CHAPTER

3 Matrices

CASE STUDY / PASSAGE BASED QUESTIONS

Syllabus

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices. Operation on matrices: Addition and multiplication
and multiplication
with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices, Invertible matrices; (Here all matrices will have real entries).

1

In a city there are two factories A and B. Each factory produces sports clothes for boys and girls. There are three types of clothes produced in both the factories, type I, II and III. For boys the number of units of types I, II and III respectively are 80, 70 and 65 in factory A and 85, 65 and 72 are in factory B. For girls the number of units of types I, II and III respectively are 80, 75, 90 in factory A and 50, 55, 80 are in factory B.

Based on the above information, answer the following questions.

(i) If P represents the matrix of number of units of each type produced by factory A for both boys and girls, then P is given by

(a) $\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \text{Boys} \quad \text{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array}\begin{array}{c} \text{I} \quad \text{II} \quad \text{III} \\ \begin{bmatrix} 50 & 55 & 80 \\ 85 & 65 & 72 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array}\begin{array}{c} \text{I} \quad \text{II} \quad \text{III} \\ \begin{bmatrix} 80 & 75 & 90 \\ 80 & 70 & 65 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \text{Boys} \quad \text{Girls} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} \end{array}$

(ii) If Q represents the matrix of number of units of each type produced by factory B for both boys and girls, then Q is given by

(a) $\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \text{Boys} \quad \text{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array}\begin{array}{c} \text{I} \quad \text{II} \quad \text{III} \\ \begin{bmatrix} 50 & 55 & 80 \\ 85 & 65 & 72 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ \text{Boys} \\ \text{Girls} \end{array}\begin{array}{c} \begin{matrix} \text{I} & \text{II} & \text{III} \end{matrix} \\ \begin{bmatrix} 80 & 75 & 90 \\ 80 & 70 & 65 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \end{array}\begin{array}{c} \begin{matrix} \text{Boys} & \text{Girls} \end{matrix} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} \end{array}$

(iii) The total production of sports clothes of each type for boys is given by the matrix

(a) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [165 & 130 & 137] \end{matrix}$ (b) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [130 & 165 & 137] \end{matrix}$ (c) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [165 & 135 & 137] \end{matrix}$ (d) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [137 & 135 & 165] \end{matrix}$

(iv) The total production of sports clothes of each type for girls is given by the matrix

(a) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [130 & 130 & 170] \end{matrix}$ (b) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [170 & 130 & 130] \end{matrix}$ (c) $\begin{matrix} \text{I} & \text{II} & \text{III} \\ [130 & 170 & 130] \end{matrix}$ (d) none of these

(v) Let R be a 3×2 matrix that represent the total production of sports clothes of each type for boys and girls, then transpose of R is

(a) $\begin{bmatrix} 165 & 135 & 137 \\ 130 & 130 & 170 \end{bmatrix}$ (b) $\begin{bmatrix} 130 & 130 & 170 \\ 165 & 135 & 138 \end{bmatrix}$ (c) $\begin{bmatrix} 165 & 132 \\ 135 & 130 \\ 137 & 170 \end{bmatrix}$ (d) $\begin{bmatrix} 130 & 168 \\ 130 & 135 \\ 170 & 137 \end{bmatrix}$

2

To promote the making of toilets for women, an organisation tried to generate awareness through (i) house calls (ii) emails and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40

The number of attempts made in the villages X, Y and Z are given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also, the chance of making of toilets corresponding to one attempt of given modes is

(i) 2% (ii) 4% (iii) 20%

Based on the above information, answer the following questions.

(i) The cost incurred by the organisation on village X is

(a) ₹ 10000 (b) ₹ 15000 (c) ₹ 30000 (d) ₹ 20000

(ii) The cost incurred by the organisation on village Y is

(a) ₹ 25000 (b) ₹ 18000 (c) ₹ 23000 (d) ₹ 28000

(iii) The cost incurred by the organisation on village Z is

(a) ₹ 19000 (b) ₹ 39000 (c) ₹ 45000 (d) ₹ 50000

(iv) The total number of toilets that can be expected after the promotion in village X, is

(a) 20 (b) 30 (c) 40 (d) 50

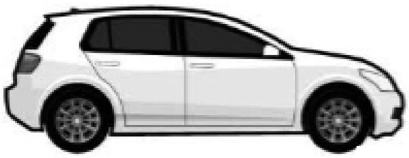
(v) The total number of toilets that can be expected after the promotion in village Z, is

(a) 26 (b) 36 (c) 46 (d) 56

3

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and

300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



Based on the above information, answer the following questions.

(i) The matrix summarizing sales data of 2019 is

(a) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 30 & 5 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 100 & 30 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 200 & 50 & 6 \\ 100 & 30 & 5 \\ 300 & 150 & 20 \end{bmatrix} \end{array}$

(ii) The matrix summarizing sales data of 2020 is

(a) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 60 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 100 & 60 & 5 \\ 120 & 50 & 10 \\ 90 & 40 & 2 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 200 & 50 & 6 \\ 100 & 60 & 5 \\ 300 & 150 & 20 \end{bmatrix} \end{array}$

(iii) The total number of cars sold in two given years, by each dealer, is given by the matrix

(a) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 190 & 100 & 7 \\ 300 & 80 & 11 \\ 420 & 200 & 30 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 300 & 80 & 11 \\ 190 & 100 & 7 \\ 420 & 200 & 30 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix} \end{array}$

(d) None of these

(iv) The increase in sales from 2019 to 2020 is given by the matrix

(a) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 180 & 100 & 10 \\ 10 & 20 & 1 \\ 100 & 20 & 3 \end{bmatrix} \end{array}$

(b) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\ \begin{bmatrix} 10 & 20 & 3 \\ 100 & 20 & 1 \\ 180 & 100 & 10 \end{bmatrix} \end{array}$

(c) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \text{Hatchback} & \text{Sedan} & \text{SUV} \end{array} \\ \begin{bmatrix} 180 & 100 & 10 \\ 100 & 20 & 1 \\ 10 & 20 & 3 \end{bmatrix} \end{array}$

(d) $\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} \text{Hatchback} & \text{Sedan} & \text{SUV} \end{array} \\ \begin{bmatrix} 100 & 20 & 3 \\ 180 & 100 & 10 \\ 10 & 20 & 3 \end{bmatrix} \end{array}$

(v) If each dealer receive profit of ₹ 50000 on sale of a Hatchback, ₹ 100000 on sale of a Sedan and ₹ 200000 on sale of a SUV, then amount of profit received in the year 2020 by each dealer is given by the matrix.

(a) $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 30000000 \\ 15000000 \\ 12000000 \end{bmatrix}$ (b) $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 12000000 \\ 16200000 \\ 34000000 \end{bmatrix}$ (c) $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$ (d) $\begin{array}{c} A \\ B \\ C \end{array}\begin{bmatrix} 15000000 \\ 30000000 \\ 12000000 \end{bmatrix}$

4

A trust fund has ₹ 35000 that must be invested in two different types of bonds, say X and Y. The first bond pays 10% interest p.a. which will be given to an old age home and second one pays 8% interest p.a. which will be given to WWA (Women Welfare Association).

Let A be a 1×2 matrix and B be a 2×1 matrix, representing the investment and interest rate on each bond respectively.

Based on the above information, answer the following questions.

(i) If ₹ 15000 is invested in bond X, then

(a) $A = \begin{array}{c} X \\ Y \end{array}\overset{\text{Investment}}{\begin{bmatrix} 15000 \\ 20000 \end{bmatrix}}; B = \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 0.1 & 0.08 \end{bmatrix} \end{array}$ Interest rate

(b) $A = \text{Investment} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 15000 & 20000 \end{bmatrix} \end{array}; B = \begin{array}{c} X \\ Y \end{array}\overset{\text{Interest rate}}{\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}}$

(c) $A = \text{Investment} \begin{array}{c} \begin{array}{cc} X & Y \end{array} \\ \begin{bmatrix} 20000 & 15000 \end{bmatrix} \end{array}; B = \begin{array}{c} X \\ Y \end{array}\overset{\text{Interest rate}}{\begin{bmatrix} 0.08 \\ 0.1 \end{bmatrix}}$

(d) None of these

(ii) If ₹ 15000 is invested in bond X, then total amount of interest received on both bonds is

(a) ₹ 2000 (b) ₹ 2100 (c) ₹ 3100 (d) ₹ 4000

(iii) If the trust fund obtains an annual total interest of ₹ 3200, then the investment in two bonds is

(a) ₹ 15000 in X, ₹ 20000 in Y (b) ₹ 17000 in X, ₹ 18000 in Y
(c) ₹ 20000 in X, ₹ 15000 in Y (d) ₹ 18000 in X, ₹ 17000 in Y

(iv) The total amount of interest received on both bonds is given by

(a) AB (b) $A'B$ (c) $B'A$ (d) none of these

(v) If the amount of interest given to old age home is ₹ 500, then the amount of investment in bond Y is

(a) ₹ 20000 (b) ₹ 30000 (c) ₹ 15000 (d) ₹ 25000

5

Three schools A, B and C organized a mela for collecting funds for helping the rehabilitation of flood victims. They sold hand made fans, mats and plates from recycled material at a cost of ₹ 25, ₹ 100 and ₹ 50 each. The number of articles sold by school A, B, C are given below.

School / Article	A	B	C
Fans	40	25	35
Mats	50	40	50
Plates	20	30	40

Based on above information, answer the following questions.

(i) If P be a 3×3 matrix represent the sale of handmade fans, mats and plates by three schools A, B and C, then

(a) $P = \begin{matrix} & \text{Fans} & \text{Mats} & \text{Plates} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}$

(b) $P = \begin{matrix} & \text{Fans} & \text{Mats} & \text{Plates} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 25 & 40 & 20 \\ 35 & 40 & 30 \\ 40 & 50 & 20 \end{bmatrix}$

(c) $P = \begin{matrix} & \text{Fans} & \text{Mats} & \text{Plates} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 40 & 25 & 35 \\ 50 & 40 & 50 \\ 20 & 30 & 40 \end{bmatrix}$

(d) $P = \begin{matrix} & \text{Fans} & \text{Mats} & \text{Plates} \\ A \\ B \\ C \end{matrix}\begin{bmatrix} 25 & 35 & 40 \\ 40 & 40 & 50 \\ 20 & 30 & 20 \end{bmatrix}$

(ii) If Q be a 3×1 matrix represent the sale prices (in ₹) of given products per unit, then

(a) $Q = \begin{bmatrix} 25 \\ 50 \\ 100 \end{bmatrix}\begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$ (b) $Q = \begin{matrix} \text{Fans} & \text{Mats} & \text{Plates} \\ [25 & 50 & 100] \end{matrix}$ (c) $Q = \begin{matrix} \text{Fans} & \text{Mats} & \text{Plates} \\ [25 & 100 & 50] \end{matrix}$ (d) $Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}\begin{matrix} \text{Fans} \\ \text{Mats} \\ \text{Plates} \end{matrix}$

(iii) The funds collected by school A by selling the given articles is

(a) ₹ 7000 (b) ₹ 6125 (c) ₹ 7875 (d) ₹ 8000

(iv) The funds collected by school B by selling the given articles is

(a) ₹ 5125 (b) ₹ 6125 (c) ₹ 7125 (d) ₹ 8125

(v) The total funds collected for the required purpose is

(a) ₹ 20000 (b) ₹ 21000 (c) ₹ 30000 (d) ₹ 35000

6

Two farmers Shyam and Balwan Singh cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in ₹) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in ₹)

$$A = \begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

October sales (in ₹)

$$B = \begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$$

Using algebra of matrices, answer the following questions.

(i) The combined sales of Masoor in September and October, for farmer Balwan Singh, is
(a) ₹ 80000 (b) ₹ 90000 (c) ₹ 40000 (d) ₹ 135000

(ii) The combined sales of Urad in September and October, for farmer Shyam is
(a) ₹ 20000 (b) ₹ 30000 (c) ₹ 36000 (d) ₹ 15000

(iii) Find the decrease in sales of Mung from September to October, for the farmer Shyam.
(a) ₹ 24000 (b) ₹ 10000 (c) ₹ 30000 (d) No change

(iv) If both farmers receive 2% profit on gross sales, compute the profit for each farmer and for each variety sold in October.

(a) $\begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 100 & 200 & 220 \\ 400 & 300 & 200 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$

(b) $\begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$

(c) $\begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 150 & 200 & 220 \\ 400 & 200 & 280 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$

(d) $\begin{array}{c} \text{Urad} \quad \text{Masoor} \quad \text{Mung} \\ \begin{bmatrix} 100 & 200 & 120 \\ 250 & 200 & 220 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh} \end{array}$

(v) Which variety of pulse has the highest selling value in the month of September for the farmer Balwan Singh?
(a) Urad (b) Masoor
(c) Mung (d) All of these have the same price

7

A manufacturer produces three types of bolts, x, y and z which he sells in two markets. Annual sales (in ₹) are indicated below :

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

If unit sales prices of x, y and z are ₹ 2.50, ₹ 1.50 and ₹ 1.00 respectively, then answer the following questions using the concept of matrices.

(i) Find the total revenue collected from the Market-I.
(a) ₹ 44000 (b) ₹ 48000 (c) ₹ 46000 (d) ₹ 53000

(ii) Find the total revenue collected from the Market-II.
(a) ₹ 51000 (b) ₹ 53000 (c) ₹ 46000 (d) ₹ 49000

(iii) If the unit costs of the above three commodities are ₹ 2.00, ₹ 1.00 and 50 paise respectively, then find the gross profit from both the markets.
(a) ₹ 53000 (b) ₹ 46000 (c) ₹ 34000 (d) ₹ 32000

(iv) If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = 1$, if $i \neq j$ and $a_{ij} = 0$ if $i = j$, then A^2 is equal to
(a) I (b) A (c) O (d) none of these

(v) If A and B are matrices of same order, then $(AB' - BA')$ is a
(a) skew-symmetric matrix (b) null matrix (c) symmetric matrix (d) unit matrix

8

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$ are two matrices, then $A \pm B$ is of order $m \times n$ and is defined as $(A \pm B)_{ij} = a_{ij} \pm b_{ij}$, where $i = 1, 2, ..., m$ and $j = 1, 2, ..., n$

If $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ are two matrices, then AB is of order $m \times p$ and is defined as

$$(AB)_{ik} = \sum_{r=1}^{n} a_{ir}\, b_{rk} = a_{i1}b_{1k} + a_{i2}b_{2k} + ... + a_{in}b_{nk}$$

Consider $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ and $D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Using the concept of matrices answer the following questions.

(i) Find the product AB.
(a) $\begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 3 \\ 22 & 43 \end{bmatrix}$ (c) $\begin{bmatrix} 43 & 22 \\ 0 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 22 & 43 \\ 3 & 0 \end{bmatrix}$

(ii) If A and B are any other two matrices such that AB exists, then
(a) BA does not exist (b) BA will be equal to AB
(c) BA may or may not exist (d) None of these

(iii) Find the values of a and c in the matrix D such than $CD - AB = 0$.
(a) $a = 77, c = -191$ (b) $a = -191, c = 77$ (c) $a = 191, c = 77$ (d) $a = 91, c = 70$

(iv) Find the values of b and d in the matrix D such that $CD - AB = 0$.
(a) $b = 44, d = -110$ (b) $b = 110, d = 44$ (c) $b = -110, d = 44$ (d) $b = -44, d = 110$

(v) Find $B + D$.
(a) $\begin{bmatrix} 80 & 200 \\ 115 & 105 \end{bmatrix}$ (b) $\begin{bmatrix} 84 & 48 \\ 180 & 181 \end{bmatrix}$ (c) $\begin{bmatrix} 186 & 108 \\ -84 & -48 \end{bmatrix}$ (d) $\begin{bmatrix} -186 & -108 \\ 84 & 48 \end{bmatrix}$

9

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a children and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for children.

Based on the above information, answer the following questions.

(i) The requirement of calories and proteins for each person in matrix form can be represented as

(a)
$$\begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

(b)
$$\begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 1900 & 55 \\ 2400 & 45 \\ 1800 & 33 \end{bmatrix} \end{array}$$

(c)
$$\begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 1800 & 33 \\ 1900 & 55 \\ 2400 & 45 \end{bmatrix} \end{array}$$

(d)
$$\begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 33 \\ 1900 & 55 \\ 1800 & 45 \end{bmatrix} \end{array}$$

(ii) Requirement of calories of family A is

(a) 24000 (b) 24400 (c) 15000 (d) 15800

(iii) Requirement of proteins for family B is

(a) 560 grams (b) 332 grams (c) 266 grams (d) 300 grams

(iv) If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

(v) If $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$ and $C = (c_{ij})_{p \times q}$, then the product $(BC)A$ is possible only when

(a) $m = q$ (b) $n = q$ (c) $p = q$ (d) $m = p$

10

Three shopkeepers A, B and C go to a store to buy stationary. A purchase 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs ₹ 40, a pen costs ₹ 12 and a pencil costs ₹ 3.

Based on the above information, answer the following questions.

(i) The number of items purchased by shopkeepers A, B and C represented in matrix form as

(a) $\begin{array}{c} \text{Notebooks} \quad \text{pens} \quad \text{pencils} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$

(b) $\begin{array}{c} \text{Notebooks} \quad \text{pens} \quad \text{pencils} \\ \begin{bmatrix} 144 & 72 & 60 \\ 120 & 84 & 72 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$

(c) $\begin{array}{c} \text{Notebooks} \quad \text{pens} \quad \text{pencils} \\ \begin{bmatrix} 144 & 72 & 72 \\ 120 & 156 & 84 \\ 132 & 84 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$

(d) $\begin{array}{c} \text{Notebooks} \quad \text{pens} \quad \text{pencils} \\ \begin{bmatrix} 144 & 60 & 60 \\ 120 & 84 & 72 \\ 132 & 156 & 96 \end{bmatrix} \begin{matrix} A \\ B \\ C \end{matrix} \end{array}$

(ii) If Y represents the matrix formed by the cost of each item, then XY equals

(a) $\begin{bmatrix} 5740 \\ 6780 \\ 8040 \end{bmatrix}$ (b) $\begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$ (c) $\begin{bmatrix} 5916 \\ 6696 \\ 7440 \end{bmatrix}$ (d) $\begin{bmatrix} 6740 \\ 5740 \\ 8140 \end{bmatrix}$

(iii) Bill of A is equal to

(a) ₹ 6740 (b) ₹ 8140 (c) ₹ 5740 (d) ₹ 6696

(iv) If $A^2 = A$, then $(A + I)^3 - 7A =$

(a) A (b) $A - I$ (c) I (d) $A + I$

(v) If A and B are 3×3 matrices such that $A^2 - B^2 = (A - B)(A + B)$, then

(a) either A or B is zero matrix (b) either A or B is unit matrix

(c) $A = B$ (d) $AB = BA$

HINTS & EXPLANATIONS

1. **(i) (d):** In factory A, number of units of types I, II and III for boys are 80, 70, 65 respectively and for girls number of units of types I, II and III are 80, 75, 90 respectively.

$$\therefore \quad P = \begin{matrix} \\ \text{I} \\ \text{II} \\ \text{III} \end{matrix} \begin{matrix} \text{Boys} & \text{Girls} \\ \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} \end{matrix}$$

(ii) (a): In factory B, number of units of types I, II and III for boys are 85, 65, 72 respectively and for girls number of units of types I, II and III are 50, 55, 80 respectively.

$$\therefore \quad Q = \begin{matrix} \\ \text{I} \\ \text{II} \\ \text{III} \end{matrix} \begin{matrix} \text{Boys} & \text{Girls} \\ \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} \end{matrix}$$

(iii) (c) : Let X be the matrix that represent the number of units of each type produced by factory A for boys, and Y be the matrix that represent the number of units of each type produced by factory B for boys.

Then, $X = \begin{bmatrix} \overset{\text{I}}{80} & \overset{\text{II}}{70} & \overset{\text{III}}{65} \end{bmatrix}$ and $Y = \begin{bmatrix} \overset{\text{I}}{85} & \overset{\text{II}}{65} & \overset{\text{III}}{72} \end{bmatrix}$

Now, required matrix $= X + Y = [80\ 70\ 65] + [85\ 65\ 72]$
$= [165\ 135\ 137]$

(iv) (a): Required matrix $= [80\ 75\ 90] + [50\ 55\ 80]$
$= [130\ 130\ 170]$

(v) (a): Clearly, $R = P + Q$

$$= \begin{bmatrix} 80 & 80 \\ 70 & 75 \\ 65 & 90 \end{bmatrix} + \begin{bmatrix} 85 & 50 \\ 65 & 55 \\ 72 & 80 \end{bmatrix} = \begin{bmatrix} 165 & 130 \\ 135 & 130 \\ 137 & 170 \end{bmatrix}$$

$$\therefore \quad R' = \begin{bmatrix} 165 & 135 & 137 \\ 130 & 130 & 170 \end{bmatrix}$$

2. **(i) (c) :** Let ₹ A, ₹ B and ₹ C be the cost incurred by the organisation for villages X, Y and Z respectively. Then A, B, C will be given by the following matrix equation.

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} A \\ B \\ C \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 400\times 50+300\times 20+100\times 40 \\ 300\times 50+250\times 20+75\times 40 \\ 500\times 50+400\times 20+150\times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 20000+6000+4000 \\ 15000+5000+3000 \\ 25000+8000+6000 \end{bmatrix} = \begin{bmatrix} 30000 \\ 23000 \\ 39000 \end{bmatrix}$$

(ii) **(c)** **(iii)** **(b)**

(iv) **(c) :** Total number of toilets that can be expected in each village is given by the following matrix

$$\begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}\begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

$$= \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 8+12+20 \\ 6+10+15 \\ 10+16+30 \end{bmatrix} = \begin{matrix} X \\ Y \\ Z \end{matrix}\begin{bmatrix} 40 \\ 31 \\ 56 \end{bmatrix}$$

(v) **(d)**

3. **(i)** **(b):** In 2019,

dealer A sold 120 Hatchback, 50 Sedan and 10 SUV;
dealer B sold 100 Hatchback, 30 Sedan and 5 SUV
and dealer C sold 90 Hatchback, 40 Sedan and 2 SUV

$\therefore$ Required matrix, say P, is given by

$$P = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix} \end{matrix}$$

(ii) **(a):** In 2020,

dealer A sold 300 Hatchback, 150 Sedan, 20 SUV
dealer B sold 200 Hatchback, 50 sedan, 6 SUV
dealer C sold 100 Hatchback, 60 sedan, 5 SUV

$\therefore$ Required matrix, say Q, is given by

$$Q = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{matrix}$$

(iii) **(c) :** Total number of cars sold in two given years, by each dealer, is given by

$$P+Q = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 120+300 & 50+150 & 10+20 \\ 100+200 & 30+50 & 5+6 \\ 90+100 & 40+60 & 2+5 \end{bmatrix} \end{matrix}$$

$$= \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 420 & 200 & 30 \\ 300 & 80 & 11 \\ 190 & 100 & 7 \end{bmatrix} \end{matrix}$$

(iv) **(c):** The increase in sales from 2019 to 2020 is given by

$$Q-P = \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 300-120 & 150-50 & 20-10 \\ 200-100 & 50-30 & 6-5 \\ 100-90 & 60-40 & 5-2 \end{bmatrix} \end{matrix}$$

$$= \begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 180 & 100 & 10 \\ 100 & 20 & 1 \\ 10 & 20 & 3 \end{bmatrix} \end{matrix}$$

(v) **(c):** The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{matrix} \\ A \\ B \\ C \end{matrix}\begin{matrix} \text{Hatchback} & \text{Sedan} & \text{SUV} \\ \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix} \end{matrix} \cdot \begin{bmatrix} 50000 \\ 100000 \\ 200000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 15000000+15000000+4000000 \\ 10000000+5000000+1200000 \\ 5000000+6000000+1000000 \end{bmatrix}$$

$$= \begin{matrix} A \\ B \\ C \end{matrix}\begin{bmatrix} 34000000 \\ 16200000 \\ 12000000 \end{bmatrix}$$

4. **(i)** **(b):** If ₹ 15000 is invested in bond X, then the amount invested in bond Y = ₹ (35000 – 15000) = ₹ 20000.

$$A = \text{Investment} \begin{matrix} X & Y \\ [15000 & 20000] \end{matrix}$$

$$\text{and } B = \begin{matrix} X \\ Y \end{matrix}\begin{matrix} \text{Interest rate} \\ \begin{bmatrix} 10\% \\ 8\% \end{bmatrix} \end{matrix} = \begin{matrix} X \\ Y \end{matrix}\begin{matrix} \text{Interest rate} \\ \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} \end{matrix}$$

(ii) **(c):** The amount of interest received on each bond is given by

$$AB = [15000 \quad 20000] \times \begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix}$$

$= [15000 \times 0.1 + 20000 \times 0.08] = [1500 + 1600] = 3100$

(iii) **(c):** Let ₹ x be invested in bond X and then ₹ $(35000 - x)$ will be invested in bond Y.
Now, total amount of interest is given by

$$[x \quad 35000 - x]\begin{bmatrix} 0.1 \\ 0.08 \end{bmatrix} = [0.1x + (35000 - x)0.08]$$

But, it is given that total amount of interest = ₹ 3200

$\therefore \quad 0.1x + 2800 - 0.08x = 3200$

$\Rightarrow \quad 0.02x = 400 \Rightarrow x = 20000$

Thus, ₹ 20000 invested in bond X and ₹ 35000 – ₹ 20000 = ₹ 15000 invested in bond Y.

(iv) (a): AB will give the total amount of interest received on both bonds.

(v) (b): Let ₹ x invested in bond X, then we have

$$x \times \frac{10}{100} = 500 \Rightarrow x = 5000$$

Thus, amount invested in bond X is ₹ 5000 and so investment in bond Y be ₹ (35000 – 5000) = ₹ 30000

5. (i) (a): Clearly,
$$P = \begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc}\text{Fans} & \text{Mats} & \text{Plates}\end{array} \\ \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}\end{array}$$

(ii) (d): Since Q is a 3×1 matrix, therefore

$$Q = \begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}\begin{array}{l}\text{Fans} \\ \text{Maths} \\ \text{Plates}\end{array}$$

(iii) (a) : Clearly, total funds collected by each school is given by the matrix

$$PQ = \begin{bmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{bmatrix}\begin{bmatrix} 25 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 1000+5000+1000 \\ 625+4000+1500 \\ 875+5000+2000 \end{bmatrix} = \begin{bmatrix} 7000 \\ 6125 \\ 7875 \end{bmatrix}$$

∴ Funds collected by school A is ₹ 7000
Funds collected by school B is ₹ 6125
Funds collected by school C is ₹ 7875

(iv) (b)

(v) (b): Total funds collected for the required purpose = ₹ (7000 + 6125 + 7875) = ₹ 21000

6. Combined sales in September and October for each farmer in each variety is given by

$$A + B = \begin{array}{c}\begin{array}{ccc}\text{Urad} & \text{Masoor} & \text{Mung}\end{array} \\ \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix}\end{array}\begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh}\end{array}$$

(i) (c) : Combined sales of Masoor in September and October for farmer Balwan Singh = ₹ 40000

(ii) (d) : Combined sales of Urad in September and October for farmer Shyam = ₹ 15000

(iii) (a) : Change in sales from September to October is given by

$$A - B = \begin{array}{c}\begin{array}{ccc}\text{Urad} & \text{Masoor} & \text{Mung}\end{array} \\ \begin{bmatrix} 5000 & 10000 & 24000 \\ 30000 & 20000 & 0 \end{bmatrix}\end{array}\begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh}\end{array}$$

∴ Decrease in sales of Mung from September to October for farmer Shyam = ₹ 24000.

(iv) (b) : Required profit is given by

$$2\% \text{ of } B = \frac{2}{100} \times B = 0.02 \times B$$

$$= 0.02\begin{array}{c}\begin{array}{ccc}\text{Urad} & \text{Masoor} & \text{Mung}\end{array} \\ \begin{bmatrix} 5000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix}\end{array}\begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh}\end{array}$$

$$= \begin{array}{c}\begin{array}{ccc}\text{Urad} & \text{Masoor} & \text{Mung}\end{array} \\ \begin{bmatrix} 100 & 200 & 120 \\ 400 & 200 & 200 \end{bmatrix}\end{array}\begin{array}{l} \\ \text{Shyam} \\ \text{Balwan Singh}\end{array}$$

Thus, in October Shyam receives ₹ 100, ₹ 200 and ₹ 120 as profit in the sale of each variety of pulses, respectively and Balwan Singh receives a profit of ₹ 400, ₹ 200 and ₹ 200 in the sale of each variety of pulses respectively.

(v) (a)

7. Let A be the 2×3 matrix representing the annual sales of products in two markets.

$$\therefore \quad A = \begin{array}{c}\begin{array}{ccc} x & y & z\end{array} \\ \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\end{array}\begin{array}{l} \\ \text{Market I} \\ \text{Market II}\end{array}$$

Let B be the column matrix representing the sale price of each unit of products x, y, z.

$$\therefore \quad B = \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

Now, revenue = sale price × number of items sold

$$= \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25000+3000+18000 \\ 15000+30000+8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

Therefore, the revenue collected from Market I = ₹ 46000 and the revenue collected from Market II = ₹ 53000.

(i) (c) **(ii) (b)**

(iii) (d): Let C be the column matrix representing cost price of each unit of products x, y, z.

$$\text{Then, } C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

∴ Total cost in each market is given by

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 20000+2000+9000 \\ 12000+20000+4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Now, Profit matrix = Revenue matrix – Cost matrix

$$= \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets
= ₹ 15000 + ₹ 17000 = ₹ 32000

(iv) **(a) :** We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(v) **(a) :** We have, $(AB' - BA')' = (B')'A' - (A')'B'$
$= BA' - AB' = -(AB' - BA')$
Thus, $AB' - BA'$ is a skew-symmetric matrix.

8. **(i)** **(a) :** $AB = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

(ii) **(c)**

(iii) **(b) :** We have, $CD - AB = O$

$$\Rightarrow \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a+5c-3 & 2b+5d \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

By equality of matrices, we get $2a + 5c - 3 = 0$ ...(i)
$3a + 8c - 43 = 0$ ...(ii)
$2b + 5d = 0$ ...(iii)
$3b + 8d - 22 = 0$ ...(iv)
Solving (i) and (ii), we get $a = -191$, $c = 77$

(iv) **(c) :** Solving (iii) and (iv), we get $b = -110$, $d = 44$

(v) **(d) :** We have, $B + D = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} + \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

$$= \begin{bmatrix} -186 & -108 \\ 84 & 48 \end{bmatrix}$$

9. **(i)** **(a) :** Let F be the matrix representing the number of family members and R be the matrix representing the requirement of calories and proteins for each person. Then

$$F = \begin{matrix} \text{Family } A \\ \text{Family } B \end{matrix} \overset{\begin{matrix} \text{Men} & \text{Women} & \text{Children} \end{matrix}}{\begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix}}$$

$$R = \begin{matrix} \text{Man} \\ \text{Woman} \\ \text{Children} \end{matrix} \overset{\begin{matrix} \text{Calories} & \text{Proteins} \end{matrix}}{\begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}}$$

(ii) **(b) :** The requirement of calories and proteins for each of the two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400+1900+1800) & 4(45+55+33) \\ 2(2400+1900+1800) & 2(45+55+33) \end{bmatrix}$$

$$FR = \overset{\begin{matrix} \text{Calories} & \text{Proteins} \end{matrix}}{\begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix}} \begin{matrix} \text{Family } A \\ \text{Family } B \end{matrix}$$

(iii) **(c)**

(iv) **(c) :** Since, $AB = B$ ...(i) and $BA = A$ ...(ii)

$\therefore \quad A^2 + B^2 = A \cdot A + B \cdot B$
$= A(BA) + B(AB)$ [using (i) and (ii)]
$= (AB)A + (BA)B$ [Associative law]
$= BA + AB$ [using (i) and (ii)]
$= A + B$

(v) **(a) :** $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$, $C = (c_{ij})_{p \times q}$
$BC = (b_{ij})_{n \times p} \times (c_{ij})_{p \times q} = (d_{ij})_{n \times q}$
$(BC)A = (d_{ij})_{n \times q} \times (a_{ij})_{m \times n}$
Hence, $(BC)A$ is possible only when $m = q$

10. **(i)** **(a) :** Number of items purchased by shopkeepers A, B and C can be written in matrix form as

$$X = \overset{\begin{matrix} \text{Notebooks} & \text{pens} & \text{pencils} \end{matrix}}{\begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix}} \begin{matrix} A \\ B \\ C \end{matrix}$$

(ii) **(b) :** Since, $Y = \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix} \begin{matrix} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{matrix}$

$$\therefore \quad XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 40 \\ 12 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5760+720+216 \\ 4800+864+252 \\ 5280+1872+288 \end{bmatrix} = \begin{bmatrix} 6696 \\ 5916 \\ 7440 \end{bmatrix}$$

(iii) **(d) :** Bill of A is ₹ 6696.

(iv) **(c) :** $(A + I)^2 = A^2 + 2A + I = 3A + I$
$\Rightarrow \quad (A + I)^3 = (3A + I)(A + I)$
$= 3A^2 + 4A + I = 7A + I$
$\therefore \quad (A + I)^3 - 7A = I$

(v) **(d) :** $A^2 - B^2 = (A - B)(A + B) = A^2 + AB - BA - B^2$
$\therefore \quad AB = BA.$