

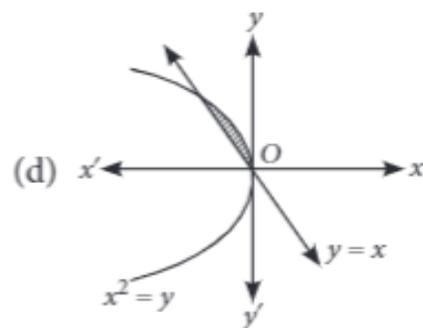
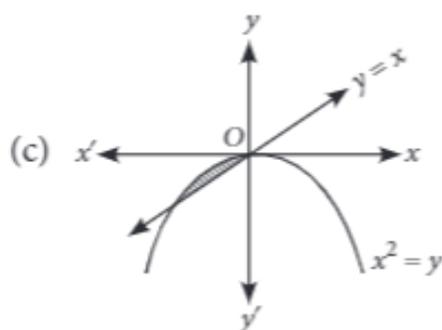
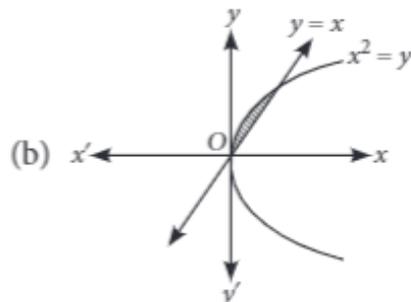
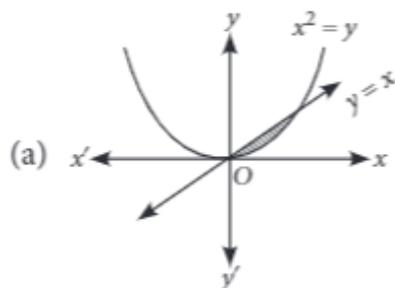
# Application of Integrals

## CASE STUDY / PASSAGE BASED QUESTIONS

1

Consider the following equations of curves  $x^2 = y$  and  $y = x$ .  
On the basis of above information, answer the following questions.

- (i) The point(s) of intersection of both the curves is (are)  
 (a)  $(0, 0), (2, 2)$     (b)  $(0, 0), (1, 1)$     (c)  $(0, 0), (-1, -1)$     (d)  $(0, 0), (-2, -2)$
- (ii) Area bounded by the curves is represented by which of the following graph?



(iii) The value of the integral  $\int_0^1 x \, dx$  is

- (a)  $1/4$     (b)  $1/3$     (c)  $1/2$     (d) 1

(iv) The value of the integral  $\int_0^1 x^2 \, dx$  is

- (a)  $1/4$     (b)  $1/3$     (c)  $1/2$     (d) 1

(v) The value of area bounded by the curves  $x^2 = y$  and  $x = y$  is

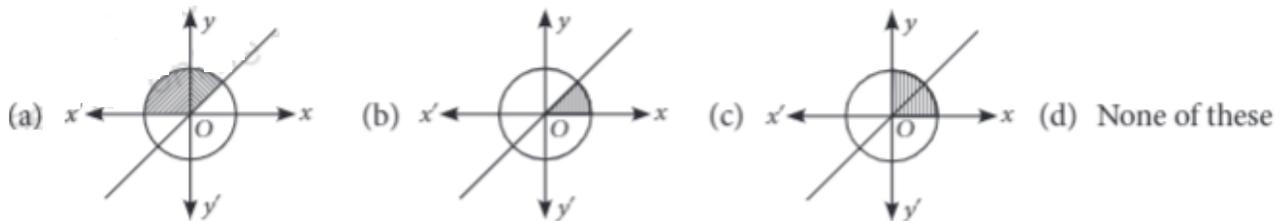
- (a)  $\frac{1}{6}$  sq. unit    (b)  $\frac{1}{3}$  sq. unit    (c)  $\frac{1}{2}$  sq. unit    (d) 1 sq. unit

### Syllabus

Applications in finding the area under simple curves, especially lines, parabolas; area of circles /ellipses (in standard form only) (the region should be clearly identifiable).

Consider the curve  $x^2 + y^2 = 16$  and line  $y = x$  in the first quadrant. Based on the above information, answer the following questions.

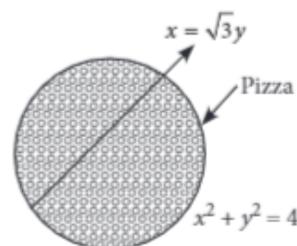
- (i) Point of intersection of both the given curves is  
 (a)  $(0, 4)$  (b)  $(0, 2\sqrt{2})$  (c)  $(2\sqrt{2}, 2\sqrt{2})$  (d)  $(2\sqrt{2}, 4)$
- (ii) Which of the following shaded portion represent the area bounded by given two curves?



- (iii) The value of the integral  $\int_0^{2\sqrt{2}} x \, dx$  is  
 (a) 0 (b) 1 (c) 2 (d) 4
- (iv) The value of the integral  $\int_{2\sqrt{2}}^4 \sqrt{16 - x^2} \, dx$  is  
 (a)  $2(\pi - 2)$  (b)  $2(\pi - 8)$  (c)  $4(\pi - 2)$  (d)  $4(\pi + 2)$
- (v) Area bounded by the two given curves is  
 (a)  $3\pi$  sq. units (b)  $\frac{\pi}{2}$  sq. units (c)  $\pi$  sq. units (d)  $2\pi$  sq. units

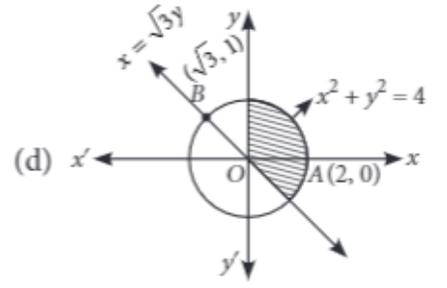
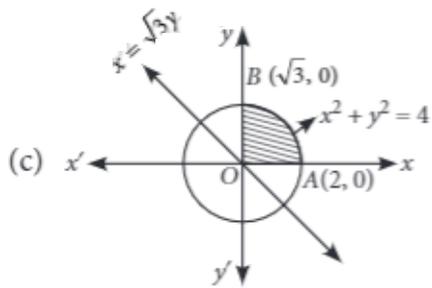
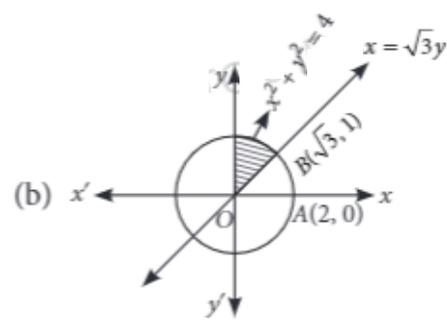
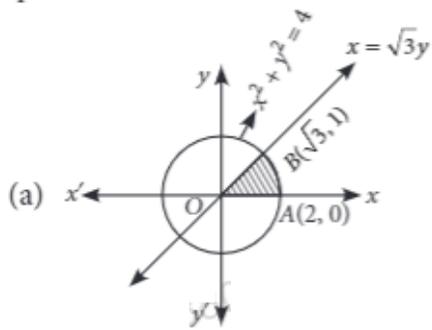
A child cut a pizza with a knife. Pizza is circular in shape which is represented by  $x^2 + y^2 = 4$  and sharp edge of knife represents a straight line given by  $x = \sqrt{3}y$ .

Based on the above information, answer the following questions.



- (i) The point(s) of intersection of the edge of knife (line) and pizza shown in the figure is (are)  
 (a)  $(1, \sqrt{3}), (-1, -\sqrt{3})$  (b)  $(\sqrt{3}, 1), (-\sqrt{3}, -1)$   
 (c)  $(\sqrt{2}, 0), (0, \sqrt{3})$  (d)  $(-\sqrt{3}, 1), (1, -\sqrt{3})$

(ii) Which of the following shaded portion represent the smaller area bounded by pizza and edge of knife in first quadrant?



(iii) Value of area of the region bounded by circular pizza and edge of knife in first quadrant is

- (a)  $\frac{\pi}{2}$  sq. units      (b)  $\frac{\pi}{3}$  sq. units      (c)  $\frac{\pi}{5}$  sq. units      (d)  $\pi$  sq. units

(iv) Area of each slice of pizza when child cut the pizza into 4 equal pieces is

- (a)  $\pi$  sq. units      (b)  $\frac{\pi}{2}$  sq. units      (c)  $3\pi$  sq. units      (d)  $2\pi$  sq. units

(v) Area of whole pizza is

- (a)  $3\pi$  sq. units      (b)  $2\pi$  sq. units      (c)  $5\pi$  sq. units      (d)  $4\pi$  sq. units

**4**

Consider the following equation of curve  $y^2 = 4x$  and straight line  $x + y = 3$ .

Based on the above information, answer the following questions.

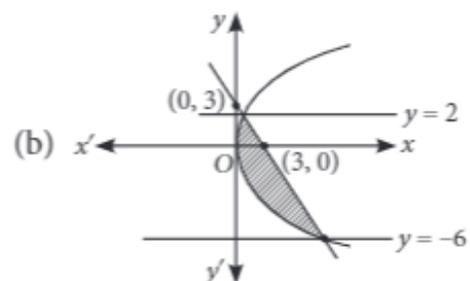
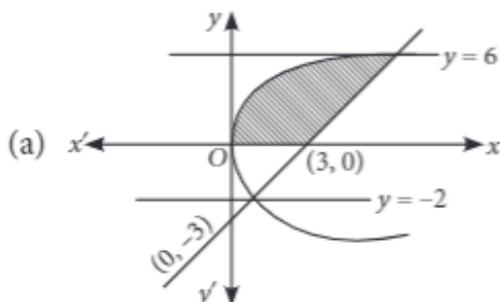
(i) The line  $x + y = 3$  cuts the  $x$ -axis and  $y$ -axis respectively at

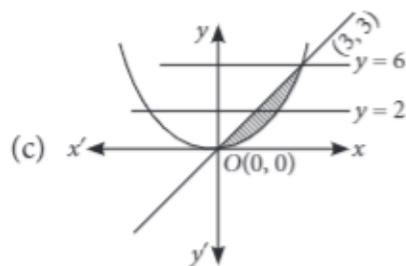
- (a)  $(0, 2), (2, 0)$       (b)  $(3, 3), (0, 0)$       (c)  $(0, 3), (3, 0)$       (d)  $(3, 0), (0, 3)$

(ii) Point(s) of intersection of two given curves is (are)

- (a)  $(1, -2), (-9, 6)$       (b)  $(2, 1), (-6, 9)$       (c)  $(1, 2), (9, -6)$       (d) None of these

(iii) Which of the following shaded portion represent the area bounded by given curves?





(d) None of these

(iv) Value of the integral  $\int_{-6}^2 (3-y) dy$  is

(a) 10

(b) 20

(c) 30

(d) 40

(v) Value of area bounded by given curves is

(a) 56 sq. units

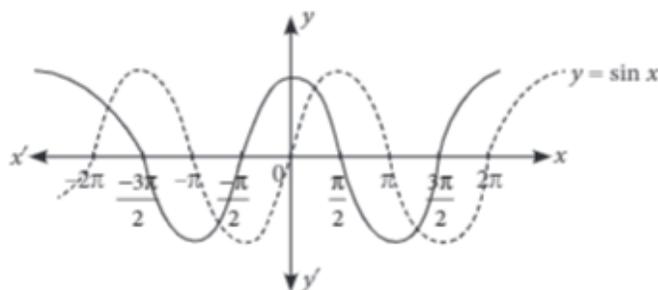
(b)  $\frac{63}{5}$  sq. units

(c)  $\frac{64}{3}$  sq. units

(d) 31 sq. units

## 5

In a classroom, teacher explains the properties of a particular curve by saying that this particular curve has beautiful up and downs. It starts at 1 and heads down until  $\pi$  radian, and then heads up again and closely related to sine function and both follow each other, exactly  $\frac{\pi}{2}$  radians apart as shown in figure.



Based on the above information, answer the following questions.

(i) Name the curve, about which teacher explained in the classroom.

(a) cosine

(b) sine

(c) tangent

(d) cotangent

(ii) Area of curve explained in the passage from 0 to  $\frac{\pi}{2}$  is

(a)  $\frac{1}{3}$  sq. unit

(b)  $\frac{1}{2}$  sq. unit

(c) 1 sq. unit

(d) 2 sq. units

(iii) Area of curve discussed in classroom from  $\frac{\pi}{2}$  to  $\frac{3\pi}{2}$  is

(a) -2 sq. units

(b) 2 sq. units

(c) 3 sq. units

(d) -3 sq. units

(iv) Area of curve discussed in classroom from  $\frac{3\pi}{2}$  to  $2\pi$  is

(a) 1 sq. unit

(b) 2 sq. units

(c) 3 sq. units

(d) 4 sq. units

(v) Area of explained curve from 0 to  $2\pi$  is

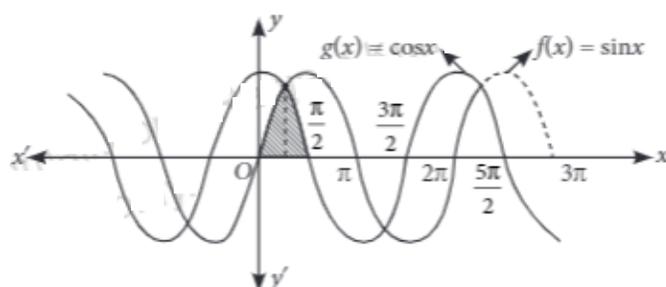
(a) 1 sq. unit

(b) 2 sq. units

(c) 3 sq. units

(d) 4 sq. units

Graphs of two function  $f(x) = \sin x$  and  $g(x) = \cos x$  is given below :



Based on the above information, answer the following questions.

(i) In  $[0, \pi]$ , the curves  $f(x) = \sin x$  and  $g(x) = \cos x$  intersect at  $x =$

- (a)  $\frac{\pi}{2}$                       (b)  $\frac{\pi}{3}$                       (c)  $\frac{\pi}{4}$                       (d)  $\pi$

(ii) Value of  $\int_0^{\pi/4} \sin x \, dx$  is

- (a)  $1 - \frac{1}{\sqrt{2}}$                       (b)  $1 + \frac{1}{\sqrt{2}}$                       (c)  $2 - \frac{1}{\sqrt{2}}$                       (d)  $2 + \frac{1}{\sqrt{2}}$

(iii) Value of  $\int_{\pi/4}^{\pi/2} \cos x \, dx$  is

- (a)  $1 + \frac{1}{\sqrt{2}}$                       (b)  $1 - \frac{1}{\sqrt{2}}$                       (c)  $2 - \sqrt{2}$                       (d)  $2 + \sqrt{2}$

(iv) Value of  $\int_0^{\pi} \sin x \, dx$  is

- (a) 0                      (b) 1                      (c) 2                      (d) -2

(v) Value of  $\int_0^{\pi/2} \sin x \, dx$  is

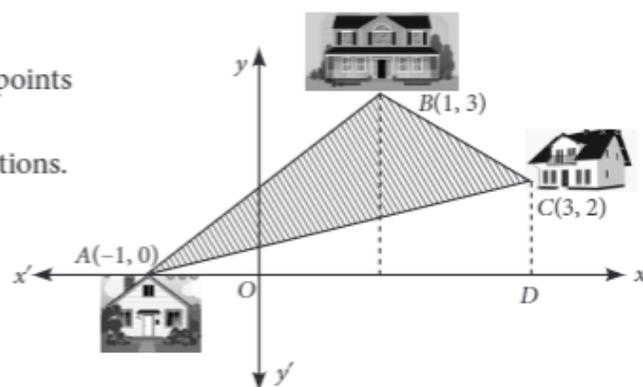
- (a) 0                      (b) 1                      (c) 3                      (d) 4

Location of three houses of a society is represented by the points  $A(-1, 0)$ ,  $B(1, 3)$  and  $C(3, 2)$  as shown in figure.

Based on the above information, answer the following questions.

(i) Equation of line  $AB$  is

- (a)  $y = \frac{3}{2}(x+1)$                       (b)  $y = \frac{3}{2}(x-1)$   
 (c)  $y = \frac{1}{2}(x+1)$                       (d)  $y = \frac{1}{2}(x-1)$



(ii) Equation of line  $BC$  is

(a)  $y = \frac{1}{2}x - \frac{7}{2}$

(b)  $y = \frac{3}{2}x - \frac{7}{2}$

(c)  $y = \frac{-1}{2}x + \frac{7}{2}$

(d)  $y = \frac{3}{2}x + \frac{7}{2}$

(iii) Area of region  $ABCD$  is

(a) 2 sq. units

(b) 4 sq. units

(c) 6 sq. units

(d) 8 sq. units

(iv) Area of  $\triangle ADC$  is

(a) 4 sq. units

(b) 8 sq. units

(c) 16 sq. units

(d) 32 sq. units

(v) Area of  $\triangle ABC$  is

(a) 3 sq. units

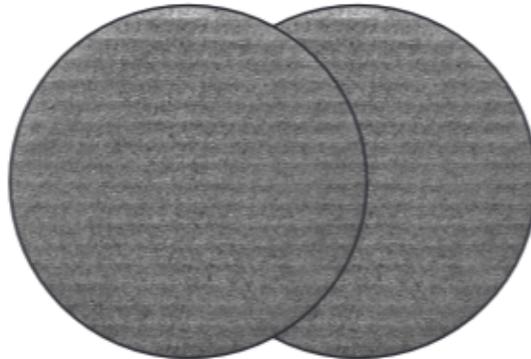
(b) 4 sq. units

(c) 5 sq. units

(d) 6 sq. units

8

Ajay cut two circular pieces of cardboard and placed one upon other as shown in figure. One of the circle represents the equation  $(x - 1)^2 + y^2 = 1$ , while other circle represents the equation  $x^2 + y^2 = 1$ .



Based on the above information, answer the following questions.

(i) Both the circular pieces of cardboard meet each other at

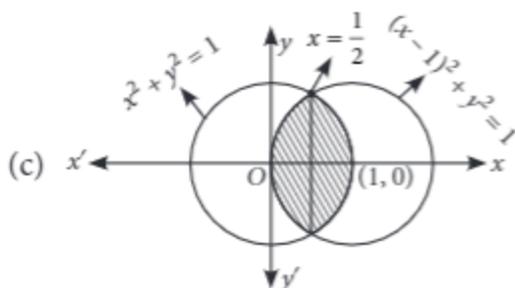
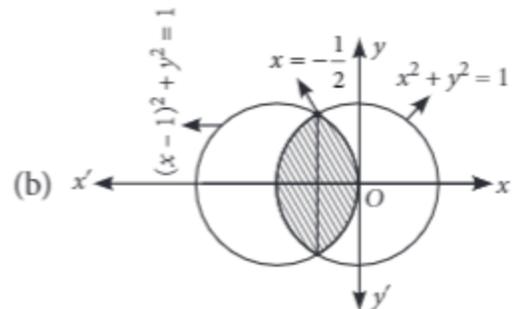
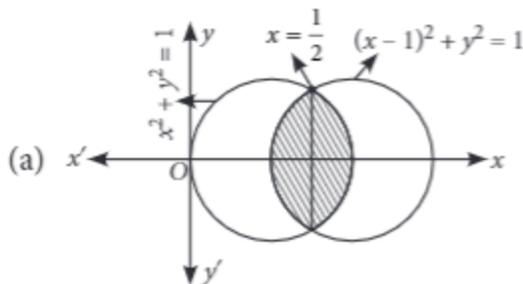
(a)  $x = 1$

(b)  $x = \frac{1}{2}$

(c)  $x = \frac{1}{3}$

(d)  $x = \frac{1}{4}$

(ii) Graph of given two curves can be drawn as



(d) None of these

(iii) Value of  $\int_0^{1/2} \sqrt{1-(x-1)^2} dx$  is

- (a)  $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$       (b)  $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$       (c)  $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$       (d)  $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

(iv) Value of  $\int_{1/2}^1 \sqrt{1-x^2} dx$  is

- (a)  $\frac{\pi}{2} + \frac{\sqrt{3}}{4}$       (b)  $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$       (c)  $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$       (d)  $\frac{\pi}{2} - \frac{\sqrt{3}}{4}$

(v) Area of hidden portion of lower circle is

- (a)  $\left(\frac{2\pi}{3} + \frac{\sqrt{3}}{2}\right)$  sq. units      (b)  $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{8}\right)$  sq. units  
 (c)  $\left(\frac{\pi}{3} + \frac{\sqrt{3}}{8}\right)$  sq. units      (d)  $\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$  sq. units

9

A mirror in the shape of an ellipse represented by  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  was hanging on the wall. Arun and his sister were playing with ball inside the house, even their mother refused to do so. All of sudden, ball hit the mirror and got a scratch in the shape of line represented by  $\frac{x}{3} + \frac{y}{2} = 1$ .

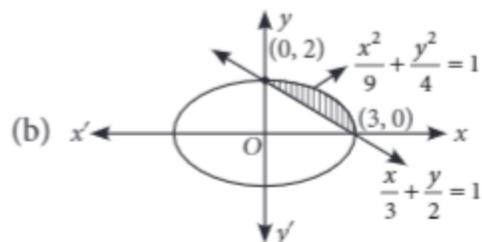
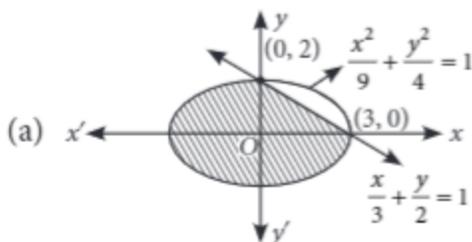


Based on the above information, answer the following questions.

(i) Point(s) of intersection of ellipse and scratch (straight line) is (are)

- (a) (0, 2), (3, 0)      (b) (2, 0), (0, 3)  
 (c) (2, 3), (0, 0)      (d) (0, 3), (3, 0)

(ii) Area of smaller region bounded by the ellipse and line is represented by





## HINTS & EXPLANATIONS

1. (i) (b): We have,  $x^2 = y$  ... (i) and  $x = y$  ... (ii)

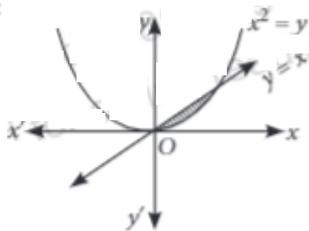
From (i) and (ii),  $x^2 = x \Rightarrow x^2 - x = 0$

$$\Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$$

From (ii),  $y = 0, 1$

Required points of intersection are  $(0, 0)$ ,  $(1, 1)$ .

(ii) (a):



$$(iii) (c): \int_0^1 x \, dx = \left[ \frac{x^2}{2} \right]_0^1 = \frac{1}{2} - 0 = \frac{1}{2}$$

$$(iv) (b): \int_0^1 x^2 \, dx = \left[ \frac{x^3}{3} \right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$$

$$(v) (a): \text{Required area} = \int_0^1 x \, dx - \int_0^1 x^2 \, dx \\ = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units.}$$

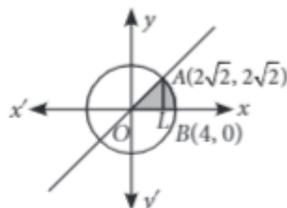
2. (i) (c): We have,  $x^2 + y^2 = 16$  ... (i)

and  $y = x$  ... (ii)

From (i) and (ii),  $2x^2 = 16 \Rightarrow x^2 = 8 \Rightarrow x = 2\sqrt{2}$   
 ( $\because x$  lies in first quadrant)

$\therefore$  Point of intersection of (i) and (ii) in first quadrant is  $(2\sqrt{2}, 2\sqrt{2})$ .

(ii) (b): The shaded region which represent the area bounded by two given curves in first quadrant is shown below.



$$(iii) (d): \int_0^{2\sqrt{2}} x \, dx = \left[ \frac{x^2}{2} \right]_0^{2\sqrt{2}} = \frac{(2\sqrt{2})^2}{2} = \frac{8}{2} = 4$$

$$(iv) (a): \int_{2\sqrt{2}}^4 \sqrt{16-x^2} \, dx = \left[ \frac{x}{2} \sqrt{16-x^2} + \frac{16}{2} \cdot \sin^{-1} \left( \frac{x}{4} \right) \right]_{2\sqrt{2}}^4 \\ = 8 \sin^{-1}(1) - 4 - 8 \sin^{-1} \left( \frac{1}{\sqrt{2}} \right)$$

$$= 8 \left( \frac{\pi}{2} \right) - 4 - 8 \left( \frac{\pi}{4} \right) = 4\pi - 4 - 2\pi = 2\pi - 4 = 2(\pi - 2)$$

(v) (d): Required area = Area (OLA) + Area (BAL)

$$= \int_0^{2\sqrt{2}} x \, dx + \int_{2\sqrt{2}}^4 \sqrt{16-x^2} \, dx$$

$$= 4 + 2(\pi - 2) = 2\pi \text{ sq. units.}$$

3. (i) (b): We have,  $x^2 + y^2 = 4$  ... (i)

and  $x = \sqrt{3}y$  ... (ii)

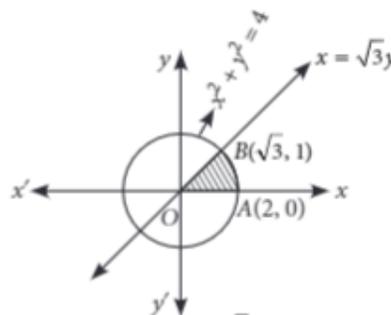
From (i) and (ii), we get

$$3y^2 + y^2 = 4 \Rightarrow 4y^2 = 4 \Rightarrow y^2 = 1 \Rightarrow y = \pm 1$$

From (ii),  $x = \sqrt{3}, -\sqrt{3}$

$\therefore$  Points of intersection of pizza and edge of knife are  $(\sqrt{3}, 1)$ ,  $(-\sqrt{3}, -1)$ .

(ii) (a):



$$(iii) (b): \text{Required area} = \int_0^{\sqrt{3}} \frac{x}{\sqrt{3}} \, dx + \int_{\sqrt{3}}^2 \sqrt{4-x^2} \, dx \\ = \frac{1}{\sqrt{3}} \left[ \frac{x^2}{2} \right]_0^{\sqrt{3}} + \left[ \frac{x}{2} \sqrt{4-x^2} + \frac{4}{2} \sin^{-1} \left( \frac{x}{2} \right) \right]_{\sqrt{3}}^2 \\ = \frac{1}{\sqrt{3}} \left[ \frac{3}{2} - 0 \right] + \left[ 2 \sin^{-1}(1) - \left( \frac{\sqrt{3}}{2} + 2 \sin^{-1} \frac{\sqrt{3}}{2} \right) \right] \\ = \frac{\sqrt{3}}{2} + \frac{2\pi}{2} - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \frac{\pi}{3} \text{ sq. units}$$

(iv) (a): We have,  $x^2 + y^2 = 4$

$$\Rightarrow (x-0)^2 + (y-0)^2 = (2)^2$$

$\therefore$  Radius = 2

Area of  $\frac{1}{4}$  th slice of pizza =  $\frac{1}{4} \pi (2)^2 = \pi$  sq. units

(v) (d): Area of whole pizza =  $\pi (2)^2 = 4\pi$  sq. units

4. (i) (d): Line  $x + y = 3$  cuts the  $x$ -axis and  $y$ -axis at  $(3, 0)$  and  $(0, 3)$  respectively.

[Since, at  $x$ -axis,  $y = 0$  and at  $y$ -axis,  $x = 0$ ]

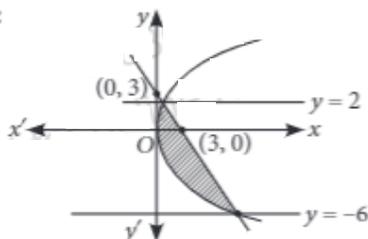
(ii) (c) : We have,  $y^2 = 4x$  ... (i)  
and  $x + y = 3$  ... (ii)

$$\begin{aligned} \text{From (i) and (ii), we have } y^2 &= 4(3 - y) \\ \Rightarrow y^2 + 4y - 12 &= 0 \Rightarrow y^2 + 6y - 2y - 12 = 0 \\ \Rightarrow y(y + 6) - 2(y + 6) &= 0 \\ \Rightarrow (y + 6)(y - 2) &= 0 \Rightarrow y = 2, y = -6 \end{aligned}$$

From (ii),  $x = 3 - 2 = 1$  or  $x = 3 + 6 = 9$

$\therefore$  Required points of intersection are  $(1, 2), (9, -6)$

(iii) (b):



$$\begin{aligned} \text{(iv) (d): } \int_{-6}^2 (3 - y) dy &= \left[ 3y - \frac{y^2}{2} \right]_{-6}^2 \\ &= \left[ 6 - \frac{4}{2} - \left[ 3(-6) - \frac{(-6)^2}{2} \right] \right] = 4 + 36 = 40 \end{aligned}$$

$$\begin{aligned} \text{(v) (c): Required area} &= \int_{-6}^2 (3 - y) dy - \int_{-6}^2 \frac{y^2}{4} dy \\ &= 40 - \frac{1}{4} \left[ \frac{y^3}{3} \right]_{-6}^2 = 40 - \frac{1}{4} \left[ \frac{8}{3} - \frac{(-6)^3}{3} \right] \\ &= 40 - \frac{2}{3} - \frac{216}{12} = \frac{480 - 8 - 216}{12} = \frac{256}{12} = \frac{64}{3} \text{ sq. units} \end{aligned}$$

5. (i) (a) : Here, teacher explained about cosine curve.

$$\begin{aligned} \text{(ii) (c): Required area} &= \int_0^{\pi/2} \cos x dx \\ &= [\sin x]_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1 - 0 = 1 \text{ sq. unit} \end{aligned}$$

$$\begin{aligned} \text{(iii) (b): Required area} &= \left| \int_{\pi/2}^{3\pi/2} \cos x dx \right| = \left| [\sin x]_{\pi/2}^{3\pi/2} \right| \\ &= \left| \sin \frac{3\pi}{2} - \sin \frac{\pi}{2} \right| = |-1 - 1| = |-2| \\ &= 2 \text{ sq. units} \quad [\text{Since, area can't be negative}] \end{aligned}$$

$$\begin{aligned} \text{(iv) (a): Required area} &= \int_{3\pi/2}^{2\pi} \cos x dx = [\sin x]_{3\pi/2}^{2\pi} \\ &= \sin 2\pi - \sin \frac{3\pi}{2} = 0 - (-1) = 1 \text{ sq. unit} \end{aligned}$$

$$\begin{aligned} \text{(v) (d): Required area} &= \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{3\pi/2} \cos x dx \right| + \int_{3\pi/2}^{2\pi} \cos x dx \\ &= 1 + 2 + 1 = 4 \text{ sq. units} \end{aligned}$$

6. (i) (c) : For point of intersection, we have  $\sin x = \cos x$

$$\Rightarrow \frac{\sin x}{\cos x} = 1 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}$$

$$\begin{aligned} \text{(ii) (a): } \int_0^{\pi/4} \sin x dx &= [-\cos x]_0^{\pi/4} = -\cos \frac{\pi}{4} + \cos 0 \\ &= 1 - \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} \text{(iii) (b): } \int_{\pi/4}^{\pi/2} \cos x dx &= [\sin x]_{\pi/4}^{\pi/2} = \sin \frac{\pi}{2} - \sin \frac{\pi}{4} \\ &= 1 - \frac{1}{\sqrt{2}} \end{aligned}$$

$$\text{(iv) (c): } \int_0^{\pi} \sin x dx = [-\cos x]_0^{\pi} = [-\cos \pi + \cos 0] = 2$$

$$\begin{aligned} \text{(v) (b): } \int_0^{\pi/2} \sin x dx &= [-\cos x]_0^{\pi/2} = \left[ -\cos \frac{\pi}{2} + \cos 0 \right] \\ &= 0 + 1 = 1 \end{aligned}$$

7. (i) (a) : Equation of line AB is

$$y - 0 = \frac{3 - 0}{1 + 1}(x + 1) \Rightarrow y = \frac{3}{2}(x + 1)$$

(ii) (c) : Equation of line BC is  $y - 3 = \frac{2 - 3}{3 - 1}(x - 1)$

$$\Rightarrow y = -\frac{1}{2}x + \frac{1}{2} + 3 \Rightarrow y = -\frac{1}{2}x + \frac{7}{2}$$

(iii) (d) : Area of region ABCD

= Area of  $\triangle ABE$  + Area of region BCDE

$$\begin{aligned} &= \int_{-1}^1 \frac{3}{2}(x + 1) dx + \int_1^3 \left( -\frac{1}{2}x + \frac{7}{2} \right) dx \\ &= \frac{3}{2} \left[ \frac{x^2}{2} + x \right]_{-1}^1 + \left[ -\frac{x^2}{4} + \frac{7}{2}x \right]_1^3 \\ &= \frac{3}{2} \left[ \frac{1}{2} + 1 - \frac{1}{2} + 1 \right] + \left[ -\frac{9}{4} + \frac{21}{2} + \frac{1}{4} - \frac{7}{2} \right] \\ &= 3 + 5 = 8 \text{ sq. units} \end{aligned}$$

(iv) (a) : Equation of line AC is  $y - 0 = \frac{2 - 0}{3 + 1}(x + 1)$

$$\Rightarrow y = \frac{1}{2}(x + 1)$$

$$\therefore \text{Area of } \triangle ADC = \int_{-1}^3 \frac{1}{2}(x + 1) dx = \left[ \frac{x^2}{4} + \frac{1}{2}x \right]_{-1}^3$$

$$= \frac{9}{4} + \frac{3}{2} - \frac{1}{4} - \frac{1}{2} = 4 \text{ sq. units}$$

(v) (b) : Area of  $\triangle ABC$  = Area of region ABCD - Area of  $\triangle ACD$  =  $8 - 4 = 4$  sq. units

8. (i) (b): We have,  $(x-1)^2 + y^2 = 1$

$\Rightarrow y = \sqrt{1-(x-1)^2}$

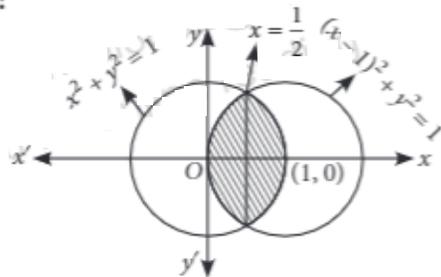
Also,  $x^2 + y^2 = 1 \Rightarrow y = \sqrt{1-x^2}$

From (i) and (ii), we get

$$\sqrt{1-(x-1)^2} = \sqrt{1-x^2}$$

$\Rightarrow (x-1)^2 = x^2 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

(ii) (c):



(iii) (a):  $\int_0^{1/2} \sqrt{1-(x-1)^2} dx$

$$= \left[ \frac{x-1}{2} \sqrt{1-(x-1)^2} + \frac{1}{2} \sin^{-1} \left( \frac{x-1}{1} \right) \right]_0^{1/2}$$

$$= \frac{1}{2} \left( \frac{1}{2} - 1 \right) \sqrt{1 - \frac{1}{4}} + \frac{1}{2} \sin^{-1} \left( -\frac{1}{2} \right) - \left( -\frac{1}{2} \right) (0) - \frac{1}{2} \sin^{-1}(-1)$$

$$= \left[ \frac{-1}{4} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\pi}{6} + 0 + \frac{1}{2} \cdot \frac{\pi}{2} \right] = \frac{-\sqrt{3}}{8} - \frac{\pi}{12} + \frac{\pi}{4}$$

$$= \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

(iv) (c):  $\int_{1/2}^1 \sqrt{1-x^2} dx = \left[ \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x \right]_{1/2}^1$

$$= 0 + \frac{1}{2} \sin^{-1}(1) - \frac{1}{4} \sqrt{1 - \frac{1}{4}} - \frac{1}{2} \sin^{-1} \left( \frac{1}{2} \right)$$

$$= \frac{\pi}{4} - \frac{\sqrt{3}}{8} - \frac{\pi}{12} = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

(v) (d): Required area

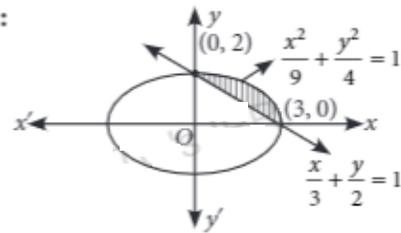
$$= 2 \left[ \int_0^{1/2} \sqrt{1-(x-1)^2} dx + \int_{1/2}^1 \sqrt{1-x^2} dx \right]$$

$$= 2 \left[ \frac{\pi}{6} - \frac{\sqrt{3}}{8} + \frac{\pi}{6} - \frac{\sqrt{3}}{8} \right]$$

$$= 2 \left[ \frac{\pi}{3} - \frac{\sqrt{3}}{4} \right] = \left( \frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) \text{sq. units}$$

9. (i) (a): Points (0, 2) and (3, 0) pass through both line and ellipse.

(ii) (b):



(iii) (c):  $\frac{2}{3} \int_0^3 \sqrt{9-x^2} dx = \frac{2}{3} \int_0^3 \sqrt{(3)^2 - x^2} dx$

$$= \frac{2}{3} \left[ \frac{1}{2} x \sqrt{9-x^2} + \frac{9}{2} \sin^{-1} \left( \frac{x}{3} \right) \right]_0^3$$

$$= \frac{2}{3} \left[ \frac{3}{2} \sqrt{0} + \frac{9}{2} \sin^{-1}(1) - \frac{1}{2} (0) - \frac{9}{2} \sin^{-1}(0) \right]$$

$$= \frac{2}{3} \left[ \frac{9}{2} \cdot \frac{\pi}{2} \right] = \frac{3\pi}{2}$$

(iv) (d):  $2 \int_0^3 \left( 1 - \frac{x}{3} \right) dx = 2 \left[ x - \frac{x^2}{6} \right]_0^3$

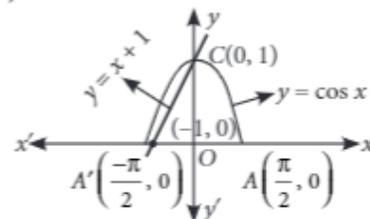
$$= 2 \left( 3 - \frac{9}{6} - 0 - 0 \right) = 2 \times \frac{3}{2} = 3$$

(v) (d): Area of smaller region bounded by the mirror and scratch

$$= \frac{2}{3} \int_0^3 \sqrt{9-x^2} dx - 2 \int_0^3 \left( 1 - \frac{x}{3} \right) dx$$

$$= \frac{3\pi}{2} - 3 = 3 \left( \frac{\pi}{2} - 1 \right) \text{sq. units}$$

10. (i) (b): Curves  $y = \cos x$  and  $y = x + 1$  meet at point C(0, 1).



(ii) (c): Curve  $y = \cos x$  meet the x-axis at  $A' \left( -\frac{\pi}{2}, 0 \right)$  and  $A \left( \frac{\pi}{2}, 0 \right)$ .

(iii) (a):  $\int_{-1}^0 (x+1) dx = \left[ \frac{x^2}{2} + x \right]_{-1}^0 = 0 - \left( \frac{1}{2} - 1 \right) = \frac{1}{2}$

(iv) (d):  $\int_0^{\pi/2} \cos x dx = [\sin x]_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0 = 1$

(v) (b): Required area =  $\int_{-1}^0 (x+1) dx + \int_0^{\pi/2} \cos x dx$

$$= \frac{1}{2} + 1 = \frac{3}{2} \text{sq. units}$$