

CASE STUDY / PASSAGE BASED QUESTIONS

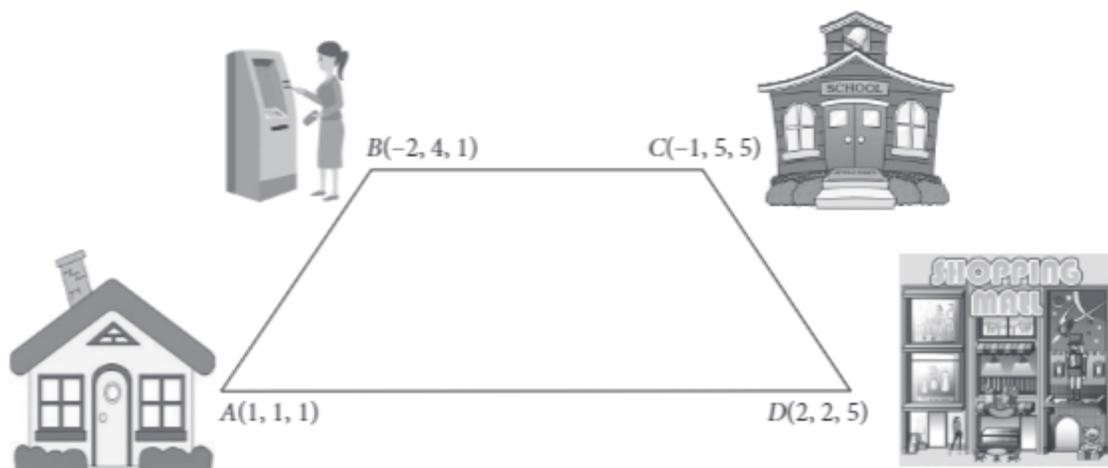
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### Syllabus

Vectors and scalars, magnitude and direction of a vector.

Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and application of scalar (dot) product of vectors, vector (cross) product of vectors.

Ritika starts walking from his house to shopping mall. Instead of going to the mall directly, she first goes to a ATM, from there to her daughter's school and then reaches the mall. In the diagram,  $A$ ,  $B$ ,  $C$  and  $D$  represent the coordinates of House, ATM, School and Mall respectively.

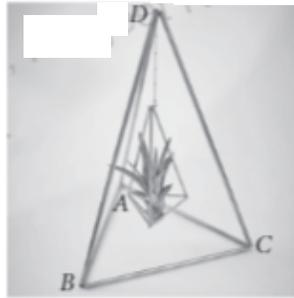


Based on the above information, answer the following questions.

- (i) Distance between House ( $A$ ) and ATM ( $B$ ) is
  - (a) 3 units
  - (b)  $3\sqrt{2}$  units
  - (c)  $\sqrt{2}$  units
  - (d)  $4\sqrt{2}$  units
- (ii) Distance between ATM ( $B$ ) and School ( $C$ ) is
  - (a)  $\sqrt{2}$  units
  - (b)  $2\sqrt{2}$  units
  - (c)  $3\sqrt{2}$  units
  - (d)  $4\sqrt{2}$  units
- (iii) Distance between School ( $C$ ) and Shopping mall ( $D$ ) is
  - (a)  $3\sqrt{2}$  units
  - (b)  $5\sqrt{2}$  units
  - (c)  $7\sqrt{2}$  units
  - (d)  $10\sqrt{2}$  units
- (iv) What is the total distance travelled by Ritika?
  - (a)  $4\sqrt{2}$  units
  - (b)  $6\sqrt{2}$  units
  - (c)  $8\sqrt{2}$  units
  - (d)  $9\sqrt{2}$  units
- (v) What is the extra distance travelled by Ritika in reaching the shopping mall?
  - (a)  $3\sqrt{2}$  units
  - (b)  $5\sqrt{2}$  units
  - (c)  $6\sqrt{2}$  units
  - (d)  $7\sqrt{2}$  units

Ginni purchased an air plant holder which is in the shape of a tetrahedron.

Let  $A, B, C$  and  $D$  are the coordinates of the air plant holder where  $A \equiv (1, 1, 1)$ ,  $B \equiv (2, 1, 3)$ ,  $C \equiv (3, 2, 2)$  and  $D \equiv (3, 3, 4)$ .



Based on the above information, answer the following questions.

(i) Find the position vector of  $\overline{AB}$ .

- (a)  $-\hat{i} - 2\hat{k}$       (b)  $2\hat{i} + \hat{k}$       (c)  $\hat{i} + 2\hat{k}$       (d)  $-2\hat{i} - \hat{k}$

(ii) Find the position vector of  $\overline{AC}$ .

- (a)  $2\hat{i} - \hat{j} - \hat{k}$       (b)  $2\hat{i} + \hat{j} + \hat{k}$       (c)  $-2\hat{i} - \hat{j} + \hat{k}$       (d)  $\hat{i} + 2\hat{j} + \hat{k}$

(iii) Find the position vector of  $\overline{AD}$ .

- (a)  $2\hat{i} - 2\hat{j} - 3\hat{k}$       (b)  $\hat{i} + \hat{j} - 3\hat{k}$       (c)  $3\hat{i} + 2\hat{j} + 2\hat{k}$       (d)  $2\hat{i} + 2\hat{j} + 3\hat{k}$

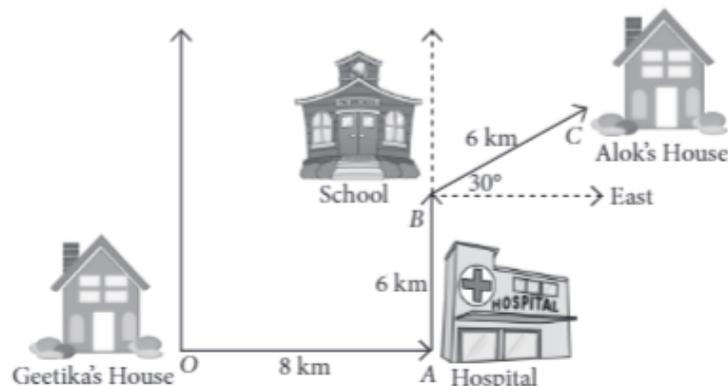
(iv) Area of  $\Delta ABC =$

- (a)  $\frac{\sqrt{11}}{2}$  sq. units      (b)  $\frac{\sqrt{14}}{2}$  sq. units      (c)  $\frac{\sqrt{13}}{2}$       (d)  $\frac{\sqrt{17}}{2}$  sq. units

(v) Find the unit vector along  $\overline{AD}$ .

- (a)  $\frac{1}{\sqrt{17}}(2\hat{i} + 2\hat{j} + 3\hat{k})$       (b)  $\frac{1}{\sqrt{17}}(3\hat{i} + 3\hat{j} + 2\hat{k})$       (c)  $\frac{1}{\sqrt{11}}(2\hat{i} + 2\hat{j} + 3\hat{k})$       (d)  $(2\hat{i} + 2\hat{j} + 3\hat{k})$

Geetika's house is situated at Shalimar Bagh at point  $O$ , for going to Alok's house she first travels 8 km by bus in the East. Here at point  $A$ , a hospital is situated. From Hospital, Geetika takes an auto and goes 6 km in the North, here at point  $B$  school is situated. From school, she travels by bus to reach Alok's house which is at  $30^\circ$  East, 6 km from point  $B$ .



Based on the above information, answer the following questions.

- (i) What is the vector distance between Geetika's house and school?  
 (a)  $8\hat{i} - 6\hat{j}$  (b)  $8\hat{i} + 6\hat{j}$  (c)  $8\hat{i}$  (d)  $6\hat{j}$
- (ii) How much distance Geetika travels to reach school?  
 (a) 14 km (b) 15 km (c) 16 km (d) 17 km
- (iii) What is the vector distance from school to Alok's house?  
 (a)  $\sqrt{3}\hat{i} + \hat{j}$  (b)  $3\sqrt{3}\hat{i} + 3\hat{j}$  (c)  $6\hat{i}$  (d)  $6\hat{j}$
- (iv) What is the vector distance from Geetika's house to Alok's house?  
 (a)  $(8 + 3\sqrt{3})\hat{i} + 9\hat{j}$  (b)  $4\hat{i} + 6\hat{j}$  (c)  $15\hat{i}$  (d)  $16\hat{j}$
- (v) What is the total distance travelled by Geetika from her house to Alok's house?  
 (a) 19 km (b) 20 km (c) 21 km (d) 22 km

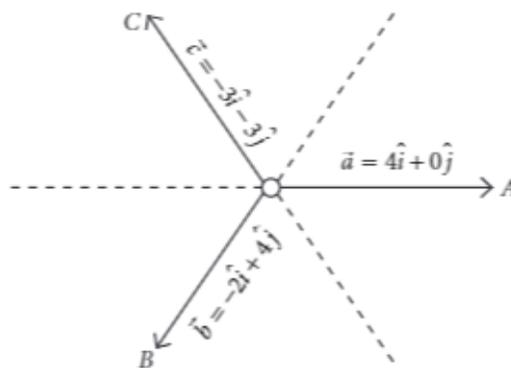
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Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below).

Team A pulls with force  $F_1 = 4\hat{i} + 0\hat{j}$  KN

Team B  $\rightarrow F_2 = -2\hat{i} + 4\hat{j}$  KN

Team C  $\rightarrow F_3 = -3\hat{i} - 3\hat{j}$  KN

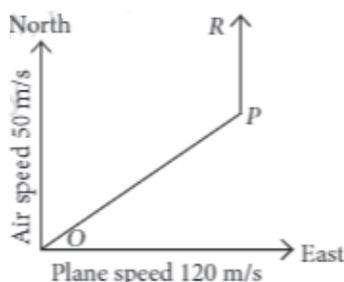


Based on the above information, answer the following questions.

- (i) Which team will win the game?  
 (a) Team B (b) Team A (c) Team C (d) No one
- (ii) What is the magnitude of the teams combined force?  
 (a) 7 KN (b) 1.4 KN (c) 1.5 KN (d) 2 KN
- (iii) In what direction is the ring getting pulled?  
 (a) 2.0 radian (b) 2.5 radian (c) 2.4 radian (d) 3 radian
- (iv) What is the magnitude of the force of Team B?  
 (a)  $2\sqrt{5}$  KN (b) 6 KN (c) 2 KN (d)  $\sqrt{6}$  KN
- (v) How many KN force is applied by Team A?  
 (a) 5 KN (b) 4 KN (c) 2 KN (d) 16 KN

A plane started from airport situated at  $O$  with a velocity of 120 m/s towards east. Air is blowing at a velocity of 50 m/s towards the north as shown in the figure.

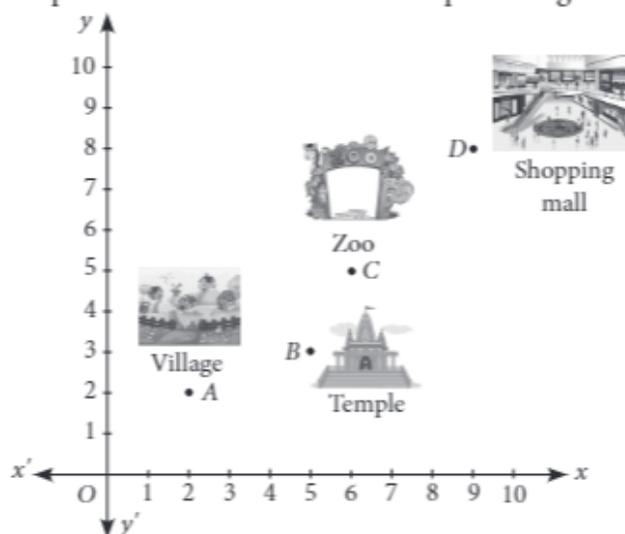
The plane travelled 1 hr in  $OP$  direction with the resultant velocity. From  $P$  to  $R$  the plane travelled 1 hr keeping velocity of 120 m/s and finally landed at  $R$ .



Based on the above information, answer the following questions.

- (i) What is the resultant velocity from  $O$  to  $P$ ?
- (a) 100 m/s                      (b) 130 m/s                      (c) 126 m/s                      (d) 180 m/s
- (ii) What is the direction of travel of plane from  $O$  to  $P$  with East?
- (a)  $\tan^{-1}\left(\frac{5}{12}\right)$                       (b)  $\tan^{-1}\left(\frac{12}{3}\right)$                       (c) 50                      (d) 80
- (iii) What is the displacement from  $O$  to  $P$ ?
- (a) 600 km                      (b) 468 km                      (c) 532 km                      (d) 500 km
- (iv) What is the resultant velocity from  $P$  to  $R$ ?
- (a) 120 m/s                      (b) 70 m/s                      (c) 170 m/s                      (d) 200 m/s
- (v) What is the displacement from  $P$  to  $R$ ?
- (a) 450 km                      (b) 532 km                      (c) 610 km                      (d) 612 km

Ishaan left from his village on weekend. First, he travelled up to temple. After this, he left for the zoo. After this he left for shopping in a mall. The positions of Ishaan at different places is given in the following graph.



Based on the above information, answer the following questions.

(i) Position vector of  $B$  is

- (a)  $3\hat{i} + 5\hat{j}$                       (b)  $5\hat{i} + 3\hat{j}$                       (c)  $-5\hat{i} - 3\hat{j}$                       (d)  $-5\hat{i} + 3\hat{j}$

(ii) Position vector of  $D$  is

- (a)  $5\hat{i} + 3\hat{j}$                       (b)  $3\hat{i} + 5\hat{j}$                       (c)  $8\hat{i} + 9\hat{j}$                       (d)  $9\hat{i} + 8\hat{j}$

(iii) Find the vector  $\overline{BC}$  in terms of  $\hat{i}, \hat{j}$ .

- (a)  $\hat{i} - 2\hat{j}$                       (b)  $\hat{i} + 2\hat{j}$                       (c)  $2\hat{i} + \hat{j}$                       (d)  $2\hat{i} - \hat{j}$

(iv) Length of vector  $\overline{AD}$  is

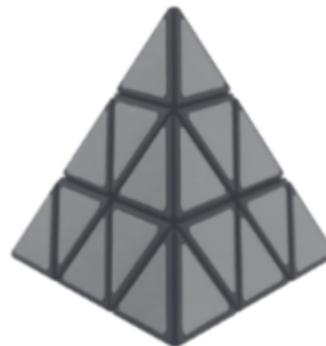
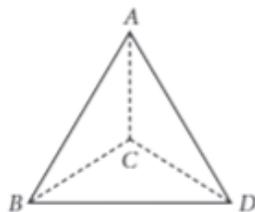
- (a)  $\sqrt{67}$  units                      (b)  $\sqrt{85}$  units                      (c) 90 units                      (d) 100 units

(v) If  $\vec{M} = 4\hat{j} + 3\hat{k}$ , then its unit vector is

- (a)  $\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$                       (b)  $\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k}$                       (c)  $-\frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$                       (d)  $-\frac{4}{5}\hat{j} - \frac{3}{5}\hat{k}$

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A building is to be constructed in the form of a triangular pyramid,  $ABCD$  as shown in the figure.



Let its angular points are  $A(0, 1, 2)$ ,  $B(3, 0, 1)$ ,  $C(4, 3, 6)$  and  $D(2, 3, 2)$  and  $G$  be the point of intersection of the medians of  $\triangle BCD$ .

Based on the above information, answer the following questions.

(i) The coordinates of point  $G$  are

- (a)  $(2, 3, 3)$                       (b)  $(3, 3, 2)$                       (c)  $(3, 2, 3)$                       (d)  $(0, 2, 3)$

(ii) The length of vector  $\overline{AG}$  is

- (a)  $\sqrt{17}$  units                      (b)  $\sqrt{11}$  units                      (c)  $\sqrt{13}$  units                      (d)  $\sqrt{19}$  units

(iii) Area of  $\triangle ABC$  (in sq. units) is

- (a)  $\sqrt{10}$                       (b)  $2\sqrt{10}$                       (c)  $3\sqrt{10}$                       (d)  $5\sqrt{10}$

(iv) The sum of lengths of  $\overline{AB}$  and  $\overline{AC}$  is

- (a) 5 units                      (b) 9.32 units                      (c) 10 units                      (d) 11 units

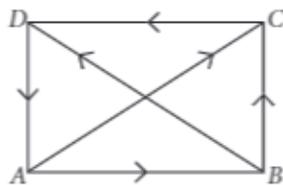
(v) The length of the perpendicular from the vertex  $D$  on the opposite face is

- (a)  $\frac{6}{\sqrt{10}}$  units                      (b)  $\frac{2}{\sqrt{10}}$  units                      (c)  $\frac{3}{\sqrt{10}}$  units                      (d)  $8\sqrt{10}$  units

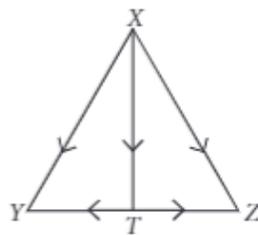
If two vectors are represented by the two sides of a triangle taken in order, then their sum is represented by the third side of the triangle taken in opposite order and this is known as triangle law of vector addition.

Based on the above information, answer the following questions.

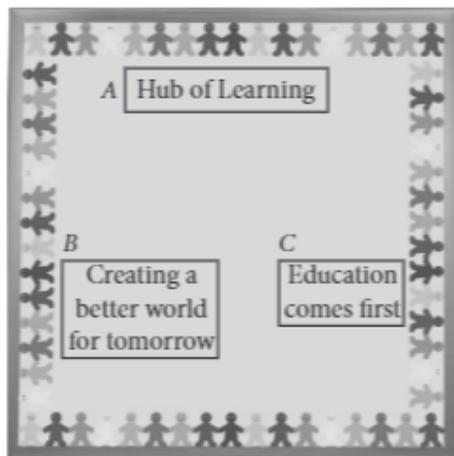
- (i) If  $\vec{p}, \vec{q}, \vec{r}$  are the vectors represented by the sides of a triangle taken in order, then  $\vec{q} + \vec{r} =$   
 (a)  $\vec{p}$  (b)  $2\vec{p}$  (c)  $-\vec{p}$  (d) None of these
- (ii) If  $ABCD$  is a parallelogram and  $AC$  and  $BD$  are its diagonals, then  $\vec{AC} + \vec{BD} =$   
 (a)  $2\vec{DA}$  (b)  $2\vec{AB}$  (c)  $2\vec{BC}$  (d)  $2\vec{BD}$
- (iii) If  $ABCD$  is a parallelogram, where  $\vec{AB} = 2\vec{a}$  and  $\vec{BC} = 2\vec{b}$ , then  $\vec{AC} - \vec{BD} =$   
 (a)  $3\vec{a}$  (b)  $4\vec{a}$  (c)  $2\vec{b}$  (d)  $4\vec{b}$
- (iv) If  $ABCD$  is a quadrilateral whose diagonals are  $\vec{AC}$  and  $\vec{BD}$ , then  $\vec{BA} + \vec{CD} =$



- (a)  $\vec{AC} + \vec{DB}$  (b)  $\vec{AC} + \vec{BD}$  (c)  $\vec{BC} + \vec{AD}$  (d)  $\vec{BD} + \vec{CA}$
- (v) If  $T$  is the mid point of side  $YZ$  of  $\triangle XYZ$ , then  $\vec{XY} + \vec{XZ} =$   
 (a)  $2\vec{YT}$   
 (b)  $2\vec{XT}$   
 (c)  $2\vec{TZ}$   
 (d) None of these



Three slogans on chart papers are to be placed on a school bulletin board at the points  $A$ ,  $B$  and  $C$  displaying  $A$  (Hub of Learning),  $B$  (Creating a better world for tomorrow) and  $C$  (Education comes first). The coordinates of these points are  $(1, 4, 2)$ ,  $(3, -3, -2)$  and  $(-2, 2, 6)$  respectively.

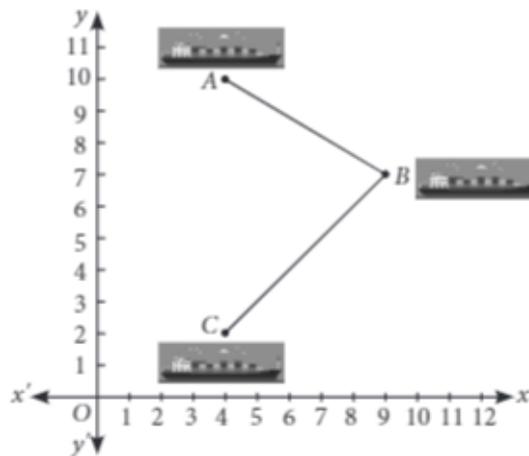


Based on the above information, answer the following questions.

- (i) Let  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  be the position vectors of points A, B and C respectively, then  $\vec{a} + \vec{b} + \vec{c}$  is equal to  
 (a)  $2\hat{i} + 3\hat{j} + 6\hat{k}$  (b)  $2\hat{i} - 3\hat{j} - 6\hat{k}$  (c)  $2\hat{i} + 8\hat{j} + 3\hat{k}$  (d)  $2(7\hat{i} + 8\hat{j} + 3\hat{k})$
- (ii) Which of the following is not true?  
 (a)  $\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}$  (b)  $\vec{AB} + \vec{BC} - \vec{AC} = \vec{0}$  (c)  $\vec{AB} + \vec{BC} - \vec{CA} = \vec{0}$  (d)  $\vec{AB} - \vec{CB} + \vec{CA} = \vec{0}$
- (iii) Area of  $\Delta ABC$  is  
 (a) 19 sq. units (b)  $\sqrt{1937}$  sq. units (c)  $\frac{1}{2}\sqrt{1937}$  sq. units (d)  $\sqrt{1837}$  sq. units
- (iv) Suppose, if the given slogans are to be placed on a straight line, then the value of  $|\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$  will be equal to  
 (a) -1 (b) -2 (c) 2 (d) 0
- (v) If  $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ , then unit vector in the direction of vector  $\vec{a}$  is  
 (a)  $\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k}$  (b)  $\frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$  (c)  $\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$  (d) None of these

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A barge is pulled into harbour by two tug boats as shown in the figure.



Based on the above information, answer the following questions.

- (i) Position vector of A is  
 (a)  $4\hat{i} + 2\hat{j}$  (b)  $4\hat{i} + 10\hat{j}$  (c)  $4\hat{i} - 10\hat{j}$  (d)  $4\hat{i} - 2\hat{j}$
- (ii) Position vector of B is  
 (a)  $4\hat{i} + 4\hat{j}$  (b)  $6\hat{i} + 6\hat{j}$  (c)  $9\hat{i} + 7\hat{j}$  (d)  $3\hat{i} + 3\hat{j}$
- (iii) Find the vector  $\vec{AC}$  in terms of  $\hat{i}$ ,  $\hat{j}$ .  
 (a)  $8\hat{j}$  (b)  $-8\hat{j}$  (c)  $8\hat{i}$  (d) None of these
- (iv) If  $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ , then its unit vector is  
 (a)  $\frac{\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{3\hat{k}}{\sqrt{14}}$  (b)  $\frac{3\hat{i}}{\sqrt{14}} + \frac{2\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$  (c)  $\frac{2\hat{i}}{\sqrt{14}} + \frac{3\hat{j}}{\sqrt{14}} + \frac{\hat{k}}{\sqrt{14}}$  (d) None of these
- (v) If  $\vec{A} = 4\hat{i} + 3\hat{j}$  and  $\vec{B} = 3\hat{i} + 4\hat{j}$ , then  $|\vec{A}| + |\vec{B}| =$  \_\_\_\_\_.  
 (a) 12 (b) 13 (c) 14 (d) 10

## HINTS & EXPLANATIONS

1. (i) (b):  $\overline{AB} = (-2\hat{i} + 4\hat{j} + \hat{k}) - (\hat{i} + \hat{j} + \hat{k}) = -3\hat{i} + 3\hat{j}$

$\therefore |\overline{AB}| = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$

Distance between House (A) and ATM (B) is  $3\sqrt{2}$  units.

(ii) (c):  $\overline{BC} = (-\hat{i} + 5\hat{j} + 5\hat{k}) - (-2\hat{i} + 4\hat{j} + \hat{k}) = \hat{i} + \hat{j} + 4\hat{k}$

$\therefore |\overline{BC}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{1+1+16}$   
 $= \sqrt{18} = 3\sqrt{2}$

Distance between ATM (B) and School (C) is  $3\sqrt{2}$  units.

(iii) (a):  $\overline{CD} = (2\hat{i} + 2\hat{j} + 5\hat{k}) - (-\hat{i} + 5\hat{j} + 5\hat{k}) = 3\hat{i} - 3\hat{j}$

$\therefore |\overline{CD}| = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = 3\sqrt{2}$

Distance between School (C) and Shopping mall (D) is  $3\sqrt{2}$  units.

(iv) (d): Total distance travelled by Ritika  
 $= |\overline{AB}| + |\overline{BC}| + |\overline{CD}| = (3\sqrt{2} + 3\sqrt{2} + 3\sqrt{2})$  units  
 $= 9\sqrt{2}$  units

(v) (c): Distance between house and shopping mall is  $|\overline{AD}|$ .

Now,  $\overline{AD} = \hat{i} + \hat{j} + 4\hat{k}$

$\therefore |\overline{AD}| = \sqrt{1^2 + 1^2 + 4^2} = \sqrt{1+1+16} = \sqrt{18} = 3\sqrt{2}$

Thus, extra distance travelled by Ritika in reaching shopping mall =  $(9\sqrt{2} - 3\sqrt{2})$  units =  $6\sqrt{2}$  units

2. (i) (c): Position vector of  $\overline{AB}$   
 $= (2-1)\hat{i} + (1-1)\hat{j} + (3-1)\hat{k} = \hat{i} + 2\hat{k}$

(ii) (b): Position vector of  $\overline{AC}$   
 $= (3-1)\hat{i} + (2-1)\hat{j} + (2-1)\hat{k} = 2\hat{i} + \hat{j} + \hat{k}$

(iii) (d): Position vector of  $\overline{AD}$   
 $= (3-1)\hat{i} + (3-1)\hat{j} + (4-1)\hat{k} = 2\hat{i} + 2\hat{j} + 3\hat{k}$

(iv) (b): Area of  $\Delta ABC = \frac{1}{2} |\overline{AB} \times \overline{AC}|$

$$\overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 2 & 1 & 1 \end{vmatrix} = \hat{i}(0-2) - \hat{j}(1-4) + \hat{k}(1-0)$$

$$= -2\hat{i} + 3\hat{j} + \hat{k}$$

$\Rightarrow |\overline{AB} \times \overline{AC}| = \sqrt{(-2)^2 + 3^2 + 1^2}$   
 $= \sqrt{4+9+1} = \sqrt{14}$

$\therefore$  Area of  $\Delta ABC = \frac{1}{2} \sqrt{14}$  sq. units

(v) (a): Unit vector along  $\overline{AD} = \frac{\overline{AD}}{|\overline{AD}|}$   
 $= \frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{2^2 + 2^2 + 3^2}} = \frac{2\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{4+4+9}} = \frac{1}{\sqrt{17}} (2\hat{i} + 2\hat{j} + 3\hat{k})$

3. (i) (b): We have,  $\overline{OA} = 8\hat{i}$  and  $\overline{AB} = 6\hat{j}$

$\therefore \overline{OB} = \overline{OA} + \overline{AB} = 8\hat{i} + 6\hat{j}$

(ii) (a): To reach school Geetika travels  
 $= (8 + 6)$  km = 14 km

(iii) (b): Vector distance from school to Alok's house  
 $= 6 \cos 30^\circ \hat{i} + 6 \sin 30^\circ \hat{j}$   
 $= 6 \times \frac{\sqrt{3}}{2} \hat{i} + 6 \times \frac{1}{2} \hat{j} = 3\sqrt{3} \hat{i} + 3\hat{j}$

(iv) (a): Vector distance from Geetika's house to Alok's house =  $8\hat{i} + 6\hat{j} + 3\sqrt{3} \hat{i} + 3\hat{j} = (8 + 3\sqrt{3})\hat{i} + 9\hat{j}$

(v) (b): Total distance travelled by Geetika from her house to Alok's house =  $(8 + 6 + 6)$  km = 20 km

4. Here,  $|\vec{F}_1| = \sqrt{(4)^2 + 0^2} = 4$  KN

$|\vec{F}_2| = \sqrt{(-2)^2 + 4^2} = \sqrt{20}$  KN

$|\vec{F}_3| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18}$  KN

(i) (a): Since,  $\sqrt{20}$  is larger. So, team B will win the game.

(ii) (b): Let  $\vec{F}$  be the combined force.

$\therefore \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 4\hat{i} + 0\hat{j} - 3\hat{i} - 3\hat{j} - 2\hat{i} + 4\hat{j}$   
 $= -\hat{i} + \hat{j}$

$\therefore |\vec{F}| = \sqrt{(-1)^2 + 1^2} = \sqrt{2} = 1.4$  KN

(iii) (c): We have,  $\vec{F} = -\hat{i} + \hat{j}$

$\therefore \theta = \tan^{-1} \left( \frac{F_y}{F_x} \right) = \tan^{-1} \left( \frac{1}{-1} \right) = \frac{3\pi}{4}$  radian

$= 0.75 \times 3.14$  radian = 2.3555 radian  $\approx 2.4$  radian

(iv) (a): Magnitude of force of Team B =  $\sqrt{20}$  KN  
 $= 2\sqrt{5}$  KN

(v) (b): 4 KN force is applied by team A.

5. (i) (b): Resultant velocity from  $O$  to  $P$

$$= \sqrt{120^2 + 50^2}$$

$$= \sqrt{14400 + 2500} = \sqrt{16900} = 130 \text{ m/s}$$

(ii) (a): Direction of travel of plane from  $O$  to  $P$  with east is  $\tan^{-1}\left(\frac{5}{12}\right)$ .

(iii) (b): Resultant velocity from  $O$  to  $A = 130 \text{ m/s}$

$$= \left(\frac{130 \times 3600}{1000}\right) \text{ km/h}$$

Time = 1 hr

$$\therefore \text{Displacement} = \frac{130 \times 3600}{1000} = 468 \text{ km}$$

(iv) (c): Resultant velocity from  $P$  to  $R$   
 $= (120 + 50) = 170 \text{ m/s}$

(v) (d): Displacement from  $P$  to  $R = \left(\frac{170 \times 3600}{1000}\right)$   
 $= 612 \text{ km}$

6. (i) (b): Here  $(5, 3)$  are the coordinates of  $B$ .

$$\therefore \text{P.V. of } B = 5\hat{i} + 3\hat{j}$$

(ii) (d): Here  $(9, 8)$  are the coordinates of  $D$ .

$$\therefore \text{P.V. of } D = 9\hat{i} + 8\hat{j}$$

(iii) (b): P.V. of  $B = 5\hat{i} + 3\hat{j}$  and P.V. of  $C = 6\hat{i} + 5\hat{j}$

$$\therefore \overline{BC} = (6 - 5)\hat{i} + (5 - 3)\hat{j} = \hat{i} + 2\hat{j}$$

(iv) (b): Since P.V. of  $A = 2\hat{i} + 2\hat{j}$ , P.V. of  $D = 9\hat{i} + 8\hat{j}$

$$\therefore \overline{AD} = (9 - 2)\hat{i} + (8 - 2)\hat{j} = 7\hat{i} + 6\hat{j}$$

$$|\overline{AD}|^2 = 7^2 + 6^2 = 49 + 36 = 85$$

$$\Rightarrow |\overline{AD}| = \sqrt{85} \text{ units}$$

(v) (a): We have,  $\vec{M} = 4\hat{j} + 3\hat{k}$ ,

$$\therefore |\vec{M}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\therefore \hat{M} = \frac{\vec{M}}{|\vec{M}|} = \frac{4\hat{j} + 3\hat{k}}{5} = \frac{4}{5}\hat{j} + \frac{3}{5}\hat{k}$$

7. (i) (c): Clearly,  $G$  be the centroid of  $\triangle BCD$ , therefore coordinates of  $G$  are

$$\left(\frac{3 + 4 + 2}{3}, \frac{0 + 3 + 3}{3}, \frac{1 + 6 + 2}{3}\right) = (3, 2, 3)$$

(ii) (b): Since,  $A = (0, 1, 2)$  and  $G = (3, 2, 3)$

$$\therefore \overline{AG} = (3 - 0)\hat{i} + (2 - 1)\hat{j} + (3 - 2)\hat{k} = 3\hat{i} + \hat{j} + \hat{k}$$

$$\Rightarrow |\overline{AG}|^2 = 3^2 + 1^2 + 1^2 = 9 + 1 + 1 = 11$$

$$\Rightarrow |\overline{AG}| = \sqrt{11}$$

(iii) (c): Clearly, area of  $\triangle ABC = \frac{1}{2} |\overline{AB} \times \overline{AC}|$

$$\text{Here, } \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 - 0 & 0 - 1 & 1 - 2 \\ 4 - 0 & 3 - 1 & 6 - 2 \end{vmatrix}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$$

$$= \hat{i}(-4 + 2) - \hat{j}(12 + 4) + \hat{k}(6 + 4) = -2\hat{i} - 16\hat{j} + 10\hat{k}$$

$$\therefore |\overline{AB} \times \overline{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$$

$$= \sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$$

Hence, area of  $\triangle ABC = \frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$  sq. units

(iv) (b): Here,  $\overline{AB} = 3\hat{i} - \hat{j} - \hat{k}$

$$\Rightarrow |\overline{AB}| = \sqrt{9 + 1 + 1} = \sqrt{11}$$

$$\text{Also, } \overline{AC} = 4\hat{i} + 2\hat{j} + 4\hat{k}$$

$$\Rightarrow |\overline{AC}| = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

$$\text{Now, } |\overline{AB}| + |\overline{AC}| = \sqrt{11} + 6 = 3.32 + 6 = 9.32 \text{ units}$$

(v) (a): The length of the perpendicular from the vertex  $D$  on the opposite face

$$= |\text{Projection of } \overline{AD} \text{ on } \overline{AB} \times \overline{AC}|$$

$$= \left| \frac{(2\hat{i} + 2\hat{j}) \cdot (-2\hat{i} - 16\hat{j} + 10\hat{k})}{\sqrt{(-2)^2 + (-16)^2 + 10^2}} \right|$$

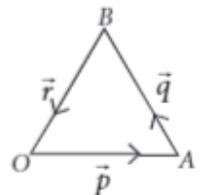
$$= \left| \frac{-4 - 32}{\sqrt{360}} \right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}} \text{ units}$$

8. (i) (c): Let  $OAB$  be a triangle such that

$$\overline{AO} = -\vec{p}, \overline{AB} = \vec{q}, \overline{BO} = \vec{r}$$

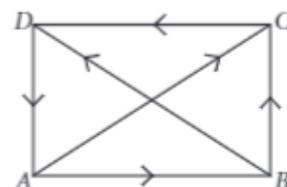
$$\text{Now, } \vec{q} + \vec{r} = \overline{AB} + \overline{BO}$$

$$= \overline{AO} = -\vec{p}$$



(ii) (c): From triangle law of vector addition,

$$\overline{AC} + \overline{BD} = \overline{AB} + \overline{BC} + \overline{BC} + \overline{CD}$$

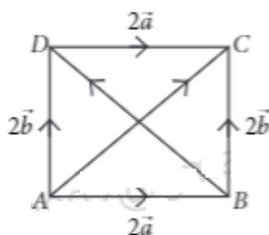


$$= \overline{AB} + 2\overline{BC} + \overline{CD}$$

$$= \overline{AB} + 2\overline{BC} - \overline{AB} = 2\overline{BC}$$

$$[\because \overline{AB} = -\overline{CD}]$$

$$(iii) (b): \text{ In } \triangle ABC, \overline{AC} = 2\vec{a} + 2\vec{b} \quad \dots(i)$$



$$\text{and in } \triangle ABD, 2\vec{b} = 2\vec{a} + \overline{BD} \quad \dots(ii)$$

[By triangle law of addition]

Adding (i) and (ii), we have

$$\overline{AC} + 2\vec{b} = 4\vec{a} + \overline{BD} + 2\vec{b}$$

$$\Rightarrow \overline{AC} - \overline{BD} = 4\vec{a}$$

$$(iv) (d): \text{ In } \triangle ABC, \overline{BA} + \overline{AC} = \overline{BC} \quad \dots(i)$$

[By triangle law]

$$\text{In } \triangle BCD, \overline{BC} + \overline{CD} = \overline{BD} \quad \dots(ii)$$

$$\text{From (i) and (ii), } \overline{BA} + \overline{AC} = \overline{BD} - \overline{CD}$$

$$\Rightarrow \overline{BA} + \overline{CD} = \overline{BD} - \overline{AC} = \overline{BD} + \overline{CA}$$

(v) (b): Since  $T$  is the mid point of  $YZ$ .

$$\text{So, } \overline{YT} = \overline{TZ}$$

$$\text{Now, } \overline{XY} + \overline{XZ} = (\overline{XT} + \overline{TY}) + (\overline{XT} + \overline{TZ})$$

[By triangle law]

$$\text{Now, } \overline{XY} + \overline{XZ} = (\overline{XT} + \overline{TY}) + (\overline{XT} + \overline{TZ})$$

[By triangle law]

$$= 2\overline{XT} + \overline{TY} + \overline{TZ} = 2\overline{XT} \quad [\because \overline{TY} = -\overline{TZ}]$$

$$9. (i) (a): \vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 3\hat{j} - 2\hat{k}$$

$$\text{and } \vec{c} = -2\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\therefore \vec{a} + \vec{b} + \vec{c} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

(ii) (c): Using triangle law of addition in  $\triangle ABC$ , we get

$$\overline{AB} + \overline{BC} + \overline{CA} = \vec{0}, \text{ which can be rewritten as}$$

$$\overline{AB} + \overline{BC} - \overline{AC} = \vec{0} \text{ or } \overline{AB} - \overline{CB} + \overline{CA} = \vec{0}$$

(iii) (c): We have,  $A(1, 4, 2)$ ,  $B(3, -3, -2)$  and  $C(-2, 2, 6)$

$$\text{Now, } \overline{AB} = \vec{b} - \vec{a} = 2\hat{i} - 7\hat{j} - 4\hat{k}$$

$$\text{and } \overline{AC} = \vec{c} - \vec{a} = -3\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\therefore \overline{AB} \times \overline{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -7 & -4 \\ -3 & -2 & 4 \end{vmatrix}$$

$$= \hat{i}(-28 - 8) - \hat{j}(8 - 12) + \hat{k}(-4 - 21) = -36\hat{i} + 4\hat{j} - 25\hat{k}$$

$$\text{Now, } |\overline{AB} \times \overline{AC}| = \sqrt{(-36)^2 + 4^2 + (-25)^2}$$

$$= \sqrt{1296 + 16 + 625} = \sqrt{1937}$$

$$\therefore \text{ Area of } \triangle ABC = \frac{1}{2} |\overline{AB} \times \overline{AC}| = \frac{1}{2} \sqrt{1937} \text{ sq. units}$$

(iv) (d): If the given points lie on the straight line, then the points will be collinear and so area of  $\triangle ABC = 0$ .

$$\Rightarrow |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}| = 0$$

[ $\because$  If  $\vec{a}, \vec{b}, \vec{c}$  are the position vectors of the three vertices  $A, B$  and  $C$  of  $\triangle ABC$ , then area of triangle

$$= \frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|]$$

$$(v) (b): \text{ Here, } |\vec{a}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

$\therefore$  Unit vector in the direction of vector  $\vec{a}$  is

$$\hat{a} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7} = \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

10. (i) (b): Here,  $(4, 10)$  are the coordinates of  $A$ .

$$\therefore \text{ P.V. of } A = 4\hat{i} + 10\hat{j}$$

(ii) (c): Here,  $(9, 7)$  are the coordinates of  $B$ .

$$\therefore \text{ P.V. of } B = 9\hat{i} + 7\hat{j}$$

(ii) (c): Here,  $(9, 7)$  are the coordinates of  $B$ .

$$\therefore \text{ P.V. of } B = 9\hat{i} + 7\hat{j}$$

(iii) (b): Here, P.V. of  $A = 4\hat{i} + 10\hat{j}$  and P.V. of

$$C = 4\hat{i} + 2\hat{j}$$

$$\therefore \overline{AC} = (4 - 4)\hat{i} + (2 - 10)\hat{j} = -8\hat{j}$$

(iv) (a): Here  $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\therefore |\vec{A}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\therefore \hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}} = \frac{1}{\sqrt{14}}\hat{i} + \frac{2}{\sqrt{14}}\hat{j} + \frac{3}{\sqrt{14}}\hat{k}$$

(v) (d): We have,  $\vec{A} = 4\hat{i} + 3\hat{j}$  and  $\vec{B} = 3\hat{i} + 4\hat{j}$

$$\therefore |\vec{A}| = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$

$$\text{and } |\vec{B}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

$$\text{Thus, } |\vec{A}| + |\vec{B}| = 5 + 5 = 10$$