

1

Units and Measurements

Multiple Choice Questions (MCQs)

Q. 1 The number of significant figures in 0.06900 is

- (a) 5 (b) 4 (c) 2 (d) 3

💡 **Thinking Process**

If the number is less than 1, the zero(s) on the right of decimal point and before the first non-zero digit are not significant.

Ans. (b) In 0.06900, the underlined zeroes are not significant. Hence, number of significant figures are **four** (6900).

Q. 2 The sum of the numbers 436.32, 227.2 and 0.301 in appropriate significant figures is

- (a) 663.821 (b) 664 (c) 663.8 (d) 663.82

Ans. (b) The sum of the numbers can be calculated as 663.821 arithmetically. The number with least decimal places is 227.2 is correct to only one decimal place.

The final result should, therefore be rounded off to one decimal place *i.e.*, 664.

Note *In calculating the sum, we should not confuse with the number of decimal places and significant figures. The result should have least number of decimal places.*

Q. 3 The mass and volume of a body are 4.237 g and 2.5 cm³, respectively. The density of the material of the body in correct significant figures is

- (a) 1.6048 g cm⁻³ (b) 1.69 g cm⁻³ (c) 1.7 g cm⁻³ (d) 1.695 g cm⁻³

💡 **Thinking Process**

In multiplication or division, the final result should retain as many significant figures as are there in the original number with the least significant figures.

Ans. (c) In this question, density should be reported to two significant figures.

$$\text{Density} = \frac{4.237\text{g}}{2.5 \text{ cm}^3} = 1.6948$$

As rounding off the number, we get density = 1.7

Q. 7 Measure of two quantities along with the precision of respective measuring instrument is

$A = 2.5 \text{ ms}^{-1} \pm 0.5 \text{ ms}^{-1}$, $B = 0.10 \text{ s} \pm 0.01 \text{ s}$. The value of AB will be

- (a) $(0.25 \pm 0.08) \text{ m}$ (b) $(0.25 \pm 0.5) \text{ m}$ (c) $(0.25 \pm 0.05) \text{ m}$ (d) $(0.25 \pm 0.135) \text{ m}$

Ans. (a) Given,

$$A = 2.5 \text{ ms}^{-1} \pm 0.5 \text{ ms}^{-1}, B = 0.10 \text{ s} \pm 0.01 \text{ s}$$

$$\begin{aligned} x &= AB = (2.5)(0.10) = 0.25 \text{ m} \\ \frac{\Delta x}{x} &= \frac{\Delta A}{A} + \frac{\Delta B}{B} \\ &= \frac{0.5}{2.5} + \frac{0.01}{0.10} = \frac{0.05 + 0.025}{0.25} = \frac{0.075}{0.25} \end{aligned}$$

$\Delta x = 0.075 = 0.08 \text{ m}$, rounding off to two significant figures.

$$AB = (0.25 \pm 0.08) \text{ m}$$

Q. 8 You measure two quantities as $A = 1.0 \text{ m} \pm 0.2 \text{ m}$, $B = 2.0 \text{ m} \pm 0.2 \text{ m}$. We should report correct value for \sqrt{AB} as

- (a) $1.4 \text{ m} \pm 0.4 \text{ m}$ (b) $1.41 \text{ m} \pm 0.15 \text{ m}$ (c) $1.4 \text{ m} \pm 0.3 \text{ m}$ (d) $1.4 \text{ m} \pm 0.2 \text{ m}$

Ans. (d) Given,

$$A = 1.0 \text{ m} \pm 0.2 \text{ m}, B = 2.0 \text{ m} \pm 0.2 \text{ m}$$

Let, $Y = \sqrt{AB} = \sqrt{(1.0)(2.0)} = 1.414 \text{ m}$

Rounding off to two significant digit $Y = 1.4 \text{ m}$

$$\frac{\Delta Y}{Y} = \frac{1}{2} \left[\frac{\Delta A}{A} + \frac{\Delta B}{B} \right] = \frac{1}{2} \left[\frac{0.2}{1.0} + \frac{0.2}{2.0} \right] = \frac{0.6}{2 \times 2.0}$$

$$\Rightarrow \Delta Y = \frac{0.6Y}{2 \times 2.0} = \frac{0.6 \times 1.4}{2 \times 2.0} = 0.212$$

Rounding off to one significant digit $\Delta Y = 0.2 \text{ m}$

Thus, correct value for $\sqrt{AB} = r + \Delta r = 1.4 \pm 0.2 \text{ m}$

Q. 9 Which of the following measurement is most precise?

- (a) 5.00 mm (b) 5.00 cm (c) 5.00 m (d) 5.00 km

Ans. (a) All given measurements are correct upto two decimal places. As here 5.00 mm has the smallest unit and the error in 5.00 mm is least (commonly taken as 0.01 mm if not specified), hence, 5.00 mm is most precise.

Note In solving these type of questions, we should be careful about units although their magnitude is same.

Q. 10 The mean length of an object is 5 cm. Which of the following measurements is most accurate?

- (a) 4.9 cm (b) 4.805 cm (c) 5.25 cm (d) 5.4 cm

Ans. (a) Given length $l = 5 \text{ cm}$

Now, checking the errors with each options one by one, we get

$$\Delta l_1 = 5 - 4.9 = 0.1 \text{ cm}$$

$$\Delta l_2 = 5 - 4.805 = 0.195 \text{ cm}$$

$$\Delta l_3 = 5.25 - 5 = 0.25 \text{ cm}$$

$$\Delta l_4 = 5.4 - 5 = 0.4 \text{ cm}$$

Error Δl_1 is least.

Hence, 4.9 cm is most precise.

Q. 11 Young's modulus of steel is $1.9 \times 10^{11} \text{ N/m}^2$. When expressed in CGS units of dyne/cm^2 , it will be equal to ($1\text{N} = 10^5 \text{ dyne}$, $1 \text{ m}^2 = 10^4 \text{ cm}^2$)

- (a) 1.9×10^{10} (b) 1.9×10^{11} (c) 1.9×10^{12} (d) 1.9×10^{13}

Ans. (c) Given, Young's modulus $Y = 1.9 \times 10^{11} \text{ N/m}^2$
 $1\text{N} = 10^5 \text{ dyne}$
 Hence, $Y = 1.9 \times 10^{11} \times 10^5 \text{ dyne/m}^2$
 We know that $1\text{m} = 100\text{cm}$
 $\therefore Y = 1.9 \times 10^{11} \times 10^5 \text{ dyne}/(100)^2 \text{ cm}^2$
 $= 1.9 \times 10^{16-4} \text{ dyne/cm}^2$
 $Y = 1.9 \times 10^{12} \text{ dyne/cm}^2$

Note While we are going through units conversion, we should keep in mind that proper relation between units are mentioned.

Q. 12 If momentum (p), area (A) and time (T) are taken to be fundamental quantities, then energy has the dimensional formula

- (a) $[pA^{-1}T^1]$ (b) $[p^2AT]$ (c) $[pA^{-1/2}T]$ (d) $[pA^{1/2}T]$

Ans. (d) Given, fundamental quantities are momentum (p), area (A) and time (T).
 We can write energy E as

$$E \propto p^a A^b T^c$$

$$E = k p^a A^b T^c$$

where k is dimensionless constant of proportionality.

Dimensions of $E = [E] = [\text{ML}^2\text{T}^{-2}]$ and $[p] = [\text{MLT}^{-1}]$

$$[A] = [\text{L}^2]$$

$$[T] = [T]$$

$$[E] = [K][p]^a[A]^b[T]^c$$

Putting all the dimensions, we get

$$\text{ML}^2\text{T}^{-2} = [\text{MLT}^{-1}]^a[\text{L}^2]^b[\text{T}]^c$$

$$= \text{M}^a\text{L}^{2b+a}\text{T}^{-a+c}$$

By principle of homogeneity of dimensions,

$$a = 1, 2b + a = 2$$

$$\Rightarrow 2b + 1 = 2$$

$$\Rightarrow b = 1/2 \quad -a + c = -2$$

$$\Rightarrow c = -2 + a = -2 + 1 = -1$$

Hence, $E = pA^{1/2}T^{-1}$

Multiple Choice Questions (More Than One Options)

Q. 13 On the basis of dimensions, decide which of the following relations for the displacement of a particle undergoing simple harmonic motion is not correct?

$$(a) y = a \sin 2\pi t/T$$

$$(b) y = a \sin vt$$

$$(c) y = \frac{a}{T} \sin\left(\frac{t}{a}\right)$$

$$(d) y = a\sqrt{2}\left(\sin\frac{2\pi t}{T} - \cos\frac{2\pi t}{T}\right)$$

💡 Thinking Process

We know that angle is dimensionless. Here, a is displacement and y is also displacement, hence they are having same dimensions.

Ans. (b, c)

Now, by principle of homogeneity of dimensions LHS and RHS of (a) and (d) will be same and is L .

For (c)

$$[\text{LHS}] = L$$

$$[\text{RHS}] = \frac{L}{T} = LT^{-1}$$

$$[\text{LHS}] \neq [\text{RHS}]$$

Hence, (c) is not correct option.

In option (b) dimension of angle is $[vt]$ i.e., L

$$\Rightarrow \text{RHS} = L.L = L^2 \text{ and LHS} = L$$

$$\Rightarrow \text{LHS} \neq \text{RHS.}$$

So, option (b) is also not correct.

Q. 14 If P , Q , R are physical quantities, having different dimensions, which of the following combinations can never be a meaningful quantity?

$$(a) (P - Q)/R$$

$$(b) PQ - R$$

$$(c) PQ/R$$

$$(d) (PR - Q^2)/R$$

$$(e) (R + Q)/P$$

💡 Thinking Process

We should keep in mind that when two physical quantities are added or subtracted they should have same dimensions.

Ans. (a, e)

In this question, it is given that P , Q and R are having different dimensions, hence they cannot be added or subtracted, so we can say that (a) and (e) are not meaningful. We cannot say about the dimension of product of these quantities, hence (b), (c) and (d) may be meaningful.

Note In this question, we are certain about the quantity which is never meaningful but we should keep in mind that others may or may not be meaningful.

Q. 15 Photon is quantum of radiation with energy $E = h\nu$, where ν is frequency and h is Planck's constant. The dimensions of h are the same as that of

$$(a) \text{ linear impulse}$$

$$(b) \text{ angular impulse}$$

$$(c) \text{ linear momentum}$$

$$(d) \text{ angular momentum}$$

Ans. (b, d)

We know that energy of radiation, $E = h\nu$

$$[h] = \frac{[E]}{[\nu]} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

Dimension of linear impulse = Dimension of momentum = $[MLT^{-1}]$

As we know that linear impulse $J = \Delta P$

\Rightarrow Angular impulse = $\tau dt = \Delta L =$ Change in angular momentum

Hence, dimension of angular impulse

$$= \text{Dimension of angular momentum} \\ = [ML^2T^{-1}].$$

This is similar to the dimension of Planck's constant h .

Q. 16 If Planck's constant (h) and speed of light in vacuum (c) are taken as two fundamental quantities, which one of the following can, in addition, be taken to express length, mass and time in terms of the three chosen fundamental quantities?

- (a) Mass of electron (m_e) (b) Universal gravitational constant (G)
 (c) Charge of electron (e) (d) Mass of proton (m_p)

Ans. (a, b, d)

We know that dimension of $h = [h] = [ML^2T^{-1}]$

$$[c] = [LT^{-1}], [m_e] = M$$

$$[G] = [M^{-1}L^3T^{-2}]$$

$$[e] = [AT], [m_p] = [M]$$

$$\left[\frac{hc}{G} \right] = \frac{[ML^2T^{-1}][LT^{-1}]}{[M^{-1}L^3T^{-2}]} = [M^2]$$

$$M = \sqrt{\frac{hc}{G}}$$

Similarly,

$$\frac{h}{c} = \frac{[ML^2T^{-1}]}{[LT^{-1}]} = [ML]$$

$$L = \frac{h}{cM} = \frac{h}{c} \sqrt{\frac{G}{hc}} = \frac{\sqrt{Gh}}{c^{3/2}}$$

As,

$$c = LT^{-1}$$

\Rightarrow

$$[T] = \frac{[L]}{[c]} = \frac{\sqrt{Gh}}{c^{3/2} \cdot c} = \frac{\sqrt{Gh}}{c^{5/2}}$$

Hence, (a), (b) or (d) any can be used to express L , M and T in terms of three chosen fundamental quantities.

Q. 17 Which of the following ratios express pressure?

- (a) Force/Area (b) Energy/Volume (c) Energy/Area (d) Force/Volume

Thinking Process

While solving this type of questions, we should first write an expression and try to express it in terms of quantities given in the option.

Ans. (a, b)

We know that pressure = $\frac{\text{Force}}{\text{Area}}$

$$\text{Pressure} = \frac{\text{Force} \times \text{Distance}}{\text{Area} \times \text{Distance}} = \frac{\text{Work}}{\text{Volume}} = \frac{\text{Energy}}{\text{Volume}}$$

Note Here, we should keep it in mind that above values are not exactly equal but these are equivalent with respect to their units.

Q. 18 Which of the following are not a unit of time?

- (a) Second (b) Parsec (c) Year (d) Light year

Ans. (b, d)

We know that 1 light year = 9.46×10^{11} m

= distance that light travels in 1 year with speed 3×10^8 m/s.

1 parsec = 3.08×10^{16} m

= Distance at which average radius of earth's orbit subtends an angle of 1 parsecond

Here, second and year represent time.

Very Short Answer Type Questions

Q. 19 Why do we have different units for the same physical quantity?

Ans. The value of any given physical quantity may vary over a wide range, therefore, different units of same physical quantity are required.

e.g., The length of a pen can be easily measured in cm, the height of a tree can be measured in metres, the distance between two cities can be measured in kilometres and distance between two heavenly bodies can be measured in light year.

Q. 20 The radius of atom is of the order of 1 \AA and radius of nucleus is of the order of fermi. How many magnitudes higher is the volume of atom as compared to the volume of nucleus?

Ans.

$$\text{Radius of atom} = 1 \text{ \AA} = 10^{-10} \text{ m}$$

$$\text{Radius of nucleus} = 1 \text{ fermi} = 10^{-15} \text{ m}$$

$$\text{Volume of atom} = V_A = \frac{4}{3} \pi R_A^3$$

$$\text{Volume of nucleus } V_N = \frac{4}{3} \pi R_N^3$$

$$\frac{V_A}{V_N} = \frac{\frac{4}{3} \pi R_A^3}{\frac{4}{3} \pi R_N^3} = \left(\frac{R_A}{R_N} \right)^3 = \left(\frac{10^{-10}}{10^{-15}} \right)^3 = 10^{15}$$

Note In such type of questions, always change the value in same unit.

Q. 21 Name the device used for measuring the mass of atoms and molecules.

Ans. A mass spectrograph is used for measuring the mass of atoms and molecules.

Q. 22 Express unified atomic mass unit in kg.

Ans. One atomic mass unit is the $\frac{1}{12}$ of the mass of a ${}_{6}\text{C}^{12}$ atom.

Mass of one mole of ${}_{6}\text{C}^{12}$ atom = 12 g

Number of atoms in one mole = Avogadro's number
= 6.023×10^{23}

$$\therefore \text{Mass of one } {}_6\text{C}^{12} \text{ atom} = \frac{12}{6.023 \times 10^{23}} \text{ g}$$

$$1 \text{ amu} = \frac{1}{12} \times \text{mass of one } {}_6\text{C}^{12} \text{ atom}$$

$$\therefore 1 \text{ amu} = \left(\frac{1}{12} \times \frac{12}{6.023 \times 10^{23}} \right) \text{ g} = 1.67 \times 10^{-24} \text{ g}$$

$$= 1.67 \times 10^{-27} \text{ kg} \quad (\because 1 \text{ g} = 10^{-3} \text{ kg})$$

Q. 23 A function $f(\theta)$ is defined as $f(\theta) = 1 - \theta + \frac{\theta^2}{2!} - \frac{\theta^3}{3!} + \frac{\theta^4}{4!} + \dots$

Why is it necessary for $f(\theta)$ to be a dimensionless quantity?

Ans. Since, $f(\theta)$ is a sum of different powers of θ and it is a dimensionless quantity. By principle of homogeneity as RHS is dimensionless, hence LHS should also be dimensionless.

Q. 24 Why length, mass and time are chosen as base quantities in mechanics?

Ans. Length, mass and time are chosen as base quantities in mechanics because

- Length, mass and time cannot be derived from one another, that is these quantities are independent.
- All other quantities in mechanics can be expressed in terms of length, mass and time.

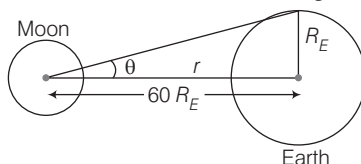
Short Answer Type Questions

- Q. 25** (a) The earth-moon distance is about 60 earth radius. What will be the diameter of the earth (approximately in degrees) as seen from the moon?
- (b) Moon is seen to be of $(1/2)^\circ$ diameter from the earth. What must be the relative size compared to the earth?
- (c) From parallax measurement, the sun is found to be at a distance of about 400 times the earth-moon distance. Estimate the ratio of sun-earth diameters.

Thinking Process

To solve this question, we have to treat radius of earth as an arc as seen from the moon.

Ans. (a) Angle subtended at distance r due to an arc of length l is



$$\theta = \frac{l}{r}$$

- Q. 28** The vernier scale of a travelling microscope has 50 divisions which coincide with 49 main scale divisions. If each main scale division is 0.5 mm, calculate the minimum inaccuracy in the measurement of distance.

Thinking Process

Inaccuracy will be measured by difference of 1MSD and 1VSD, where MSD = main scale division and VSD = vernier scale division.

Ans. By question, it is given that

$$50\text{VSD} = 49\text{MSD}$$

$$1\text{MSD} = \frac{50}{49}\text{VSD}$$

$$1\text{VSD} = \frac{49}{50}\text{MSD}$$

$$\text{Minimum inaccuracy} = 1\text{MSD} - 1\text{VSD}$$

$$= 1\text{MSD} - \frac{49}{50}\text{MSD} = \frac{1}{50}\text{MSD}$$

Given, $1\text{MSD} = 0.5\text{ mm}$

Hence, $\text{minimum inaccuracy} = \frac{1}{50} \times 0.5\text{ mm} = \frac{1}{100} = 0.01\text{ mm}$

- Q. 29** During a total solar eclipse the moon almost entirely covers the sphere of the sun. Write the relation between the distances and sizes of the sun and moon.

Ans. Consider the diagram given below

R_{me} = Distance of moon from earth

R_{se} = Distance of sun from earth

Let angle made by sun and moon is θ , we can write

$$\theta = \frac{A_{\text{sun}}}{R_{se}^2} = \frac{A_{\text{moon}}}{R_{me}^2}$$

Here,

A_{sun} = Area of the sun

A_{moon} = Area of the moon

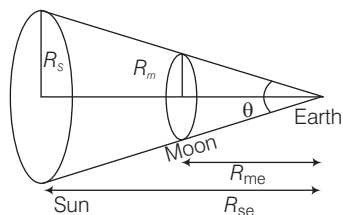
$$\Rightarrow \theta = \frac{\pi R_s^2}{R_{se}^2} = \frac{\pi R_m^2}{R_{me}^2}$$

$$\Rightarrow \left(\frac{R_s}{R_{se}}\right)^2 = \left(\frac{R_m}{R_{me}}\right)^2$$

$$\Rightarrow \frac{R_s}{R_{se}} = \frac{R_m}{R_{me}}$$

$$\Rightarrow \frac{R_s}{R_m} = \frac{R_{se}}{R_{me}}$$

(Here, radius of sun and moon represents their sizes respectively)



- Q. 30** If the unit of force is 100 N, unit of length is 10 m and unit of time is 100 s, what is the unit of mass in this system of units?

Thinking Process

First write dimension of each quantity and then relate them.

Ans. Dimension of force $F = [\text{MLT}^{-2}] = 100\text{ N}$... (i)

Length (L) = $[L] = 10\text{ m}$... (ii)

Time (t) = $[T] = 100\text{ s}$... (iii)

Substituting values of L and T from Eqs. (ii) and (iii) in Eq. (i), we get

$$M \times 10 \times (100)^{-2} = 100$$

$$\Rightarrow \frac{M \times 10}{100 \times 100} = 100$$

$$\Rightarrow M = 100 \times 1000 \text{ kg}$$

$$M = 10^5 \text{ kg}$$

Q. 31 Give an example of

- (a) a physical quantity which has a unit but no dimensions
- (b) a physical quantity which has neither unit nor dimensions
- (c) a constant which has a unit
- (d) a constant which has no unit

Ans. (a) Plane angle $\theta = \frac{L}{r}$ radian

its unit is radian but has no dimensions

(b) Strain = $\frac{\Delta L}{L} = \frac{\text{Change in length}}{\text{length}}$

It has neither unit nor dimensions

(c) Gravitational constant (G) = $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$

(d) Reynold's number is a constant which has no unit.

Q. 32 Calculate the length of the arc of a circle of radius 31.0 cm which subtends an angle of $\frac{\pi}{6}$ at the centre.

Ans. We know that angle $\theta = \frac{l}{r}$ radian

Given,
$$\theta = \frac{\pi}{6} = \frac{l}{31} \text{ cm}$$

Hence,
$$\text{length} = l = 31 \times \frac{\pi}{6} \text{ cm} = \frac{31 \times 3.14}{6} \text{ cm} = 16.22 \text{ cm}$$

Rounding off to three significant figures it would be 16.2 cm.

Q. 33 Calculate the solid angle subtended by the periphery of an area of 1 cm^2 at a point situated symmetrically at a distance of 5 cm from the area.

Ans. We know that solid angle $\Omega = \frac{\text{Area}}{(\text{Distance})^2}$

$$= \frac{1 \text{ cm}^2}{(5 \text{ cm})^2} = \frac{1}{25} = 4 \times 10^{-2} \text{ steradian}$$

(\because Area = 1 cm^2 , distance = 5 cm)

Note We should not confuse, solid angle with plane angle $\theta = \frac{l}{r}$ radian.

Q. 34 The displacement of a progressive wave is represented by $y = A \sin(\omega t - kx)$, where x is distance and t is time. Write the dimensional formula of (i) ω and (ii) k .

Thinking Process

In solving these type of questions, we should apply principle of homogeneity of dimensions.

Ans. Now, by the principle of homogeneity, i.e., dimensions of LHS and RHS should be equal, hence

$$[\text{LHS}] = [\text{RHS}]$$

$$\Rightarrow [L] = [A] = L$$

As $\omega t - kx$ should be dimensionless, $[\omega t] = [kx] = 1$

$$\Rightarrow [\omega]T = [k]L = 1$$

$$\Rightarrow [\omega] = T^{-1} \text{ and } [k] = L^{-1}$$

Q. 35 Time for 20 oscillations of a pendulum is measured as $t_1 = 39.6$ s; $t_2 = 39.9$ s and $t_3 = 39.5$ s. What is the precision in the measurements? What is the accuracy of the measurement?

Thinking Process

We will apply formula for mean value, absolute error as well as mean absolute error.

Ans. Given, $t_1 = 39.6$ s, $t_2 = 39.9$ s and $t_3 = 39.5$ s
 Least count of measuring instrument = 0.1 s
 (As measurements have only one decimal place)
 Precision in the measurement = Least count of the measuring instrument = 0.1 s
 Mean value of time for 20 oscillations is given by

$$t = \frac{t_1 + t_2 + t_3}{3}$$

$$= \frac{39.6 + 39.9 + 39.5}{3} = 39.7 \text{ s}$$

Absolute errors in the measurements

$$\Delta t_1 = t - t_1 = 39.7 - 39.6 = 0.1 \text{ s}$$

$$\Delta t_2 = t - t_2 = 39.7 - 39.9 = -0.2 \text{ s}$$

$$\Delta t_3 = t - t_3 = 39.7 - 39.5 = 0.2 \text{ s}$$

$$\text{Mean absolute error} = \frac{|\Delta t_1| + |\Delta t_2| + |\Delta t_3|}{3}$$

$$= \frac{0.1 + 0.2 + 0.2}{3}$$

$$= \frac{0.5}{3} = 0.17 \approx 0.2 \quad (\text{rounding off upto one decimal place})$$

\therefore Accuracy of measurement = ± 0.2 s

Long Answer Type Questions

Q. 36 A new system of units is proposed in which unit of mass is α kg, unit of length β m and unit of time γ s. How much will 5J measure in this new system?

Thinking Process

For solving this question, we will apply the formula for a system of unit $u, nu = \text{constant}$.

Ans. We know that dimension of energy = $[ML^2T^{-2}]$

Let M_1, L_1, T_1 and M_2, L_2, T_2 are units of mass, length and time in given two systems.

\therefore $M_1 = 1 \text{ kg}, L_1 = 1 \text{ m}, T_1 = 1 \text{ s}$

$M_2 = \alpha \text{ kg}, L_2 = \beta \text{ m}, T_2 = \gamma \text{ s}$

The magnitude of a physical quantity remains the same, whatever be the system of units of its measurement i.e.,

$n_1 u_1 = n_2 u_2$

\Rightarrow
$$n_2 = n_1 \frac{u_1}{u_2} = n_1 \frac{[M_1 L_1^2 T_1^{-2}]}{[M_2 L_2^2 T_2^{-2}]} = 5 \left[\frac{M_1}{M_2} \right] \times \left[\frac{L_1}{L_2} \right]^2 \times \left[\frac{T_1}{T_2} \right]^{-2}$$

$$= 5 \left[\frac{1}{\alpha} \text{ kg} \right] \times \left[\frac{1}{\beta} \text{ m} \right]^2 \times \left[\frac{1}{\gamma} \text{ s} \right]^{-2}$$

$$= 5 \times \frac{1}{\alpha} \times \frac{1}{\beta^2} \times \frac{1}{\gamma^{-2}}$$

$$n_2 = \frac{5\gamma^2}{\alpha\beta^2}$$

Thus, new unit of energy will be $\frac{\gamma^2}{\alpha\beta^2}$.

Q. 37 The volume of a liquid flowing out per second of a pipe of length l and radius r is written by a student as $V = \frac{\pi pr^4}{8 \eta l}$ where p is the pressure

difference between the two ends of the pipe and η is coefficient of viscosity of the liquid having dimensional formula $[ML^{-1}T^{-1}]$. Check whether the equation is dimensionally correct.

Thinking Process

If dimensions of LHS of an equation is equal to dimensions of RHS, then equation is said to be dimensionally correct.

Ans. The volume of a liquid flowing out per second of a pipe is given by $V = \frac{\pi pr^4}{8 \eta l}$

Dimension of $V = \frac{\text{Dimension of volume}}{\text{Dimension of time}} = \frac{[L^3]}{[T]} = [L^3T^{-1}]$

($\therefore V$ is the volume of liquid flowing out per second)

Dimension of $p = [ML^{-1}T^{-2}]$

Dimension of $\eta = [ML^{-1}T^{-1}]$

Dimension of $l = [L]$

Dimension of $r = [L]$

$$\text{Dimensions of LHS, } [V] = \frac{[L^3]}{[T]} = [L^3T^{-1}]$$

$$\text{Dimensions of RHS, } \frac{[ML^{-1}T^{-2}] \times [L^4]}{[ML^{-1}T^{-1}] \times [L]} = [L^3T^{-1}]$$

As dimensions of LHS is equal to the dimensions of RHS.
Therefore, equation is correct dimensionally.

- Q. 38** A physical quantity X is related to four measurable quantities a, b, c and d as follows $X = a^2b^3c^{5/2}d^{-2}$. The percentage error in the measurement of a, b, c and d are 1%, 2%, 3% and 4%, respectively. What is the percentage error in quantity X ? If the value of X calculated on the basis of the above relation is 2.763, to what value should you round off the result.

Thinking Process

We will apply the formula for percentage error in quantity x , as $\frac{\Delta x}{x} \times 100$.

- Ans.** Given, physical quantity is $X = a^2b^3c^{5/2}d^{-2}$

Maximum percentage error in X is

$$\begin{aligned} \frac{\Delta X}{X} \times 100 &= \pm \left[2 \left(\frac{\Delta a}{a} \times 100 \right) + 3 \left(\frac{\Delta b}{b} \times 100 \right) + \frac{5}{2} \left(\frac{\Delta c}{c} \times 100 \right) + 2 \left(\frac{\Delta d}{d} \times 100 \right) \right] \\ &= \pm \left[2(1) + 3(2) + \frac{5}{2}(3) + 2(4) \right] \% \\ &= \pm \left[2 + 6 + \frac{15}{2} + 8 \right] = \pm 23.5\% \end{aligned}$$

\therefore Percentage error in quantity $X = \pm 23.5\%$

Mean absolute error in $X = \pm 0.235 = \pm 0.24$ (rounding-off upto two significant digits)

The calculated value of x should be round-off upto two significant digits.

$\therefore X = 2.8$

- Q. 39** In the expression $P = El^2m^{-5}G^{-2}$, E, m, l and G denote energy, mass, angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity.

Thinking Process

A dimensionless quantity will have dimensional formula as $[M^0 L^0 T^0]$.

- Ans.** Given, expression is $P = El^2m^{-5}G^{-2}$

where E is energy $[E] = [ML^2T^{-2}]$

m is mass $[m] = [M]$

L is angular momentum $[L] = [ML^2T^{-1}]$

G is gravitational constant $[G] = [M^{-1}L^3T^{-2}]$

Substituting dimensions of each term in the given expression,

$$\begin{aligned} [P] &= [ML^2T^{-2}] \times [ML^2T^{-1}]^2 \times [M]^{-5} \times [M^{-1}L^3T^{-2}]^{-2} \\ &= [M^{1+2-5+2} L^{2+4-6} T^{-2-2+4}] = [M^0 L^0 T^0] \end{aligned}$$

Therefore, P is a dimensionless quantity.

Q. 40 If velocity of light c , Planck's constant h and gravitational constant G are taken as fundamental quantities, then express mass, length and time in terms of dimensions of these quantities.

Thinking Process

In this problem, we have to apply principle of homogeneity of dimensions that is LHS and RHS of an equation will have same dimensions.

Ans. We know that, dimensions of $(h) = [ML^2T^{-1}]$
 Dimensions of $(c) = [LT^{-1}]$
 Dimensions of gravitational constant $(G) = [M^{-1}L^3T^{-2}]$

(i) Let $m \propto c^x h^y G^z$
 $\Rightarrow m = kc^x h^y G^z$... (i)

where, k is a dimensionless constant of proportionality.

Substituting dimensions of each term in Eq. (i), we get

$$[ML^0T^0] = [LT^{-1}]^x \times [ML^2T^{-1}]^y \times [M^{-1}L^3T^{-2}]^z$$

$$= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}]$$

Comparing powers of same terms on both sides, we get

$$y - z = 1 \quad \dots (ii)$$

$$x + 2y + 3z = 0 \quad \dots (iii)$$

$$-x - y - 2z = 0 \quad \dots (iv)$$

Adding Eqs. (ii), (iii) and (iv), we get

$$2y = 1 \Rightarrow y = \frac{1}{2}$$

Substituting value of y in Eq. (ii), we get

$$z = -\frac{1}{2}$$

From Eq. (iv)

$$x = -y - 2z$$

Substituting values of y and z , we get

$$x = -\frac{1}{2} - 2\left(-\frac{1}{2}\right) = \frac{1}{2}$$

Putting values of x , y and z in Eq. (i), we get

$$m = kc^{1/2} h^{1/2} G^{-1/2}$$

$\Rightarrow m = k \sqrt{\frac{ch}{G}}$

(ii) Let $L \propto c^x h^y G^z$
 $\Rightarrow L = kc^x h^y G^z$... (v)

where, k is a dimensionless constant.

Substituting dimensions of each term in Eq. (v), we get

$$[M^0 L T^0] = [LT^{-1}]^x \times [ML^2T^{-1}]^y \times [M^{-1}L^3T^{-2}]^z$$

$$= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}]$$

On comparing powers of same terms, we get

$$y - z = 0 \quad \dots (vi)$$

$$x + 2y + 3z = 1 \quad \dots (vii)$$

$$-x - y - 2z = 0 \quad \dots (viii)$$

Adding Eqs. (vi), (vii) and (viii), we get

$$\begin{aligned} 2y &= 1 \\ \Rightarrow y &= \frac{1}{2} \end{aligned}$$

Substituting value of y in Eq. (vi), we get

$$z = \frac{1}{2}$$

From Eq. (viii),

$$x = -y - 2z$$

Substituting values of y and z , we get

$$x = -\frac{1}{2} - 2\left(\frac{1}{2}\right) = -\frac{3}{2}$$

Putting values of x , y and z in Eq. (v), we get

$$L = kc^{-3/2}h^{1/2}G^{1/2}$$

$$L = k\sqrt{\frac{hG}{c^3}}$$

(iii) Let $T \propto c^x h^y G^z$

$$\Rightarrow T = kc^x h^y G^z \quad \dots \text{(ix)}$$

where, k is a dimensionless constant.

Substituting dimensions of each term in Eq. (ix), we get

$$\begin{aligned} [M^0 L^0 T] &= [LT^{-1}]^x \times [ML^2 T^{-1}]^y \times [M^{-1} L^3 T^{-2}]^z \\ &= [M^{y-z} L^{x+2y+3z} T^{-x-y-2z}] \end{aligned}$$

On comparing powers of same terms, we get

$$y - z = 0 \quad \dots \text{(x)}$$

$$x + 2y + 3z = 0 \quad \dots \text{(xi)}$$

$$-x - y - 2z = 1 \quad \dots \text{(xii)}$$

Adding Eqs. (x), (xi) and (xii), we get

$$\begin{aligned} 2y &= 1 \\ \Rightarrow y &= \frac{1}{2} \end{aligned}$$

Substituting value of y in Eq. (x), we get

$$z = y = \frac{1}{2}$$

From Eq. (xii),

$$x = -y - 2z - 1$$

Substituting values of y and z , we get

$$x = -\frac{1}{2} - 2\left(\frac{1}{2}\right) - 1 = -\frac{5}{2}$$

Putting values of x , y and z in Eq. (ix), we get

$$T = kc^{-5/2}h^{1/2}G^{1/2}$$

$$T = k\sqrt{\frac{hG}{c^5}}$$

- Q. 41** An artificial satellite is revolving around a planet of mass M and radius R , in a circular orbit of radius r . From Kepler's third law about the period of a satellite around a common central body, square of the period of revolution T is proportional to the cube of the radius of the orbit r . Show using dimensional analysis, that $T = \frac{k}{R} \sqrt{\frac{r^3}{g}}$, where k is a dimensionless constant and g is acceleration due to gravity.

Thinking Process

In this problem, we have to apply Kepler's third law, $T^2 \propto a^3$ i.e., square of time period (T^2) of a satellite revolving around a planet, is proportional to cube of the radius of the orbit (a^3).

Ans. By Kepler's third law, $T^2 \propto r^3 \Rightarrow T \propto r^{3/2}$

We know that T is a function of R and g .

Let $T \propto r^{3/2} R^a g^b$

$\Rightarrow T = k r^{3/2} R^a g^b \quad \dots(i)$

where, k is a dimensionless constant of proportionality.

Substituting the dimensions of each term in Eq. (i), we get

$$[M^0 L^0 T] = k [L]^{3/2} [L]^a [L T^{-2}]^b$$

$$= k [L^{a+b+3/2} T^{-2b}]$$

On comparing the powers of same terms, we get

$$a + b + 3/2 = 0 \quad \dots(ii)$$

$$-2b = 1 \Rightarrow b = -1/2 \quad \dots(iii)$$

From Eq. (ii), we get

$$a - 1/2 + 3/2 = 0 \Rightarrow a = -1$$

Substituting the values of a and b in Eq. (i), we get

$$T = k r^{3/2} R^{-1} g^{-1/2}$$

$\Rightarrow T = \frac{k}{R} \sqrt{\frac{r^3}{g}}$

Note When we are applying formulae, we should be careful about r (radius of orbit) and R (radius of planet).

- Q. 42** In an experiment to estimate the size of a molecule of oleic acid 1 mL of oleic acid is dissolved in 19 mL of alcohol. Then 1 mL of this solution is diluted to 20 mL by adding alcohol. Now, 1 drop of this diluted solution is placed on water in a shallow trough. The solution spreads over the surface of water forming one molecule thick layer. Now, lycopodium powder is sprinkled evenly over the film and its diameter is measured. Knowing the volume of the drop and area of the film we can calculate the thickness of the film which will give us the size of oleic acid molecule.

Read the passage carefully and answer the following questions

- Why do we dissolve oleic acid in alcohol?
- What is the role of lycopodium powder?
- What would be the volume of oleic acid in each mL of solution prepared?
- How will you calculate the volume of n drops of this solution of oleic acid?
- What will be the volume of oleic acid in one drop of this solution?

Ans. (a) Oleic acid does not dissolve in water hence, it is dissolved in alcohol.

(b) Lycopodium powder spreads over the entire surface of water when it is sprinkled evenly. When a drop of prepared solution is dropped on water, oleic acid does not dissolve in water. Instead it spreads on the water surface pushing the lycopodium powder away to clear a circular area where the drop falls. We can therefore, measure the area over which oleic acid spreads.

(c) In each mL of solution prepared volume of oleic acid = $\frac{1}{20}$ mL \times $\frac{1}{20}$ = $\frac{1}{400}$ mL

(d) Volume of n drops of this solution of oleic acid can be calculated by means of a burette and measuring cylinder and measuring the number of drops.

(e) If n drops of the solution make 1 mL, the volume of oleic acid in one drop will be $\frac{1}{(400)n}$ mL.

Q. 43 (a) How many astronomical units (AU) make 1 parsec?

(b) Consider a sunlike star at a distance of 2 parsecs. When it is seen through a telescope with 100 magnification, what should be the angular size of the star? Sun appears to be $(1/2)^\circ$ from the earth. Due to atmospheric fluctuations, eye cannot resolve objects smaller than 1 arc minute.

(c) Mars has approximately half of the earth's diameter. When it is closest to the earth it is at about $1/2$ AU from the earth. Calculate what size it will appear when seen through the same telescope.

Ans. (a) By definition,

1 parsec = Distance at which 1 AU long arc subtends an angle of 1 s.

$$\therefore 1 \text{ parsec} = \left(\frac{1 \text{ AU}}{1 \text{ arc sec}} \right)$$

$$1 \text{ deg} = 3600 \text{ arc sec}$$

$$\therefore 1 \text{ parsec} = \frac{\pi}{3600 \times 180} \text{ rad}$$

$$\therefore 1 \text{ parsec} = \frac{3600 \times 180}{\pi} \text{ AU} = 206265 \text{ AU} \approx 2 \times 10^5 \text{ AU}$$

(b) Sun's diameter is $\left(\frac{1}{2}\right)^\circ$ at 1 AU.

Therefore, at 1 parsec, star is $\frac{1/2}{2 \times 10^5}$ degree in diameter = 15×10^{-5} arc min.

With 100 magnification, it should look 15×10^{-3} arcmin. However, due to atmospheric fluctuations, it will still look of about 1 arcmin. It cannot be magnified using telescope.

(c) Given that $\frac{D_{\text{mars}}}{D_{\text{earth}}} = \frac{1}{2}$... (i)

where D represents diameter.

From answer 25(e)

we know that,

$$\frac{D_{\text{earth}}}{D_{\text{sun}}} = \frac{1}{100}$$

$\therefore \frac{D_{\text{mars}}}{D_{\text{sun}}} = \frac{1}{2} \times \frac{1}{100}$ [from Eq. (i)]

At 1 AU sun's diameter = $\left(\frac{1}{2}\right)^\circ$

\therefore mar's diameter = $\frac{1}{2} \times \frac{1}{200} = \frac{1}{400}$

At $\frac{1}{2}$ AU, mar's diameter = $\frac{1}{400} \times 2^\circ = \left(\frac{1}{200}\right)^\circ$

With 100 magnification, Mar's diameter = $\frac{1}{200} \times 100^\circ = \left(\frac{1}{2}\right)^\circ = 30'$

This is larger than resolution limit due to atmospheric fluctuations. Hence, it looks magnified.

Q. 44 Einstein's mass-energy relation emerging out of his famous theory of relativity relates mass (m) to energy (E) as $E = mc^2$, where c is speed of light in vacuum. At the nuclear level, the magnitudes of energy are very small. The energy at nuclear level is usually measured in MeV, where $1\text{MeV} = 1.6 \times 10^{-13}\text{J}$; the masses are measured in unified atomic mass unit (u) where, $1u = 1.67 \times 10^{-27}\text{ kg}$.

(a) Show that the energy equivalent of $1u$ is 931.5 MeV.

(b) A student writes the relation as $1u = 931.5\text{ MeV}$. The teacher points out that the relation is dimensionally incorrect. Write the correct relation.

Thinking Process

In this problem, we have to apply Einstein's mass-energy relation. $E = mc^2$, to calculate the energy equivalent of the given mass.

Ans. (a) We know that

$$1\text{ amu} = 1u = 1.67 \times 10^{-27}\text{kg}$$

Applying $E = mc^2$

$$\text{Energy} = E = (1.67 \times 10^{-27})(3 \times 10^8)^2 \text{ J}$$

$$= 1.67 \times 9 \times 10^{-11}\text{J}$$

$$E = \frac{1.67 \times 9 \times 10^{-11}}{1.6 \times 10^{-13}} \text{MeV} = 939.4 \text{MeV} \approx 931.5 \text{MeV}$$

(b) The dimensionally correct relation is

$$1\text{ amu} \times c^2 = 1u \times c^2 = 931.5 \text{MeV}$$