

PROBABILITY

It is remarkable that a science, which began with the consideration of games of chance, should be elevated to the rank of the most important subject of human knowledge.

—**Pierre Simon Laplace**

15.1 Introduction

In everyday life, we come across statements such as

- (1) It will **probably** rain today.
- (2) I **doubt** that he will pass the test.
- (3) **Most probably**, Kavita will stand first in the annual examination.
- (4) **Chances** are high that the prices of diesel will go up.
- (5) There is a 50-50 **chance** of India winning a toss in today's match.

The words 'probably', 'doubt', 'most probably', 'chances', etc., used in the statements above involve an element of uncertainty. For example, in (1), 'probably rain' will mean it may rain or may not rain today. We are predicting rain today based on our past experience when it rained under similar conditions. Similar predictions are also made in other cases listed in (2) to (5).

The uncertainty of 'probably' etc can be measured numerically by means of 'probability' in many cases.

Though probability started with gambling, it has been used extensively in the fields of Physical Sciences, Commerce, Biological Sciences, Medical Sciences, Weather Forecasting, etc.

15.2 Probability – an Experimental Approach

In earlier classes, you have had a glimpse of probability when you performed experiments like tossing of coins, throwing of dice, etc., and observed their *outcomes*. You will now learn to measure the chance of occurrence of a particular *outcome* in an experiment.



Blaise Pascal
(1623–1662)

Fig. 15.1

The concept of probability developed in a very strange manner. In 1654, a gambler Chevalier de Mere, approached the well-known 17th century French philosopher and mathematician Blaise Pascal regarding certain dice problems. Pascal became interested in these problems, studied them and discussed them with another French mathematician, Pierre de Fermat. Both Pascal and Fermat solved the problems independently. This work was the beginning of Probability Theory.



Pierre de Fermat
(1601–1665)

Fig. 15.2

The first book on the subject was written by the Italian mathematician, J. Cardan (1501–1576). The title of the book was ‘Book on Games of Chance’ (Liber de Ludo Aleae), published in 1663. Notable contributions were also made by mathematicians J. Bernoulli (1654–1705), P. Laplace (1749–1827), A.A. Markov (1856–1922) and A.N. Kolmogorov (born 1903).

Activity 1 : (i) Take any coin, toss it ten times and note down the number of times a head and a tail come up. Record your observations in the form of the following table

Table 15.1

Number of times the coin is tossed	Number of times head comes up	Number of times tail comes up
10	—	—

Write down the values of the following fractions:

$$\frac{\text{Number of times a head comes up}}{\text{Total number of times the coin is tossed}}$$

and

$$\frac{\text{Number of times a tail comes up}}{\text{Total number of times the coin is tossed}}$$

- (ii) Toss the coin twenty times and in the same way record your observations as above. Again find the values of the fractions given above for this collection of observations.
- (iii) Repeat the same experiment by increasing the number of tosses and record the number of heads and tails. Then find the values of the corresponding fractions.

You will find that as the number of tosses gets larger, the values of the fractions come closer to 0.5. To record what happens in more and more tosses, the following group activity can also be performed:

Activity 2 : Divide the class into groups of 2 or 3 students. Let a student in each group toss a coin 15 times. Another student in each group should record the observations regarding heads and tails. [Note that coins of the same denomination should be used in all the groups. It will be treated as if only one coin has been tossed by all the groups.]

Now, on the blackboard, make a table like Table 15.2. First, Group 1 can write down its observations and calculate the resulting fractions. Then Group 2 can write down its observations, but will calculate the fractions for the combined data of Groups 1 and 2, and so on. (We may call these fractions as *cumulative fractions*.) We have noted the first three rows based on the observations given by one class of students.

Table 15.2

Group (1)	Number of heads (2)	Number of tails (3)	Cumulative number of heads	Cumulative number of tails
			Total number of times the coin is tossed (4)	Total number of times the coin is tossed (5)
1	3	12	$\frac{3}{15}$	$\frac{12}{15}$
2	7	8	$\frac{7+3}{15+15} = \frac{10}{30}$	$\frac{8+12}{15+15} = \frac{20}{30}$
3	7	8	$\frac{7+10}{15+30} = \frac{17}{45}$	$\frac{8+20}{15+30} = \frac{28}{45}$
4	M	M	M	M

What do you observe in the table? You will find that as the total number of tosses of the coin increases, the values of the fractions in Columns (4) and (5) come nearer and nearer to 0.5.

Activity 3 : (i) Throw a die* 20 times and note down the number of times the numbers

*A die is a well balanced cube with its six faces marked with numbers from 1 to 6, one number on one face. Sometimes dots appear in place of numbers.

1, 2, 3, 4, 5, 6 come up. Record your observations in the form of a table, as in Table 15.3:

Table 15.3

Number of times a die is thrown	Number of times these scores turn up					
	1	2	3	4	5	6
20						

Find the values of the following fractions:

$$\frac{\text{Number of times 1 turned up}}{\text{Total number of times the die is thrown}}$$

$$\frac{\text{Number of times 2 turned up}}{\text{Total number of times the die is thrown}}$$

M
M

$$\frac{\text{Number of times 6 turned up}}{\text{Total number of times the die is thrown}}$$

(ii) Now throw the die 40 times, record the observations and calculate the fractions as done in (i).

As the number of throws of the die increases, you will find that the value of each fraction calculated in (i) and (ii) comes closer and closer to $\frac{1}{6}$.

To see this, you could perform a group activity, as done in Activity 2. Divide the students in your class, into small groups. One student in each group should throw a die ten times. Observations should be noted and cumulative fractions should be calculated.

The values of the fractions for the number 1 can be recorded in Table 15.4. This table can be extended to write down fractions for the other numbers also or other tables of the same kind can be created for the other numbers.

Table 15.4

Group (1)	Total number of times a die is thrown in a group (2)	<u>Cumulative number of times 1 turned up</u> <u>Total number of times the die is thrown</u> (3)
1	—	—
2	—	—
3	—	—
4	—	—

The dice used in all the groups should be almost the same in size and appearance. Then all the throws will be treated as throws of the same die.

What do you observe in these tables?

You will find that as the total number of throws gets larger, the fractions in Column (3) move closer and closer to $\frac{1}{6}$.

Activity 4 : (i) Toss two coins simultaneously ten times and record your observations in the form of a table as given below:

Table 15.5

Number of times the two coins are tossed	Number of times no head comes up	Number of times one head comes up	Number of times two heads come up
10	—	—	—

Write down the fractions:

$$A = \frac{\text{Number of times no head comes up}}{\text{Total number of times two coins are tossed}}$$

$$B = \frac{\text{Number of times one head comes up}}{\text{Total number of times two coins are tossed}}$$

$$C = \frac{\text{Number of times two heads come up}}{\text{Total number of times two coins are tossed}}$$

Calculate the values of these fractions.

Now increase the number of tosses (as in Activity 2). You will find that the more the number of tosses, the closer are the values of A, B and C to 0.25, 0.5 and 0.25, respectively.

In Activity 1, each toss of a coin is called a *trial*. Similarly in Activity 3, each throw of a die is a *trial*, and each simultaneous toss of two coins in Activity 4 is also a *trial*.

So, a *trial* is an action which results in one or several outcomes. The possible outcomes in Activity 1 were Head and Tail; whereas in Activity 3, the possible outcomes were 1, 2, 3, 4, 5 and 6.

In Activity 1, the getting of a head in a particular throw is an *event with outcome 'head'*. Similarly, *getting a tail is an event with outcome 'tail'*. In Activity 2, the getting of a particular number, say 1, is an *event with outcome 1*.

If our experiment was to throw the die for getting an even number, then the event would consist of three outcomes, namely, 2, 4 and 6.

So, an *event* for an experiment is the collection of some outcomes of the experiment. In Class X, you will study a more formal definition of an event.

So, can you now tell what the events are in Activity 4?

With this background, let us now see what probability is. Based on what we directly observe as the outcomes of our trials, we find the *experimental or empirical* probability.

Let n be the total number of trials. The *empirical probability* $P(E)$ of an event E happening, is given by

$$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{The total number of trials}}$$

In this chapter, we shall be finding the empirical probability, though we will write 'probability' for convenience.

Let us consider some examples.

To start with let us go back to Activity 2, and Table 15.2. In Column (4) of this table, what is the fraction that you calculated? Nothing, but it is the empirical probability of getting a head. Note that this probability kept changing depending on the number of trials and the number of heads obtained in these trials. Similarly, the empirical probability of getting a tail is obtained in Column (5) of Table 15.2. This is $\frac{12}{15}$ to start with, then it is $\frac{2}{3}$, then $\frac{28}{45}$, and so on.

So, the empirical probability depends on the number of trials undertaken, and the number of times the outcomes you are looking for coming up in these trials.

Activity 5 : Before going further, look at the tables you drew up while doing Activity 3. Find the probabilities of getting a 3 when throwing a die a certain number of times. Also, show how it changes as the number of trials increases.

Now let us consider some other examples.

Example 1 : A coin is tossed 1000 times with the following frequencies:

Head : 455, Tail : 545

Compute the probability for each event.

Solution : Since the coin is tossed 1000 times, the total number of trials is 1000. Let us call the events of getting a head and of getting a tail as E and F, respectively. Then, the number of times E happens, i.e., the number of times a head come up, is 455.

So, the probability of E = $\frac{\text{Number of heads}}{\text{Total number of trials}}$

$$\text{i.e., } P(E) = \frac{455}{1000} = 0.455$$

Similarly, the probability of the event of getting a tail = $\frac{\text{Number of tails}}{\text{Total number of trials}}$

$$\text{i.e., } P(F) = \frac{545}{1000} = 0.545$$

Note that in the example above, $P(E) + P(F) = 0.455 + 0.545 = 1$, and E and F are the only two possible outcomes of each trial.

Example 2 : Two coins are tossed simultaneously 500 times, and we get

Two heads : 105 times

One head : 275 times

No head : 120 times

Find the probability of occurrence of each of these events.

Solution : Let us denote the events of getting two heads, one head and no head by E_1 , E_2 and E_3 , respectively. So,

$$P(E_1) = \frac{105}{500} = 0.21$$

$$P(E_2) = \frac{275}{500} = 0.55$$

$$P(E_3) = \frac{120}{500} = 0.24$$

Observe that $P(E_1) + P(E_2) + P(E_3) = 1$. Also E_1, E_2 and E_3 cover all the outcomes of a trial.

Example 3 : A die is thrown 1000 times with the frequencies for the outcomes 1, 2, 3, 4, 5 and 6 as given in the following table :

Table 15.6

Outcome	1	2	3	4	5	6
Frequency	179	150	157	149	175	190

Find the probability of getting each outcome.

Solution : Let E_i denote the event of getting the outcome i , where $i = 1, 2, 3, 4, 5, 6$. Then

$$\begin{aligned} \text{Probability of the outcome 1} = P(E_1) &= \frac{\text{Frequency of 1}}{\text{Total number of times the die is thrown}} \\ &= \frac{179}{1000} = 0.179 \end{aligned}$$

$$\text{Similarly, } P(E_2) = \frac{150}{1000} = 0.15, \quad P(E_3) = \frac{157}{1000} = 0.157,$$

$$P(E_4) = \frac{149}{1000} = 0.149, \quad P(E_5) = \frac{175}{1000} = 0.175$$

$$\text{and } P(E_6) = \frac{190}{1000} = 0.19.$$

$$\text{Note that } P(E_1) + P(E_2) + P(E_3) + P(E_4) + P(E_5) + P(E_6) = 1$$

Also note that:

- (i) The probability of each event lies between 0 and 1.
- (ii) The sum of all the probabilities is 1.
- (iii) $E_1, E_2, \dots, E_6$ cover all the possible outcomes of a trial.

Example 4 : On one page of a telephone directory, there were 200 telephone numbers. The frequency distribution of their unit place digit (for example, in the number 25828573, the unit place digit is 3) is given in Table 15.7 :

Table 15.7

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	22	26	22	22	20	10	14	28	16	20

Without looking at the page, the pencil is placed on one of these numbers, i.e., the number is chosen at *random*. What is the probability that the digit in its unit place is 6?

Solution : The probability of digit 6 being in the unit place

$$\begin{aligned}
 &= \frac{\text{Frequency of 6}}{\text{Total number of selected telephone numbers}} \\
 &= \frac{14}{200} = 0.07
 \end{aligned}$$

You can similarly obtain the empirical probabilities of the occurrence of the numbers having the other digits in the unit place.

Example 5 : The record of a weather station shows that out of the past 250 consecutive days, its weather forecasts were correct 175 times.

- What is the probability that on a given day it was correct?
- What is the probability that it was not correct on a given day?

Solution : The total number of days for which the record is available = 250

- (i) P(the forecast was correct on a given day)

$$\begin{aligned}
 &= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the record is available}} \\
 &= \frac{175}{250} = 0.7
 \end{aligned}$$

- (ii) The number of days when the forecast was not correct = $250 - 175 = 75$

$$\text{So, P(the forecast was not correct on a given day)} = \frac{75}{250} = 0.3$$

Notice that:

$$\begin{aligned}
 &\text{P(forecast was correct on a given day)} + \text{P(forecast was not correct on a given day)} \\
 &= 0.7 + 0.3 = 1
 \end{aligned}$$

Example 6 : A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table shows the results of 1000 cases.

Table 15.8

Distance (in km)	less than 4000	4000 to 9000	9001 to 14000	more than 14000
Frequency	20	210	325	445

If you buy a tyre of this company, what is the probability that :

- (i) it will need to be replaced before it has covered 4000 km?
- (ii) it will last more than 9000 km?
- (iii) it will need to be replaced after it has covered somewhere between 4000 km and 14000 km?

Solution : (i) The total number of trials = 1000.

The frequency of a tyre that needs to be replaced before it covers 4000 km is 20.

$$\text{So, } P(\text{tyre to be replaced before it covers 4000 km}) = \frac{20}{1000} = 0.02$$

(ii) The frequency of a tyre that will last more than 9000 km is $325 + 445 = 770$

$$\text{So, } P(\text{tyre will last more than 9000 km}) = \frac{770}{1000} = 0.77$$

(iii) The frequency of a tyre that requires replacement between 4000 km and 14000 km is $210 + 325 = 535$.

$$\text{So, } P(\text{tyre requiring replacement between 4000 km and 14000 km}) = \frac{535}{1000} = 0.535$$

Example 7 : The percentage of marks obtained by a student in the monthly unit tests are given below:

Table 15.9

Unit test	I	II	III	IV	V
Percentage of marks obtained	69	71	73	68	74

Based on this data, find the probability that the student gets more than 70% marks in a unit test.

Solution : The total number of unit tests held is 5.

The number of unit tests in which the student obtained more than 70% marks is 3.

$$\text{So, } P(\text{scoring more than 70\% marks}) = \frac{3}{5} = 0.6$$

Example 8 : An insurance company selected 2000 drivers at random (i.e., without any preference of one driver over another) in a particular city to find a relationship between age and accidents. The data obtained are given in the following table:

Table 15.10

Age of drivers (in years)	Accidents in one year				
	0	1	2	3	over 3
18 - 29	440	160	110	61	35
30 - 50	505	125	60	22	18
Above 50	360	45	35	15	9

Find the probabilities of the following events for a driver chosen at random from the city:

- being 18-29 years of age *and* having exactly 3 accidents in one year.
- being 30-50 years of age *and* having one or more accidents in a year.
- having no accidents in one year.

Solution : Total number of drivers = 2000.

- The number of drivers who are 18-29 years old and have exactly 3 accidents in one year is 61.

$$\begin{aligned} \text{So, } P(\text{driver is 18-29 years old with exactly 3 accidents}) &= \frac{61}{2000} \\ &= 0.0305 \approx 0.031 \end{aligned}$$

- The number of drivers 30-50 years of age and having one or more accidents in one year = $125 + 60 + 22 + 18 = 225$

$$\begin{aligned} \text{So, } P(\text{driver is 30-50 years of age and having one or more accidents}) &= \frac{225}{2000} = 0.1125 \approx 0.113 \end{aligned}$$

- The number of drivers having no accidents in one year = $440 + 505 + 360 = 1305$

Therefore, $P(\text{drivers with no accident}) = \frac{1305}{2000} = 0.653$

Example 9 : Consider the frequency distribution table (Table 14.3, Example 4, Chapter 14), which gives the weights of 38 students of a class.

- Find the probability that the weight of a student in the class lies in the interval 46-50 kg.
- Give two events in this context, one having probability 0 and the other having probability 1.

Solution : (i) The total number of students is 38, and the number of students with weight in the interval 46 - 50 kg is 3.

So, $P(\text{weight of a student is in the interval 46 - 50 kg}) = \frac{3}{38} = 0.079$

- For instance, consider the event that a student weighs 30 kg. Since no student has this weight, the probability of occurrence of this event is 0. Similarly, the probability of a student weighing more than 30 kg is $\frac{38}{38} = 1$.

Example 10 : Fifty seeds were selected at random from each of 5 bags of seeds, and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Table 15.11

Bag	1	2	3	4	5
Number of seeds germinated	40	48	42	39	41

What is the probability of germination of

- more than 40 seeds in a bag?
- 49 seeds in a bag?
- more than 35 seeds in a bag?

Solution : Total number of bags is 5.

- Number of bags in which more than 40 seeds germinated out of 50 seeds is 3.

$$P(\text{germination of more than 40 seeds in a bag}) = \frac{3}{5} = 0.6$$

- (ii) Number of bags in which 49 seeds germinated = 0.

$$P(\text{germination of 49 seeds in a bag}) = \frac{0}{5} = 0.$$

- (iii) Number of bags in which more than 35 seeds germinated = 5.

So, the required probability = $\frac{5}{5} = 1$.

Remark : In all the examples above, you would have noted that the probability of an event can be any fraction from 0 to 1.

EXERCISE 15.1

1. In a cricket match, a batswoman hits a boundary 6 times out of 30 balls she plays. Find the probability that she did not hit a boundary.
2. 1500 families with 2 children were selected randomly, and the following data were recorded:

Number of girls in a family	2	1	0
Number of families	475	814	211

Compute the probability of a family, chosen at random, having

- (i) 2 girls (ii) 1 girl (iii) No girl

Also check whether the sum of these probabilities is 1.

3. Refer to Example 5, Section 14.4, Chapter 14. Find the probability that a student of the class was born in August.
4. Three coins are tossed simultaneously 200 times with the following frequencies of different outcomes:

Outcome	3 heads	2 heads	1 head	No head
Frequency	23	72	77	28

If the three coins are simultaneously tossed again, compute the probability of 2 heads coming up.

5. An organisation selected 2400 families at random and surveyed them to determine a relationship between income level and the number of vehicles in a family. The

information gathered is listed in the table below:

Monthly income (in `)	Vehicles per family			
	0	1	2	Above 2
Less than 7000	10	160	25	0
7000 – 10000	0	305	27	2
10000 – 13000	1	535	29	1
13000 – 16000	2	469	59	25
16000 or more	1	579	82	88

Suppose a family is chosen. Find the probability that the family chosen is

- earning ` 10000 – 13000 per month and owning exactly 2 vehicles.
- earning ` 16000 or more per month and owning exactly 1 vehicle.
- earning less than ` 7000 per month and does not own any vehicle.
- earning ` 13000 – 16000 per month and owning more than 2 vehicles.
- owning not more than 1 vehicle.

6. Refer to Table 14.7, Chapter 14.

- Find the probability that a student obtained less than 20% in the mathematics test.
- Find the probability that a student obtained marks 60 or above.

7. To know the opinion of the students about the subject *statistics*, a survey of 200 students was conducted. The data is recorded in the following table.

Opinion	Number of students
like	135
dislike	65

Find the probability that a student chosen at random

- likes statistics,
- does not like it.

8. Refer to Q.2, Exercise 14.2. What is the empirical probability that an engineer lives:

- less than 7 km from her place of work?
- more than or equal to 7 km from her place of work?
- within $\frac{1}{2}$ km from her place of work?

9. **Activity :** Note the frequency of two-wheelers, three-wheelers and four-wheelers going past during a time interval, in front of your school gate. Find the probability that any one vehicle out of the total vehicles you have observed is a two-wheeler.
10. **Activity :** Ask all the students in your class to write a 3-digit number. Choose any student from the room at random. What is the probability that the number written by her/him is divisible by 3? Remember that a number is divisible by 3, if the sum of its digits is divisible by 3.
11. Eleven bags of wheat flour, each marked 5 kg, actually contained the following weights of flour (in kg):
4.97 5.05 5.08 5.03 5.00 5.06 5.08 4.98 5.04 5.07 5.00
Find the probability that any of these bags chosen at random contains more than 5 kg of flour.
12. In Q.5, Exercise 14.2, you were asked to prepare a frequency distribution table, regarding the concentration of sulphur dioxide in the air in parts per million of a certain city for 30 days. Using this table, find the probability of the concentration of sulphur dioxide in the interval 0.12 - 0.16 on any of these days.
13. In Q.1, Exercise 14.2, you were asked to prepare a frequency distribution table regarding the blood groups of 30 students of a class. Use this table to determine the probability that a student of this class, selected at random, has blood group AB.

15.3 Summary

In this chapter, you have studied the following points:

1. An event for an experiment is the collection of some outcomes of the experiment.
2. The empirical (or experimental) probability $P(E)$ of an event E is given by

$$P(E) = \frac{\text{Number of trials in which } E \text{ has happened}}{\text{Total number of trials}}$$

3. The Probability of an event lies between 0 and 1 (0 and 1 inclusive).

ANSWERS/HINTS

EXERCISE 1.1

1. Yes. $0 = \frac{0}{1} = \frac{0}{2} = \frac{0}{3}$ etc., denominator q can also be taken as negative integer.
2. There can be infinitely many rationals between numbers 3 and 4, one way is to take them
 $3 = \frac{21}{6+1}, 4 = \frac{28}{6+1}$. Then the six numbers are $\frac{22}{7}, \frac{23}{7}, \frac{24}{7}, \frac{25}{7}, \frac{26}{7}, \frac{27}{7}$.
3. $\frac{3}{5} = \frac{30}{50}, \frac{4}{5} = \frac{40}{50}$. Therefore, five rationals are : $\frac{31}{50}, \frac{32}{50}, \frac{33}{50}, \frac{34}{50}, \frac{35}{50}$.
4. (i) True, since the collection of whole numbers contains all the natural numbers.
(ii) False, for example -2 is not a whole number.
(iii) False, for example $\frac{1}{2}$ is a rational number but not a whole number.

EXERCISE 1.2

1. (i) True, since collection of real numbers is made up of rational and irrational numbers.
(ii) False, no negative number can be the square root of any natural number.
(iii) False, for example 2 is real but not irrational.
2. No. For example, $\sqrt{4} = 2$ is a rational number.
3. Repeat the procedure as in Fig. 1.8 several times. First obtain $\sqrt{4}$ and then $\sqrt{5}$.

EXERCISE 1.3

1. (i) 0.36, terminating. (ii) $0.\overline{09}$, non-terminating repeating.
 (iii) 4.125, terminating. (iv) $0.\overline{230769}$, non-terminating repeating.
 (v) $0.\overline{18}$ non-terminating repeating. (vi) 0.8225 terminating.
2. $\frac{2}{7} = 2 \times \frac{1}{7} = 0.\overline{285714}$, $\frac{3}{7} = 3 \times \frac{1}{7} = 0.\overline{428571}$, $\frac{4}{7} = 4 \times \frac{1}{7} = 0.\overline{571428}$,
 $\frac{5}{7} = 5 \times \frac{1}{7} = 0.\overline{714285}$, $\frac{6}{7} = 6 \times \frac{1}{7} = 0.\overline{857142}$
3. (i) $\frac{2}{3}$ [Let $x = 0.666\dots$ So $10x = 6.666\dots$ or, $10x = 6 + x$ or, $x = \frac{6}{9} = \frac{2}{3}$]
 (ii) $\frac{43}{90}$ (iii) $\frac{1}{999}$
4. 1 [Let $x = 0.9999\dots$ So $10x = 9.999\dots$ or, $10x = 9 + x$ or, $x = 1$]
5. $0.\overline{0588235294117647}$
6. The prime factorisation of q has only powers of 2 or powers of 5 or both.
7. $0.01001000100001\dots$, $0.202002000200002\dots$, $0.003000300003\dots$
8. $0.75075007500075000075\dots$, $0.767076700767000767\dots$, $0.808008000800008\dots$
9. (i) and (v) irrational; (ii), (iii) and (iv) rational.

EXERCISE 1.4

1. Proceed as in Section 1.4 for 2.665.
2. Proceed as in Example 11.

EXERCISE 1.5

1. (i) Irrational (ii) Rational (iii) Rational (iv) Irrational
 (v) Irrational
2. (i) $6 + 3\sqrt{2} + 2\sqrt{3} + \sqrt{6}$ (ii) 6 (iii) $7 + 2\sqrt{10}$ (iv) 3
3. There is no contradiction. Remember that when you measure a length with a scale or any other device, you only get an approximate rational value. So, you may not realise that either c or d is irrational.

4. Refer Fig. 1.17.

5. (i) $\frac{\sqrt{7}}{7}$ (ii) $\sqrt{7} + \sqrt{6}$ (iii) $\frac{\sqrt{5} - \sqrt{2}}{3}$ (iv) $\frac{\sqrt{7} + 2}{3}$

EXERCISE 1.6

1. (i) 8 (ii) 2 (iii) 5 2. (i) 27 (ii) 4 (iii) 8 (iv) $\frac{1}{5} \left[(125)^{-\frac{1}{3}} = (5^3)^{-\frac{1}{3}} = 5^{-1} \right]$

3. (i) $2^{\frac{13}{15}}$ (ii) 3^{-21} (iii) $11^{\frac{1}{4}}$ (iv) $56^{\frac{1}{2}}$

EXERCISE 2.1

1. (i) and (ii) are polynomials in one variable, (v) is a polynomial in three variables, (iii), (iv) are not polynomials, because in each of these exponent of the variable is not a whole number.

2. (i) 1 (ii) -1 (iii) $\frac{\pi}{2}$ (iv) 0

3. $3x^{35} - 4; \sqrt{2}y^{100}$ (You can write some more polynomials with different coefficients.)

4. (i) 3 (ii) 2 (iii) 1 (iv) 0

5. (i) quadratic (ii) cubic (iii) quadratic (iv) linear
(v) linear (vi) quadratic (vii) cubic

EXERCISE 2.2

1. (i) 3 (ii) -6 (iii) -3

2. (i) 1, 1, 3 (ii) 2, 4, 4 (iii) 0, 1, 8 (iv) -1, 0, 3

3. (i) Yes (ii) No (iii) Yes (iv) Yes
(v) Yes (vi) Yes

(vii) $-\frac{1}{\sqrt{3}}$ is a zero, but $\frac{2}{\sqrt{3}}$ is not a zero of the polynomial (viii) No

4. (i) -5 (ii) 5 (iii) $-\frac{5}{2}$ (iv) $\frac{2}{3}$
(v) 0 (vi) 0 (vii) $-\frac{d}{c}$

EXERCISE 2.3

1. (i) 0 (ii) $\frac{27}{8}$ (iii) 1 (iv) $-\pi^3 + 3\pi^2 - 3\pi + 1$ (v) $-\frac{27}{8}$
2. $5a$ 3. No, since remainder is not zero.

EXERCISE 2.4

1. $(x+1)$ is a factor of (i), but not the factor of (ii), (iii) and (iv).
2. (i) Yes (ii) No (iii) Yes
3. (i) -2 (ii) $-(2 + \sqrt{2})$ (iii) $\sqrt{2} - 1$ (iv) $\frac{3}{2}$
4. (i) $(3x-1)(4x-1)$ (ii) $(x+3)(2x+1)$ (iii) $(2x+3)(3x-2)$ (iv) $(x+1)(3x-4)$
5. (i) $(x-2)(x-1)(x+1)$ (ii) $(x+1)(x+1)(x-5)$
 (iii) $(x+1)(x+2)(x+10)$ (iv) $(y-1)(y+1)(2y+1)$

EXERCISE 2.5

1. (i) $x^2 + 14x + 40$ (ii) $x^2 - 2x - 80$ (iii) $9x^2 - 3x - 20$
 (iv) $y^4 - \frac{9}{4}$ (v) $9 - 4x^2$
2. (i) 11021 (ii) 9120 (iii) 9984
3. (i) $(3x+y)(3x+y)$ (ii) $(2y-1)(2y-1)$ (iii) $\left(x + \frac{y}{10}\right)\left(x - \frac{y}{10}\right)$
4. (i) $x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$
 (ii) $4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$
 (iii) $4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$
 (iv) $9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$
 (v) $4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$
 (vi) $\frac{a^2}{16} + \frac{b^2}{4} + 1 - \frac{ab}{4} - b + \frac{a}{2}$
5. (i) $(2x+3y-4z)(2x+3y-4z)$ (ii) $(-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$
6. (i) $8x^3 + 12x^2 + 6x + 1$ (ii) $8a^3 - 27b^3 - 36a^2b + 54ab^2$

$$(iii) \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

$$(iv) x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4xy^2}{3}$$

$$7. (i) 970299$$

$$(ii) 1061208$$

$$(iii) 994011992$$

$$8. (i) (2a+b)(2a+b)(2a+b)$$

$$(ii) (2a-b)(2a-b)(2a-b)$$

$$(iii) (3-5a)(3-5a)(3-5a)$$

$$(iv) (4a-3b)(4a-3b)(4a-3b)$$

$$(v) \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)$$

$$10. (i) (3y+5z)(9y^2+25z^2-15yz)$$

$$(ii) (4m-7n)(16m^2+49n^2+28mn)$$

$$11. (3x+y+z)(9x^2+y^2+z^2-3xy-yz-3xz)$$

12. Simplify RHS.

13. Put $x+y+z=0$ in Identity VIII.

14. (i) -1260 . Let $a=-12$, $b=7$, $c=5$. Here $a+b+c=0$. Use the result given in Q13.

(ii) 16380

15. (i) One possible answer is : Length = $5a-3$, Breadth = $5a-4$

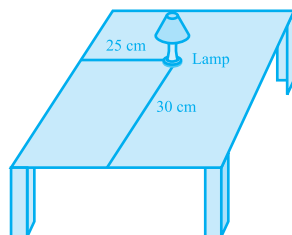
(ii) One possible answer is : Length = $7y-3$, Breadth = $5y+4$

16. (i) One possible answer is : 3 , x and $x-4$.

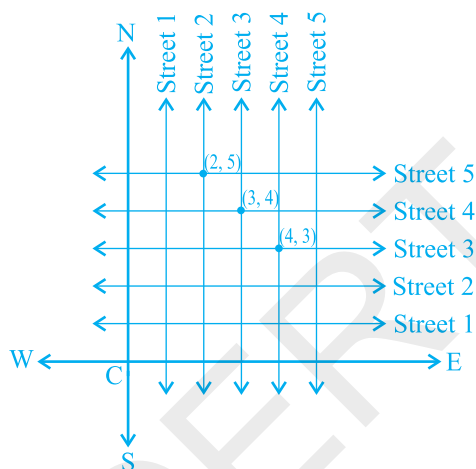
(ii) One possible answer is : $4k$, $3y+5$ and $y-1$.

EXERCISE 3.1

- Consider the lamp as a point and table as a plane. Choose any two perpendicular edges of the table. Measure the distance of the lamp from the longer edge, suppose it is 25 cm. Again, measure the distance of the lamp from the shorter edge, and suppose it is 30 cm. You can write the position of the lamp as (30, 25) or (25, 30), depending on the order you fix.



2. The Street plan is shown in figure given below.



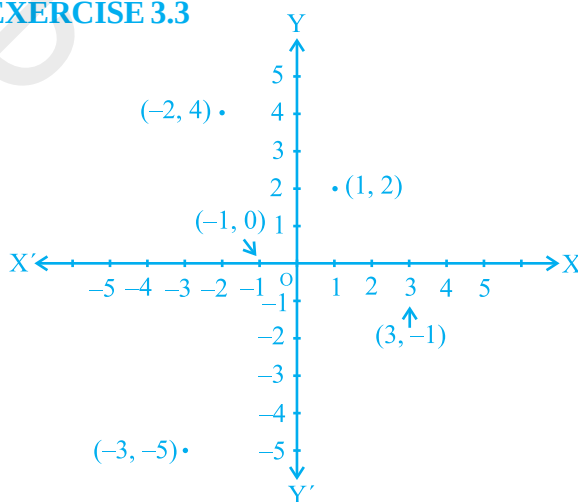
Both the cross-streets are marked in the figure above. They are *uniquely* found because of the two reference lines we have used for locating them.

EXERCISE 3.2

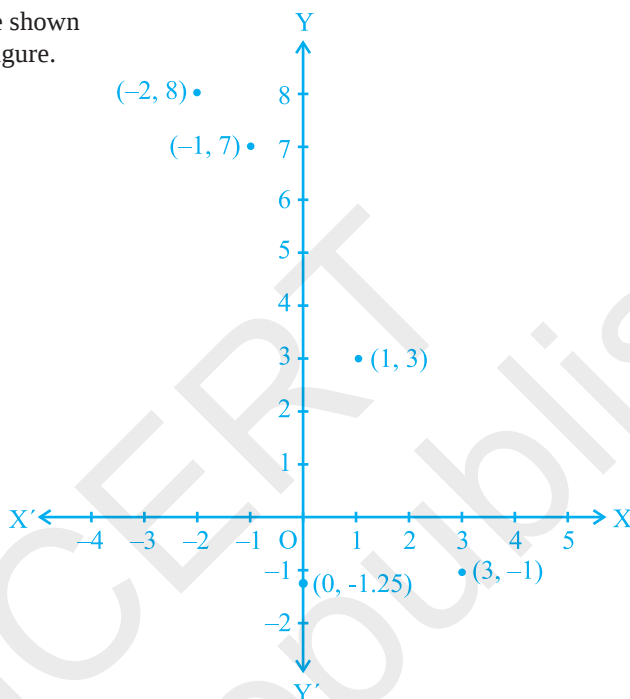
- (i) The x - axis and the y - axis (ii) Quadrants (iii) The origin
- (i) $(-5, 2)$ (ii) $(5, -5)$ (iii) E (iv) G (v) 6 (vi) -3 (vii) $(0, 5)$ (viii) $(-3, 0)$

EXERCISE 3.3

- The point $(-2, 4)$ lies in quadrant II, the point $(3, -1)$ lies in the quadrant IV, the point $(-1, 0)$ lies on the negative x - axis, the point $(1, 2)$ lies in the quadrant I and the point $(-3, -5)$ lies in the quadrant III. Locations of the points are shown in the adjoining figure.



2. Positions of the points are shown by dots in the adjoining figure.



EXERCISE 4.1

1. $x - 2y = 0$
2. (i) $2x + 3y - 9.35 = 0$; $a = 2$, $b = 3$, $c = -9.35$
 (ii) $x - \frac{y}{5} - 10 = 0$; $a = 1$, $b = \frac{-1}{5}$, $c = -10$
 (iii) $-2x + 3y - 6 = 0$; $a = -2$, $b = 3$, $c = -6$
 (iv) $1.x - 3y + 0 = 0$; $a = 1$, $b = -3$, $c = 0$
 (v) $2x + 5y + 0 = 0$; $a = 2$, $b = 5$, $c = 0$
 (vi) $3x + 0.y + 2 = 0$; $a = 3$, $b = 0$, $c = 2$
 (vii) $0.x + 1.y - 2 = 0$; $a = 0$, $b = 1$, $c = -2$
 (viii) $-2x + 0.y + 5 = 0$; $a = -2$, $b = 0$, $c = 5$

EXERCISE 4.2

1. (iii), because for every value of x , there is a corresponding value of y and vice-versa.

2. (i) $(0, 7), (1, 5), (2, 3), (4, -1)$

(ii) $(1, 9 - \pi), (0, 9), (-1, 9 + \pi), \left(\frac{9}{\pi}, 0\right)$

(iii) $(0, 0), (4, 1), (-4, 1), \left(2, \frac{1}{2}\right)$

3. (i) No

(ii) No

(iii) Yes

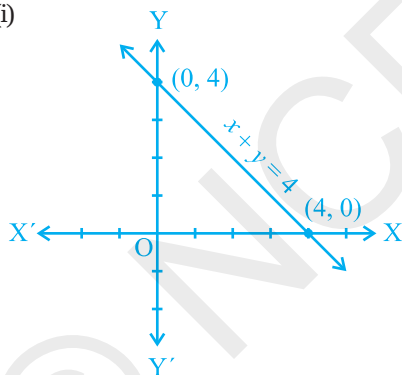
(iv) No

(v) No

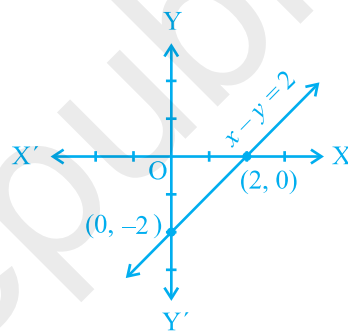
4. 7

EXERCISE 4.3

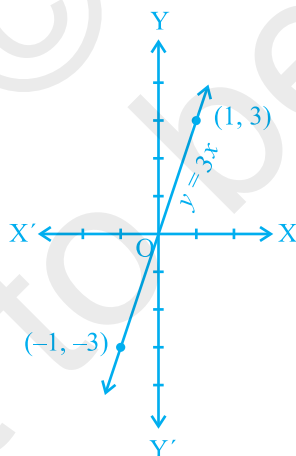
1. (i)



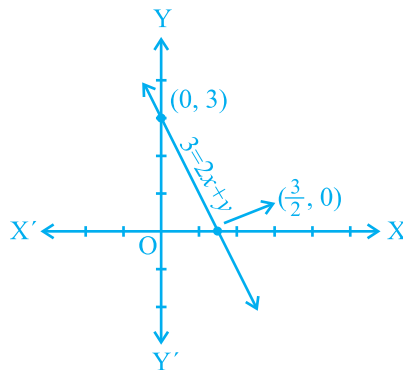
(ii)



(iii)



(iv)



2. $7x - y = 0$ and $x + y = 16$; infinitely many [Through a point infinitely many lines can be drawn]

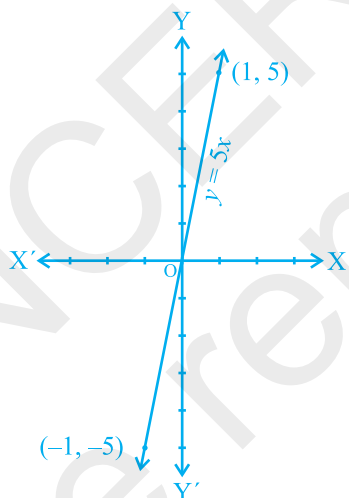
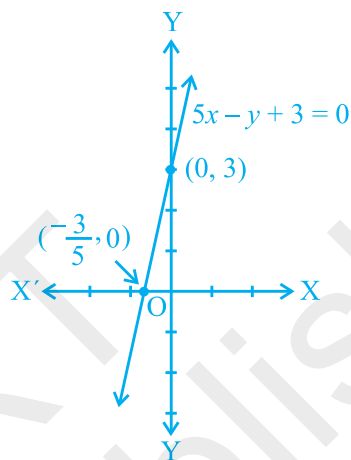
3. $\frac{5}{3}$

4. $5x - y + 3 = 0$

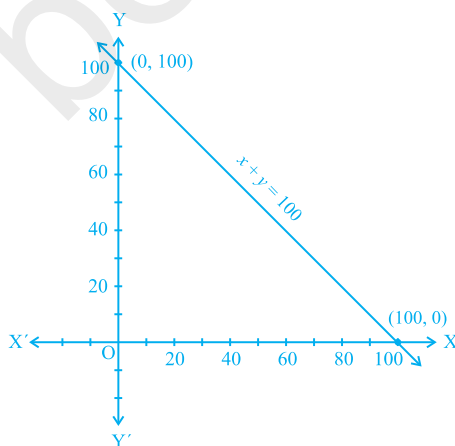
5. For Fig. 4.6, $x + y = 0$ and for Fig. 4.7, $y = -x + 2$.
 6. Supposing x is the distance and y is the work done. Therefore according to the problem the equation will be $y = 5x$.

(i) 10 units

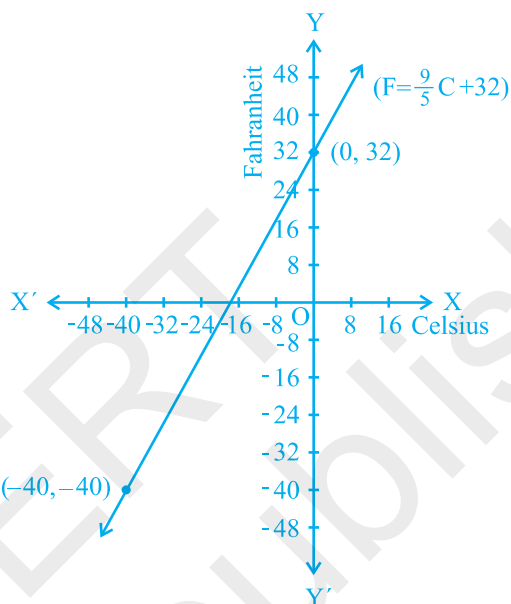
(ii) 0 unit



7. $x + y = 100$



8. (i) See adjacent figure.
 (ii) 86°F
 (iii) 35°C
 (iv) 32°F , -17.8°C (approximately)
 (v) Yes, -40° (both in F and C)

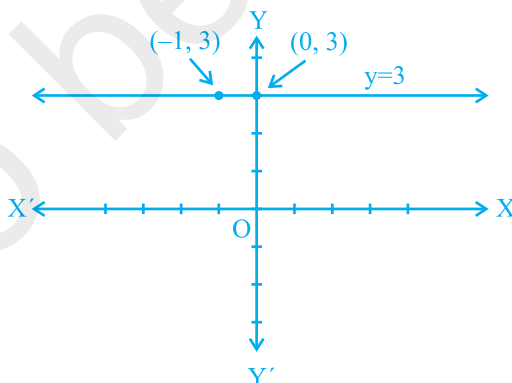


EXERCISE 4.4

1. (i)



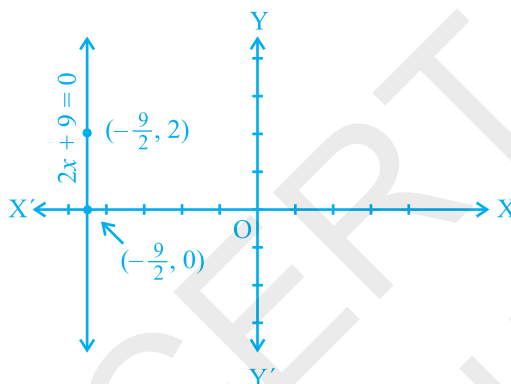
- (ii)



2. (i)



(ii)

**EXERCISE 5.1**

1. (i) False. This can be seen visually by the student.
 (ii) False. This contradicts Axiom 5.1.
 (iii) True. (Postulate 2)
 (iv) True. If you superimpose the region bounded by one circle on the other, then they coincide. So, their centres and boundaries coincide. Therefore, their radii will coincide.
 (v) True. The first axiom of Euclid.
3. There are several undefined terms which the student should list. They are consistent, because they deal with two different situations — (i) says that given two points A and B, there is a point C lying on the line in between them; (ii) says that given A and B, you can take C not lying on the line through A and B.

These 'postulates' do not follow from Euclid's postulates. However, they follow from Axiom 5.1.

4.

$$AC = BC$$

So,

$$AC + AC = BC + AC$$

(Equals are added to equals)

i.e.,

$$2AC = AB$$

(BC + AC coincides with AB)

Therefore,

$$AC = \frac{1}{2} AB$$

5. Make a temporary assumption that different points C and D are two mid-points of AB. Now, you show that points C and D are not two different points.
6. $AC = BD$ (Given) (1)
 $AC = AB + BC$ (Point B lies between A and C) (2)
 $BD = BC + CD$ (Point C lies between B and D) (3)
- Substituting (2) and (3) in (1), you get
 $AB + BC = BC + CD$
- So, $AB = CD$ (Subtracting equals from equals)
7. Since this is true for any thing in any part of the world, this is a universal truth.

EXERCISE 5.2

- Any formulation the student gives should be discussed in the class for its validity.
- If a straight line l falls on two straight lines m and n such that sum of the interior angles on one side of l is two right angles, then by Euclid's fifth postulate the line will not meet on this side of l . Next, you know that the sum of the interior angles on the other side of line l will also be two right angles. Therefore, they will not meet on the other side also. So, the lines m and n never meet and are, therefore, parallel.

EXERCISE 6.1

- $30^\circ, 250^\circ$
- 126°
- Sum of all the angles at a point = 360°
- $\angle QOS = \angle SOR + \angle ROQ$ and $\angle POS = \angle POR - \angle SOR$.
- $122^\circ, 302^\circ$

EXERCISE 6.2

- $130^\circ, 130^\circ$
- 126°
- $126^\circ, 36^\circ, 54^\circ$
- 60°
- $50^\circ, 77^\circ$
- Angle of incidence = Angle of reflection. At point B, draw $BE \perp PQ$ and at point C, draw $CF \perp RS$.

EXERCISE 6.3

- 65°
- $32^\circ, 121^\circ$
- 92°
- 60°
- $37^\circ, 53^\circ$
- Sum of the angles of $\triangle PQR =$ Sum of the angles of $\triangle QTR$ and $\angle PRS = \angle QPR + \angle PQR$.

EXERCISE 7.1

- They are equal.
- $\angle BAC = \angle DAE$

EXERCISE 7.2

6. $\angle BCD = \angle BCA + \angle DCA = \angle B + \angle D$ 7. each is of 45°

EXERCISE 7.3

3. (ii) From (i), $\angle ABM = \angle PQN$

EXERCISE 7.4

4. Join BD and show $\angle B > \angle D$. Join AC and show $\angle A > \angle C$.
5. $\angle Q + \angle QPS > \angle R + \angle RPS$, etc.

EXERCISE 8.1

1. 36° , 60° , 108° and 156° .
6. (i) From $\triangle DAC$ and $\triangle BCA$, show $\angle DAC = \angle BCA$ and $\angle ACD = \angle CAB$, etc.
(ii) Show $\angle BAC = \angle BCA$, using Theorem 8.4.

EXERCISE 8.2

2. Show PQRS is a parallelogram. Also show $PQ \parallel AC$ and $PS \parallel BD$. So, $\angle P = 90^\circ$.
5. AECF is a parallelogram. So, $AF \parallel CE$, etc.

EXERCISE 9.1

1. (i) Base DC, parallels DC and AB; (iii) Base QR, parallels QR and PS;
(v) Base AD, parallels AD and BQ

EXERCISE 9.2

1. 12.8 cm. 2. Join EG; Use result of Example 2.
6. Wheat in $\triangle APQ$ and pulses in other two triangles or pulses in $\triangle APQ$ and wheat in other two triangles.

EXERCISE 9.3

4. Draw $CM \perp AB$ and $DN \perp AB$. Show $CM = DN$. 12. See Example 4.

EXERCISE 9.4 (Optional)

7. Use result of Example 3 repeatedly.

EXERCISE 10.1

- | | | |
|-----------------|---------------|----------------|
| 1. (i) Interior | (ii) Exterior | (iii) Diameter |
| (iv) Semicircle | (v) The chord | (vi) Three |
| 2. (i) True | (ii) False | (iii) False |
| (iv) True | (v) False | (vi) True |

EXERCISE 10.2

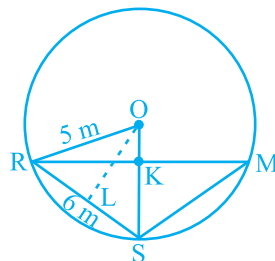
1. Prove exactly as Theorem 10.1 by considering chords of congruent circles.
2. Use SAS axiom of congruence to show the congruence of the two triangles.

EXERCISE 10.3

1. 0, 1, 2. Two
2. Proceed as in Example 1.
3. Join the centres O, O' of the circles to the mid-point M of the common chord AB. Then, show $\angle OMA = 90^\circ$ and $\angle O'MA = 90^\circ$.

EXERCISE 10.4

1. 6 cm. First show that the line joining centres is perpendicular to the radius of the smaller circle and then that common chord is the diameter of the smaller circle.
2. If AB, CD are equal chords of a circle with centre O intersecting at E, draw perpendiculars OM on AB and ON on CD and join OE. Show that right triangles OME and ONE are congruent.
3. Proceed as in Example 2.
4. Draw perpendicular OM on AD.
5. Represent Reshma, Salma and Mandip by R, S and M respectively. Let $KR = x$ m (see figure).
 Area of $\triangle ORS = \frac{1}{2} \times x \times 5$. Also, area of $\triangle ORS = \frac{1}{2} \times RS \times OL = \frac{1}{2} \times 6 \times 4$.
 Find x and hence RM.
6. Use the properties of an equilateral triangle and also Pythagoras Theorem.



EXERCISE 10.5

1. 45°
2. $150^\circ, 30^\circ$
3. 10°
4. 80°
5. 110°
6. $\angle BCD = 80^\circ$ and $\angle ECD = 50^\circ$
8. Draw perpendiculars AM and BN on CD ($AB \parallel CD$ and $AB < CD$). Show $\triangle AMD \cong \triangle BNC$. This gives $\angle C = \angle D$ and, therefore, $\angle A + \angle C = 180^\circ$.

EXERCISE 10.6 (Optional)

2. Let O be the centre of the circle. Then perpendicular bisector of both the chords will be same and passes through O. Let r be the radius, then $r^2 = \left(\frac{11}{2}\right)^2 + x^2 = \left(\frac{5}{2}\right)^2 + (6-x)^2$, where x is length of the perpendicular from O on the chord of length 11 cm. This gives $x = 1$. So, $r = \frac{5\sqrt{5}}{2}$ cm.
3. 3 cm.
4. Let $\angle AOC = x$ and $\angle DOE = y$. Let $\angle AOD = z$. Then $\angle EOC = z$ and $x + y + 2z = 360^\circ$.
 $\angle ODB = \angle OAD + \angle DOA = 90^\circ - \frac{1}{2}z + z = 90^\circ + \frac{1}{2}z$. Also $\angle OEB = 90^\circ + \frac{1}{2}z$
8. $\angle ABE = \angle ADE$, $\angle ADF = \angle ACF = \frac{1}{2} \angle C$.
 Therefore, $\angle EDF = \angle ABE + \angle ADF = \frac{1}{2}(\angle B + \angle C) = \frac{1}{2}(180^\circ - \angle A) = 90^\circ - \frac{1}{2} \angle A$.
9. Use Q. 1, Ex. 10.2 and Theorem 10.8.
10. Let angle-bisector of $\angle A$ intersect circumcircle of $\triangle ABC$ at D. Join DC and DB. Then $\angle BCD = \angle BAD = \frac{1}{2} \angle A$ and $\angle DBC = \angle DAC = \frac{1}{2} \angle A$. Therefore, $\angle BCD = \angle DBC$ or, $DB = DC$. So, D lies on the perpendicular bisector of BC.

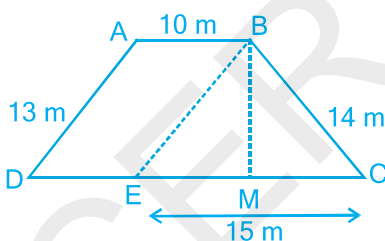
EXERCISE 12.1

1. $\frac{\sqrt{3}}{4}a^2$, $900, 3\text{cm}^2$
2. ₹ 1650000
3. $20\sqrt{2}\text{ m}^2$
4. $21\sqrt{11}\text{cm}^2$
5. 9000cm^2
6. $9\sqrt{15}\text{ cm}^2$

EXERCISE 12.2

1. 65.5 m^2 (approx.) 2. 15.2 cm^2 (approx.) 3. 19.4 cm^2 (approx.)
4. 12 cm 5. 48 m^2 6. $1000\sqrt{6} \text{ cm}^2$, $1000\sqrt{6} \text{ cm}^2$
7. Area of shade I = Area of shade II = 256 cm^2 and area of shade III = 17.92 cm^2
8. ₹ 705.60 9. 196 m^2

[See the figure. Find area of $\triangle BEC = 84 \text{ m}^2$, then find the height BM.]



EXERCISE 13.1

1. (i) 5.45 m^2 (ii) ₹ 109 2. ₹ 555 3. 6 m 4. 100 bricks.
5. (i) Lateral surface area of cubical box is greater by 40 cm^2 .
(ii) Total surface area of cuboidal box is greater by 10 cm^2 .
6. (i) 4250 cm^2 of glass (ii) 320 cm of tape. [Calculate the sum of all the edges (The 12 edges consist of 4 lengths, 4 breadths and 4 heights)].
7. ₹ 2184 8. 47 m^2

EXERCISE 13.2

1. 2 cm 2. 7.48 m^2 3. (i) 968 cm^2 (ii) 1064.8 cm^2 (iii) 2038.08 cm^2

[Total surface area of a pipe is (inner curved surface area + outer curved surface area + areas of the two bases). Each base is a ring of area given by $\pi (R^2 - r^2)$, where R = outer radius and r = inner radius].

4. 1584 m^2 5. Rs 68.75 6. 1 m
7. (i) 110 m^2 (ii) Rs 4400 8. 4.4 m^2
9. (i) 59.4 m^2 (ii) 95.04 m^2

[Let the actual area of steel used be $x \text{ m}^2$. Since $\frac{1}{12}$ of the actual steel used was

wasted, the area of steel which has gone into the tank = $\frac{11}{12}$ of x . This means that the

actual area of steel used = $\frac{12}{11} \times 87.12 \text{ m}^2$]

10. 2200 cm^2 ; Height of the cylinder should be treated as $(30 + 2.5 + 2.5) \text{ cm}$

11. 7920 cm^2

EXERCISE 13.3

- | | | |
|---------------------------------------|--------------------------|---|
| 1. 165 cm^2 | 2. 1244.57 m^2 | 3. (i) 7 cm (ii) 462 cm^2 |
| 4. (i) 26 m (ii) ₹ 137280 | 5. 63 m | 6. ₹ 1155 |
| 7. 5500 cm^2 | 8. ₹ 384.34 (approx.) | |

EXERCISE 13.4

- | |
|---|
| 1. (i) 1386 cm^2 (ii) 394.24 cm^2 (iii) 2464 cm^2 |
| 2. (i) 616 cm^2 (ii) 1386 cm^2 (iii) 38.5 m^2 |
| 3. 942 cm^2 4. $1:4$ 5. ₹ 27.72 |
| 6. 3.5 cm 7. $1:16$ 8. 173.25 cm^2 |
| 9. (i) $4\pi r^2$ (ii) $4\pi r^2$ (iii) $1:1$ |

EXERCISE 13.5

- | | | | | |
|-----------------------|----------------------------|------------------------|-----------------------|------------------|
| 1. 180 cm^3 | 2. 135000 litres | 3. 4.75 m | 4. ₹ 4320 | 5. 2 m |
| 6. 3 days | 7. 16000 | 8. $6 \text{ cm}, 4:1$ | 9. 4000 m^3 | |

EXERCISE 13.6

- 34.65 litres
- 3.432 kg [Volume of a pipe = $\pi h \times (R^2 - r^2)$, where R is the outer radius and r is the inner radius].
- The cylinder has the greater capacity by 85 cm^3 .
- (i) 3 cm (ii) 141.3 cm^3
- (i) 110 m^2 (ii) 1.75 m (iii) 96.25 kl 6. 0.4708 m^2
- Volume of wood = 5.28 cm^3 , Volume of graphite = 0.11 cm^3 .
- 38500 cm^3 or 38.5 l of soup

EXERCISE 13.7

1. (i) 264 cm^3 (ii) 154 cm^3
2. (i) 1.232 l (ii) $\frac{11}{35} \text{ l}$
3. 10 cm 4. 8 cm 5. 38.5 kl
6. (i) 48 cm (ii) 50 cm (iii) 2200 cm^2 7. $100\pi \text{ cm}^3$ 8. $240\pi \text{ cm}^3; 5 : 12$
9. $86.625 \text{ m}^3, 99.825 \text{ m}^2$

EXERCISE 13.8

1. (i) $1437 \frac{1}{3} \text{ cm}^3$ (ii) 1.05 m^3 (approx.)
2. (i) $11498 \frac{2}{3} \text{ cm}^3$ (ii) 0.004851 m^3 3. 345.39 g (approx.) 4. $\frac{1}{64}$
5. 0.303 l (approx.) 6. 0.06348 m^3 (approx.)
7. $179 \frac{2}{3} \text{ cm}^3$ 8. (i) 249.48 m^2 (ii) 523.9 m^3 (approx.) 9. (i) $3r$ (ii) $1 : 9$
10. 22.46 mm^3 (approx.)

EXERCISE 13.9 (Optional)

1. ₹ 6275
2. ₹ 2784.32 (approx.) [Remember to subtract the part of the sphere that is resting on the support while calculating the cost of silver paint]. 3. 43.75%

EXERCISE 14.1

1. Five examples of data that we can gather from our day-to-day life are :
 - (i) Number of students in our class.
 - (ii) Number of fans in our school.
 - (iii) Electricity bills of our house for last two years.
 - (iv) Election results obtained from television or newspapers.
 - (v) Literacy rate figures obtained from Educational Survey.

Of course, remember that there can be many more different answers.

2. Primary data; (i), (ii) and (iii)
Secondary data; (iv) and (v)

EXERCISE 14.2

1.

Blood group	Number of students
A	9
B	6
O	12
AB	3
Total	30

Most common – O , Rarest – AB

2.

Distances (in km)	Tally Marks	Frequency
0 - 5		5
5 - 10		11
10 - 15		11
15 - 20		9
20 - 25		1
25 - 30		1
30 - 35		2
Total		40

3. (i)

Relative humidity (in %)	Frequency
84 - 86	1
86 - 88	1
88 - 90	2
90 - 92	2
92 - 94	7
94 - 96	6
96 - 98	7
98 - 100	4
Total	30

- (ii) The data appears to be taken in the rainy season as the relative humidity is high.
 (iii) $\text{Range} = 99.2 - 84.9 = 14.3$

4. (i)

Heights (in cm)	Frequency
150 - 155	12
155 - 160	9
160 - 165	14
165 - 170	10
170 - 175	5
Total	50

- (ii) One conclusion that we can draw from the above table is that more than 50% of students are shorter than 165 cm.

5. (i)

Concentration of Sulphur dioxide (in ppm)	Frequency
0.00 - 0.04	4
0.04 - 0.08	9
0.08 - 0.12	9
0.12 - 0.16	2
0.16 - 0.20	4
0.20 - 0.24	2
Total	30

- (ii) The concentration of sulphur dioxide was more than 0.11 ppm for 8 days.

6.

Number of heads	Frequency
0	6
1	10
2	9
3	5
Total	30

7. (i)

Digits	Frequency
0	2
1	5
2	5
3	8
4	4
5	5
6	4
7	4
8	5
9	8
Total	50

(ii) The most frequently occurring digits are 3 and 9. The least occurring is 0.

8. (i)

Number of hours	Frequency
0 - 5	10
5 - 10	13
10 - 15	5
15 - 20	2
Total	30

(ii) 2 children.

9.

Life of batteries (in years)	Frequency
2.0 - 2.5	2
2.5 - 3.0	6
3.0 - 3.5	14
3.5 - 4.0	11
4.0 - 4.5	4
4.5 - 5.0	3
Total	40

EXERCISE 14.3

1. (ii) Reproductive health conditions.
 3. (ii) Party A 4. (ii) Frequency polygon (iii) No 5. (ii) 184
 8.

Age (in years)	Frequency	Width	Length of the rectangle
1 - 2	5	1	$\frac{5}{1} \times 1 = 5$
2 - 3	3	1	$\frac{3}{1} \times 1 = 3$
3 - 5	6	2	$\frac{6}{2} \times 1 = 3$
5 - 7	12	2	$\frac{12}{2} \times 1 = 6$
7 - 10	9	3	$\frac{9}{3} \times 1 = 3$
10 - 15	10	5	$\frac{10}{5} \times 1 = 2$
15 - 17	4	2	$\frac{4}{2} \times 1 = 2$

Now, you can draw the histogram, using these lengths.

9. (i)

Number of letters	Frequency	Width of interval	Length of rectangle
1 - 4	6	3	$\frac{6}{3} \times 2 = 4$
4 - 6	30	2	$\frac{30}{2} \times 2 = 30$
6 - 8	44	2	$\frac{44}{2} \times 2 = 44$
8 - 12	16	4	$\frac{16}{4} \times 2 = 8$
12 - 20	4	8	$\frac{4}{8} \times 2 = 1$

Now, draw the histogram.

- (ii) 6 - 8

EXERCISE 14.4

1. Mean = 2.8; Median = 3; Mode = 3
2. Mean = 54.8; Median = 52; Mode = 52
3. $x = 62$ 4. 14
5. Mean salary of 60 workers is Rs 5083.33.

EXERCISE 15.1

1. $\frac{24}{30}$, i.e., $\frac{4}{5}$ 2. (i) $\frac{19}{60}$ (ii) $\frac{407}{750}$ (iii) $\frac{211}{1500}$ 3. $\frac{3}{20}$ 4. $\frac{9}{25}$
5. (i) $\frac{29}{2400}$ (ii) $\frac{579}{2400}$ (iii) $\frac{1}{240}$ (iv) $\frac{1}{96}$ (v) $\frac{1031}{1200}$ 6. (i) $\frac{7}{90}$ (ii) $\frac{23}{90}$
7. (i) $\frac{27}{40}$ (ii) $\frac{13}{40}$ 8. (i) $\frac{9}{40}$ (ii) $\frac{31}{40}$ (iii) 0 11. $\frac{7}{11}$ 12. $\frac{1}{15}$ 13. $\frac{1}{10}$

EXERCISE A1.1

1. (i) False. There are 12 months in a year.
 (ii) Ambiguous. In a given year, Diwali may or may not fall on a Friday.
 (iii) Ambiguous. At some time in the year, the temperature in Magadi, may be 26°C .
 (iv) Always true.
 (v) False. Dogs cannot fly.
 (vi) Ambiguous. In a leap year, February has 29 days.
2. (i) False. The sum of the interior angles of a quadrilateral is 360° .
 (ii) True (iii) True (iv) True
 (v) False, for example, $7 + 5 = 12$, which is not an odd number.
3. (i) All prime numbers greater than 2 are odd. (ii) Two times a natural number is always even. (iii) For any $x > 1$, $3x + 1 > 4$. (iv) For any $x \geq 0$, $x^3 \geq 0$.
 (v) In an equilateral triangle, a median is also an angle bisector.

EXERCISE A1.2

1. (i) Humans are vertebrates. (ii) No. Dinesh could have got his hair cut by anybody else. (iii) Gulag has a red tongue. (iv) We conclude that the gutters will have to be cleaned tomorrow. (v) All animals having tails need not be dogs. For example, animals such as buffaloes, monkeys, cats, etc. have tails but are not dogs.
2. You need to turn over B and 8. If B has an even number on the other side, then the rule

has been broken. Similarly, if 8 has a consonant on the other side, then the rule has been broken.

EXERCISE A1.3

- Three possible conjectures are:
 - The product of any three consecutive even numbers is even.
 - The product of any three consecutive even numbers is divisible by 4.
 - The product of any three consecutive even numbers is divisible by 6.
- Line 4: $1\ 3\ 3\ 1 = 11^3$; Line 5: $1\ 4\ 6\ 4\ 1 = 11^4$; the conjecture holds for Line 4 and Line 5; No, because $11^5 \neq 15101051$.
- $T_4 + T_5 = 25 = 5^2$; $T_{n-1} + T_n = n^2$.
- $111111^2 = 12345654321$; $1111111^2 = 1234567654321$
- Student's own answer. For example, Euclid's postulates.

EXERCISE A1.4

- You can give any two triangles with the same angles but of different sides.
 - A rhombus has equal sides but may not be a square.
 - A rectangle has equal angles but may not be a square.
 - For $a = 3$ and $b = 4$, the statement is not true.
 - For $n = 11$, $2n^2 + 11 = 253$ which is not a prime.
 - For $n = 41$, $n^2 - n + 41$ is not a prime.
- Student's own answer.
- Let x and y be two odd numbers. Then $x = 2m + 1$ for some natural number m and $y = 2n + 1$ for some natural number n .
 $x + y = 2(m + n + 1)$. Therefore, $x + y$ is divisible by 2 and is even.
- See Q.3. $xy = (2m + 1)(2n + 1) = 2(2mn + m + n) + 1$.
 Therefore, xy is not divisible by 2, and so it is odd.
- Let $2n$, $2n + 2$ and $2n + 4$ be three consecutive even numbers. Then their sum is $6(n + 1)$, which is divisible by 6.
- Let your original number be n . Then we are doing the following operations:

$$n \rightarrow 2n \rightarrow 2n + 9 \rightarrow 2n + 9 + n = 3n + 9 \rightarrow \frac{3n + 9}{3} = n + 3 \rightarrow n + 3 + 4 = n + 7 \rightarrow n + 7 - n = 7.$$
 - Note that $7 \times 11 \times 13 = 1001$. Take any three digit number say, abc . Then $abc \times 1001 = abcabc$. Therefore, the six digit number $abcabc$ is divisible by 7, 11 and 13.

EXERCISE A2.1

1. **Step 1: Formulation :**

The relevant factors are the time period for hiring a computer, and the two costs given to us. We assume that there is no significant change in the cost of purchasing or hiring the computer. So, we treat any such change as irrelevant. We also treat all brands and generations of computers as the same, i.e. these differences are also irrelevant.

The expense of hiring the computer for x months is ₹ $2000x$. If this becomes more than the cost of purchasing a computer, we will be better off buying a computer. So, the equation is

$$2000x = 25000 \quad (1)$$

Step 2 : Solution : Solving (1), $x = \frac{25000}{2000} = 12.5$

Step 3 : Interpretation : Since the cost of hiring a computer becomes more **after** 12.5 months, it is cheaper to buy a computer, if you have to use it for more than 12 months.

2. **Step 1 : Formulation :** We will assume that cars travel at a constant speed. So, any change of speed will be treated as irrelevant. If the cars meet after x hours, the first car would have travelled a distance of $40x$ km from A and the second car would have travelled $30x$ km, so that it will be at a distance of $(100 - 30x)$ km from A. So the equation will be $40x = 100 - 30x$, i.e., $70x = 100$.

Step 2 : Solution : Solving the equation, we get $x = \frac{100}{70}$.

Step 3 : Interpretation : $\frac{100}{70}$ is approximately 1.4 hours. So, the cars will meet after 1.4 hours.

3. **Step 1: Formulation :** The speed at which the moon orbits the earth is

$$\frac{\text{Length of the orbit}}{\text{Time taken}}$$

Step 2 : Solution : Since the orbit is nearly circular, the length is $2 \times \pi \times 384000$ km = 2411520 km

The moon takes 24 hours to complete one orbit.

$$\text{So, speed} = \frac{2411520}{24} = 100480 \text{ km/hour.}$$

Step 3 : Interpretation : The speed is 100480 km/h.

4. **Formulation :** An assumption is that the difference in the bill is only because of using the water heater.

Let the average number of hours for which the water heater is used = x

Difference per month due to using water heater = ₹ 1240 – ₹ 1000 = ₹ 240

Cost of using water heater for one hour = ₹ 8

So, the cost of using the water heater for 30 days = $8 \times 30 \times x$

Also, the cost of using the water heater for 30 days = Difference in bill due to using water heater

So, $240x = 240$

Solution : From this equation, we get $x = 1$.

Interpretation : Since $x = 1$, the water heater is used for an average of 1 hour in a day.

EXERCISE A2.2

1. We will not discuss any particular solution here. You can use the same method as we used in last example, or any other method you think is suitable.

EXERCISE A2.3

1. We have already mentioned that the formulation part could be very detailed in real-life situations. Also, we do not validate the answer in word problems. Apart from this word problem have a 'correct answer'. This need not be the case in real-life situations.
2. The important factors are (ii) and (iii). Here (i) is not an important factor although it can have an effect on the number of vehicles sold.