3

Matrices

Short Answer Type Questions

Q. 1 If a matrix has 28 elements, what are the possible orders it can have? What if it has 13 elements?

Sol. We know that, if a matrix is of order $m \times n$, it has mn elements, where m and n are natural numbers.

We have, $m \times n = 28$

$\Rightarrow \quad (m, n) = \{(1, 28), (2, 14), (4, 7), (7, 4), (14, 2), (28, 1)\}$

So, the possible orders are $1\times28, 2\times14, 4\times7, 7\times4, 14\times2, 28\times1$.

Also, if it has 13 elements, then $m \times n = 13$

$\Rightarrow \quad (m, n) = \{(1,13), (13,1)\}$

Hence, the possible orders are $1\times13, 13\times1$.

Q. 2 In the matrix $A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$, write

(i) the order of the matrix A.

(ii) the number of elements.

(iii) elements a_{23}, a_{31} and a_{12}.

Sol. We have, $$A = \begin{bmatrix} a & 1 & x \\ 2 & \sqrt{3} & x^2 - y \\ 0 & 5 & \frac{-2}{5} \end{bmatrix}$$

(i) the order of matrix $A = 3\times3$

(ii) the number of elements $= 3\times3 = 9$

[since, the number of elements in an $m \times n$ matrix will be equal to $m \times n = mn$]

(iii) $a_{23} = x^2 - y$, $a_{31} = 0$, $a_{12} = 1$

[since, we know that a_{ij}, is a representation of element lying in the ith row and jth column]

Q. 3 Construct $a_{2\times2}$ matrix, where

(i) $a_{ij} = \frac{(i-2j)^2}{2}$ (ii) $a_{ij} = |-2i+3j|$

Sol. We know that, the notation, namely $A = [a_{ij}]_{m\times n}$ indicates that A is a matrix of order $m \times n$, also $1 \le i \le m, 1 \le j \le n;\ i, j \in N$.

(i) Here, $A = [a_{ij}]_{2\times2}$

$$\Rightarrow \quad A = \frac{(i-2j)^2}{2}, 1 \le i \le 2 ; 1 \le j \le 2 \quad \text{...(i)}$$

$$\therefore \quad a_{11} = \frac{(1-2)^2}{2} = \frac{1}{2}$$

$$a_{12} = \frac{(1-2\times2)^2}{2} = \frac{9}{2}$$

$$a_{21} = \frac{(2-2\times1)^2}{2} = 0$$

$$a_{22} = \frac{(2-2\times2)^2}{2} = 2$$

$$\text{Thus,} \quad A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}_{2\times2}$$

(ii) Here, $A = [a_{ij}]_{2\times2} = |-2i+3j|,\ 1 \le i \le 2;\ 1 \le j \le 2$

$$\therefore \quad a_{11} = |-2\times1+3\times1| = 1$$

$$a_{12} = |-2\times1+3\times2| = 4 \quad [\because |-1| = 1]$$

$$a_{21} = |-2\times2+3\times1| = 1$$

$$a_{22} = |-2\times2+3\times2| = 2$$

$$\therefore \quad A = \begin{vmatrix} 1 & 4 \\ 1 & 2 \end{vmatrix}_{2\times2}$$

Q. 4 Construct a 3×2 matrix whose elements are given by $a_{ij} = e^{i\cdot x} = \sin jx$.

Sol. Since, $A = [a_{ij}]_{m\times n}$ $1 \le i \le m$ and $1 \le j \le n, i, j \in N$

$$\therefore \quad A = [e^{i\cdot x}\sin jx]_{3\times2};\ 1 \le i \le 3;\ 1 \le j \le 2$$

$$\Rightarrow \quad a_{11} = e^{1\cdot x}\cdot\sin 1\cdot x = e^{x}\sin x$$

$$a_{12} = e^{1\cdot x}\cdot\sin 2\cdot x = e^{x}\sin 2x$$

$$a_{21} = e^{2\cdot x}\cdot\sin 1\cdot x = e^{2x}\sin x$$

$$a_{22} = e^{2\cdot x}\cdot\sin 2\cdot x = e^{2x}\sin 2x$$

$$a_{31} = e^{3\cdot x}\cdot\sin 1\cdot x = e^{3x}\sin x$$

$$a_{32} = e^{3\cdot x}\cdot\sin 2\cdot x = e^{3x}\sin 2x$$

$$\therefore \quad A = \begin{bmatrix} e^{x}\sin x & e^{x}\sin 2x \\ e^{2x}\sin x & e^{2x}\sin 2x \\ e^{3x}\sin x & e^{3x}\sin 2x \end{bmatrix}_{3\times2}$$

Q. 5 Find the values of a and b, if $A = B$, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Thinking Process

By using equality of two matrices, we know that each element of A is equal to corresponding element of B.

Sol. We have, $A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}_{2\times 2}$

Also, $A = B$

By equality of matrices we know that each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j.

$\therefore$ $a_{11} = b_{11} \Rightarrow a+4 = 2a+2 \Rightarrow a = 2$

$a_{12} = b_{12} \Rightarrow 3b = b^2+2 \Rightarrow b^2 = 3b-2$

and $a_{22} = b_{22} \Rightarrow -6 = b^2 - 5b$

$\Rightarrow$ $-6 = 3b-2-5b$ $[\because b^2 = 3b-2]$

$\Rightarrow$ $2b = 4 \Rightarrow b = 2$

$\therefore$ $a = 2$ and $b = 2$

Q. 6 If possible, find the sum of the matrices A and B, where $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix}$.

Thinking Process

We know that, two matrices are added, if they have same order.

Sol. We have, $A = \begin{bmatrix} \sqrt{3} & 1 \\ 2 & 3 \end{bmatrix}_{2\times 2}$ and $B = \begin{bmatrix} x & y & z \\ a & b & 6 \end{bmatrix}_{2\times 3}$

Here, A and B are of different orders. Also, we know that the addition of two matrices A and B is possible only if order of both the matrices A and B should be same.

Hence, the sum of matrices A and B is not possible.

Q. 7 If $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}$, then find

(i) $X + Y$.

(ii) $2X - 3Y$.

(iii) a matrix Z such that $X + Y + Z$ is a zero matrix.

Sol. We have, $X = \begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix}_{2\times 3}$ and $Y = \begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix}_{2\times 3}$

(i) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & 1 \end{bmatrix}$

(ii) $\because\ 2X = 2\begin{bmatrix} 3 & 1 & -1 \\ 5 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 2 & -2 \\ 10 & -4 & -6 \end{bmatrix}$

and $$3Y = 3\begin{bmatrix} 2 & 1 & -1 \\ 7 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 & -3 \\ 21 & 6 & 12 \end{bmatrix}$$

$\therefore$ $$2X - 3Y = \begin{bmatrix} 6-6 & 2-3 & -2+3 \\ 10-21 & -4-6 & -6-12 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 1 \\ -11 & -10 & -18 \end{bmatrix}$$

(iii) $X + Y = \begin{bmatrix} 3+2 & 1+1 & -1-1 \\ 5+7 & -2+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 5 & 2 & -2 \\ 12 & 0 & +1 \end{bmatrix}$

Also, $$X + Y + Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We see that Z is the additive inverse of $(X+Y)$ or negative of $(X+Y)$.

$\therefore$ $$Z = \begin{bmatrix} -5 & -2 & 2 \\ -12 & 0 & -1 \end{bmatrix} \qquad [\because Z = -(X+Y)]$$

Q. 8 Find non-zero values of x satisfying the matrix equation

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ (10) & 6x \end{bmatrix}.$$

Sol. Given that,

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} (x^2+8) & 24 \\ 10 & 6x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2+16 & 2x+10x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\Rightarrow 2x + 10x = 48$$

$$\Rightarrow 12x = 48$$

$$\therefore x = \frac{48}{12} = 4$$

Q. 9 If $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then show that

$$(A+B)\ (A-B) \neq A^2 - B^2.$$

Sol. We have, $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\therefore$ $$(A+B) = \begin{bmatrix} 0+0 & 1-1 \\ 1+1 & 1+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 1 \end{bmatrix}_{2\times 2}$$

and $$(A-B) = \begin{bmatrix} 0-0 & 1+1 \\ 1-1 & 1-0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

Since, $(A+B)\cdot(A-B)$ is defined, if the number of columns of $(A+B)$ is equal to the number of rows of $(A-B)$, so here multiplication of matrices $(A+B)\cdot(A-B)$ is possible.

Now, $$(A+B)_{2\times 2}\cdot(A-B)_{2\times 2} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 4+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \quad \ldots(\text{i})$$

Also, $A^2 = A \cdot A$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+1 \\ 0+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

and $B^2 = B \cdot B$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-1 & 0+0 \\ 0+0 & -1+0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\therefore$ $$A^2 - B^2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \quad \text{...(ii)}$$

Thus, we see that

$$(A + B) \cdot (A - B) \neq A^2 - B^2$$ [using Eqs. (i) and (ii)]

$\Rightarrow$ $$\begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$ **Hence proved.**

Q. 10 Find the value of x, if $[1\ x\ 1] \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$

Sol. We have, $$[1\ x\ 1]_{1\times 3} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [1+2x+15 \quad 3+5x+3 \quad 2+x+2]_{1\times 3} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [16+2x \quad 5x+6 \quad x+4]_{1\times 3} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix}_{3\times 1} = 0$$

$$\Rightarrow [16+2x+(5x+6)\cdot 2+(x+4)\cdot x]_{1\times 1} = 0$$

$$\Rightarrow [16+2x+10x+12+x^2+4x] = 0$$

$$\Rightarrow [x^2+16x+28] = 0$$

$$\Rightarrow [x^2+2x+14x+28] = 0$$

$$\Rightarrow (x+2)(x+14) = 0$$

$$\therefore x = -2, -14$$

Q. 11 Show that $A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$ satisfies the equation $A^2 - 3A - 7I = O$ and hence find the value of A^{-1}.

Sol. We have, $$A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\therefore A^2 = A \cdot A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 25-3 & 15-6 \\ -5+2 & -3+4 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$3A = 3\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix}$$

and $$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$\therefore$ $$A^2 - 3A - 7I = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 22-15-7 & 9-9-0 \\ -3+3-0 & 1+6-7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$ **Hence proved.**

Since, $$A^2 - 3A - 7I = 0$$

$\Rightarrow$ $$A^{-1}[(A^2) - 3A - 7I] = A^{-1}0$$

$\Rightarrow$ $$A^{-1}A \cdot A - 3A^{-1}A - 7A^{-1}I = 0 \qquad [\because A^{-1}0 = 0]$$

$\Rightarrow$ $$IA - 3I - 7A^{-1} = 0 \qquad [\because A^{-1}A = I]$$

$\Rightarrow$ $$A - 3I - 7A^{-1} = 0 \qquad [\because A^{-1}I = A^{-1}]$$

$\Rightarrow$ $$-7A^{-1} = -A + 3I$$

$$= \begin{bmatrix} -5 & -3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \frac{-1}{7}\begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix}$$

Q. 12 Find the matrix A satisfying the matrix equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Thinking Process

We know that, if two matrices A and B of order $m \times n$ and $p \times q$ respectively are multiplied, then necessity condition to multiplication of $A \cdot B$ is $n = p$. So, by taking a matrix of correct order we can get the desired elements of the required matrix.

Sol. We have, $$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}_{2\times 2} A \cdot \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

Let $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}_{2\times 2}$$

$\therefore$ $$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 2a+c & 2b+d \\ 3a+2c & 3b+2d \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} -6a-3c+10b+5d & 4a+2c-6b-3d \\ -9a-6c+15b+10d & 6a+4c-9b-6d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$-6a - 3c + 10b + 5d = 1 \qquad \ldots(\text{i})$$

$$\Rightarrow \quad 4a+2c-6b-3d=0 \quad \ldots(ii)$$
$$\Rightarrow \quad -9a-6c+15b+10d=0 \quad \ldots(iii)$$
$$\Rightarrow \quad 6a+4c-9b-6d=1 \quad \ldots(iv)$$

On adding Eqs. (i) and (iv), we get

$$c+b-d=2 \Rightarrow d=c+b-2 \quad \ldots(v)$$

On adding Eqs. (ii) and (iii), we get

$$-5a-4c+9b+7d=0 \quad \ldots(vi)$$

On adding Eqs. (vi) and (iv), we get

$$a+0+0+d=1 \Rightarrow d=1-a \quad \ldots(vii)$$

From Eqs. (v) and (vii),

$$c+b-2=1-a \Rightarrow a+b+c=3 \quad \ldots(viii)$$
$$\Rightarrow \quad a=3-b-c$$

Now, using the values of a and d in Eq. (iii), we get

$$-9(3-b-c)-6c+15b+10(-2+b+c)=0$$
$$\Rightarrow \quad -27+9b+9c-6c+15b-20+10b+10c=0$$
$$\Rightarrow \quad 34b+13c=47 \quad \ldots(ix)$$

Now, using the values of a and d in Eq. (ii), we get

$$4(3-b-c)+2c-6b-3(b+c-2)=0$$
$$\Rightarrow \quad 12-4b-4c+2c-6b-3b-3c+6=0$$
$$\Rightarrow \quad -13b-5c=-18 \quad \ldots(x)$$

On multiplying Eq. (ix) by 5 and Eq. (x) by 13, then adding, we get

$$\begin{aligned} -169b-65c &= -234 \\ 170b+65c &= 235 \\ \hline b &= 1 \end{aligned}$$

$$\Rightarrow \quad -13\times 1-5c=-18 \quad \text{[from Eq. (x)]}$$
$$\Rightarrow \quad -5c=-18+13=-5 \Rightarrow c=1$$
$$\therefore \quad a=3-1-1=1 \text{ and } d=1-1=0$$
$$\therefore \quad A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}$$

Q. 13 Find A, if $\begin{bmatrix}4\\1\\3\end{bmatrix}A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$.

Sol. We have,

$$\begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times 1} A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times 3}$$

Let $\quad A=[x\ y\ z]$

$$\therefore \quad \begin{bmatrix}4\\1\\3\end{bmatrix}_{3\times 1}[x\ y\ z]_{1\times 3}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}_{3\times 3}$$

$$\Rightarrow \quad \begin{bmatrix}4x & 4y & 4z\\ x & y & z\\ 3x & 3y & 3z\end{bmatrix}=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$$

$\Rightarrow$ $4x = -4 \Rightarrow x = -1,\ 4y = 8$

$\Rightarrow$ $y = 2$ and $4z = 4$

$\Rightarrow$ $z = 1$

$\therefore$ $A = [-1\ 2\ 1]$

Q. 14 If $A \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, then verify $(BA)^2 \neq B^2 A^2$.

Sol. We have, $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times2}$ and $B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$

$\therefore$
$$BA = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \begin{bmatrix} 3 & -4 \\ 1 & 1 \\ 2 & 0 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 6+1+4 & -8+1+0 \\ 3+2+8 & -4+2+0 \end{bmatrix} = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

and
$$(BA)\cdot(BA) = \begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}\begin{bmatrix} 11 & -7 \\ 13 & -2 \end{bmatrix}$$

$\Rightarrow$
$$(BA)^2 = \begin{bmatrix} 121-91 & -77+14 \\ 143-26 & -91+4 \end{bmatrix} = \begin{bmatrix} 30 & -63 \\ 117 & -87 \end{bmatrix} \quad \text{...(i)}$$

Also,
$$B^2 = B\cdot B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$$

So, B^2 is not possible, since the B is not a square matrix.
Hence, $(BA)^2 \neq B^2 A^2$.

Q. 15 If possible, find the value of BA and AB, where

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}.$$

Sol. We have, $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3}$ and $B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times2}$

So, AB and BA both are possible.

[since, in both $A \cdot B$ and $B \cdot A$, the number of columns of first is equal to the number of rows of second.]

$\therefore$
$$AB = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2\times3} \cdot \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3\times2}$$

$$= \begin{bmatrix} 8+2+2 & 2+3+4 \\ 4+4+4 & 1+6+8 \end{bmatrix} = \begin{bmatrix} 12 & 9 \\ 12 & 15 \end{bmatrix}$$

and $$BA = \begin{bmatrix} 4 & 1 \\ 2 & 3 \\ 1 & 2 \end{bmatrix}_{3 \times 2} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}_{2 \times 3}$$

$$= \begin{bmatrix} 4 \times 2 + 1 & 4 + 2 & 8 + 4 \\ 4 + 3 & 2 + 6 & 4 + 12 \\ 2 + 2 & 1 + 4 & 2 + 8 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 12 \\ 7 & 8 & 16 \\ 4 & 5 & 10 \end{bmatrix}$$

Q. 16 Show by an example that for $A \neq 0$, $B \neq 0$ and $AB = 0$.

Sol. Let $$A = \begin{bmatrix} 0 & -4 \\ 0 & 2 \end{bmatrix} \neq 0 \text{ and } B = \begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix} \neq 0$$

$\therefore$ $$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ **Hence proved.**

Q. 17 Given, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}$. is $(AB)' = B'\ A'$?

Sol. We have, $A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 9 & 6 \end{bmatrix}_{2 \times 3}$ and $B = \begin{bmatrix} 1 & 4 \\ 2 & 8 \\ 1 & 3 \end{bmatrix}_{3 \times 2}$

$\therefore$ $$AB = \begin{bmatrix} 2 + 8 + 0 & 8 + 32 + 0 \\ 3 + 18 + 6 & 12 + 72 + 18 \end{bmatrix} = \begin{bmatrix} 10 & 40 \\ 27 & 102 \end{bmatrix}$$

and $$(AB)' = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \text{...(i)}$$

Also, $$B' = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 8 & 3 \end{bmatrix}_{2 \times 3} \text{ and } A' = \begin{bmatrix} 2 & 3 \\ 4 & 9 \\ 0 & 6 \end{bmatrix}_{3 \times 2}$$

$\therefore$ $$B'A' = \begin{bmatrix} 2 + 8 + 0 & 3 + 18 + 6 \\ 8 + 32 + 0 & 12 + 72 + 18 \end{bmatrix} = \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix} \quad \text{...(ii)}$$

Thus, we see that, $(AB)' = B'A'$ [using Eqs. (i) and (ii)]

Q. 18 Solve for x and y, $x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0.$

Sol. We have, $$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$\Rightarrow$ $$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3 \cdot y \\ 5 \cdot y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = 0$$

$\Rightarrow$ $$\begin{bmatrix} 2x & 3y & -8 \\ x & 5y & -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ $$2x + 3y - 8 = 0$$

$\Rightarrow$ $$4x + 6y = 16 \quad \text{...(i)}$$

and $$x + 5y - 11 = 0$$

$\Rightarrow$ $$4x + 20y = 44 \quad \text{...(ii)}$$

On subtracting Eq. (i) from Eq. (ii), we get

$$14y = 28 \Rightarrow y = 2$$

$\therefore$ $$2x + 3 \times 2 - 8 = 0$$

$\Rightarrow$ $$2x = 2 \Rightarrow x = 1$$

$\therefore$ $$x = 1 \text{ and } y = 2$$

Q. 19 If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}.$$

Sol. We have,

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \quad \ldots(i)$$

and $$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \quad \ldots(ii)$$

On subtracting Eq. (i) from Eq. (ii), we get

$\therefore$ $$(3X + 2Y) - (2X + 3Y) = \begin{bmatrix} -2-2 & 2-3 \\ 1-4 & -5-0 \end{bmatrix}$$

$$(X - Y) = \begin{bmatrix} -4 & -1 \\ -3 & -5 \end{bmatrix} \quad \ldots(iii)$$

On adding Eqs. (i) and (ii), we get

$$(5X + 5Y) = \begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix}$$

$\Rightarrow$ $$(X + Y) = \frac{1}{5}\begin{bmatrix} 0 & 5 \\ 5 & -5 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad \ldots(iv)$$

On adding Eqs. (iii) and (iv), we get

$$(X - Y) + (X + Y) = \begin{bmatrix} -4 & 0 \\ -2 & -6 \end{bmatrix}$$

$\Rightarrow$ $$2X = 2\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

$\therefore$ $$X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

From Eq. (iv),

$$\begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix} + Y = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$\therefore$ $$Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix} \text{ and } X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Q. 20 **If $A = [3\ \ 5]$ and $B = [7\ \ 3]$, then find a non-zero matrix C such that $AC = BC$.**

Sol. We have, $A = [3\ \ 5]_{1 \times 2}$ and $B = [7\ \ 3]_{1 \times 2}$

Let $C = \begin{bmatrix} x \\ y \end{bmatrix}_{2\times1}$ is a non-zero matrix of order 2×1.

$$\therefore \quad AC = [3\ \ 5]\begin{bmatrix} x \\ y \end{bmatrix} = [3x + 5y]$$

and
$$BC = [7\ \ 3]\begin{bmatrix} x \\ y \end{bmatrix} = [7x + 3y]$$

For $AC = BC$,

$$[3x + 5y] = [7x + 3y]$$

On using equality of matrix, we get

$$3x + 5y = 7x + 3y$$

$$\Rightarrow \quad 4x = 2y$$

$$\Rightarrow \quad x = \frac{1}{2}y$$

$$\Rightarrow \quad y = 2x$$

$$\therefore \quad C = \begin{bmatrix} x \\ 2x \end{bmatrix}$$

We see that on taking C of order $2 \times 1, 2 \times 2, 2 \times 3, \ldots$, we get

$$C = \begin{bmatrix} x \\ 2x \end{bmatrix}, \begin{bmatrix} x & x \\ 2x & 2x \end{bmatrix}, \begin{bmatrix} x & x & x \\ 2x & 2x & 2x \end{bmatrix} \ldots$$

In general,

$$C = \begin{bmatrix} k \\ 2k \end{bmatrix}, \begin{bmatrix} k & k \\ 2k & 2k \end{bmatrix} \text{ etc...}$$

where, k is any real number.

Q. 21 **Give an example of matrices A, B and C, such that $AB = AC$, where A is non-zero matrix but $B \neq C$.**

Sol. Let
$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} \quad [\because B \neq C]$$

$$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \quad \ldots\text{(i)}$$

and
$$AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} \quad \ldots\text{(ii)}$$

Thus, we see that $AB = AC$ [using Eqs. (i) and (ii)]

where, A is non-zero matrix but $B \neq C$.

Q. 22 If $A=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}$, verify

(i) $(AB)\,C = A(BC)$.

(ii) $A\,(B+C) = AB + AC$.

Sol. We have, $A=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$, $B=\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}$ and $C=\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}$

(i) $(AB)=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}=\begin{bmatrix}2+6 & 3-8\\ -4+3 & -6-4\end{bmatrix}=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}$

and $(AB)C=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}$

$=\begin{bmatrix}8+5 & 0\\ -1+10 & 0\end{bmatrix}=\begin{bmatrix}13 & 0\\ 9 & 0\end{bmatrix}$ ...(i)

Again, $(BC)=\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}$

$=\begin{bmatrix}2-3 & 0\\ 3+4 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 7 & 0\end{bmatrix}$

and $A\,(BC)=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}-1 & 0\\ 7 & 0\end{bmatrix}$

$=\begin{bmatrix}-1+14 & 0\\ +2+7 & 0\end{bmatrix}=\begin{bmatrix}13 & 0\\ 9 & 0\end{bmatrix}$ ...(ii)

$\therefore$ $(AB)\,C = A(BC)$ [using Eqs. (i) and (ii)]

(ii) $(B+C)=\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}+\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}=\begin{bmatrix}3 & 3\\ 2 & -4\end{bmatrix}$

and $A\cdot(B+C)=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}3 & 3\\ 2 & -4\end{bmatrix}$

$=\begin{bmatrix}3+4 & 3-8\\ -6+2 & -6-4\end{bmatrix}=\begin{bmatrix}7 & -5\\ -4 & -10\end{bmatrix}$ ...(iii)

Also, $AB=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\cdot\begin{bmatrix}2 & 3\\ 3 & -4\end{bmatrix}$

$=\begin{bmatrix}2+6 & 3-8\\ -4+3 & -6-4\end{bmatrix}=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}$

and $AC=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ -1 & 0\end{bmatrix}=\begin{bmatrix}1-2 & 0\\ -2-1 & 0\end{bmatrix}=\begin{bmatrix}-1 & 0\\ -3 & 0\end{bmatrix}$

$\therefore$ $AB+AC=\begin{bmatrix}8 & -5\\ -1 & -10\end{bmatrix}+\begin{bmatrix}-1 & 0\\ -3 & 0\end{bmatrix}$

$\Rightarrow$ $AB+AC=\begin{bmatrix}7 & -5\\ -4 & -10\end{bmatrix}$ ...(iv)

From Eqs. (iii) and (iv),

$A\,(B+C)=AB+AC$

Q. 23 If $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, then prove that

$$PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Sol. $PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$ ...(i)

and $QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$ ...(ii)

Thus, we see that, $PQ = QP$ [using Eqs. (i) and (ii)]

Hence proved.

Q. 24 If $\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$, then find the value of A.

Sol. We have, $[2\ 1\ 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

$\therefore \quad [2\ 1\ 3] \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = [-2-1+0 \quad 0+1+3 \quad -2+0+3]$

$= [-3 \quad 4 \quad 1]$

Now, $[-3\ 4\ 1] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$

$\therefore \quad A = [-3\ 4\ 1] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$= [-3 + 0 - 1] = [-4]$

Q. 25 If $A = [2\ 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then verify that $A(B + C) = (AB + AC)$.

Sol. We have to verify that, $A(B + C) = AB + AC$

We have, $A = [2 \quad 1]$, $B = \begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$\therefore \quad A(B+C) = [2 \quad 1]\begin{bmatrix} 5-1 & 3+2 & 4+1 \\ 8+1 & 7+0 & 6+2 \end{bmatrix}$

$= [2 \ 1]\begin{bmatrix} 4 & 5 & 5 \\ 9 & 7 & 8 \end{bmatrix}$

$= [8+9 \quad 10+7 \quad 10+8]$

$= [17 \ 17 \ 18]$ ...(i)

Also, $AB = [2 \ 1]\begin{bmatrix} 5 & 3 & 4 \\ 8 & 7 & 6 \end{bmatrix}$

$= [10+8 \quad 6+7 \quad 8+6] = [18 \quad 13 \quad 14]$

and $AC = [2 \ 1]\begin{bmatrix} -1 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$= [-2+1 \quad 4+0 \quad 2+2] = [-1 \quad 4 \quad 4]$

$\therefore \quad AB + AC = [18 \quad 13 \quad 14] + [-1 \quad 4 \quad 4]$

$= [17 \quad 17 \quad 18]$ ...(ii)

$\therefore \quad A(B+C) = (AB + AC)$ [using Eqs. (i) and (ii)]

Hence proved.

Q. 26 If $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = (A + I)$, where I is 3×3 unit matrix.

Sol. We have, $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A$

$= \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix}$

$\therefore \quad A^2 + A = \begin{bmatrix} 1 & -1 & -2 \\ 4 & 4 & 4 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix}$ ...(i)

Now, $A + I = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

and $A(A + I) = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & -1 \\ 2 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -3 \\ 6 & 5 & 7 \\ 2 & 3 & 5 \end{bmatrix}$ ...(ii)

Thus, we see that $A^2 + A = A(A + I)$ [using Eqs. (i) and (ii)]

Q. 27 If $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$, then verify that

(i) $(A')' = A$
(ii) $(AB)' = B'A'$
(iii) $(kA)' = (kA')$.

Sol. We have, $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$

(i) We have to verify that, $A' = A$

$\therefore \quad A' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix}$

and $\quad A' = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix} = A$ **Hence proved.**

(ii) We have to verify that, $AB' = B'A'$

$\therefore \quad AB = \begin{bmatrix} 3 & 9 \\ 11 & -15 \end{bmatrix}$

$\Rightarrow \quad (AB)' = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$

and $\quad B'A' = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0 & 4 \\ -1 & 3 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 3 & 11 \\ 9 & -15 \end{bmatrix}$

$= (AB)'$ **Hence proved.**

(iii) We have to verify that, $(kA)' = (kA')$

Now, $\quad (kA) = \begin{bmatrix} 0 & -k & 2k \\ 4k & 3k & -4k \end{bmatrix}$

and $\quad (kA)' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

Also, $\quad kA' = \begin{bmatrix} 0 & 4k \\ -k & 3k \\ 2k & -4k \end{bmatrix}$

$= (kA)'$ **Hence proved.**

Q. 28 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$, then verify that

(i) $(2A + B)' = 2AA + B'$.

(ii) $(A - B)' = A' - B'$.

Sol. We have,
$$A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix}$$

(i) $\therefore$ $(2A + B) = \begin{bmatrix} 2 & 4 \\ 8 & 2 \\ 10 & 12 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 14 & 6 \\ 17 & 15 \end{bmatrix}$

and $(2A + B)' = \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix}$

Also, $2A' + B' = 2\begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 3 & 14 & 17 \\ 6 & 6 & 15 \end{bmatrix} = (2A + B)'$ **Hence proved.**

(ii) $(A - B) = \begin{bmatrix} 1 & 2 \\ 4 & 1 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 6 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -2 & -3 \\ -2 & 3 \end{bmatrix}$

and $(A - B)' = \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

Also, $A' - B' = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 1 & 6 \end{bmatrix} - \begin{bmatrix} 1 & 6 & 7 \\ 2 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 & -2 \\ 0 & -3 & 3 \end{bmatrix}$

$= (A - B)'$ **Hence proved.**

Q. 29 Show that $A'A$ and AA' are both symmetric matrices for any matrix A.

Thinking Process

We know that, for a matrix A to be symmetric matrix, $A' = A$. Also by using the result $(AB)' = BA'$, we can prove that $A'A$ and AA' are both symmetric matrices for any matrix A.

Sol. Let $P = A'A$

$\therefore$ $P' = (AA')'$

$= A'(A')'$ $[\because (AB')' = B'A']$

$= A'A = P$

So, $A'A$ is symmetric matrix for any matrix A.

Similarly, let $Q = AA'$

$\therefore$ $Q' = (AA')' = (A')'(A)'$

$= A(A')' = Q$

So, AA' is symmetric matrix for any matrix A.

Q. 30 Let A and B be square matrices of the order 3×3. Is $(AB)^2 = A^2B^2$? Give reasons.

Sol. Since, A and B are square matrices of order 3×3.

$$\therefore \quad AB^2 = AB \cdot AB$$
$$= ABAB$$
$$= AABB \qquad [\because AB = BA]$$
$$= A^2B^2$$

So, $AB^2 = A^2B^2$ is true when $AB = BA$.

Q. 31 Show that, if A and B are square matrices such that $AB = BA$, then $(A+B)^2 = A^2 + 2AB + B^2$.

Sol. Since, A and B are square matrices such that $AB = BA$.

$$\therefore \quad (A+B)^2 = (A+B)\cdot(A+B)$$
$$= A^2 + AB + BA + B^2$$
$$= A^2 + AB + AB + B^2 \qquad [\because AB = BA]$$
$$= A^2 + 2AB + B^2$$

Hence proved.

Q. 32 If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}, a = 4,$ and $b = -2,$ then show that

(i) $A + (B + C) = (A + B) + C$

(ii) $A\ (BC) = (AB)\ C$

(iii) $(a + b)B = aB + bB$

(iv) $a\ (C - A) = aC - aA$

(v) $(A^T)^T = A$

(vi) $(bA)^T = b\ A^T$

(vii) $(AB)^T = B^T\ A^T$

(viii) $(A - B)C = AC - BC$

(ix) $(A - B)^T = A^T - B^T$

Sol. We have,
$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } a = 4,\ b = -2$$

(i) $A + (B + C) = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix}$

and
$$(A + B) + C = \begin{bmatrix} 5 & 2 \\ 0 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 7 & 2 \\ 1 & 6 \end{bmatrix} = A + (B + C)$$

Hence proved.

(ii) $(BC)=\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix}\begin{bmatrix}2 & 0\\1 & -2\end{bmatrix}=\begin{bmatrix}8 & 0\\7 & -10\end{bmatrix}$

and $A\ (BC)=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}\begin{bmatrix}8 & 0\\7 & -10\end{bmatrix}$

$=\begin{bmatrix}8+14 & 0-20\\-8+21 & 0-30\end{bmatrix}=\begin{bmatrix}22 & -20\\13 & -30\end{bmatrix}$

Also, $(AB)=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}\cdot\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix}=\begin{bmatrix}6 & 10\\-1 & 15\end{bmatrix}$

$(AB)\ C=\begin{bmatrix}6 & 10\\-1 & 15\end{bmatrix}\begin{bmatrix}2 & 0\\1 & -2\end{bmatrix}$

$=\begin{bmatrix}22 & -20\\13 & -30\end{bmatrix}=A(BC)$ **Hence proved.**

(iii) $(a+b)\ B=(4-2)\begin{bmatrix}4 & 0\\1 & 5\end{bmatrix}$ $[\because a=4, b=-2]$

$=\begin{bmatrix}8 & 0\\2 & 10\end{bmatrix}$

and $aB+bB=4B-2B$

$=\begin{bmatrix}16 & 0\\4 & 20\end{bmatrix}-\begin{bmatrix}8 & 0\\2 & 10\end{bmatrix}$

$=\begin{bmatrix}8 & 0\\2 & 10\end{bmatrix}$

$=(a+b)B$ **Hence proved.**

(iv) $(C-A)=\begin{bmatrix}2-1 & 0-2\\1+1 & -2-3\end{bmatrix}=\begin{bmatrix}1 & -2\\2 & -5\end{bmatrix}$

and $a(C-A)=\begin{bmatrix}4 & -8\\8 & -20\end{bmatrix}$ $[\because a=4]$

Also, $aC-aA=\begin{bmatrix}8 & 0\\4 & -8\end{bmatrix}-\begin{bmatrix}4 & 8\\-4 & 12\end{bmatrix}=\begin{bmatrix}4 & -8\\8 & -20\end{bmatrix}$

$=a(C-A)$ **Hence proved.**

(v) $A^T=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}^T=\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

Now, $(A^T)^T=\begin{bmatrix}1 & 2\\-1 & 3\end{bmatrix}^T$

$=A$ **Hence proved.**

(vi) $(bA)^T=\begin{bmatrix}-2 & -4\\2 & -6\end{bmatrix}^T$ $[\because b=-2]$

$=\begin{bmatrix}-2 & 2\\-4 & -6\end{bmatrix}$

and $A^T=\begin{bmatrix}1 & -1\\2 & 3\end{bmatrix}$

$\therefore$ $bA^T=\begin{bmatrix}-2 & 2\\-4 & -6\end{bmatrix}=(bA)^T$ **Hence proved.**

(vii) $AB=\begin{bmatrix}1&2\\-1&3\end{bmatrix}\begin{bmatrix}4&0\\1&5\end{bmatrix}=\begin{bmatrix}4+2&0+10\\-4+3&0+15\end{bmatrix}=\begin{bmatrix}6&10\\-1&15\end{bmatrix}$

$\therefore \quad (AB)^T=\begin{bmatrix}6&-1\\10&15\end{bmatrix}$

Now, $\quad B^TA^T=\begin{bmatrix}4&1\\0&5\end{bmatrix}\begin{bmatrix}1&-1\\2&3\end{bmatrix}=\begin{bmatrix}6&-1\\10&15\end{bmatrix}$

$=(AB)^T$ **Hence proved.**

(viii) $(A-B)=\begin{bmatrix}1-4&2-0\\-1-1&3-5\end{bmatrix}=\begin{bmatrix}-3&2\\-2&-2\end{bmatrix}$

$(A-B)\ C=\begin{bmatrix}-3&2\\-2&-2\end{bmatrix}\begin{bmatrix}2&0\\1&-2\end{bmatrix}=\begin{bmatrix}-4&-4\\-6&4\end{bmatrix}$ ...(i)

Now, $\quad AC=\begin{bmatrix}1&2\\-1&3\end{bmatrix}\begin{bmatrix}2&0\\1&-2\end{bmatrix}=\begin{bmatrix}4&-4\\1&-6\end{bmatrix}$ ...(ii)

and $\quad BC=\begin{bmatrix}4&0\\1&5\end{bmatrix}\begin{bmatrix}2&0\\1&-2\end{bmatrix}=\begin{bmatrix}8&0\\7&-10\end{bmatrix}$ ...(iii)

$\therefore \quad AC-BC=\begin{bmatrix}4-8&-4-0\\1-7&-6+10\end{bmatrix}$ [using Eqs. (ii) and (iii)]

$=\begin{bmatrix}-4&-4\\-6&4\end{bmatrix}$

$=(A-B)C$ [using Eq. (i)] **Hence proved.**

(ix) $(A-B)^T=\begin{bmatrix}1-4&2-0\\-1-1&3-5\end{bmatrix}^T$

$=\begin{bmatrix}-3&2\\-2&-2\end{bmatrix}^T=\begin{bmatrix}-3&-2\\2&-2\end{bmatrix}$

$A^T-B^T=\begin{bmatrix}1&-1\\2&3\end{bmatrix}-\begin{bmatrix}4&1\\0&5\end{bmatrix}$

$=\begin{bmatrix}-3&-2\\2&-2\end{bmatrix}=(A-B)^T$ **Hence proved.**

Q. 33 If $A=\begin{bmatrix}\cos q&\sin q\\-\sin q&\cos q\end{bmatrix}$, then show that $A^2=\begin{bmatrix}\cos 2q&\sin 2q\\-\sin 2q&\cos 2q\end{bmatrix}$.

Sol. We have, $A=\begin{bmatrix}\cos q&\sin q\\-\sin q&\cos q\end{bmatrix}$

$\therefore \quad A^2=A\cdot A=\begin{bmatrix}\cos q&\sin q\\-\sin q&\cos q\end{bmatrix}\cdot\begin{bmatrix}\cos q&\sin q\\-\sin q&\cos q\end{bmatrix}$

$=\begin{bmatrix}\cos^2 q-\sin^2 q&\cos q\cdot\sin q+\sin q\cos q\\-\sin q\cos q-\cos q\sin q&-\sin^2 q+\cos^2 q\end{bmatrix}$

$=\begin{bmatrix}\cos 2q&2\sin q\cos q\\-2\sin q\cos q&\cos 2q\end{bmatrix}$ $[\because \cos^2\theta-\sin^2\theta=\cos 2\theta]$

$=\begin{bmatrix}\cos 2q&\sin 2q\\-\sin 2q&\cos 2q\end{bmatrix}$ $[\because \sin 2\theta=2\sin\theta\cdot\cos\theta]$ **Hence proved.**

Q. 34 If $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$, then show that $(A+B)^2=A^2+B^2$.

Sol. We have, $A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}$, $B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and $x^2=-1$

$\therefore$ $(A+B)=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

and $(A+B)^2=\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}\begin{bmatrix}0 & -x+1\\ x+1 & 0\end{bmatrix}$

$=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}$ ...(i)

Also, $A^2=A\cdot A=\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}\begin{bmatrix}0 & -x\\ x & 0\end{bmatrix}=\begin{bmatrix}-x^2 & 0\\ 0 & -x^2\end{bmatrix}$

and $B^2=B\cdot B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

Now, $A^2+B^2=\begin{bmatrix}-x^2+1 & 0\\ 0 & -x^2+1\end{bmatrix}=\begin{bmatrix}1-x^2 & 0\\ 0 & 1-x^2\end{bmatrix}$ [using Eq. (i)]

$=(A+B)^2$ **Hence proved.**

Q. 35 Verify that $A^2=I$, when $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$.

Sol. We have, $A=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$

$\therefore$ $A^2=\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}\cdot\begin{bmatrix}0 & 1 & -1\\ 4 & -3 & 4\\ 3 & -3 & 4\end{bmatrix}$ $[\because A^2=A\cdot A]$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=I$ **Hence proved.**

Q. 36 Prove by mathematical induction that $(A')^n=(A^n)'$ where $n\in N$ for any square matrix A.

Sol. Let $P(n): (A')^n=(A^n)'$

$\therefore$ $P(1): (A')^1=(A)'$

$\Rightarrow$ $A'=A' \Rightarrow P(1)$ is true.

Now, $P(k): (A')^k=(A^k)'$,

where $k\in N$

and $P(k+1): (A')^{k+1}=(A^{k+1})'$

where $P(k+1)$ is true whenever $P(k)$ is true.

$\therefore$ $$P(k+1) : (A')^k . (A')^1 = [A^{k+1}]'$$

$$(A^k)' . (A)' = [A^{k+1}]'$$

$$(A \cdot A^k)' = [A^{k+1}]' \quad [\because (A')^k = (A^k)' \text{ and } (AB) = B'A']$$

$$(A^{k+1})' = [A^{k+1}]'$$

Hence proved.

Q. 37 Find inverse, by elementary row operations (if possible), of the following matrices.

(i) $\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

Thinking Process

To find the inverse of a matrix A, we know that $A = IA$ is used for elementary row operations. So, with the help of this method we can get the desired result.

Sol. **(i)** Let $A = \begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix}$

In order to use elementary row operations we may write $A = IA$.

$\therefore$ $$\begin{bmatrix} 1 & 3 \\ -5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ $$\begin{bmatrix} 1 & 3 \\ 0 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + 5R_1]$$

$\Rightarrow$ $$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5/22 & 1/22 \end{bmatrix} A \quad \left[\because R_2 \to \frac{1}{22} R_2\right]$$

$\Rightarrow$ $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7/22 & -3/22 \\ 5/22 & 1/22 \end{bmatrix} A \quad [\because R_1 \to R_1 - 3R_2]$$

$\Rightarrow$ $$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & 1 \end{bmatrix} A$$

$\Rightarrow$ $I = BA$, where B is the inverse of A.

$\therefore$ $$B = \frac{1}{22} \begin{bmatrix} 7 & -3 \\ 5 & -1 \end{bmatrix}$$

(ii) Let $A = \begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix}$

In order to use elementary row operations, we write $A = IA$

$\Rightarrow$ $$\begin{bmatrix} 1 & -3 \\ -2 & 6 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ $$\begin{bmatrix} 1 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + 2R_1]$$

Since, we obtain all zeroes in a row of the matrix A on LHS, so A^{-1} does not exist.

Q. 38 If $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$, then find the values of x, y, z and w.

Sol. We have, $\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$

By equality of matrix, $x + y = 6$ and $xy = 8$

$\Rightarrow$ $x = 6 - y$ and $(6 - y) \cdot y = 8$

$\Rightarrow$ $y^2 - 6y + 8 = 0$

$\Rightarrow$ $y^2 - 4y - 2y + 8 = 0$

$\Rightarrow$ $(y - 2)(y - 4) = 0$

$\Rightarrow$ $y = 2$ or $y = 4$

$\therefore$ $x = 6 - 2 = 4$

or $x = 6 - 4 = 2$ $[\because x = 6 - y]$

Also, $z + 6 = 0$

$\Rightarrow$ $z = -6$ and $w = 4$

$\therefore$ $x = 2, y = 4$ or $x = 4, y = 2, z = -6$ and $w = 4$

Q. 39 If $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, then find a matrix C such that $3A + 5B + 2C$ is a null matrix.

Sol. We have, $A = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$ and $B = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$

Let $C = \begin{bmatrix} a & b \\ C & d \end{bmatrix}$

$\therefore$ $3A + 5B + 2C = 0$

$\Rightarrow$ $\begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ $\begin{bmatrix} 48 + 2a & 20 + 2b \\ 56 + 2c & 76 + 2d \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow$ $2a + 48 = 0 \Rightarrow a = -24$

Also, $20 + 2b = 0 \Rightarrow b = -10$

$56 + 2c = 0 \Rightarrow c = -28$

and $76 + 2d = 0 \Rightarrow d = -38$

$\therefore$ $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

Q. 40 If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Sol. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ ...(i)

$\therefore$ $A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$ ...(ii)

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now, $A^2 - 5A - 14\,I = 0$

$\Rightarrow \quad A \cdot A^2 - 5\,A \cdot A - 14\,A\,I = 0$

$\Rightarrow \quad A^3 - 5A^2 - 14A = 0 \qquad [\because A\,I = A]$

$$\Rightarrow \quad A^3 = 5A^2 = 14A$$

$$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \qquad \text{[using Eqs. (i) and (ii)]}$$

$$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$= \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Q. 41 Find the values of a, b, c and d, if

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2\,d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix} z.$$

Sol. We have,

$$3\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & 6 \\ -1 & 2d \end{bmatrix} + \begin{bmatrix} 4 & a+b \\ c+d & 3 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 3a & 3b \\ 3c & 3d \end{bmatrix} = \begin{bmatrix} a+4 & 6+a+b \\ c+d-1 & 3+2\,d \end{bmatrix}$$

$\Rightarrow \quad 3a = a + 4 \Rightarrow a = 2;$

$3b = 6 + a + b$

$\Rightarrow \quad 3b - b = 8 \Rightarrow b = 4;$

$3d = 3 + 2d \Rightarrow d = 3$

and $\Rightarrow \quad 3c = c + d - 1$

$\Rightarrow \quad 2c = 3 - 1\; c = 1$

$\therefore \quad a = 2,\; b = 4,\; c = 1$ and $d = 3$

Q. 42 Find the matrix A such that $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$.

Sol. We have, $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}_{3 \times 2} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}_{3 \times 3}$

From the given equation, it is clear that order of A should be 2×3.

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

$$\therefore \quad \begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a+0d & b+0\cdot e & c+0\cdot f \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2a-d & 2b-e & 2c-f \\ a & b & c \\ -3a+4d & -3b+4e & -3c+4f \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$$

By equality of matrices, we get

$$a = 1, b = -2, c = -5$$

and $\quad 2a - d = -1 \Rightarrow d = 2a + 1 = 3;$

$\Rightarrow \quad 2b - e = -8 \Rightarrow e = 2(-2) + 8 = 4$

$2c - f = -10 \Rightarrow f = 2c + 10 = 0$

$$\therefore \quad A = \begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$$

Q. 43 If $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$, then find $A^2 + 2A + 7I$.

Sol. We have, $A = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix} \qquad [\because A^2 = A \cdot A]$$

$$= \begin{bmatrix} 1+8 & 2+2 \\ 4+4 & 8+1 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix}$$

$$\therefore \quad A^2 + 2A + 7I = \begin{bmatrix} 9 & 4 \\ 8 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 18 & 8 \\ 16 & 18 \end{bmatrix}$$

Q. 44 If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ and $A^{-1} = A'$, then find the value of α.

Sol. We have,

$$A = \begin{bmatrix} \cos a & \sin a \\ -\sin a & \cos a \end{bmatrix} \text{ and } A' = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

Also, $\quad A^{-1} = A'$

$\Rightarrow \quad AA^{-1} = AA'$

$$\Rightarrow \quad I = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & 0 \\ 0 & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

By using equality of matrices, we get

$$\cos^2\alpha + \sin^2\alpha = 1$$

which is true for all real values of α.

Q. 45 If matrix $\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then find the values of a, b and c.

Thinking Process

We know that, a matrix A is skew-symmetric matrix, if A′= – A, so by using this we can get the values of a, b and c.

Sol. Let $A = \begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$

Since, A is skew-symmetric matrix.

$$\therefore \quad A' = -A$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 0 & 2 & c \\ a & b & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -a & -3 \\ -2 & -b & +1 \\ -c & -1 & 0 \end{bmatrix}$$

By equality of matrices, we get

$$a = -2,\ c = -3 \text{ and } b = -b \Rightarrow b = 0$$

$$\therefore \quad a = -2,\ b = 0 \text{ and } c = -3$$

Q. 46 If $P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x) \cdot P(y) = P(x + y) = P(y) \cdot P(x)$.

Sol. We have,

$$P(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\therefore \quad P(y) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

Now, $$P(x) \cdot P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cdot \cos y - \sin x \cdot \sin y & \cos x \cdot \sin y + \sin x \cdot \cos y \\ -\sin x \cdot \cos y - \cos x \cdot \sin y & -\sin x \cdot \sin y + \cos x \cdot \cos y \end{bmatrix}$$

$$= \begin{bmatrix} \cos (x + y) & \sin (x + y) \\ -\sin (x + y) & \cos (x + y) \end{bmatrix} \quad \text{...(i)}$$

$$\left[\because \cos (x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y \text{ and } \sin (x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y\right]$$

and $$P(x + y) = \begin{bmatrix} \cos (x + y) & \sin (x + y) \\ -\sin (x + y) & \cos (x + y) \end{bmatrix} \quad \text{...(ii)}$$

Also, $P(y)\cdot P(x)=\begin{bmatrix}\cos y & \sin y\\ -\sin y & \cos y\end{bmatrix}\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$

$$=\begin{bmatrix}\cos y\cdot\cos x-\sin y\cdot\sin x & \cos y\cdot\sin x+\sin y\cdot\cos x\\ -\sin y\cdot\cos x-\sin x\cdot\cos y & -\sin y\cdot\sin x+\cos y\cdot\cos x\end{bmatrix}$$

$$=\begin{bmatrix}\cos(x+y) & \sin(x+y)\\ -\sin(x+y) & \cos(x+y)\end{bmatrix} \quad ...(iii)$$

Thus, we see from the Eqs. (i), (ii) and (iii) that,

$$P(x)\cdot P(y)=P(x+y)=P(y)\cdot P(x)$$ **Hence proved.**

Q. 47 If A is square matrix such that $A^2=A$, then show that $(I+A)^3=7A+I$.

Sol. Since, $A^2=A$ and $(I+A)\cdot(I+A)=I^2+IA+AI+A^2$

$=I^2+2AI+A^2$

$=I+2A+A=I+3A$

and $(I+A)\cdot(I+A)(I+A)=(I+A)(I+3A)$

$=I^2+3AI+AI+3A^2$

$=I+4AI+3A$

$=I+7A=7A+I$ **Hence proved.**

Q. 48 If A, B are square matrices of same order and B is a skew-symmetric matrix, then show that $A'BA$ is skew-symmetric.

Sol. Since, A and B are square matrices of same order and B is a skew-symmetric matrix *i.e.,* $B'=-B$.

Now, we have to prove that $A'BA$ is a skew-symmetric matrix.

$\therefore$ $A'BA'=A'BA'=BA'A'$ $[\because AB'=B'A']$

$=A'B'A=A'-BA=-A'BA$

Hence, $A'BA$ is a skew-symmetric matrix.

Long Answer Type Questions

Q. 49 If $AB=BA$ for any two square matrices, then prove by mathematical induction that $(AB)^n=A^nB^n$.

Sol. Let $P(n):(AB)^n=A^nB^n$

$\therefore$ $P(1):(AB)^1=A^1B^1\Rightarrow AB=AB$

So, $P(1)$ is true.

Now, $P(k):(AB)^k=A^kB^k,\ k\in N$

So, $P(K)$ is true, whenever $P(k+1)$ is true.

$\therefore$ $P(K+1:AB)^{k+1}=A^{k+1}B^{k+1}$ ...(i)

$\Rightarrow$ $AB^k\cdot AB^1$ $[\because AB=BA]$

$\Rightarrow$ $A^kB^k\cdot BA\Rightarrow A^kB^{k+1}A$

$\Rightarrow$ $A^k\cdot A\cdot B^{k+1}\Rightarrow A^{k+1}B^{k+1}$

$\Rightarrow$ $(A\cdot B)^{k+1}=A^{k+1}B^{k+1}$

So, $P(k+1)$ is true for all $n\in N$, whenever $P(k)$ is true.

By mathematical induction $(AB)=A^nB^n$ is true for all $n\in N$.

Q. 50 Find x, y and z, if $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A' = A^{-1}$.

Sol. We have, $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ and $A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$

By using elementary row transformations, we get

$$A = IA$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ 0 & -2y & 2z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} A \qquad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 0 & 2y & z \\ x & 3y & 0 \\ 0 & 0 & 3z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix} A \qquad \begin{bmatrix} \because R_3 \to R_3 + R_1 \\ \text{and } R_2 \to R_2 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -x & -y & z \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to R_1 - R_2 \\ \text{and } R_3 \to \frac{1}{3} R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ x & 3y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ 1 & 1 & 0 \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because R_1 \to R_1 - R_3]$$

$$\Rightarrow \begin{bmatrix} -x & -y & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} \frac{-1}{3} & \frac{-2}{3} & \frac{-1}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad [\because R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} -x & 0 & 0 \\ 0 & 2y & 0 \\ 0 & 0 & z \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-1}{2} \\ \frac{2}{3} & \frac{1}{3} & \frac{-1}{3} \\ \frac{1}{3} & \frac{-1}{3} & \frac{1}{3} \end{bmatrix} A \qquad \left[\because R_1 \to R_1 + \frac{1}{2} R_2\right]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} A \qquad \begin{bmatrix} \because R_1 \to \frac{-1}{x} R_1, \\ R_2 \to \frac{1}{2y} R_2 \\ \text{and } R_3 \to \frac{1}{z} R_3 \end{bmatrix}$$

$\therefore$ $$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2x} & \frac{1}{2x} \\ \frac{1}{3y} & \frac{1}{6y} & \frac{-1}{6y} \\ \frac{1}{3z} & \frac{-1}{3z} & \frac{1}{3z} \end{bmatrix} = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

$\Rightarrow$ $$\frac{1}{2x} = x \quad \Rightarrow \quad = \pm \frac{1}{\sqrt{2}}$$

$\Rightarrow$ $$\frac{1}{6y} = y \quad \Rightarrow \quad y = \pm \frac{1}{\sqrt{6}}$$

and $$\frac{1}{3z} = z \quad \Rightarrow \quad z = \pm \frac{1}{\sqrt{3}}$$

Alternate Method

We have,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \text{ and } A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Also, $$A' = A^{-1}$$

$\Rightarrow$ $$AA' = AA^{-1} \qquad [\because AA^{-1} = I]$$

$\Rightarrow$ $$AA' = I$$

$\Rightarrow$ $$\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$\begin{bmatrix} 4y^2 + z^2 & 2y^2 - z^2 & -2y^2 + z^2 \\ 2y^2 - z^2 & x^2 + y^2 + z^2 & x^2 - y^2 - z^2 \\ -2y^2 + z^2 & x^2 - y^2 - z^2 & x^2 + y^2 + z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $$2y^2 - z^2 = 0 \Rightarrow 2y^2 = z^2$$

$\Rightarrow$ $$4y^2 + z^2 = 1$$

$\Rightarrow$ $$2 \cdot z^2 + z^2 = 1$$

$$z = \pm \frac{1}{\sqrt{3}}$$

$\therefore$ $$y^2 = \frac{z^2}{2} \Rightarrow y = \pm \frac{1}{\sqrt{6}}$$

Also, $$x^2 + y^2 + z^2 = 1$$

$\Rightarrow$ $$x^2 = 1 - y^2 - z^2 = 1 - \frac{1}{6} - \frac{1}{3}$$

$$= 1 - \frac{3}{6} = \frac{1}{2}$$

$\Rightarrow$ $$x = \pm \frac{1}{\sqrt{2}}$$

$\therefore$ $$x = \pm, \frac{1}{\sqrt{2}}, y = \pm \frac{1}{\sqrt{6}}$$

and $$z = \pm \frac{1}{\sqrt{3}}$$

Q. 51 If possible, using elementary row transformations, find the inverse of the following matrices.

(i) $\begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Sol. For getting the inverse of the given matrix A by row elementary operations we may write the given matrix as

$$A = IA$$

(i) $\because \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ -3 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & 3 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ -3 & 2 & 4 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_1 \to R_1 + R_2]$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & -17 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ -5 & -2 & 0 \\ -1 & -1 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 - 3R_1]$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & -10 \\ 0 & -1 & -17 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 & 0 \\ -5 & -2 & 0 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to R_1 + R_2 \\ \text{and } R_3 \to -1 \cdot R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 9 & -10 \\ 12 & 15 & -17 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to R_1 + 10R_3 \\ \text{and } R_2 \to R_2 + 17R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to -1R_1 \\ \text{and } R_2 \to -1R_2 \end{bmatrix}$$

So, the inverse of A is $\begin{bmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{bmatrix}$.

(ii) $\therefore$ $$\begin{bmatrix} 2 & 3 & -3 \\ -1 & -2 & 2 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} A \quad \begin{bmatrix} \because R_2 \to R_2 + R_3 \\ \text{and } R_1 \to R_1 - 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 + R_1]$$

Since, second row of the matrix A on LHS is containing all zeroes, so we can say that inverse of matrix A does not exist.

(iii) $\therefore$ $$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_2 \to R_2 - R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \quad \begin{bmatrix} \because R_2 \to R_2 - R_1 \\ \text{and } R_3 \to R_3 + R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 4 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \begin{bmatrix} \because R_3 \to R_3 + R_1 \\ \text{and } R_2 \to R_2 - \frac{1}{2}R_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - 2R_1]$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A \quad [\because R_3 \to R_3 - R_2]$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to \frac{1}{2}R_1 \\ \text{and } R_3 \to 2R_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \quad \begin{bmatrix} \because R_1 \to R_1 + \frac{1}{2}R_3 \\ \text{and } R_2 \to R_2 - \frac{5}{2}R_3 \end{bmatrix}$$

Hence, $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ is the inverse of given matrix A.

Q. 52 Express the matrix $\begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Thinking Process

We know that, any square matrix A can be expressed as the sum of a symmetric matrix and skew-symmetric matrix, i.e., $A = \frac{A + A'}{2} + \frac{A - A'}{2}$, *where* $A + A'$ *and* $A - A'$ *are a symmetric matrix and a skew-symmetric matrix, respectively.*

Sol. We have,
$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & -1 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

$\therefore$
$$A' = \begin{bmatrix} 2 & 1 & 4 \\ 3 & -1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

Now,
$$\frac{A + A'}{2} = \frac{1}{2}\begin{bmatrix} 4 & 4 & 5 \\ 4 & -2 & 3 \\ 5 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix}$$

and
$$\frac{A - A'}{2} = \frac{1}{2}\begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

$\therefore$
$$\frac{A + A'}{2} + \frac{A - A'}{2} = \begin{bmatrix} 2 & 2 & \frac{5}{2} \\ 2 & -1 & \frac{3}{2} \\ \frac{5}{2} & \frac{3}{2} & 2 \end{bmatrix} + \begin{bmatrix} 0 & 1 & \frac{-3}{2} \\ -1 & 0 & \frac{1}{2} \\ \frac{3}{2} & \frac{-1}{2} & 0 \end{bmatrix}$$

which is the required expression.

Objective Type Questions

Q. 53 The matrix $P=\begin{bmatrix}0&0&4\\0&4&0\\4&0&0\end{bmatrix}$ is a

(a) square matrix (b) diagonal matrix
(c) unit matrix (d) None of these

Sol. *(a)* We know that, in a square matrix number of rows are equal to the number of columns, so the matrix $P=\begin{bmatrix}0&0&4\\0&4&0\\4&0&0\end{bmatrix}$ is a square matrix.

Q. 54 Total number of possible matrices of order 3×3 with each entry 2 or 0 is

(a) 9 (b) 27
(c) 81 (d) 512

Sol. *(d)* Total number of possible matrices of order 3×3 with each entry 2 or 0 is 2^9 *i.e.,* 512.

Q. 55 $\begin{bmatrix}2x+y & 4x\\5x-7 & 4x\end{bmatrix}=\begin{bmatrix}7 & 7y-13\\y & x+6\end{bmatrix}$, then the value of $x+y$ is

(a) $x=3, y=1$ (b) $x=2, y=3$
(c) $x=2, y=4$ (d) $x=3, y=3$

Sol. *(b)* We have, $4x=x+6 \Rightarrow x=2$

and $4x=7y-13 \Rightarrow 8=7y-13$

$\Rightarrow$ $7y=21 \Rightarrow y=3$

$\therefore$ $x+y=2+3=5$

Q. 56 If $A=\dfrac{1}{\pi}\begin{bmatrix}\sin^{-1}(x\pi) & \tan^{-1}\left(\dfrac{x}{\pi}\right)\\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & \cot^{-1}(\pi x)\end{bmatrix}$ and $B=\dfrac{1}{\pi}\begin{bmatrix}-\cos^{-1}(x\pi) & \tan^{-1}\left(\dfrac{x}{\pi}\right)\\ \sin^{-1}\left(\dfrac{x}{\pi}\right) & -\tan^{-1}(\pi x)\end{bmatrix}$,

then $A-B$ is equal to

(a) I (b) 0 (c) $2I$ (d) $\dfrac{1}{2}I$

Sol. *(d)* We have, $A=\begin{bmatrix}\dfrac{1}{\pi}\sin^{-1}x\pi & \dfrac{1}{\pi}\tan^{-1}\dfrac{x}{\pi}\\ \dfrac{1}{\pi}\sin^{-1}\dfrac{x}{\pi} & \dfrac{1}{\pi}\cot^{-1}\pi x\end{bmatrix}$

and $B=\begin{bmatrix}\dfrac{-1}{\pi}\cos^{-1}x\pi & \dfrac{1}{\pi}\tan^{-1}\dfrac{x}{\pi}\\ \dfrac{1}{\pi}\sin^{-1}\dfrac{x}{\pi} & \dfrac{-1}{\pi}\tan^{-1}\pi x\end{bmatrix}$

$$\therefore \quad A-B=\begin{bmatrix} \frac{1}{\pi}(\sin^{-1}x\pi+\cos^{-1}x\pi) & \frac{1}{\pi}\left(\tan^{-1}\frac{x}{\pi}-\tan^{-1}\frac{x}{\pi}\right) \\ \frac{1}{\pi}\left(\sin^{-1}\frac{x}{\pi}-\sin^{-1}\frac{x}{\pi}\right) & \frac{1}{\pi}\cot^{-1}\pi x+\tan^{-1}\pi x \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1}{\pi}\cdot\frac{\pi}{2} & 0 \\ 0 & \frac{1}{\pi}\cdot\frac{\pi}{2} \end{bmatrix} \qquad \begin{bmatrix} \because \sin^{-1}x+\cos^{-1}x=\frac{\pi}{2} \\ \text{and } \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2} \end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\frac{1}{2}I$$

Q. 57 If A and B are two matrices of the order $3\times m$ and $3\times n$, respectively and $m=n$, then order of matrix $(5A-2B)$ is

(a) $m\times 3$ (b) 3×3
(c) $m\times n$ (d) $3\times n$

Sol. ***(d)*** $A_{3\times m}$ and $B_{3\times n}$ are two matrices. If $m=n$, then A and B have same orders as $3\times n$ each, so the order of $(5A-2B)$ should be same as $3\times n$.

Q. 58 If $A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, then A^2 is equal to

(a) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Sol. ***(d)*** $\because A^2=A\cdot A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q. 59 If matrix $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=1$, if $i\neq j=0$ and if $i=j$, then A^2 is equal to

(a) I (b) A
(c) 0 (d) None of these

Sol. ***(a)*** We have, $A=[a_{ij}]_{2\times 2}$, where $a_{ij}=1$, if $i\neq j=0$ and if $i=j$

$$\therefore \quad A=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

and

$$A^2=\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}=I$$

Q. 60 The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a

(a) identity matrix (b) symmetric matrix
(c) skew-symmetric matrix (d) None of these

Sol. *(b)* Let

$$A=\begin{bmatrix}1&0&0\\0&2&0\\0&0&4\end{bmatrix}$$

$$\therefore \quad A'=\begin{bmatrix}1&0&0\\0&2&0\\0&0&4\end{bmatrix}=A$$

So, the given matrix is a symmetric matrix.

[since, in a square matrix A, if $A'=A$, then A is called symmetric matrix]

Q. 61 **The matrix** $\begin{bmatrix}0&-5&8\\5&0&12\\-8&-12&0\end{bmatrix}$ **is a**

(a) diagonal matrix (b) symmetric matrix

(c) skew-symmetric matrix (d) scalar matrix

Sol. *(c)* We know that, in a square matrix, if $b_{ij}=0$, when $i \neq j$, then it is said to be a diagonal matrix. Here, $b_{12}, b_{13}, \ldots \neq 0$, so the given matrix is not a diagonal matrix.

Now,

$$B=\begin{bmatrix}0&-5&8\\5&0&12\\-8&-12&0\end{bmatrix}$$

$$\therefore \quad B'=\begin{bmatrix}0&5&-8\\-5&0&-12\\8&12&0\end{bmatrix}=-\begin{bmatrix}0&-5&8\\5&0&12\\-8&-12&0\end{bmatrix}=-B$$

So, the given matrix is a skew-symmetric matrix, since we know that in a square matrix B, if $B'=-B$, then it is called skew-symmetric matrix.

Q. 62 **If** A **is matrix of order** $m \times n$ **and** B **is a matrix such that** AB' **and** $B'A$ **are both defined, then order of matrix** B **is**

(a) $m \times m$ (b) $n \times n$ (c) $n \times m$ (d) $m \times n$

Sol. *(d)* Let $A=[a_{ij}]_{m\times n}$ and $B=[b_{ij}]_{p\times q}$

$\therefore \quad B'=[b_{ji}]_{q\times p}$

Now, AB' is defined, so $n=q$

and $B'A$ is also defined, so $p=m$

$\therefore \quad$ Order of $B'=[b_{ji}]_{n\times m}$

and order of $B=[b_{ij}]_{m\times n}$

Q. 63 **If** A **and** B **are matrices of same order, then** $(AB'-BA')$ **is a**

(a) skew-symmetric matrix (b) null matrix

(c)symmetric matrix (d) unit matrix

Sol. *(a)* We have matrices A and B of same order.

Let $P=(AB'-BA')$

Then,
$$P'=(AB'-BA')'=(AB')'-(BA')'$$
$$=(B')'(A)'-(A')'B'=BA'-AB'$$
$$=-(AB'-BA')=-P$$

Hence, $(AB'-BA')$ is a skew-symmetric matrix.

Q. 64 If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) $I + A$ (d) $3A$

Sol. *(a)* We have, $A^2 = I$

$\therefore (A - I)^3 + (A + I)^3 - 7A = [(A - I) + (A + I)\{(A - I)^2 + (A + I)^2 - (A - I)(A + I)\}] - 7A$

$[\because a^3 + b^3 = (a + b)(a^2 + b^2 - ab)]$

$= [(2A)\{A^2 + I^2 - 2AI + A^2 + I^2 + AI - (A^2 - I^2)\}] - 7A$

$= 2A[I + I^2 + I + I^2 - A^2 + I^2] - 7A$ $[\because A^2 = AI]$

$= 2A[5I - I] - 7A$

$= 8AI - 7AI$ $[\because A = AI]$

$= AI = A$

Q. 65 For any two matrices A and B, we have

(a) $AB = BA$ (b) $AB \neq BA$ (c) $AB = O$ (d) None of these

Sol. *(d)* For any two matrices A and B, we may have $AB = BA = I$, $AB \neq BA$ and $AB = O$ but it is not always true.

Q. 66 On using elementary column operations $C_2 \to C_2 - 2C_1$ in the following matrix equation $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$, we have

(a) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -0 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -2 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Sol. *(d)* Given that, $\begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

On using $C_2 \to C_2 - 2C_1$, $\begin{bmatrix} 1 & -5 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 0 \end{bmatrix}$

Since, on using elementary column operation on $X = AB$, we apply these operations simultaneously on X and on the second matrix B of the product AB on RHS.

Q. 67 On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$, we have

(a) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} -1 & -3 \\ 1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -7 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 4 & 2 \\ -5 & -7 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -3 & -3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Sol. *(a)* We have, $\begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$

Using elementary row operation $R_1 \to R_1 - 3R_2$,

$$\begin{bmatrix} -5 & -7 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

Since, on using elementary row operation on $X = AB$, we apply these operation simultaneously on X and on the first matrix A of the product AB on RHS.

Fillers

Q. 68 .... matrix is both symmetric and skew-symmetric matrix.

Sol. Null matrix is both symmetric and skew-symmetric matrix.

Q. 69 Sum of two skew-symmetric matrices is always .... matrix.

Sol. Let A is a given matrix, then $(-A)$ is a skew-symmetric matrix.
Similarly, for a given matrix $-B$ is *a* skew-symmetric matrix.
Hence, $-A - B = -(A + B) \Rightarrow$ sum of two skew-symmetric matrices is always skew-symmetric matrix.

Q. 70 The negative of a matrix is obtained by multiplying it by ....

Sol. Let A is a given matrix.

$\therefore$ $$-A = -1\,[A]$$

So, the negative of a matrix is obtained by multiplying it by -1.

Q. 71 The product of any matrix by the scalar .... is the null matrix.

Sol. The product of any matrix by the scalar 0 is the null matrix. *i.e.,* $0 \cdot A = 0$.
[where, A is any matrix]

Q. 72 A matrix which is not a square matrix is called a .... matrix.

Sol. A matrix which is not a square matrix is called a rectangular matrix. For example a rectangular matrix is $A = [a_{ij}]_{m \times n}$, where $m \neq n$.

Q. 73 Matrix multiplication is ...... over addition.

Sol. Matrix multiplication is distributive over addition.
e.g., For three matrices A, B and C,

(i) $A(B + C) = AB + AC$

(ii) $(A + B)C = AC + BC$

Q. 74 If A is a symmetric matrix, then A^3 is a .... matrix.

Sol. If A is a symmetric matrix, then A^3 is a symmetric matrix.

$\because$ $A' = A$

$\therefore$ $(A^3)' = A'^3$

$= A^3$ $\quad [\because (A')^n = (A^n)']$

Q. 75 If A is a skew-symmetric matrix, then A^2 is a ....

Sol. If A is a skew-symmetric matrix, then A^2 is a symmetric matrix.

$\because$ $A' = -A$

$\therefore$ $(A^2)' = (A')^2$

$= (-A)^2$ $\quad [\because A' = -A]$

$= A^2$

So, A^2 is a symmetric matrix.

Q. 76 If A and B are square matrices of the same order, then

(i) $(AB)' =$ ....

(ii) $(kA)' =$ .... (where, k is any scalar)

(iii) $[k(A-B)]' =$ ....

Sol. **(i)** $(AB)' = B'A'$

(ii) $(kA)' = k\,A'$

(iii) $[k(A-B)]' = k(A' - B')$

Q. 77 If A is a skew-symmetric, then kA is a .... (where, k is any scalar).

Sol. If A is a skew-symmetric, then kA is a skew-symmetric matrix (where, k is any scalar).

$[\because A' = -A \Rightarrow (kA)' = k(A)' = -(kA)]$

Q. 78 If A and B are symmetric matrices, then

(i) $AB - BA$ is a ....

(ii) $BA - 2AB$ is a ....

Sol. **(i)** $AB - BA$ is a skew-symmetric matrix.

Since, $[AB - BA]' = (AB)' - (BA)'$

$= B'A' - A'B'$ $\quad [\because (AB)' = B'A']$

$= BA - AB$ $\quad [\because A' = A$ and $B' = B]$

$= -[AB - BA]$

So, $[AB - BA]$ is a skew-symmetric matrix.

(ii) $[BA - 2AB]$ is a neither symmetric nor skew-symmetric matrix.

$\therefore$ $(BA - 2AB)' = (BA)' - 2(AB)'$

$= A'B' - 2B'A'$

$= AB - 2BA$

$= -(2BA - AB)$

So, $[BA - 2AB]$ is neither symmetric nor skew-symmetric matrix.

Q. 79 If A is symmetric matrix, then $B'AB$ is ....

Sol. If A is a symmetric matrix, then $B'AB$ is a symmetric metrix.

$\because$

$$[B'AB]' = [B'(AB)]'$$
$$= (AB)'(B')' \quad [\because (AB)' = B'A']$$
$$= B'A'B$$
$$= [B'A'B] \quad [\because A' = A]$$

So, $B'AB$ is a symmetric matrix.

Q. 80 If A and B are symmetric matrices of same order, then AB is symmetric if and only if......... .

Sol. If A and B are symmetric matrices of same order, then AB is symmetric if and only if $AB = BA$.

$\therefore$

$$(AB)'$$
$$= B'A' = BA$$
$$= AB \quad [\because AB = BA]$$

Q. 81 In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeroes in one or more, then A^{-1} ....

Sol. In applying one or more row operations while finding A^{-1} by elementary row operations, we obtain all zeroes in one or more, then A^{-1} does not exist.

True/False

Q. 82 A matrix denotes a number.

Sol. *False*

A matrix is an ordered rectangular array of numbers or functions.

Q. 83 Matrices of any order can be added.

Sol. *False*

Two matrices are added, if they are of the same order.

Q. 84 Two matrices are equal, if they have same number of rows and same number of columns.

Sol. *False*

If two matrices have same number of rows and same number of columns, then they are said to be square matrix and if two square matrices have same elements in both the matrices, only then they are called equal.

Q. 85 Matrices of different order cannot be subtracted.

Sol. *True*

Two matrices of same order can be subtracted

Q. 86 Matrix addition is associative as well as commutative.

Sol. ***True***

Matrix addition is associative as well as commutative *i.e.,*
$(A + B) + C = A + (B + C)$ and $A + B = B + A$, where A, B and C are matrices of same order.

Q. 87 Matrix multiplication is commutative.

Sol. ***False***

Since, $AB \neq BA$ is possible when AB and BA are both defined.

Q. 88 A square matrix where every element is unity is called an identity matrix.

Sol. ***False***

Since, in an identity matrix, the diagonal elements are all one and rest are all zero.

Q. 89 If A and B are two square matrices of the same order, then $A + B = B + A$.

Sol. ***True***

Since, matrix addition is commutative *i.e.,* $A + B = B + A$, where A and B are two square matrices.

Q. 90 If A and B are two matrices of the same order, then $A - B = B - A$.

Sol. ***False***

Since, the addition of two matrices of same order are commutative.

$$\therefore \quad A + (-B) = A - B = -[B - A] \neq B - A$$

Q. 91 If matrix $AB = 0$, then $A = 0$ or $B = 0$ or both A and B are null matrices.

Sol. ***False***

Since, for two non-zero matrices A and B of same order, it can be possible that $A \cdot B = 0 =$ null matrix

Q. 92 Transpose of a column matrix is a column matrix.

Sol. ***False***

Transpose of a column matrix is a row matrix.

Q. 93 If A and B are two square matrices of the same order, then $AB = BA$.

Sol. ***False***

For two square matrices of same order it is not always true that $AB = BA$.

Q. 94 If each of the three matrices of the same order are symmetric, then their sum is a symmetric matrix.

Sol. ***True***

Let A, B and C are three matrices of same order

$$\therefore \quad A' = A, B' = B \text{ and } C' = C$$

$$\therefore \quad (A + B + C)' = A' + B' + C'$$

$$= (A + B + C)$$

Q. 95 If A and B are any two matrices of the same order, then $(AB)' = A'B'$.

Sol. ***False***

$\because \quad (AB)' = B'A'$

Q. 96 If $(AB)' = B'\ A'$, where A and B are not square matrices, then number of rows in A is equal to number of columns in B and number of columns in A is equal to number of rows in B.

Sol. ***True***

Let A is of order $m \times n$ and B is of order $p \times q$.

Since, $(AB)' = B'A'$

$\therefore \quad A_{(m \times n)} B_{(p \times q)}$ is defined $\Rightarrow n = p$ ...(i)

and AB is of order $m \times q$.

$\Rightarrow \quad (AB)'$ is of order $q \times m$ ...(ii)

Also, B' is of order $q \times p$ and A' is of order $n \times m$

$\therefore \quad B'A'$ is defined $\Rightarrow p = n$

and $B'A'$ is of order $q \times m$. ...(iii)

Also, equality of matrices $(AB)' = B'A'$, we get the given statement as true.

e.g., If A is of order (3×1) and B is of order (1×3), we get

Order of $(AB)'$ = Order of $(B'A') = 3 \times 3$

Q. 97 If A, B and C are square matrices of same order, then $AB = AC$ always implies that $B = C$.

Sol. ***False***

If $AB = AC = 0$, then it can be possible that B and C are two non-zero matrices such that $B \neq C$.

$\therefore \quad A \cdot B = 0 = A \cdot C$

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}$

and $C = \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix}$

$\therefore \quad AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

and $AC = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \quad AB = AC$ but $B \neq C$

Q. 98 AA' is always a symmetric matrix for any matrix A.

Sol. ***True***

$\because \ [AA']' = (A')' A' = [AA']$

Q. 99 If $A = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{vmatrix}$ and $B = \begin{vmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{vmatrix}$, then AB and BA are defined and equal.

Sol. ***False***

Since, AB is defined.

$$\therefore \quad AB = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 14 & 20 \\ 22 & 25 \end{bmatrix}$$

Also, BA is defined.

$$\therefore \quad BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -1 \\ 1 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 18 & 4 \\ 13 & 32 & 6 \\ 5 & 10 & 0 \end{bmatrix}$$

$$\therefore \quad AB \neq BA$$

Q. 100 If A is skew-symmetric matrix, then A^2 is a symmetric matrix.

Sol. ***True***

$$\because \quad [A^2]' = [A']^2$$

$$= [-A]^2 \quad [\because A' = -A]$$

$$= A^2$$

Hence, A^2 is symmetric matrix.

Q. 101 $(AB)^{-1} = A^{-1} \cdot B^{-1}$, where A and B are invertible matrices satisfying commutative property with respect to multiplication.

Sol. ***True***

We know that, if A and B are invertible matrices of the same order, then

$$(AB)^{-1} = (BA)^{-1} \quad [\because AB = BA]$$

Here, $(AB)^{-1} = (AB)^{-1}$

$$\Rightarrow \quad B^{-1}A^{-1} = A^{-1}B^{-1}$$

[since, A and B are satisfying commutative property with respect to multiplications].