

Application Integrals

Short Answer Type Questions

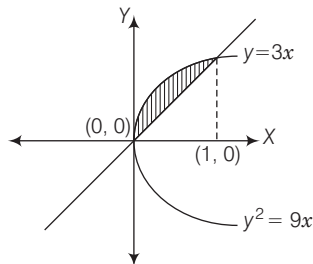
Q. 1 Find the area of the region bounded by the curves $y^2 = 9x$ and $y = 3x$.

Thinking Process

On solving both the equation of curves, get the values of x and then at those values, find the area of the shaded region.

Sol. We have,

$$\begin{aligned} & y^2 = 9x \text{ and } y = 3x \\ \Rightarrow & (3x)^2 = 9x \\ \Rightarrow & 9x^2 - 9x = 0 \\ \Rightarrow & 9x(x - 1) = 0 \\ \Rightarrow & x = 1, 0 \end{aligned}$$



$$\begin{aligned} \therefore \text{ Required area, } A &= \int_0^1 \sqrt{9x} \, dx - \int_0^1 3x \, dx \\ &= 3 \int_0^1 x^{1/2} \, dx - 3 \int_0^1 x \, dx \\ &= 3 \left[\frac{x^{3/2}}{3/2} \right]_0^1 - 3 \left[\frac{x^2}{2} \right]_0^1 \\ &= 3 \left(\frac{2}{3} - 0 \right) - 3 \left(\frac{1}{2} - 0 \right) \\ &= 2 - \frac{3}{2} = \frac{1}{2} \text{ sq units} \end{aligned}$$

Q. 2 Find the area of the region bounded by the parabola $y^2 = 2px$ and $x^2 = 2py$.

Sol. We have,

$$y^2 = 2px \text{ and } x^2 = 2py$$

\therefore

$$y = \sqrt{2px}$$

\Rightarrow

$$x^2 = 2p \cdot \sqrt{2px}$$

\Rightarrow

$$x^4 = 4p^2 \cdot (2px)$$

\Rightarrow

$$x^4 = 8p^3x$$

\Rightarrow

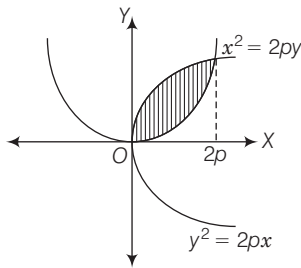
$$x^4 - 8p^3x = 0$$

\Rightarrow

$$x^3(x - 8p^3) = 0$$

\Rightarrow

$$x = 0, 2p$$

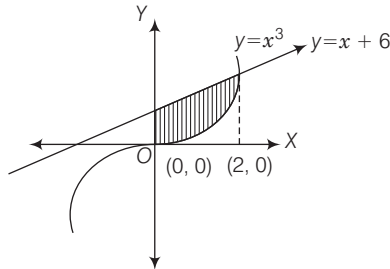


\therefore

$$\begin{aligned} \text{Required area} &= \int_0^{2p} \sqrt{2px} \, dx - \int_0^{2p} \frac{x^2}{2p} \, dx \\ &= \sqrt{2p} \int_0^{2p} x^{1/2} \, dx - \frac{1}{2p} \int_0^{2p} x^2 \, dx \\ &= \sqrt{2p} \left[\frac{2(x)^{3/2}}{3} \right]_0^{2p} - \frac{1}{2p} \left[\frac{x^3}{3} \right]_0^{2p} \\ &= \sqrt{2p} \left[\frac{2}{3} \cdot (2p)^{3/2} - 0 \right] - \frac{1}{2p} \left[\frac{1}{3} (2p)^3 - 0 \right] \\ &= \sqrt{2p} \left(\frac{2}{3} \cdot 2\sqrt{2}p^{3/2} \right) - \frac{1}{2p} \left(\frac{1}{3} 8p^3 \right) \\ &= \sqrt{2p} \left(\frac{4\sqrt{2}}{3} p^{3/2} \right) - \frac{1}{2p} \left(\frac{8}{3} p^3 \right) \\ &= \frac{4\sqrt{2}}{3} \cdot \sqrt{2}p^2 - \frac{8}{6}p^2 \\ &= \frac{(16 - 8)p^2}{6} = \frac{8p^2}{6} \\ &= \frac{4p^2}{3} \text{ sq units} \end{aligned}$$

Q. 3 Find the area of the region bounded by the curve $y = x^3$, $y = x + 6$ and $x = 0$.

Sol. We have, $y = x^3$, $y = x + 6$ and $x = 0$



$$\begin{aligned} \therefore & x^3 = x + 6 \\ \Rightarrow & x^3 - x = 6 \\ \Rightarrow & x^3 - x - 6 = 0 \\ \Rightarrow & x^2(x - 2) + 2x(x - 2) + 3(x - 2) = 0 \\ \Rightarrow & (x - 2)(x^2 + 2x + 3) = 0 \\ \Rightarrow & x = 2, \text{ with two imaginary points} \end{aligned}$$

$$\begin{aligned} \therefore \text{ Required area of shaded region} &= \int_0^2 (x + 6 - x^3) dx \\ &= \left[\frac{x^2}{2} + 6x - \frac{x^4}{4} \right]_0^2 \\ &= \left[\frac{4}{2} + 12 - \frac{16}{4} - 0 \right] \\ &= [2 + 12 - 4] = 10 \text{ sq units} \end{aligned}$$

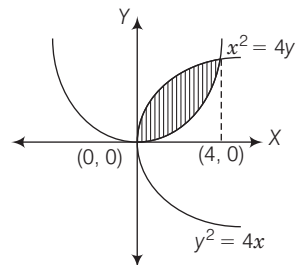
Q. 4 Find the area of the region bounded by the curve $y^2 = 4x$ and $x^2 = 4y$.

💡 Thinking Process

First, by using both the equation get the values of x and then find the shaded region by using these value of x in the equation of curve in x only.

Sol. Given equation of curves are

$$\begin{aligned} & y^2 = 4x && \dots(i) \\ \text{and} & x^2 = 4y && \dots(ii) \\ \Rightarrow & \left(\frac{x^2}{4}\right)^2 = 4x \\ \Rightarrow & \frac{x^4}{4 \cdot 4} = 4x \\ \Rightarrow & x^4 = 64x \\ \Rightarrow & x^4 - 64x = 0 \\ \Rightarrow & x(x^3 - 4^3) = 0 \\ \Rightarrow & x = 4, 0 \end{aligned}$$

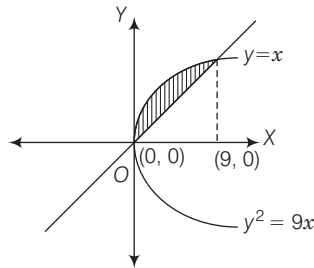


$$\begin{aligned}
 \therefore \text{Area of shaded region, } A &= \int_0^4 \left(\sqrt{4x} - \frac{x^2}{4} \right) dx \\
 &= \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4} \right) dx = \left[\frac{2x^{3/2} \cdot 2}{3} - \frac{1}{4} \cdot \frac{x^3}{3} \right]_0^4 \\
 &= \frac{2 \cdot 2}{3} \cdot 8 - \frac{1}{4} \cdot \frac{64}{3} - 0 = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq units}
 \end{aligned}$$

Q. 5 Find the area of the region included between $y^2 = 9x$ and $y = x$.

Sol. We have, $y^2 = 9x$ and $y = x$

$$\begin{aligned}
 \Rightarrow & x^2 = 9x \\
 \Rightarrow & x^2 - 9x = 0 \\
 \Rightarrow & x(x - 9) = 0 \\
 \Rightarrow & x = 0, 9
 \end{aligned}$$

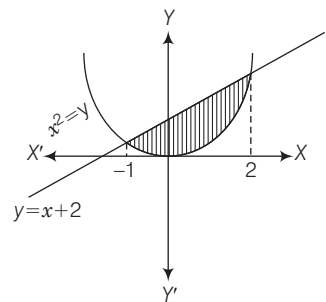


$$\begin{aligned}
 \therefore \text{Area of shaded region, } A &= \int_0^9 (\sqrt{9x} - x) dx = \int_0^9 3x^{1/2} dx - \int_0^9 x dx \\
 &= \left[3 \cdot \frac{x^{3/2}}{3} \cdot 2 \right]_0^9 - \left[\frac{x^2}{2} \right]_0^9 \\
 &= \left[\frac{3 \cdot 3^{\frac{3}{2} \times 2}}{3} \cdot 2 - 0 \right] - \left[\frac{81}{2} - 0 \right] \\
 &= 54 - \frac{81}{2} = \frac{108 - 81}{2} = \frac{27}{2} \text{ sq units}
 \end{aligned}$$

Q. 6 Find the area of the region enclosed by the parabola $x^2 = y$ and the line $y = x + 2$.

Sol. We have, $x^2 = y$ and $y = x + 2$

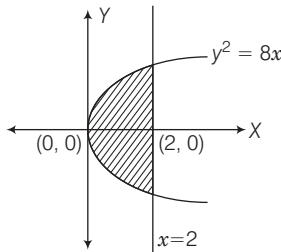
$$\begin{aligned}
 \Rightarrow & x^2 = x + 2 \\
 \Rightarrow & x^2 - x - 2 = 0 \\
 \Rightarrow & x^2 - 2x + x - 2 = 0 \\
 \Rightarrow & x(x - 2) + 1(x - 2) = 0 \\
 \Rightarrow & (x + 1)(x - 2) = 0 \\
 \Rightarrow & x = -1, 2
 \end{aligned}$$



$$\begin{aligned}
 \therefore \text{ Required area of shaded region} &= \int_{-1}^2 (x + 2 - x^2) dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^2 \\
 &= \left[\frac{4}{2} + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} \right] \\
 &= 6 + \frac{3}{2} - \frac{9}{3} = \frac{36 + 9 - 18}{6} = \frac{27}{6} = \frac{9}{2} \text{ sq units}
 \end{aligned}$$

Q. 7 Find the area of the region bounded by line $x = 2$ and parabola $y^2 = 8x$.

Sol. We have, $y^2 = 8x$ and $x = 2$

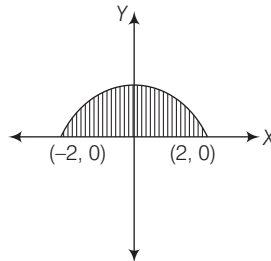


$$\begin{aligned}
 \therefore \text{ Area of shaded region, } A &= 2 \int_0^2 \sqrt{8x} dx = 2 \cdot 2\sqrt{2} \int_0^2 x^{1/2} dx \\
 &= 4 \cdot \sqrt{2} \cdot \left[2 \cdot \frac{x^{3/2}}{3} \right]_0^2 = 4\sqrt{2} \left[\frac{2}{3} \cdot 2\sqrt{2} - 0 \right] \\
 &= \frac{32}{3} \text{ sq units}
 \end{aligned}$$

Q. 8 Sketch the region $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X -axis. Find the area of the region using integration.

Sol. Given region is $\{(x, 0) : y = \sqrt{4 - x^2}\}$ and X -axis.

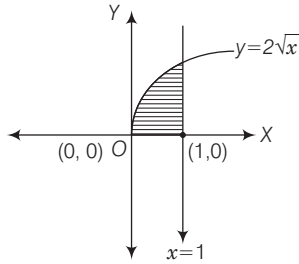
We have, $y = \sqrt{4 - x^2} \Rightarrow y^2 = 4 - x^2 \Rightarrow x^2 + y^2 = 4$



$$\begin{aligned}
 \therefore \text{ Area of shaded region, } A &= \int_{-2}^2 \sqrt{4 - x^2} dx = \int_{-2}^2 \sqrt{2^2 - x^2} dx \\
 &= \left[\frac{x}{2} \sqrt{2^2 - x^2} + \frac{2^2}{2} \cdot \sin^{-1} \frac{x}{2} \right]_{-2}^2 \\
 &= \frac{2}{2} \cdot 0 + 2 \cdot \frac{\pi}{2} + \frac{2}{2} \cdot 0 - 2 \sin^{-1}(-1) = 2 \cdot \frac{\pi}{2} + 2 \cdot \frac{\pi}{2} \\
 &= 2\pi \text{ sq units}
 \end{aligned}$$

Q. 9 Calculate the area under the curve $y = 2\sqrt{x}$ included between the lines $x = 0$ and $x = 1$.

Sol. We have, $y = 2\sqrt{x}$, $x = 0$ and $x = 1$

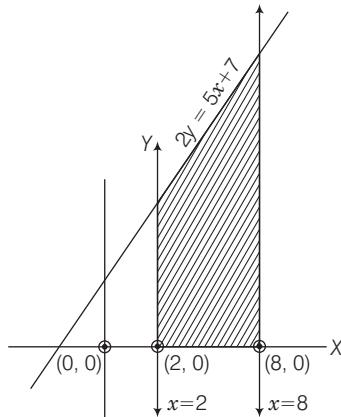


$$\begin{aligned} \therefore \text{Area of shaded region, } A &= \int_0^1 (2\sqrt{x}) dx \\ &= 2 \cdot \left[\frac{x^{3/2}}{3} \cdot 2 \right]_0^1 \\ &= 2 \left(\frac{2}{3} \cdot 1 - 0 \right) = \frac{4}{3} \text{ sq units} \end{aligned}$$

Q. 10 Using integration, find the area of the region bounded by the line $2y = 5x + 7$, X -axis and the lines $x = 2$ and $x = 8$.

Sol. We have,

$$\begin{aligned} 2y &= 5x + 7 \\ \Rightarrow y &= \frac{5x}{2} + \frac{7}{2} \end{aligned}$$



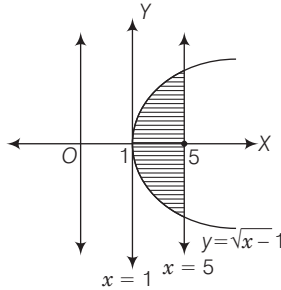
$$\begin{aligned} \therefore \text{Area of shaded region} &= \frac{1}{2} \int_2^8 (5x + 7) dx = \frac{1}{2} \left[5 \cdot \frac{x^2}{2} + 7x \right]_2^8 \\ &= \frac{1}{2} [5 \cdot 32 + 7 \cdot 8 - 10 - 14] = \frac{1}{2} [160 + 56 - 24] \\ &= \frac{192}{2} = 96 \text{ sq units} \end{aligned}$$

Q. 11 Draw a rough sketch of the curve $y = \sqrt{x-1}$ in the interval $[1, 5]$.

Find the area under the curve and between the lines $x = 1$ and $x = 5$.

Sol. Given equation of the curve is $y = \sqrt{x-1}$.

$$\Rightarrow y^2 = x - 1$$

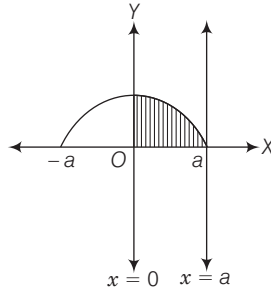


$$\begin{aligned} \therefore \text{Area of shaded region, } A &= \int_1^5 (x-1)^{1/2} dx = \left[\frac{2 \cdot (x-1)^{3/2}}{3} \right]_1^5 \\ &= \left[\frac{2}{3} \cdot (5-1)^{3/2} - 0 \right] = \frac{16}{3} \text{ sq units} \end{aligned}$$

Q. 12 Determine the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines $x = 0$ and $x = a$.

Sol. Given equation of the curve is $y = \sqrt{a^2 - x^2}$.

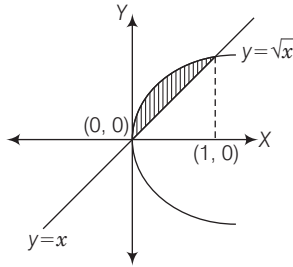
$$\Rightarrow y^2 = a^2 - x^2 \Rightarrow y^2 + x^2 = a^2$$



$$\begin{aligned} \therefore \text{Required area of shaded region, } A &= \int_0^a \sqrt{a^2 - x^2} dx \\ &= \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \\ &= \left[0 + \frac{a^2}{2} \sin^{-1}(1) - 0 - \frac{a^2}{2} \sin^{-1} 0 \right] \\ &= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{\pi a^2}{4} \text{ sq units} \end{aligned}$$

Q. 13 Find the area of the region bounded by $y = \sqrt{x}$ and $y = x$.

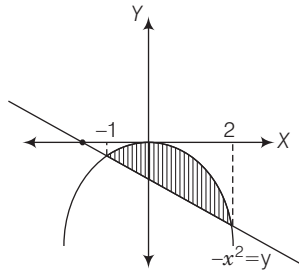
Sol. Given equation of curves are $y = \sqrt{x}$ and $y = x$.
 $\Rightarrow x = \sqrt{x} \Rightarrow x^2 = x$
 $\Rightarrow x^2 - x = 0 \Rightarrow x(x - 1) = 0$
 $\Rightarrow x = 0, 1$



$$\begin{aligned} \therefore \text{ Required area of shaded region, } A &= \int_0^1 (\sqrt{x}) dx - \int_0^1 x dx \\ &= \left[2 \cdot \frac{x^{3/2}}{3} \right]_0^1 - \left[\frac{x^2}{2} \right]_0^1 \\ &= \frac{2}{3} \cdot 1 - \frac{1}{2} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6} \text{ sq units} \end{aligned}$$

Q. 14 Find the area enclosed by the curve $y = -x^2$ and the straight line $x + y + 2 = 0$.

Sol. We have, $y = -x^2$ and $x + y + 2 = 0$



$$\begin{aligned} \Rightarrow -x - 2 &= -x^2 \Rightarrow x^2 - x - 2 = 0 \\ \Rightarrow x^2 + x - 2x - 2 &= 0 \Rightarrow x(x + 1) - 2(x + 1) = 0 \\ \Rightarrow (x - 2)(x + 1) &= 0 \Rightarrow x = 2, -1 \\ \therefore \text{ Area of shaded region, } A &= \left| \int_{-1}^2 (-x - 2 + x^2) dx \right| = \left| \int_{-1}^2 (x^2 - x - 2) dx \right| \\ &= \left| \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-1}^2 \right| = \left| \left[\frac{8}{3} - \frac{4}{2} - 4 + \frac{1}{3} + \frac{1}{2} - 2 \right] \right| \\ &= \left| \frac{16 - 12 - 24 + 2 + 3 - 12}{6} \right| = \left| -\frac{27}{6} \right| = \frac{9}{2} \text{ sq units} \end{aligned}$$

Q. 15 Find the area bounded by the curve $y = \sqrt{x}$, $x = 2y + 3$ in the first quadrant and X -axis.

Sol. Given equation of the curves are $y = \sqrt{x}$ and $x = 2y + 3$ in the first quadrant.

On solving both the equations for y , we get

$$\Rightarrow y = \sqrt{2y + 3}$$

$$\Rightarrow y^2 = 2y + 3$$

$$\Rightarrow y^2 - 2y - 3 = 0$$

$$\Rightarrow y^2 - 3y + y - 3 = 0$$

$$\Rightarrow y(y - 3) + 1(y - 3) = 0$$

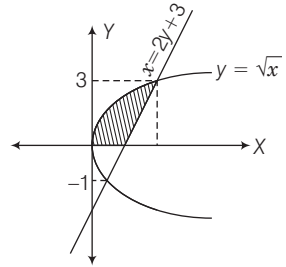
$$\Rightarrow (y + 1)(y - 3) = 0$$

$$\Rightarrow y = -1, 3$$

\therefore Required area of shaded region,

$$A = \int_0^3 (2y + 3 - y^2) dy = \left[\frac{2y^2}{2} + 3y - \frac{y^3}{3} \right]_0^3$$

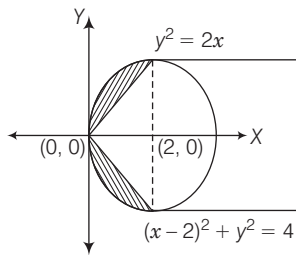
$$= \left[\frac{18}{2} + 9 - 9 - 0 \right] = 9 \text{ sq units}$$



Long Answer Type Questions

Q. 16 Find the area of the region bounded by the curve $y^2 = 2x$ and $x^2 + y^2 = 4x$.

Sol. We have, $y^2 = 2x$ and $x^2 + y^2 = 4x$



$$\Rightarrow x^2 + 2x = 4x$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0, 2$$

Also, $x^2 + y^2 = 4x$

$$\Rightarrow x^2 - 4x = -y^2$$

$$\Rightarrow x^2 - 4x + 4 = -y^2 + 4$$

$$\Rightarrow (x - 2)^2 - 2^2 = -y^2$$

$$\begin{aligned}
 \therefore \text{ Required area} &= 2 \cdot \int_0^2 \left[\sqrt{2^2 - (x-2)^2} - \sqrt{2x} \right] dx \\
 &= 2 \left[\left[\frac{x-2}{2} \cdot \sqrt{2^2 - (x-2)^2} + \frac{2^2}{2} \sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^2 - \left[\sqrt{2} \cdot \frac{x^{3/2}}{3/2} \right]_0^2 \right] \\
 &= 2 \left[\left(0 + 0 - 1 \cdot 0 + 2 \cdot \frac{\pi}{2} \right) - \frac{2\sqrt{2}}{3} (2^{3/2} - 0) \right] \\
 &= \frac{4\pi}{2} - \frac{8 \cdot 2}{3} = 2\pi - \frac{16}{3} = 2 \left(\pi - \frac{8}{3} \right) \text{ sq units}
 \end{aligned}$$

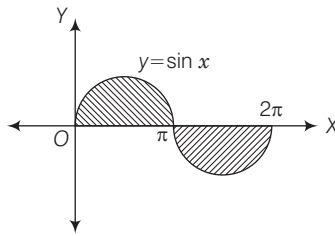
Q. 17 Find the area bounded by the curve $y = \sin x$ between $x = 0$ and $x = 2\pi$.

Thinking Process

We know that, $\sin x$ curve has positive region from $[0, \pi]$ and negative region in $[\pi, 2\pi]$.

Sol. Required area = $\int_0^{2\pi} \sin x \, dx = \int_0^{\pi} \sin x \, dx + \left| \int_{\pi}^{2\pi} \sin x \, dx \right|$

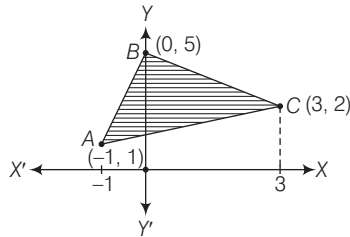
$$\begin{aligned}
 &= -[\cos x]_0^{\pi} + \left| -[\cos x]_{\pi}^{2\pi} \right| \\
 &= -[\cos \pi - \cos 0] + \left| -[\cos 2\pi - \cos \pi] \right|
 \end{aligned}$$



$$\begin{aligned}
 &= -[-1 - 1] + \left| -(1 + 1) \right| \\
 &= 2 + 2 = 4 \text{ sq units}
 \end{aligned}$$

Q. 18 Find the area of region bounded by the triangle whose vertices are $(-1, 1)$, $(0, 5)$ and $(3, 2)$, using integration.

Sol. Let us have the vertices of a ΔABC as $A(-1, 1)$, $B(0, 5)$ and $C(3, 2)$.



\therefore Equation of AB is $y - 1 = \left(\frac{5-1}{0+1} \right) (x + 1)$

$\Rightarrow y - 1 = 4x + 4$

$\Rightarrow y = 4x + 5$

and equation of BC is $y - 5 = \left(\frac{2-5}{3-0} \right) (x - 0)$

... (i)

$$\Rightarrow y - 5 = \frac{-3}{3}(x)$$

$$\Rightarrow y = 5 - x \quad \dots(ii)$$

Similarly, equation of AC is $y - 1 = \left(\frac{2-1}{3+1}\right)(x+1)$

$$\Rightarrow y - 1 = \frac{1}{4}(x+1)$$

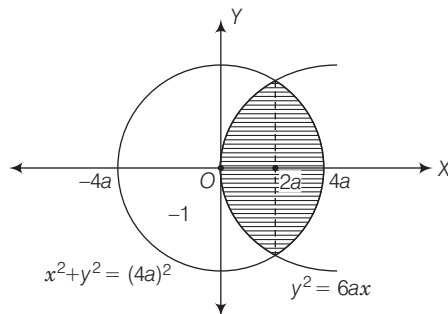
$$\Rightarrow 4y = x + 5 \quad \dots(iii)$$

$$\begin{aligned} \therefore \text{Area of shaded region} &= \int_{-1}^0 (y_1 - y_2) dx + \int_0^3 (y_1 - y_2) dx \\ &= \int_{-1}^0 \left[4x + 5 - \frac{x+5}{4} \right] dx + \int_0^3 \left[5 - x - \frac{x+5}{4} \right] dx \\ &= \left[\frac{4x^2}{2} + 5x - \frac{x^2}{8} - \frac{5x}{4} \right]_{-1}^0 + \left[5x - \frac{x^2}{2} - \frac{x^2}{8} - \frac{5x}{4} \right]_0^3 \\ &= \left[0 - \left(4 \cdot \frac{1}{2} + 5(-1) - \frac{1}{8} + \frac{5}{4} \right) \right] + \left[\left(15 - \frac{9}{2} - \frac{9}{8} - \frac{15}{4} \right) - 0 \right] \\ &= \left[-2 + 5 + \frac{1}{8} - \frac{5}{4} + 15 - \frac{9}{2} - \frac{9}{8} - \frac{15}{4} \right] \\ &= 18 + \left(\frac{1 - 10 - 36 - 9 - 30}{8} \right) \\ &= 18 + \left(-\frac{84}{8} \right) = 18 - \frac{21}{2} = \frac{15}{2} \text{ sq units} \end{aligned}$$

Q. 19 Draw a rough sketch of the region $\{(x, y) : y^2 \leq 6ax \text{ and } x^2 + y^2 \leq 16a^2\}$. Also, find the area of the region sketched using method of integration.

Sol. We have,

$$\begin{aligned} \Rightarrow y^2 &= 6ax \text{ and } x^2 + y^2 = 16a^2 \\ \Rightarrow x^2 + 6ax &= 16a^2 \\ \Rightarrow x^2 + 6ax - 16a^2 &= 0 \\ \Rightarrow x^2 + 8ax - 2ax - 16a^2 &= 0 \\ \Rightarrow x(x + 8a) - 2a(x + 8a) &= 0 \\ \Rightarrow (x - 2a)(x + 8a) &= 0 \\ \Rightarrow x &= 2a, -8a \end{aligned}$$



$$\begin{aligned}
\therefore \text{Area of required region} &= 2 \left[\int_0^{2a} \sqrt{6ax} \, dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} \, dx \right] \\
&= 2 \left[\int_0^{2a} \sqrt{6a} x^{1/2} dx + \int_{2a}^{4a} \sqrt{(4a)^2 - x^2} dx \right] \\
&= 2 \left[\sqrt{6a} \left[\frac{x^{3/2}}{3/2} \right]_0^{2a} + \left(\frac{x}{2} \sqrt{(4a)^2 - x^2} + \frac{(4a)^2}{2} \sin^{-1} \frac{x}{4a} \right) \Bigg|_{2a}^{4a} \right] \\
&= 2 \left[\sqrt{6a} \cdot \frac{2}{3} ((2a)^{3/2} - 0) + \frac{4a}{2} \cdot 0 + \frac{16a^2}{2} \cdot \frac{\pi}{2} - \frac{2a}{2} \sqrt{16a^2 - 4a^2} - \frac{16a^2}{2} \cdot \sin^{-1} \frac{2a}{4a} \right] \\
&= 2 \left[\sqrt{6a} \cdot \frac{2}{3} \cdot 2\sqrt{2} a^{3/2} + 0 + 4\pi a^2 - \frac{2a}{2} \cdot 2\sqrt{3}a - 8a^2 \cdot \frac{\pi}{6} \right] \\
&= 2 \left[\sqrt{12} \cdot \frac{4}{3} a^2 + 4\pi a^2 - 2\sqrt{3}a^2 - \frac{4a^2\pi}{3} \right] \\
&= 2 \left[\frac{8\sqrt{3}a^2 + 12\pi a^2 - 6\sqrt{3}a^2 - 4a^2\pi}{3} \right] \\
&= \frac{2}{3} a^2 [8\sqrt{3} + 12\pi - 6\sqrt{3} - 4\pi] \\
&= \frac{2}{3} a^2 [2\sqrt{3} + 8\pi] = \frac{4}{3} a^2 [\sqrt{3} + 4\pi]
\end{aligned}$$

Q. 20 Compute the area bounded by the lines $x + 2y = 2$, $y - x = 1$ and $2x + y = 7$.

Sol. We have,

$$x + 2y = 2 \quad \dots(i)$$

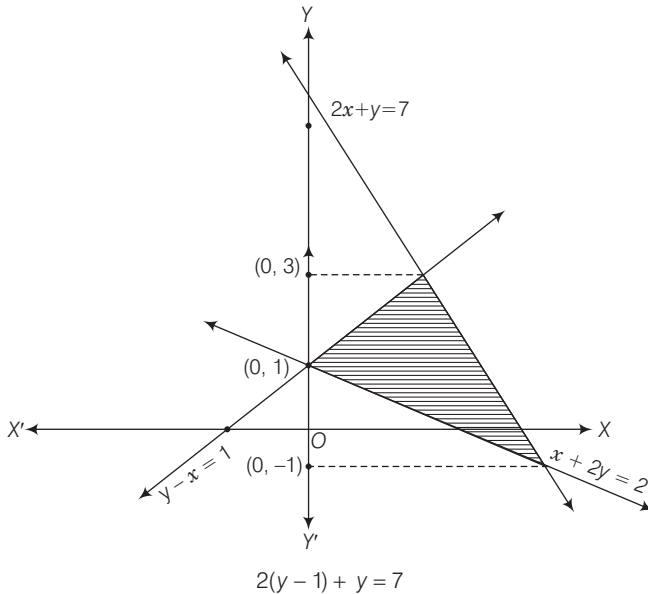
$$y - x = 1 \quad \dots(ii)$$

and

$$2x + y = 7 \quad \dots(iii)$$

On solving Eqs. (i) and (ii), we get

$$y - (2 - 2y) = 1 \Rightarrow 3y - 2 = 1 \Rightarrow y = 1$$



On solving Eqs. (ii) and (iii), we get

$$\begin{aligned} \Rightarrow 2y - 2 + y &= 7 \\ \Rightarrow y &= 3 \end{aligned}$$

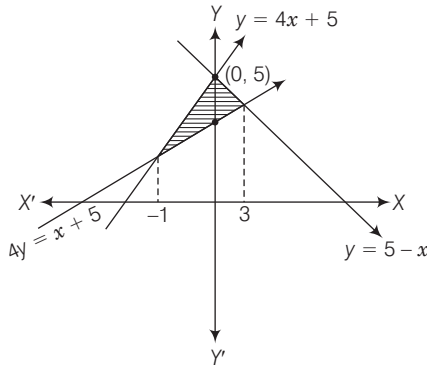
On solving Eqs. (i) and (iii), we get

$$\begin{aligned} \Rightarrow 2(2 - 2y) + y &= 7 \\ \Rightarrow 4 - 4y + y &= 7 \\ \Rightarrow -3y &= 3 \\ \Rightarrow y &= -1 \end{aligned}$$

$$\begin{aligned} \therefore \text{Required area} &= \int_{-1}^1 (2 - 2y) dy + \int_{-1}^3 \frac{(7 - y)}{2} dy - \int_1^3 (y - 1) dy \\ &= \left[-2y + \frac{2y^2}{2} \right]_{-1}^1 + \left[\frac{7y}{2} - \frac{y^2}{2 \cdot 2} \right]_{-1}^3 - \left[\frac{y^2}{2} - y \right]_1^3 \\ &= \left[-2 + \frac{2}{2} - 2 - \frac{2}{2} \right] + \left[\frac{21}{2} - \frac{9}{4} + \frac{7}{2} + \frac{1}{4} \right] - \left[\frac{9}{2} - 3 - \frac{1}{2} + 1 \right] \\ &= [-4] + \left[\frac{42 - 9 + 14 + 1}{4} \right] - \left[\frac{9 - 6 - 1 + 2}{2} \right] \\ &= -4 + 12 - 2 = 6 \text{ sq units} \end{aligned}$$

Q. 21 Find the area bounded by the lines $y = 4x + 5$, $y = 5 - x$ and $4y = x + 5$.

Sol.



Given equations of lines are

$$y = 4x + 5 \quad \dots(i)$$

$$y = 5 - x \quad \dots(ii)$$

and

$$4y = x + 5 \quad \dots(iii)$$

On solving Eqs. (i) and (ii), we get

$$\begin{aligned} 4x + 5 &= 5 - x \\ \Rightarrow x &= 0 \end{aligned}$$

On solving Eqs. (i) and (iii), we get

$$\begin{aligned} \Rightarrow 4(4x + 5) &= x + 5 \\ \Rightarrow 16x + 20 &= x + 5 \\ \Rightarrow 15x &= -15 \\ \Rightarrow x &= -1 \end{aligned}$$

On solving Eqs. (ii) and (iii), we get

$$\begin{aligned} \Rightarrow 4(5 - x) &= x + 5 \\ \Rightarrow 20 - 4x &= x + 5 \end{aligned}$$

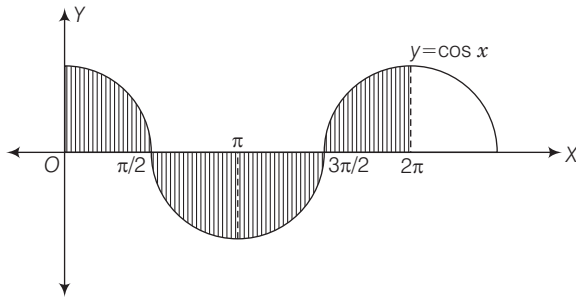
⇒

$$\begin{aligned}
 \therefore \text{ Required area} &= \int_{-1}^0 (4x + 5) dx + \int_0^3 (5 - x) dx - \frac{1}{4} \int_{-1}^3 (x + 5) dx \\
 &= \left[\frac{4x^2}{2} + 5x \right]_{-1}^0 + \left[5x - \frac{x^2}{2} \right]_0^3 - \frac{1}{4} \left[\frac{x^2}{2} + 5x \right]_{-1}^3 \\
 &= [0 - 2 + 5] + \left[15 - \frac{9}{2} - 0 \right] - \frac{1}{4} \left[\frac{9}{2} + 15 - \frac{1}{2} + 5 \right] \\
 &= 3 + \frac{21}{2} - \frac{1}{4} \cdot 24 \\
 &= -3 + \frac{21}{2} = \frac{15}{2} \text{ sq units}
 \end{aligned}$$

Q. 22 Find the area bounded by the curve $y = 2\cos x$ and the X -axis from $x = 0$ to $x = 2\pi$.

Sol. Required area of shaded region = $\int_0^{2\pi} 2\cos x dx$

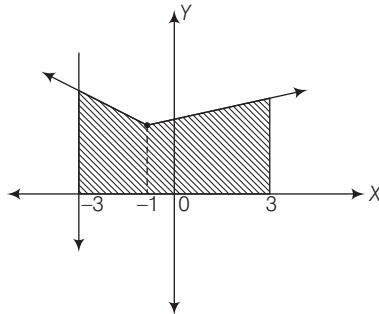
$$= \int_0^{\pi/2} 2\cos x dx + \left| \int_{\pi/2}^{3\pi/2} 2\cos x dx \right| + \int_{3\pi/2}^{2\pi} 2\cos x dx$$



$$\begin{aligned}
 &= 2[\sin x]_0^{\pi/2} + \left| 2(\sin x)_{\pi/2}^{3\pi/2} \right| + 2[\sin x]_{3\pi/2}^{2\pi} \\
 &= 2 + 4 + 2 = 8 \text{ sq units}
 \end{aligned}$$

Q. 23 Draw a rough sketch of the given curve $y = 1 + |x + 1|$, $x = -3$, $x = 3$, $y = 0$ and find the area of the region bounded by them, using integration.

Sol. We have, $y = 1 + |x + 1|$, $x = -3$, $x = 3$ and $y = 0$



$$\begin{aligned} \therefore y &= \begin{cases} -x, & \text{if } x < -1 \\ x + 2, & \text{if } x \geq -1 \end{cases} \\ \therefore \text{Area of shaded region, } A &= \int_{-3}^{-1} -x \, dx + \int_{-1}^3 (x + 2) \, dx \\ &= -\left[\frac{x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^3 \\ &= -\left[\frac{1}{2} - \frac{9}{2}\right] + \left[\frac{9}{2} + 6 - \frac{1}{2} + 2\right] \\ &= -[-4] + [8 + 4] \\ &= 12 + 4 = 16 \text{ sq units} \end{aligned}$$

Objective Type Questions

Q. 24 The area of the region bounded by the Y-axis $y = \cos x$ and $y = \sin x$, where $0 \leq x \leq \frac{\pi}{2}$, is

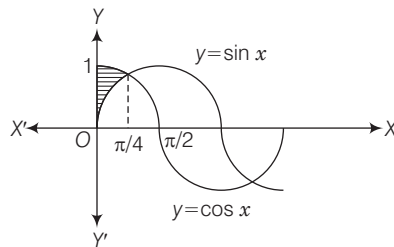
(a) $\sqrt{2}$ sq units

(b) $(\sqrt{2} + 1)$ sq units

(c) $(\sqrt{2} - 1)$ sq units

(d) $(2\sqrt{2} - 1)$ sq units

Sol. (c) We have, Y-axis i.e., $x = 0$, $y = \cos x$ and $y = \sin x$, where $0 \leq x \leq \frac{\pi}{2}$.



$$\begin{aligned} \therefore \text{Required area} &= \int_0^{\pi/4} (\cos x - \sin x) \, dx \\ &= [\sin x]_0^{\pi/4} + [\cos x]_0^{\pi/4} \\ &= \left(\sin \frac{\pi}{4} - \sin 0\right) + \left(\cos \frac{\pi}{4} - \cos 0\right) \\ &= \left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - 1\right) \\ &= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 \\ &= -1 + \frac{2}{\sqrt{2}} = \frac{-\sqrt{2} + 2}{\sqrt{2}} \\ &= \frac{-2 + 2\sqrt{2}}{2} = (\sqrt{2} - 1) \text{ sq units} \end{aligned}$$

Q. 25 The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

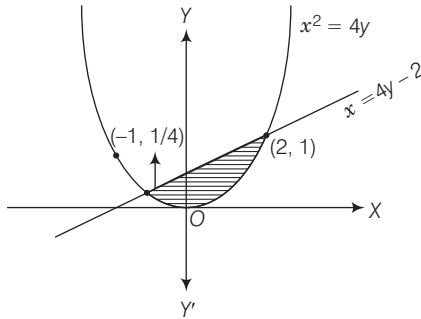
(a) $\frac{3}{8}$ sq unit

(b) $\frac{5}{8}$ sq unit

(c) $\frac{7}{8}$ sq unit

(d) $\frac{9}{8}$ sq units

Sol. (d) Given equation of curve is $x^2 = 4y$ and the straight line $x = 4y - 2$.



For intersection point, put $x = 4y - 2$ in equation of curve, we get

$$(4y - 2)^2 = 4y$$

$$\Rightarrow 16y^2 + 4 - 16y = 4y$$

$$\Rightarrow 16y^2 - 20y + 4 = 0$$

$$\Rightarrow 4y^2 - 5y + 1 = 0$$

$$\Rightarrow 4y^2 - 4y - y + 1 = 0$$

$$\Rightarrow 4y(y - 1) - 1(y - 1) = 0$$

$$\Rightarrow (4y - 1)(y - 1) = 0$$

$$\therefore y = 1, \frac{1}{4}$$

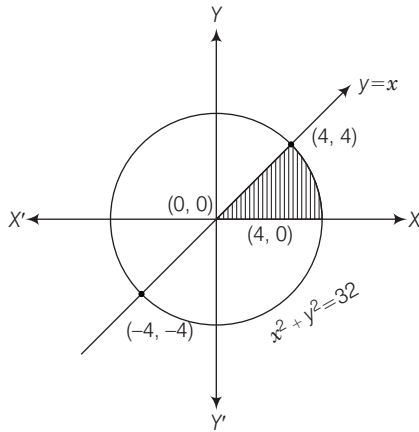
For $y = 1$, $x = \sqrt{4 \cdot 1} = 2$ [since, negative value does not satisfy the equation of line]

For $y = \frac{1}{4}$, $x = \sqrt{4 \cdot \frac{1}{4}} = -1$ [positive value does not satisfy the equation of line]

So, the intersection points are $(2, 1)$ and $(-1, \frac{1}{4})$.

$$\begin{aligned} \therefore \text{Area of shaded region} &= \int_{-1}^2 \left(\frac{x+2}{4} \right) dx - \int_{-1}^2 \frac{x^2}{4} dx \\ &= \frac{1}{4} \left[\frac{x^2}{2} + 2x \right]_{-1}^2 - \frac{1}{4} \left[\frac{x^3}{3} \right]_{-1}^2 \\ &= -\frac{1}{4} \left[\frac{4}{2} + 4 - \frac{1}{2} + 2 \right] - \frac{1}{4} \left[\frac{8}{3} + \frac{1}{3} \right] \\ &= \frac{1}{4} \cdot \frac{15}{2} - \frac{1}{4} \cdot \frac{9}{3} = \frac{45 - 18}{24} \\ &= \frac{27}{24} = \frac{9}{8} \text{ sq units} \end{aligned}$$

So, the circle intersects the X-axis at $(\pm 4\sqrt{2}, 0)$.

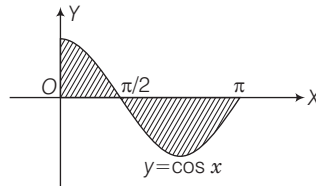


$$\begin{aligned}
 \text{Area of shaded region} &= \int_0^4 x dx + \int_4^{4\sqrt{2}} \sqrt{(4\sqrt{2})^2 - x^2} dx \\
 &= \left[\frac{x^2}{2} \right]_0^4 + \left[\frac{x}{2} \sqrt{(4\sqrt{2})^2 - x^2} + \frac{(4\sqrt{2})^2}{2} \sin^{-1} \frac{x}{4\sqrt{2}} \right]_4^{4\sqrt{2}} \\
 &= \frac{16}{2} + \left[\frac{4\sqrt{2}}{2} \cdot 0 + 16 \sin^{-1} \frac{(4\sqrt{2})}{(4\sqrt{2})} - \frac{4}{2} \sqrt{(4\sqrt{2})^2 - 16} - 16 \sin^{-1} \frac{4}{4\sqrt{2}} \right] \\
 &= 8 + \left[16 \cdot \frac{\pi}{2} - 2 \cdot \sqrt{16} - 16 \cdot \frac{\pi}{4} \right] \\
 &= 8 + [8\pi - 8 - 4\pi] = 4\pi \text{ sq units}
 \end{aligned}$$

Q. 28 Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = \pi$ is

- (a) 2 sq units (b) 4 sq units
(c) 3 sq units (d) 1 sq unit

Sol. (a) Required area enclosed by the curve $y = \cos x$, $x = 0$ and $x = \pi$ is

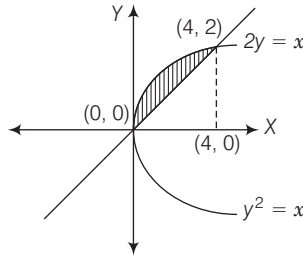


$$\begin{aligned}
 A &= \int_0^{\pi/2} \cos x dx + \left| \int_{\pi/2}^{\pi} \cos x dx \right| \\
 &= \left[\sin \frac{\pi}{2} - \sin 0 \right] + \left| \sin \frac{\pi}{2} - \sin \pi \right| \\
 &= 1 + 1 = 2 \text{ sq units}
 \end{aligned}$$

Q. 29 The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is

- (a) $\frac{4}{3}$ sq units (b) 1 sq unit (c) $\frac{2}{3}$ sq unit (d) $\frac{1}{3}$ sq unit

Sol. (a) We have to find the area enclosed by parabola $y^2 = x$ and the straight line $2y = x$.



$$\begin{aligned} \therefore \left(\frac{x}{2}\right)^2 &= x \\ \Rightarrow x^2 &= 4x \Rightarrow x(x-4) = 0 \\ \Rightarrow x &= 4 \Rightarrow y = 2 \text{ and } x = 0 \Rightarrow y = 0 \end{aligned}$$

So, the intersection points are (0, 0) and (4, 2).

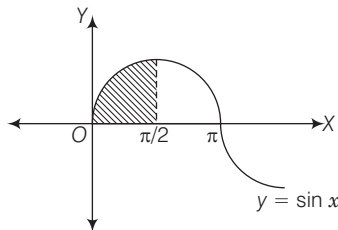
Area enclosed by shaded region,

$$\begin{aligned} A &= \int_0^4 \left[\sqrt{x} - \frac{x}{2} \right] dx \\ &= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{1}{2} \cdot \frac{x^2}{2} \right]_0^4 = \left[2 \cdot \frac{x^{3/2}}{3} - \frac{x^2}{4} \right]_0^4 \\ &= \frac{2}{3} 4^{3/2} - \frac{16}{4} - \frac{2}{3} \cdot 0 + \frac{1}{4} \cdot 0 \\ &= \frac{16}{3} - \frac{16}{4} = \frac{64 - 48}{12} = \frac{16}{12} = \frac{4}{3} \text{ sq units} \end{aligned}$$

Q. 30 The area of the region bounded by the curve $y = \sin x$ between the ordinates $x = 0$, $x = \frac{\pi}{2}$ and the X-axis is

- (a) 2 sq units (b) 4 sq units (c) 3 sq units (d) 1 sq unit

Sol. (d) Area of the region bounded by the curve $y = \sin x$ between the ordinates $x = 0$, $x = \frac{\pi}{2}$ and the X-axis is



$$\begin{aligned}
 A &= \int_0^{\pi/2} \sin x dx \\
 &= -[\cos x]_0^{\pi/2} = -\left[\cos \frac{\pi}{2} - \cos 0\right] \\
 &= -[0 - 1] = 1 \text{ sq unit}
 \end{aligned}$$

Q. 31 The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is

- (a) 20π sq units (b) $20\pi^2$ sq units (c) $16\pi^2$ sq units (d) 25π sq units

Sol. (a) We have,

$$\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$$

Here,

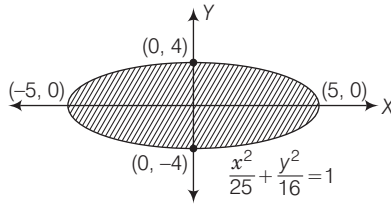
$$a = \pm 5 \text{ and } b = \pm 4$$

and

$$\frac{y^2}{4^2} = 1 - \frac{x^2}{5^2}$$

\Rightarrow

$$y^2 = 16 \left(1 - \frac{x^2}{25}\right)$$



\Rightarrow

$$y = \sqrt{\frac{16}{25}(25 - x^2)}$$

\Rightarrow

$$y = \frac{4}{5}\sqrt{5^2 - x^2}$$

$$\therefore \text{Area enclosed by ellipse, } A = 2 \cdot \frac{4}{5} \int_{-5}^5 \sqrt{5^2 - x^2} dx$$

$$= 2 \cdot \frac{8}{5} \int_0^5 \sqrt{5^2 - x^2} dx$$

$$= 2 \cdot \frac{8}{5} \left[\frac{x}{2} \sqrt{5^2 - x^2} + \frac{5^2}{2} \sin^{-1} \frac{x}{5} \right]_0^5$$

$$= 2 \cdot \frac{8}{5} \left[\frac{5}{2} \sqrt{5^2 - 5^2} + \frac{5^2}{2} \sin^{-1} \frac{5}{5} - 0 - \frac{25}{2} \cdot 0 \right]$$

$$= 2 \cdot \frac{8}{5} \left[\frac{25}{2} \cdot \frac{\pi}{2} \right]$$

$$= \frac{16}{5} \cdot \frac{25\pi}{4}$$

$$= 20\pi \text{ sq units}$$

Q. 34 The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$, $y = -1$ is

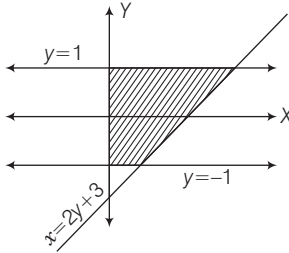
(a) 4 sq units

(b) $\frac{3}{2}$ sq units

(c) 6 sq units

(d) 8 sq units

Sol. (c) Required area, $A = \int_{-1}^1 (2y + 3) dy$



$$\begin{aligned} &= \left[\frac{2y^2}{2} + 3y \right]_{-1}^1 \\ &= [y^2 + 3y]_{-1}^1 \\ &= [1 + 3 - 1 + 3] = 6 \text{ sq units} \end{aligned}$$