Activity 11

Objective

To verify Rolle's Theorem.

Material Required

A piece of plywood, wires of different lengths, white paper, sketch pen.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent x-axis and y-axis (see Fig. 11).
3. Take a piece of wire of 15 cm length and bend it in the shape of a curve and fix it on the plywood as shown in the figure.

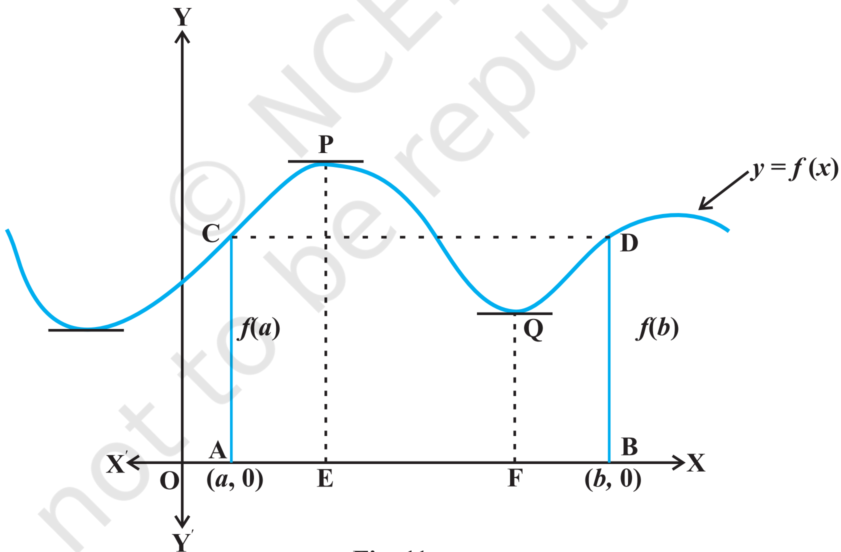

Fig. 11

4. Take two straight wires of the same length and fix them in such way that they are perpendicular to x-axis at the points A and B and meeting the curve at the points C and D (see Fig.11).

DEMONSTRATION

1. In the figure, let the curve represent the function $y = f(x)$. Let OA = a units and OB = b units.
2. The coordinates of the points A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. There is no break in the curve in the interval $[a, b]$. So, the function f is continuous on $[a, b]$.
4. The curve is smooth between $x = a$ and $x = b$ which means that at each point, a tangent can be drawn which in turn gives that the function f is differentiable in the interval (a, b).
5. As the wires at A and B are of equal lengths, i.e., AC = BD, so $f(a) = f(b)$.
6. In view of steps (3), (4) and (5), conditions of Rolle's theorem are satisfied. From Fig.11, we observe that tangents at P as well as Q are parallel to x-axis, therefore, $f'(x)$ at P and also at Q are zero.

Thus, there exists at least one value c of x in (a,b) such that $f'(c) = 0$.

Hence, the Rolle's theorem is verified.

OBSERVATION

From Fig. 11.

a = _______________, b = ______________

$f(a)$ = ____________, $f(b)$ = _________ Is $f(a) = f(b)$? (Yes/No)

Slope of tangent at P = __________, so, $f(x)$ (at P) =

APPLICATION

This theorem may be used to find the roots of an equation.

Activity 12

OBJECTIVE

To verify Lagrange's Mean Value Theorem.

MATERIAL REQUIRED

A piece of plywood, wires, white paper, sketch pens, wires.

METHOD OF CONSTRUCTION

1. Take a piece of plywood and paste a white paper on it.
2. Take two wires of convenient size and fix them on the white paper pasted on the plywood to represent x-axis and y-axis (see Fig. 12).
3. Take a piece of wire of about 10 cm length and bend it in the shape of a curve as shown in the figure. Fix this curved wire on the white paper pasted on the plywood.

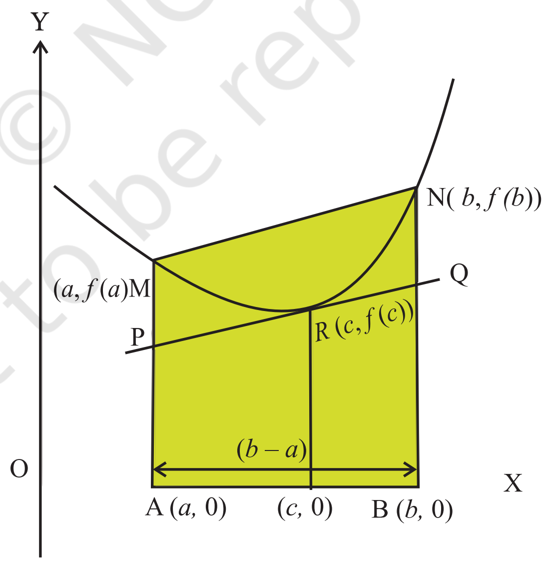

Fig. 12

4. Take two straight wires of lengths 10 cm and 13 cm and fix them at two different points of the curve parallel to y-axis and their feet touching the x-axis. Join the two points, where the two vertical wires meet the curve, using another wire.
5. Take one more wire of a suitable length and fix it in such a way that it is tangential to the curve and is parallel to the wire joining the two points on the curve (see Fig. 12).

Demonstration

1. Let the curve represent the function $y = f(x)$. In the figure, let OA = a units and OB = b units.
2. The coordinates of A and B are $(a, 0)$ and $(b, 0)$, respectively.
3. MN is a chord joining the points M $(a, f(a))$ and N $(b, f(b))$.
4. PQ represents a tangent to the curve at the point R $(c, f(c))$, in the interval (a, b).
5. $f'(c)$ is the slope of the tangent PQ at $x = c$.
6. $\dfrac{f(b)-f(a)}{b-a}$ is the slope of the chord MN.
7. MN is parallel to PQ, therefore, $f'(c) = \dfrac{f(b)-f(a)}{b-a}$. Thus, the Langrange's Mean Value Theorem is verified.

Observation

1. a = ___________, b = _______________,

   $f(a)$ = ________, $f(b)$= ____________.
2. $f(a) - f(b)$ = ________,

   $b - a$ = ________,

3. $\frac{f(b)-f(a)}{b-a}$ = ________ = Slope of MN.

4. Since PQ || MN $\Rightarrow$ Slope of PQ $= f'(c) = \frac{f(a)-f(a)}{b-a}$.

APPLICATION

Langrange's Mean Value Theorem has significant applications in calculus. For example this theorem is used to explain concavity of the graph.

Activity 13

OBJECTIVE

To understand the concepts of decreasing and increasing functions.

MATERIAL REQUIRED

Pieces of wire of different lengths, piece of plywood of suitable size, white paper, adhesive, geometry box, trigonometric tables.

METHOD OF CONSTRUCTION

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires of length say 20 cm each and fix them on the white paper to represent *x*-axis and *y*-axis.
3. Take two more pieces of wire each of suitable length and bend them in the shape of curves representing two functions and fix them on the paper as shown in the Fig. 13.

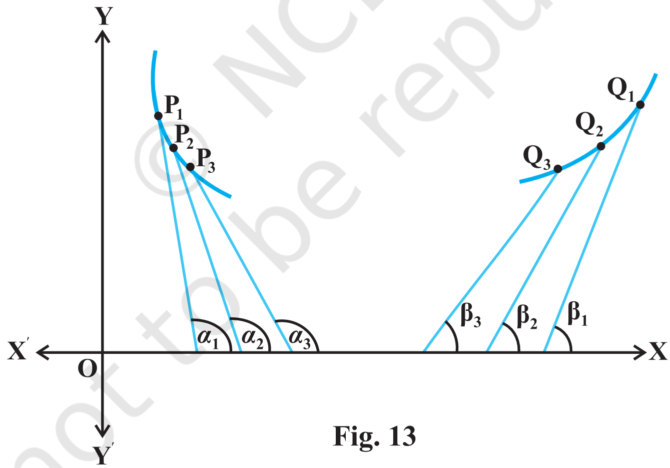

Fig. 13

4. Take two straight wires each of suitable length for the purpose of showing tangents to the curves at different points on them.

DEMONSTRATION

1. Take one straight wire and place it on the curve (on the left) such that it is

tangent to the curve at the point say P_1 and making an angle α_1 with the positive direction of x-axis.

2. α_1 is an obtuse angle, so $\tan\alpha_1$ is negative, i.e., the slope of the tangent at P_1 (derivative of the function at P_1) is negative.

3. Take another two points say P_2 and P_3 on the same curve, and make tangents, using the same wire, at P_2 and P_3 making angles α_2 and α_3, respectively with the positive direction of x-axis.

4. Here again α_2 and α_3 are obtuse angles and therefore slopes of the tangents $\tan \alpha_2$ and $\tan \alpha_3$ are both negative, i.e., derivatives of the function at P_2 and P_3 are negative.

5. The function given by the curve (on the left) is a decreasing function.

6. On the curve (on the right), take three point Q_1, Q_2, Q_3, and using the other straight wires, form tangents at each of these points making angles β_1, β_2, β_3, respectively with the positive direction of x-axis, as shown in the figure. β_1, β_2, β_3 are all acute angles.

   So, the derivatives of the function at these points are positive. Thus, the function given by this curve (on the right) is an increasing function.

Observation

1. α_1 = _______ , > 90° α_2 = _______ > _______, α_3 = _______> _______, tan α_1 = _______, (negative) tan α_2 = _______, (_______), tan α_3 = _______, (_______). Thus the function is _______.

2. β_1 = _______< 90°, β_2 = _______, < _______, β_3 = _______ , < _______

   tan β_1 = _______ , (positive), tan β_2 = _______, (_______), tan β_3 = _______(_______). Thus, the function is _______.

Application

This activity may be useful in explaining the concepts of decreasing and increasing functions.

Activity 14

Objective

To understand the concepts of local maxima, local minima and point of inflection.

Material Required

A piece of plywood, wires, adhesive, white paper.

Method of Construction

1. Take a piece of plywood of a convenient size and paste a white paper on it.
2. Take two pieces of wires each of length 40 cm and fix them on the paper on plywood in the form of x-axis and y-axis.
3. Take another wire of suitable length and bend it in the shape of curve. Fix this curved wire on the white paper pasted on plywood, as shown in Fig. 14.

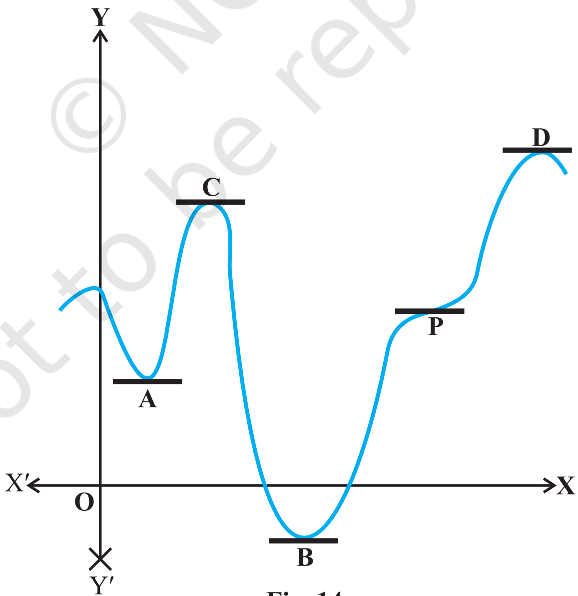

Fig. 14

4. Take five more wires each of length say 2 cm and fix them at the points A, C, B, P and D as shown in figure.

Demonstration

1. In the figure, wires at the points A, B, C and D represent tangents to the curve and are parallel to the axis. The slopes of tangents at these points are zero, i.e., the value of the first derivative at these points is zero. The tangent at P intersects the curve.
2. At the points A and B, sign of the first derivative changes from negative to positive. So, they are the points of local minima.
3. At the point C and D, sign of the first derivative changes from positive to negative. So, they are the points of local maxima.
4. At the point P, sign of first derivative does not change. So, it is a point of inflection.

Observation

1. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate left of A is _______.
2. Sign of the slope of the tangent (first derivative) at a point on the curve to the immediate right of A is_______.
3. Sign of the first derivative at a point on the curve to immediate left of B is _______.
4. Sign of the first derivative at a point on the curve to immediate right of B is _______.
5. Sign of the first derivative at a point on the curve to immediate left of C is _______.
6. Sign of the first derivative at a point on the curve to immediate right of C is _______.
7. Sign of the first derivative at a point on the curve to immediate left of D is _______.

8. Sign of the first derivative at a point on the curve to immediate right of D is _______.
9. Sign of the first derivative at a point immediate left of P is _______ and immediate right of P is_______.
10. A and B are points of local _______.
11. C and D are points of local _______.
12. P is a point of _______.

APPLICATION

1. This activity may help in explaining the concepts of points of local maxima, local minima and inflection.
2. The concepts of maxima/minima are useful in problems of daily life such as making of packages of maximum capacity at minimum cost.

Activity 15

OBJECTIVE

To understand the concepts of absolute maximum and minimum values of a function in a given closed interval through its graph.

MATERIAL REQUIRED

Drawing board, white chart paper, adhesive, geometry box, pencil and eraser, sketch pens, ruler, calculator.

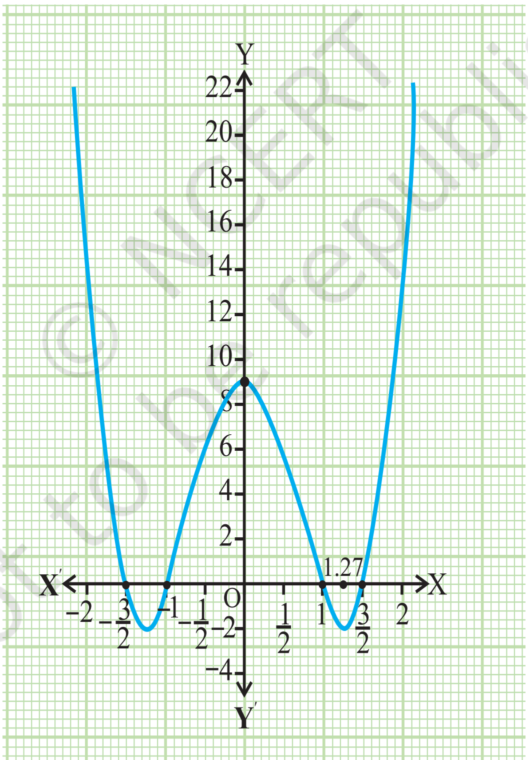

Fig 15

METHOD OF CONSTRUCTION

1. Fix a white chart paper of convenient size on a drawing board using adhesive.
2. Draw two perpendicular lines on the squared paper as the two rectangular axes.
3. Graduate the two axes as shown in Fig.15.
4. Let the given function be $f(x) = (4x^2 - 9)(x^2 - 1)$ in the interval [–2, 2].
5. Taking different values of x in [–2, 2], find the values of $f(x)$ and plot the ordered pairs $(x, f(x))$.
6. Obtain the graph of the function by joining the plotted points by a free hand curve as shown in the figure.

DEMONSTRATION

1. Some ordered pairs satisfying $f(x)$ are as follows:

x	0	± 0.5	± 1.0	1.25	1.27	± 1.5	± 2
$f(x)$	9	6	0	– 1.55	–1.56	0	21

2. Plotting these points on the chart paper and joining the points by a free hand curve, the curve obtained is shown in the figure.

OBSERVATION

1. The absolute maximum value of $f(x)$ is ________ at x = ________.
2. Absolute minimum value of $f(x)$ is ________ at x = ________.

APPLICATION

The activity is useful in explaining the concepts of absolute maximum / minimum value of a function graphically.

NOTE

Consider $f(x) = (4x^2 - 9)(x^2 - 1)$

$f(x) = 0$ gives the values of x as $\pm\frac{3}{2}$ and ± 1. Both these values of x lie in the given closed interval [–2, 2].

$f'(x) = (4x^2 - 9)\,2x + 8x(x^2 - 1) = 16x^3 - 26x = 2x(8x^2 - 13)$

$f'(x) = 0$ gives $x = 0,\ x = \pm\sqrt{\frac{13}{8}} = \pm 1.27$. These two values of x lie in [–2, 2].

The function has local maxima/minima at $x = 0$ and $x = \pm\ 1.27$, respectively.

Activity 16

Objective

To construct an open box of maximum volume from a given rectangular sheet by cutting equal squares from each corner.

Material Required

Chart papers, scissors, cellotape, calculator.

Method of Construction

1. Take a rectangular chart paper of size 20 cm × 10 cm and name it as ABCD.
2. Cut four equal squares each of side x cm from each corner A, B, C and D.
3. Repeat the process by taking the same size of chart papers and different values of x.
4. Make an open box by folding its flaps using cellotape/adhesive.

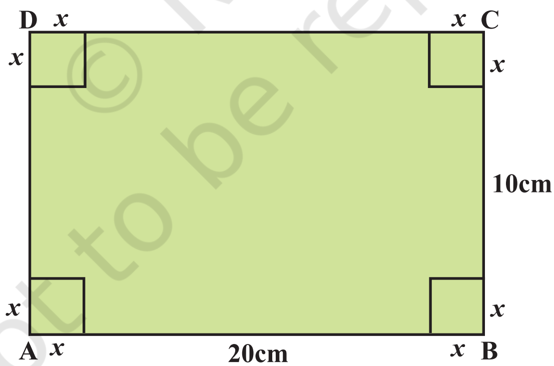

Fig. 16

Demonstration

1. When $x = 1$, Volume of the box = 144 cm^3
2. When $x = 1.5$, Volume of the box = 178.5 cm^3

3. When $x = 1.8$, Volume of the box = 188.9 cm^3.

4. When $x = 2$, Volume of the box = 192 cm^3.

5. When $x = 2.1$, Volume of the box = 192.4 cm^3.

6. When $x = 2.2$, Volume of the box = 192.2 cm^3.

7. When $x = 2.5$, Volume of the box = 187.5 cm^3.

8. When $x = 3$, Volume of the box = 168 cm^3.

Clearly, volume of the box is maximum when $x = 2.1$.

OBSERVATION

1. V_1 = Volume of the open box (when $x = 1.6$) = ...
2. V_2 = Volume of the open box (when $x = 1.9$) = ...
3. V = Volume of the open box (when $x = 2.1$) = ...
4. V_3 = Volume of the open box (when $x = 2.2$) = ...
5. V_4 = Volume of the open box (when $x = 2.4$) = ...
6. V_5 = Volume of the open box (when $x = 3.2$) = ...
7. Volume V_1 is ____________ than volume V.
8. Volume V_2 is ____________ than volume V.
9. Volume V_3 is ____________ than volume V.
10. Volume V_4 is ____________ than volume V.
11. Volume V_5 is ____________ than volume V.

So, Volume of the open box is maximum when x = ________.

APPLICATION

This activity is useful in explaining the concepts of maxima/minima of functions. It is also useful in making packages of maximum volume with minimum cost.

NOTE

Let V denote the volume of the box.

Now V = (20 – 2x) (10 – 2x) x

or V = 200x – 60x^2 + 4x^3

$\frac{dV}{dx} = 200 - 120x + 12x^2$. For maxima or minima, we have,

$\frac{dV}{dx} = 0$, i.e., $3x^2 - 30x + 50 = 0$

i.e., $x = \frac{30 \pm \sqrt{900 - 600}}{6} = 7.9$ or 2.1

Reject $x = 7.9$.

$\frac{d^2V}{dx^2} = -120 + 24x$

When $x = 2.1$, $\frac{d^2V}{dx^2}$ is negative.

Hence, V should be maximum at $x = 2.1$.

Activity 17

OBJECTIVE

To find the time when the area of a rectangle of given dimensions become maximum, if the length is decreasing and the breadth is increasing at given rates.

MATERIAL REQUIRED

Chart paper, paper cutter, scale, pencil, eraser, cardboard.

METHOD OF CONSTRUCTION

1. Take a rectangle R_1 of dimensions 16 cm × 8 cm.
2. Let the length of the rectangle is decreasing at the rate of 1cm/second and the breadth is increasing at the rate of 2 cm/second.
3. Cut other rectangle R_2, R_3, R_4, R_5, R_6, R_7, R_8, R_9, etc. of dimensions 15 cm × 10 cm, 14 cm × 12 cm, 13 cm × 14 cm, 12 cm × 16 cm, 11 cm × 18 cm, 10 cm × 20 cm, 9 cm × 22 cm, 8 cm × 24 cm (see Fig.17).
4. Paste these rectangles on card board.

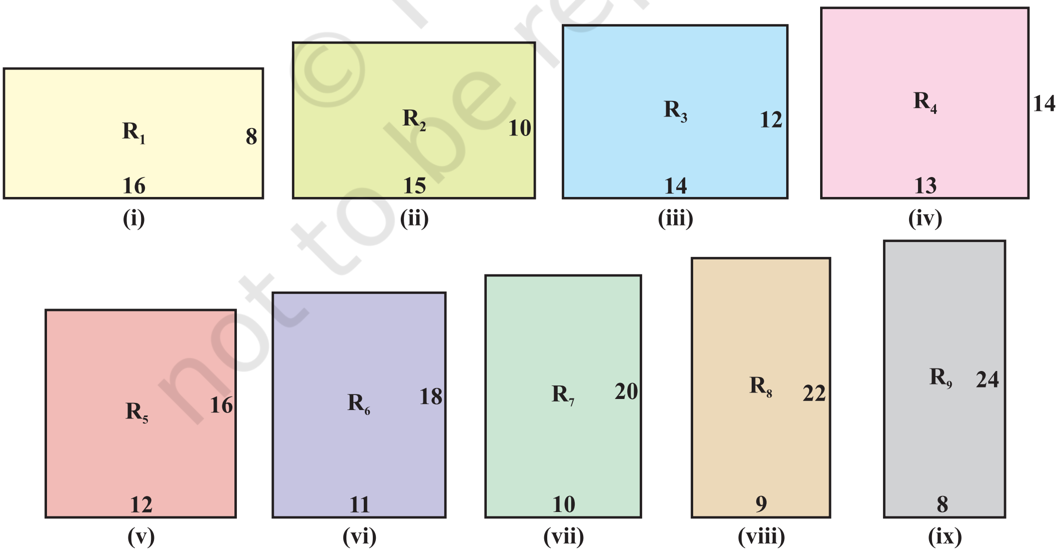

Fig. 17

Demonstration

1. Length of the rectangle is decreasing at the rate of 1cm/s and the breadth is increasing at the rate of 2cm/s.
2. (i) Area of the given rectangle $R_1 = 16 \times 8 = 128$ cm^2.
   (ii) Area of rectangle $R_2 = 15 \times 10 = 150$ cm^2 (after 1 sec).
   (iii) Area of rectangle $R_3 = 168$ cm^2 (after 2 sec).
   (iv) Area of rectangle $R_4 = 182$ cm^2 (after 3 sec).
   (v) Area of rectangle $R_5 = 192$ cm^2 (after 4 sec).
   (vi) Area of rectangle $R_6 = 198$ cm^2 (after 5 sec).
   (vii) Area of rectangle $R_7 = 200$ cm^2 (after 6 sec).
   (viii) Area of rectangle $R_8 = 198$ cm^2 (after 7 sec) and so on.

Thus the area of the rectangle is maximum after 6 sec.

Observation

1. Area of the rectangle R_2 (after 1 sec) = __________.
2. Area of the rectangle R_4 (after 3 sec) = __________.
3. Area of the rectangle R_6 (after 5 sec) = __________.
4. Area of the rectangle R_7 (after 6 sec) = __________.
5. Area of the rectangle R_8 (after 7 sec) = __________.
6. Area of the rectangle R_9 (after 8 sec) = __________.
7. Rectangle of Maximum area (after ..... seconds) = ________.
8. Area of the rectangle is maximum after _________ sec.
9. Maximum area of the rectangle is _________.

APPLICATION

This activity can be used in explaining the concept of rate of change and optimisation of a function.

The function has local maxima/minima at $x = 0$ and $x = \pm 1.27$, respectively.

NOTE

Let the length and breadth of rectangle be a and b.

The length of rectangle after t seconds = $a - t$.

The breadth of rectangle after t seconds = $b + 2t$.

Area of the rectangle (after t sec) = $A(t) = (a - t)(b + 2t) = ab - bt + 2at - 2t^2$

$A'(t) = -b + 2a - 4t$

For maxima or minima, $A'(t) = 0$.

$$A'(t)=0 \quad t \quad \frac{2a-b}{4}$$

$$A''(t)=-4$$

$A'' \quad \frac{2a-b}{4} \quad =-4$, which is negative

Thus, $A(t)$ is maximum at $t=\frac{2a-b}{4}$ seconds.

Here, a = 16 cm, b = 8 cm.

Thus, $t=\frac{32-8}{4}=\frac{24}{4}=6$ seconds

Hence, after 6 second, the area will become maximum.

Activity 18

Objective

To verify that amongst all the rectangles of the same perimeter, the square has the maximum area.

Material Required

Chart paper, paper cutter, scale, pencil, eraser cardboard, glue.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Make rectangles each of perimeter say 48 cm on a chart paper. Rectangles of different dimensions are as follows:

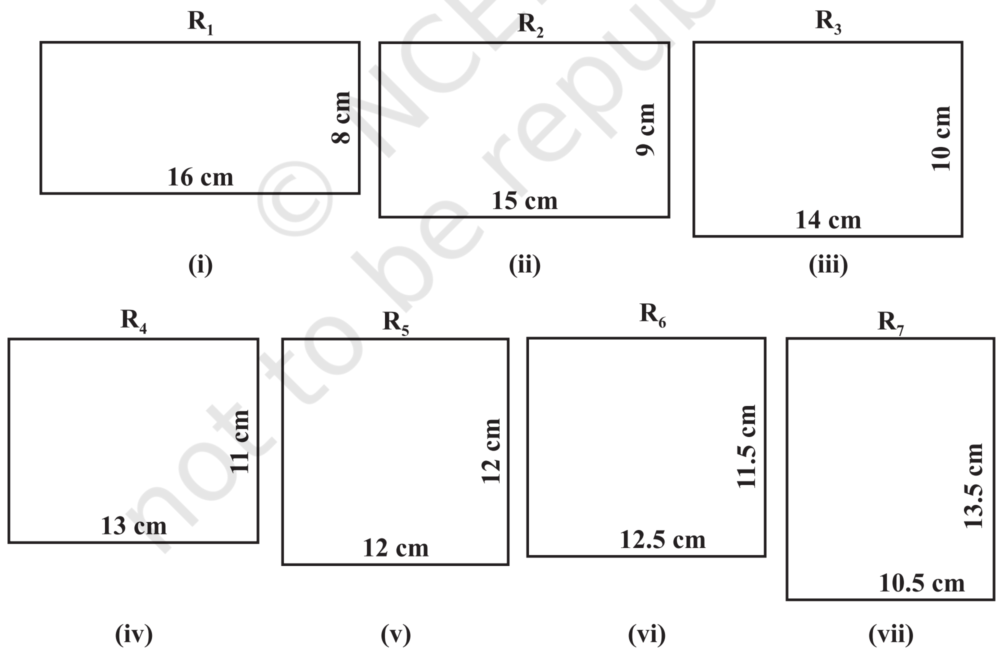

Fig. 18

R_1 : 16 cm × 8 cm, R_2 : 15 cm × 9 cm

R_3 : 14 cm × 10 cm, R_4 : 13 cm × 11 cm

R_5 : 12 cm × 12 cm, R_6 : 12.5 cm × 11.5 cm

R_7 : 10.5 cm × 13.5 cm

3. Cut out these rectangles and paste them on the white paper on the cardboard (see Fig. 18 (i) to (vii)).
4. Repeat step 2 for more rectangles of different dimensions each having perimeter 48 cm.
5. Paste these rectangles on cardboard.

Demonstration

1. Area of rectangle of R_1 = 16 cm × 8 cm = 128 cm^2

   Area of rectangle R_2 = 15 cm × 9 cm = 135 cm^2

   Area of R_3 = 140 cm^2

   Area of R_4 = 143 cm^2

   Area of R_5 = 144 cm^2

   Area of R_6 = 143.75 cm^2

   Area of R_7 = 141.75 cm^2

2. Perimeter of each rectangle is same but their area are different. Area of rectangle R_5 is the maximum. It is a square of side 12 cm. This can be verified using theoretical description given in the note.

Observation

1. Perimeter of each rectangle R_1, R_2, R_3, R_4, R_4, R_6, R_7 is _________.
2. Area of the rectangle R_3 ________ than the area of rectangle R_5.

3. Area of the rectangle R_6 _______ than the area of rectangle R_5.

4. The rectangle R_5 has the diamensions ______ × ______ and hence it is a ________.

5. Of all the rectangles with same perimeter, the ________ has the maximum area.

Application

This activity is useful in explaining the idea of Maximum of a function. The result is also useful in preparing economical packages.

Note

Let the length and breadth of rectangle be x and y.

The perimeter of the rectangle P = 48 cm.

$2(x + y) = 48$

or $x + y = 24$ or $y = 24 - x$

Let A (x) be the area of rectangle, then

$A(x) = xy$

$= x(24 - x)$

$= 24x - x^2$

$A'(x) = 24 - 2x$

$A'(x) = \Rightarrow 24 - 2x = 0 \Rightarrow x = 12$

$A''(x) = -2$

$A''(12) = -2$, which is negative

Therefore, area is maximum when $x = 12$

$y = x = 24 - 12 = 12$

So, $x = y = 12$

Hence, amongst all rectangles, the square has the maximum area.

Activity 19

Objective

To evaluate the definite integral $\int_a^b \sqrt{(1-x^2)}\, dx$ as the limit of a sum and verify it by actual integration.

Material Required

Cardboard, white paper, scale, pencil, graph paper

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw two perpendicular lines to represent coordinate axes XOX′ and YOY′.
3. Draw a quadrant of a circle with O as centre and radius 1 unit (10 cm) as shown in Fig.19.

The curve in the 1st quadrant represents the graph of the function $\sqrt{1-x^2}$ in the interval [0, 1].

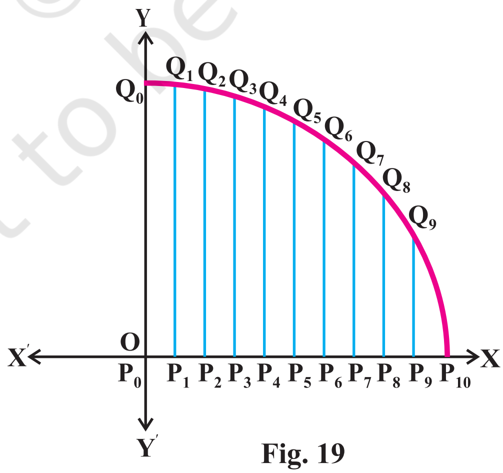

Fig. 19

DEMONSTRATION

1. Let origin O be denoted by P_0 and the points where the curve meets the x-axis and y-axis be denoted by P_{10} and Q, respectively.
2. Divide P_0P_{10} into 10 equal parts with points of division as, P_1, P_2, P_3, ..., P_9.
3. From each of the points, P_i , $i = 1, 2, ..., 9$ draw perpendiculars on the x-axis to meet the curve at the points, Q_1, Q_2, Q_3 ,..., Q_9. Measure the lengths of P_0Q_0, P_1 Q_1, ..., P_9Q_9 and call them as y_0, y_1 , ..., y_9 whereas width of each part, P_0P_1, P_1P_2, ..., is 0.1 units.
4. $y_0 = P_0Q_0 = 1$ units

   $y_1 = P_1Q_1 = 0.99$ units

   $y_2 = P_2Q_2 = 0.97$ units

   $y_3 = P_3Q_3 = 0.95$ units

   $y_4 = P_4Q_4 = 0.92$ units

   $y_5 = P_5Q_5 = 0.87$ units

   $y_6 = P_6Q_6 = 0.8$ units

   $y_7 = P_7Q_7 = 0.71$ units

   $y_8 = P_8Q_8 = 0.6$ units

   $y_9 = P_9Q_9 = 0.43$ units

   $y_{10} = P_{10}Q_{10}$ = which is very small near to 0.
5. Area of the quadrant of the circle (area bounded by the curve and the two axis) = sum of the areas of trapeziums.

$$=\frac{1}{2}\times 0.1\begin{bmatrix}(1+0.99)+(0.99+0.97)+(0.97+0.95)+(0.95+0.92)\\ +(0.92+0.87)+(0.87+0.8)+(0.8+0.71)+(0.71+0.6)\\ +(0.6+0.43)+(0.43)\end{bmatrix}$$

$= 0.1\ [0.5 + 0.99 + 0.97 + 0.95 + 0.92 + 0.87 + 0.80 + 0.71 + 0.60 + 0.43]$

$= 0.1 \times 7.74 = 0.774$ sq. units.(approx.)

6. Definite integral $= \int_0^1 \sqrt{1-x^2}\, dx$

$$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1} x \right]_0^1 = \frac{1}{2} \times \frac{\pi}{2} = \frac{3.14}{4} = 0.785 \text{ sq.units}$$

Thus, the area of the quadrant as a limit of a sum is nearly the same as area obtained by actual integration.

Observation

1. Function representing the arc of the quadrant of the circle is $y =$ ______.
2. Area of the quadrant of a circle with radius 1 unit $= \int_0^1 \sqrt{1-x^2}\, dx =$ ________. sq. units
3. Area of the quadrant as a limit of a sum = _______ sq. units.
4. The two areas are nearly _________.

Application

This activity can be used to demonstrate the concept of area bounded by a curve. This activity can also be applied to find the approximate value of π.

Note

Demonstrate the same activity by drawing the circle $x^2 + y^2 = 9$ and find the area between $x = 1$ and $x = 2$.

24/04/18

Activity 20

Objective

To verify geometrically that

$\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$

Material Required

Geometry box, cardboard, white paper, cutter, sketch pen, cellotape.

Method of Construction

1. Fix a white paper on the cardboard.
2. Draw a line segment OA (= 6 cm, say) and let it represent $\vec{c}$.
3. Draw another line segment OB (= 4 cm, say) at an angle (say 60°) with OA.

   Let $\overrightarrow{OB} = \vec{a}$

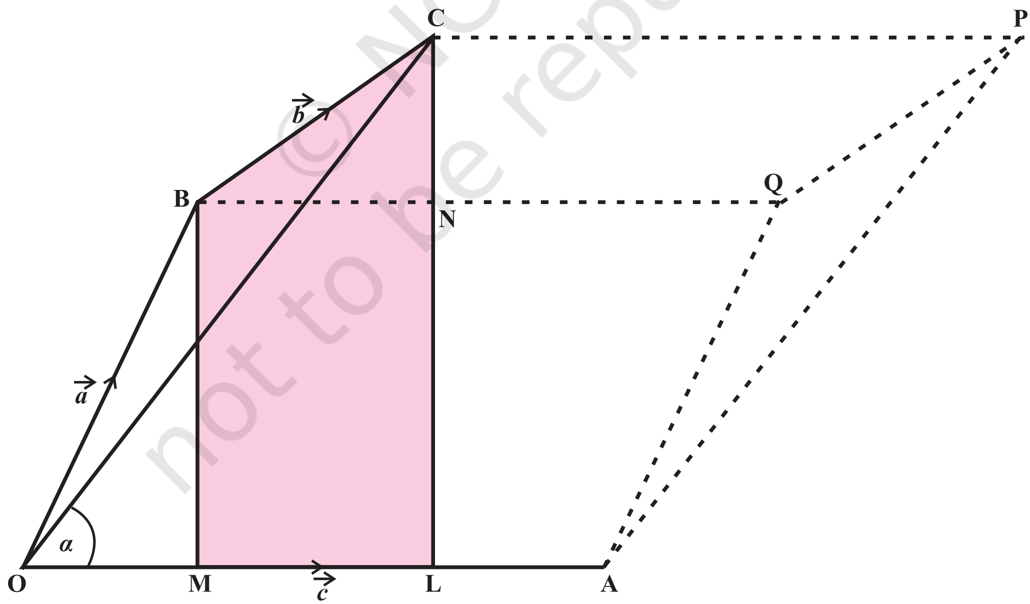

Fig. 20

4. Draw BC (= 3 cm, say) making an angle (say 30°) with $\overrightarrow{OA}$. Let $\overrightarrow{BC} = \vec{b}$

5. Draw perpendiculars BM, CL and BN.

6. Complete parallelograms OAPC, OAQB and BQPC.

Demonstration

1. $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \vec{a} + \vec{b}$, and let $\angle COA = \alpha$.

2. $\left|\vec{c} \times \left(\vec{a} + \vec{b}\right)\right| = \left|\vec{c}\right|\left|\vec{a} + \vec{b}\right|$ sin α = area of parallelogram OAPC.

3. $\left|\vec{c} \times \vec{a}\right|$ = area of parallelogram OAQB.

4. $\left|\vec{c} \times \vec{b}\right|$ = area of parallelogram BQPC.

5. Area of parallelogram OAPC = (OA) (CL)

= (OA) (LN + NC) = (OA) (BM + NC)

= (OA) (BM) + (OA) (NC)

= Area of parallelogram OAQB + Area of parallelogram BQPC

$= \left|\vec{c} + \vec{a}\right| + \left|\vec{c} \times \vec{b}\right|$

So, $\left|\vec{c} \times (\vec{a} + \vec{b})\right| = \left|\vec{c} \times \vec{b}\right| + \left|\vec{c} \times \vec{b}\right|$

Direction of each of these vectors $\vec{c} \times (\vec{a} + \vec{b})$, $\vec{c} \times \vec{a}$ and $\vec{c} \times \vec{b}$ is perpendicular to the same plane.

So, $\vec{c} \times (\vec{a} + \vec{b}) = \vec{c} \times \vec{a} + \vec{c} \times \vec{b}$.

Observation

$\left|\vec{c}\right| = \left|\overrightarrow{OA}\right| = OA =$ _______

$\left|\vec{a}+\vec{b}\right| = \left|\overrightarrow{OC}\right| = OC =$ ______

CL = ______

$\left|\vec{c}\times(\vec{a}+\vec{b})\right|$ = Area of parallelogram OAPC

= (OA) (CL) = _____________ sq. units (i)

$\left|\vec{c}\times\vec{a}\right|$ = Area of parallelogram OAQB

= (OA) (BM) = _____ × _____ = ______ (ii)

$\left|\vec{c}\times\vec{b}\right|$ = Area of parallelogram BQPC

= (OA) (CN) = _____ × _____ = ______ (iii)

From (i), (ii) and (iii),

Area of parallelogram OAPC = Area of parallelgram OAQB + Area of Parallelgram ________.

Thus $\left|\vec{c}\times(\vec{a}+\vec{b})\right| = \left|\vec{c}\times\vec{a}\right| + \left|\vec{c}\times\vec{b}\right|$

$\vec{c}\times\vec{a}$, $\vec{c}\times\vec{b}$ and $\vec{c}\times\left(\vec{a}+\vec{b}\right)$ are all in the direction of _______ to the plane of paper.

Therefore $\vec{c}\times\left(\vec{a}+\vec{b}\right) = \vec{c}\times\vec{a}$ + ________.

APPLICATION

Through the activity, distributive property of vector multiplication over addition can be explained.

NOTE

This activity can also be performed by taking rectangles instead of parallelograms.