1

Sets

Short Answer Type Questions

Q. 1 Write the following sets in the roaster form.

(i) $A = \{x : x \in R, 2x + 11 = 15\}$

(ii) $B = \{x \mid x^2 = x, x \in R\}$

(iii) $C = \{x \mid x$ is a positive factor of a prime number $p\}$

Thinking Process

Solve the equation and get the value of x.

Sol. **(i)** We have, $A = \{x : x \in R, 2x + 11 = 15\}$

$\therefore \quad 2x + 11 = 15$

$\Rightarrow \quad 2x = 15 - 11 \Rightarrow 2x = 4$

$\Rightarrow \quad x = 2$

$\therefore \quad A = \{2\}$

(ii) We have, $B = \{x \mid x^2 = x, x \in R\}$

$\therefore \quad x^2 = x$

$\Rightarrow \quad x^2 - x = 0 \Rightarrow x(x - 1) = 0$

$\Rightarrow \quad x = 0, 1$

$\therefore \quad B = \{0, 1\}$

(iii) We have, $C = \{x \mid x$ is a positive factor of prime number $p\}$.

Since, positive factors of a prime number are 1 and the number itself.

$\therefore \quad C = \{1, p\}$

Q. 2 Write the following sets in the roaster form.

(i) $D = \{t \mid t^3 = t, t \in R\}$

(ii) $E = \{w \mid \dfrac{w - 2}{w + 3} = 3, w \in R\}$

(iii) $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

Thinking Process

Solve the given equation and get the value of respective variable.

Sol. **(i)** We have, $D = \{t \mid t^3 = t, t \in R\}$

$\therefore \quad t^3 = t$

$\Rightarrow \quad t^3 - t = 0 \Rightarrow t(t^2 - 1) = 0$

$\Rightarrow \quad t(t-1)(t+1) = 0 \Rightarrow t = 0, 1, -1$

$\therefore \quad D = \{-1, 0, 1\}$

(ii) We have, $E = \{w \mid \frac{w-2}{w+3} = 3, w \in R\}$

$\therefore \quad \frac{w-2}{w+3} = 3$

$\Rightarrow \quad w - 2 = 3w + 9 \Rightarrow w - 3w = 9 + 2$

$\Rightarrow \quad -2w = 11 \Rightarrow w = \frac{-11}{2}$

$\therefore \quad E = \left\{\frac{-11}{2}\right\}$

(iii) We have, $F = \{x \mid x^4 - 5x^2 + 6 = 0, x \in R\}$

$\therefore \quad x^4 - 5x^2 + 6 = 0$

$\Rightarrow \quad x^4 - 3x^2 - 2x^2 + 6 = 0$

$\Rightarrow \quad x^2(x^2 - 3) - 2(x^2 - 3) = 0$

$\Rightarrow \quad (x^2 - 3)(x^2 - 2) = 0$

$\Rightarrow \quad x = \pm\sqrt{3}, \pm\sqrt{2}$

$\therefore \quad F = \{-\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}\}$

Note *In roaster form, the order in which elements are listed is immaterial. Thus, we can also write* $F = \{-\sqrt{3}, \sqrt{2}, -\sqrt{2}, \sqrt{3}\}$.

Q. 3 If $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$. Write Y in the roaster form.

Thinking Process

First, write all the factors of 2^{p-1}, *where* $p = 1, 2, 3, \ldots, p$ *and then get y.*

Sol. $Y = \{x \mid x$ is a positive factor of the number $2^{p-1}(2^p - 1)$, where $2^p - 1$ is a prime number$\}$.

So, the factor of 2^{p-1} are $1, 2, 2^2, 2^3, \ldots, 2^{p-1}$.

$\therefore \quad Y = \{1, 2, 2^2, 2^3, \ldots, 2^{p-1}, 2^p - 1\}$

Q. 4 State which of the following statements are true and which are false. Justify your answer.

(i) $35 \in \{x \mid x$ has exactly four positive factors$\}$.

(ii) $128 \in \{y \mid$ the sum of all the positive factors of y is $2y\}$.

(iii) $3 \notin \{x \mid x^4 - 5x^3 + 2x^2 - 112x + 6 = 0\}$.

(iv) $496 \notin \{y \mid$ the sum of all the positive factors of y is $2y\}$.

Sol. **(i)** Since, the factors of 35 are 1, 5, 7 and 35. So, statement (i) is true.

(ii) Since, the factors of 128 are 1, 2, 4, 8, 16, 32, 64 and 128.

$\therefore$ Sum of factors $= 1 + 2 + 4 + 8 + 16 + 32 + 64 + 128$

$= 255 \neq 2 \times 128$

So, statement (ii) is false.

(iii) We have, $x^4 - 5x^3 + 2x^2 - 112x + 6 = 0$

$\therefore$ For $x = 3$,

$$(3)^4 - 5(3)^3 + 2(3)^2 - 112(3) + 6 = 0$$

$\Rightarrow$ $81 - 135 + 18 - 336 + 6 = 0$

$\Rightarrow$ $-346 = 0$

which is not true.

Hence, statement (iii) is true.

(iv) $\because$ $496 = 2^4 \times 31$

So, the factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248 and 496.

$\therefore$ Sum of factors $= 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 + 496$

$= 992 = 2(496)$

So, $496 \in \{y \mid$ the sum of all the positive factor of y is $2y\}$.

Hence, statement (iv) is false.

Q. 5 If $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$, then verify that $L - (M \cup N) = (L - M) \cap (L - N)$.

Sol. Given, $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6\}$ and $N = \{1, 3, 5\}$

$\therefore$ $M \cup N = \{1, 3, 4, 5, 6\}$

$L - (M \cup N) = \{2\}$

Now, $L - M = \{1, 2\}$, $L - N = \{2, 4\}$

$\therefore$ $(L - M) \cap (L - N) = \{2\}$

Hence, $L - (M \cup N) = (L - M) \cap (L - N)$.

Q. 6 If A and B are subsets of the universal set U, then show that

(i) $A \subset A \cup B$ (ii) $A \subset B \Leftrightarrow A \cup B = B$

(iii) $(A \cap B) \subset A$

Sol. **(i)** Let $x \in A$

$\Rightarrow$ $x \in A$ or $x \in B$ $\Rightarrow$ $x \in A \cup B$

Hence, $\subset A \cup B$

(ii) If $A \subset B$

Let $x \in A \cup B$

$\Rightarrow$ $x \in A$ or $x \in B$ $\Rightarrow$ $x \in B$ $[\because A \subset B]$

$\Rightarrow$ $A \cup B \subset B$ ... (i)

But $B \subset A \cup B$ ... (ii)

From Eqs. (i) and (ii),

$A \cup B = B$

If $A \cup B = B$

Let $y \in A$

$\Rightarrow$ $y \in A \cup B$ $\Rightarrow$ $y \in B$ $[\because A \cup B = B]$

$\Rightarrow$ $A \subset B$

Hence, $A \subset B \Leftrightarrow A \cup B = B$

(iii) Let $x \in A \cap B$

$\Rightarrow$ $x \in A$ and $x \in B$ $\Rightarrow$ $x \in A$

Hence, $A \cap B \subset A$

Q. 7 Given that $N = \{1, 2, 3, \ldots, 100\}$. Then, write

(i) the subset of N whose elements are even numbers.

(ii) the subset of N whose elements are perfect square numbers.

Sol. We have, $N = \{1, 2, 3, 4, \ldots, 100\}$

(i) Required subset $= \{2, 4, 6, 8, \ldots, 100\}$

(ii) Required subset $= \{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

Q. 8 If $X = \{1, 2, 3\}$, if n represents any member of X, write the following sets containing all numbers represented by

(i) $4n$ (ii) $n + 6$ (iii) $\frac{n}{2}$ (iv) $n - 1$

Sol. Given, $X = \{1, 2, 3\}$

(i) $\{4n \mid n \in X\} = \{4, 8, 12\}$

(ii) $\{n + 6 \mid n \in X\} = \{7, 8, 9\}$

(iii) $\left\{\frac{n}{2} \mid n \in X\right\} = \left\{\frac{1}{2}, 1, \frac{3}{2}\right\}$

(iv) $\{n - 1 \mid n \in X\} = \{0, 1, 2\}$

Q. 9 If $Y = \{1, 2, 3, \ldots, 10\}$ and a represents any element of Y, write the following sets, containing all the elements satisfying the given conditions.

(i) $a \in Y$ but $a^2 \notin Y$

(ii) $a + 1 = 6, a \in Y$

(iii) a is less than 6 and $a \in Y$

Sol. Given, $Y = \{1, 2, 3, \ldots, 10\}$

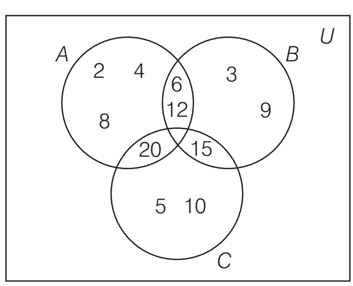
(i) $\{a : a \in Y \text{ and } a^2 \notin Y\} = \{4, 5, 6, 7, 8, 9, 10\}$

(ii) $\{a : a + 1 = 6, a \in Y\} = \{5\}$

(iii) is less than 6 and $a \in Y\} = \{1, 2, 3, 4, 5,\}$

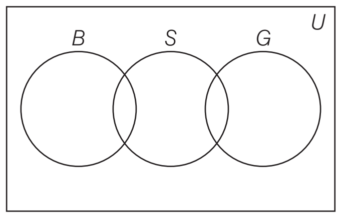
Q. 10 A, B and C are subsets of universal set U. If $A = \{2, 4, 6, 8, 12, 20\}$, $B = \{3, 6, 9, 12, 15\}$, $C = \{5, 10, 15, 20\}$ and U is the set of all whole numbers, draw a Venn diagram showing the relation of U, A, B and C.

Sol.



Q. 11 Let U be the set of all boys and girls in a school, G be the set of all girls in the school, B be the set of all boys in the school and S be the set of all students in the school who take swimming. Some but not all, students in the school take swimming. Draw a Venn diagram showing one of the possible interrelationship among sets U, G, B and S.

Sol.



Q. 12 For all sets A, B and C, show that $(A - B) \cap (A - C) = A - (B \cup C)$.

Thinking Process

To prove this we have to show that $(A-B) \cap (A-C) \subseteq A-(B \cup C)$ and $A-(B \cup C) \subseteq (A-B) \cap (A-C)$.

Sol. Let $x \in (A - B) \cap (A - C)$

$\Rightarrow$ $x \in (A - B)$ and $x \in (A - C)$

$\Rightarrow$ $(x \in A$ and $x \notin B)$ and $(x \in A$ and $x \notin C)$

$\Rightarrow$ $x \in A$ and $(x \notin B$ and $x \notin C)$

$\Rightarrow$ $x \in A$ and $x \notin (B \cup C)$

$\Rightarrow$ $x \in A - (B \cup C)$

$\Rightarrow$ $(A - B) \cap (A - C) \subset A - (B \cup C)$ ... (i)

Now, let $y \in A - (B \cup C)$

$\Rightarrow$ $y \in A$ and $y \notin (B \cup C)$

$\Rightarrow$ $y \in A$ and $(y \notin B$ and $y \notin C)$

$\Rightarrow$ $(y \in A$ and $y \notin B)$ and $(y \in A$ and $y \notin C)$

$\Rightarrow$ $y \in (A - B)$ and $y \in (A - C)$

$\Rightarrow$ $y \in (A - B) \cap (A - C)$

$\Rightarrow$ $A - (B \cup C) \subset (A - B) \cap (A - C)$ ...(ii)

From Eqs. (i) and (ii),

$$A - (B \cup C) = (A - B) \cap (A - C)$$

Q. 13 For all sets A and B, $(A - B) \cup (A \cap B) = A$.

Thinking Process

To solve the above problem, use distributive law on sets
i.e., $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

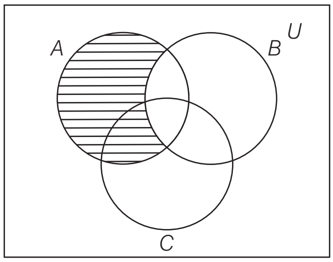
Sol.

$$\begin{aligned} \text{LHS} &= (A - B) \cup (A \cap B) \\ &= [(A - B) \cup A] \cap [(A - B) \cup B] \\ &= A \cap (A \cup B) = A = \text{RHS} \end{aligned}$$

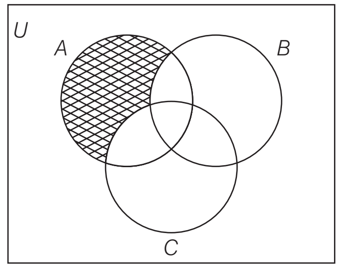
Hence, given statement is true.

Q. 14 For all sets A, B and C, $A - (B - C) = (A - B) - C$.

Sol. See the Venn diagrams given below, where shaded portions are representing $A-(B-C)$ and $(A-B)-C$ respectively.



$A - (B - C)$



$(A - B) - C$

Clearly, $A - (B - C) \neq (A - B) - C$.

Hence, given statement is false.

Q. 15 For all sets A, B and C, if $A \subset B$, then $A \cap C \subset B \cap C$.

Sol. Let $x \in A \cap C$

$\Rightarrow$ $x \in A$ and $x \in C$

$\Rightarrow$ $x \in B$ and $x \in C$ $\quad [\because A \subset B]$

$\Rightarrow$ $x \in (B \cap C) \Rightarrow (A \cap C) \subset (B \cap C)$

Hence, given statement is true.

Q. 16 For all sets A, B and C, if $A \subset B$, then $A \cup C \subset B \cup C$.

Sol. Let $x \in A \cup C$

$\Rightarrow$ $x \in A$ and $x \in C$

$\Rightarrow$ $x \in B$ and $x \in C$ $\quad [\because A \subset B]$

$\Rightarrow$ $x \in B \cup C \Rightarrow A \cup C \subset B \cup C$

Hence, given statement is true.

Q. 17 For all sets A, B and C, if $A \subset C$ and $B \subset C$, then $A \cup B \subset C$.

Sol. Let $x \in A \cup B$

$\Rightarrow$ $x \in A$ and $x \in B$

$\Rightarrow$ $x \in C$ and $x \in C$ $\quad [\because A \subset C \text{ and } B \subset C]$

$\Rightarrow$ $x \in C \Rightarrow A \cup B \subset C$

Hence, given statement is true.

Q. 18 For all sets A and B, $A \cup (B - A) = A \cup B$.

Thinking Process

To solve the above problem, use distributive law i.e., $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Sol. $\because$ LHS $= A \cup (B - A) = A \cup (B \cap A')$ $\quad [\because A - B = A \cap B']$

$= (A \cup B) \cap (A \cup A') = (A \cup B) \cap U$ $\quad [\because A \cup A' = U]$

$= A \cup B =$ RHS $\quad [\because A \cap U = A]$

Q. 19 For all sets A and B, $A - (A - B) = A \cap B$.

Sol.
$$\begin{aligned} \text{LHS} &= A - (A - B) = A - (A \cap B') && [\because A - B = A \cap B'] \\ &= A \cap (A \cap B')' = A \cap [A' \cup (B')'] && [\because (A \cap B)' = A' \cup B'] \\ &= A \cap (A' \cup B) && [\because (A')' = A] \\ &= (A \cap A') \cup (A \cap B) = \phi \cup (A \cap B) \\ &= A \cap B = \text{RHS} \end{aligned}$$

Q. 20 For all sets A and B, $A - (A \cap B) = A - B$.

Sol.
$$\begin{aligned} \text{LHS} &= A - (A \cap B) = A \cap (A \cap B)' && [\because A - B = A \cap B'] \\ &= A \cap (A' \cup B') && [\because (A \cap B)' = A' \cup B'] \\ &= (A \cap A') \cup (A \cap B') = \phi \cup (A \cap B') \\ &= A \cap B' && [\because \phi \cup A = A] \\ &= A - B = \text{RHS} \end{aligned}$$

Q. 21 For all sets A and B, $(A \cup B) - B = A - B$.

Sol.
$$\begin{aligned} \text{LHS} &= (A \cup B) - B = (A \cup B) \cap B' && [\because A - B = A \cap B'] \\ &= (A \cap B') \cup (B \cap B') = (A \cap B') \cup \phi && [\because B \cap B' = \phi] \\ &= A \cap B' && [\because A \cup \phi = A] \\ &= A - B = \text{RHS} \end{aligned}$$

Q. 22 Let $T = \left\{x \mid \dfrac{x+5}{x-7} - 5 = \dfrac{4x-40}{13-x}\right\}$. Is T an empty set? Justify your answer.

Thinking Process

First of all solve the given equation and get the value of x.

Sol. Since,
$$T = \left\{x \mid \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x}\right\}$$

$$\begin{aligned} \because \quad & \frac{x+5}{x-7} - 5 = \frac{4x-40}{13-x} \\ \Rightarrow \quad & \frac{x+5-5(x-7)}{x-7} = \frac{4x-40}{13-x} \\ \Rightarrow \quad & \frac{x+5-5x+35}{x-7} = \frac{4x-40}{13-x} \\ \Rightarrow \quad & \frac{-4x+40}{x-7} = \frac{4x-40}{13-x} \\ \Rightarrow \quad & -(4x-40)(13-x) = (4x-40)(x-7) \\ \Rightarrow \quad & (4x-40)(x-7) + (4x-40)(13-x) = 0 \\ \Rightarrow \quad & (4x-40)(x-7+13-x) = 0 \\ \Rightarrow \quad & 4(x-10)6 = 0 \\ \Rightarrow \quad & 24(x-10) = 0 \\ \Rightarrow \quad & x = 10 \\ \therefore \quad & T = \{10\} \end{aligned}$$

Hence, T is not an empty set.

Long Answer Type Questions

Q. 23 If A, B and C be sets. Then, show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Sol. Let $x \in A \cap (B \cup C)$

$\Rightarrow$ $x \in A$ and $x \in (B \cup C)$

$\Rightarrow$ $x \in A$ and ($x \in B$ or $x \in C$)

$\Rightarrow$ ($x \in A$ and $x \in B$) or ($x \in A$ and $x \in C$)

$\Rightarrow$ $x \in A \cap B$ or $x \in A \cap C$

$x \in (A \cap B) \cup (A \cap C)$

$\Rightarrow$ $A \cap (B \cup C) \subset (A \cap B) \cup (A \cap C)$ ...(i)

Again, let $y \in (A \cap B) \cup (A \cap C)$

$\Rightarrow$ $y \in (A \cap B)$ or $y \in (A \cap C)$

$\Rightarrow$ ($y \in A$ and $y \in B$) or ($y \in A$ and $y \in C$)

$\Rightarrow$ $y \in A$ and ($y \in B$ or $y \in C$)

$\Rightarrow$ $y \in A$ and $y \in B \cup C$

$\Rightarrow$ $y \in A \cap (B \cup C)$

$\Rightarrow$ $(A \cap B) \cup (A \cap C) \subset A \cap (B \cup C)$ ...(ii)

From Eqs.(i) and(ii),

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Q. 24 Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed

(i) in English and Mathematics but not in Science.

(ii) in Mathematics and Science but not in English.

(iii) in Mathematics only.

(iv) in more than one subject only.

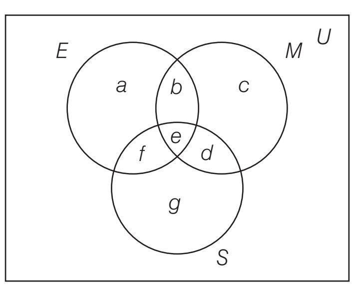
Sol. Let M be the set of students who passed in Mathematics, E be the set of students who passed in English and S be the set of students who passed in Science.

Then, $n(u) = 100$,

$n(E) = 15$, $n(M) = 12$, $n(S) = 8$, $n(E \cap M) = 6$, $n(M \cap S) = 7$,

$n(E \cap S) = 4$, and $n(E \cap M \cap S) = 4$,

$\because$ $n(E) = 15$



$\Rightarrow$ $a + b + e + f = 15$ ...(i)

and $n(M) = 12$

$\Rightarrow$ $b + c + e + d = 12$ ...(ii)

Also, $n(S) = 8$

$\Rightarrow \quad d + e + f + g = 8 \quad \text{...(iii)}$

$n(E \cap M) = 6$

$\Rightarrow \quad b + e = 6 \quad \text{...(iv)}$

$n(M \cap S) = 7$

$\Rightarrow \quad e + d = 7 \quad \text{... (v)}$

$n(E \cap S) = 4$

$\Rightarrow \quad e + f = 4 \quad \text{... (vi)}$

$n(E \cap M \cap S) = 4$

$\Rightarrow \quad e = 4 \quad \text{... (vii)}$

From Eqs. (vi) and (vii), $f = 0$

From Eqs. (v) and (vii), $d = 3$

From Eqs. (iv) and (vii), $b = 2$

On substituting the values of d, e and f in Eq. (iii), we get

$$3 + 4 + 0 + g = 8$$

$\Rightarrow \quad g = 1$

On substituting the value of b, e and d in Eq. (ii), we get

$$2 + c + 4 + 3 = 12$$

$\Rightarrow \quad c = 3$

On substituting b, e, and f in Eq. (i), we get

$$a + 2 + 4 + 0 = 15$$

$\Rightarrow \quad a = 9$

(i) Number of students who passed in English and Mathematics but not in Science

$$= b = 2$$

(ii) Number of students who passed in Mathematics and Science but not in English

$$= d = 3$$

(iii) Number of students who passed in Mathematics only $= c = 3$

(iv) Number of students who passed in more than one subject

$$= b + e + d + f$$
$$= 2 + 4 + 3 + 0 = 9$$

Alternate Method

Let E denotes the set of student who passed in English. M denotes the set of students who passed in Mathematics. S denotes the set of students who passed in Science.

Now, $n(U) = 100, \; n(E) = 15, n(m) = 12, \; n(S) = 8,$

$n(E \cap M) = 6, n(M \cap S) = 7,$

$n(E \cap S) = 4, n(E \cap M \cap S) = 4$

(i) Number of students passed in English and Mathematics but not in Science

i.e., $n(E \cap M \cap S') = n(E \cap M) - n(E \cap M \cap S) \quad [\because A \cap B' = A - (A \cap B)]$

$$= 6 - 4 = 2$$

(ii) Number of students passed in Mathematics and Science but not in English.

i.e., $n(M \cap S \cap E') = n(M \cap S) - n(M \cap S \cap E)$

$$= 7 - 4 = 3$$

(iii) Number of students passed in mathematics only

i.e., $n(M \cap S' \cap E') = n(M) - n(M \cap S) - n(M \cap E) + n(M \cap S \cap E)$

$$= 12 - 7 - 6 + 4 = 3$$

(iv) Number of students passed in more than one subject only

i.e., $n(E \cap M) + n(M \cap S) + n(E \cap S) - 3\,n(E \cap M \cap S) + n(E \cap M \cap S)$

$$= 6 + 7 + 4 - 4 \times 3 + 4$$
$$= 17 - 12 + 4 = 5 + 4 = 9$$

Q. 25 In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games. Find the number of students who play neither.

Sol. Let C be the set of students who play cricket and T be the set of students who play tennis.

Then, $n(U) = 60,\ n(C) = 25,\ n(T) = 20,$ and $n(C \cap T) = 10$

$\therefore\ n(C \cup T) = n(C) + n(T) - n(C \cap T)$

$= 25 + 20 - 10 = 35$

$\therefore$ Number of students who play neither $= n(U) - n(C \cup T)$

$= 60 - 35 = 25$

Q. 26 In a survey of 200 students of a school, it was found that 120 study Mathematics, 90 study Physics and 70 study Chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. Find the number of students who study all the three subjects.

Thinking Process

To solve this problem, use the formula for all the three subjects
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C).$

Sol. Let M be the set of students who study Mathematics, P be the set of students who study Physics and C be the set of students who study Chemistry.

Then, $n(U) = 200,\ n(M) = 120,\ n(P) = 90,$

$n(C) = 70,\ n(M \cap P) = 40,\ n(P \cap C) = 30,$

$n(C \cap M) = 50,\ n(M' \cap P' \cap C') = 20,$

$n(U) - n(M \cup P \cup C) = 20,$

$n(M \cup P \cup C) = 200 - 20 = 180$

$\because\ n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(C \cap M) + n(M \cap P \cap C)$

$\Rightarrow\ 180 = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap P \cap C)$

$\Rightarrow\ 180 = 160 + n(M \cap P \cap C)$

$\Rightarrow\ n(M \cap P \cap C) = 180 - 160 = 20$

So, the number of students who study all the three subjects is 20.

Q. 27 In a town of 10000 families, it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C, 5% families buy A and B, 3% buy B and C and 4% buy A and C. If 2% families buy all the three newspaper. Find

(i) the number of families which buy newspaper A only.

(ii) the number of families which buy none of A, B and C.

Sol. Let A be the set of families which buy newspaper A, B be the set of families which buy newspaper B and C be the set of families which buy newspaper C.

Then, $n(U) = 10000,\ n(A) = 40\%\ n(B) = 20\%$ and $n(C) = 10\%$

$n(A \cap B) = 5\%,$

$n(B \cap C) = 3\%$

$n(A \cap C) = 4\%$

$n(A \cap B \cap C) = 2\%$

(i) Number of families which buy newspaper A only

$$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$
$$= (40 - 5 - 4 + 2)\% = 33\%$$
$$10000 \times 33/100 = 3300$$

(ii) Number of families which buy none of A, B and C

$$= n(U) - n(A \cup B \cup C)$$
$$= n(U) - [n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)]$$
$$= 100 - [40 + 20 + 10 - 5 - 3 - 4 + 2]$$
$$= 100 - 60\% = 40\%$$
$$= 10000 \times \frac{40}{100} = 4000$$

Q. 28 In a group of 50 students, the number of students studying French, English, Sanskrit were found to be as follows French = 17, English = 13, Sanskrit = 15 French and English = 09, English and Sanskrit = 4, French and Sanskrit = 5, English, French and Sanskrit = 3. Find the number of students who study

(i) only French.
(ii) only English.
(iii) only Sanskrit.
(iv) English and Sanskrit but not French.
(v) French and Sanskrit but not English.
(vi) French and English but not Sanskrit.
(vii) atleast one of the three languages.
(viii) none of the three languages.

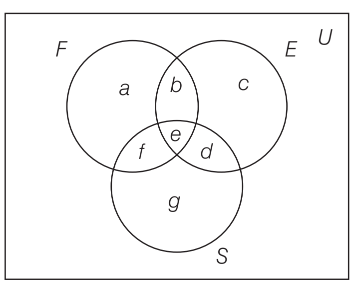
Sol. Let F be the set of students who study French, E be the set of students who study English and S be the set of students who study Sanskrit.

Then, $n(U) = 50$, $n(F) = 17$, $n(E) = 13$, and $n(S) = 15$,

$n(F \cap E) = 9$, $n(E \cap S) = 4$, $n(F \cap S) = 5$,

$n(F \cap E \cap S) = 3$,

$\because \quad n(F) = 17$



$\Rightarrow \quad a + b + e + f = 17 \quad$ ...(i)

$n(E) = 13$

$\Rightarrow \quad b + c + d + e = 13 \quad$ ... (ii)

$n(S) = 15$

$\Rightarrow \quad d + e + f + g = 15 \quad$ ... (iii)

$n(F \cap E) = 9$

$\Rightarrow \quad b + e = 9 \quad$ ... (iv)

$n(E \cap S) = 4$

$$\Rightarrow \quad e + d = 4 \quad \text{... (v)}$$
$$n(F \cap S) = 5$$
$$\Rightarrow \quad f + e = 5 \quad \text{... (vi)}$$
$$n(F \cap E \cap S) = 3$$
$$\Rightarrow \quad e = 3 \quad \text{... (vii)}$$

From Eqs. (vi) and (vii), $f = 2$
From Eqs. (v)and (vii), $d = 1$
From Eqs. (iv) and (vii), $b = 6$
On substituting the values of e, f and d in Eq. (iii), we get

$$1 + 3 + 2 + g = 15$$
$$\Rightarrow \quad g = 9$$

On substituting the values of b, d and e in Eq. (ii), we get

$$6 + c + 1 + 3 = 13$$
$$\Rightarrow \quad c = 3$$

On substituting the values of b, e and f in Eq. (i), we get

$$a + 6 + 3 + 2 = 17$$
$$\Rightarrow \quad a = 6$$

(i) Number of students who study French only, $a = 6$
(ii) Number of students who study English only, $c = 3$
(iii) Number of students who study Sanskrit only, $g = 9$
(iv) Number of students who study English and Sanskrit but not French, $d = 1$
(v) Number of students who study French and Sanskrit but not English, $f = 2$
(vi) Number of students who study French and English but not Sanskrit, $b = 6$
(vii) Number of students who study atleast one of the three languages

$$= a + b + c + d + e + f + g$$
$$= 6 + 6 + 3 + 1 + 3 + 2 + 9 = 30$$

(viii) Number of students who study none of three languages = Total students – Students who study atleast one of the three languages

$$= 50 - 30 = 20$$

Objective Type Questions

Q. 29 Suppose, $A_1, A_2, \ldots, A_{30}$ are thirty sets each having 5 elements and B_1, B_2, B_n are n sets each with 3 elements, let $\bigcup_{i=1}^{30} A_i = \bigcup_{j=1}^{n} B_j = S$ and each element of S belongs to exactly 10 of the A_i's and exactly 9 of the B_j's. Then, n is equal to

(a) 15 (b) 3
(c) 45 (d) 35

Thinking Process

First find the total number of elements for the both sets, then compare them.

Sol. *(c)* If elements are not repeated, then number of elements in $A_1 \cup A_2 \cup A_3, \dots \cup A_{30}$ is 30×5.

But each element is used 10 times, so

$$S = \frac{30 \times 5}{10} = 15$$

If elements in $B_1, B_2, \dots, B_n$ are not repeated, then total number of elements is $3n$ but each element is repeated 9 times, so

$$S = \frac{3n}{9} \Rightarrow 15 = \frac{3n}{9}$$

$$\therefore \quad n = 45$$

Q. 30 Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n are, respectively

(a) 4, 7 (b) 7, 4 (c) 4, 4 (d) 7, 7

Thinking Process

We know that, if a set A contains n elements, then the number of subsets of A is equal to 2^n.

Sol. *(a)* Since, number of subsets of a set containing m elements is 112 more than the subsets of the set containing n elements.

$$\because \quad 2^m - 2^n = 112$$

$$\Rightarrow \quad 2^n \cdot (2^{m-n} - 1) = 2^4 \cdot 7$$

$$\Rightarrow \quad 2^n = 2^4 \text{ and } 2^{m-n} - 1 = 7$$

$$\Rightarrow \quad n = 4 \text{ and } 2^{m-n} = 8$$

$$\Rightarrow \quad 2^{m-n} = 2^3 \Rightarrow m - n = 3$$

$$\Rightarrow \quad m - 4 = 3 \Rightarrow m = 4 + 3$$

$$\therefore \quad m = 7$$

Q. 31 The set $(A \cap B')' \cup (B \cap C)$ is equal to

(a) $A' \cup B \cup C$ (b) $A' \cup B$ (c) $A' \cup C'$ (d) $A' \cap B$

Sol. *(b)* We know that, $(A \cap B)' = (A' \cup B')$ and $(A')' = A$

$$\therefore \quad = (A \cap B')' \cup (B \cap C)$$

$$= [A' \cup (B')'] \cup (B \cap C)$$

$$= (A' \cup B) \cup (B \cap C) = A' \cup B$$

Q. 32 Let F_1 be the set of parallelograms, F_2 the set of rectangles, F_3 the set of rhombuses, F_4 the set of squares and F_5 the set of trapeziums in a plane. Then, F_1 may be equal to

(a) $F_2 \cap F_3$ (b) $F_3 \cap F_4$

(c) $F_2 \cup F_5$ (d) $F_2 \cup F_3 \cup F_4 \cup F_1$

Sol. *(d)* Every rectangle, rhombus, square in a plane is a parallelogram but every trapezium is not a parallelogram.

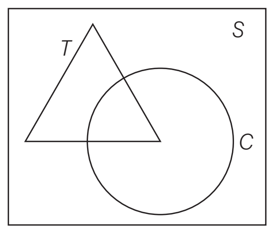
So, F_1 is either of F_1, F_2, F_3 and F_4.

$$\therefore \quad F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$$

Q. 33 Let S = set of points inside the square, T = set of points inside the triangle and C = set of points inside the circle. If the triangle and circle intersect each other and are contained in a square. Then,

(a) $S \cap T \cap C = \phi$ (b) $S \cup T \cup C = C$
(c) $S \cup T \cup C = S$ (d) $S \cup T = S \cap C$

Sol. ***(c)*** The given sets can be represented in Venn diagram as shown below



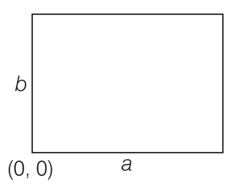
It is clear from the diagram that, $S \cup T \cup C = S$.

Q. 34 If R be the set of points inside a rectangle of sides a and b ($a, b > 1$) with two sides along the positive direction of X-axis and Y-axis. Then,

(a) $R = \{(x, y) : 0 \le x \le a, 0 \le y \le b\}$
(b) $R = \{(x, y) : 0 \le x < a, 0 \le y \le b\}$
(c) $R = \{(x, y) : 0 \le x \le a, 0 < y < b\}$
(d) $R = \{(x, y) : 0 < x < a, 0 < y < b\}$

Sol. ***(d)*** Since, R be the set of points inside the rectangle.

$\therefore \quad R = \{(x, y) : 0 < x < a \text{ and } 0 < y < b\}$



Q. 35 In a town of 840 persons, 450 persons read Hindi, 300 read English and 200 read both. Then, the number of persons who read neither, is

(a) 210 (b) 290 (c) 180 (d) 260

Sol. ***(b)*** Let H be the set of persons who read Hindi and E be the set of persons who read English.

Then, $n(U) = 840$, $n(H) = 450$, $n(E) = 300$, $n(H \cap E) = 200$

Number of persons who read neither $= n(H' \cap F')$

$= n(H \cup E)'$

$= n(U) - n(H \cup E)$

$= 840 - [n(H) + n(E) - n(H \cap E)]$

$= 840 - (450 + 300 - 200)$

$= 840 - 550 = 290$

Q. 36 If $X = \{8^n - 7n - 1 | n \in N\}$ and $y = \{49n - 49 | n \in N\}$. Then,

(a) $X \subset Y$ (b) $Y \subset X$ (c) $X = Y$ (d) $X \cap Y = \phi$

Thinking Process

If every element of A is an elements of B, then $A \subseteq B$.

Sol. *(a)*
$$X = \{8^n - 7n - 1 | n \in N\} = \{0, 49, 490, ...\}$$
$$Y = \{49n - 49 | n \in N\} = \{0, 49, 98, 147, ...\}$$

Clearly, every elements of X is in Y but every element of Y is not in X.

$\therefore \quad X \subset Y$

Q. 37 A survey shows that 63% of the people watch a news channel whereas 76%watch another channel. If x% of the people watch both channel, then

(a) $x = 35$ (b) $x = 63$ (c) $39 \le x \le 63$ (d) $x = 39$

Sol. *(c)* Let A be the set of percentage of those people who watch a news channel and B be the set of percentage of those people who watch another channel.

$$n(A) = 63, \; n(B) = 76, \text{ and } n(A \cap B) = x$$

$\because \quad n(A \cup B) \le 100$

$\Rightarrow \quad n(A) + n(B) - n(A \cap B) \le 100$

$\Rightarrow \quad 63 + 76 - x \le 100 \quad \Rightarrow \quad 139 - x \le 100$

$\Rightarrow \quad 139 - 100 \le x \quad \Rightarrow \quad 39 \le x$

$\because \quad n(A) = 63$

$\Rightarrow \quad x(A \cap B) \le n(A) \quad \Rightarrow \quad x \le 63$

$\therefore \quad 39 \le x \le 63$

Q. 38 If sets A and B are defined as

$$A = \{(x, y) | y = \frac{1}{x}, 0 \ne x \in R\}, B = \{(x, y) | y = -x, x \in R,\}. \text{ Then,}$$

(a) $A \cap B = A$ (b) $A \cap B = B$ (c) $A \cap B = \phi$ (d) $A \cup B = A$

Sol. *(c)* Let $x \in R$

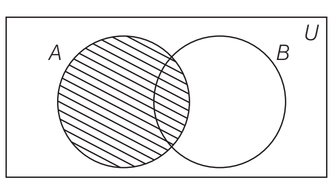
We know that, $\quad -x \ne \dfrac{1}{x}$

$\therefore \quad A \cap B = \phi$

Q. 39 If A and B are two sets, then $A \cap (A \cup B)$ equals to

(a) A (b) B (c) ϕ (d) $A \cap B$

Sol. *(a)* $\because \quad A \cap (A \cup B) = A$



$A \cap (A \cup B)$

Q. 40 If $A = \{1, 3, 5, 7, 9, 11, 13, 15, 17\}$, $B = \{2, 4, \ldots, 18\}$ and N the set of natural numbers is the universal set, then $(A' \cup (A \cup B) \cap B')$ is

(a) ϕ (b) N (c) A (d) B

Thinking Process

To solve this problem, use the distributive law i.e., $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Sol. *(b)*

$$A' \cup [(A \cup B) \cap B'] \quad [\because A \cap (B \cup C) = (A \cap B) \cup (A \cap C)]$$
$$= A' \cup [(A \cap B') \cup (B \cap B')]$$
$$= A' \cup [(A \cap B') \cup \phi] = A' \cup (A \cap B')$$
$$= (A' \cup A) \cap (A' \cup B')$$
$$= A' \cup B' = N \cap (A' \cup B')$$
$$= A' \cup B' = (A \cap B)' \quad [\because A \cap B = \phi]$$
$$= \phi = N$$

Q. 41 If $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$ and $P = \{x \mid x$ is a prime number less than 20$\}$. Then, $n(S) + n(P)$ is equal to

(a) 34 (b) 31 (c) 33 (d) 41

Sol. *(d)* $\because$ $S = \{x \mid x$ is a positive multiple of 3 less than 100$\}$

$\therefore$ $n(S) = 33$

and $P = \{x \mid x$ is a prime number less than 20$\}$

$\therefore$ $n(P) = 8$

$$n(S) + n(P) = 33 + 8 = 41$$

Q. 42 If X and Y are two sets and X' denotes the complement of X, then $X \cap (X \cup Y)'$ is equal to

(a) X (b) Y (c) ϕ (d) $X \cap Y$

Sol. *(c)*

$$X \cap (X \cup Y)' = X \cap (X' \cap Y') \quad [\because (A \cup B)' = A' \cap B']$$
$$= (X \cap X') \cap (X \cap Y')$$
$$= \phi \cap (X \cap Y') = \phi \quad [\because \phi \cap A = \phi]$$

Fillers

Q. 43 The set $\{x \in R : 1 \le x < 2\}$ can be written as ....

Sol. The set $\{x \in R : 1 \le x < 2\}$ can be written as (1, 2).

Q. 44 When $A = \phi$, then number of elements in $P(A)$ is ....

Sol. $\therefore$ $A = \phi \Rightarrow n(A) = 0$

$$n\{P(A)\} = 2^{n(A)} = 2^0 = 1$$

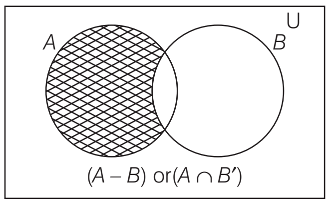
So, number of element in *P(A)* is 1.

Q. 45 If A and B are finite sets ,such that $A \subset B$, then $n(A \cup B)$ is equal to ....

Sol. If A and B are two finite sets such that $A \subset B$, then $n(A \cup B) = n(B)$.

Q. 46 If A and B are any two sets, then $A - B$ is equal to ....

Sol. If A and B are any two sets, then $A - B = A \cap B'$



$\therefore \quad A - B = A \cap B'$

Q. 47 Power set of the set $A = \{1, 2\}$ is ....

Thinking Process

We know that, the power set is a collection of all the subset of a set. To solve this problem, write the all subset of the given set.

Sol. $\therefore A = \{1, 2\}$

So, the subsets of A are ϕ, $\{1\}$, $\{2\}$ and $\{1, 2\}$.

$\therefore \quad P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

Q. 48 If the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$. Then, the universal set of all the three sets A, B and C can be ....

Sol. Universal set for A, B and C is given by $U = \{0, 1, 2, 3, 4, 5, 6, 8\}$

Q. 49 If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$. Then,

(i) $(B \cup C)'$ is .... (ii) $(C - A)'$ is ....

Sol. If $U = \{1, 2, 3, 4, 5, \ldots, 10\}$,

$A = \{1, 2, 3, 5\}$, $B = \{2, 4, 6, 7\}$ and $C = \{2, 3, 4, 8\}$

$\therefore \quad B \cup C = \{2, 3, 4, 6, 7, 8\}$

(i) $(B \cup C)' = U - (B \cup C) = \{1, 5, 9, 10\}$

(ii) $C - A = \{4, 8\}$

$\therefore \quad (C - A)' = U - (C - A) = \{1, 2, 3, 5, 6, 7, 9, 10\}$

Q. 50 For all sets A and B, $A - (A \cap B)$ is equal to ....

Sol. $A - (A \cap B) = A - B = A \cap B'$

Q. 51 Match the following sets for all sets A, B and C

	Column I		Column II
(i)	$((A' \cup B') - A)'$	(a)	$A - B$
(ii)	$[(B' \cup (B' - A)]'$	(b)	A
(iii)	$(A - B) - (B - C)$	(c)	B
(iv)	$(A - B) \cap (C - B)$	(d)	$(A \times B) \cap (A \times C)$
(v)	$A \times (B \cap C)$	(e)	$(A \times B) \cup (A \times C)$
(vi)	$A \times (B \cup C)$	(f)	$(A \cap C) - B$

Sol. **(i)** $[(A' \cup B') - A]' = [(A' \cup B') \cap A']'$ $[\because A - B = A \cap B']$

$= [(A \cap B)' \cap A']'$ $[\because (A \cap B)' = A' \cup B']$

$= [(A \cap B)']' \cup (A')' = (A \cap B) \cup A$ $[\because (A')' = A]$

$= A$

(ii) $[B' \cup (B' - A)]' = [B' \cup (B' \cap A')]'$ $[\because A - B = A \cap B']$

$= [B' \cup (B \cup A)']'$ $[\because A' \cap B' = (A \cup B)']$

$= (B')' \cap [(B \cup A)']'$ $[\because (A \cup B)' = A' \cap B']$

$= B \cap (B \cup A)$ $[\because (A')' = A]$

$= B$

(iii) $(A - B) - (B - C) = (A \cap B') - (B \cap C')$ $[\because A - B = A \cap B']$

$= (A \cap B') \cap (B \cap C')'$

$= (A \cap B') \cap [B' \cup (C')']$

$= (A \cap B') \cap (B' \cup C)$ $[\because (A')' = A]$

$= [A \cap (B' \cup C)] \cap [B' \cap (B' \cup C)]$

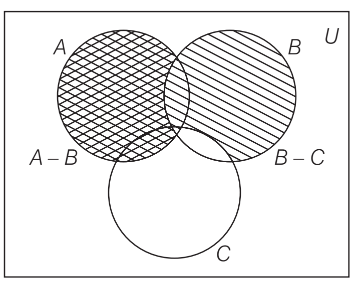
$= [A \cap (B' \cup C)] \cap B'$

$= (A \cap B') \cap [(B' \cup C) \cap B']$

$= (A \cap B') \cap B' = A \cap B' = A - B$

Alternate Method

It is clear from the diagram, $(A - B) - (B - C) = A - B$.



(iv) $(A - B) \cap (C - B)$

$\Rightarrow$ $(A \cap B') \cap (C \cap B')$ $[\because A - B = A \cap B']$

$\Rightarrow$ $(A \cap C) \cap B'$

$\Rightarrow$ $(A \cap C) - B$ $[\because A \cap B' = A - B]$

(v) $A \times B \cap C = (A \times B) \cap (A \times C)$

(vi) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Hence, the correct matches are

(i) $\leftrightarrow$ (b), (ii) $\leftrightarrow$ (c), (iii) $\leftrightarrow$ (a), (iv) $\leftrightarrow$ (f), (v) $\leftrightarrow$ (d), (vi) $\leftrightarrow$ (e)

True/False

Q. 52 If A is any set, then $A \subset A$.

Sol. ***True***

Since, every set is the subset of itself.
Therefore, for any set A, $A \subset A$.

Q. 53 If $M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, then $B \not\subset M$.

Sol. ***False***

$$M = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$
$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

Since, every elements of B is also in M.

$\therefore$ $B \subset M$

Q. 54 The sets {1, 2, 3, 4} and {3, 4, 5, 6} are equal

Sol. ***False***

Since, $2 \in \{1, 2, 3, 4\}$

But $2 \notin \{3, 4, 5, 6\}$

$\therefore$ $\{1, 2, 3, 4\} \neq \{3, 4, 5, 6\}$

Q. 55 $Q \cup Z = Q$, where Q is the set of rational numbers and Z is the set of integers.

Sol. ***True***

Since, every integer is also a rational number, then $Z \subset Q$
where, Z is the set of integer and Q is the set of rational number.

$\therefore$ $Q \cup Z = Q$

Q. 56 Let sets R and T be defined as

$$R = \{x \in Z \mid x \text{ is divisible by } 2\}$$
$$T = \{x \in Z \mid x \text{ is divisible by } 6\}. \text{ Then, } T \subset R$$

Sol. ***True***

$$R = \{x \in Z \mid x \text{ is divisible by } 2\} = \{\ldots -6, -4, -2, 0, 2, 4, 6, \ldots\}$$
$$T = \{x \in Z \mid x \text{ is divisible by } 6\} = \{\ldots, -12, -6, 0, 6, 12, \ldots\}$$

Thus, this every elements of T is also in R.

$\therefore$ $T \subset R$

Q. 57 Given $A = \{0, 1, 2\}$, $B = \{x \in R \mid 0 \le x \le 2\}$. Then, $A = B$.

Sol. ***False***

$$A = \{0, 1, 2\}, \text{ and } B = \{x \in R \mid 0 \le x \le 2\}$$

$\Rightarrow$ $n(A) = 3$

So, A is finite. Since, there are infinite real numbers from 0 to 2. So, B is infinite.

$\therefore$ $A \neq B$