

# 4

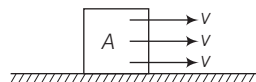
## Laws of Motion

### Multiple Choice Questions (MCQS)

- Q. 1** A ball is travelling with uniform translatory motion. This means that
- (a) it is at rest
  - (b) the path can be a straight line or circular and the ball travels with uniform speed
  - (c) all parts of the ball have the same velocity (magnitude and direction) and the velocity is constant
  - (d) the centre of the ball moves with constant velocity and the ball spins about its centre uniformly

**Ans. (c)** In a uniform translatory motion, all parts of the ball have the same velocity in magnitude and direction and this velocity is constant.

The situation is shown in adjacent diagram where a body A is in uniform translatory motion.



- Q. 2** A metre scale is moving with uniform velocity. This implies
- (a) the force acting on the scale is zero, but a torque about the centre of mass can act on the scale
  - (b) the force acting on the scale is zero and the torque acting about centre of mass of the scale is also zero
  - (c) the total force acting on it need not be zero but the torque on it is zero
  - (d) neither the force nor the torque need to be zero

**Ans. (b)** To solve this question we have to apply Newton's second law of motion, in terms of force and change in momentum.

We know that 
$$F = \frac{dp}{dt}$$

given that meter scale is moving with uniform velocity, hence,  $dp = 0$

$$\text{Force} = F = 0.$$

As all part of the scale is moving with uniform velocity and total force is zero, hence, torque will also be zero.

**Q. 3** A cricket ball of mass 150 g has an initial velocity  $\mathbf{u} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\text{ms}^{-1}$  and a final velocity  $\mathbf{v} = -(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\text{ms}^{-1}$ , after being hit. The change in momentum (final momentum–initial momentum) is (in  $\text{kgms}^{-1}$ )

- (a) zero (b)  $-(0.45\hat{\mathbf{i}} + 0.6\hat{\mathbf{j}})$   
 (c)  $-(0.9\hat{\mathbf{i}} + 1.2\hat{\mathbf{j}})$  (d)  $-5(\hat{\mathbf{i}} + \hat{\mathbf{j}})\hat{\mathbf{i}}$

**Ans. (c)** Given,  $\mathbf{u} = (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\text{m/s}$   
 and  $\mathbf{v} = -(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})\text{m/s}$

Mass of the ball = 150 g = 0.15 kg

$\Delta\mathbf{p}$  = Change in momentum

$$\begin{aligned} &= \text{Final momentum} - \text{Initial momentum} \\ &= m\mathbf{v} - m\mathbf{u} \\ &= m(\mathbf{v} - \mathbf{u}) = (0.15) [-(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) - (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})] \\ &= (0.15) [-6\hat{\mathbf{i}} - 8\hat{\mathbf{j}}] \\ &= -[0.15 \times 6\hat{\mathbf{i}} + 0.15 \times 8\hat{\mathbf{j}}] \\ &= -[0.9\hat{\mathbf{i}} + 1.20\hat{\mathbf{j}}] \end{aligned}$$

Hence,  $\Delta\mathbf{p} = -[0.9\hat{\mathbf{i}} + 1.2\hat{\mathbf{j}}]$

**Q. 4** In the previous problem (3), the magnitude of the momentum transferred during the hit is

- (a) zero (b)  $0.75 \text{ kg}\cdot\text{m s}^{-1}$  (c)  $1.5 \text{ kg}\cdot\text{m s}^{-1}$  (d)  $14 \text{ kg}\cdot\text{m s}^{-1}$

**Ans. (c)** By previous solution  $\Delta\mathbf{p} = -(0.9\hat{\mathbf{i}} + 1.2\hat{\mathbf{j}})$

$$\begin{aligned} \text{Magnitude} &= |\Delta\mathbf{p}| = \sqrt{(0.9)^2 + (1.2)^2} \\ &= \sqrt{0.81 + 1.44} = 1.5 \text{ kg}\cdot\text{m s}^{-1}. \end{aligned}$$

**Q. 5** Conservation of momentum in a collision between particles can be understood from

- (a) conservation of energy (b) Newton's first law only  
 (c) Newton's second law only (d) both Newton's second and third law

**Thinking Process**

*For conservation of momentum we have to see whether net external force is acting on a system or not.*

**Ans. (d)** We know that for a system  $F_{\text{ext}} = \frac{dp}{dt}$  (from Newton's second law)

If  $F_{\text{ext}} = 0, dp = 0 \Rightarrow p = \text{constant}$

Hence, momentum of a system will remain conserve if external force on the system is zero.

In case of collision' between particles equal and opposite forces will act on individual particles by Newtons third law,

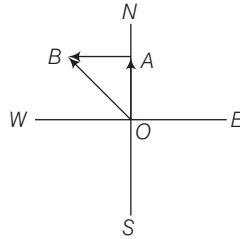
Hence total force on the system will be zero.

**Note** *We should not confuse with system and individual particles. As total force on the system of both particles is zero but force acts on individual particles.*

**Q. 6** A hockey player is moving northward and suddenly turns westward with the same speed to avoid an opponent. The force that acts on the player is

- frictional force along westward
- muscle force along southward
- frictional force along south-West
- muscle force along south-West

**Ans. (c)** Consider the adjacent diagram



Let

$$\begin{aligned} \mathbf{OA} &= \mathbf{p}_1 \\ &= \text{Initial momentum of player northward} \end{aligned}$$

$$\mathbf{AB} = \mathbf{p}_2 = \text{Final momentum of player towards west.}$$

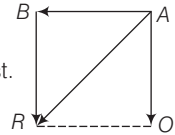
Clearly

$$\mathbf{OB} = \mathbf{OA} + \mathbf{AB}$$

Change in momentum =  $\mathbf{p}_2 - \mathbf{p}_1$

$$= \mathbf{AB} - \mathbf{OA} = \mathbf{AB} + (-\mathbf{OA})$$

$$= \text{Clearly resultant } \mathbf{AR} \text{ will be along south-west.}$$



**Q. 7** A body of mass 2kg travels according to the law  $x(t) = pt + qt^2 + rt^3$  where,  $q = 4\text{ms}^{-2}$ ,  $p = 3\text{ms}^{-1}$  and  $r = 5\text{ms}^{-3}$ . The force acting on the body at  $t = 2\text{s}$  is

- 136 N
- 134 N
- 158 N
- 68 N

### Thinking Process

We have to apply differentiations to calculate acceleration and then Newton's second law will be applied.

**Ans. (a)** Given, mass = 2 kg

$$x(t) = pt + qt^2 + rt^3$$

$$v = \frac{dx}{dt} = p + 2qt + 3rt^2$$

$$a = \frac{dv}{dt} = 0 + 2q + 6rt$$

$$\text{at } t = 2\text{s}; a = 2q + 6 \times 2 \times r$$

$$= 2q + 12r$$

$$= 2 \times 4 + 12 \times 5$$

$$= 8 + 60 = 68\text{m/s}$$

$$\text{Force} = F = ma$$

$$= 2 \times 68 = 136\text{N}$$

**Q. 8** A body with mass 5 kg is acted upon by a force  $\mathbf{F} = (-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$  N. If its initial velocity at  $t = 0$  is  $\mathbf{v} = (6\hat{\mathbf{i}} - 12\hat{\mathbf{j}})\text{ms}^{-1}$ , the time at which it will just have a velocity along the Y-axis is

- (a) never                      (b) 10 s                      (c) 2 s                      (d) 15 s

**Ans. (b)** Given, mass =  $m = 5$  kg

Acting force =  $\mathbf{F} = (-3\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$  N

Initial velocity at  $t = 0$ ,  $\mathbf{u} = (6\hat{\mathbf{i}} - 12\hat{\mathbf{j}})$  m / s

Retardation,  $\hat{\mathbf{a}} = \frac{\mathbf{F}}{m} = \left(-\frac{3\hat{\mathbf{i}}}{5} + \frac{4\hat{\mathbf{j}}}{5}\right) \text{m/s}^2$

As final velocity is along Y-axis only, its x-component must be zero.

From  $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ , for X-component only,  $0 = 6\hat{\mathbf{i}} - \frac{3\hat{\mathbf{i}}}{5}t$

$$t = \frac{5 \times 6}{3} = 10 \text{ s}$$

**Q. 9** A car of mass  $m$  starts from rest and acquires a velocity along east,  $\mathbf{v} = v\hat{\mathbf{i}}$  ( $v > 0$ ) in two seconds. Assuming the car moves with uniform acceleration, the force exerted on the car is

- (a)  $\frac{mv}{2}$  eastward and is exerted by the car engine  
 (b)  $\frac{mv}{2}$  eastward and is due to the friction on the tyres exerted by the road  
 (c) more than  $\frac{mv}{2}$  eastward exerted due to the engine and overcomes the friction of the road  
 (d)  $\frac{mv}{2}$  exerted by the engine

**Ans. (b)** Given, mass of the car =  $m$

As car starts from rest,  $u = 0$

Velocity acquired along east =  $v\hat{\mathbf{i}}$

Duration =  $t = 2$ s.

We know that

$$v = u + at$$

⇒

$$v\hat{\mathbf{i}} = 0 + a \times 2$$

⇒

$$\mathbf{a} = \frac{v}{2}\hat{\mathbf{i}}$$

Force,

$$\mathbf{F} = m\mathbf{a} = \frac{mv}{2}\hat{\mathbf{i}}$$

Hence, force acting on the car is  $\frac{mv}{2}$  towards east. As external force on the system is

only friction hence, the force  $\frac{mv}{2}$  is by friction. Hence, force by engine is internal force.

## Multiple Choice Questions (More Than One Options)

**Q. 10** The motion of a particle of mass  $m$  is given by  $x = 0$  for  $t < 0$ s,  $x(t) = A \sin 4\pi t$  for  $0 < t < (1/4)$ s ( $A > 0$ ), and  $x = 0$  for  $t > (1/4)$  s. Which of the following statements is true?

- (a) The force at  $t = (1/8)$ s on the particle is  $-16\pi^2 A m$
- (b) The particle is acted upon by an impulse of magnitude  $4\pi^2 A m$  at  $t = 0$  s and  $t = (1/4)$  s
- (c) The particle is not acted upon by any force
- (d) The particle is not acted upon by a constant force
- (e) There is no impulse acting on the particle

### Thinking Process

Here, position of the particle is given for different time intervals. Hence, we have to find velocity and acceleration corresponding to the intervals.

**Ans. (a, b, d)**

Given,

$$x = 0 \text{ for } t < 0 \text{ s.}$$

$$x(t) = A \sin 4\pi t; \text{ for } 0 < t < \frac{1}{4} \text{ s}$$

$$x = 0; \text{ for } t > \frac{1}{4} \text{ s}$$

For,  $0 < t < \frac{1}{4}$  s

$$v(t) = \frac{dx}{dt} = 4\pi A \cos 4\pi t$$

$$a(t) = \text{acceleration}$$

$$= \frac{dv(t)}{dt} = -16\pi^2 A \sin 4\pi t$$

$$\text{At } t = \frac{1}{8} \text{ s, } a(t) = -16\pi^2 A \sin 4\pi \times \frac{1}{8} = -16\pi^2 A$$

$$F = ma(t) = -16\pi^2 A \times m = -16\pi^2 mA$$

$$\text{Impulse} = \text{Change in linear momentum}$$

$$= F \times t = (-16\pi^2 Am) \times \frac{1}{4}$$

$$= -4\pi^2 Am$$

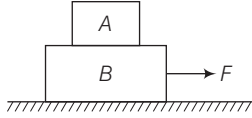
The impulse (Change in linear momentum)

$$\text{at } t = 0 \text{ is same as, } t = \frac{1}{4} \text{ s.}$$

Clearly, force depends upon  $A$  which is not constant. Hence, force is also not constant.

**Note** We have to keep in mind that the force is varying for different time intervals. Hence, we should apply differential formulae for each interval separately.

**Q. 11** In figure the coefficient of friction between the floor and the body  $B$  is 0.1. The coefficient of friction between the bodies  $B$  and  $A$  is 0.2. A force  $F$  is applied as shown on  $B$ . The mass of  $A$  is  $m/2$  and of  $B$  is  $m$ . Which of the following statements are true?



- (a) The bodies will move together if  $F = 0.25\text{ mg}$
- (b) The body  $A$  will slip with respect to  $B$  if  $F = 0.5\text{ mg}$
- (c) The bodies will move together if  $F = 0.5\text{ mg}$
- (d) The bodies will be at rest if  $F = 0.1\text{ mg}$
- (e) The maximum value of  $F$  for which the two bodies will move together is  $0.45\text{ mg}$

**Thinking Process**

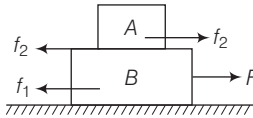
*In this problem we have to find frictional forces on each surface and accordingly we will decide maximum force.*

**Ans. (a, b, d, e)**

Consider the adjacent diagram. Frictional force on  $B(f_1)$  and frictional force on  $A(f_2)$  will be as shown.

$$\text{Let } A \text{ and } B \text{ are moving together } a_{\text{common}} = \frac{F - f_1}{m_A + m_B} = \frac{F - f_1}{(m/2) + m} = \frac{2(F - f_1)}{3m}$$

$$\begin{aligned} \text{Pseudo force on } A &= (m_A) \times a_{\text{common}} \\ &= m_A \times \frac{2(F - f_1)}{3m} = \frac{m}{2} \times \frac{2(F - f_1)}{3m} = \frac{(F - f_1)}{3} \end{aligned}$$



The force ( $F$ ) will be maximum when

Pseudo force on  $A =$  Frictional force on  $A$

$$\begin{aligned} \Rightarrow \frac{F_{\text{max}} - f_1}{3} &= \mu m_A g \\ &= 0.2 \times \frac{m}{2} \times g = 0.1\text{ mg} \end{aligned}$$

$$\begin{aligned} \Rightarrow F_{\text{max}} &= 0.3\text{ mg} + f_1 \\ &= 0.3\text{ mg} + (0.1) \frac{3}{2}\text{ mg} = 0.45\text{ mg} \end{aligned}$$

$\Rightarrow$  Hence, maximum force upto which bodies will move together is  $F_{\text{max}} = 0.45\text{ mg}$

- (a) Hence, for  $F = 0.25\text{ mg} < F_{\text{max}}$  bodies will move together.
- (b) For  $F = 0.5\text{ mg} > F_{\text{max}}$ , body  $A$  will slip with respect to  $B$ .
- (c) For  $F = 0.5\text{ mg} > F_{\text{max}}$ , bodies slip.

$$(f_1)_{\text{max}} = \mu m_B g = (0.1) \times \frac{3}{2} m \times g = 0.15\text{ mg}$$

$$(f_2)_{\text{max}} = \mu m_A g = (0.2) \left(\frac{m}{2}\right) (g) = 0.1\text{ mg}$$

Hence, minimum force required for movement of the system (A + B)

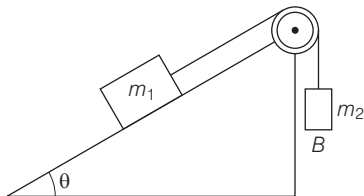
$$F_{\min} = (f_1)_{\max} + (f_2)_{\max} \\ = 0.15 mg + 0.1 mg = 0.25 mg$$

(d) Given, force  $F = 0.1 mg < F_{\min}$ .

Hence, the bodies will be at rest.

(e) Maximum force for combined movement  $F_{\max} = 0.45 mg$ .

**Q. 12** Mass  $m_1$  moves on a slope making an angle  $\theta$  with the horizontal and is attached to mass  $m_2$  by a string passing over a frictionless pulley as shown in figure. The coefficient of friction between  $m_1$  and the sloping surface is  $\mu$ . Which of the following statements are true?



- (a) If  $m_2 > m_1 \sin \theta$ , the body will move up the plane  
 (b) If  $m_2 > m_1 (\sin \theta + \mu \cos \theta)$ , the body will move up the plane  
 (c) If  $m_2 < m_1 (\sin \theta + \mu \cos \theta)$ , the body will move up the plane  
 (d) If  $m_2 < m_1 (\sin \theta - \mu \cos \theta)$ , the body will move down the plane

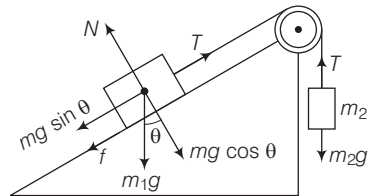
**Thinking Process**

*The friction force always have tendency to oppose the motion. Consider the adjacent diagram.*

**Ans. (b, d)**

Let  $m_1$  moves up the plane. Different forces involved are shown in the diagram.

- $N$  = Normal reaction  
 $f$  = Frictional force  
 $T$  = Tension in the string  
 $f = \mu N = \mu m_1 g \cos \theta$



For the system ( $m_1 + m_2$ ) to move up

$$m_2 g - (m_1 g \sin \theta + f) > 0 \\ \Rightarrow m_2 g - (m_1 g \sin \theta + \mu m_1 g \cos \theta) > 0 \\ \Rightarrow m_2 > m_1 (\sin \theta + \mu \cos \theta)$$

Hence, option (b) is correct.

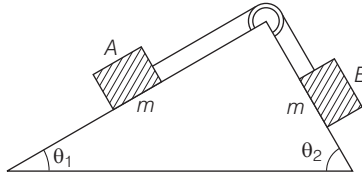
Let the body moves down the plane, in this case  $f$  acts up the plane.

Hence,

$$m_1 g \sin \theta - f > m_2 g \\ \Rightarrow m_1 g \sin \theta - \mu m_1 g \cos \theta > m_2 g \\ \Rightarrow m_1 (\sin \theta - \mu \cos \theta) > m_2 \\ \Rightarrow m_2 < m_1 (\sin \theta - \mu \cos \theta)$$

Hence, option (d) is correct.

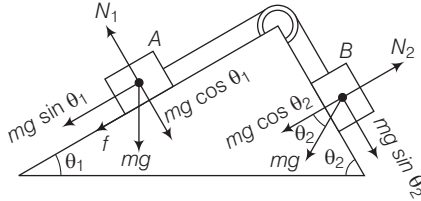
**Q. 13** In figure a body  $A$  of mass  $m$  slides on plane inclined at angle  $\theta_1$  to the horizontal and  $\mu$  is the coefficient of friction between  $A$  and the plane.  $A$  is connected by a light string passing over a frictionless pulley to another body  $B$ , also of mass  $m$ , sliding on a frictionless plane inclined at an angle  $\theta_2$  to the horizontal. Which of the following statements are true?



- (a)  $A$  will never move up the plane
- (b)  $A$  will just start moving up the plane when  $\mu = \frac{\sin \theta_2 - \sin \theta_1}{\cos \theta_1}$
- (c) For  $A$  to move up the plane,  $\theta_2$  must always be greater than  $\theta_1$
- (d)  $B$  will always slide down with constant speed

**Ans. (b, c)**

Let  $A$  moves up the plane frictional force on  $A$  will be downward as shown.



When  $A$  just starts moving up

$$mg \sin \theta_1 + f = mg \sin \theta_2$$

$$\Rightarrow mg \sin \theta_1 + \mu mg \cos \theta_1 = mg \sin \theta_2$$

$$\Rightarrow \mu = \frac{\sin \theta_2 - \sin \theta_1}{\cos \theta_1}$$

When  $A$  moves upwards

$$f = mg \sin \theta_2 - mg \sin \theta_1 > 0$$

$$\Rightarrow \sin \theta_2 > \sin \theta_1 \Rightarrow \theta_2 > \theta_1$$

**Q. 14** Two billiard balls  $A$  and  $B$ , each of mass  $50\text{g}$  and moving in opposite directions with speed of  $5\text{ m s}^{-1}$  each, collide and rebound with the same speed. If the collision lasts for  $10^{-3}\text{ s}$ , which of the following statements are true?

- (a) The impulse imparted to each ball is  $0.25\text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$  and the force on each ball is  $250\text{ N}$
- (b) The impulse imparted to each ball is  $0.25\text{ kg}\cdot\text{m}\cdot\text{s}^{-1}$  and the force exerted on each ball is  $25 \times 10^{-5}\text{ N}$
- (c) The impulse imparted to each ball is  $0.5\text{ N}\cdot\text{s}$
- (d) The impulse and the force on each ball are equal in magnitude and opposite in directions



**Ans. (c, d)**

Given,  $m_1 = m_2 = 50(\text{g}) = \frac{50}{1000} \text{ kg} = \frac{1}{20} \text{ kg}$

Initial velocity ( $u$ ) =  $u_1 = u_2 = 5 \text{ m/s}$

Final velocity ( $v$ ) =  $v_1 = v_2 = -5 \text{ m/s}$

Time duration of collision =  $10^{-3} \text{ s}$ .

Change in linear momentum =  $m(v - u)$

$$= \frac{1}{20} [-5 - 5] = -0.5 \text{ N-s.}$$

$$\text{Force} = \frac{\text{Impulse}}{\text{Time}} = \frac{\text{Change in momentum}}{10^{-3} \text{ s}}$$

$$= \frac{0.5}{10^{-3}} = 500 \text{ N}$$

Impulse and force are opposite in directions.

**Q. 15** A body of mass 10 kg is acted upon by two perpendicular forces, 6N and 8N. The resultant acceleration of the body is

(a)  $1 \text{ m s}^{-2}$  at an angle of  $\tan^{-1}\left(\frac{4}{3}\right)$  w.r.t. 6N force

(b)  $0.2 \text{ m s}^{-2}$  at an angle of  $\tan^{-1}\left(\frac{4}{3}\right)$  w.r.t. 6N force

(c)  $1 \text{ m s}^{-2}$  at an angle of  $\tan^{-1}\left(\frac{3}{4}\right)$  w.r.t. 8N force

(d)  $0.2 \text{ m s}^{-2}$  at an angle of  $\tan^{-1}\left(\frac{3}{4}\right)$  w.r.t. 8N force

**Thinking Process**

*In this problem, we have to use the concept of resultant of two vectors, when they are perpendicular.*

**Ans. (a, c)**

Consider the adjacent diagram

Given, mass =  $m = 10 \text{ kg}$ .

$$F_1 = 6\text{N}, F_2 = 8\text{N}$$

$$\text{Resultant force} = F = \sqrt{F_1^2 + F_2^2} = \sqrt{36 + 64}$$

$$= 10 \text{ N}$$

$$a = \frac{F}{m} = \frac{10}{10} = 1 \text{ m/s}^2; \text{ along } R.$$

Let  $\theta_1$  be angle between  $R$  and  $F_1$

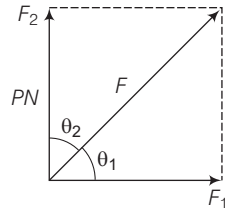
$$\tan \theta_1 = \frac{8}{6} = \frac{4}{3}$$

$$\theta_1 = \tan^{-1}(4/3) \text{ w.r.t. } F_1 = 6\text{N}$$

Let  $\theta_2$  be angle between  $F$  and  $F_2$

$$\tan \theta_2 = \frac{6}{8} = \frac{3}{4}$$

$$\theta_2 = \tan^{-1}\left(\frac{3}{4}\right) \text{ w.r.t. } F_2 = 8\text{N}$$



## Very Short Answer Type Questions

- Q. 16** A girl riding a bicycle along a straight road with a speed of  $5 \text{ ms}^{-1}$  throws a stone of mass  $0.5 \text{ kg}$  which has a speed of  $15 \text{ ms}^{-1}$  with respect to the ground along her direction of motion. The mass of the girl and bicycle is  $50 \text{ kg}$ . Does the speed of the bicycle change after the stone is thrown? What is the change in speed, if so?

### 💡 Thinking Process

*In this problem, we have to apply conservation of linear momentum.*

- Ans.** Given, total mass of girl, bicycle and stone =  $m_1 = (50 + 0.5) \text{ kg} = 50.5 \text{ kg}$ .

Velocity of bicycle  $u_1 = 5 \text{ m/s}$ , Mass of stone  $m_2 = 0.5 \text{ kg}$

Velocity of stone  $u_2 = 15 \text{ m/s}$ , Mass of girl and bicycle  $m = 50 \text{ kg}$

Yes, the speed of the bicycle changes after the stone is thrown.

Let after throwing the stone the speed of bicycle be  $v \text{ m/s}$ .

According to law of conservation of linear momentum,

$$m_1 u_1 = m_2 u_2 + mv$$

$$50.5 \times 5 = 0.5 \times 15 + 50 \times v$$

$$252.5 - 7.5 = 50v$$

or 
$$v = \frac{245.0}{50}$$

$$v = 4.9 \text{ m/s}$$

$$\text{Change in speed} = 5 - 4.9 = 0.1 \text{ m/s.}$$

- Q. 17** A person of mass  $50 \text{ kg}$  stands on a weighing scale on a lift. If the lift is descending with a downward acceleration of  $9 \text{ ms}^{-2}$ , what would be the reading of the weighing scale? ( $g = 10 \text{ ms}^{-2}$ )

- Ans.** When a lift descends with a downward acceleration  $a$  the apparent weight of a body of mass  $m$  is given by

$$w' = R = m(g - a)$$

Mass of the person  $m = 50 \text{ kg}$

Descending acceleration  $a = 9 \text{ m/s}^2$

Acceleration due to gravity  $g = 10 \text{ m/s}^2$

Apparent weight of the person,

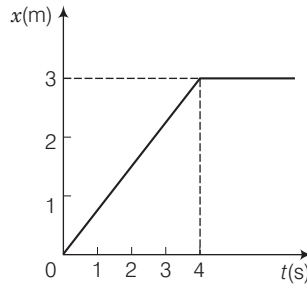
$$R = m(g - a)$$

$$= 50(10 - 9)$$

$$= 50 \text{ N}$$

$$\therefore \text{Reading of the weighing scale} = \frac{R}{g} = \frac{50}{10} = 5 \text{ kg.}$$

- Q. 18** The position-time graph of a body of mass 2 kg is as given in figure. What is the impulse on the body at  $t = 0$  s and  $t = 4$  s.



- Ans.** Given, mass of the body ( $m$ ) = 2 kg  
 From the position-time graph, the body is at  $x = 0$  when  $t = 0$ , i.e., body is at rest.  
 $\therefore$  Impulse at  $t = 0$ ,  $s = 0$ , is zero  
 From  $t = 0$  s to  $t = 4$  s, the position-time graph is a straight line, which shows that body moves with uniform velocity.  
 Beyond  $t = 4$  s, the graph is a straight line parallel to time axis, i.e., body is at rest ( $v = 0$ ).  
 Velocity of the body = slope of position-time graph

$$= \tan \theta = \frac{3}{4} \text{ m/s}$$

Impulse (at  $t = 4$  s) = change in momentum

$$= mv - mu$$

$$= m(v - u)$$

$$= 2 \left( 0 - \frac{3}{4} \right)$$

$$= -\frac{3}{2} \text{ kg-m/s} = -1.5 \text{ kg-m/s}$$

- Q. 19** A person driving a car suddenly applies the brakes on seeing a child on the road ahead. If he is not wearing seat belt, he falls forward and hits his head against the steering wheel. Why?

- Ans.** When a person driving a car suddenly applies the brakes, the lower part of the body slows down with the car while upper part of the body continues to move forward due to inertia of motion.

If driver is not wearing seat belt, then he falls forward and his head hits against the steering wheel.

- Q. 20** The velocity of a body of mass 2 kg as a function of  $t$  is given by  $\mathbf{v}(t) = 2t \hat{i} + t^2 \hat{j}$ . Find the momentum and the force acting on it, at time  $t = 2$  s.

- Ans.** Given, mass of the body  $m = 2$  kg.  
 Velocity of the body  $\mathbf{v}(t) = 2t \hat{i} + t^2 \hat{j}$

$\therefore$  Velocity of the body at  $t = 2$  s

$$\mathbf{v} = 2 \times 2\hat{i} + (2)^2 \hat{j} = (4\hat{i} + 4\hat{j})$$

Momentum of the body ( $p$ ) =  $m\mathbf{v}$

$$= 2(4\hat{i} + 4\hat{j}) = (8\hat{i} + 8\hat{j}) \text{ kg-m/s}$$

Acceleration of the body  $(a) = \frac{dv}{dt}$

$$= \frac{d}{dt} (2t \hat{i} + t^2 \hat{j})$$

$$= (2\hat{i} + 2t \hat{j})$$

At  $t = 2$  s

$$\mathbf{a} = (2\hat{i} + 2 \times 2\hat{j})$$

$$= (2\hat{i} + 4\hat{j})$$

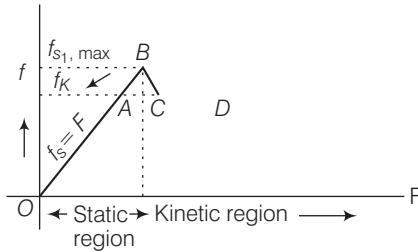
Force acting on the body  $(\mathbf{F}) = m\mathbf{a}$

$$= 2(2\hat{i} + 4\hat{j})$$

$$= (4\hat{i} + 8\hat{j})\text{N}$$

**Q. 21** A block placed on a rough horizontal surface is pulled by a horizontal force  $F$ . Let  $f$  be the force applied by the rough surface on the block. Plot a graph of  $f$  versus  $F$ .

**Ans.** The approximate graph is shown in the diagram



The frictional force  $f$  is shown on vertical axis and the applied force  $F$  is shown on the horizontal axis. The portion  $OA$  of graph represents static friction which is self adjusting. In this portion,  $f = F$ .

The point  $B$  corresponds to force of limiting friction which is the maximum value of static friction.  $CD \parallel OX$  represents kinetic friction, when the body actually starts moving. The force of kinetic friction does not increase with applied force, and is slightly less than limiting friction.

**Q. 22** Why are porcelain objects wrapped in paper or straw before packing for transportation?

**Ans.** Porcelain object are wrapped in paper or straw before packing to reduce the chances of damage during transportation. During transportation sudden jerks or even fall takes place, the force takes longer time to reach the porcelain objects through paper or straw for same change in momentum as  $F = \frac{\Delta p}{\Delta t}$  and therefore, a lesser force acts on object.

**Q. 23** Why does a child feel more pain when she falls down on a hard cement floor, than when she falls on the soft muddy ground in the garden?

**Ans.** When a child falls on a cement floor, her body comes to rest instantly. But  $F \times \Delta t = \text{change in momentum} = \text{constant}$ . As time of stopping  $\Delta t$  decreases, therefore  $F$  increases and hence, child feel more pain. When she falls on a soft muddy ground in the garden the time of stopping increases and hence,  $F$  decreases and she feels lesser pain.

**Q. 24** A woman throws an object of mass 500 g with a speed of  $25 \text{ ms}^{-1}$ .

(a) What is the impulse imparted to the object?

(b) If the object hits a wall and rebounds with half the original speed, what is the change in momentum of the object?

**Ans.** Mass of the object ( $m$ ) = 500 g = 0.5 kg

Speed of the object ( $v$ ) = 25 m/s

(a) Impulse imparted to the object = change in momentum

$$= mv - mu$$

$$= m(v - u)$$

$$= 0.5 (25 - 0) = 12.5 \text{ N-s}$$

(b) Velocity of the object after rebounding

$$= -\frac{25}{2} \text{ m/s}$$

$$v' = -12.5 \text{ m/s}$$

$$\therefore \text{Change in momentum} = m(v' - v)$$

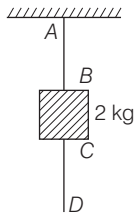
$$= 0.5 (-12.5 - 25) = -18.75 \text{ N-s}$$

**Q. 25** Why are mountain roads generally made winding upwards rather than going straight up?

**Ans.** While going up a mountain, the force of friction acting on a vehicle of mass  $m$  is  $f = \mu R = \mu mg \cos \theta$ , where  $\theta$  is the angle of slope of the road with the horizontal. To avoid skidding force of friction ( $f$ ) should be large and therefore,  $\cos \theta$  should be large and hence,  $\theta$  should be small.

That's why mountain roads are generally made winding upwards rather than going straight upto avoid skidding.

**Q. 26** A mass of 2 kg is suspended with thread  $AB$  (figure). Thread  $CD$  of the same type is attached to the other end of 2 kg mass. Lower thread is pulled gradually, harder and harder in the downward direction, so as to apply force on  $AB$ . Which of the threads will break and why?



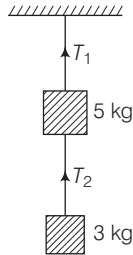
**Ans.** The thread  $AB$  will break earlier than the thread  $CD$ . This is because force acting on thread  $CD$  = applied force and force acting on thread  $AB$  = (applied force + weight of 2 kg mass). Hence, force acting on thread  $AB$  is larger than the force acting on thread  $CD$ .

**Q. 27** In the above given problem if the lower thread is pulled with a jerk, what happens?

**Ans.** When the lower thread  $CD$  is pulled with a jerk, the thread  $CD$  itself break. Because pull on thread  $CD$  is not transmitted to the thread  $AB$  instantly.

### Short Answer Type Questions

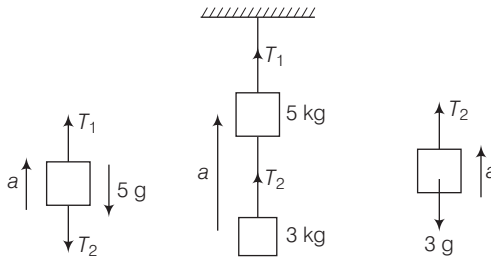
**Q. 28** Two masses of 5 kg and 3 kg are suspended with help of massless inextensible strings as shown in figure. Calculate  $T_1$  and  $T_2$  when whole system is going upwards with acceleration =  $2\text{ m/s}^2$  (use  $g = 9.8\text{ ms}^{-2}$ ).



**Thinking Process**

As the whole system is going upward with an acceleration we have to apply Newton's laws.

**Ans.** Given,  $m_1 = 5\text{ kg}$ ,  $m_2 = 3\text{ kg}$   
 $g = 9.8\text{ m/s}^2$  and  $a = 2\text{ m/s}^2$



For the upper block

$$T_1 - T_2 - 5g = 5a$$

$\Rightarrow$

$$T_1 - T_2 = 5(g + a)$$

... (i)

For the lower block

$$T_2 - 3g = 3a$$

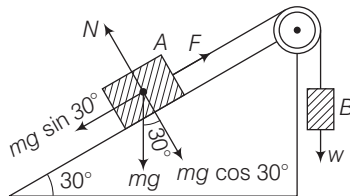
$\Rightarrow$

$$T_2 = 3(g + a) = 3(9.8 + 2) = 35.4\text{ N}$$

From Eq. (i)

$$T_1 = T_2 + 5(g + a) = 35.4 + 5(9.8 + 2) = 94.4\text{ N}$$

**Q. 29** Block A of weight 100 N rests on a frictionless inclined plane of slope angle  $30^\circ$ . A flexible cord attached to A passes over a frictionless pulley and is connected to block B of weight  $w$ . Find the weight  $w$  for which the system is in equilibrium.



**Ans.** In equilibrium, the force  $mg \sin \theta$  acting on block A parallel to the plane should be balanced by the tension in the string, i.e.,

$$mg \sin \theta = T = F \quad [\because T = F \text{ given}] \dots(i)$$

and for block B,

$$w = T = F \quad \dots(ii)$$

where,  $w$  is the weight of block B.

From Eqs. (i) and (ii), we get,

$$\begin{aligned} \therefore w &= mg \sin \theta \\ &= 100 \times \sin 30^\circ \quad (\because mg = 100 \text{ N}) \\ &= 100 \times \frac{1}{2} \text{ N} = 50 \text{ N} \end{aligned}$$

**Note** While finding normal reaction in such cases, we should be careful it will be  $N = mg \cos \theta$ , where  $\theta$  is angle of inclination.

**Q. 30** A block of mass  $M$  is held against a rough vertical wall by pressing it with a finger. If the coefficient of friction between the block and the wall is  $\mu$  and the acceleration due to gravity is  $g$ , calculate the minimum force required to be applied by the finger to hold the block against the wall.

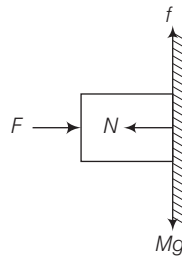
**Ans.** Given, mass of the block =  $M$

Coefficient of friction between the block and the wall =  $\mu$

Let a force  $F$  be applied on the block to hold the block against the wall. The normal reaction of mass be  $N$  and force of friction acting upward be  $f$ . In equilibrium, vertical and horizontal forces should be balanced separately.

$$\therefore f = Mg \quad \dots(i)$$

$$\text{and } F = N \quad \dots(ii)$$



But force of friction  $(f) = \mu N$

$$= \mu F \quad \text{[using Eq. (ii)] } \dots(iii)$$

From Eqs. (i) and (iii), we get

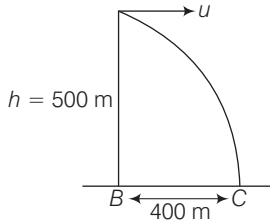
$$\mu F = Mg$$

or

$$F = \frac{Mg}{\mu}$$

**Q. 31** A 100 kg gun fires a ball of 1 kg horizontally from a cliff of height 500 m. It falls on the ground at a distance of 400 m from the bottom of the cliff. Find the recoil velocity of the gun.  
(acceleration due to gravity =  $10 \text{ ms}^{-2}$ )

**Ans.** Given, mass of the gun ( $m_1$ ) = 100 kg



Mass of the ball ( $m_2$ ) = 1 kg

Height of the cliff ( $h$ ) = 500 m

Horizontal distance travelled by the ball ( $x$ ) = 400 m

From  $h = \frac{1}{2}gt^2$  ( $\because$  Initial velocity in downward direction is zero)

$$500 = \frac{1}{2} \times 10t^2$$

$$t = \sqrt{100} = 10 \text{ s}$$

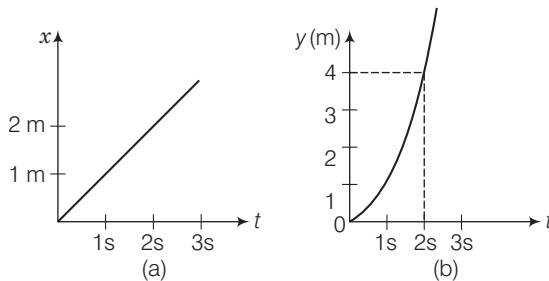
From  $x = ut, u = \frac{x}{t} = \frac{400}{10} = 40 \text{ m/s}$

If  $v$  is recoil velocity of gun, then according to principle of conservation of linear momentum,

$$m_1 v = m_2 u$$

$$v = \frac{m_2 u}{m_1} = \frac{1}{100} \times 40 = 0.4 \text{ m/s}$$

**Q. 32** Figure shows  $(x, t), (y, t)$  diagram of a particle moving in 2-dimensions.



If the particle has a mass of 500 g, find the force (direction and magnitude) acting on the particle.

**Thinking Process**

To solve this question, we have to find the relation for  $x$  and time ( $t$ ),  $y$  and time ( $t$ ) from the given diagram.

**Ans.** Clearly from diagram (a), the variation can be related as

$$x = t \Rightarrow \frac{dx}{dt} = 1 \text{ m/s}$$

$$a_x = 0$$



From diagram (b)  $y = t^2$

$\Rightarrow \frac{dy}{dt} = 2t$  or  $a_y = \frac{d^2y}{dt^2} = 2 \text{ m/s}^2$

Hence,  $I_y = ma_y = 500 \times 10^{-3} \times 2 = 1\text{N}$  ( $\because m = 500\text{g}$ )

$F_x = ma_x = 0$

Hence, net force,  $F = \sqrt{F_x^2 + F_y^2} = F_y = 1\text{N}$  (along y-axis)

**Q. 33** A person in an elevator accelerating upwards with an acceleration of  $2 \text{ ms}^{-2}$ , tosses a coin vertically upwards with a speed of  $20 \text{ ms}^{-1}$ . After how much time will the coin fall back into his hand? ( $g = 10 \text{ ms}^{-2}$ )

**Ans.** Here, initial speed of the coin ( $u$ ) =  $20 \text{ m/s}$   
 Acceleration of the elevator ( $a$ ) =  $2 \text{ m/s}^2$  (upwards)

Acceleration due to gravity ( $g$ ) =  $10 \text{ m/s}^2$

$\therefore$  Effective acceleration  $a' = g + a = 10 + 2 = 12 \text{ m/s}^2$  (here, acceleration is w.r.t. the lift)

If the time of ascent of the coin is  $t$ , then

$$v = u + at$$

$$0 = 20 + (-12) \times t$$

or

$$t = \frac{20}{12} = \frac{5}{3} \text{ s}$$

Time of ascent = Time of descent

$\therefore$  Total time after which the coin fall back into hand =  $\left(\frac{5}{3} + \frac{5}{3}\right) \text{ s} = \frac{10}{3} \text{ s} = 3.33 \text{ s}$

**Note** While calculating net acceleration we should be aware that if lift is going upward net acceleration is  $(g + a)$  and for downward net acceleration is  $(g - a)$ .

## Long Answer Type Questions

**Q. 34** There are three forces  $F_1$ ,  $F_2$  and  $F_3$  acting on a body, all acting on a point  $P$  on the body. The body is found to move with uniform speed.

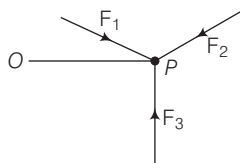
- Show that the forces are coplanar.
- Show that the torque acting on the body about any point due to these three forces is zero.

### Thinking Process

As the body is found to move with uniform velocity hence, we can say that total force acting will be zero.

**Ans.** As the body is moving with uniform speed (velocity) its acceleration  $a = 0$ .

$\therefore$  The sum of the forces is zero,  $F_1 + F_2 + F_3 = 0$



(a) Let  $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3$  be the three forces passing through a point. Let  $\mathbf{F}_1$  and  $\mathbf{F}_2$  be in the plane A (one can always draw a plane having two intersecting lines such that the two lines lie on the plane). Then  $\mathbf{F}_1 + \mathbf{F}_2$  must be in the plane A.

Since,  $\mathbf{F}_3 = -(\mathbf{F}_1 + \mathbf{F}_2)$ ,  $\mathbf{F}_3$  is also in the plane A.

(b) Consider the torque of the forces about P. Since, all the forces pass through P, the torque is zero. Now, consider torque about another point O. Then torque about O is

$$\text{Torque} = \mathbf{OP} \times (\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3)$$

Since,  $\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$ , torque = 0

**Q. 35** When a body slides down from rest along a smooth inclined plane making an angle of  $45^\circ$  with the horizontal, it takes time  $T$ . When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time  $pT$ , where  $p$  is some number greater than 1. Calculate the coefficient of friction between the body and the rough plane.

**Ans.** Consider the diagram where a body slides down from along an inclined plane of inclination  $\theta (= 45^\circ)$ .

**On smooth inclined plane** Acceleration of a body sliding down a smooth inclined plane

$$a = g \sin \theta$$

Here,

$$\theta = 45^\circ$$

$\therefore$

$$a = g \sin 45^\circ = \frac{g}{\sqrt{2}}$$

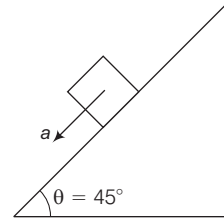
Let the travelled distance be  $s$ .

Using equation of motion,  $s = ut + \frac{1}{2}at^2$ , we get

$$s = 0.t + \frac{1}{2} \frac{g}{\sqrt{2}} T^2$$

or

$$s = \frac{gT^2}{2\sqrt{2}} \quad \dots(i)$$



**On rough inclined plane** Acceleration of the body  $a = g(\sin \theta - \mu \cos \theta)$

$$= g(\sin 45^\circ - \mu \cos 45^\circ)$$

$$= \frac{g(1 - \mu)}{\sqrt{2}} \quad \left( \text{As, } \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}} \right)$$

Again using equation of motion,  $s = ut + \frac{1}{2}at^2$ , we get

$$s = 0(pT) + \frac{1}{2} \frac{g(1 - \mu)}{\sqrt{2}} (pT)^2$$

or

$$s = \frac{g(1 - \mu)p^2T^2}{2\sqrt{2}} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{gT^2}{2\sqrt{2}} = \frac{g(1 - \mu)p^2T^2}{2\sqrt{2}}$$

or

$$(1 - \mu)p^2 = 1$$

or

$$1 - \mu = \frac{1}{p^2}$$

or

$$\mu = \left( 1 - \frac{1}{p^2} \right)$$

**Q. 36** Figure shows  $(v_x, t)$ , and  $(v_y, t)$  diagrams for a body of unit mass. Find the force as a function of time.

**Ans.** Consider figure (a)

$$\begin{aligned} v_x &= 2t \text{ for } 0 < t < 1\text{s} \\ &= 2(2-t) \text{ for } 1 < t < 2\text{s} \\ &= 0 \text{ for } t > 2\text{s}. \end{aligned}$$

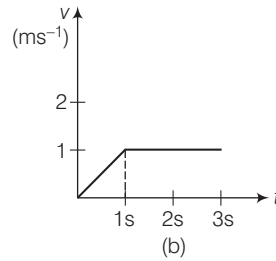
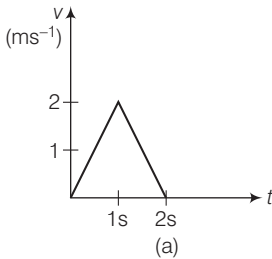
From figure (b)

$$\begin{aligned} v_y &= t \text{ for } 0 < t < 1\text{s} \\ &= 1 \text{ for } t > 1\text{s} \end{aligned}$$

$\therefore$

$$\begin{aligned} F_x &= ma_x = m \frac{dv_x}{dt} \\ &= 1 \times 2 \\ &= 1(-2) \end{aligned}$$

for  $0 < t < 1\text{s}$   
for  $1 < t < 2\text{s}$   
for  $2 < t$



$$F_y = ma_y = m \frac{dv_y}{dt}$$

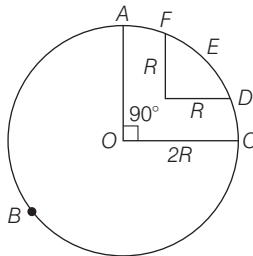
$$= 1 \times 1 \text{ for } 0 < t < 1\text{s} = 0 \text{ for } 1 < t < 2\text{s}$$

Hence,

$$\begin{aligned} \mathbf{F} &= F_x \hat{i} + F_y \hat{j} \\ &= 2\hat{i} + \hat{j} \\ &= -2\hat{i} \\ &= 0 \end{aligned}$$

for  $0 < t < 1\text{s}$   
for  $1 < t < 2\text{s}$   
for  $t > 2\text{s}$ .

**Q. 37** A racing car travels on a track (without banking)  $ABCDEFA$ .  $ABC$  is a circular arc of radius  $2R$ .  $CD$  and  $FA$  are straight paths of length  $R$  and  $DEF$  is a circular arc of radius  $R = 100$  m. The coefficient of friction on the road is  $\mu = 0.1$ . The maximum speed of the car is  $50 \text{ ms}^{-1}$ . Find the minimum time for completing one round.



### Thinking Process

The necessary centripetal force required for the circular motion will be provided by the frictional force.

**Ans.** Balancing frictional force for centripetal force  $\frac{mv^2}{r} = f = \mu N = \mu mg$

where,  $N$  is normal reaction.

$$\therefore v = \sqrt{\mu rg} \quad (\text{where, } r \text{ is radius of the circular track})$$

For path  $ABC$  Path length =  $\frac{3}{4}(2\pi R) = 3\pi R = 3\pi \times 100$   
 $= 300\pi\text{m}$

$$v_1 = \sqrt{\mu 2Rg} = \sqrt{0.1 \times 2 \times 100 \times 10}$$

$$= 14.14 \text{ m/s}$$

$$\therefore t_1 = \frac{300\pi}{14.14} = 66.6 \text{ s}$$

For path  $DEF$  Path length =  $\frac{1}{4}(2\pi R) = \frac{\pi \times 100}{2} = 50\pi$

$$v_2 = \sqrt{\mu Rg} = \sqrt{0.1 \times 100 \times 10} = 10 \text{ m/s}$$

$$t_2 = \frac{50\pi}{10} = 5\pi \text{ s} = 15.7 \text{ s}$$

For paths,  $CD$  and  $FA$

$$\text{Path length} = R + R = 2R = 200 \text{ m}$$

$$t_3 = \frac{200}{50} = 4.0 \text{ s.}$$

$\therefore$  Total time for completing one round

$$t = t_1 + t_2 + t_3 = 66.6 + 15.7 + 4.0 = 86.3 \text{ s}$$

**Q. 38** The displacement vector of a particle of mass  $m$  is given by  $\mathbf{r}(t) = \hat{\mathbf{i}} A \cos \omega t + \hat{\mathbf{j}} B \sin \omega t$ .

(a) Show that the trajectory is an ellipse.

(b) Show that  $F = -m\omega^2 \mathbf{r}$ .

**Thinking Process**

To find trajectory, we will relate  $x$  and  $y$  in terms of constants  $A$  and  $B$ .

**Ans. (a)** Displacement vector of the particle of mass  $m$  is given by

$$\mathbf{r}(t) = \hat{\mathbf{i}} A \cos \omega t + \hat{\mathbf{j}} B \sin \omega t$$

$\therefore$  Displacement along  $x$ -axis is,

$$x = A \cos \omega t$$

or  $\frac{x}{A} = \cos \omega t$  ...(i)

Displacement along  $y$ -axis is,

and  $y = B \sin \omega t$

or  $\frac{y}{B} = \sin \omega t$

Squaring and then adding Eqs. (i) and (ii), we get

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = \cos^2 \omega t + \sin^2 \omega t = 1$$

This is an equation of ellipse.

Therefore, trajectory of the particle is an ellipse.

(b) Velocity of the particle

$$\begin{aligned}\mathbf{v} &= \frac{d\mathbf{r}}{dt} = \hat{i} \frac{d}{dt} (A \cos \omega t) + \hat{j} \frac{d}{dt} (B \sin \omega t) \\ &= \hat{i} [A (-\sin \omega t) \cdot \omega] + \hat{j} [B (\cos \omega t) \cdot \omega] \\ &= -\hat{i} A \omega \sin \omega t + \hat{j} B \omega \cos \omega t\end{aligned}$$

$$\text{Acceleration of the particle } (\mathbf{a}) = \frac{d\mathbf{v}}{dt}$$

$$\begin{aligned}\text{or } \mathbf{a} &= -\hat{i} A \omega \frac{d}{dt} (\sin \omega t) + \hat{j} B \omega \frac{d}{dt} (\cos \omega t) \\ &= -\hat{i} A \omega [\cos \omega t] \cdot \omega + \hat{j} B \omega [-\sin \omega t] \cdot \omega \\ &= -\hat{i} A \omega^2 \cos \omega t - \hat{j} B \omega^2 \sin \omega t \\ &= -\omega^2 [\hat{i} A \cos \omega t + \hat{j} B \sin \omega t] \\ &= -\omega^2 \mathbf{r}\end{aligned}$$

$\therefore$  Force acting on the particle,

$$F = m\mathbf{a} = -m \omega^2 \mathbf{r},$$

Hence proved.

**Q. 39** A cricket bowler releases the ball in two different ways

- giving it only horizontal velocity, and
- giving it horizontal velocity and a small downward velocity.

The speed  $v_s$  at the time of release is the same. Both are released at a height  $H$  from the ground. Which one will have greater speed when the ball hits the ground? Neglect air resistance.

**Thinking Process**

*The horizontal component of velocity will remain unaffected by gravity.*

**Ans.** (a) When ball is given only horizontal velocity Horizontal velocity at the time of release ( $u_x$ ) =  $v_s$

During projectile motion, horizontal velocity remains unchanged,

Therefore,

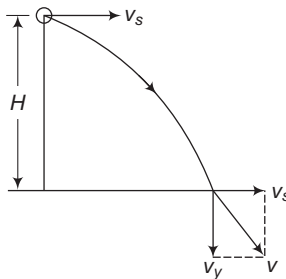
$$v_x = u_x = v_s$$

In vertical direction,

$$v_y^2 = u_y^2 + 2gH$$

$$v_y = \sqrt{2gH}$$

$$(\because u_y = 0)$$

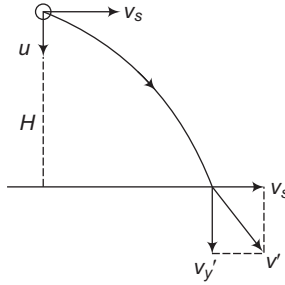


$\therefore$  Resultant speed of the ball at bottom,

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{v_s^2 + 2gH}\end{aligned}$$

... (i)

(b) When ball is given horizontal velocity and a small downward velocity



Let the ball be given a small downward velocity  $u$ .

In horizontal direction  $v'_x = u_x = v_s$

In vertical direction  $v_y^2 = u^2 + 2gH$

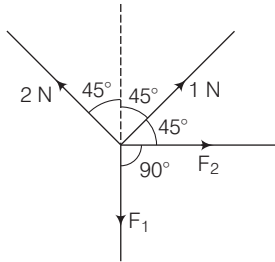
or  $v'_y = \sqrt{u^2 + 2gH}$

$\therefore$  Resultant speed of the ball at the bottom

$$v' = \sqrt{v_x^2 + v_y^2} = \sqrt{v_s^2 + u^2 + 2gH} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get  $v' > v$

**Q. 40** There are four forces acting at a point  $P$  produced by strings as shown in figure. Which is at rest? Find the forces  $F_1$  and  $F_2$ .



**Thinking Process**

To balance the forces, we have to resolve them along two mutually perpendicular directions.

**Ans.** Consider the adjacent diagram, in which forces are resolved.

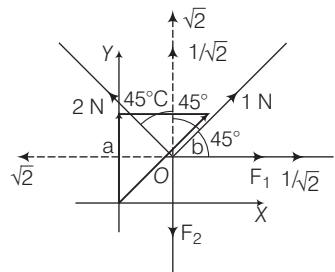
On resolving forces into rectangular components, in equilibrium forces  $\left(F_1 + \frac{1}{\sqrt{2}}\right)$  N are equal to  $\sqrt{2}$  N and

$F_2$  is equal to  $\left(\sqrt{2} + \frac{1}{\sqrt{2}}\right)$  N.

$$\therefore F_1 + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$F_1 = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}} = 0.707 \text{ N}$$

and  $F_2 = \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{2+1}{\sqrt{2}} \text{ N} = \frac{3}{\sqrt{2}} \text{ N} = 2.121 \text{ N}$



**Q. 41** A rectangular box lies on a rough inclined surface. The coefficient of friction between the surface and the box is  $\mu$ . Let the mass of the box be  $m$ .

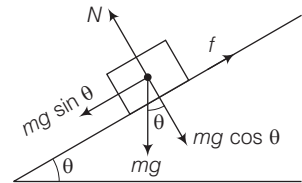
- At what angle of inclination  $\theta$  of the plane to the horizontal will the box just start to slide down the plane?
- What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to  $\alpha > \theta$ ?
- What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed?
- What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration  $a$ ?

**Ans. (a)** Consider the adjacent diagram, force of friction on the box will act up the plane.

For the box to just start sliding down  $mg$

$$\sin \theta = f = \mu N = \mu mg \cos \theta$$

or  $\tan \theta = \mu \Rightarrow \theta = \tan^{-1}(\mu)$



- (b) When angle of inclination is increased to  $\alpha > \theta$ , then net force acting on the box, down the plane is

$$F_1 = mg \sin \alpha - f = mg \sin \alpha - \mu N \\ = mg (\sin \alpha - \mu \cos \alpha).$$

- (c) To keep the box either stationary or just move it up with uniform speed, upward force needed,  $F_2 = mg \sin \alpha + f = mg (\sin \alpha + \mu \cos \alpha)$  (In this case, friction would act down the plane).
- (d) If the box is to be moved with an upward acceleration  $a$ , then upward force needed,  $F_3 = mg (\sin \alpha + \mu \cos \alpha) + ma$ .

**Q. 42** A helicopter of mass 2000 kg rises with a vertical acceleration of  $15 \text{ ms}^{-2}$ . The total mass of the crew and passengers is 500 kg. Give the magnitude and direction of the ( $g = 10 \text{ ms}^{-2}$ )

- force on the floor of the helicopter by the crew and passengers.
- action of the rotor of the helicopter on the surrounding air.
- force on the helicopter due to the surrounding air.

**Ans.** Given, mass of helicopter ( $m_1$ ) = 2000 kg

Mass of the crew and passengers  $m_2 = 500 \text{ kg}$

Acceleration in vertical direction  $a = 15 \text{ m/s}^2$  ( $\uparrow$ ) and  $g = 10 \text{ m/s}^2$  ( $\downarrow$ )

- (a) Force on the floor of the helicopter by the crew and passengers

$$m_2(g + a) = 500(10 + 15) \text{ N}$$

$$500 \times 25 \text{ N} = 12500 \text{ N}$$

- (b) Action of the rotor of the helicopter on the surrounding air =  $(m_1 + m_2)(g + a)$

$$= (2000 + 500) \times (10 + 15) = 2500 \times 25$$

$$= 62500 \text{ N (downward)}$$

- (c) Force on the helicopter due to the surrounding air

$$= \text{reaction of force applied by helicopter.}$$

$$= 62500 \text{ N (upward)}$$

**Note** We should be very clear when we are balancing action and reaction forces. We must know that which part is action and which part is reaction due to the action.