

Application of Derivatives

CASE STUDY / PASSAGE BASED QUESTIONS

1

Megha wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of box, she takes a square piece of cardboard of side 20 cm.



Based on the above information, answer the following questions.

- (i) If x cm be the length of each side of the square cardboard which is to be cut off from corners of the square piece of side 20 cm, then possible value of x will be given by the interval
- (a) $[0, 20]$ (b) $(0, 10)$ (c) $(0, 3)$ (d) None of these
- (ii) Volume of the open box formed by folding up the cutting corner can be expressed as
- (a) $V = x(20 - 2x)(20 - 2x)$ (b) $V = \frac{x}{2}(20 + x)(20 - x)$
 (c) $V = \frac{x}{3}(20 - 2x)(20 + 2x)$ (d) $V = x(20 - 2x)(20 - x)$
- (iii) The values of x for which $\frac{dV}{dx} = 0$, are
- (a) 3, 4 (b) $0, \frac{10}{3}$ (c) 0, 10 (d) $10, \frac{10}{3}$
- (iv) Megha is interested in maximising the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?
- (a) 12 cm (b) 8 cm (c) $\frac{10}{3}$ cm (d) 2 cm
- (v) The maximum value of the volume is
- (a) $\frac{17000}{27}$ cm³ (b) $\frac{11000}{27}$ cm³
 (c) $\frac{8000}{27}$ cm³ (d) $\frac{16000}{27}$ cm³

Syllabus

Applications of derivatives: increasing/decreasing functions, tangents and normals, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

Shobhit's father wants to construct a rectangular garden using a brick wall on one side of the garden and wire fencing for the other three sides as shown in figure. He has 200 ft of wire fencing.



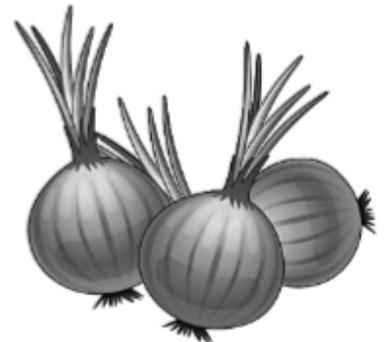
Based on the above information, answer the following questions.

- (i) To construct a garden using 200 ft of fencing, we need to maximise its
 (a) volume (b) area (c) perimeter (d) length of the side
- (ii) If x denote the length of side of garden perpendicular to brick wall and y denote the length of side parallel to brick wall, then find the relation representing total amount of fencing wire.
 (a) $x + 2y = 150$ (b) $x + 2y = 50$ (c) $y + 2x = 200$ (d) $y + 2x = 100$
- (iii) Area of the garden as a function of x , say $A(x)$, can be represented as
 (a) $200 + 2x^2$ (b) $x - 2x^2$ (c) $200x - 2x^2$ (d) $200 - x^2$
- (iv) Maximum value of $A(x)$ occurs at x equals
 (a) 50 ft (b) 30 ft (c) 26 ft (d) 31 ft
- (v) Maximum area of garden will be
 (a) 2500 sq. ft (b) 4000 sq. ft (c) 5000 sq. ft (d) 6000 sq. ft

The Government declare that farmers can get ₹ 300 per quintal for their onions on 1st July and after that, the price will be dropped by ₹ 3 per quintal per extra day. Shyam's father has 80 quintal of onions in the field on 1st July and he estimates that crop is increasing at the rate of 1 quintal per day.

Based on the above information, answer the following questions.

- (i) If x is the number of days after 1st July, then price and quantity of onion respectively can be expressed as
 (a) ₹ $(300 - 3x)$, $(80 + x)$ quintals (b) ₹ $(300 - 3x)$, $(80 - x)$ quintals
 (c) ₹ $(300 + x)$, 80 quintals (d) None of these
- (ii) Revenue R as a function of x can be represented as
 (a) $R(x) = 3x^2 - 60x - 24000$ (b) $R(x) = -3x^2 + 60x + 24000$
 (c) $R(x) = 3x^2 + 40x - 16000$ (d) $R(x) = 3x^2 - 60x - 14000$



(iii) Find the number of days after 1st July, when Shyam's father attain maximum revenue.

- (a) 10 (b) 20 (c) 12 (d) 22

(iv) On which day should Shyam's father harvest the onions to maximise his revenue?

- (a) 11th July (b) 20th July (c) 12th July (d) 22nd July

(v) Maximum revenue is equal to

- (a) ₹ 20,000 (b) ₹ 24,000 (c) ₹ 24,300 (d) ₹ 24,700

4

An owner of an electric bike rental company have determined that if they charge customers ₹ x per day to rent a bike, where $50 \leq x \leq 200$, then number of bikes (n), they rent per day can be shown by linear function $n(x) = 2000 - 10x$. If they charge ₹ 50 per day or less, they will rent all their bikes. If they charge ₹ 200 or more per day, they will not rent any bike.



Based on the above information, answer the following questions.

(i) Total revenue R as a function of x can be represented as

- (a) $2000x - 10x^2$ (b) $2000x + 10x^2$
(c) $2000 - 10x$ (d) $2000 - 5x^2$

(ii) If $R(x)$ denote the revenue, then maximum value of $R(x)$ occur when x equals

- (a) 10 (b) 100 (c) 1000 (d) 50

(iii) At $x = 260$, the revenue collected by the company is

- (a) ₹ 10 (b) ₹ 500 (c) ₹ 0 (d) ₹ 1000

(iv) The number of bikes rented per day, if $x = 105$ is

- (a) 850 (b) 900 (c) 950 (d) 1000

(v) Maximum revenue collected by company is

- (a) ₹ 40,000 (b) ₹ 50,000 (c) ₹ 75,000 (d) ₹ 1,00,000

5

Mr. Sahil is the owner of a high rise residential society having 50 apartments. When he set rent at ₹ 10000/month, all apartments are rented. If he increases rent by ₹ 250/month, one fewer apartment is rented. The maintenance cost for each occupied unit is ₹ 500/month.

Based on the above information answer the following questions.

(i) If P is the rent price per apartment and N is the number of rented apartment, then profit is given by

- (a) NP (b) $(N - 50)P$ (c) $N(P - 500)$ (d) none of these

(ii) If x represent the number of apartments which are not rented, then the profit expressed as a function of x is

- (a) $(50 - x)(38 + x)$ (b) $(50 + x)(38 - x)$ (c) $250(50 - x)(38 + x)$ (d) $250(50 + x)(38 - x)$



(iii) If $P = 10500$, then $N =$

- (a) 47 (b) 48 (c) 49 (d) 50

(iv) If $P = 11,000$, then the profit is

- (a) ₹ 4,83,000 (b) ₹ 5,00,000 (c) ₹ 5,05,000 (d) ₹ 6,50,000

(v) The rent that maximizes the total amount of profit is

- (a) ₹ 11000 (b) ₹ 11500 (c) ₹ 15800 (d) ₹ 16500

6

Western music concert is organised every year in the stadium that can hold 36000 spectators. With ticket price of ₹ 10, the average attendance has been 24000. Some financial expert estimated that price of a ticket should be determined by the function $p(x) = 15 - \frac{x}{3000}$, where x is the number of tickets sold.



Based on the above information, answer the following questions.

(i) The revenue, R as a function of x can be represented as

- (a) $15x - \frac{x^2}{3000}$ (b) $15 - \frac{x^2}{3000}$ (c) $15x - \frac{1}{30000}$ (d) $15x - \frac{x}{3000}$

(ii) The range of x is

- (a) [24000, 36000] (b) [0, 24000] (c) [0, 36000] (d) none of these

(iii) The value of x for which revenue is maximum, is

- (a) 20000 (b) 21000 (c) 22500 (d) 25000

(iv) When the revenue is maximum, the price of the ticket is

- (a) ₹ 5 (b) ₹ 5.5 (c) ₹ 7 (d) ₹ 7.5

(v) How many spectators should be present to maximize the revenue?

- (a) 21500 (b) 21000 (c) 22000 (d) 22500

7

A tin can manufacturer designs a cylindrical tin can for a company making sanitizer and disinfectant. The tin can is made to hold 3 litres of sanitizer or disinfectant.



Based on the above information, answer the following questions.

(i) If r cm be the radius and h cm be the height of the cylindrical tin can, then the surface area expressed as a function of r as

- (a) $2\pi r^2$ (b) $2\pi r^2 + 6000$ (c) $2\pi r^2 + \frac{5000}{r}$ (d) $2\pi r^2 + \frac{6000}{r}$

(ii) The radius that will minimize the cost of the material to manufacture the tin can is

- (a) $\sqrt[3]{\frac{600}{\pi}}$ cm (b) $\sqrt{\frac{500}{\pi}}$ cm (c) $\sqrt[3]{\frac{1500}{\pi}}$ cm (d) $\sqrt{\frac{1500}{\pi}}$ cm

(iii) The height that will minimize the cost of the material to manufacture the tin can is

- (a) $\sqrt[3]{\frac{1500}{\pi}}$ cm (b) $2\sqrt[3]{\frac{1500}{\pi}}$ cm (c) $\sqrt{\frac{1500}{\pi}}$ (d) $2\sqrt{\frac{1500}{\pi}}$

(iv) If the cost of material used to manufacture the tin can is ₹ 100/m² and $\sqrt[3]{\frac{1500}{\pi}} \approx 7.8$, then minimum cost is approximately

- (a) ₹ 11.538 (b) ₹ 12 (c) ₹ 13 (d) ₹ 14

(v) To minimize the cost of the material used to manufacture the tin can, we need to minimize the

- (a) volume (b) curved surface area
(c) total surface area (d) surface area of the base

8

A poster is to be formed for a company advertisement. The top and bottom margins of poster should be 9 cm and the side margins should be 6 cm. Also, the area for printing the advertisement should be 864 cm².

Based on the above information, answer the following questions.

(i) If a cm be the width and b cm be the height of poster, then the area of poster, expressed in terms of a and b , is given by

- (a) $648 + 18a + 12b$ (b) $18a + 12b$
(c) $584 + 18a + 12b$ (d) none of these

(ii) The relation between a and b is given by

- (a) $a = \frac{648+12b}{b-18}$ (b) $a = \frac{12b}{b-18}$ (c) $a = \frac{12b}{b+18}$ (d) none of these

(iii) Area of poster in terms of b is given by

- (a) $\frac{12b^2}{b-18}$ (b) $\frac{648b+12b^2}{b-18}$ (c) $\frac{648b+12b^2}{b+18}$ (d) $\frac{12b^2}{b+18}$

(iv) The value of b , so that area of the poster is minimized, is

- (a) 54 (b) 36 (c) 27 (d) 22

(v) The value of a , so that area of the poster is minimized, is

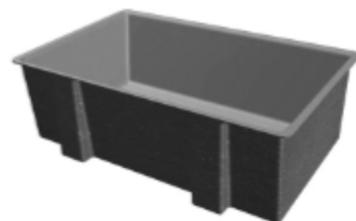
- (a) 24 (b) 36 (c) 40 (d) 22



9

Nitin wants to construct a rectangular plastic tank for his house that can hold 80 ft³ of water. The top of the tank is open. The width of tank will be 5 ft but the length and heights are variables. Building the tank cost ₹ 20 per sq. foot for the base and ₹ 10 per sq. foot for the side.

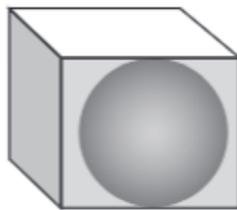
Based on the above information, answer the following questions.



- (i) In order to make a least expensive water tank, Nitin need to minimize its
- (a) Volume (b) Base (c) Curved surface area (d) Cost
- (ii) Total cost of tank as a function of h can be represented as
- (a) $c(h) = 100h - 320 - 1600/h$ (b) $c(h) = 100h - 320h - 720h^2$
- (c) $c(h) = 100 + 320h + 1600h^2$ (d) $c(h) = 100h + 320 + \frac{1600}{h}$
- (iii) Range of h is
- (a) (3, 5) (b) (0, ∞) (c) (0, 8) (d) (0, 3)
- (iv) Value of h at which $c(h)$ is minimum, is
- (a) 4 (b) 5 (c) 6 (d) 6.7
- (v) The cost of least expensive tank is
- (a) ₹ 1020 (b) ₹ 1100 (c) ₹ 1120 (d) ₹ 1220

10

Shreya got a rectangular parallelepiped shaped box and spherical ball inside it as return gift. Sides of the box are x , $2x$, and $x/3$, while radius of the ball is r .



Based on the above information, answer the following questions.

- (i) If S represents the sum of volume of parallelepiped and sphere, then S can be written as
- (a) $\frac{4x^3}{3} + \frac{2}{3}\pi r^2$ (b) $\frac{2x^2}{3} + \frac{4}{3}\pi r^2$ (c) $\frac{2x^3}{3} + \frac{4}{3}\pi r^3$ (d) $\frac{2}{3}x + \frac{4}{3}\pi r$
- (ii) If sum of the surface areas of box and ball are given to be constant k^2 , then x is equal to
- (a) $\sqrt{\frac{k^2 - 4\pi r^2}{6}}$ (b) $\sqrt{\frac{k^2 - 4\pi r}{6}}$ (c) $\sqrt{\frac{k^2 - 4\pi}{6}}$ (d) none of these
- (iii) The radius of the ball, when S is minimum, is
- (a) $\sqrt{\frac{k^2}{54 + \pi}}$ (b) $\sqrt{\frac{k^2}{54 + 4\pi}}$ (c) $\sqrt{\frac{k^2}{64 + 3\pi}}$ (d) $\sqrt{\frac{k^2}{4\pi + 3}}$
- (iv) Relation between length of the box and radius of the ball can be represented as
- (a) $x = 2r$ (b) $x = \frac{r}{2}$ (c) $x = \frac{r}{2}$ (d) $x = 3r$
- (v) Minimum value of S is
- (a) $\frac{k^2}{2(3\pi + 54)^{2/3}}$ (b) $\frac{k}{(3\pi + 54)^{3/2}}$ (c) $\frac{k^3}{3(4\pi + 54)^{1/2}}$ (d) none of these

A real estate company is going to build a new residential complex. The land they have purchased can hold at most 4500 apartments. Also, if they make x apartments, then the monthly maintenance cost for the whole complex would be as follows : Fixed cost = ₹ 50,00,000. Variable cost = ₹(160x - 0.04x²)



Based on the above information, answer the following questions.

- (i) The maintenance cost as a function of x will be
- (a) $160x - 0.04x^2$ (b) 5000000
(c) $5000000 + 160x - 0.04x^2$ (d) None of these
- (ii) If $C(x)$ denote the maintenance cost function, then maximum value of $C(x)$ occur at $x =$
- (a) 0 (b) 2000 (c) 4500 (d) 5000
- (iii) The maximum value of $C(x)$ would be
- (a) ₹ 5225000 (b) ₹ 5160000 (c) ₹ 5000000 (d) ₹ 4000000
- (iv) The number of apartments, that the complex should have in order to minimize the maintenance cost, is
- (a) 4500 (b) 5000 (c) 1750 (d) 3500
- (v) If the minimum maintenance cost is attain, then the maintenance cost for each apartment would be
- (a) ₹ 1091.11 (b) ₹ 1200 (c) ₹ 1000 (d) ₹ 2000

Kyra has a rectangular painting canvas having a total area of 24 ft² which includes a border of 0.5 ft on the left, right and a border of 0.75 ft on the bottom, top inside it.



Based on the above information, answer the following questions.

- (i) If Kyra wants to paint in the maximum area, then she needs to maximize
- (a) Area of outer rectangle (b) Area of inner rectangle
(c) Area of top border (d) None of these
- (ii) If x is the length of the outer rectangle, then area of inner rectangle in terms of x is
- (a) $(x+3)\left(\frac{24}{x}-2\right)$ (b) $(x-1)\left(\frac{24}{x}+1.5\right)$ (c) $(x-1)\left(\frac{24}{x}-1.5\right)$ (d) $(x-1)\left(\frac{24}{x}\right)$
- (iii) Find the range of x .
- (a) $(1, \infty)$ (b) $(1, 16)$ (c) $(-\infty, 16)$ (d) $(-1, 16)$
- (iv) If area of inner rectangle is maximum, then x is equal to
- (a) 2 ft (b) 3 ft (c) 4 ft (d) 5 ft
- (v) If area of inner rectangle is maximum, then length and breadth of this rectangle are respectively
- (a) 3 ft, 4.5 ft (b) 4.5 ft, 5 ft (c) 1 ft, 2 ft (d) 2 ft, 4 ft

13

A magazine company in a town has 5000 subscribers on its list and collects fix charges of ₹ 3000 per year from each subscriber. The company proposes to increase the annual charges and it is believed that for every increase of ₹ 1, one subscriber will discontinue service.



Based on the above information, answer the following questions.

- (i) If x denote the amount of increase in annual charges, then revenue R , as a function of x can be represented as
- (a) $R(x) = 3000 \times 5000 \times x$ (b) $R(x) = (3000 - 2x)(5000 + 2x)$
(c) $R(x) = (5000 + x)(3000 - x)$ (d) $R(x) = (3000 + x)(5000 - x)$
- (ii) If magazine company increases ₹ 500 as annual charges, then R is equal to
- (a) ₹ 15750000 (b) ₹ 16750000 (c) ₹ 17500000 (d) ₹ 15000000
- (iii) If revenue collected by the magazine company is ₹ 15640000, then value of amount increased as annual charges for each subscriber, is
- (a) 400 (b) 1600 (c) Both (a) and (b) (d) None of these
- (iv) What amount of increase in annual charges will bring maximum revenue?
- (a) ₹ 1000 (b) ₹ 2000 (c) ₹ 3000 (d) ₹ 4000
- (v) Maximum revenue is equal to
- (a) ₹ 15000000 (b) ₹ 16000000 (c) ₹ 20500000 (d) ₹ 25000000

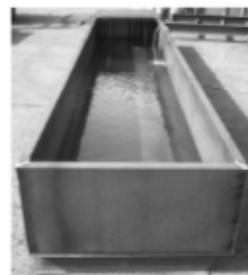
In a street two lamp posts are 600 feet apart. The light intensity at a distance d from the first (stronger) lamp post is $\frac{1000}{d^2}$, the light intensity at distance d from the second (weaker) lamp post is $\frac{125}{d^2}$ (in both cases the light intensity is inversely proportional to the square of the distance to the light source). The combined light intensity is the sum of the two light intensities coming from both lamp posts.



Based on the above information, answer the following questions.

- (i) If you are in between the lamp posts, at distance x feet from the stronger light, then the formula for the combined light intensity coming from both lamp posts as function of x , is
- (a) $\frac{1000}{x^2} + \frac{125}{x^2}$ (b) $\frac{1000}{(600-x)^2} + \frac{125}{x^2}$ (c) $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$ (d) None of these
- (ii) The maximum value of x can not be
- (a) 100 (b) 200 (c) 600 (d) None of these
- (iii) The minimum value of x can not be
- (a) 0 (b) 100 (c) 200 (d) None of these
- (iv) If $I(x)$ denote the combined light intensity, then $I(x)$ will be minimum when $x =$
- (a) 200 (b) 400 (c) 600 (d) 800
- (v) The darkest spot between the two lights is
- (a) at a distance of 200 feet from the weaker lamp post.
 (b) at distance of 200 feet from the stronger lamp post.
 (c) at a distance of 400 feet from the weaker lamp post.
 (d) None of these

An open water tank of aluminium sheet of negligible thickness, with a square base and vertical sides, is to be constructed in a farm for irrigation. It should hold 32000 l of water, that comes out from a tube well.

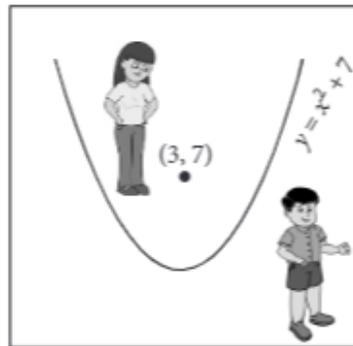


Based on above information, answer the following questions.

- (i) If the length, width and height of the open tank be x , x and y m respectively, then total surface area of tank is
 (a) $(x^2 + 2xy) \text{ m}^2$ (b) $(2x^2 + 4xy) \text{ m}^2$ (c) $(2x^2 + 2xy) \text{ m}^2$ (d) $(2x^2 + 8xy) \text{ m}^2$
- (ii) The relation between x and y is
 (a) $x^2y = 32$ (b) $xy^2 = 32$ (c) $x^2y^2 = 32$ (d) $xy = 32$
- (iii) The outer surface area of tank will be minimum when depth of tank is equal to
 (a) half of its width (b) its width (c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its width (d) $\left(\frac{1}{3}\right)^{\text{rd}}$ of its width
- (iv) The cost of material will be least when width of tank is equal to
 (a) half of its depth (b) twice of its depth (c) $\left(\frac{1}{4}\right)^{\text{th}}$ of its depth (d) thrice of its depth
- (v) If cost of aluminium sheet is ₹ $360/\text{m}^2$, then the minimum cost for the construction of tank will be
 (a) ₹ 15,000 (b) ₹ 16280 (c) ₹ 17280 (d) ₹ 18280

16

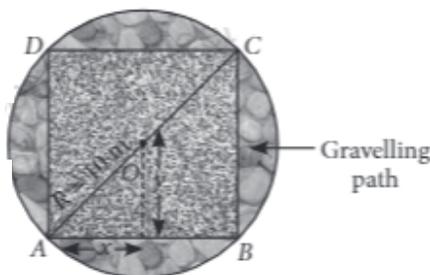
A student Arun is running on a playground along the curve given by $y = x^2 + 7$. Another student Manita standing at point $(3, 7)$ on playground wants to hit Arun by paper ball when Arun is nearest to Manita.



Based on above information, answer the following questions.

- (i) Arun's position at any value of x will be
 (a) $(x^2, y - 7)$ (b) $(x^2, y + 7)$ (c) $(x, x^2 + 7)$ (d) $(x^2, x - 7)$
- (ii) Distance (say D) between Arun and Manita will be
 (a) $(x - 1)(2x^2 + 2x + 3)$ (b) $(x - 3)^2 + x^4$
 (c) $\sqrt{(x - 3)^2 + x^4}$ (d) $\sqrt{(x - 1)(2x^2 + 2x + 3)}$
- (iii) For which real value(s) of x , first derivative of D^2 w.r.t. ' x ' will Vanish?
 (a) 1 (b) 2 (c) 3 (d) 4
- (iv) Find the position of Arun when Manita will hit the paper ball.
 (a) $(5, 32)$ (b) $(1, 8)$ (c) $(3, 7)$ (d) $(3, 16)$
- (v) The minimum value of D is
 (a) 3 (b) $\sqrt{3}$ (c) 5 (d) $\sqrt{5}$

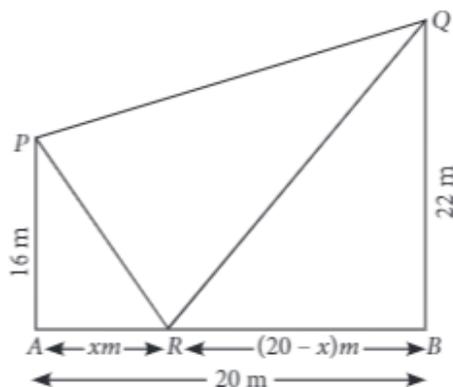
In a society there is a garden in the shape of rectangle inscribed in a circle of radius 10 m as shown in given figure.



Based on the above information, answer the following questions.

- (i) If $2x$ and $2y$ denotes the length and breadth in metres, of the rectangular part, then the relation between the variables is
- (a) $x^2 - y^2 = 10$ (b) $x^2 + y^2 = 10$ (c) $x^2 + y^2 = 100$ (d) $x^2 - y^2 = 100$
- (ii) The area (A) of green grass, in terms of x , is given by
- (a) $2x\sqrt{100-x^2}$ (b) $4x\sqrt{100-x^2}$ (c) $2x\sqrt{100+x^2}$ (d) $4x\sqrt{100+x^2}$
- (iii) The maximum value of A is
- (a) 100 m^2 (b) 200 m^2 (c) 400 m^2 (d) 1600 m^2
- (iv) The value of length of rectangle, when A is maximum, is
- (a) $10\sqrt{2} \text{ m}$ (b) $20\sqrt{2} \text{ m}$ (c) 20 m (d) $5\sqrt{2} \text{ m}$
- (v) The area of gravelling path is
- (a) $100(\pi + 2) \text{ m}^2$ (b) $100(\pi - 2) \text{ m}^2$ (c) $200(\pi + 2) \text{ m}^2$ (d) $200(\pi - 2) \text{ m}^2$

Two multi-storey buildings (represented by AP and BQ) are on opposite side of a 20 m wide road at point A and B respectively. There is a point R on road as shown in figure.



Based on the above information, answer the following questions.

- (i) Area of trapezium $ABQP$ is
- (a) 380 sq. m (b) 280 sq. m (c) 320 sq. m (d) 430 sq. m

An architecture design a auditorium for a school for its cultural activities. The floor of the auditorium is rectangular in shape and has a fixed perimeter P .



Based on the above information, answer the following questions.

- (i) If x and y represents the length and breadth of the rectangular region, then relation between the variable is
 (a) $x + y = P$ (b) $x^2 + y^2 = P^2$ (c) $2(x + y) = P$ (d) $x + 2y = P$
- (ii) The area (A) of the rectangular region, as a function of x , can be expressed as
 (a) $A = Px + \frac{x}{2}$ (b) $A = \frac{Px + x^2}{2}$ (c) $A = \frac{Px - 2x^2}{2}$ (d) $A = \frac{x^2}{2} + Px^2$
- (iii) School's manager is interested in maximising the area of floor 'A' for this to be happen, the value of x should be
 (a) P (b) $\frac{P}{2}$ (c) $\frac{P}{3}$ (d) $\frac{P}{4}$
- (iv) The value of y , for which the area of floor is maximum is
 (a) $\frac{P}{2}$ (b) $\frac{P}{3}$ (c) $\frac{P}{4}$ (d) $\frac{P}{16}$
- (v) Maximum area of floor is
 (a) $\frac{P^2}{16}$ (b) $\frac{P^2}{64}$ (c) $\frac{P^2}{4}$ (d) $\frac{P^2}{28}$

HINTS & EXPLANATIONS

1. (i) (b): Since, side of square is of length 20 cm, therefore $x \in (0, 10)$.

(ii) (a) : Clearly, height of open box = x cm
 Length of open box = $20 - 2x$
 and width of open box = $20 - 2x$

\therefore Volume (V) of the open box
 $= x \times (20 - 2x) \times (20 - 2x)$

(iii) (d) : We have, $V = x(20 - 2x)^2$

$$\therefore \frac{dV}{dx} = x \cdot 2(20 - 2x)(-2) + (20 - 2x)^2$$

$$= (20 - 2x)(-4x + 20 - 2x) = (20 - 2x)(20 - 6x)$$

$$\text{Now, } \frac{dV}{dx} = 0 \Rightarrow 20 - 2x = 0 \text{ or } 20 - 6x = 0$$

$$\Rightarrow x = 10 \text{ or } \frac{10}{3}$$

(iv) (c) : We have, $V = x(20 - 2x)^2$

$$\text{and } \frac{dV}{dx} = (20 - 2x)(20 - 6x)$$

$$\Rightarrow \frac{d^2V}{dx^2} = (20 - 2x)(-6) + (20 - 6x)(-2)$$

$$= (-2)[60 - 6x + 20 - 6x] = (-2)[80 - 12x] = 24x - 160$$

$$\text{For } x = \frac{10}{3}, \frac{d^2V}{dx^2} < 0$$

$$\text{and for } x = 10, \frac{d^2V}{dx^2} > 0$$

So, volume will be maximum when $x = \frac{10}{3}$.

(v) (d) : We have, $V = x(20 - 2x)^2$, which will be maximum when $x = \frac{10}{3}$,

$$\therefore \text{Maximum volume} = \frac{10}{3} \left(20 - 2 \times \frac{10}{3} \right)^2$$

$$= \frac{10}{3} \times \frac{40}{3} \times \frac{40}{3} = \frac{16000}{27} \text{ cm}^3$$

2. (i) (b) : To create a garden using 200 ft fencing, we need to maximise its area.

(ii) (c) : Required relation is given by $2x + y = 200$.

(iii) (c) : Area of garden as a function of x can be represented as

$$A(x) = x \cdot y = x(200 - 2x) = 200x - 2x^2$$

$$(iv) (a) : A(x) = 200x - 2x^2 \Rightarrow A'(x) = 200 - 4x$$

For the area to be maximum $A'(x) = 0$

$$\Rightarrow 200 - 4x = 0 \Rightarrow x = 50 \text{ ft.}$$

(v) (c) : Maximum area of the garden

$$= 200(50) - 2(50)^2 = 10000 - 5000 = 5000 \text{ sq. ft}$$

3. (i) (a) : Let x be the number of extra days after 1st July.

$$\therefore \text{Price} = ₹(300 - 3 \times x) = ₹(300 - 3x)$$

$$\text{Quantity} = 80 \text{ quintals} + x(1 \text{ quintal per day})$$

$$= (80 + x) \text{ quintals}$$

(ii) (b) : $R(x) = \text{Quantity} \times \text{Price}$

$$= (80 + x)(300 - 3x) = 24000 - 240x + 300x - 3x^2$$

$$= 24000 + 60x - 3x^2$$

(iii) (a) : We have, $R(x) = 24000 + 60x - 3x^2$

$$\Rightarrow R'(x) = 60 - 6x \Rightarrow R''(x) = -6$$

For $R(x)$ to be maximum, $R'(x) = 0$ and $R''(x) < 0$

$$\Rightarrow 60 - 6x = 0 \Rightarrow x = 10$$

(iv) (a) : Shyam's father will attain maximum revenue after 10 days.

So, he should harvest the onions after 10 days of 1st July i.e., on 11th July.

(v) (c) : Maximum revenue is collected by Shyam's father when $x = 10$

$$\therefore \text{Maximum revenue} = R(10)$$

$$= 24000 + 60(10) - 3(10)^2 = 24000 + 600 - 300 = 24300$$

4. (i) (a) : Let x be the charges per bike per day and n be the number of bikes rented per day.

$$R(x) = n \times x = (2000 - 10x)x = -10x^2 + 2000x$$

(ii) (b) : We have, $R(x) = 2000x - 10x^2$

$$\Rightarrow R'(x) = 2000 - 20x$$

For $R(x)$ to be maximum or minimum, $R'(x) = 0$

$$\Rightarrow 2000 - 20x = 0 \Rightarrow x = 100$$

Also, $R''(x) = -20 < 0$

Thus, $R(x)$ is maximum at $x = 100$

(iii) (c) : If company charge ₹ 200 or more, they will not rent any bike. Therefore, revenue collected by him will be zero.

(iv) (c) : If $x = 105$, number of bikes rented per day is given by

$$n = 2000 - 10 \times 105 = 950$$

(v) (d) : At $x = 100$, $R(x)$ is maximum.

$$\therefore \text{Maximum revenue} = R(100)$$

$$= -10(100)^2 + 2000(100) = ₹ 1,00,000$$

5. (i) (c) : If P is the rent price per apartment and N is the number of rented apartment, the profit is given by

$$NP - 500N = N(P - 500)$$

[\because ₹ 500/month is the maintenance charges for each occupied unit]

(ii) (c) : Now, if x be the number of non-rented apartments, then $N = 50 - x$ and $P = 10000 + 250x$

$$\text{Thus, profit} = N(P - 500) = (50 - x)(10000 + 250x - 500)$$

$$= (50 - x)(9500 + 250x) = 250(50 - x)(38 + x)$$

(iii) (b) : Clearly, if $P = 10500$, then

$$10500 = 10000 + 250x \Rightarrow x = 2 \Rightarrow N = 48$$

(iv) (a) : Also, if $P = 11000$, then

$$11000 = 10000 + 250x \Rightarrow x = 4 \text{ and so profit}$$

$$P(4) = 250(50 - 4)(38 + 4) = ₹ 4,83,000$$

(v) (b) : We have, $P(x) = 250(50 - x)(38 + x)$

$$\text{Now, } P'(x) = 250[50 - x - (38 + x)] = 250[12 - 2x]$$

For maxima/minima, put $P'(x) = 0$

$$\Rightarrow 12 - 2x = 0 \Rightarrow x = 6$$

Thus, price per apartment is, $P = 10000 + 1500 = 11500$

Hence, the rent that maximizes the profit is ₹ 11500.

6. (i) (a) : Let p be the price per ticket and x be the number of tickets sold.

$$\text{Then, revenue function } R(x) = p \times x = \left(15 - \frac{x}{3000} \right) x$$

$$= 15x - \frac{x^2}{3000}$$

(ii) (c) : Since, more than 36000 tickets cannot be sold.

So, range of x is $[0, 36000]$.

(iii) (c) : We have, $R(x) = 15x - \frac{x^2}{3000}$

$$\Rightarrow R'(x) = 15 - \frac{x}{1500}$$

For maxima/minima, put $R'(x) = 0$

$$\Rightarrow 15 - \frac{x}{1500} = 0 \Rightarrow x = 22500$$

$$\text{Also, } R''(x) = -\frac{1}{1500} < 0.$$

(iv) (d) : Maximum revenue will be at $x = 22500$

$$\therefore \text{ Price of a ticket} = 15 - \frac{22500}{3000} = 15 - 7.5 = ₹ 7.5$$

(v) (d) : Number of spectators will be equal to number of tickets sold.

$$\therefore \text{ Required number of spectators} = 22500$$

7. (i) (d) : Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume = $3 \text{ l} = 3000 \text{ cm}^3$
($\because 1 \text{ l} = 1000 \text{ cm}^3$)

$$\Rightarrow \pi r^2 h = 3000 \Rightarrow h = \frac{3000}{\pi r^2}$$

Now, the surface area, as a function of r is given by

$$\begin{aligned} S(r) &= 2\pi r^2 + 2\pi r h = 2\pi r^2 + 2\pi r \left(\frac{3000}{\pi r^2} \right) \\ &= 2\pi r^2 + \frac{6000}{r} \end{aligned}$$

(ii) (c) : Now, $S(r) = 2\pi r^2 + \frac{6000}{r}$

$$\Rightarrow S'(r) = 4\pi r - \frac{6000}{r^2}$$

To find critical points, put $S'(r) = 0$

$$\Rightarrow \frac{4\pi r^3 - 6000}{r^2} = 0$$

$$\Rightarrow r^3 = \frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi} \right)^{1/3}$$

$$\text{Also, } S''(r) \Big|_{r = \sqrt[3]{\frac{1500}{\pi}}} = 4\pi + \frac{12000 \times \pi}{1500}$$

$$= 4\pi + 8\pi = 12\pi > 0$$

Thus, the critical point is the point of minima.

(iii) (b) : The cost of material for the tin can is minimized when $r = \sqrt[3]{\frac{1500}{\pi}}$ cm and the height is

$$\frac{3000}{\pi \left(\sqrt[3]{\frac{1500}{\pi}} \right)^2} = 2\sqrt[3]{\frac{1500}{\pi}} \text{ cm.}$$

(iv) (a) : We have, minimum surface area = $\frac{2\pi r^3 + 6000}{r}$

$$= \frac{2\pi \cdot \frac{1500}{\pi} + 6000}{\sqrt[3]{\frac{1500}{\pi}}} = \frac{9000}{7.8} = 1153.84 \text{ cm}^2$$

Cost of 1 m^2 material = ₹100

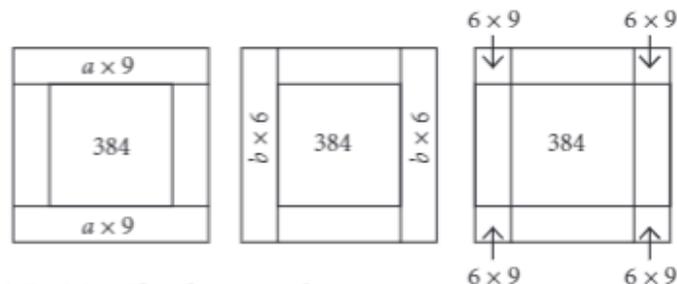
$$\therefore \text{ Cost of } 1 \text{ cm}^2 \text{ material} = ₹ \frac{1}{100}$$

$$\therefore \text{ Minimum cost} = ₹ \frac{1153.84}{100} = ₹ 11.538$$

(v) (c) : To minimize the cost we need to minimize the total surface area.

8. (i) (a) : Let A be the area of the poster, then

$$\begin{aligned} A &= 864 + 2(a \cdot 9) + 2(b \cdot 6) - 4(6 \cdot 9) \\ &= 864 + 18a + 12b - 216 = 648 + 18a + 12b \end{aligned}$$



(ii) (a) : Clearly, $A = a \cdot b$

$$\therefore 648 + 18a + 12b = ab$$

$$\Rightarrow ab - 18a = 648 + 12b \Rightarrow a(b - 18) = 648 + 12b$$

$$\Rightarrow a = \frac{648 + 12b}{b - 18}$$

(iii) (b) : Since, $A = a \cdot b$, therefore

$$A = \left(\frac{648 + 12b}{b - 18} \right) \cdot b = \frac{648b + 12b^2}{b - 18} \quad \left[\because a = \frac{648 + 12b}{b - 18} \right]$$

(iv) (a) : Clearly,

$$\begin{aligned} A'(b) &= \frac{(b - 18)(648 + 24b) - (648b + 12b^2)}{(b - 18)^2} \\ &= \frac{12[b^2 - 36b - 972]}{(b - 18)^2} \end{aligned}$$

For minimum, consider $A'(b) = 0$

$$\Rightarrow b^2 - 36b - 972 = 0$$

$$\Rightarrow b^2 - 54b + 18b - 972 = 0$$

$$\Rightarrow b(b - 54) + 18(b - 54) = 0$$

$$\Rightarrow b = -18 \text{ or } b = 54$$

$\therefore b$ is height, therefore can't be negative.

So, $b = 54$.

(v) (b) : Since, $a = \frac{648 + 12b}{b - 18}$

$$\therefore a = \frac{648 + 12 \times 54}{54 - 18} = \frac{648 + 648}{36} = 36$$

9. (i) (d) : In order to make least expensive water tank, Nitin need to minimize its cost.

(ii) (d) : Let l ft be the length and h ft be the height of the tank. Since breadth is equal to 5 ft. (Given)

\therefore Two sides will be $5h$ sq. feet and two sides will be lh sq. feet. So, the total area of the sides is $(10h + 2lh) \text{ ft}^2$

(v) (a) : The minimum maintenance cost for each apartment would be ₹ 1091.11

12. (i) (b): In order to paint in the maximum area, Kyra needs to maximize the area of inner rectangle.

(ii) (c) : Let x be the length and y be the breadth of outer rectangle.

\therefore Length of inner rectangle = $x - 1$

and breadth of inner rectangle = $y - 1.5$

$\therefore A(x) = (x - 1)(y - 1.5)$ [$\because xy = 24$ (given)]

$$= (x - 1) \left(\frac{24}{x} - 1.5 \right)$$

(iii) (b) : Dimensions of rectangle (outer/inner) should be positive.

$$\therefore x - 1 > 0 \text{ and } \frac{24}{x} - 1.5 > 0$$

$$\Rightarrow x > 1 \text{ and } x < 16$$

\therefore Range of x is (1, 16).

(iv) (c) : We have, $A(x) = (x - 1) \left(\frac{24}{x} - 1.5 \right)$

$$\Rightarrow A'(x) = (x - 1) \left(\frac{-24}{x^2} \right) + \left(\frac{24}{x} - 1.5 \right) = \frac{24}{x^2} - 1.5$$

$$\text{and } A''(x) = \frac{-48}{x^3}$$

For $A(x)$ to be maximum or minimum, $A'(x) = 0$

$$\Rightarrow -1.5 + \frac{24}{x^2} = 0 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$$

$\therefore x = 4$ [Since, length can't be negative]

$$\text{Also, } A''(4) = \frac{-48}{4^3} < 0$$

Thus, at $x = 4$, area is maximum.

(v) (a) : If area of inner rectangle is maximum, then Length of inner rectangle = $x - 1 = 4 - 1 = 3$ ft

$$\begin{aligned} \text{And breadth of inner rectangle} &= y - 1.5 = \frac{24}{x} - 1.5 \\ &= \frac{24}{4} - 1.5 = 6 - 1.5 = 4.5 \text{ ft} \end{aligned}$$

13. (i) (d): If x be the amount of increase in annual charges, then number of subscriber reduces to $5000 - x$

$$\begin{aligned} \therefore \text{Revenue, } R(x) &= (3000 + x)(5000 - x) \\ &= 15000000 + 2000x - x^2, 0 < x < 5000 \end{aligned}$$

(ii) (a) : Clearly, at $x = 500$

$$\begin{aligned} R(500) &= 15000000 + 2000(500) - (500)^2 \\ &= 15000000 + 1000000 - 250000 = ₹ 15750000 \end{aligned}$$

(iii) (c) : Since, $15000000 + 2000x - x^2 = 15640000$ (Given)

$$\Rightarrow x^2 - 2000x + 640000 = 0$$

$$\Rightarrow x^2 - 1600x - 400x + 640000 = 0$$

$$\Rightarrow x(x - 1600) - 400(x - 1600) = 0 \Rightarrow x = 400, 1600$$

$$(iv) (a) : \frac{dR}{dx} = 2000 - 2x \text{ and } \frac{d^2R}{dx^2} = -2 < 0$$

For maximum revenue, $\frac{dR}{dx} = 0 \Rightarrow x = 1000$

\therefore Required amount = ₹ 1000

(v) (b) : Maximum revenue = $R(1000)$

$$= (3000 + 1000)(5000 - 1000)$$

$$= 4000 \times 4000 = ₹ 16000000$$

14. (i) (c): Since, the distance is x feet from the stronger light, therefore the distance from the weaker light will be $600 - x$.

So, the combined light intensity from both lamp posts is given by $\frac{1000}{x^2} + \frac{125}{(600-x)^2}$.

(ii) (c) : Since, the person is in between the lamp posts, therefore x will lie in the interval (0, 600).

So, maximum value of x can't be 600.

(iii) (a) : Since, $0 < x < 600$, therefore minimum value of x can't be 0.

(iv) (b) : We have, $I(x) = \frac{1000}{x^2} + \frac{125}{(600-x)^2}$

$$\Rightarrow I'(x) = \frac{-2000}{x^3} + \frac{250}{(600-x)^3} \text{ and}$$

$$\Rightarrow I''(x) = \frac{6000}{x^4} + \frac{750}{(600-x)^4}$$

For maxima/minima, $I'(x) = 0$

$$\Rightarrow \frac{2000}{x^3} = \frac{250}{(600-x)^3} \Rightarrow 8(600-x)^3 = x^3$$

Taking cube root on both sides, we get

$$2(600 - x) = x \Rightarrow 1200 = 3x \Rightarrow x = 400$$

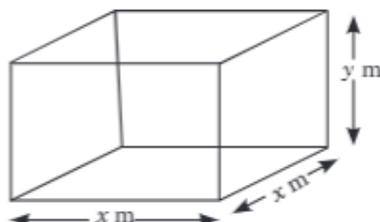
Thus, $I(x)$ is minimum when you are at 400 feet from the strong intensity lamp post.

(v) (a) : Since, $I(x)$ is minimum when $x = 400$ feet, therefore the darkest spot between the two light is at a distance of 400 feet from stronger lamp post, i.e., at a distance of $600 - 400 = 200$ feet from the weaker lamp post.

15. (i) (d): Since the tank is open from the top, therefore the total surface area is

= (Outer + Inner) surface area

$$= 2(x \times x + 2(xy + yx)) = 2(x^2 + 2(2xy)) = (2x^2 + 8xy) \text{ m}^2$$



(ii) (a) : Since, volume of tank should be 32000 l.
 $\therefore x^2y m^3 = 32000 l = 32 m^3$ [$\because 1 \text{ litre} = 0.001 m^3$]
 So, $x^2y = 32$

(iii) (a) : Let S be the outer surface area of tank.
 Then, $S = x^2 + 4xy$

$$\Rightarrow S(x) = x^2 + 4x \cdot \frac{32}{x^2} = x^2 + \frac{128}{x} \quad [\because x^2y = 32]$$

$$\Rightarrow \frac{dS}{dx} = 2x - \frac{128}{x^2} \text{ and } \frac{d^2S}{dx^2} = 2 + \frac{256}{x^3}$$

For maximum or minimum values of S , consider

$$\frac{dS}{dx} = 0$$

$$\Rightarrow 2x = \frac{128}{x^2} \Rightarrow x^3 = 64 \Rightarrow x = 4 \text{ m}$$

$$\text{At } x = 4, \frac{d^2S}{dx^2} = 2 + \frac{256}{4^3} = 2 + 4 = 6 > 0$$

$\therefore S$ is minimum when $x = 4$

Now as $x^2y = 32$, therefore $y = 2$

Thus, $x = 2y$

(iv) (b) : Since, surface area is minimum when $x = 2y$, therefore cost of material will be least when $x = 2y$.

Thus, cost of material will be least when width is equal to twice of its depth.

(v) (c) : Since, minimum surface area
 $= x^2 + 4xy = 4^2 + 4 \times 4 \times 2 = 48 m^2$ and
 cost per $m^2 = ₹ 360$

\therefore Minimum cost is $= ₹ (48 \times 360) = ₹ 17280$

16. (i) (c) : For all values of x , $y = x^2 + 7$

\therefore Arun's position at any point of x will be $(x, x^2 + 7)$

(ii) (c) : Distance between Arun and Manita, i.e., D

$$= \sqrt{(x-3)^2 + (x^2+7-7)^2}$$

$$= \sqrt{(x-3)^2 + x^4}$$

(iii) (a) : We have, $D = \sqrt{(x-3)^2 + x^4}$

$$\therefore D^2 = (x-3)^2 + x^4$$

$$\text{Now, } \frac{d}{dx} (D^2) = 2(x-3) + 4x^3 = 0$$

$$\Rightarrow 4x^3 + 2x - 6 = 0 \Rightarrow 2x^3 + x - 3 = 0$$

$$\Rightarrow (x-1)(2x^2 + 2x + 3) = 0$$

$$\therefore x = 1$$

($\because 2x^2 + 2x + 3 = 0$ will give imaginary values)

(iv) (b) : We have, $D = \sqrt{(x-3)^2 + x^4}$

$$D'(x) = \frac{2(x-3) + 4x^3}{2\sqrt{(x-3)^2 + x^4}} = 0$$

$$\Rightarrow 2x^3 + x - 3 = 0 \quad \begin{array}{ccc} \text{Arun} & & \text{Manita} \\ (1, 8) & 0 \text{ (min.)} & (3, 7) \end{array}$$

Clearly, $D''(x)$ at $x = 1$ is > 0

\therefore Value of x for which D will be minimum is 1

For $x = 1, y = 8$.

Thus, the required position is $(1, 8)$.

$$\begin{aligned} \text{(v) (d) : Minimum value of } D &= \sqrt{(1-3)^2 + (1)^4} \\ &= \sqrt{4+1} = \sqrt{5} \end{aligned}$$

17. (i) (c) : Length, $AB = 2x$

Breadth, $BC = 2y$

Also, radius, $OA = 10$

$\therefore AC = 20$

In $\triangle ABC$, $AB^2 + BC^2 = AC^2$

$$\Rightarrow (2x)^2 + (2y)^2 = (20)^2$$

$$\Rightarrow x^2 + y^2 = 100$$

(ii) (b) : Area of green grass = Area of rectangular part

$\therefore A = 2x \cdot 2y$ [\because Area of rectangle = length \times breadth]

$$= 4xy = 4x \sqrt{100 - x^2} \quad [\because x^2 + y^2 = 100]$$

(iii) (b) : We have, $A = 4x \sqrt{100 - x^2}$

$$\frac{dA}{dx} = \frac{4x(-2x)}{2\sqrt{100 - x^2}} + \sqrt{100 - x^2} \cdot 4$$

$$= \frac{-4x^2 + 4(100 - x^2)}{\sqrt{100 - x^2}}$$

For maximum value, $\frac{dA}{dx} = 0$

$$\Rightarrow -4x^2 + 400 - 4x^2 = 0$$

$$\Rightarrow -8x^2 + 400 = 0$$

$$\Rightarrow x^2 = 50 \Rightarrow x = 5\sqrt{2}$$

At $x = 5\sqrt{2}$,

$$A = 4x \sqrt{100 - x^2}$$

$$= 4 \times 5\sqrt{2} \cdot \sqrt{100 - 50} = 4 \times 5\sqrt{2} \times 5\sqrt{2} = 200 m^2$$

(iv) (a) : Length of rectangle for which A is maximum

$$= 2 \times 5\sqrt{2} = 10\sqrt{2}$$

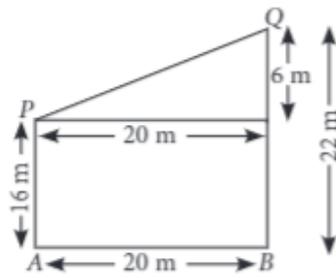
$$\begin{aligned} \text{(v) (b) : Area of gravelling path} &= \pi(10)^2 - 200 \\ &= 100(\pi - 2) m^2 \end{aligned}$$

18. (i) (a) : Area of trapezium $= \frac{1}{2} \times$ (sum of parallel sides) \times distance between parallel sides

$$= \frac{1}{2} \times (16 + 22) \times 20 = 380 m^2$$

(ii) (c) : $PQ^2 = 6^2 + (20)^2 = 36 + 400 = 436$

$$\therefore PQ = \sqrt{436} = 20.88 \text{ m}$$



(iii) (c) : We have, $S = RP^2 + RQ^2$
 Since, $RP^2 = (16)^2 + x^2 = 256 + x^2$
 and $RQ^2 = (22)^2 + (20 - x)^2 = 484 + 400 + x^2 - 40x$
 $\therefore S = 2x^2 - 40x + 1140$

(iv) (a) : We have, $S(x) = 2x^2 - 40x + 1140$
 $\therefore S'(x) = 4x - 40$

For minimum value of x , $S'(x) = 0$
 $\Rightarrow 4x - 40 = 0 \Rightarrow x = 10$

Clearly, at $x = 10$, $S''(x) = 4 > 0$

(v) (b) : At $x = 10$, $PR^2 = 16^2 + x^2$
 $= (16)^2 + (10)^2 = 256 + 100 = 356$

$\therefore PR = \sqrt{356} = 18.86 \text{ m}$

Also, $RQ^2 = (22)^2 + (20 - 10)^2 = 484 + 100 = 584$

$\therefore RQ = \sqrt{584} = 24.17 \text{ m}$

19. (i) (b) : Given, perimeter of window = 10 m

$\therefore x + y + y + \text{perimeter of semicircle} = 10$

$\Rightarrow x + 2y + \pi \frac{x}{2} = 10$

(ii) (b) : $A = x \cdot y + \frac{1}{2} \pi \left(\frac{x}{2}\right)^2$

$= x \left(5 - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2} \pi \frac{x^2}{4} \left[\because \text{From (i), } y = 5 - \frac{x}{2} - \frac{\pi x}{4} \right]$

$= 5x - \frac{x^2}{2} - \frac{\pi x^2}{4} + \frac{\pi x^2}{8} = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$

(iii) (c) : We have, $A = 5x - \frac{x^2}{2} - \frac{\pi x^2}{8}$

$\Rightarrow \frac{dA}{dx} = 5 - x - \frac{\pi x}{4}$

Now, $\frac{dA}{dx} = 0 \Rightarrow 5 = x + \frac{\pi x}{4}$

$\Rightarrow x(4 + \pi) = 20 \Rightarrow x = \frac{20}{4 + \pi}$

$\left[\text{Clearly, } \frac{d^2A}{dx^2} < 0 \text{ at } x = \frac{20}{4 + \pi} \right]$

(iv) (d) : At $x = \frac{20}{4 + \pi}$

$A = 5 \left(\frac{20}{4 + \pi}\right) - \left(\frac{20}{4 + \pi}\right)^2 \frac{1}{2} - \frac{\pi}{8} \left(\frac{20}{4 + \pi}\right)^2$
 $= \frac{100}{4 + \pi} - \frac{200}{(4 + \pi)^2} - \frac{50\pi}{(4 + \pi)^2}$

$= \frac{(4 + \pi)(100) - 200 - 50\pi}{(4 + \pi)^2} = \frac{400 + 100\pi - 200 - 50\pi}{(4 + \pi)^2}$

$= \frac{200 + 50\pi}{(4 + \pi)^2} = \frac{50(4 + \pi)}{(4 + \pi)^2} = \frac{50}{4 + \pi}$

(v) (a) : We have, $y = 5 - \frac{x}{2} - \frac{\pi x}{4} = 5 - x \left(\frac{1}{2} + \frac{\pi}{4}\right)$

$= 5 - x \left(\frac{2 + \pi}{4}\right) = 5 - \left(\frac{20}{4 + \pi}\right) \left(\frac{2 + \pi}{4}\right)$

$= 5 - 5 \frac{(2 + \pi)}{4 + \pi} = \frac{20 + 5\pi - 10 - 5\pi}{4 + \pi} = \frac{10}{4 + \pi}$

20. (i) (c) : Perimeter of floor = 2(length + breadth)

$\Rightarrow P = 2(x + y)$

(ii) (c) : Area, $A = \text{length} \times \text{breadth}$

$\Rightarrow A = xy$

Since, $P = 2(x + y)$

$\Rightarrow \frac{P - 2x}{2} = y$

(iii) (d) : We have, $A = \frac{1}{2} (Px - 2x^2)$

$\frac{dA}{dx} = \frac{1}{2} (P - 4x) = 0$

$\Rightarrow P - 4x = 0 \Rightarrow x = \frac{P}{4}$

Clearly, at $x = \frac{P}{4}$, $\frac{d^2A}{dx^2} = -2 < 0$

\therefore Area is maximum at $x = \frac{P}{4}$.

(iv) (c) : We have, $y = \frac{P - 2x}{2} = \frac{P}{2} - \frac{P}{4} = \frac{P}{4}$

(v) (a) : We have, $A = xy = \frac{P}{4} \cdot \frac{P}{4} = \frac{P^2}{16}$