

# Relations and Functions

## Short Answer Type Questions

**Q. 1** Let  $A = \{a, b, c\}$  and the relation  $R$  be defined on  $A$  as follows

$$R = \{(a, a), (b, c), (a, b)\}$$

Then, write minimum number of ordered pairs to be added in  $R$  to make  $R$  reflexive and transitive.

**Sol.** Given relation,  $R = \{(a, a), (b, c), (a, b)\}$ .

To make  $R$  is reflexive we must add  $(b, b)$  and  $(c, c)$  to  $R$ . Also, to make  $R$  is transitive we must add  $(a, c)$  to  $R$ .

So, minimum number of ordered pair is to be added are  $(b, b), (c, c), (a, c)$ .

**Q. 2** Let  $D$  be the domain of the real valued function  $f$  defined by  $f(x) = \sqrt{25 - x^2}$ . Then, write  $D$ .

**Sol.** Given function is,  $f(x) = \sqrt{25 - x^2}$

For real valued of  $f(x)$   $25 - x^2 \geq 0$

$$x^2 \leq 25$$

$$-5 \leq x \leq 5$$

$\therefore D = [-5, 5]$

**Q. 3** If  $f, g : R \rightarrow R$  be defined by  $f(x) = 2x + 1$  and  $g(x) = x^2 - 2, \forall x \in R$ , respectively. Then, find  $gof$ .

### Thinking Process

If  $f, g : R \rightarrow R$  be two functions, then  $gof(x) = g\{f(x)\} \forall x \in R$ .

**Sol.** Given that,  $f(x) = 2x + 1$  and  $g(x) = x^2 - 2, \forall x \in R$

$$\begin{aligned} \therefore gof &= g\{f(x)\} \\ &= g(2x + 1) = (2x + 1)^2 - 2 \\ &= 4x^2 + 4x + 1 - 2 \\ &= 4x^2 + 4x - 1 \end{aligned}$$

**Q. 4** Let  $f:R \rightarrow R$  be the function defined by  $f(x) = 2x - 3, \forall x \in R$ . Write  $f^{-1}$ .

**Sol.** Given that,  $f(x) = 2x - 3, \forall x \in R$   
 Now, let  $y = 2x - 3$   
 $2x = y + 3$   
 $x = \frac{y + 3}{2}$   
 $\therefore f^{-1}(x) = \frac{x + 3}{2}$

**Q. 5** If  $A = \{a, b, c, d\}$  and the function  $f = \{(a, b), (b, d), (c, a), (d, c)\}$ , write  $f^{-1}$ .

**Sol.** Given that,  $A = \{a, b, c, d\}$   
 and  $f = \{(a, b), (b, d), (c, a), (d, c)\}$   
 $f^{-1} = \{(b, a), (d, b), (a, c), (c, d)\}$

**Q. 6** If  $f:R \rightarrow R$  is defined by  $f(x) = x^2 - 3x + 2$ , write  $f\{f(x)\}$ .

**Thinking Process**

To solve this problem use the formula i.e.,  $(a + b + c)^2 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca)$

**Sol.** Given that,  $f(x) = x^2 - 3x + 2$   
 $\therefore f\{f(x)\} = f(x^2 - 3x + 2)$   
 $= (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$   
 $= x^4 + 9x^2 + 4 - 6x^3 - 12x + 4x^2 - 3x^2 + 9x - 6 + 2$   
 $= x^4 + 10x^2 - 6x^3 - 3x$   
 $f\{f(x)\} = x^4 - 6x^3 + 10x^2 - 3x$

**Q. 7** Is  $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$  a function? If  $g$  is described by  $g(x) = \alpha x + \beta$ , then what value should be assigned to  $\alpha$  and  $\beta$ ?

**Sol.** Given that,  $g = \{(1, 1), (2, 3), (3, 5), (4, 7)\}$ .  
 Here, each element of domain has unique image. So,  $g$  is a function.

Now given that,  $g(x) = \alpha x + \beta$   
 $g(1) = \alpha + \beta$   
 $\alpha + \beta = 1 \quad \dots(i)$   
 $g(2) = 2\alpha + \beta$   
 $2\alpha + \beta = 3 \quad \dots(ii)$

From Eqs. (i) and (ii),

$2(1 - \beta) + \beta = 3$   
 $\Rightarrow 2 - 2\beta + \beta = 3$   
 $\Rightarrow 2 - \beta = 3$   
 $\beta = -1$   
 If  $\beta = -1$ , then  $\alpha = 2$   
 $\alpha = 2, \beta = -1$

**Q. 8** Are the following set of ordered pairs functions? If so examine whether the mapping is injective or surjective.

(i)  $\{(x, y) : x \text{ is a person, } y \text{ is the mother of } x\}$ .

(ii)  $\{(a, b) : a \text{ is a person, } b \text{ is an ancestor of } a\}$ .

**Sol.** (i) Given set of ordered pair is  $\{(x, y) : x \text{ is a person, } y \text{ is the mother of } x\}$ .

It represent a function. Here, the image of distinct elements of  $x$  under  $f$  are not distinct, so it is not a injective but it is a surjective.

(ii) Set of ordered pairs =  $\{(a, b) : a \text{ is a person, } b \text{ is an ancestor of } a\}$

Here, each element of domain does not have a unique image. So, it does not represent function.

**Q. 9** If the mappings  $f$  and  $g$  are given by  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(2, 3), (5, 1), (1, 3)\}$ , write  $fog$ .

**Sol.** Given that,

$$f = \{(1, 2), (3, 5), (4, 1)\}$$

and

$$g = \{(2, 3), (5, 1), (1, 3)\}$$

Now,

$$fog(2) = f\{g(2)\} = f(3) = 5$$

$$fog(5) = f\{g(5)\} = f(1) = 2$$

$$fog(1) = f\{g(1)\} = f(3) = 5$$

$$fog = \{(2, 5), (5, 2), (1, 5)\}$$

**Q. 10** Let  $C$  be the set of complex numbers. Prove that the mapping  $f : C \rightarrow R$  given by  $f(z) = |z|, \forall z \in C$ , is neither one-one nor onto.

**Sol.** The mapping

$$f : C \rightarrow R$$

Given,

$$f(z) = |z|, \forall z \in C$$

$$f(1) = |1| = 1$$

$$f(-1) = |-1| = 1$$

$$f(1) = f(-1)$$

But

$$1 \neq -1$$

So,  $f(z)$  is not one-one. Also,  $f(z)$  is not onto as there is no pre-image for any negative element of  $R$  under the mapping  $f(z)$ .

**Q. 11** Let the function  $f : R \rightarrow R$  be defined by  $f(x) = \cos x, \forall x \in R$ . Show that  $f$  is neither one-one nor onto.

**Sol.** Given function,  $f(x) = \cos x, \forall x \in R$

Now,

$$f\left(\frac{\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\Rightarrow f\left(\frac{-\pi}{2}\right) = \cos \frac{\pi}{2} = 0$$

$$\Rightarrow f\left(\frac{\pi}{2}\right) = f\left(\frac{-\pi}{2}\right)$$

But

$$\frac{\pi}{2} \neq \frac{-\pi}{2}$$

So,  $f(x)$  is not one-one.

Now,  $f(x) = \cos x, \forall x \in R$  is not onto as there is no pre-image for any real number. Which does not belonging to the intervals  $[-1, 1]$ , the range of  $\cos x$ .

**Q. 12** Let  $X = \{1, 2, 3\}$  and  $Y = \{4, 5\}$ . Find whether the following subsets of  $X \times Y$  are functions from  $X$  to  $Y$  or not.

- (i)  $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$       (ii)  $g = \{(1, 4), (2, 4), (3, 4)\}$   
 (iii)  $h = \{(1, 4), (2, 5), (3, 5)\}$       (iv)  $k = \{(1, 4), (2, 5)\}$

**Sol.** Given that,  $X = \{1, 2, 3\}$  and  $Y = \{4, 5\}$   
 $X \times Y = \{(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)\}$

(i)  $f = \{(1, 4), (1, 5), (2, 4), (3, 5)\}$   
 $f$  is not a function because  $f$  has not unique image.

(ii)  $g = \{(1, 4), (2, 4), (3, 4)\}$   
 Since,  $g$  is a function as each element of the domain has unique image.

(iii)  $h = \{(1, 4), (2, 5), (3, 5)\}$   
 It is clear that  $h$  is a function.

(iv)  $k = \{(1, 4), (2, 5)\}$   
 $k$  is not a function as 3 has not any image under the mapping.

**Q. 13** If functions  $f : A \rightarrow B$  and  $g : B \rightarrow A$  satisfy  $gof = I_A$ , then show that  $f$  is one-one and  $g$  is onto.

**Sol.** Given that,  $f : A \rightarrow B$  and  $g : B \rightarrow A$  satisfy  $gof = I_A$   
 $\therefore gof = I_A$   
 $\Rightarrow gof\{f(x_1)\} = gof\{f(x_2)\}$   
 $\Rightarrow g(x_1) = g(x_2)$  [ $\because gof = I_A$ ]  
 $\therefore x_1 = x_2$   
 Hence,  $f$  is one-one and  $g$  is onto.

**Q. 14** Let  $f : R \rightarrow R$  be the function defined by  $f(x) = \frac{1}{2 - \cos x}$ ,  $\forall x \in R$ .

Then, find the range of  $f$ .

**Thinking Process**

Range of  $f = \{y \in Y : y = f(x) \text{ for some } x\}$  and use range of  $\cos x$  is  $[-1, 1]$

**Sol.** Given function,  $f(x) = \frac{1}{2 - \cos x}$ ,  $\forall x \in R$

Let  $y = \frac{1}{2 - \cos x}$

$\Rightarrow 2y - y \cos x = 1$

$\Rightarrow y \cos x = 2y - 1$

$\Rightarrow \cos x = \frac{2y - 1}{y} = 2 - \frac{1}{y} \Rightarrow \cos x = 2 - \frac{1}{y}$

$\Rightarrow -1 \leq \cos x \leq 1 \Rightarrow -1 \leq 2 - \frac{1}{y} \leq 1$

$\Rightarrow -3 \leq -\frac{1}{y} \leq -1 \Rightarrow 1 \leq \frac{1}{y} \leq 3$

$\Rightarrow \frac{1}{3} \leq \frac{1}{y} \leq 1$

So,  $y$  range is  $\left[\frac{1}{3}, 1\right]$ .

**Q. 15** Let  $n$  be a fixed positive integer. Define a relation  $R$  in  $Z$  as follows  $\forall a, b \in Z, aRb$  if and only if  $a - b$  is divisible by  $n$ . Show that  $R$  is an equivalence relation.

**Sol.** Given that,  $\forall a, b \in Z, aRb$  if and only if  $a - b$  is divisible by  $n$ .  
Now,

**I. Reflexive**

$aRa \Rightarrow (a - a)$  is divisible by  $n$ , which is true for any integer  $a$  as '0' is divisible by  $n$ .  
Hence,  $R$  is reflexive.

**II. Symmetric**

$aRb$   
 $\Rightarrow a - b$  is divisible by  $n$ .  
 $\Rightarrow -b + a$  is divisible by  $n$ .  
 $\Rightarrow -(b - a)$  is divisible by  $n$ .  
 $\Rightarrow (b - a)$  is divisible by  $n$ .  
 $\Rightarrow bRa$

Hence,  $R$  is symmetric.

**III. Transitive**

Let  $aRb$  and  $bRc$

$\Rightarrow (a - b)$  is divisible by  $n$  and  $(b - c)$  is divisible by  $n$   
 $\Rightarrow (a - b) + (b - c)$  is divisible by  $n$   
 $\Rightarrow (a - c)$  is divisible by  $n$   
 $\Rightarrow aRc$

Hence,  $R$  is transitive.

So,  $R$  is an equivalence relation.

## Long Answer Type Questions

**Q. 16** If  $A = \{1, 2, 3, 4\}$ , define relations on  $A$  which have properties of being

- (i) reflexive, transitive but not symmetric.
- (ii) symmetric but neither reflexive nor transitive.
- (iii) reflexive, symmetric and transitive.

**Sol.** Given that,  $A = \{1, 2, 3, 4\}$

(i) Let  $R_1 = \{(1, 1), (1, 2), (2, 3), (2, 2), (1, 3), (3, 3)\}$

$R_1$  is reflexive, since,  $(1, 1)$   $(2, 2)$   $(3, 3)$  lie in  $R_1$ .

Now,  $(1, 2) \in R_1, (2, 3) \in R_1 \Rightarrow (1, 3) \in R_1$

Hence,  $R_1$  is also transitive but  $(1, 2) \in R_1 \Rightarrow (2, 1) \notin R_1$ .

So, it is not symmetric.

(ii) Let  $R_2 = \{(1, 2), (2, 1)\}$

Now,  $(1, 2) \in R_2, (2, 1) \in R_2$

So, it is symmetric.

(iii) Let  $R_3 = \{(1, 2), (2, 1), (1, 1), (2, 2), (3, 3), (1, 3), (3, 1), (2, 3)\}$

Hence,  $R_3$  is reflexive, symmetric and transitive.

**Q. 17** Let  $R$  be relation defined on the set of natural number  $N$  as follows,  $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$ . Find the domain and range of the relation  $R$ . Also verify whether  $R$  is reflexive, symmetric and transitive.

**Sol.** Given that,  $R = \{(x, y) : x \in N, y \in N, 2x + y = 41\}$ .  
 Domain =  $\{1, 2, 3, \dots, 20\}$   
 Range =  $\{1, 3, 5, 7, \dots, 39\}$   
 $R = \{(1, 39), (2, 37), (3, 35), \dots, (19, 3), (20, 1)\}$   
 $R$  is not reflexive as  $(2, 2) \notin R$   
 $2 \times 2 + 2 \neq 41$   
 So,  $R$  is not symmetric.  
 As  $(1, 39) \in R$  but  $(39, 1) \notin R$   
 So,  $R$  is not transitive.  
 As  $(11, 19) \in R, (19, 3) \in R$   
 But  $(11, 3) \notin R$   
 Hence,  $R$  is neither reflexive, nor symmetric and nor transitive.

**Q. 18** Given,  $A = \{2, 3, 4\}, B = \{2, 5, 6, 7\}$ . Construct an example of each of the following

- (i) an injective mapping from  $A$  to  $B$ .
- (ii) a mapping from  $A$  to  $B$  which is not injective.
- (iii) a mapping from  $B$  to  $A$ .

**Sol.** Given that,  $A = \{2, 3, 4\}, B = \{2, 5, 6, 7\}$   
 (i) Let  $f : A \rightarrow B$  denote a mapping  
 $f = \{(x, y) : y = x + 3\}$   
*i.e.*,  $f = \{(2, 5), (3, 6), (4, 7)\}$ , which is an injective mapping.  
 (ii) Let  $g : A \rightarrow B$  denote a mapping such that  $g = \{(2, 2), (3, 5), (4, 5)\}$ , which is not an injective mapping.  
 (iii) Let  $h : B \rightarrow A$  denote a mapping such that  $h = \{(2, 2), (5, 3), (6, 4), (7, 4)\}$ , which is a mapping from  $B$  to  $A$ .

**Q. 19** Give an example of a map

- (i) which is one-one but not onto.
- (ii) which is not one-one but onto.
- (iii) which is neither one-one nor onto.

**Sol.** (i) Let  $f : N \rightarrow N$ , be a mapping defined by  $f(x) = 2x$  which is one-one.  
 For  $f(x_1) = f(x_2)$   
 $\Rightarrow 2x_1 = 2x_2$   
 $x_1 = x_2$   
 Further  $f$  is not onto, as for  $1 \in N$ , there does not exist any  $x$  in  $N$  such that  $f(x) = 2x + 1$ .  
 (ii) Let  $f : N \rightarrow N$ , given by  $f(1) = f(2) = 1$  and  $f(x) = x - 1$  for every  $x > 2$  is onto but not one-one.  $f$  is not one-one as  $f(1) = f(2) = 1$ . But  $f$  is onto.  
 (iii) The mapping  $f : R \rightarrow R$  defined as  $f(x) = x^2$ , is neither one-one nor onto.

**Q. 20** Let  $A = R - \{3\}$ ,  $B = R - \{1\}$ . If  $f : A \rightarrow B$  be defined by  $f(x) = \frac{x-2}{x-3}$ ,

$\forall x \in A$ . Then, show that  $f$  is bijective.

**Thinking Process**

A function  $f : x \rightarrow y$  is said to be bijective, if  $f$  is both one-one and onto.

**Sol.** Given that,  $A = R - \{3\}$ ,  $B = R - \{1\}$ .

$f : A \rightarrow B$  is defined by  $f(x) = \frac{x-2}{x-3}$ ,  $\forall x \in A$

**For injectivity**

Let  $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

$$\begin{aligned} \Rightarrow & (x_1-2)(x_2-3) = (x_2-2)(x_1-3) \\ \Rightarrow & x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 3x_2 - 2x_1 + 6 \\ \Rightarrow & -3x_1 - 2x_2 = -3x_2 - 2x_1 \\ \Rightarrow & -x_1 = -x_2 \Rightarrow x_1 = x_2 \end{aligned}$$

So,  $f(x)$  is an injective function.

**For surjectivity**

Let  $y = \frac{x-2}{x-3} \Rightarrow x-2 = xy-3y$

$\Rightarrow x(1-y) = 2-3y \Rightarrow x = \frac{2-3y}{1-y}$

$\Rightarrow x = \frac{3y-2}{y-1} \in A, \forall y \in B$  [codomain]

So,  $f(x)$  is surjective function.

Hence,  $f(x)$  is a bijective function.

**Q. 21** Let  $A = [-1, 1]$ , then, discuss whether the following functions defined on  $A$  are one-one onto or bijective.

(i)  $f(x) = \frac{x}{2}$

(ii)  $g(x) = |x|$

(iii)  $h(x) = x|x|$

(iv)  $k(x) = x^2$

**Sol.** Given that,  $A = [-1, 1]$

(i)  $f(x) = \frac{x}{2}$

Let  $f(x_1) = f(x_2)$   
 $\Rightarrow \frac{x_1}{2} = \frac{x_2}{2} \Rightarrow x_1 = x_2$

So,  $f(x)$  is one-one.

Now, let  $y = \frac{x}{2}$

$\Rightarrow x = 2y \notin A, \forall y \in A$

As for  $y = 1 \in A, x = 2 \notin A$

So,  $f(x)$  is not onto.

Also,  $f(x)$  is not bijective as it is not onto.

(ii)  $g(x) = |x|$

Let

$$g(x_1) = g(x_2)$$

$\Rightarrow$

$$|x_1| = |x_2| \Rightarrow x_1 = \pm x_2$$

So,  $g(x)$  is not one-one.

Now,

$$y = |x| \Rightarrow x = \pm y \notin A, \forall y \in A$$

So,  $g(x)$  is not onto, also,  $g(x)$  is not bijective.

(iii)  $h(x) = x|x|$

Let

$$h(x_1) = h(x_2)$$

$\Rightarrow$

$$x_1|x_1| = x_2|x_2| \Rightarrow x_1 = x_2$$

So,  $h(x)$  is one-one.

Now, let

$$y = x|x|$$

$\Rightarrow$

$$y = x^2 \in A, \forall x \in A$$

So,  $h(x)$  is onto also,  $h(x)$  is a bijective.

(iv)  $k(x) = x^2$

Let

$$k(x_1) = k(x_2)$$

$\Rightarrow$

$$x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$$

Thus,  $k(x)$  is not one-one.

Now, let

$$y = x^2$$

$\Rightarrow$

$$x = \sqrt{y} \notin A, \forall y \in A$$

As for  $y = -1$ ,  $x = \sqrt{-1} \notin A$

Hence,  $k(x)$  is neither one-one nor onto.

**Q. 22** Each of the following defines a relation of  $N$

(i)  $x$  is greater than  $y$ ,  $x, y \in N$ .

(ii)  $x + y = 10$ ,  $x, y \in N$ .

(iii)  $xy$  is square of an integer  $x, y \in N$ .

(iv)  $x + 4y = 10$ ,  $x, y \in N$

Determine which of the above relations are reflexive, symmetric and transitive.

**Sol.** (i)  $x$  is greater than  $y$ ,  $x, y \in N$

$$(x, x) \in R$$

For  $xRx$   $x > x$  is not true for any  $x \in N$ .

Therefore,  $R$  is not reflexive.

Let

$$(x, y) \in R \Rightarrow xRy$$

$$x > y$$

but  $y > x$  is not true for any  $x, y \in N$

Thus,  $R$  is not symmetric.

Let

$$xRy \text{ and } yRz$$

$$x > y \text{ and } y > z \Rightarrow x > z$$

$\Rightarrow$

$$xRz$$

So,  $R$  is transitive.

(ii)  $x + y = 10, x, y \in N$

$$R = \{(x, y) : x + y = 10, x, y \in N\}$$

$$R = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\} \quad (1, 1) \notin R$$

So,  $R$  is not reflexive.

$$(x, y) \in R \Rightarrow (y, x) \in R$$

Therefore,  $R$  is symmetric.

$$(1, 9) \in R, (9, 1) \in R \Rightarrow (1, 1) \notin R$$

Hence,  $R$  is not transitive.

(iii) Given  $xy$ , is square of an integer  $x, y \in N$ .

$$\Rightarrow R = \{(x, y) : xy \text{ is a square of an integer } x, y \in N\}$$

$$(x, x) \in R, \forall x \in N$$

As  $x^2$  is square of an integer for any  $x \in N$ .

Hence,  $R$  is reflexive.

$$\text{If } (x, y) \in R \Rightarrow (y, x) \in R$$

Therefore,  $R$  is symmetric.

$$\text{If } (x, y) \in R, (y, z) \in R$$

So,  $xy$  is square of an integer and  $yz$  is square of an integer.

Let  $xy = m^2$  and  $yz = n^2$  for some  $m, n \in Z$

$$x = \frac{m^2}{y} \text{ and } z = \frac{x^2}{y}$$

$$xz = \frac{m^2 n^2}{y^2}, \text{ which is square of an integer.}$$

So,  $R$  is transitive.

(iv)  $x + 4y = 10, x, y \in N$

$$R = \{(x, y) : x + 4y = 10, x, y \in N\}$$

$$R = \{(2, 2), (6, 1)\}$$

$$(1, 1), (3, 3), \dots, \notin R$$

Thus,  $R$  is not reflexive.

$$(6, 1) \in R \text{ but } (1, 6) \notin R$$

Hence,  $R$  is not symmetric.

$$(x, y) \in R \Rightarrow x + 4y = 10 \text{ but } (y, z) \in R$$

$$y + 4z = 10 \Rightarrow (x, z) \in R$$

So,  $R$  is transitive.

**Q. 23** Let  $A = \{1, 2, 3, \dots, 9\}$  and  $R$  be the relation in  $A \times A$  defined by  $(a, b) R (c, d)$  if  $a + d = b + c$  for  $(a, b), (c, d)$  in  $A \times A$ . Prove that  $R$  is an equivalence relation and also obtain the equivalent class  $[(2, 5)]$ .

**Sol.** Given that,  $A = \{1, 2, 3, \dots, 9\}$  and  $(a, b) R (c, d)$  if  $a + d = b + c$  for  $(a, b) \in A \times A$  and  $(c, d) \in A \times A$ .

Let  $(a, b) R (a, b)$

$$\Rightarrow a + b = b + a, \forall a, b \in A$$

which is true for any  $a, b \in A$ .

Hence,  $R$  is reflexive.

Let  $(a, b) R (c, d)$   $a + d = b + c$

$$c + b = d + a \Rightarrow (c, d) R (a, b)$$

So,  $R$  is symmetric.

Let  $(a, b) R (c, d)$  and  $(c, d) R (e, f)$

$$a + d = b + c \text{ and } c + f = d + e$$

$$a + d = b + c \text{ and } d + e = c + f$$

$$(a + d) - (d + e) = (b + c) - (c + f)$$

$$(a - e) = b - f$$

$$a + f = b + e$$

$$(a, b) R (e, f)$$

So,  $R$  is transitive.

Hence,  $R$  is an equivalence relation.

Now, equivalence class containing  $[(2, 5)]$  is  $\{(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)\}$ .

**Q. 24** Using the definition, prove that the function  $f : A \rightarrow B$  is invertible if and only if  $f$  is both one-one and onto.

**Sol.** A function  $f : X \rightarrow Y$  is defined to be invertible, if there exist a function  $g : Y \rightarrow X$  such that  $gof = I_X$  and  $fog = I_Y$ . The function is called the inverse of  $f$  and is denoted by  $f^{-1}$ .

A function  $f : X \rightarrow Y$  is invertible iff  $f$  is a bijective function.

**Q. 25** Functions  $f, g : R \rightarrow R$  are defined, respectively, by  $f(x) = x^2 + 3x + 1$ ,  $g(x) = 2x - 3$ , find

(i)  $fog$                       (ii)  $gof$                       (iii)  $fof$                       (iv)  $gog$

**Sol.** Given that,  $f(x) = x^2 + 3x + 1$ ,  $g(x) = 2x - 3$

(i)  $fog = f\{g(x)\} = f(2x - 3)$   
 $= (2x - 3)^2 + 3(2x - 3) + 1$   
 $= 4x^2 + 9 - 12x + 6x - 9 + 1 = 4x^2 - 6x + 1$

(ii)  $gof = g\{f(x)\} = g(x^2 + 3x + 1)$   
 $= 2(x^2 + 3x + 1) - 3$   
 $= 2x^2 + 6x + 2 - 3 = 2x^2 + 6x - 1$

(iii)  $fof = f\{f(x)\} = f(x^2 + 3x + 1)$   
 $= (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1$   
 $= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$   
 $= x^4 + 6x^3 + 14x^2 + 15x + 5$

(iv)  $gog = g\{g(x)\} = g(2x - 3)$   
 $= 2(2x - 3) - 3$   
 $= 4x - 6 - 3 = 4x - 9$

**Q. 26** Let  $*$  be the binary operation defined on  $Q$ . Find which of the following binary operations are commutative

(i)  $a * b = a - b, \forall a, b \in Q$                       (ii)  $a * b = a^2 + b^2, \forall a, b \in Q$   
 (iii)  $a * b = a + ab, \forall a, b \in Q$                       (iv)  $a * b = (a - b)^2, \forall a, b \in Q$

**Sol.** Given that  $*$  be the binary operation defined on  $Q$ .

(i)  $a * b = a - b, \forall a, b \in Q$  and  $b * a = b - a$

So,  $a * b \neq b * a$  [ $\because b - a \neq a - b$ ]

Hence,  $*$  is not commutative.

(ii)  $a * b = a^2 + b^2$

$$b * a = b^2 + a^2$$

So,  $*$  is commutative.

[since, '+' is on rational is commutative]

(iii)  $a * b = a + ab$

$$b * a = b + ab$$

Clearly,  $a + ab \neq b + ab$

So,  $*$  is not commutative.

(iv)  $a * b = (a - b)^2, \forall a, b \in Q$

$$b * a = (b - a)^2$$

$$\therefore (a - b)^2 = (b - a)^2$$

Hence,  $*$  is commutative.

**Q. 27** If  $*$  be binary operation defined on  $R$  by  $a * b = 1 + ab, \forall a, b \in R$ .

Then, the operation  $*$  is

(i) commutative but not associative.

(ii) associative but not commutative.

(iii) neither commutative nor associative.

(iv) both commutative and associative.

**Sol.** (i) Given that,  $a * b = 1 + ab, \forall a, b \in R$

$$a * b = ab + 1 = b * a$$

So,  $*$  is a commutative binary operation.

Also,  $a * (b * c) = a * (1 + bc) = 1 + a(1 + bc)$

$$a * (b * c) = 1 + a + abc \quad \dots(i)$$

$$(a * b) * c = (1 + ab) * c$$

$$= 1 + (1 + ab)c = 1 + c + abc \quad \dots(ii)$$

From Eqs. (i) and (ii),

$$a * (b * c) \neq (a * b) * c$$

So,  $*$  is not associative

Hence,  $*$  is commutative but not associative.

## Objective Type Questions

**Q. 28** Let  $T$  be the set of all triangles in the Euclidean plane and let a relation  $R$  on  $T$  be defined as  $aRb$ , if  $a$  is congruent to  $b, \forall a, b \in T$ .

Then,  $R$  is

(a) reflexive but not transitive

(b) transitive but not symmetric

(c) equivalence

(d) None of these

**Sol.** (c) Consider that  $aRb$ , if  $a$  is congruent to  $b, \forall a, b \in T$ .

Then,  $aRa \Rightarrow a \cong a$ ,

which is true for all  $a \in T$

So,  $R$  is reflexive,

... (i)

Let  $aRb \Rightarrow a \cong b$   
 $\Rightarrow b \cong a \Rightarrow bRa$  ... (ii)  
 $\Rightarrow bRa$   
 So,  $R$  is symmetric.  
 Let  $aRb$  and  $bRc$   
 $\Rightarrow a \cong b$  and  $b \cong c$   
 $\Rightarrow a \cong c \Rightarrow aRc$  ... (iii)  
 So,  $R$  is transitive.  
 Hence,  $R$  is equivalence relation.

**Q. 29** Consider the non-empty set consisting of children in a family and a relation  $R$  defined as  $aRb$ , if  $a$  is brother of  $b$ . Then,  $R$  is

- (a) symmetric but not transitive
- (b) transitive but not symmetric
- (c) neither symmetric nor transitive
- (d) both symmetric and transitive

**Sol. (b)** Given,  $aRb \Rightarrow a$  is brother of  $b$   
 $\therefore aRa \Rightarrow a$  is brother of  $a$ , which is not true.  
 So,  $R$  is not reflexive.

$$aRb \Rightarrow a \text{ is brother of } b.$$

This does not mean  $b$  is also a brother of  $a$  and  $b$  can be a sister of  $a$ .

Hence,  $R$  is not symmetric.

$$aRb \Rightarrow a \text{ is brother of } b$$

and  $bRc \Rightarrow b$  is a brother of  $c$ .

So,  $a$  is brother of  $c$ .

Hence,  $R$  is transitive.

**Q. 30** The maximum number of equivalence relations on the set  $A = \{1, 2, 3\}$  are

- (a) 1
- (b) 2
- (c) 3
- (d) 5

**Sol. (d)** Given that,  $A = \{1, 2, 3\}$   
 Now, number of equivalence relations as follows

$$R_1 = \{(1, 1), (2, 2), (3, 3)\}$$

$$R_2 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)\}$$

$$R_3 = \{(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)\}$$

$$R_4 = \{(1, 1), (2, 2), (3, 3), (2, 3), (3, 2)\}$$

$$R_5 = \{(1, 2, 3) \leftrightarrow A \times A = A^2\}$$

$\therefore$  Maximum number of equivalence relation on the set  $A = \{1, 2, 3\} = 5$

**Q. 31** If a relation  $R$  on the set  $\{1, 2, 3\}$  be defined by  $R = \{(1, 2)\}$ , then  $R$  is

- (a) reflexive
- (b) transitive
- (c) symmetric
- (d) None of these

**Sol. (b)**  $R$  on the set  $\{1, 2, 3\}$  be defined by  $R = \{(1, 2)\}$   
 It is clear that  $R$  is transitive.

**Q. 32** Let us define a relation  $R$  in  $R$  as  $aRb$  if  $a \geq b$ . Then,  $R$  is

- (a) an equivalence relation
- (b) reflexive, transitive but not symmetric
- (c) symmetric, transitive but not reflexive
- (d) neither transitive nor reflexive but symmetric

**Sol. (b)** Given that,  $aRb$  if  $a \geq b$   
 $\Rightarrow aRa \Rightarrow a \geq a$  which is true.  
 Let  $aRb$ ,  $a \geq b$ , then  $b \geq a$  which is not true  $R$  is not symmetric.  
 But  $aRb$  and  $bRc$   
 $\Rightarrow a \geq b$  and  $b \geq c$   
 $\Rightarrow a \geq c$   
 Hence,  $R$  is transitive.

**Q. 33** If  $A = \{1, 2, 3\}$  and consider the relation

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$$

Then,  $R$  is

- (a) reflexive but not symmetric
- (b) reflexive but not transitive
- (c) symmetric and transitive
- (d) neither symmetric nor transitive

**Sol. (a)** Given that,  $A = \{1, 2, 3\}$   
 and  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$   
 $\therefore (1, 1), (2, 2), (3, 3) \in R$   
 Hence,  $R$  is reflexive.  
 $(1, 2) \in R$  but  $(2, 1) \notin R$   
 Hence,  $R$  is not symmetric.  
 $(1, 2) \in R$  and  $(2, 3) \in R$   
 $\Rightarrow (1, 3) \in R$   
 Hence,  $R$  is transitive.

**Q. 34** The identity element for the binary operation  $*$  defined on  $Q - \{0\}$  as

$$a * b = \frac{ab}{2}, \forall a, b \in Q - \{0\} \text{ is}$$

- (a) 1
- (b) 0
- (c) 2
- (d) None of these

**Thinking Process**

For given binary operation  $*$  :  $A \times A \rightarrow A$ , an element  $e \in A$ , if it exists, is called identity for the operation  $*$ , if  $a * e = a = e * a, \forall a \in A$ .

**Sol. (c)** Given that,  $a * b = \frac{ab}{2}, \forall a, b \in Q - \{0\}$ .  
 Let  $e$  be the identity element for  $*$ .  
 $\therefore a * e = \frac{ae}{2}$   
 $\Rightarrow a = \frac{ae}{2} \Rightarrow e = 2$

**Q. 35** If the set  $A$  contains 5 elements and the set  $B$  contains 6 elements, then the number of one-one and onto mappings from  $A$  to  $B$  is

- (a) 720                      (b) 120                      (c) 0                      (d) None of these

**Sol. (c)** We know that, if  $A$  and  $B$  are two non-empty finite set containing  $m$  and  $n$  elements respectively, then the number of one-one and onto mapping from  $A$  to  $B$  is

$$n! \text{ if } m = n$$

$$0 \text{ if } m \neq n$$

Given that,

$$m = 5 \text{ and } n = 6$$

$\therefore$

$$m \neq n$$

Number of mapping = 0

**Q. 36** If  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$ . Then, the number of surjections from  $A$  into  $B$  is

- (a)  ${}^n P_2$                       (b)  $2^n - 2$                       (c)  $2^n - 1$                       (d) None of these

**Sol. (d)** Given that,  $A = \{1, 2, 3, \dots, n\}$  and  $B = \{a, b\}$ .

We know that, if  $A$  and  $B$  are two non-empty finite sets containing  $m$  and  $n$  elements respectively, then the number of surjection from  $A$  into  $B$  is

$${}^n C_m \times m!, \text{ if } n \geq m$$

$$0 \text{ if } n < m$$

Here,  $m = 2$

$$\begin{aligned} \therefore \text{ Number of surjection from } A \text{ into } B \text{ is } {}^n C_2 \times 2! &= \frac{n!}{2!(n-2)!} \times 2! \\ &= \frac{n(n-1)(n-2)!}{2 \times 1(n-2)!} \times 2! = n^2 - n \end{aligned}$$

**Q. 37** If  $f : R \rightarrow R$  be defined by  $f(x) = \frac{1}{x}, \forall x \in R$ . Then,  $f$  is

- (a) one-one                      (b) onto                      (c) bijective                      (d)  $f$  is not defined

**Thinking Process**

In the given function at  $x=0, f(x)=\infty$ . So, the function is not define.

**Sol. (d)** Given that,  $f(x) = \frac{1}{x}, \forall x \in R$

For

$$x = 0,$$

$f(x)$  is not defined.

Hence,  $f(x)$  is a not define function.

**Q. 38** If  $f : R \rightarrow R$  be defined by  $f(x) = 3x^2 - 5$  and  $g : R \rightarrow R$  by

$g(x) = \frac{x}{x^2 + 1}$ . Then,  $gof$  is

(a)  $\frac{3x^2 - 5}{9x^4 - 30x^2 + 26}$

(b)  $\frac{3x^2 - 5}{9x^4 - 6x^2 + 26}$

(c)  $\frac{3x^2}{x^4 + 2x^2 - 4}$

(d)  $\frac{3x^2}{9x^4 + 30x^2 - 2}$

**Sol. (a)** Given that,  $f(x) = 3x^2 - 5$  and  $g(x) = \frac{x}{x^2 + 1}$

$$\begin{aligned}gof &= g\{f(x)\} = g(3x^2 - 5) \\ &= \frac{3x^2 - 5}{(3x^2 - 5)^2 + 1} = \frac{3x^2 - 5}{9x^4 - 30x^2 + 25 + 1} \\ &= \frac{3x^2 - 5}{9x^4 - 30x^2 + 26}\end{aligned}$$

**Q. 39** Which of the following functions from  $Z$  into  $Z$  are bijections?

(a)  $f(x) = x^3$       (b)  $f(x) = x + 2$       (c)  $f(x) = 2x + 1$       (d)  $f(x) = x^2 + 1$

**Sol. (b)** Here,  $f(x) = x + 2 \Rightarrow f(x_1) = f(x_2)$

$$x_1 + 2 = x_2 + 2 \Rightarrow x_1 = x_2$$

Let  $y = x + 2$

$$x = y - 2 \in Z, \forall y \in x$$

Hence,  $f(x)$  is one-one and onto.

**Q. 40** If  $f : R \rightarrow R$  be the functions defined by  $f(x) = x^3 + 5$ , then  $f^{-1}(x)$  is

(a)  $(x + 5)^{\frac{1}{3}}$       (b)  $(x - 5)^{\frac{1}{3}}$       (c)  $(5 - x)^{\frac{1}{3}}$       (d)  $5 - x$

**Sol. (b)** Given that,  $f(x) = x^3 + 5$

Let  $y = x^3 + 5 \Rightarrow x^3 = y - 5$

$$x = (y - 5)^{\frac{1}{3}} \Rightarrow f(x)^{-1} = (x - 5)^{\frac{1}{3}}$$

**Q. 41** If  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be the bijective functions, then  $(gof)^{-1}$  is

(a)  $f^{-1}og^{-1}$       (b)  $fog$       (c)  $g^{-1}of^{-1}$       (d)  $gof$

**Sol. (a)** Given that,  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be the bijective functions.

$$(gof)^{-1} = f^{-1}og^{-1}$$

**Q. 42** If  $f : R - \left\{ \frac{3}{5} \right\} \rightarrow R$  be defined by  $f(x) = \frac{3x + 2}{5x - 3}$ , then

(a)  $f^{-1}(x) = f(x)$       (b)  $f^{-1}(x) = -f(x)$       (c)  $(fof)x = -x$       (d)  $f^{-1}(x) = \frac{1}{19}f(x)$

**Sol. (a)** Given that,  $f(x) = \frac{3x + 2}{5x - 3}$

Let  $y = \frac{3x + 2}{5x - 3}$

$$3x + 2 = 5xy - 3y \Rightarrow x(3 - 5y) = -3y - 2$$

$$x = \frac{3y + 2}{5y - 3} \Rightarrow f^{-1}(x) = \frac{3x + 2}{5x - 3}$$

$\therefore f^{-1}(x) = f(x)$

**Q. 43** If  $f: [0, 1] \rightarrow [0, 1]$  be defined by  $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$

then  $(f \circ f)x$  is

- (a) constant                      (b)  $1 + x$                       (c)  $x$                       (d) None of these

**Sol. (c)** Given that,  $f: [0, 1] \rightarrow [0, 1]$  be defined by

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1 - x, & \text{if } x \text{ is irrational} \end{cases}$$

$$\therefore (f \circ f)x = f(f(x)) = x$$

**Q. 44** If  $f: [2, \infty) \rightarrow R$  be the function defined by  $f(x) = x^2 - 4x + 5$ , then the range of  $f$  is

- (a)  $R$                       (b)  $[1, \infty)$                       (c)  $[4, \infty)$                       (d)  $[5, \infty)$

● **Thinking Process**

Range of  $f = \{y \in Y : y = f(x) \text{ for some } x \in X\}$

**Sol. (b)** Given that,  $f(x) = x^2 - 4x + 5$

Let

$$y = x^2 - 4x + 5$$

$\Rightarrow$

$$y = x^2 - 4x + 4 + 1 = (x - 2)^2 + 1$$

$\Rightarrow$

$$(x - 2)^2 = y - 1 \Rightarrow x - 2 = \sqrt{y - 1}$$

$\Rightarrow$

$$x = 2 + \sqrt{y - 1}$$

$\therefore$

$$y - 1 \geq 0, y \geq 1$$

$$\text{Range} = [1, \infty)$$

**Q. 45** If  $f: N \rightarrow R$  be the function defined by  $f(x) = \frac{2x - 1}{2}$  and  $g: Q \rightarrow R$

be another function defined by  $g(x) = x + 2$ . Then,  $(g \circ f)\frac{3}{2}$  is

- (a) 1                      (b) 1                      (c)  $\frac{7}{2}$                       (d) None of these

**Sol. (d)** Given that,  $f(x) = \frac{2x - 1}{2}$  and  $g(x) = x + 2$

$$\begin{aligned} (g \circ f)\frac{3}{2} &= g\left[f\left(\frac{3}{2}\right)\right] = g\left(\frac{2 \times \frac{3}{2} - 1}{2}\right) \\ &= g(1) = 1 + 2 = 3 \end{aligned}$$

**Q. 46** If  $f : R \rightarrow R$  be defined by  $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$

Then,  $f(-1) + f(2) + f(4)$  is

- (a) 9                      (b) 14                      (c) 5                      (d) None of these

**Sol. (a)** Given that,  $f(x) = \begin{cases} 2x : x > 3 \\ x^2 : 1 < x \leq 3 \\ 3x : x \leq 1 \end{cases}$

$$\begin{aligned} f(-1) + f(2) + f(4) &= 3(-1) + (2)^2 + 2 \times 4 \\ &= -3 + 4 + 8 = 9 \end{aligned}$$

**Q. 47** If  $f : R \rightarrow R$  be given by  $f(x) = \tan x$ , then  $f^{-1}(1)$  is

- (a)  $\frac{\pi}{4}$                       (b)  $\left\{n\pi + \frac{\pi}{4} : n \in Z\right\}$   
 (c) Does not exist                      (d) None of these

**Sol. (a)** Given that,  $f(x) = \tan x$   
 Let  $y = \tan x \Rightarrow x = \tan^{-1} y$   
 $\Rightarrow f^{-1}(x) = \tan^{-1} x \Rightarrow f^{-1}(1) = \tan^{-1} 1$   
 $\Rightarrow \tan^{-1} \tan \frac{\pi}{4} = \frac{\pi}{4}$  [  $\because \tan \frac{\pi}{4} = 1$  ]

## Fillers

**Q. 48** Let the relation  $R$  be defined in  $N$  by  $aRb$ , if  $2a + 3b = 30$ . Then,  $R = \dots$

**Sol.** Given that,  $2a + 3b = 30$   
 $3b = 30 - 2a$   
 $b = \frac{30 - 2a}{3}$   
 For  $a = 3, b = 8$   
 $a = 6, b = 6$   
 $a = 9, b = 4$   
 $a = 12, b = 2$   
 $R = \{(3, 8), (6, 6), (9, 4), (12, 2)\}$

**Q. 49** If the relation  $R$  be defined on the set  $A = \{1, 2, 3, 4, 5\}$  by  $R = \{(a, b) : |a^2 - b^2| < 8\}$ . Then,  $R$  is given by .....

**Sol.** Given,  $A = \{1, 2, 3, 4, 5\}$ ,  
 $R = \{(a, b) : |a^2 - b^2| < 8\}$   
 $R = \{(1, 1), (1, 2), (2, 1), (2, 2), (2, 3), (3, 2), (3, 3), (4, 3), (3, 4), (4, 4), (5, 5)\}$

**Q. 50** If  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(2, 3), (5, 1), (1, 3)\}$ , then  
 $gof = \dots\dots\dots$  and  $fog = \dots\dots\dots$  .

**Sol.** Given that,  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(2, 3), (5, 1), (1, 3)\}$   
 $gof(1) = g\{f(1)\} = g(2) = 3$   
 $gof(3) = g\{f(3)\} = g(5) = 1$   
 $gof(4) = g\{f(4)\} = g(1) = 3$   
 $gof = \{(1, 3), (3, 1), (4, 3)\}$   
 Now,  
 $fog(2) = f\{g(2)\} = f(3) = 5$   
 $fog(5) = f\{g(5)\} = f(1) = 2$   
 $fog(1) = f\{g(1)\} = f(3) = 5$   
 $fog = \{(2, 5), (5, 2), (1, 5)\}$

**Q. 51** If  $f : R \rightarrow R$  be defined by  $f(x) = \frac{x}{\sqrt{1+x^2}}$ , then  $(fofof)(x) = \dots\dots\dots$  .

**Sol.** Given that,  $f(x) = \frac{x}{\sqrt{1+x^2}}$   
 $(fofof)(x) = f[f\{f(x)\}]$   
 $= f \left[ f \left( \frac{x}{\sqrt{1+x^2}} \right) \right] = f \left( \frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1 + \frac{x^2}{1+x^2}}} \right)$   
 $= f \left[ \frac{x\sqrt{1+x^2}}{\sqrt{1+x^2}(\sqrt{2x^2+1})} \right] = f \left( \frac{x}{\sqrt{1+2x^2}} \right)$   
 $= \frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{1 + \frac{x^2}{1+2x^2}}} = \frac{x\sqrt{1+2x^2}}{\sqrt{1+2x^2}\sqrt{1+3x^2}}$   
 $= \frac{x}{\sqrt{1+3x^2}} = \frac{x}{\sqrt{3x^2+1}}$

**Q. 52** If  $f(x) = [4 - (x - 7)^3]$ , then  $f^{-1}(x) = \dots\dots\dots$  .

**Sol.** Given that,  $f(x) = [4 - (x - 7)^3]$   
 Let  $y = [4 - (x - 7)^3]$   
 $(x - 7)^3 = 4 - y$   
 $(x - 7) = (4 - y)^{1/3}$   
 $\Rightarrow x = 7 + (4 - y)^{1/3}$   
 $f^{-1}(x) = 7 + (4 - x)^{1/3}$

## True/False

**Q. 53** Let  $R = \{(3, 1), (1, 3), (3, 3)\}$  be a relation defined on the set  $A = \{1, 2, 3\}$ . Then,  $R$  is symmetric, transitive but not reflexive.

**Sol. False**

Given that,  $R = \{(3, 1), (1, 3), (3, 3)\}$  be defined on the set  $A = \{1, 2, 3\}$

$$(1, 1) \notin R$$

So,  $R$  is not reflexive.

$$(3, 1) \in R, (1, 3) \in R$$

Hence,  $R$  is symmetric.

Since,

$$(3, 1) \in R, (1, 3) \in R$$

But

$$(1, 1) \notin R$$

Hence,  $R$  is not transitive.

**Q. 54** If  $f : R \rightarrow R$  be the function defined by  $f(x) = \sin(3x + 2) \forall x \in R$ . Then,  $f$  is invertible.

**Sol. False**

Given that,  $f(x) = \sin(3x + 2), \forall x \in R$  is not one-one function for all  $x \in R$ .

So,  $f$  is not invertible.

**Q. 55** Every relation which is symmetric and transitive is also reflexive.

**Sol. False**

Let  $R$  be a relation defined by

$$R = \{(1, 2), (2, 1), (1, 1), (2, 2)\} \text{ on the set } A = \{1, 2, 3\}$$

It is clear that  $(3, 3) \notin R$ . So, it is not reflexive.

**Q. 56** An integer  $m$  is said to be related to another integer  $n$ , if  $m$  is a integral multiple of  $n$ . This relation in  $Z$  is reflexive, symmetric and transitive.

**Sol. False**

The given relation is reflexive and transitive but not symmetric.

**Q. 57** If  $A = \{0, 1\}$  and  $N$  be the set of natural numbers. Then, the mapping  $f : N \rightarrow A$  defined by  $f(2n - 1) = 0, f(2n) = 1, \forall n \in N$ , is onto.

**Sol. True**

Given,

$$A = \{0, 1\}$$

$$f(2n - 1) = 0, f(2n) = 1, \forall n \in N$$

So, the mapping  $f : N \rightarrow A$  is onto.

**Q. 58** The relation  $R$  on the set  $A = \{1, 2, 3\}$  defined as  $R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$  is reflexive, symmetric and transitive.

**Sol. False**

Given that,

$$R = \{(1, 1), (1, 2), (2, 1), (3, 3)\}$$
$$(2, 2) \notin R$$

So,  $R$  is not reflexive.

**Q. 59** The composition of function is commutative.

**Sol.** *False*

Let

$$f(x) = x^2$$

and

$$g(x) = x + 1$$

$$f \circ g(x) = f\{g(x)\} = f(x + 1)$$

$$= (x + 1)^2 = x^2 + 2x + 1$$

$$g \circ f(x) = g\{f(x)\} = g(x^2) = x^2 + 1$$

$\therefore$

$$f \circ g(x) \neq g \circ f(x)$$

**Q. 60** The composition of function is associative.

**Sol.** *True*

Let

$$f(x) = x, g(x) = x + 1$$

and

$$h(x) = 2x - 1$$

Then,

$$f \circ \{g \circ h(x)\} = f\{g\{h(x)\}\}$$

$$= f\{g(2x - 1)\}$$

$$= f(2x - 1 + 1)$$

$$= f(2x) = 2x$$

$\therefore$

$$(f \circ g) \circ h(x) = (f \circ g)\{h(x)\}$$

$$= (f \circ g)(2x - 1)$$

$$= f\{g(2x - 1)\}$$

$$= f(2x - 1 + 1)$$

$$= f(2x) = 2x$$

**Q. 61** Every function is invertible.

**Sol.** *False*

Only bijective functions are invertible.

**Q. 62** A binary operation on a set has always the identity element.

**Sol.** *False*

'+' is a binary operation on the set  $N$  but it has no identity element.