

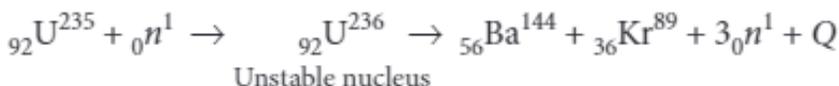
CASE STUDY / PASSAGE BASED QUESTIONS

Questions 1-6 are Case Study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

1

Nuclear Fission

In the year 1939, German scientist Otto Hahn and Strassmann discovered that when an uranium isotope was bombarded with a neutron, it breaks into two intermediate mass fragments. It was observed that, the sum of the masses of new fragments formed were less than the mass of the original nuclei. This difference in the mass appeared as the energy released in the process. Thus, the phenomenon of splitting of a heavy nucleus (usually $A > 230$) into two or more lighter nuclei by the bombardment of proton, neutron, α -particle, etc with liberation of energy is called nuclear fission.



- (i) Nuclear fission can be explained on the basis of
- Millikan's oil drop method
 - Liquid drop model
 - Shell model
 - Bohr's model.
- (ii) For sustaining the nuclear fission chain reaction in a sample (of small size) of ${}_{92}^{235}\text{U}$, it is desirable to slow down fast neutrons by
- friction
 - elastic damping/scattering
 - absorption
 - none of these.
- (iii) Which of the following is/are fission reaction(s)?
- ${}_0^1n + {}_{92}^{235}\text{U} \rightarrow {}_{92}^{236}\text{U} \rightarrow {}_{51}^{133}\text{Sb} + {}_{41}^{99}\text{Nb} + 4{}_0^1n$
 - ${}_0^1n + {}_{92}^{235}\text{U} \rightarrow {}_{54}^{140}\text{Xe} + {}_{38}^{94}\text{Sr} + 2{}_0^1n$
 - ${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0^1n$
- Both II and III
 - Both I and III
 - Only II
 - Both I and II

Syllabus

Composition and size of nucleus
Nuclear force :
Mass-energy relation,
mass defect,
nuclear fission,
nuclear fusion.

- (iv) A nuclear fission is said to be critical when multiplication factor or K
- (a) $K = 1$ (b) $K > 1$ (c) $K < 1$ (d) $K = 0$
- (v) Einstein's mass-energy conversion relation $E = mc^2$ is illustrated by
- (a) nuclear fission (b) β -decay (c) rocket propulsion (d) steam engine

4

Nuclear Force

Neutrons and protons are identical particles in the sense that their masses are nearly the same and the force, called nuclear force, does not distinguish them. Nuclear force is the strongest force. Stability of nucleus is determined by the neutron-proton ratio or mass defect or packing fraction. Shape of nucleus is calculated by quadrupole moment and spin of nucleus depends on even or odd mass number. Volume of nucleus depends on the mass number. Whole mass of the atom (nearly 99%) is centred at the nucleus.

- (i) The correct statements about the nuclear force is/are
- (a) change independent (b) short range force
(c) non-conservative force (d) all of these.
- (ii) The range of nuclear force is the order of
- (a) 2×10^{-10} m (b) 1.5×10^{-20} m (c) 1.2×10^{-4} m (d) 1.4×10^{-15} m
- (iii) A force between two protons is same as the force between proton and neutron. The nature of the force is
- (a) electrical force (b) weak nuclear force (c) gravitational force (d) strong nuclear force.
- (iv) Two protons are kept at a separation of 40 \AA . F_n is the nuclear force and F_e is the electrostatic force between them. Then
- (a) $F_n \ll F_e$ (b) $F_n = F_e$ (c) $F_n \gg F_e$ (d) $F_n \approx F_e$
- (v) All the nucleons in an atom are held by
- (a) nuclear forces (b) van der Waal's forces
(c) tensor forces (d) coulomb forces

5

Nuclear Density

The density of nuclear matter is the ratio of the mass of a nucleus to its volume. As the volume of a nucleus is directly proportional to its mass number A , so the density of nuclear matter is independent of the size of the nucleus. Thus, the nuclear matter behaves like a liquid of constant density. Different nuclei are like drops of this liquid, of different sizes but of same density.

Let A be the mass number and R be the radius of a nucleus. If m is the average mass of a nucleon, then

$$\text{Mass of nucleus} = mA$$

$$\text{Volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi(R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\therefore \text{Nuclear density, } \rho_{\text{nu}} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}} \text{ or } \rho_{\text{nu}} = \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

Clearly, nuclear density is independent of mass number A or the size of the nucleus.

The nuclear mass density is of the order $10^{17} \text{ kg m}^{-3}$. This density is very large as compared to the density of ordinary matter, say water, for which $\rho = 1.0 \times 10^3 \text{ kg m}^{-3}$.

- (i) The nuclear radius of $^{16}_8\text{O}$ is 3×10^{-15} m. The density of nuclear matter is
 (a) $2.9 \times 10^{34} \text{ kg m}^{-3}$ (b) $1.2 \times 10^{17} \text{ kg m}^{-3}$ (c) $16 \times 10^{27} \text{ kg m}^{-3}$ (d) $2.4 \times 10^{17} \text{ kg m}^{-3}$
- (ii) What is the density of hydrogen nucleus in SI units? Given $R_0 = 1.1$ fermi and $m_p = 1.007825$ amu.
 (a) $2.98 \times 10^{17} \text{ kg m}^{-3}$ (b) $3.0 \times 10^{34} \text{ kg m}^{-3}$ (c) $1.99 \times 10^{11} \text{ kg m}^{-3}$ (d) $7.85 \times 10^{17} \text{ kg m}^{-3}$
- (iii) Density of a nucleus is
 (a) more for lighter elements and less for heavier elements
 (b) more for heavier elements and less for lighter elements
 (c) very less compared to ordinary matter
 (d) a constant.
- (iv) The nuclear mass of $^{56}_{26}\text{Fe}$ is 55.85 amu. The its nuclear density is
 (a) $5.0 \times 10^{19} \text{ kg m}^{-3}$ (b) $1.5 \times 10^{19} \text{ kg m}^{-3}$ (c) $2.9 \times 10^{17} \text{ kg m}^{-3}$ (d) $9.2 \times 10^{26} \text{ kg m}^{-3}$
- (v) If the nucleus of $^{27}_{13}\text{Al}$ has a nuclear radius of about 3.6 fm, then $^{125}_{52}\text{Te}$ would have its radius approximately as
 (a) 9.6 fm (b) 12 fm (c) 4.8 fm (d) 6 fm

6

Mass-Energy

When subatomic particles undergo reactions, energy is conserved, but mass is not necessarily conserved. However, a particle's mass "contributes" to its total energy, in accordance with Einstein's famous equation, $E = mc^2$. In this equation, E denotes the energy carried by a particle because of its mass. The particle can also have additional energy due to its motion and its interactions with other particles. Consider a neutron at rest and well separated from other particles. It decays into a proton, an electron and an undetected third particle as given here :
 Neutron \rightarrow proton + electron + ???

The given table summarizes some data from a single neutron decay. Electron volt is a unit of energy. Column 2 shows the rest mass of the particle times the speed of light squared.

Particle	Mass $\times c^2$ (MeV)	Kinetic energy (MeV)
Neutron	940.97	0.00
Proton	939.67	0.01
Electron	0.51	0.39

- (i) From the given table, which properties of the undetected third particle can be calculate?
 (a) Total energy, but not kinetic energy (b) Kinetic energy, but not total energy
 (c) Both total energy and kinetic energy (d) Neither total energy nor kinetic energy
- (ii) Assuming the table contains no major errors, what can we conclude about the (mass $\times c^2$) of the undetected third particle?
 (a) It is 0.79 MeV
 (b) It is 0.39 MeV
 (c) It is less than or equal to 0.79 MeV; but we cannot be more precise.
 (d) It is less than or equal to 0.40 MeV; but we cannot be more precise.
- (iii) Could this reaction occur?
 Proton \rightarrow neutron + other particles

- (a) Yes, if the other particles have much more kinetic energy than mass energy.
 (b) Yes, but only if the proton has potential energy (due to interactions with other particles).
 (c) No, because a neutron is more massive than a proton.
 (d) No, because a proton is positively charged while a neutron is electrically neutral.
- (iv) How much mass has to be converted into energy to produce electric power of 500 MW for one hour?
 (a) 2×10^{-5} kg (b) 1×10^{-5} kg (c) 3×10^{-5} kg (d) 4×10^{-5} kg
- (v) The equivalent energy of 1 g of substance is
 (a) 9×10^{13} J (b) 6×10^{12} J (c) 3×10^{13} J (d) 6×10^{13} J

ASSERTION & REASON

For question numbers 7-20, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

- (a) Both A and R are true and R is the correct explanation of A
 (b) Both A and R are true but R is NOT the correct explanation of A
 (c) A is true but R is false
 (d) A is false and R is also false
7. **Assertion (A)** : Rydberg's constant varies with the mass number of a given element.
Reason (R) : The reduced mass of the electron depends on the mass of the nucleus only.
8. **Assertion (A)** : Isotopes of an element can be separated by using a mass spectrometer.
Reason (R) : Separation of isotopes is possible because of the difference in electron numbers of isotopes.
9. **Assertion (R)** : ${}^{14}_7\text{N}$ is stable.
Reason (R) : Nuclei having an odd number of protons and an odd number of neutrons are generally less stable than the one having even number of protons and even number of neutrons.
10. **Assertion (A)** : Nuclear density is extremely higher than atomic density.
Reason (R) : Most of the mass of the atom is concentrated in the nucleus.
11. **Assertion (A)** : Two protons can attract each other.
Reason (R) : The distance between the protons within the nucleus is about 10^{-15} m.
12. **Assertion (A)** : The nuclear force becomes weak if the nucleus contains too many protons compared to neutrons.
Reason (R) : The electrostatic forces weaken the nuclear force.
13. **Assertion (A)** : For the fission of heavy nuclei, neutrons are more effective than protons.
Reason (R) : Neutrons are heavier than protons.
14. **Assertion (A)** : Energy is released in a nuclear reaction.
Reason (R) : In any nuclear reaction the reactants and resultant products obey the law of conservation of charge and mass only.
15. **Assertion (A)** : Density of all the nuclei is same.
Reason (R) : Radius of nucleus is directly proportional to the cube root of mass number.
16. **Assertion (A)** : There is a chain reaction when uranium is bombarded with slow neutrons.
Reason (R) : When uranium is bombarded with slow neutrons more neutrons are produced.
17. **Assertion (A)** : Cadmium rods used in a nuclear reactor, control the rate of fission.
Reason (R) : Cadmium rods speed up the slow neutrons.

18. **Assertion (A)** : A fission reaction can be more easily controlled than a fission reaction.
Reason (R) : The percentage of mass converted to energy in a fission reaction is 0.1% whereas in a fusion reaction it is 0.4%.
19. **Assertion (A)** : The ratio for time taken for light emission from an atom to that for release of nuclear energy in fission is 1 : 100.
Reason (R) : Time taken for the light emission from an atom is of the order of 10^{-8} s.
20. **Assertion (A)** : Thermonuclear fusion reactions may become the source of unlimited power for the mankind.
Reason (R) : A single fusion event involving isotopes of hydrogen produces more energy than energy from nuclear fission of a single uranium.

HINTS & EXPLANATIONS

1. (i) (b)

(ii) (b): Fast neutrons are slowed down by elastic scattering with light nuclei as each collision takes away nearly 50% of energy.

(iii) (d): Reactions I and II represent fission of uranium isotope ${}_{92}^{235}\text{U}$, when bombarded with neutrons that breaks it into two intermediate mass nuclear fragments. However, reaction III represents two deuterons fuses together to form the light isotope of helium.

(iv) (c) : On an average 2.5 neutrons are released per fission of the uranium atom.

The energy of the neutron released per fission of the uranium atom is 2 MeV.

(v) (a): In fission process, when a parent nucleus breaks into daughter products, then some mass is lost in the form of energy. Thus,
 mass of fission products < mass of parent nucleus.

$$\Rightarrow \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$$

2. (i) (a) : As nearly 99.9% mass of atom is in nucleus

$$\therefore \frac{\text{Mass of nucleus}}{\text{Mass of atom}} = \frac{99.9}{100} = 0.99 \approx 1$$

(ii) (a): Since, the nuclei of deuterium and tritium are isotopes of hydrogen, they must contain only one proton each. But the masses of the nuclei of hydrogen, deuterium and tritium are in the ratio of 1 : 2 : 3, because of presence of neutral matter in deuterium and tritium nuclei.

(iii) (c)

$$(iv) (a) : R = R_0 A^{1/3}$$

$$\log R = \log R_0 + \frac{1}{3} \log A$$

On comparing the above equation of straight line; $y = mx + c$. So, the graph between $\log A$ and $\log R$ is a straight line also.

$$(v) (a) : \text{Here, } A_1 = 197 \text{ and } A_2 = 107$$

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2} \right)^{1/3} = \left(\frac{197}{107} \right)^{1/3} = 1.225 \approx 1.23$$

3. (i) (a) : Let the number of fissions per second be n .

Energy released per second

$$= n \times 200 \text{ MeV} = n \times 200 \times 1.6 \times 10^{-13} \text{ J}$$

Energy required per second = power \times time

$$= 1 \text{ kW} \times 1 \text{ s} = 1000 \text{ J}$$

$$\therefore n \times 200 \times 1.6 \times 10^{-13} = 1000$$

$$\text{or } n = \frac{1000}{3.2 \times 10^{-11}} = \frac{10}{3.2} \times 10^{13} = 3.125 \times 10^{13}$$

(ii) (a)

(iii) (b): As only 0.1% of the original mass is converted into energy, hence out of 1 kg mass 1 g is converted into energy.

$$\therefore \text{Energy released during fission, } E = \Delta mc^2 \\ = 1 \text{ g} \times (3 \times 10^8 \text{ m s}^{-1})^2 = 10^{-3} \times 9 \times 10^{16} \text{ J} = 9 \times 10^{13} \text{ J}$$

(iv) (a)

(v) (a)

4. (i) (d) : All options are basic properties of nuclear forces. So, all options are correct.

(ii) (d): The nuclear force is of short range and the range of nuclear force is the order of 1.4×10^{-15} m.

Now, volume $\propto R^3 \propto A$

(iii) (d)

(iv) (a): Nuclear force is much stronger than the electrostatic force inside the nucleus *i.e.*, at distances of the order of fermi. At 40 Å, nuclear force is ineffective and only electrostatic force of repulsion is present. This is very high at this distance because nuclear force is not acting now and the gravitational force is very feeble. $F_{\text{nuclear}} \ll F_{\text{electrostatic}}$ in this case.

(v) (a)

5. (i) (d) : Here $R = 3 \times 10^{-15}$ m

Nuclear mass = 16 amu = $16 \times 1.66 \times 10^{-27}$ kg

$$\rho_{\text{nu}} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{16 \times 1.66 \times 10^{-27}}{\frac{4}{3} \pi (3 \times 10^{-15})^3}$$

$$= 2.359 \times 10^{17} \text{ kg m}^{-3} \approx 2.4 \times 10^{17} \text{ kg m}^{-3}$$

(ii) (a): Density,

$$\rho = \frac{3m_p}{4\pi R_0^3} = \frac{3 \times 1.007825 \times 1.66 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.1 \times 10^{-15})^3}$$

$$= 2.98 \times 10^{17} \text{ kg m}^{-3}$$

(iii) (d): Density = $\frac{\text{Mass}}{\text{Volume}} = \frac{Am_p}{\frac{4}{3} \pi (R_0 A^{1/3})^3}$

$$= \frac{m_p}{\frac{4}{3} \pi R_0^3},$$

where $m_p = 1.6 \times 10^{-27}$ kg³

= 2.3×10^{17} kg m⁻³, which is a constant.

(iv) (c) : Given, mass of $m_{\text{Fe}} = 55.85$ amu

$$= 55.85 \times 1.66 \times 10^{-27} \text{ kg} = 9.27 \times 10^{-26} \text{ kg}$$

Nuclear radius = $R_0 A^{1/3} = 1.1 \times 10^{-15} \times (56)^{1/3}$ m

[∵ $A = 56$]

$$\rho_{\text{nu}} = \frac{\text{Nuclear mass}}{\text{Nuclear volume}} = \frac{m_{\text{Fe}}}{\frac{4}{3} \pi R^3}$$

$$= \frac{9.27 \times 10^{-26}}{\frac{4\pi}{3} \times (1.1 \times 10^{-15})^3 \times 56} = 2.9 \times 10^{17} \text{ kg m}^{-3}$$

(v) (d): Here, $A_1 = 27, A_2 = 125, R_1 = 3.6$ fm

$$\text{As, } \frac{R_2}{R_1} = \left(\frac{A_2}{A_1} \right)^{1/3} = \left(\frac{125}{27} \right)^{1/3} = \frac{5}{3}$$

$$\therefore R_2 = \frac{5}{3} R_1 = \frac{5}{3} \times 3.6 = 6 \text{ fm}$$

6. (i) (a) : As just shown, energy conservation allows us to calculate the third particle's total energy. But we do not know what percentage of that total is mass energy.

(ii) (d): According to the passage, subatomic reactions do not conserve mass. So, we cannot find the third particle's mass by setting m_{neutron} equal to $m_{\text{proton}} + m_{\text{electron}} + m_{\text{third particle}}$

The neutron has energy 940.97 MeV. The proton has energy 939.67 MeV + 0.01 MeV = 939.69 MeV. The electron has energy 0.51 MeV + 0.39 MeV = 0.90 MeV. Therefore, the third particle has energy

$$E_{\text{third particle}} = E_{\text{neutron}} - E_{\text{proton}} - E_{\text{electron}}$$

$$= 940.97 - 939.67 - 0.90 = 0.40 \text{ MeV}$$

We just found the third particle's total energy, the sum of its mass energy and kinetic energy. Without more information, we cannot figure out how much of that energy is mass energy.

(iii) (b)

(iv) (a) : Here, $P = 500 \text{ MW} = 5 \times 10^8 \text{ W}$,

$t = 1 \text{ h} = 3600 \text{ s}$

Energy produced, $E = P \times t = 5 \times 10^8 \times 3600$

$$= 18 \times 10^{11} \text{ J}$$

As $E = \Delta mc^2$

$$\therefore \Delta m = \frac{E}{c^2} = \frac{18 \times 10^{11}}{(3 \times 10^8)^2} = \frac{18 \times 10^{11}}{9 \times 10^{16}} = 2 \times 10^{-5} \text{ kg}$$

(v) (a): Using, $E = mc^2$

Here, $m = 1 \text{ g} = 1 \times 10^{-3} \text{ kg}$, $c = 3 \times 10^8 \text{ m s}^{-1}$

$$\therefore E = 10^{-3} \times 9 \times 10^{16} = 9 \times 10^{13} \text{ J}$$

7. (d): The Rydberg constant is given by $R = \frac{13.6 \text{ eV}}{hc}$,

which is independent of the mass number of an element. Reduced mass of the system depends on the masses of both the electron as well as nucleus.

8. (c) : Isotopes have same number of electrons and protons but different neutron number. That's why the mass number of isotopes are different and can be separated by using a mass spectrometer.

9. (b): Nitrogen is a stable element. Those elements which have an odd number of protons and an odd number of neutrons are generally unstable. ${}^{14}_7\text{N}$ has 7 protons and 7 neutrons *i.e.*, odd number of both protons and neutrons but it is an exception. It is a stable element.

10. (a): According to the planetary model of the atom the mass of the atom is concentrated at the centre, the nucleus. The electrons orbit around the nucleus in circular paths of different radii. The radius of the outermost orbit gives the size of atom. The size of the atom is $\sim 10^{-10}$ m as compared to the size of the nucleus 10^{-15} m. Electrons being extremely light ($m_e = 5.49 \times 10^{-4}$ u = 9.11×10^{-31} kg) as compared to protons and neutrons have a negligible contribution to the weight of the atom. They mainly increase the volume of the atom. Therefore, the nuclear density is much higher than the atomic density (10^3 kg/m³).

11. (a): Due to electrostatic forces between two protons (like charges) there is a force of repulsion. However, when the distance between them is $\sim 10^{-15}$ m they come under the influence of the short range, strong nuclear forces. (The range of the nuclear forces is $\sim 10^{-15}$ m). These forces are attractive forces and charge independent. The net force on the protons is attractive as nuclear forces are much stronger than electrostatic forces. The protons attract each other.

12. (c): Nuclear forces are strongest when the number of protons equals the number of neutrons. An excess of protons compared to neutrons weakens the nuclear force. Also too many neutrons compared to protons inside the nucleus weaken the nuclear forces. The electrostatic force which is a hundred times less than the nuclear force is not the cause.

13. (b): A neutron is slightly heavier than a proton. As neutrons are chargeless particles they penetrate matter more than protons. Therefore, they are more effective than protons in fission reactions.

14. (c): In both fission and fusion large amounts of energy are released. The reason is correct. Charge, mass, momentum and energy, all are conserved.

15. (a): Experimentally, it is found that the average radius of a nucleus is given by

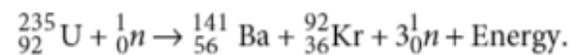
$R = R_0 A^{1/3}$ where $R_0 = 1.1 \times 10^{-15}$ m = 1.1 fm and A = mass number

The volume of a nucleus is $V = \frac{4}{3} \pi R^3 = \frac{4}{3} \pi R_0^3 A$.

Now as the masses of a proton and a neutron are roughly equal, say m , the mass of a nucleus is also roughly proportional to the mass number A , $M = mA$

Hence density within a nucleus, $\rho = \frac{M}{V} = \frac{mA}{\frac{4}{3} \pi R_0^3 A}$
 $= \frac{m}{\frac{4}{3} \pi R_0^3}$ is independent of the mass number A .

16. (a): When uranium is bombarded by slow neutrons the reaction is represented as



As more neutrons are produced, the reaction is correct. These additional neutrons strike other uranium nuclei to produce even more neutrons. Thus a chain reaction is established.

17. (c): Cadmium rods are used in a nuclear reactor to control the rate of fission. The cadmium rods do not slow down or speed up the neutrons produced in a fission reaction of ${}^{235}\text{U}$. Instead they absorb the neutrons thereby regulating the power level of the reactor.

18. (b): Percentage of mass converted to energy in a fission reaction is 0.1% whereas in a fusion reaction it is 0.4%. Consequently the amount of energy released is more in a fusion than in a fission reaction. It is not easy to control a fusion reaction.

19. (a): Time taken for the light emission from an atom $\approx 10^{-8}$ s.

Time taken for release of energy in fission $\approx 10^{-6}$ s

$$\text{Required ratio} = \frac{10^{-8}}{10^{-6}} = \frac{1}{100} = 1 : 100$$

20. (c): When fusion is achieved by raising the temperature of the system so that particles have enough kinetic energy to overcome the coulomb repulsive behaviour, it is called thermonuclear fusion. It is a clean source of energy but energy released in one fusion is much less than a single uranium fission.