (U'.VW) + (U.VW') + (U.VW)$$

Question 13:

Write the POS form of a boolean function F, which is represented in a truth table as follows :

U	V	W	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$F(U, V, W) = (U + V + W')(U + V' + W')(U' + V + W')$$

Question 14:

Write the SOP form of a Boolean function F, which is represented in a truth table as follows :

P	Q	R	G
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

x	y	z	H
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

$$G(P, Q, R) = P'QR' + PQ'R' + PQR' + PQR$$

Question 15:

Write the POS form of a boolean function H, which is represented in a truth table as follows :

Answer:

$$H(x, y, z) = (x + y + z') \cdot (x' + y + z') \cdot (x' + y' + z)$$

Question 16:

Write the SOP form of a boolean function G, which is represented in a truth table as follows :

P	Q	R	G
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$G(P, Q, R) = P'QR' + P'QR + PQ'R' + PQR' + PQR$$

Question 17:

Write the POS form of a boolean function G, which is represented in a truth table as follows :

U	V	W	G
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

Answer:

$$G(U, V, W) = (U + V + W) \cdot (U' + V + W') \cdot (U' + V' + W)$$

Question 18:

Write the POS form of a boolean function F, which is represented in a truth table as follows :

U	V	W	F
0	0	0	1

0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Answer:

$$F(U, V, W) = (U + V + W') \cdot (U + V' + W') \cdot (U' + V + W')$$

Question 19:

Reduce the following Boolean Expression to its simplest form using K-Map :

Answer:

Simplest expression is

$$= uvwz + uvwz + uvwz + uvwz + uvwz + uvwz + uvwz + uvwz$$

uv \ wz	00	01	11	10
00	1 ₀	X ₁	X ₃	1 ₂
01	X ₄	1 ₅	1 ₇	X ₆
11	X ₁₂	1 ₁₃	1 ₁₅	X ₁₄
10	1 ₈	X ₉	X ₁₁	1 ₁₀ (k-map)

Question 20:

Write the SOP form of a Boolean function G, which is represented in a truth table as

P	Q	R	G
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

follows :

Answer:

$$G(P, Q, R) = P'.Q.R' + PQ'.R' + EQ.R' + EQ.R$$

Question 21:

Write the POS equivalent of $ABC + AB'C' + ABC' + A'B'C$

Answer:

$$\Sigma (1, 4, 5, 6, 7)$$

$$\text{POS} = \pi (0, 2, 3)$$

$$= (A + B + C) (A + B' + C) (A + B' + C')$$

Short Answer Type questions-II

Question 1:

Obtain the minimal SOP form for the following Boolean expression using K-Map

$$F(A,B,C,D) = \Sigma (0,2,3,5,7,8,10,11,13,15)$$

Answer:

	CD	C'D'	C'D	CD	CD'
A	00	01	11	10	
A'B'	00	01	11	10	
A'B	01	01	11	10	
AB	11	01	11	10	
AB'	10	01	11	10	

Quad 1 = $m_0 + m_2 + m_8 + m_{10} = B'D'$
 Quad 2 = $m_3 + m_7 + m_{15} + m_{11} = CD$
 Quad 3 = $m_5 + m_7 + m_{13} + m_{11} = BD$
 Minimal SOP = $B'D' + CD + BD$

Question 2:

Reduce the following Boolean expression using K-Map:
 $F(A,B,C,D) = \pi(0,1,2,4,5,6,8,10)$

Answer:

	C+D	C+D	C'+D'	C+D
A+B	0	0		0
A+B'	0	0		0
A'+B'				
A+B	0			0

$F(A, B, C, D) = \pi(0, 1, 2, 4, 5, 6, 8, 10)$
 $F = (A + C).(A + D).(B + D)$

Question 3:

Reduce the following Boolean Expression using K-Map:

Reduce the following using K-Map:

$F(A, B,C,D) = \Sigma(1,3,4,5,6,7,12,13)$

Answer:

$$\bar{A}B + \bar{A}D + B\bar{C}.$$

	CD			
AB	$\bar{C}\bar{D}$	$\bar{C}D$	RS	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

Question 4:

Reduce the following Boolean Expression using K-Map:

$$F(P,Q,R,S) = \Sigma(0,2,4,5,6,7,8,10,13,15).$$

Answer:

$$\bar{P}Q + \bar{Q}\bar{S} + QS.$$

	RS			
PQ	$\bar{R}\bar{S}$	$\bar{R}S$	RS	$R\bar{S}$
$\bar{P}\bar{Q}$	0	1	3	2
$\bar{P}Q$	4	5	7	6
PQ	12	13	15	14
$P\bar{Q}$	8	9	11	10

Question 5:

Reduce the following Boolean Expression using K-Map:

$$F(P, Q, R, S) = \Sigma(1, 2, 3, 4, 5, 6, 7, 8, 10)$$

Answer:

	R'S'	R'S	RS	RS'
P'Q'	0	1	1	1
P'Q	1	1	1	1
PQ	0	0	0	0
PQ'	1	0	0	1

$$F(P, Q, R, S) = P'Q + P'S + P'R'S' + PQ'S'$$

Question 6:

Reduce the following Boolean Expression using K-Map:

$$F(A, B, C, D) = \Sigma (2, 3, 4, 5, 6, 7, 8, 10, 11)$$

Answer:

	A'B'	A'B	AB	AB'
C'D'		1		1
C'D		1		
CD	1	1		1
CD'	1	1		1

Or

	C'D'	C'D	CD	CD'
A'B'			1	1
A'B	1	1	1	1
AB				
AB'	1		1	1

$$F(A, B, C, D) = A'B + A'C + B'C + ABD'$$

Question 7:

Reduce the following Boolean Expression using K-Map:

$$F(C, D, A, B) = \Sigma (0, 1, 2, 4, 5, 6, 7, 10)$$

Answer:

CD \ AB		AB			
		$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	0	1	1	0	1
	1	0	1	3	2
$\bar{C}D$	4	1	1	1	1
	5			7	6
CD	12	0	0	0	0
	13			15	14
$C\bar{D}$	8	0	0	0	1
	9			11	10

Quad 1 ($m_0 + m_1 + m_4 + m_5$) reduces to D
Quad 2 ($m_4 + m_5 + m_6 + m_7$) reduces to D
Pair 1 ($m_2 + m_0$) reduces to A
Hence, $F(C, D, A, B) = D + D + A$

Question 8:

Reduce the following Boolean Expression using K-Map:

$$F(C, D, A, B) = \Sigma (0, 1, 2, 5, 7, 8, 9, 11, 12, 15)$$

Answer:

CD \ AB		AB			
		$\bar{A}\bar{B}$	$\bar{A}B$	AB	$A\bar{B}$
$\bar{C}\bar{D}$	00	1	1	0	1
	01	0	2	3	2
$\bar{C}D$	04	0	1	1	0
	05			7	6
CD	11	1	0	1	0
	12			15	14
$C\bar{D}$	10	1	1	1	0
	11			11	10

$$A'B'C + ABC + A'D' + B'C'D' + A'B'D$$

Question 9:

Reduce the following Boolean Expression using K-Map:

Reduce the following Expression to its simplest form using K-Map:

$$F(P, Q, R, S) = \Sigma(0, 4, 5, 8, 9, 10, 11, 12, 13, 15)$$

Answer:

	P'Q'	P'Q	PQ	PQ'
R'S'	1 0	1 4	1 12	1 8
R'S	1	1 5	1 13	1 9
RS	3	7	1 15	1 11
RS'	2	6	14	1 10

OR

	R'S'	R'S	RS	RS'
P'Q'	1 0	1	3	1 2
P'Q	1 4	1 5	7	1 6
PQ	1 12	1 13	1 15	1 14
PQ'	1 8	1 9	1 11	1 10

$$F(P, Q, R, S) = R'S' + PQ' + QR' + PS$$

Question 10:

Reduce the following Boolean Expression using K-Map:

$$F(X, Y, Z, W) = \Sigma(2, 6, 7, 8, 9, 10, 11, 13, 14, 15)$$

Answer:

	X'Y'	X'Y	XY	XY'
Z'W'	0	4	12	8
Z'W	1	5	13	9
ZW	3	7	15	11
ZW'	2	6	14	10

OR

	Z'W'	Z'W	ZW	ZW'
X'Y'	0	1	3	2
X'Y	4	5	7	6
XY	12	13	15	14
XY'	8	9	11	10

$$F(X, Y, Z, W) = XY' + ZW' + XW' + YZ$$

Question 11:

Reduce the following Boolean Expression using K-Map:
 $F(A,B,C,D) = \Sigma(0,1,3,5,6,7,11,13,14,15)$

Answer:

	C'D'	C'D	CD	CD'
A'B'	0	1	3	2
A'B	4	5	7	6
AB	12	13	15	14
AB'	8	9	11	10

$$A'B'C' + D + BC$$

Question 12:

Reduce the following Boolean Expression using K-Map:
 $F(R, S, P, Q) = \Sigma(0, 1, 2, 4, 5, 6, 8, 12)$

Answer:

$\Sigma (0, 1, 2, 4, 5, 6, 8, 12)$

RS \ PQ	PQ 00	PQ 01	PQ 11	PQ 10
$\bar{R}\bar{S}$ 00	1 0	1 1	0 3	1 2
$\bar{R}S$ 01	1 4	1 5	0 7	1 6
RS 11	1 12	0 13	0 15	0 14
$R\bar{S}$ 10	1 8	0 9	0 11	0 10

$$= P'R' + P'Q' + R'Q'$$

Question 13:

Simplify the following boolean expression using K-Map:

$$F(R, S, P, Q) = \Sigma (0, 2, 5, 7, 8, 10, 13, 15)$$

Answer:

	P'Q'	P'Q	PQ	PQ'
R'S'	1 0	0 1	0 3	1 2
R'S	0 4	1 5	1 7	0 6
RS	0 12	1 13	1 15	0 14
RS'	1 8	0 9	0 11	1 10

$$F(A, B, C, D) = Q'S' + QS$$

Question 14:

Reduce the following boolean expression using K-Map:

$$F(U, V, W, Z) = \Sigma (0, 1, 2, 4, 5, 6, 8, 10)$$

Answer:

$$F(W, Z, U, V) = \Sigma(0, 1, 2, 4, 5, 6, 8, 10)$$

	U'V'	U'V	UV	UV'
W'Z'	1 0	1 1	0 3	1 2
W'Z	1 4	1 5	0 7	1 6
WZ	0 12	0 13	0 15	0 14
WZ'	1 8	0 9	0 11	1 10

$$F(U, V, W, Z) = U'W' + V'Z' + W'V'$$

Question 15:

Reduce the following expression using K-map :

$$F(A, B, C, D) = \Sigma(1, 4, 5, 9, 11, 12, 13, 15)$$

Answer:

$$\bar{C}D + B\bar{C} + AD$$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	1 4	1 5	7	6
AB	1 12	1 13	1 15	14
$A\bar{B}$	8	1 9	1 11	10

Question 16:

Reduce the following Boolean Expression using K-Map:

$$F(U, V, W, Z) = \Sigma(3, 5, 7, 10, 11, 13, 15)$$

Answer:

	$\bar{W}\bar{Z}$	$\bar{W}Z$	WZ	$W\bar{Z}$
$\bar{U}\bar{V}$	0 0	0 1	1 3	0 2
$\bar{U}V$	0 4	1 5	1 7	0 6
UV	0 12	1 13	1 15	0 14
$U\bar{V}$	0 8	0 9	1 11	1 10

$$F(U,V,W,Z) = VZ + WZ + UW$$

Question 17:

Reduce the following Boolean Expression using K-Map:

$$F(A, B, C, D) = \Sigma(3, 4, 5, 6, 7, 13, 15)$$

Answer:

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
	00	01	11	10
$\bar{A}\bar{B}$ 00	0 0	0 1	1 3	0 2
$\bar{A}B$ 01	1 4	1 5	1 7	1 6
AB 11	0 12	1 13	1 15	0 14
$A\bar{B}$ 10	0 8	0 9	0 11	1 10

$$F(A, B, C, D) = A'B + BD + A'CD$$