3

Trigonometric Functions

Short Answer Type Questions

Q. 1 Prove that $\dfrac{\tan A+\sec A-1}{\tan A-\sec A+1}=\dfrac{1+\sin A}{\cos A}$.

Thinking Process

Here, use the formulae i.e., $\sec^2 A-\tan^2 A=1$ and $a^2-b^2=(a+b)(a-b)$ to solve the above problem.

Sol.

$$\text{LHS}=\frac{\tan A+\sec A-1}{\tan A-\sec A+1}$$

$$=\frac{\tan A+\sec A-(\sec^2 A-\tan^2 A)}{(\tan A-\sec A+1)} \qquad [\because \sec^2 A-\tan^2 A=1]$$

$$=\frac{(\tan A+\sec A)-(\sec A+\tan A)(\sec A-\tan A)}{(1-\sec A+\tan A)}$$

$$=\frac{(\sec A+\tan A)(1-\sec A+\tan A)}{1-\sec A+\tan A}$$

$$=\sec A+\tan A=\frac{1}{\cos A}+\frac{\sin A}{\cos A}$$

$$=\frac{1+\sin A}{\cos A}=\text{RHS}$$

Hence proved.

Q. 2 If $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$, then prove that $\dfrac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is also equal to y.

Sol. Given that, $\dfrac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}=y$

Now,
$$\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}=\frac{(1-\cos\alpha+\sin\alpha)}{(1+\sin\alpha)}\cdot\frac{(1+\cos\alpha+\sin\alpha)}{(1+\cos\alpha+\sin\alpha)}$$

$$=\frac{\{(1+\sin\alpha)-\cos\alpha\}}{(1+\sin\alpha)}\cdot\frac{\{(1+\sin\alpha)+\cos\alpha\}}{(1+\cos\alpha+\sin\alpha)}$$

$$=\frac{(1+\sin\alpha)^2-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$=\frac{(1+\sin^2\alpha+2\sin\alpha)-\cos^2\alpha}{(1+\sin\alpha)(1+\sin\alpha+\cos\alpha)}$$

$$= \frac{1 + \sin^2\alpha + 2\sin\alpha - 1 + \sin^2\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)}$$

$$= \frac{2\sin^2\alpha + 2\sin\alpha}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)}$$

$$= \frac{2\sin\alpha(1 + \sin\alpha)}{(1 + \sin\alpha)(1 + \sin\alpha + \cos\alpha)}$$

$$= \frac{2\sin\alpha}{1 + \sin\alpha + \cos\alpha} = y$$

Hence proved.

Q. 3 If $m\sin\theta = n\sin(\theta + 2\alpha)$, then prove that $\tan(\theta + \alpha)\cot\alpha = \frac{m+n}{m-n}$.

Sol. Given that, $m\sin\theta = n\sin(\theta + 2\alpha)$

$\therefore \quad \frac{\sin(\theta + 2\alpha)}{\sin\theta} = \frac{m}{n}$

Using componendo and dividendo, we get

$$\frac{\sin(\theta + 2\alpha) + \sin\theta}{\sin(\theta + 2\alpha) - \sin\theta} = \frac{m+n}{m-n}$$

$$\Rightarrow \quad \frac{2\sin\left(\frac{\theta + 2\alpha + \theta}{2}\right)\cdot\cos\left(\frac{\theta + 2\alpha - \theta}{2}\right)}{2\cos\left(\frac{\theta + 2\alpha + \theta}{2}\right)\cdot\sin\left(\frac{\theta + 2\alpha - \theta}{2}\right)} = \frac{m+n}{m-n}$$

$\left[\because \sin x + \sin y = 2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2} \text{ and } \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\cdot\frac{x-y}{2}\right]$

$$\Rightarrow \quad \frac{\sin(\theta + \alpha)\cdot\cos\alpha}{\cos(\theta + \alpha)\cdot\sin\alpha} = \frac{m+n}{m-n}$$

$$\Rightarrow \quad \tan(\theta + \alpha)\cdot\cot\alpha = \frac{m+n}{m-n}$$

Hence proved.

Q. 4 If $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$, where α lie between 0 and $\frac{\pi}{4}$, then find that value of $\tan 2\alpha$.

Sol. Given that, $\cos(\alpha + \beta) = \frac{4}{5}$ and $\sin(\alpha - \beta) = \frac{5}{13}$

$$\Rightarrow \quad \sin(\alpha + \beta) = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \pm\frac{3}{5}$$

$$\therefore \quad \sin(\alpha + \beta) = \frac{3}{5}$$

$$\text{and} \quad \cos(\alpha - \beta) = \sqrt{1 - \frac{25}{169}} = \sqrt{\frac{144}{169}} = \pm\frac{12}{13}$$

$$\therefore \quad \cos(\alpha - \beta) = \frac{12}{13}$$

$\left[\text{since, } \alpha \text{ lies between 0 and } \frac{\pi}{4}\right]$

$$\text{Now,} \quad \tan(\alpha + \beta) = \frac{\sin(\alpha + \beta)}{\cos(\alpha + \beta)}$$

$$= \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

and $\tan(\alpha - \beta) = \dfrac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \dfrac{\frac{5}{13}}{\frac{12}{13}} = \dfrac{5}{12}$

$\therefore$ $\tan 2\alpha = \tan(\alpha + \beta + \alpha - \beta)$

$$= \frac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\cdot\tan(\alpha-\beta)} \quad \left[\because \tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \cdot \tan y}\right]$$

$$= \frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\cdot\frac{5}{12}} = \frac{\frac{9+5}{12}}{\frac{16-5}{16}} = \frac{14 \times 16}{12 \times 11} = \frac{56}{33}$$

Q. 5 If $\tan x = \dfrac{b}{a}$, then find the value of $\sqrt{\dfrac{a+b}{a-b}} + \sqrt{\dfrac{a-b}{a+b}}$.

Thinking Process

First of all rationalise the given expression and used the formula, i.e., $\cos 2x = \cos^2 x - \sin^2 x$.

Sol. Given that, $\tan x = \dfrac{b}{a}$

$$\therefore \sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}} = \frac{\sqrt{(a+b)^2} + \sqrt{(a-b)^2}}{\sqrt{(a-b)(a+b)}}$$

$$= \frac{(a+b)+(a-b)}{\sqrt{a^2-b^2}} = \frac{2a}{\sqrt{a^2-b^2}} = \frac{2a}{a\sqrt{1-\left(\frac{b}{a}\right)^2}} \quad \left[\because \frac{b}{a} = \tan x\right]$$

$$= \frac{2}{\sqrt{1-\tan^2 x}} = \frac{2\cos x}{\sqrt{\cos^2 x - \sin^2 x}} \quad [\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \frac{2\cos x}{\sqrt{\cos 2x}}$$

Q. 6 Prove that $\cos\theta\cos\dfrac{\theta}{2} - \cos 3\theta\cos\dfrac{9\theta}{2} = \sin 7\theta \sin 8\theta$.

Sol. $\text{LHS} = \cos\theta\cos\dfrac{\theta}{2} - \cos 3\theta\cos\dfrac{9\theta}{2}$

$$= \frac{1}{2}\left[2\cos\theta\cdot\cos\frac{\theta}{2} - 2\cos 3\theta\cdot\cos\frac{9\theta}{2}\right]$$

$$= \frac{1}{2}\left[\cos\left(\theta + \frac{\theta}{2}\right) + \cos\left(\theta - \frac{\theta}{2}\right) - \cos\left(3\theta + \frac{9\theta}{2}\right) - \cos\left(3\theta - \frac{9\theta}{2}\right)\right]$$

$$= \frac{1}{2}(\cos\frac{3\theta}{2} + \cos\frac{\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}$$

$$= \frac{1}{2}\left[\cos\frac{\theta}{2} - \cos\frac{15\theta}{2}\right]$$

$$= -\frac{1}{2}\left[2\sin\left(\frac{\theta+15\theta}{2}\right)\cdot\sin\left(\frac{\theta-15\theta}{2}\right)\right] \quad \left[\because \cos x - \cos y = -2\sin\frac{x+y}{2}\cdot\sin\frac{x-y}{2}\right]$$

$$= +(\sin 8\theta\cdot\sin 7\theta) = \text{RHS}$$

$\therefore$ LHS = RHS **Hence proved.**

Q. 7 If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then show that $a^2 + b^2 = m^2 + n^2$.

Sol. Given that, $a\cos\theta + b\sin\theta = m$ ...(i)

and $a\sin\theta - b\cos\theta = n$ ...(ii)

On squaring and adding of Eqs. (i) and (ii), we get

$$m^2 + n^2 = (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$$

$$= a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cdot\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cdot\cos\theta$$

$\Rightarrow$ $m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)$

$\Rightarrow$ $m^2 + n^2 = a^2 + b^2$ **Hence proved.**

Q. 8 Find the value of $\tan 22°30'$.

Sol. Let $\theta = 45°$

We know that, $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} \Rightarrow \tan\frac{\theta}{2} = \frac{\sin\theta}{1+\cos\theta}$

$\therefore$ $\tan 22°30' = \frac{\sin 45°}{1 + \cos 45°}$ $[\because \theta = 45°]$

$$= \frac{\frac{1}{\sqrt{2}}}{1 + \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2} + 1}$$

Q. 9 Prove that $\sin 4A = 4\sin A\cos^3 A - 4\cos A\sin^3 A$.

Thinking Process

Here, apply the formula i.e., $\sin 2x = 2\sin x\cos x$ *and* $\cos 2x = \cos^2 x - \sin^2 x$

Sol.

$$\text{LHS} = \sin 4A$$
$$= 2\sin 2A\cdot\cos 2A$$
$$= 2(2\sin A\cdot\cos A)(\cos^2 A - \sin^2 A)$$
$$= 4\sin A\cdot\cos^3 A - 4\cos A\sin^3 A$$

$\left[\because \cos 2A = \cos^2 A - \sin^2 A \text{ and } \sin 2A = 2\sin A\cdot\cos A\right]$

$\therefore$ LHS = RHS **Hence proved.**

Q. 10 If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, then prove that $m^2 - n^2 = 4\sin\theta\tan\theta$.

Sol. Given that, $\tan\theta + \sin\theta = m$ ...(i)

and $\tan\theta - \sin\theta = n$ ...(ii)

Now, $m + n = \tan\theta + \sin\theta + \tan\theta - \sin\theta$

$m + n = 2\tan\theta$ ...(iii)

Also, $m - n = \tan\theta + \sin\theta - \tan\theta + \sin\theta$

$m - n = 2\sin\theta$ ...(iv)

From Eqs. (iii) and (iv),

$$(m + n)(m - n) = 4\sin\theta\cdot\tan\theta$$

$$m^2 - n^2 = 4\sin\theta\cdot\tan\theta$$ **Hence proved.**

Q. 11 If $\tan(A+B) = p$ and $\tan(A-B) = q$, then show that $\tan 2A = \dfrac{p+q}{1-pq}$.

Sol. Given that $\tan(A+B) = p$ ...(i)

and $\tan(A-B) = q$ ...(ii)

$$\therefore \quad \tan 2A = \tan(A+B+A-B)$$

$$= \frac{\tan(A+B)+\tan(A-B)}{1-\tan(A+B)\tan(A-B)} \quad \left[\because \tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y}\right]$$

$$= \frac{p+q}{1-pq} \quad \text{[from Eqs. (i) and (ii)]}$$

Q. 12 If $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$, then prove that $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha+\beta)$.

Sol. Given that, $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$

$$\Rightarrow \quad (\cos\alpha + \cos\beta)^2 - (\sin\alpha + \sin\beta)^2 = 0$$

$$\Rightarrow \quad \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta - \sin^2\alpha - \sin^2\beta - 2\sin\alpha\sin\beta = 0$$

$$\Rightarrow \quad \cos^2\alpha - \sin^2\alpha + \cos^2\beta - \sin^2\beta = 2(\sin\alpha\sin\beta - \cos\alpha\cos\beta)$$

$$\Rightarrow \quad \cos 2\alpha + \cos 2\beta = -2\cos(\alpha+\beta)$$

Hence proved.

Q. 13 If $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$, then show that $\dfrac{\tan x}{\tan y} = \dfrac{a}{b}$.

Sol. Given that, $\dfrac{\sin(x+y)}{\sin(x-y)} = \dfrac{a+b}{a-b}$

Using componendo and dividendo,

$$\Rightarrow \quad = \frac{\sin(x+y)+[\sin(x-y)]}{\sin(x+y)-\sin(x-y)} = \frac{a+b+a-b}{a+b-a+b}$$

$$\Rightarrow \quad = \frac{2\sin\left(\frac{x+y+x-y}{2}\right)\cdot\cos\left(\frac{x+y-x+y}{2}\right)}{2\cos\left(\frac{x+y+x-y}{2}\right)\cdot\sin\left(\frac{x+y-x+y}{2}\right)} = \frac{2a}{2b}$$

$$\left[\because \sin x + \sin y = 2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2} \text{ and } \sin x - \sin y = 2\cos\frac{x+y}{2}\cdot\sin\frac{x-y}{2}\right]$$

$$\Rightarrow \quad = \frac{\sin x \cdot \cos y}{\cos x \cdot \sin y} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{\tan x}{\tan y} = \frac{a}{b}$$

Q. 14 If $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$, then show that $\sin\alpha + \cos\alpha = \sqrt{2}\cos\theta$.

Sol. Given that, $\tan\theta = \dfrac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}$

$$\Rightarrow \quad \tan\theta = \frac{\cos\alpha(\tan\alpha - 1)}{\cos\alpha(\tan\alpha + 1)}$$

$$\Rightarrow \quad \tan\theta = \frac{\tan\alpha - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{4}\cdot\tan\alpha} \quad \left[\because \tan\frac{\pi}{4} = 1\right]$$

$$\Rightarrow \quad \tan\theta = \tan\left(\alpha - \frac{\pi}{4}\right)$$

$$\Rightarrow \quad \theta = \alpha - \frac{\pi}{4} \Rightarrow \alpha = \theta + \frac{\pi}{4}$$

$$\therefore \quad \sin\alpha + \cos\alpha = \sin\left(\theta + \frac{\pi}{4}\right) + \cos\left(\theta + \frac{\pi}{4}\right)$$

$$= \sin\theta\cdot\cos\frac{\pi}{4} + \cos\theta\cdot\sin\frac{\pi}{4} + \cos\theta\cdot\cos\frac{\pi}{4} - \sin\theta\cdot\sin\frac{\pi}{4}$$

$$= \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta \quad \left[\because \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}\right]$$

$$= \frac{2}{\sqrt{2}}\cdot\cos\theta = \sqrt{2}\cos\theta$$

Q. 15 **If $\sin\theta + \cos\theta = 1$, then find the general value of θ.**

Thinking Process

If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n\cdot\alpha$, gives general solution of the given equation.

Sol. Given that, $\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = 1$$

$$\Rightarrow \quad 1 + 2\sin\theta\cdot\cos = 1 \quad [\because \sin 2x = 2\sin x\cos x]$$

$$\Rightarrow \quad \sin 2\theta = 0 \Rightarrow 2\theta = n\pi + (-1)^n\cdot 0$$

$$\therefore \quad \theta = \frac{n\pi}{2}$$

Alternate Method

$$\sin\theta + \cos\theta = 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}}\cdot\sin\theta + \frac{1}{\sqrt{2}}\cdot\cos\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \sin\theta\cdot\cos\frac{\pi}{4} + \cos\theta\cdot\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \quad \left[\because \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}\right]$$

$$\Rightarrow \quad \sin\left(\theta + \frac{\pi}{4}\right) = \sin\frac{\pi}{4} \quad [\because \sin(x+y) = \sin x\cdot\cos y + \cos x\cdot\sin y]$$

$$\Rightarrow \quad \theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$$

$$\therefore \quad \theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$$

Q. 16 **Find the most general value of θ satisfying the equation $\tan\theta = -1$ and $\cos\theta = \frac{1}{\sqrt{2}}$.**

Sol. The given equations are

$$\tan\theta = -1 \quad \ldots(i)$$

and

$$\cos\theta = \frac{1}{\sqrt{2}} \quad \ldots(ii)$$

From Eq. (i), $\tan\theta = -\tan\frac{\pi}{4}$

$$\Rightarrow \quad \tan\theta = \tan\left(2\pi - \frac{\pi}{4}\right) \Rightarrow \tan\theta = \tan\frac{7\pi}{4}$$

$$\therefore \quad \theta = \frac{7\pi}{4}$$

From Eq. (ii), $\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \cos\theta = \cos\frac{\pi}{4}$

$\Rightarrow \cos\theta = \cos\left(2\pi - \frac{\pi}{4}\right) \Rightarrow \cos\theta = \cos\frac{7\pi}{4}$

$\therefore \theta = \frac{7\pi}{4}$

Hence, the most general value of θ *i.e.*, $\theta = 2n\pi + \frac{7\pi}{4}$.

Q. 17 If $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$, then find the general value of θ.

Sol. Given that, $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta$

$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \frac{\cos^2 + \sin^2\theta}{\sin\theta\cdot\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \frac{1}{\cos\theta} = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = 2n\pi \pm \frac{\pi}{3}$

Q. 18 If $2\sin^2\theta = 3\cos\theta$, where $0 \le \theta \le 2\pi$, then find the value of θ.

Sol. Given that, $2\sin^2\theta = 3\cos\theta$

$\Rightarrow 2 - 2\cos^2\theta = 3\cos\theta$

$\Rightarrow 2\cos^2\theta + 3\cos\theta - = 0$

$\Rightarrow 2\cos^2\theta + 4\cos\theta - \cos\theta - 2 = 0$

$\Rightarrow 2\cos\theta(\cos\theta + 2) - 1(\cos\theta + 2) = 0$

$\Rightarrow (\cos\theta + 2)(2\cos\theta - 1) = 0$

$\Rightarrow \cos\theta = -2$ not possible $[\because -1 \le \cos\theta \le 1]$

$2\cos\theta = 1$

$\Rightarrow \cos\theta = \frac{1}{2}$

$\Rightarrow \cos\theta = \cos\frac{\pi}{3}$

$\therefore \theta = \frac{\pi}{3}$

Also, $\cos\theta = \cos\left(2\pi - \frac{\pi}{3}\right)$

$\Rightarrow \cos\theta = \cos\frac{5\pi}{6}$

$\therefore \theta = \frac{5\pi}{6}$

So, the values of θ are $\frac{\pi}{3}$ and $\frac{5\pi}{6}$.

Q. 19 If $\sec x \cos 5x + 1 = 0$, where $0 < x \le \frac{\pi}{2}$, then find the value of x.

Sol. Given that, $\sec x \cos 5x + 1 = 0$

$$\frac{\cos 5x}{\cos x} + 1 = 0 \Rightarrow \cos 5x + \cos x = 0$$

$$\Rightarrow \quad 2\cos\left(\frac{5x + x}{2}\right)\cdot\cos\left(\frac{5x - x}{2}\right) = 0 \qquad \left[\because \cos x + \cos y = 2\cos\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\right]$$

$$\Rightarrow \quad 2\cos 3x \cdot \cos 2x = 0$$

$$\Rightarrow \quad \cos 3x = 0 \text{ or } \cos 2x = 0$$

$$\Rightarrow \quad \cos 3x = \cos\frac{\pi}{2} \text{ or } \cos 2x = \cos\frac{\pi}{2}$$

$$\therefore \quad 3x = \frac{\pi}{2} \Rightarrow 2x = \frac{\pi}{2}$$

and $$x = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{4}$$

Hence, the solutions are $\frac{\pi}{2}, \frac{\pi}{4}$ and $\frac{\pi}{6}$.

Long Answer Type Questions

Q. 20 If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$, then prove that $\cos(\alpha + \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2$.

Thinking Process

Express $\cos(\alpha - \beta) = \cos(\theta + \alpha) - (\theta + \beta)$.

Sol. Given that, $\sin(\theta + \alpha) = a$ ...(i)

and $\sin(\theta + \beta) = b$ ...(ii)

$$\therefore \quad \cos(\theta + \alpha) = \sqrt{1 - a^2} \text{ and } \cos(\theta + \beta) = \sqrt{1 - b^2}$$

$$\therefore \quad \cos(\alpha - \beta) = \cos\{\theta + \alpha - (\theta + \beta)\}$$

$$= \cos(\theta + \beta)\cos(\theta + \alpha) + \sin(\theta + \alpha)\sin(\theta + \beta)$$

$$= \sqrt{1 - a^2}\sqrt{1 - b^2} + a \cdot b = ab + \sqrt{(1 - a^2)(1 - b^2)}$$

$$= ab + \sqrt{1 - a^2 - b^2 + a^2b^2}$$

and $$\cos(\alpha - \beta) = ab + \sqrt{1 - a^2 - b^2 + a^2b^2}$$

$$= \cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta)$$

$$= 2\cos^2(\alpha - \beta) - 1 - 4ab\cos(\alpha - \beta)$$

$$= 2\cos(\alpha - \beta)(\cos\alpha - \beta - 2ab) - 1$$

$$= 2(ab + \sqrt{1 - a^2 - b^2 + a^2b^2})(ab + \sqrt{1 - a^2 - b^2 + a^2b^2} - 2ab) - 1$$

$$= 2[(\sqrt{1 - a^2 - b^2 + a^2b^2} + ab)(\sqrt{1 - a^2 - b^2 + a^2b^2} - ab)] - 1$$

$$= 2[1 - a^2 - b^2 + a^2b^2 - a^2b^2] - 1$$

$$= 2 - 2a^2 - 2b^2 - 1$$

$$= 1 - 2a^2 - 2b^2$$

Hence proved.

Q. 21 If $\cos(\theta+\phi) = m\cos(\theta-\phi)$, then prove that $\tan\theta = \dfrac{1-m}{1+m}\cot\phi$.

Sol. Given that, $$\cos(\theta+\phi) = m\cos(\theta-\phi)$$

$$\Rightarrow \quad \frac{\cos(\theta+\phi)}{\cos(\theta-\phi)} = \frac{m}{1}$$

Using componendo and dividendo rule,

$$\frac{\cos(\theta-\phi)-\cos(\theta+\phi)}{\cos(\theta-\phi)+\cos(\theta+\phi)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{-2\sin\left(\frac{\theta-\phi+\theta+\phi}{2}\right)\cdot\sin\left(\frac{\theta-\phi-\theta-\phi}{2}\right)}{2\cos\left(\frac{\theta-\phi+\theta+\phi}{2}\right)\cdot\cos\left(\frac{\theta-\phi-\theta-\phi}{2}\right)} = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \frac{\sin\theta\cdot\sin\phi}{\cos\theta\cdot\cos\phi} = \frac{1-m}{1+m} \qquad \left[\because \ \sin(-\theta) = -\sin\theta \text{ and} \cos(-\theta) = \cos\theta\right]$$

$$\Rightarrow \quad \tan\theta\cdot\tan\phi = \frac{1-m}{1+m}$$

$$\Rightarrow \quad \tan\theta = \left(\frac{1-m}{1+m}\right)\cot\phi$$

Q. 22 Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right].$$

Sol. Given expression,

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$$

$$= 3[\cos^4\alpha+\sin^4(\pi+\alpha)]-2[\cos^6\alpha+\sin^6(\pi-\alpha)]$$

$$= 3[\cos^4\alpha+\sin^4\alpha]-2[\cos^6\alpha+\sin^6\alpha] = 3-2 = 1$$

Q. 23 If $a\cos 2\theta + b\sin 2\theta = c$ has α and β as its roots, then prove that $\tan\alpha+\tan\beta = \dfrac{2b}{a+c}$.

Sol. Given that, $a\cos 2\theta + b\sin 2\theta = c$

$$\Rightarrow \quad a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = c \qquad \left[\because \ \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta} \text{ and } \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right]$$

$$\Rightarrow \quad a(1-\tan^2\theta)+2b\tan\theta = c(1+\tan^2\theta)$$

$$\Rightarrow \quad a-a-\tan^2\theta+2b\tan\theta = c+c\tan^2\theta$$

$$\Rightarrow \quad (a+c)\tan^2\theta-2b\tan\theta+c-a = 0$$

Since, this equation has $\tan\alpha$ and $\tan\beta$ as its roots.

$$\because \quad \tan\alpha+\tan\beta = \frac{-(-2b)}{a+c} = \frac{2b}{a+c}$$

Q. 24 If $x = \sec\phi - \tan\phi$ and $y = \operatorname{cosec}\phi + \cot\phi$, then show that $xy + x - y + 1 = 0$.

Sol. Given that, $x = \sec\phi - \tan\phi$ ...(i)

and $y = \operatorname{cosec}\phi + \cot\phi$ ...(ii)

Now, $1 \cdot xy = (\sec\phi - \tan\phi)(\operatorname{cosec}\phi + \cot\phi)$

$\Rightarrow$ $xy = \sec\phi \cdot \operatorname{cosec}\phi - \operatorname{cosec}\phi \cdot \tan\phi + \sec\phi \cdot \cot\phi - \tan\phi \cdot \cot\phi$

$\Rightarrow$ $xy = \sec\phi \cdot \operatorname{cosec}\phi - \frac{1}{\cos\phi} + \frac{1}{\sin\phi} - 1$

$\Rightarrow$ $1 + xy = \sec\phi \operatorname{cosec}\phi - \sec\phi + \operatorname{cosec}\phi$ ...(iii)

From Eqs. (i) and (ii), we get

$x - y = \sec\phi - \tan\phi - \operatorname{cosec}\phi - \cot\phi$

$\Rightarrow$ $x - y = \sec\phi - \operatorname{cosec}\phi - \frac{\sin\phi}{\cos\phi} - \frac{\cos\phi}{\sin\phi}$

$\Rightarrow$ $x - y = \sec\phi - \operatorname{cosec}\phi - \left(\frac{\sin^2\phi + \cos^2\phi}{\sin\phi \cdot \cos\phi}\right)$

$\Rightarrow$ $x - y = \sec\phi - \operatorname{cosec}\phi - \frac{1}{\sin\phi \cdot \cos\phi}$

$\Rightarrow$ $x - y = \sec\phi - \operatorname{cosec}\phi - \operatorname{cosec}\phi \cdot \sec\phi$

$\Rightarrow$ $x - y = -(\sec\phi \cdot \operatorname{cosec}\phi - \sec\phi + \operatorname{cosec}\phi)$

$\Rightarrow$ $x - y = -(xy + 1)$ [from Eq. (iii)]

$\Rightarrow$ $xy + x - y + 1 = 0$ **Hence proved**.

Q. 25 If θ lies in the first quadrant and $\cos\theta = \frac{8}{17}$, then find the value of $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$.

Sol. Given that, $\cos 3\theta = \frac{8}{17} \Rightarrow \sin\theta = \sqrt{1 - \frac{64}{289}}$

$\Rightarrow$ $\sin\theta = \sqrt{\frac{289 - 64}{289}} \Rightarrow \sin\theta = \pm\frac{15}{17}$

$\Rightarrow$ $\sin\theta = \frac{15}{17}$ [since, θ lies in first quadrant]

Now, $\cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(120^\circ - \theta)$

$= \cos(30^\circ + \theta) + \cos(45^\circ - \theta) + \cos(90^\circ + 30^\circ - \theta)$

$= \cos(30^\circ + \theta) + \cos(45^\circ - \theta) - \sin(30^\circ - \theta)$

$= \cos 30^\circ \cos\theta - \sin 30^\circ \sin\theta + \cos 45^\circ \cos\theta + \sin 45^\circ \sin\theta$
$- \sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta$

$= \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta - \frac{1}{2}\cos\theta \; \frac{\sqrt{3}}{2}\sin\theta$

$= \left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} - \frac{1}{2}\right)\cos\theta + \left(\frac{1}{\sqrt{2}} - \frac{1}{2} + \frac{\sqrt{3}}{2}\right)\sin\theta$

$= \left(\frac{\sqrt{6} + 2 - \sqrt{2}}{2\sqrt{2}}\right)\cos\theta + \left(\frac{2 - \sqrt{2} + \sqrt{6}}{2\sqrt{2}}\right)\sin\theta$

$= \left(\frac{\sqrt{6} + 2 - \sqrt{2}}{2\sqrt{2}}\right)\frac{8}{17} + \left(\frac{2 - \sqrt{2} + \sqrt{6}}{2\sqrt{2}}\right)\frac{15}{17}$

$$= \frac{1}{17(2\sqrt{2})}(8\sqrt{6} + 16 - 8\sqrt{2} + 30 - 15\sqrt{2} + 15\sqrt{6})$$
$$= \frac{1}{17(2\sqrt{2})}(23\sqrt{6} - 23\sqrt{2} + 46)$$
$$= \frac{23\sqrt{6}}{17(2\sqrt{2})} - \frac{23\sqrt{2}}{17(2\sqrt{2})} + \frac{46}{17(2\sqrt{2})}$$
$$= \frac{23\sqrt{3}}{17(2)} - \frac{23}{17(2)} + \frac{23}{17\sqrt{2}}$$
$$= \frac{23}{17}\left(\frac{\sqrt{3}-1}{2} + \frac{1}{\sqrt{2}}\right)$$

Q. 26 Find the value of $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$.

Sol. Given expression, $\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{5\pi}{8} + \cos^4 \frac{7\pi}{8}$

$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4\left(\pi - \frac{3\pi}{8}\right) + \cos^4\left(\pi - \frac{\pi}{8}\right)$$
$$= \cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4 \frac{\pi}{8}$$
$$= 2\left[\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8}\right] = 2\left[\cos^4 \frac{\pi}{8} + \cos^4\left(\frac{\pi}{2} - \frac{\pi}{8}\right)\right]$$
$$= 2\left[\cos^4 \frac{\pi}{8} + \sin^4 \frac{\pi}{8}\right]$$
$$= 2\left[\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}\right)^2 - 2\cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8}\right]$$
$$= 2\left[1 - 2\cos^2 \frac{\pi}{8} \cdot \sin^2 \frac{\pi}{8}\right] = 2 - \left(2\sin\frac{\pi}{8} \cdot \cos\frac{\pi}{8}\right)^2$$
$$= 2 - \left(\sin\frac{2\pi}{8}\right)^2 = 2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= 2 - \frac{1}{2} = \frac{3}{2}$$

Q. 27 Find the general solution of the equation $5\cos^2\theta + 7\sin^2\theta - 6 = 0$.

Sol. Given equation, $5\cos^2\theta + 7\sin^2\theta - 6 = 0$

$\Rightarrow \quad 5\cos^2 + 7(1 - \cos^2\theta) - 6 = 0$

$\Rightarrow \quad 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0$

$\Rightarrow \quad 5\cos^2\theta + 7 - 7\cos^2\theta - 6 = 0 \quad \Rightarrow \quad -2\cos^2\theta + 1 = 0$

$\Rightarrow \quad 2\cos^2\theta - 1 = 0 \qquad \left[\because \cos^2\theta = \cos^2\alpha \quad \therefore \theta = n\pi \pm \alpha\right]$

$\Rightarrow \quad \cos^2\theta = \frac{1}{2}$

$\Rightarrow \quad \cos^2\theta = \cos^2\frac{\pi}{4}$

$\therefore \quad \theta = n\pi \pm \frac{\pi}{4}$

Q. 28 Find the general of the equation $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

Sol. Given equation, $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$

$$\Rightarrow 2\sin\left(\frac{x+3x}{2}\right)\cdot\cos\left(\frac{3x-x}{2}\right) - 3\sin 2x = 2\cos\left(\frac{3x+x}{2}\right)\cdot\cos\left(\frac{3x-x}{2}\right) - 3\cos 2x$$

$$\Rightarrow 2\sin 2x\cos x - 3\sin 2x = 2\cos 2x\cdot\cos x - 3\cos 2x$$

$$\Rightarrow \sin 2x(2\cos x - 3) = \cos 2x(2\cos x - 3)$$

$$\Rightarrow \frac{\sin 2x}{\cos 2x} = 1$$

$$\Rightarrow \tan 2x = 1$$

$$\Rightarrow \tan 2x = \tan\frac{\pi}{4}$$

$$\Rightarrow 2x = n\pi + \frac{\pi}{4}$$

$$\therefore x = \frac{n\pi}{2} + \frac{\pi}{8}$$

Q.29 Find the general solution of the equation

$$(\sqrt{3} - 1)\cos\theta + (\sqrt{3} + 1)\sin\theta = 2$$

Sol. Given equation is,

$$(\sqrt{3} - 1)\cos\theta + (\sqrt{3} + 1)\sin\theta = 2 \quad \ldots(i)$$

Put $\sqrt{3} - 1 = r\sin\alpha$ and $\sqrt{3} + 1 = r\cos\alpha$

$$\therefore r^2 = (\sqrt{3} - 1)^2 + (\sqrt{3} + 1)^2$$

$$\Rightarrow = 3 + 1 - 2\sqrt{3} + 3 + 1 + 2\sqrt{3}$$

$$\Rightarrow r^2 = 8$$

$$\therefore r = 2\sqrt{2}$$

now, $$\tan\alpha = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\cdot\frac{\pi}{4}}$$

$$\Rightarrow \tan\alpha = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$\Rightarrow \tan\alpha = \tan\frac{\pi}{12}$$

$$\therefore \alpha = \frac{\pi}{12}$$

From Eq. (i), $r\sin\alpha\cos\theta + r\cos\alpha\sin\theta = 2$

$$r[\sin(\theta + \alpha)] = 2$$

$$\Rightarrow \sin(\theta + \alpha) = \frac{2}{2\sqrt{2}}$$

$$\Rightarrow \sin(\theta + \alpha) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin(\theta + \alpha) = \sin\frac{\pi}{4} \quad \theta + \alpha = n\pi + (-1)^n\frac{\pi}{4}$$

$$\theta = n\pi + (-1)^n\cdot\frac{\pi}{4} - \frac{\pi}{12}$$

Alternate Method

$$(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2 \qquad \ldots\text{(i)}$$

Put $\sqrt{3}-1 = r\cos\alpha$ and $\sqrt{3}+1 = r\sin\alpha$

$\therefore$ $r = 2\sqrt{2}$

Now, $$\tan\alpha = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{1+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}}$$

$\Rightarrow$ $$\tan\alpha = \frac{\tan\frac{\pi}{4}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{4}\cdot\tan\frac{\pi}{6}}$$

$\Rightarrow$ $$\tan\alpha = \tan\left(\frac{\pi}{4}+\frac{\pi}{6}\right) \Rightarrow \tan\alpha = \tan\frac{5\pi}{12}$$

$\therefore$ $$\alpha = \frac{5\pi}{12}$$

From Eq. (i), $r\cos\alpha\cos\theta + r\sin\alpha\sin\theta = 2$

$$r[\cos(\theta-\alpha)] = 2$$

$\Rightarrow$ $$\cos(\theta-\alpha) = \frac{2}{2\sqrt{2}}$$

$\Rightarrow$ $$\cos(\theta-\alpha) = \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\cos(\theta-\alpha) = \cos\frac{\pi}{4}$$

$\Rightarrow$ $$\theta-\alpha = 2n\pi \pm \frac{\pi}{4}$$

$\therefore$ $$\theta = 2n\pi \pm \frac{\pi}{4} + \frac{5\pi}{12}$$

Objective Type Questions

Q. 30 If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^2\theta + \operatorname{cosec}^2\theta$ is equal to

(a) 1 (b) 4 (c) 2 (d) None of these

Sol. *(c)* Given that, $\sin\theta + \operatorname{cosec}\theta = 2$

$\Rightarrow$ $$\sin^2\theta + \operatorname{cosec}^2\theta + 2\sin\theta\cdot\operatorname{cosec}\theta = 4$$

$\Rightarrow$ $$\sin^2\theta + \operatorname{cosec}^2\theta = 4-2$$

$\Rightarrow$ $$\sin^2\theta + \operatorname{cosec}^2\theta = 2$$

Q. 31 If $f(x) = \cos^2 x + \sec^2 x$, then

(a) $f(x) < 1$ (b) $f(x) = 1$ (c) $2 < f(x) < 1$ (d) $f(x) \geq 2$

Sol. *(d)* Given that, $f(x) = \cos^2 x + \sec^2 x$

We know that, AM $\geq$ GM

$$\frac{\cos^2 x + \sec^2 x}{2} \geq \sqrt{\cos^2 x\cdot\sec^2 x}$$

$\Rightarrow$ $$\cos^2 x + \sec^2 x \geq 2 \qquad [\because \cos x\cdot\sec x = 1]$$

$\Rightarrow$ $$f(x) \geq 2$$

Q. 32 If $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$, then the value of $\theta + \phi$ is

(a) $\frac{\pi}{6}$ (b) π (c) 0 (d) $\frac{\pi}{4}$

Sol. ***(d)*** Given that, $\tan\theta = \frac{1}{2}$ and $\tan\phi = \frac{1}{3}$

Now, $\tan(\theta + \phi) = \frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi}$

$\Rightarrow$ $\tan(\theta + \phi) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}} \Rightarrow \tan(\theta + \phi) = \frac{\frac{3+2}{6}}{\frac{6-1}{6}} = \frac{5}{5} = 1$

$\Rightarrow$ $\tan(\theta + \phi) = \tan\frac{\pi}{4}$

$\therefore$ $\theta + \phi = \frac{\pi}{4}$

Q. 33 Which of the following is not correct?

(a) $\sin\theta = -\frac{1}{5}$ (b) $\cos\theta = 1$ (c) $\sec\theta - \frac{1}{2}$ (d) $\tan\theta = 20$

Sol. ***(c)*** We know that, the range of $\sec\theta$ is $R - (-1, 1)$.

Hence, $\sec\theta$ cannot be equal to $\frac{1}{2}$.

Q. 34 The value of $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$ is

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) Not defined

Sol. ***(b)*** Given expression, $\tan 1° \tan 2° \tan 3° \ldots \tan 89°$

$= \tan 1° \tan 2° \ldots \tan 45° \cdot \tan(90° - 44°) \tan(90° - 43°) \ldots \tan(90° - 1°)$

$= \tan 1° \cdot \cot 1° \cdot \tan 2° \cdot \cot 2° \ldots \tan 89° \cdot \cot 89°$

$= 1 \cdot 1 \ldots 1 \cdot 1 = 1$

Q. 35 The value of $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$ is

(a) 1 (b) $\sqrt{3}$ (c) $\frac{\sqrt{3}}{2}$ (d) 2

Sol. ***(c)*** Given expression, $\frac{1 - \tan^2 15°}{1 + \tan^2 15°}$

Let $\theta = 15°$

We know that, $\cos 2\theta = \frac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$\therefore$ $\cos 30° = \frac{1 - \tan^2 15°}{1 + \tan^2 15°}$

$\Rightarrow$ $\frac{1 - \tan^2 15°}{1 + \tan^2 15°} = \frac{\sqrt{3}}{2}$ $\left[\because \cos 30° = \frac{\sqrt{3}}{2}\right]$

Q. 36 The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ$ is

(a) $\frac{1}{\sqrt{2}}$ (b) 0 (c) 1 (d) -1

Sol. *(b)* Given expression, $\cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots \cos 179^\circ$

$$= \cos 1^\circ \cos 2^\circ \ldots \cos 90^\circ \ldots \cos 179^\circ \quad [\because \cos 90^\circ = 0]$$
$$= 0$$

Q. 37 If $\tan\theta = 3$ and θ lies in third quadrant, then the value of $\sin\theta$ is

(a) $\frac{1}{\sqrt{10}}$ (b) $-\frac{1}{\sqrt{10}}$ (c) $\frac{-3}{\sqrt{10}}$ (d) $\frac{3}{\sqrt{10}}$

Sol. *(c)* Given that, $\tan\theta = 3$

$\Rightarrow$ $\sec^2\theta = 1 + \tan^2\theta$

$\Rightarrow$ $\sec\theta = \sqrt{1+9} = \pm\sqrt{10}$

$\Rightarrow$ $\sec\theta = -\sqrt{10}$

$\Rightarrow$ $\cos\theta = -\frac{1}{\sqrt{10}}$

$\Rightarrow$ $\sin\theta = \pm\sqrt{1-\frac{1}{10}} = \pm\sqrt{\frac{9}{10}} = \pm\frac{3}{\sqrt{10}}$ [since, θ lies in third quadrant]

$\therefore$ $\sin\theta = -\frac{3}{\sqrt{10}}$

Q. 38 The value of $\tan 75^\circ - \cot 75^\circ$ is

(a) $2\sqrt{3}$ (b) $2+\sqrt{3}$ (c) $2-\sqrt{3}$ (d) 1

Sol. *(a)* Given expression, $\tan 75^\circ - \cot 75^\circ$

$$= \frac{\sin 75^\circ}{\cos 75^\circ} - \frac{\cos 75^\circ}{\sin 75^\circ}$$
$$= \frac{\sin^2 75^\circ - \cos^2 75^\circ}{\sin 75^\circ \cdot \cos 75^\circ}$$
$$= \frac{-2\cos 150^\circ}{\sin 150^\circ}$$
$$= \frac{-2\cos(90^\circ + 60^\circ)}{\sin(90^\circ + 60^\circ)}$$
$$= \frac{+2\sin 60^\circ}{\cos 60^\circ}$$
$$= \frac{2\cdot\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2\sqrt{3}$$

Q. 39 Which of the following is correct?

(a) $\sin 1^\circ > \sin 1$ (b) $\sin 1^\circ < \sin 1$

(c) $\sin 1^\circ = \sin 1$ (d) $\sin 1^\circ = \frac{\pi}{18^\circ}\sin 1$

Sol. *(b)* We know that, if θ is increasing, then $\sin\theta$ is also increasing.

$\therefore$ $\sin 1^\circ < \sin 1$ $[\because 1 \text{ rad} = 57^\circ 30']$

Q. 40 If $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$, then $\alpha + \beta$ is equal to

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Sol. *(d)* Given that, $\tan\alpha = \frac{m}{m+1}$ and $\tan\beta = \frac{1}{2m+1}$

Now, $$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{\frac{m}{m+1} + \frac{1}{2m+1}}{1 - \left(\frac{m}{m+1}\right)\left(\frac{1}{2m+1}\right)}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{m(2m+1) + m + 1}{(m+1)(2m+1) - m}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{2m^2 + m + m + 1}{2m^2 + 2m + m + 1 - m}$$

$$\Rightarrow \tan(\alpha+\beta) = \frac{2m^2 + 2m + 1}{2m^2 + 2m + 1} \Rightarrow \tan(\alpha+\beta) = 1$$

$$\therefore \alpha + \beta = \frac{\pi}{4}$$

Q. 41 The minimum value of $3\cos x + 4\sin x + 8$ is

(a) 5 (b) 9 (c) 7 (d) 3

Thinking Process

For the expression $A\cos\theta + B\sin\theta$, then the minimum value is $-\sqrt{A^2 + B^2}$.

Sol. *(d)* Given expression, $3\cos x + 4\sin x + 8$

Let $y = 3\cos x + 4\sin x + 8$

$\Rightarrow y - 8 = 3\cos x + 4\sin x$

$\therefore$ Minimum value of $y - 8 = -\sqrt{9 + 16}$

$\Rightarrow y - 8 = -5 \Rightarrow y = -5 + 8$

$\therefore y = 3$

Hence, the minimum value of $3\cos x + 4\sin x + 8$ is 3.

Q. 42 The value of $\tan 3A - \tan 2A - \tan A$ is

(a) $\tan 3A \tan 2A \tan A$

(b) $-\tan 3A \tan 2A \tan A$

(c) $\tan A \tan 2A - \tan 2A \tan 3A - \tan 3A \tan A$

(d) None of the above

Sol. *(a)* Let $3A = A + 2A$

$\tan 3A = \tan(A + 2A)$

$$\Rightarrow \tan 3A = \frac{\tan A + \tan 2A}{1 - \tan A \cdot \tan 2A}$$

$\Rightarrow \tan A + \tan 2A = \tan 3A - \tan 3A \cdot \tan 2A \cdot \tan A$

$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \cdot \tan 2A \cdot \tan A$

Q. 43 The value of $\sin(45° + \theta) - \cos(45° - \theta)$ is

(a) $2\cos\theta$ (b) $2\sin\theta$ (c) 1 (d) 0

Thinking Process

Use formula i.e., $\sin(A+B) = \sin A\cos B + \cos A\sin B$

and $\cos(A-B) = \cos A\cos B + \sin A\sin B$.

Sol. ***(d)*** Given expression,

$\sin(45° + \theta) - \cos(45° - \theta)$

$$= \sin 45°\cdot\cos\theta + \cos 45°\cdot\sin\theta - \cos 45°\cdot\cos\theta - \sin 45°\cdot\sin\theta$$

$$= \frac{1}{\sqrt{2}}\cdot\cos\theta + \frac{1}{\sqrt{2}}\cdot\sin\theta - \frac{1}{\sqrt{2}}\cdot\cos\theta - \frac{1}{\sqrt{2}}\sin\theta$$

$$= 0$$

Q. 44 The value of $\cot\left(\frac{\pi}{4} + \theta\right)\cot\left(\frac{\pi}{4} - \theta\right)$ is

(a) -1 (b) 0 (c) 1 (d) Not defined

Thinking Process

Use formula i.e., $\cot(A+B) = \left(\frac{\cot A\cot B - 1}{\cot A + \cot B}\right)$ *and* $\cot(A-B) = \left(\frac{\cot A\cot B + 1}{\cot A - \cot B}\right)$.

Sol. ***(c)*** Given expression, $\cot\left(\frac{\pi}{4} + \theta\right) - \cot\left(\frac{\pi}{4} - \theta\right)$

$$= \left(\frac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\frac{\pi}{4} + \cot\theta}\right)\cdot\left(\frac{\cot\frac{\pi}{4}\cot\theta + 1}{\cot\theta - \cot\frac{\pi}{4}}\right)$$

$$= \left(\frac{\cot\theta - 1}{\cot\theta + 1}\right)\cdot\left(\frac{\cot\theta + 1}{\cot\theta - 1}\right)$$

$$= 1$$

Q. 45 $\cos 2\theta\cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$ is equal to

(a) $\sin 2(\theta + \phi)$ (b) $\cos 2(\theta + \phi)$ (c) $\sin 2(\theta - \phi)$ (d) $\cos 2(\theta - \phi)$

Sol. ***(b)*** Given expression, $\cos 2\theta\cos 2\phi + \sin^2(\theta - \phi) - \sin^2(\theta + \phi)$

$$= \cos 2\theta\cdot\cos 2\phi + \sin(\theta - \phi + \theta + \phi)\cdot\sin(\theta - \phi - \theta - \phi)$$

$$= \cos 2\theta\cdot\cos 2\phi - \sin 2\theta\cdot\sin 2\phi$$

$$= \cos(2\theta + 2\phi) = \cos 2(\theta + \phi)$$

Q. 46 The value of $\cos 12° + \cos 84° + \cos 156° + \cos 132°$ is

(a) $\frac{1}{2}$ (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{8}$

Thinking Process

Use the formula $\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$ *and*

$\cos A - \cos B = -2\sin\frac{A+B}{2}\cdot\sin\frac{A-B}{2}$ *to solve this problem.*

Sol. *(c)* Given expression, $\cos 12° + \cos 84° + \cos 150° + \cos 132°$

$$= \cos 12° + \cos 150° + \cos 84° + \cos 132°$$

$$= 2\cos\left(\frac{12° + 150°}{2}\right)\cdot\cos\left(\frac{12° - 150°}{2}\right) + 2\cos\left(\frac{84° + 132°}{2}\right)\cdot\cos\left(\frac{84° - 132°}{2}\right)$$

$$= 2\cos 84° \cos 72° + 2\cos 108° \cdot \cos 24°$$

$$= 2\cos 84° \cos(90° - 18°) + 2\cos(90° + 18°)\cdot\cos 24°$$

$$= 2\cos 84° \sin 18° - 2\sin 18° \cdot \cos 24°$$

$$= 2\sin 18°(\cos 84° - \cos 24°)$$

$$= 2\sin 18° \cdot 2\sin\left(\frac{84° + 24°}{2}\right)\cdot\sin\left(\frac{84° - 24°}{2}\right)$$

$$= -4\sin 18° \cdot \sin 54° \sin 30°$$

$$= -4\left(\frac{\sqrt{5} - 1}{4}\right)\cdot\cos 36° \cdot \frac{1}{2}$$

$$= -(\sqrt{5} - 1)\left(\frac{\sqrt{5} + 1}{4}\right)\cdot\frac{1}{2} = -\left(\frac{5 - 1}{8}\right) = \frac{-4}{8} = \frac{-1}{2}$$

Q. 47 If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $\tan(2A + B)$ is equal to

(a) 1 (b) 2 (c) 3 (d) 4

Sol. *(c)* Given that, $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

Now, $$\tan(2A + B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \cdot \tan B} \quad \text{...(i)}$$

Also, $$\tan 2A = \frac{2\tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1}{2}}{1 - \frac{1}{4}} = \frac{4}{3}$$

From Eq. (i), $$\tan(2A + B) = \frac{\frac{4}{3} + \frac{1}{3}}{1 - \frac{4}{3}\cdot\frac{1}{3}} = \frac{\frac{4}{3} + \frac{1}{3}}{\frac{9 - 4}{9}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3$$

Q. 48 The value of $\sin\frac{\pi}{10}\sin\frac{13\pi}{10}$ is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{4}$ (d) 1

Sol. *(c)* Given expression, $\sin\frac{\pi}{10}\sin\frac{13\pi}{10} = \sin\frac{\pi}{10}\sin\left(\pi + \frac{3\pi}{10}\right)$

$$= -\sin\frac{\pi}{10}\sin\frac{3\pi}{10} = -\sin 18° \cdot \sin 54°$$

$$= -\sin 18° \cdot \cos 36°$$

$$= -\left(\frac{\sqrt{5} - 1}{4}\right)\left(\frac{\sqrt{5} + 1}{4}\right) \quad \text{[since, put this value here]}$$

$$= -\left(\frac{5 - 1}{16}\right) = -\frac{1}{4}$$

Q. 49 The value of $\sin 50° - \sin 70° + \sin 10°$ is

(a) 1 (b) 0 (c) $\frac{1}{2}$ (d) 2

Thinking Process

Here, use the formula i.e., $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\cdot\sin\left(\frac{A-B}{2}\right)$ *also* $\sin(-\theta) = -\sin\theta$

.

Sol. *(b)* Given expression, $\sin 50° - \sin 70° + \sin 10°$

$$= 2\cos\left(\frac{50° + 70°}{2}\right)\cdot\sin\left(\frac{50° - 70°}{2}\right) + \sin 10°$$

$$= -2\cos 60° \sin 10° + \sin 10°$$

$$= -2\cdot\frac{1}{2}\sin 10° + \sin 10° = 0$$

Q. 50 If $\sin\theta + \cos\theta = 1$, then the value of $\sin 2\theta$ is

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) –1

Sol. *(c)* Given that, $\sin\theta + \cos\theta = 1$

On squaring both sides, we get

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = 1$$

$$\Rightarrow \quad 1 + \sin 2\theta = 1$$

$$\therefore \quad \sin 2\theta = 0$$

Q. 51 If $\alpha + \beta = \frac{\pi}{4}$, then the value of $(1 + \tan\alpha)(1 + \tan\beta)$ is

(a) 1 (b) 2 (c) –2 (d) Not defined

Thinking Process

Formula i.e., $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$ *to solve this problem.*

Sol. *(b)* Given that, $\alpha + \beta = \frac{\pi}{4}$

Now, $(1 + \tan\alpha)(1 + \tan\beta) = 1 + \tan\alpha + \tan\beta + \tan\alpha\tan\beta$ ...(i)

We know that, $\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$

$$\Rightarrow \quad 1 = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta}$$

$$\Rightarrow \quad \tan\alpha + \tan\beta = 1 - \tan\alpha\tan\beta$$

From Eq. (i),

$$(1 + \tan\alpha)(1 + \tan\beta) = 1 + 1 - \tan\alpha\cdot\tan\beta + \tan\alpha\cdot\tan\beta$$

$$= 2$$

Q. 52 If $\sin\theta = \frac{-4}{5}$ and θ lies in third quadrant, then the value of $\cos\frac{\theta}{2}$ is

(a) $\frac{1}{5}$ (b) $-\frac{1}{\sqrt{10}}$ (c) $-\frac{1}{\sqrt{5}}$ (d) $\frac{1}{\sqrt{10}}$

Thinking Process

Use $\cos\theta = \sqrt{1-\sin^2\theta}$ *and* $\cos\theta = 2\cos^2\frac{\theta}{2} - 1.$

Sol. *(c)* Given that,

$$\sin\theta = \frac{-4}{5}$$

$$\cos\theta = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{25-16}{25}} = \pm\frac{3}{5}$$

$$\cos\theta = \frac{-3}{5} \quad \text{[since, } \theta \text{ lies in third quadrant]}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2} - 1 = \frac{-3}{5}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow \quad 2\cos^2\frac{\theta}{2} = \frac{2}{5}$$

$$\therefore \quad \cos\frac{\theta}{2} = \pm\frac{1}{\sqrt{5}}$$

$$\Rightarrow \quad \cos\frac{\theta}{2} = -\frac{1}{\sqrt{5}} \quad \text{[since, } \theta \text{ lies in third quadrant]}$$

Q. 53 The number of solutions of equation $\tan x + \sec x = 2\cos x$ lying in the interval $[0, 2\pi]$ is

(a) 0 (b) 1 (c) 2 (d) 3

Sol. *(c)* Given equation,

$$\tan + \sec x = 2\cos x$$

$$\Rightarrow \quad \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$$

$$\Rightarrow \quad 1 + \sin x = 2\cos^2 x$$

$$\Rightarrow \quad 1 + \sin x = 2(1 - \sin^2 x)$$

$$\Rightarrow \quad 1 + \sin x = 2 - 2\sin^2 x$$

$$\Rightarrow \quad 2\sin^2 x + \sin x - 1 = 0$$

$$\Rightarrow \quad 2\sin^2 x + 2\sin x - \sin x - 1 = 0$$

$$\Rightarrow \quad 2\sin x(\sin x + 1) - 1(\sin x + 1) = 0$$

$$\Rightarrow \quad (\sin x + 1)(2\sin x - 1) = 0$$

$$\Rightarrow \quad \sin x + 1 = 0 \text{ or } (2\sin x - 1) = 0$$

$$\Rightarrow \quad \sin x = -1, \sin x = \frac{1}{2}$$

$$\therefore \quad x = \frac{3\pi}{2}, x = \frac{\pi}{6}$$

Hence, only two solutions possible.

Q. 54 The value of $\sin\frac{\pi}{18}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{5\pi}{18}$ is

(a) $\sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$ (b) 1

(c) $\cos\frac{\pi}{6}+\cos\frac{3\pi}{7}$ (d) $\cos\frac{\pi}{9}+\sin\frac{\pi}{9}$

Thinking Process

Here, apply the formulae i.e., $\sin A+\sin B=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$.

Sol. ***(a)*** Given expression, $\sin\frac{\pi}{18}+\sin\frac{\pi}{9}+\sin\frac{2\pi}{9}+\sin\frac{5\pi}{18}$

$$=\sin 10^\circ+\sin 20^\circ+\sin 40^\circ+\sin 50^\circ$$
$$=\sin 50^\circ+\sin 10^\circ+\sin 40^\circ+\sin 20^\circ$$
$$=\sin 130^\circ+\sin 10^\circ+\sin 140^\circ+\sin 20^\circ$$
$$=2\sin 70^\circ\cos 60^\circ+2\sin 80^\circ\cdot\cos 60^\circ \quad \left[\because \sin x+\sin y=2\sin\frac{x+y}{2}\cdot\cos\frac{x-y}{2}\right]$$
$$=2\cdot\frac{1}{2}\sin 70^\circ+2\cdot\frac{1}{2}\sin 80^\circ \quad \left[\because \cos 60^\circ=\frac{1}{2}\right]$$
$$=\sin 70^\circ+\sin 80^\circ=\sin\frac{7\pi}{18}+\sin\frac{4\pi}{9}$$

Q. 55 If A lies in the second quadrant and $3\tan A+4=0$, then the value of $2\cot A-5\cos A+\sin A$ is

(a) $\frac{-53}{10}$ (b) $\frac{23}{10}$ (c) $\frac{37}{10}$ (d) $\frac{7}{10}$

Thinking Process

Use the formulae i.e., $\sec A=\sqrt{1+\tan^2 A}$ *and* $\sin A=\sqrt{1-\cos^2 A}$, $\sec A=\frac{1}{\cos A}$ and $\tan A=\frac{1}{\cot A}$.

Sol. ***(b)*** Given equation, $3\tan A+4=0$

$\Rightarrow$ $3\tan A=-4$

$\Rightarrow$ $\tan A=\frac{-4}{3}$

$\Rightarrow$ $\cot A=\frac{-3}{4}$

$\Rightarrow$ $\sec A=\sqrt{1+\frac{16}{9}}=\sqrt{\frac{25}{9}}=\pm\frac{5}{3}$

$\Rightarrow$ $\sec A=\frac{-5}{3}$ [since, A lies in second quadrant]

$\cos A=\frac{-3}{5}$

$\sin A=\sqrt{1-\frac{9}{25}}=\frac{\sqrt{25-9}}{25}=\pm\frac{4}{5}$

$\sin A=\frac{4}{5}$ [since, A lies in second quadrant]

$$\therefore \quad 2\cot A - 5\cos A + \sin A = 2\left(\frac{-3}{4}\right) - 5\left(\frac{-3}{5}\right) + \frac{4}{5}$$
$$= \frac{-6}{4} + 3 + \frac{4}{5}$$
$$= \frac{-30 + 60 + 16}{20} = \frac{46}{20}$$
$$= \frac{23}{10}$$

Q. 56 The value of $\cos^2 48° - \sin^2 12°$ is

(a) $\frac{\sqrt{5}+1}{8}$ (b) $\frac{\sqrt{5}-1}{8}$ (c) $\frac{\sqrt{5}+1}{5}$ (d) $\frac{\sqrt{5}+1}{2\sqrt{2}}$

Sol. ***(a)*** Given expression, $\cos^2 48° - \sin^2 12°$
$$= \cos(48° + 12°) - \cos(48° - 12°)$$
$$= \cos 60° \cdot \cos 36°$$
$$= \frac{1}{2} \cdot \frac{\sqrt{5}+1}{4}$$
$$= \frac{\sqrt{5}+1}{8}$$

Q. 57 If $\tan\alpha = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$, then $\cos 2\alpha$ is equal to

(a) $\sin 2\beta$ (b) $\sin 4\beta$ (c) $\sin 3\beta$ (d) $\cos 2\beta$

Thinking Process

Use $\cos 2\alpha = \frac{1-\tan^2\alpha}{1+\tan^2\alpha}$ *and* $\sin 2\alpha = \frac{2\tan\alpha}{1+\tan^2\alpha}$

Sol. ***(b)*** Given that, $\tan = \frac{1}{7}$ and $\tan\beta = \frac{1}{3}$
$$\cos 2\alpha = \frac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \frac{\frac{48}{49}}{\frac{50}{49}}$$
$$= \frac{48}{50} = \frac{24}{25}$$
$$\Rightarrow \quad \cos 2\alpha = \frac{24}{25} \quad \text{...(i)}$$
We know that, $$\sin 4\beta = \frac{2\tan 2\beta}{1+\tan^2 2\beta} \quad \text{...(ii)}$$
and $$\tan 2\beta = \frac{2\tan\beta}{1-\tan^2\beta} = \frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}}$$
$$= \frac{\frac{2}{3}}{\frac{8}{9}} = \frac{2 \times 9}{3 \times 8} = \frac{3}{4}$$

From Eq, (ii),

$$\sin 4\beta = \frac{2 \times \frac{3}{4}}{1 + \frac{9}{16}} = \frac{\frac{6}{4}}{\frac{25}{16}} = \frac{6 \times 16}{4 \times 25}$$

$\Rightarrow$ $$\sin 4\beta = \frac{24}{25}$$

$\Rightarrow$ $$\sin 4\beta = \cos 2\alpha$$

$\therefore$ $$\cos 2\alpha = \sin 4\beta$$ [from Eq. (i)]

Q. 58 If $\tan\theta = \frac{a}{b}$, then $b\cos 2\theta + a\sin 2\theta$ is equal to

(a) a (b) b (c) $\frac{a}{b}$ (d) None of these

Sol. ***(b)*** Given that, $\tan\theta = \frac{a}{b}$

$\therefore$ $$b\cos 2\theta + a\sin 2\theta = b\left(\frac{1 - \tan^2\theta}{1 + \tan^2\theta}\right) + a\left(\frac{2\tan\theta}{1 + \tan^2\theta}\right)$$

$$= b\left(\frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}}\right) + a\left(\frac{\frac{2a}{b}}{1 + \frac{a^2}{b^2}}\right)$$

$$= b\left(\frac{b^2 - a^2}{b^2 + a^2}\right) + \frac{2a^2b}{a^2 + b^2}$$

$$= \frac{b}{a^2 + b^2}[b^2 - a^2 + 2a^2] = \frac{(a^2 + b^2)b}{(a^2 + b^2)}$$

$$= b$$

Q. 59 If for real values of x, $\cos\theta = x + \frac{1}{x}$, then

(a) θ is an acute angle (b) θ is right angle

(c) θ is an obtuse angle (d) No value of θ is possible

Thinking Process

The quadratic equation $ax^2 + bx + c = 0$ has real roots , then $b^2 - 4ac = 0$, use this condition to solve the above problem.

Sol. ***(d)*** Here, $$\cos\theta = x + \frac{1}{x}$$

$\Rightarrow$ $$\cos\theta = \frac{x^2 + 1}{x}$$

$$x^2 - x\cos\theta + 1 = 0$$

For real value of x, $$(-\cos\theta)^2 - 4 \times 1 \times 1 = 0$$

$$\cos^2\theta = 4$$

$$\cos\theta = \pm 2$$

which is not possible. $[\because -1 \le \cos\theta \le 1]$

Fillers

Q. 60 The value of $\frac{\sin 50°}{\sin 130°}$ is ....

Sol. Here,
$$\frac{\sin 50°}{\sin 130°} = \frac{\sin(180° - 130°)}{\sin 130°} = \frac{\sin 130°}{\sin 130°} = 1$$

Q. 61 If $k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$, then the numerical value of k is ....

Sol. Here,
$$k = \sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$$
$$= \sin 10° \sin 50° \sin 70°$$
$$= \sin 10° \cos 40° \cdot \cos 20°$$
$$= \frac{1}{2}\sin 10°[2\cos 40° \cdot \cos 20°]$$
$$= \frac{1}{2}\sin 10°[\cos 60° + \cos 20°] \quad [\because 2\cos x \cdot \cos y = \cos(x + y) + \cos(x - y)]$$
$$= \frac{1}{2}\sin 10° \cdot \frac{1}{2} + \frac{1}{2}\sin 10° \cos 20°$$
$$= \frac{1}{4}\sin 10° + \frac{1}{4}[\sin 30° - \sin 10°]$$
$$= \frac{1}{8}$$

Q. 62 If $\tan A = \frac{1 - \cos\theta}{\sin B}$, then $\tan 2A =$ ....

Thinking Process

Use $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$ *and* $\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$.

Sol. Given that,
$$\tan A = \frac{1 - \cos B}{\sin B}$$
$$= \frac{1 - 1 + 2\sin^2\frac{B}{2}}{2\sin\frac{B}{2} \cdot \cos\frac{B}{2}} = \tan\frac{B}{2}$$

Now,
$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$
$$\Rightarrow \quad \tan 2A = \frac{2 \cdot \tan\frac{B}{2}}{1 - \tan^2\frac{B}{2}}$$
$$\Rightarrow \quad \tan 2A = \tan B$$

Q. 63 If $\sin x + \cos x = a$, then

(i) $\sin^6 x + \cos^6 x =$ ....

(ii) $|\sin x - \cos x| =$ ....

Sol. Given that, $\sin x + \cos x = a$

On squaring both sides, we get

$$(\sin x + \cos x)^2 = (a)^2$$

$$\Rightarrow \quad \sin^2 x + \cos^2 x + 2\sin x \cos x = a^2$$

$$\Rightarrow \quad \sin x \cdot \cos x = \frac{1}{2}(a^2 - 1)$$

(i) $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$

$$= (\sin^2 x + \cos^2 x)(\sin^4 x - \sin^2 x\cos^2 x + \cos^4 x)$$

$$= \sin^4 x + \cos^4 x - \frac{1}{4}(a^2 - 1)^2$$

$$= (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x - \frac{1}{4}(a^2 - 1)^2$$

$$= 1 - 2\cdot\frac{1}{4}(a^2 - 1)^2 - \frac{1}{4}(a^2 - 1)^2 = \frac{1}{4}[4 - 3(a^2 - 1)^2]$$

(ii) $|\sin x - \cos x| = \sqrt{(\sin x - \cos x)^2}$

$$= \sqrt{\sin^2 x + \cos^2 x - 2\sin x\cos x}$$

$$= \sqrt{1 - 2\frac{1}{2}(a^2 - 1)} = \sqrt{1 - a^2 + 1} = \sqrt{2 - a^2}$$

Q. 64 In right angled ΔABC with $\angle C = 90°$ the equation whose roots are $\tan A$ and $\tan B$ is ....

Sol. In right angled $\Delta\, ABC$, $\angle C = 90°$

$$\therefore \quad \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \quad \frac{1}{0} = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \quad \tan A \tan B = 1 \quad \ldots\text{(i)}$$

$$\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B}$$

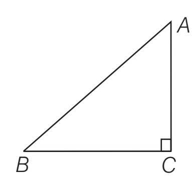
$$= \frac{\sin A}{\cos A} + \frac{\sin(90° - A)}{\cos(90° - A)} \quad [\because \angle C = 90°, \angle B = 90° - A]$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}$$

$$= \frac{1}{\sin A \cdot \cos A} = \frac{2}{2 \cdot \sin A \cdot \cos A}$$

$$= \frac{2}{\sin 2A} \quad [\because \sin 2x = 2\sin x\cos x]$$



So, the required equation is $x^2 - \left(\frac{2}{\sin A}\right)x + 1$.

Q. 65 $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x) = \ldots\ldots\ldots$.

Thinking Process

Use formulae i.e., $(a^3 + b^3) = (a+b)(a^2 - ab + b^2)$ *and* $a^2 + b^2 = (a+b)^2 - 2ab$.

Sol. Given expression, $3(\sin x - \cos x)^4 + 6(\sin x + \cos x)^2 + 4(\sin^6 x + \cos^6 x)$

$$= 3[\sin^2 x + \cos^2 x - 2\sin x \cos x]^2 + 6[\sin^2 x + \cos^2 x + 2\cdot\sin x\cdot\cos x] + 4[(\sin^2 x)^3 + (\cos^2 x)^3]$$

$$= 3(1 - \sin 2x)^2 + 6(1 + \sin 2x) + 4[(\sin^2 + \cos^2 x)(\sin^4 x - \sin x \cos^2 x + \cos^4 x)$$

$$= 3(1 + \sin^2 23x - 2\sin 2x) + 6 + 6\sin 2x + 4[(\sin^2 x + \cos^2 x)^2 3\sin x \cos^2 x]$$

$$= 3 + 3\sin^2 2x - 6\sin 2x + 6 + 6\sin 2x$$

$$= 4 - 3\sin^2 2x = 13$$

Q. 66 Given $x > 0$, the value of $f(x) = -3\cos\sqrt{3 + x + x^2}$ lie in the interval ....

Sol. Given function, $f(x) = -3\cos\sqrt{3 + x + x^2}$

We know that, $-1 \le \cos x \le 1$

$\Rightarrow$ $-3 \le 3\cos x \le 3$

$\Rightarrow$ $3 \ge -3\cos x \ge -3$

$\Rightarrow$ $-3 \le -3\cos x \le 3$

So, the value of $f(x)$ lies in $[-3, 3]$.

Q. 67 The maximum distance of a point on the graph of the function $y = \sqrt{3}\sin x + \cos x$ from X-axis is ....

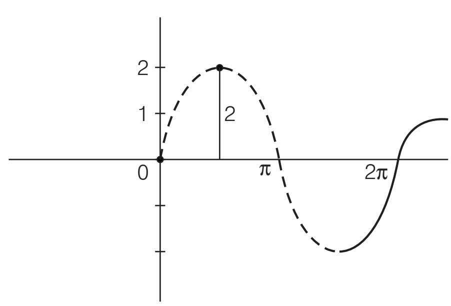
Sol. Given that, $y = \sqrt{3}\sin x + \cos x$

$$y = 2\left[\frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x\right]$$

$$= 2\left[\sin x\cdot\cos\frac{\pi}{6} + \cos x \sin\frac{\pi}{6}\right]$$

$$= 2\sin(x + \pi/6)$$

Graph of $y = 2\sin x$



Hence, the maximum distance is 2 units.

True/False

Q. 68 In each of the questions 68 to 75, state whether the statements is True or False? Also, give justification.

Thinking Process

If $\tan A = \frac{1-\cos B}{\sin B}$, *then* $\tan 2A = \tan B$

Sol. ***True***

Given that, $$\tan A = \frac{1-\cos B}{\sin B} = \frac{1-1+2\sin^2\frac{B}{2}}{2\sin\frac{B}{2}\cdot\cos\frac{B}{2}} = \tan\frac{B}{2}$$

Now, $$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\cdot\tan\frac{B}{2}}{1-\tan^2\frac{B}{2}} = \tan B$$

Q. 69 The equality $\sin A + \sin 2A + \sin 3A = 3$ holds for some real value of A.

Sol. ***False***

Given that, $\sin A + \sin 2A + \sin 3A = 3$

It is possible only if $\sin A$, $\sin 2A$, $\sin 3A$ each has a value one because maximum value of $\sin A$ is a certain angle is 1. Which is not possible because angle are different.

Q. 70 $\sin 10°$ is greater than $\cos 10°$.

Sol. ***False***

$$\sin 10° = \sin(90° - 80°)$$
$$\sin 10° = \cos 80°$$
$\therefore$ $$\cos 80° < \cos 10°$$
Hence, $$\sin 10° < \cos 10°$$

Q. 71 $\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15} = \frac{1}{16}$

Sol. ***True***

$$\text{LHS} = \cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{16\pi}{15}$$
$$= \cos 24°\cos 48°\cos 96°\cos 192°$$
$$= \frac{1}{16\sin 24°}[(2\sin 24°\cos 24°)(2\cos 48°)(2\cos 96°)(2\cos 192°)]$$
$$= \frac{1}{16\sin 24°}[2\sin 48°\cos 48°(2\cos 96°)(2\cos 192°)]$$
$$= \frac{1}{16\sin 24°}[(2\sin 96°\cos 96°)(2\cos 192°)]$$
$$= \frac{1}{16\sin 24°}[2\sin 192°\cos 192°)$$
$$= \frac{1}{16\sin 24°}\sin 384° = \frac{\sin(360° + 24°)}{16\sin 24°}$$
$$= \frac{1}{16} = \text{RHS}$$

Hence proved.

Q. 72 One value of θ which satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1$ lies between 0 and 2π.

Sol. ***False***

Given equation, $\sin^4\theta - 2\sin^2\theta - 1 = 0$

$\Rightarrow \quad \sin^2\theta = \dfrac{2 \pm \sqrt{4+4}}{2}$

$\Rightarrow \quad \sin^2\theta = \dfrac{2 \pm 2\sqrt{2}}{2}$

$\Rightarrow \quad \sin^2\theta = (1+\sqrt{2}) \text{ or } (1-\sqrt{2}) \Rightarrow -1 \le \sin\theta \le 1$

$\Rightarrow \quad \sin^2\theta \le 1$

$\because \quad \sin^2\theta = \sqrt{2+1} \text{ or } (1-\sqrt{2})$

which is not possible.

Q. 73 If $\operatorname{cosec} x = 1 + \cot x$, then $x = 2n\pi,\ 2n\pi + \dfrac{\pi}{2}$

Sol. ***True***

Given that, $\operatorname{cosec} x = 1 + \cot x$

$\Rightarrow \quad \dfrac{1}{\sin x} = 1 + \dfrac{\cos x}{\sin x} \Rightarrow \dfrac{1}{\sin x} = \dfrac{\sin x + \cos x}{\sin x}$

$\Rightarrow \quad \sin x + \cos x = 1$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} \cdot \sin x + \dfrac{1}{\sqrt{2}} \cdot \cos x = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \sin\dfrac{\pi}{4}\sin x + \cos x \cos\dfrac{\pi}{4} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \cos\left(x - \dfrac{\pi}{4}\right) = \cos\dfrac{\pi}{4}$

$\therefore \quad x - \dfrac{\pi}{4} = 2n\pi \pm \dfrac{\pi}{4}$

For positive sign, $x = 2n\pi + \dfrac{\pi}{4} + \dfrac{\pi}{4} = 2n\pi + \dfrac{\pi}{2}$

For negative sign, $x = 2n\pi - \dfrac{\pi}{4} + \dfrac{\pi}{4} = 2n\pi$

Q. 74 If $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$, then $\theta = \dfrac{n\pi}{3} + \dfrac{\pi}{9}$.

Sol. ***True***

$\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta + \tan 2\theta = \sqrt{3} - \sqrt{3}\tan\theta\tan 2\theta$

$\Rightarrow \quad \tan\theta + \tan 2\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \quad \dfrac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3}$

$\Rightarrow \quad \tan(\theta + 2\theta) = \tan\dfrac{\pi}{3} \Rightarrow \tan 3\theta = \tan\dfrac{\pi}{3}$

$\therefore \quad 3\theta = n\pi + \dfrac{\pi}{3}$

$\theta = \dfrac{n\pi}{3} + \dfrac{\pi}{9}$

Q. 75 If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then $\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}$.

Thinking Process

Use the formulae i.e., $\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$ *and* $\cos(A - B) = \cos A\cos B + \sin A\sin B$.

Sol. ***True***

We have, $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$

$\Rightarrow \tan(\pi\cos\theta) = \tan\left[\frac{\pi}{2} - (\pi\sin\theta)\right]$

$\Rightarrow \pi\cos = \frac{\pi}{2} - \pi\sin\theta$

$\Rightarrow \pi(\sin\theta + \cos\theta) = \frac{\pi}{2}$

$\Rightarrow \sin\theta + \cos\theta = \frac{1}{2}$

$\Rightarrow \frac{1}{\sqrt{2}}\cdot\sin\theta + \frac{1}{\sqrt{2}}\cdot\cos\theta = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin\theta\cdot\sin\frac{\pi}{4} + \cos\theta\cdot\cos\frac{\pi}{4} = \frac{1}{2\sqrt{2}}$

$\therefore \cos\left(\theta - \frac{\pi}{4}\right) = \frac{1}{2\sqrt{2}}$

Q. 76 In the following match each item given under the Column I to its correct answer given under the Column II.

	Column I		Column II
(i)	$\sin(x + y)\sin(x - y)$	(a)	$\cos^2 x - \sin^2 y$
(ii)	$\cos(x + y)\cos(x - y)$	(b)	$1 - \tan\theta / 1 + \tan\theta$
(iii)	$\cot\left(\frac{\pi}{4} + \theta\right)$	(c)	$1 + \tan\theta / 1 - \tan\theta$
(iv)	$\tan\left(\frac{\pi}{4} + \theta\right)$	(d)	$\sin^2 x - \sin^2 y$

Sol.

(i) $\sin(x + y)\sin(x - y) = \sin^2 x - \sin^2 y$

(ii) $\cos(x + y)\cos(x - y) = \cos^2 x - \sin^2 y$

(iii) $\cot\left(\frac{\pi}{4} + \theta\right) = \dfrac{\cot\frac{\pi}{4}\cot\theta - 1}{\cot\frac{\pi}{4} + \cot\theta}$

$= \dfrac{-1 + \cot\theta}{1 + \cot\theta} = \dfrac{1 - \tan\theta}{1 + \tan\theta}$

(iv) $\tan\left(\frac{\pi}{4} + \theta\right) = \dfrac{\tan\frac{\pi}{4} + \tan\theta}{1 - \tan\frac{\pi}{4}\tan\theta} = \dfrac{1 + \tan\theta}{1 - \tan\theta}$

Hence, the correct mathes are **(i)** → (d), **(ii)** → (a), **(iii)** → (b), **(iv)** → (c).