ANSWERS

EXERCISE 1.1

1. (i), (iv), (v), (vi), (vii) and (viii) are sets.

2. (i) $\in$ (ii) $\notin$ (iii) $\notin$ (vi) $\in$ (v) $\in$ (vi) $\notin$

3. (i) A = {–3, –2, –1, 0, 1, 2, 3, 4, 5, 6 } (ii) B = {1, 2, 3, 4, 5}
(iii) C = {17, 26, 35, 44, 53, 62, 71, 80} (iv) D = {2, 3, 5}
(v) E = {T, R, I, G, O, N, M, E, Y} (vi) F = {B, E, T, R}

4. (i) $\{ x : x = 3n, n \in \mathrm{N} \text{ and } 1 \le n \le 4 \}$ (ii) $\{ x : x = 2^n, n \in \mathrm{N} \text{ and } 1 \le n \le 5 \}$
(iii) $\{ x : x = 5^n, n \in \mathrm{N} \text{ and } 1 \le n \le 4 \}$ (iv) { x : x is an even natural number}
(v) $\{ x : x = n^2, n \in \mathrm{N} \text{ and } 1 \le n \le 10 \}$

5. (i) A = {1, 3, 5, . . . } (ii) B = {0, 1, 2, 3, 4 }
(iii) C = { –2, –1, 0, 1, 2 } (iv) D = { L, O, Y, A }
(v) E = { February, April, June, September, November }
(vi) F = {b, c, d, f, g, h, j }

6. (i) $\leftrightarrow$ (c) (ii) $\leftrightarrow$ (a) (iii) $\leftrightarrow$ (d) (iv) $\leftrightarrow$ (b)

EXERCISE 1.2

1. (i), (iii), (iv)

2. (i) Finite (ii) Infinite (iii) Finite (iv) Infinite (v) Finite

3. (i) Infinite (ii) Finite (iii) Infinite (iv) Finite (v) Infinite

4. (i) Yes (ii) No (iii) Yes (iv) No

5. (i) No (ii) Yes **6.** B= D, E = G

EXERCISE 1.3

1. (i) $\subset$ (ii) $\not\subset$ (iii) $\subset$ (iv) $\not\subset$ (v) $\not\subset$ (vi) $\subset$
(vii) $\subset$

2. (i) False (ii) True (iii) False (iv) True (v) False (vi) True

3. (i) as $\{3,4\} \in$ A, (v) as $1 \in$ A, (vii) as $\{1,2, 5\} \subset$A,
(viii) as $3 \notin$ A, (ix) as $\phi \subset$ A, (xi) as $\phi \subset$ A,

4. (i) ϕ, { a } (ii) ϕ, { a }, { b }, { a, b }
(iii) ϕ, { 1 }, { 2 }, { 3 }, { 1, 2 }, { 1, 3 }, { 2, 3 }, { 1, 2, 3 } (iv) ϕ

5. 1

6. (i) (– 4, 6] (ii) (– 12, –10) (iii) [0, 7)
(iv) [3, 4]

7. (i) $\{ x : x \in \mathrm{R}, -3 < x < 0 \}$ (ii) $\{ x : x \in \mathrm{R}, 6 \le x \le 12 \}$
(iii) $\{ x : x \in \mathrm{R}, 6 < x \le 12 \}$ (iv) $\{ x \ \mathrm{R} : -23 \le x < 5 \}$ **9.** (iii)

EXERCISE 1.4

1. (i) $X \cup Y = \{1, 2, 3, 5\}$ (ii) $A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A \cup B = \{x : x = 1, 2, 4, 5$ or a multiple of $3\}$

(iv) $A \cup B = \{x : 1 < x < 10, x \in N\}$ (v) $A \cup B = \{1, 2, 3\}$

2. Yes, $A \cup B = \{a, b, c\}$ **3.** B

4. (i) $\{1, 2, 3, 4, 5, 6\}$ (ii) $\{1, 2, 3, 4, 5, 6, 7, 8\}$ (iii) $\{3, 4, 5, 6, 7, 8\}$

(iv) $\{3, 4, 5, 6, 7, 8, 9, 10\}$ (v) $\{1, 2, 3, 4, 5, 6, 7, 8\}$

(vi) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (vii) $\{3, 4, 5, 6, 7, 8, 9, 10\}$

5. (i) $X \cap Y = \{1, 3\}$ (ii) $A \cap B = \{a\}$ (iii) $\{3\}$ (iv) ϕ (v) ϕ

6. (i) $\{7, 9, 11\}$ (ii) $\{11, 13\}$ (iii) ϕ (iv) $\{11\}$

(v) ϕ (vi) $\{7, 9, 11\}$ (vii) ϕ

(viii) $\{7, 9, 11\}$ (ix) $\{7, 9, 11\}$ (x) $\{7, 9, 11, 15\}$

7. (i) B (ii) C (iii) D (iv) ϕ

(v) $\{2\}$ (vi) $\{x : x$ is an odd prime number$\}$ **8.** (iii)

9. (i) $\{3, 6, 9, 15, 18, 21\}$ (ii) $\{3, 9, 15, 18, 21\}$ (iii) $\{3, 6, 9, 12, 18, 21\}$

(iv) $\{4, 8, 16, 20\}$ (v) $\{2, 4, 8, 10, 14, 16\}$ (vi) $\{5, 10, 20\}$

(vii) $\{20\}$ (viii) $\{4, 8, 12, 16\}$ (ix) $\{2, 6, 10, 14\}$

(x) $\{5, 10, 15\}$ (xi) $\{2, 4, 6, 8, 12, 14, 16\}$ (xii) $\{5, 15, 20\}$

10. (i) $\{a, c\}$ (ii) $\{f, g\}$ (iii) $\{b, d\}$

11. Set of irrational numbers **12.** (i) F (ii) F (iii) T (iv) T

EXERCISE 1.5

1. (i) $\{5, 6, 7, 8, 9\}$ (ii) $\{1, 3, 5, 7, 9\}$ (iii) $\{7, 8, 9\}$

(iv) $\{5, 7, 9\}$ (v) $\{1, 2, 3, 4\}$ (vi) $\{1, 3, 4, 5, 6, 7, 9\}$

2. (i) $\{d, e, f, g, h\}$ (ii) $\{a, b, c, h\}$ (iii) $\{b, d, f, h\}$

(iv) $\{b, c, d, e\}$

3. (i) $\{x : x$ is an odd natural number$\}$

(ii) $\{x : x$ is an even natural number$\}$

(iii) $\{x : x \in \mathbf{N}$ and x is not a multiple of $3\}$

(iv) $\{x : x$ is a positive composite number or $x = 1\}$

(v) $\{ x : x$ is a positive integer which is not divisible by 3 or not divisible by 5$\}$
(vi) $\{ x : x \in \mathbf{N}$ and x is not a perfect square $\}$
(vii) $\{ x : x \in \mathbf{N}$ and x is not a perfect cube $\}$
(viii) $\{ x : x \in \mathbf{N}$ and $x \neq 3 \}$ (ix) $\{ x : x \in \mathbf{N}$ and $x \neq 2 \}$
(x) $\{ x : x \in \mathbf{N}$ and $x < 7 \}$ (xi) $\{ x : x \in \mathbf{N}$ and $x \leq \frac{9}{2} \}$

6. A' is the set of all equilateral triangles.
7. (i) U (ii) A (iii) ϕ (iv) ϕ

EXERCISE 1.6

1. 2 **2.** 5 **3.** 50 **4.** 42
5. 30 **6.** 19 **7.** 25, 35 **8.** 60

Miscellaneous Exercise on Chapter 1

1. $A \subset B$, $A \subset C$, $B \subset C$, $D \subset A$, $D \subset B$, $D \subset C$
2. (i) False (ii) False (iii) True (iv) False (v) False
(vi) True
7. False **12.** We may take $A = \{ 1, 2 \}$, $B = \{ 1, 3 \}$, $C = \{ 2, 3 \}$
13. 325 **14.** 125 **15.** (i) 52, (ii) 30 **16.** 11

EXERCISE 2.1

1. $x = 2$ and $y = 1$ **2.** The number of elements in $A \times B$ is 9.
3. $G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$
$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$
4. (i) False
$P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$
(ii) True
(iii) True
5. $A \times A = \{(-1, -1), (-1, 1), (1, -1), (1, 1)\}$
$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$
6. $A = \{a, b\}$, $B = \{x, y\}$
8. $A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$
$A \times B$ will have $2^4 = 16$ subsets.
9. $A = \{x, y, z\}$ and $B = \{1,2\}$

10. A = {–1, 0, 1}, remaining elements of
A × A are (–1, –1), (–1, 1), (0, –1), (0, 0), (1, –1), (1, 0), (1, 1)

EXERCISE 2.2

1. R = {(1, 3), (2, 6), (3, 9), (4, 12)}
Domain of R = {1, 2, 3, 4}
Range of R = {3, 6, 9, 12}
Co domain of R = {1, 2, ..., 14}

2. R = {(1, 6), (2, 7), (3, 8)}
Domain of R = {1, 2, 3}
Range of R = {6, 7, 8}

3. R = {(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)}

4. (i) R = {(x, y) : $y = x - 2$ for x = 5, 6, 7}
(ii) R = {(5,3), (6,4), (7,5)}. Domain of R = {5, 6, 7}, Range of R = {3, 4, 5}

5. (i) R = {(1, 1), (1,2), (1, 3), (1, 4), (1, 6), (2 4), (2, 6), (2, 2), (4, 4), (6, 6), (3, 3), (3, 6)}
(ii) Domain of R = {1, 2, 3, 4, 6}
(iii) Range of R = {1, 2, 3, 4, 6}

6. Domain of R = {0, 1, 2, 3, 4, 5,}
Range of R = {5, 6, 7, 8, 9, 10}

7. R = {(2, 8), (3, 27), (5, 125), (7, 343)}

8. No. of relations from A into B = 2^6

9. Domain of R = **Z**
Range of R = **Z**

EXERCISE 2.3

1. (i) yes, Domain = {2, 5, 8, 11, 14, 17}, Range = {1}
(ii) yes, Domain = (2, 4, 6, 8, 10, 12, 14}, Range = {1, 2, 3, 4, 5, 6, 7}
(iii) No.

2. (i) Domain = **R**, Range = $(-\infty, 0]$
(ii) Domain of function = $\{x : -3 \le x \le 3\}$
Range of function = $\{x : 0 \le x \le 3\}$

3. (i) $f(0) = -5$ (ii) $f(7) = 9$ (iii) $f(-3) = -11$

4. (i) $t(0) = 32$ (ii) $t(28) = \frac{412}{5}$ (iii) $t(-10) = 14$ (iv) 100

5. (i) Range = $(-\infty, 2)$ (ii) Range = $[2, \infty)$ (iii) Range = **R**

Miscellaneous Exercise on Chapter 2

2. 2.1 **3.** Domain of function is set of real numbers except 6 and 2.

4. Domain $= [1, \infty)$, Range $= [0, \infty)$

5. Domain $= \mathbf{R}$, Range = non-negative real numbers

6. Range $= [0, 1)$

7. $(f + g)\, x = 3x - 2$ **8.** $a = 2$, $b = -1$ **9.** (i) No (ii) No (iii) No

$(f - g)\, x = -x + 4$

$$\left(\frac{f}{g}\right) x = \frac{x+1}{2x-3},\quad x \neq \frac{3}{2}$$

10. (i) Yes, (ii) No **11.** No **12.** Range of $f = \{3, 5, 11, 13\}$

EXERCISE 3.1

1. (i) $\frac{5\pi}{36}$ (ii) $-\frac{19\pi}{72}$ (iii) $\frac{4\pi}{3}$ (iv) $\frac{26\pi}{9}$

2. (i) $39°\, 22'\, 30''$ (ii) $-229°\, 5'\, 27''$ (iii) $300°$ (iv) $210°$

3. 12π **4.** $12°\, 36'$ **5.** $\frac{20\pi}{3}$ **6.** $5 : 4$

7. (i) $\frac{2}{15}$ (ii) $\frac{1}{5}$ (iii) $\frac{7}{25}$

EXERCISE 3.2

1. $\sin x = -\frac{\sqrt{3}}{2}, \operatorname{cosec} x = -\frac{2}{\sqrt{3}}, \sec x = -2, \tan x = \sqrt{3}, \cot x = \frac{1}{\sqrt{3}}$

2. $\operatorname{cosec} x = \frac{5}{3}, \cos x = -\frac{4}{5}, \sec x = -\frac{5}{4}, \tan x = -\frac{3}{4}, \cot x = -\frac{4}{3}$

3. $\sin x = -\frac{4}{5}, \operatorname{cosec} x = -\frac{5}{4}, \cos x = -\frac{3}{5}, \sec x = -\frac{5}{3}, \tan x = \frac{4}{3}$

4. $\sin x = -\frac{12}{13}, \operatorname{cosec} x = -\frac{13}{12}, \cos x = \frac{5}{13}, \tan x = -\frac{12}{5}, \cot x = -\frac{5}{12}$

5. $\sin x = \frac{5}{13}$, $\text{cosec } x = \frac{13}{5}$, $\cos x = -\frac{12}{13}$, $\sec x = -\frac{13}{12}$, $\cot x = -\frac{12}{5}$

6. $\frac{1}{\sqrt{2}}$ **7.** 2 **8.** $\sqrt{3}$ **9.** $\frac{\sqrt{3}}{2}$ **10.** 1

EXERCISE 3.3

5. (i) $\frac{\sqrt{3}+1}{2\sqrt{2}}$ (ii) $2 - \sqrt{3}$

EXERCISE 3.4

1. $\frac{\pi}{3}, \frac{4\pi}{3}, n\pi + \frac{\pi}{3}, n \in \mathbf{Z}$

2. $\frac{\pi}{3}, \frac{5\pi}{3}, 2n\pi \pm \frac{\pi}{3}, n \in \mathbf{Z}$

3. $\frac{5\pi}{6}, \frac{11\pi}{6}, n\pi + \frac{5\pi}{6}, n \in \mathbf{Z}$

4. $\frac{7\pi}{6}, \frac{11\pi}{6}, n\pi + (-1)^n \frac{7\pi}{6}, n \in \mathbf{Z}$

5. $x = \frac{n\pi}{3}$ or $x = n\pi$, $n \in \mathbf{Z}$

6. $x = (2n+1)\frac{\pi}{4}$, or $2n\pi \pm \frac{\pi}{3}$, $n \in \mathbf{Z}$

7. $x = n\pi + (-1)^n \frac{7\pi}{6}$ or $(2n+1)\frac{\pi}{2}$, $n \in \mathbf{Z}$

8. $x = \frac{n\pi}{2}$, or $\frac{n\pi}{2} + \frac{3\pi}{8}$, $n \in \mathbf{Z}$

9. $x = \frac{n\pi}{3}$, or $n\pi \pm \frac{\pi}{3}$, $n \in \mathbf{Z}$

Miscellaneous Exercise on Chapter 3

8. $\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}, \frac{1}{2}$

9. $\frac{\sqrt{6}}{3}, -\frac{\sqrt{3}}{3}, -\sqrt{2}$

10. $\frac{\sqrt{8+2\sqrt{15}}}{4}, \frac{\sqrt{8-2\sqrt{15}}}{4}, 4+\sqrt{15}$

EXERCISE 5.1

1. $3 + i0$
2. $0 + i\,0$
3. $0+i\,1$
4. $14 + 28\,i$
5. $2 - 7\,i$
6. $-\frac{19}{5}-\frac{21i}{10}$
7. $\frac{17}{3}+i\frac{5}{3}$
8. $-4 + i\,0$
9. $-\frac{242}{27}-26i$
10. $\frac{-22}{3}-i\frac{107}{27}$
11. $\frac{4}{25}+i\frac{3}{25}$
12. $\frac{\sqrt{5}}{14}-i\frac{3}{14}$
13. $0 + i1$
14. $0 - i\;\frac{7\sqrt{2}}{2}$

EXERCISE 5.2

1. $2, \frac{-2\pi}{3}$
2. $2, \frac{5\pi}{6}$
3. $\sqrt{2}\left(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4}\right)$
4. $\sqrt{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$
5. $\sqrt{2}\left(\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4}\right)$
6. $3\,(\cos\pi + i\sin\pi)$
7. $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
8. $\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$

EXERCISE 5.3

1. $\pm\sqrt{3}\,i$
2. $\frac{-1\pm\sqrt{7}\,i}{4}$
3. $\frac{-3\pm 3\sqrt{3}\,i}{2}$
4. $\frac{-1\pm\sqrt{7}i}{-2}$
5. $\frac{-3\pm\sqrt{11}\,i}{2}$
6. $\frac{1\pm\sqrt{7}\,i}{2}$
7. $\frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}$
8. $\frac{\sqrt{2}\pm\sqrt{34}\,i}{2\sqrt{3}}$
9. $\frac{-1\pm\sqrt{(2\sqrt{2}-1)}\,i}{2}$
10. $\frac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}$

Miscellaneous Exercise on Chapter 5

1. $2 - 2i$ **3.** $\dfrac{307+599i}{442}$

5. (i) $\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$, (ii) $\sqrt{2}\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)$

6. $\dfrac{2}{3} \pm \dfrac{4}{3}i$ **7.** $1 \pm \dfrac{\sqrt{2}}{2}i$ **8.** $\dfrac{5}{27} \pm \dfrac{\sqrt{2}}{27}i$ **9.** $\dfrac{2}{3} \pm \dfrac{\sqrt{14}}{21}i$

10. $\sqrt{2}$ **12.** (i) $\dfrac{-2}{5}$, (ii) 0 **13.** $\dfrac{1}{\sqrt{2}}, \dfrac{3\pi}{4}$ **14.** $x = 3, y = -3$

15. 2 **17.** 1 **18.** 0 **20.** 4

EXERCISE 6.1

1. (i) $\{1, 2, 3, 4\}$ (ii) $\{\ldots -3, -2, -1, 0, 1, 2, 3, 4,\}$

2. (i) No Solution (ii) $\{\ldots -4, -3\}$

3. (i) $\{\ldots -2, -1, 0, 1\}$ (ii) $(-\infty, 2)$

4. (i) $\{-1, 0, 1, 2, 3, \ldots\}$ (ii) $(-2, \infty)$

5. $(-4, \infty)$ **6.** $(-\infty, -3)$ **7.** $(-\infty, -3]$ **8.** $(-\infty, 4]$

9. $(-\infty, 6)$ **10.** $(-\infty, -6)$ **11.** $(-\infty, 2]$ **12.** $(-\infty, 120]$

13. $(4, \infty)$ **14.** $(-\infty, 2]$ **15.** $(4, \infty)$ **16.** $(-\infty, 2]$

17. $(-\infty, 3)$, 

18. $[-1, \infty)$, 

19. $(-1, \infty)$, 

20. $\left[-\dfrac{2}{7}, \infty\right)$, 

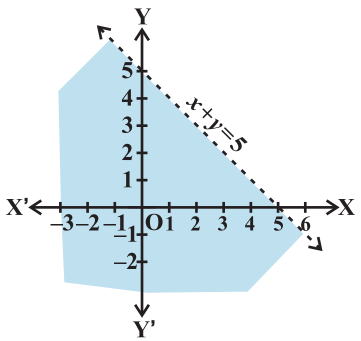
21. 35 **22.** 82

23. (5,7), (7,9) **24.** (6,8), (8,10), (10,12)

25. 9 cm **26.** Greater than or equal to 8cm but less than or equal to 22cm

EXERCISE 6.2

1.



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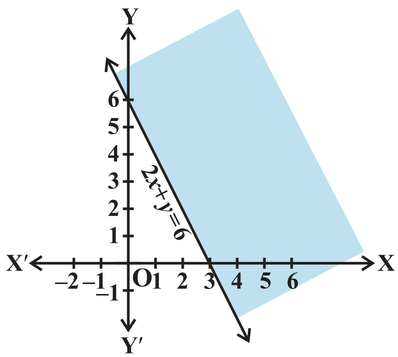

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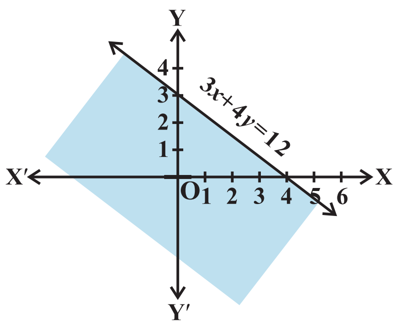

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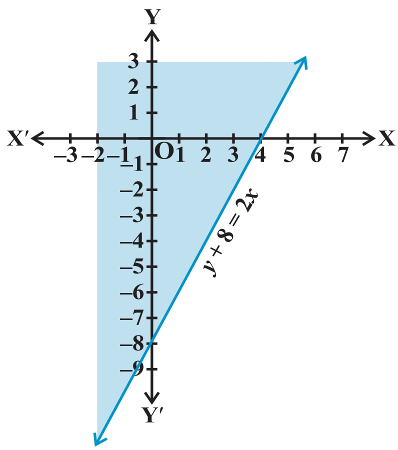

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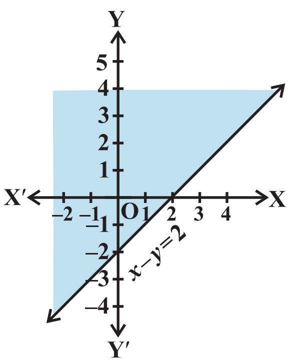

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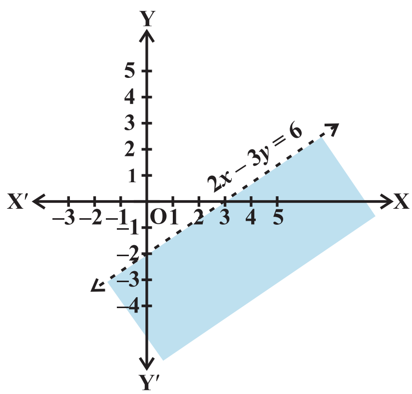

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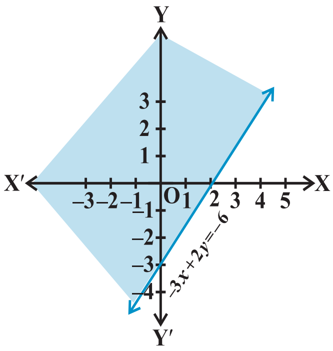

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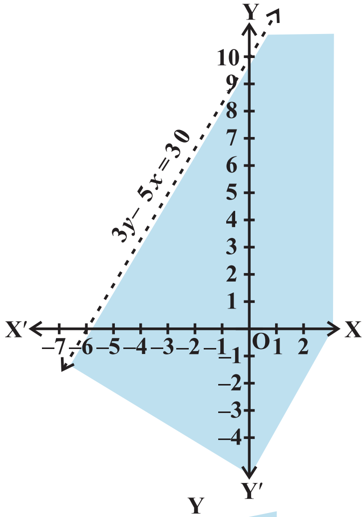

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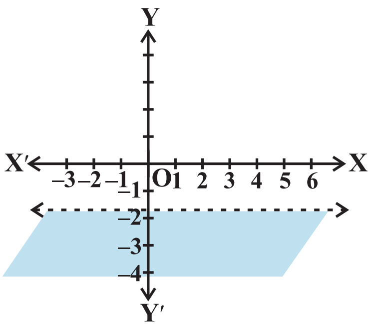

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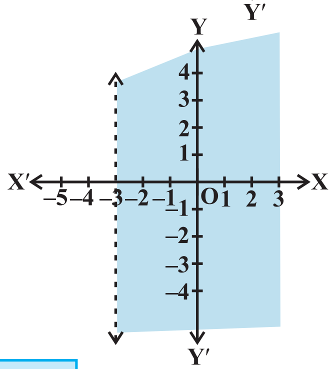
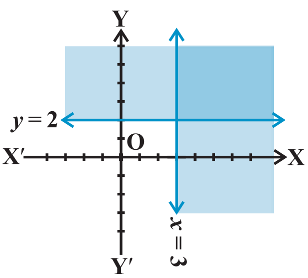


EXERCISE 6.3

1.



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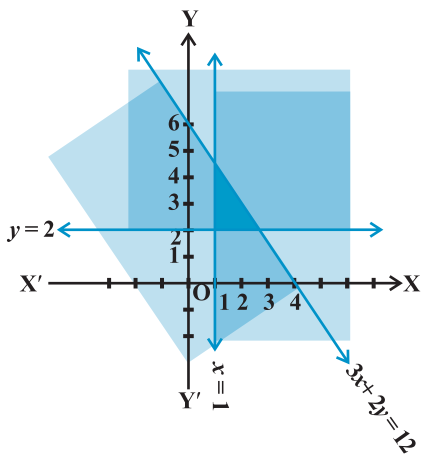

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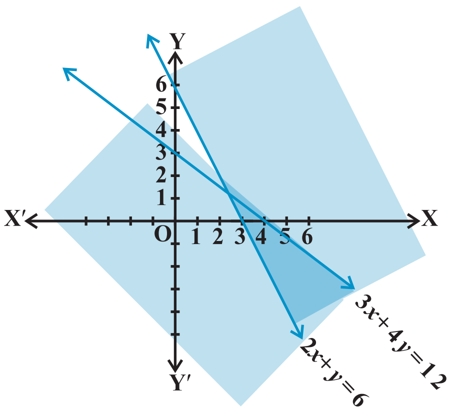
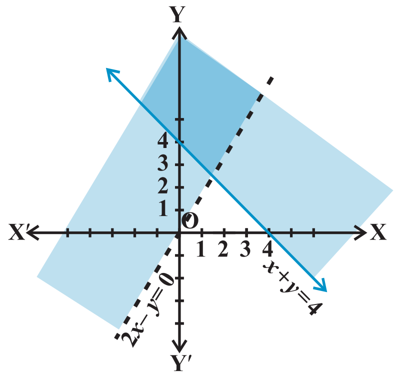
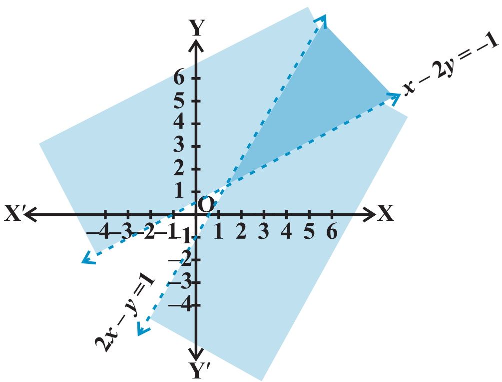

Y
X′
X
O
Y′
1 2 3 4 5 6
6 5 4 3 2 1
2x+y=6
3x+4y=12

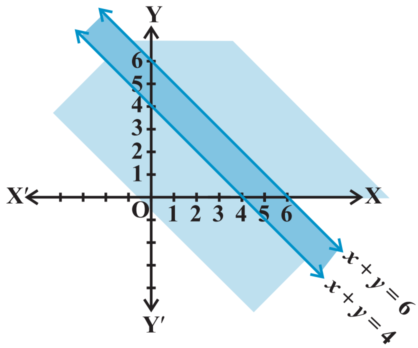
4.


Y
X′
X
O
Y′
1 2 3 4
4 3 2 1
2x – y=0
x+y=4

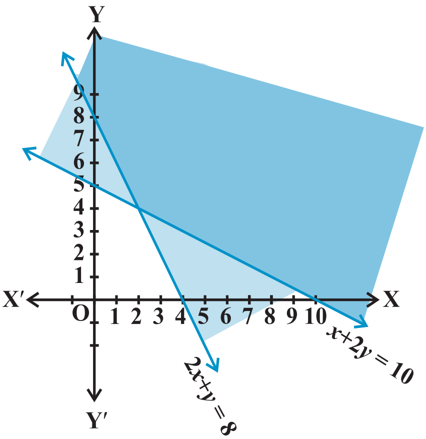
5.


Y
X′
X
O
Y′
–4 –3 –2 –1 1 2 3 4 5 6
6 5 4 3 2 1 –1 –2 –3 –4
x – 2y = –1
2x – y =1

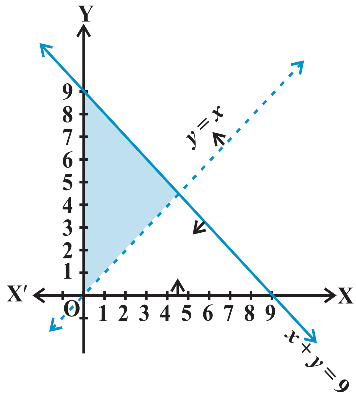
6.


Y
X′
X
O
Y′
1 2 3 4 5 6
6 5 4 3 2 1
x + y = 6
x + y = 4

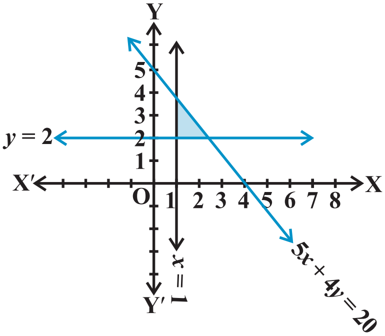
7.


Y
X′
X
O
Y′
1 2 3 4 5 6 7 8 9 10
9 8 7 6 5 4 3 2 1
2x+y = 8
x+2y = 10

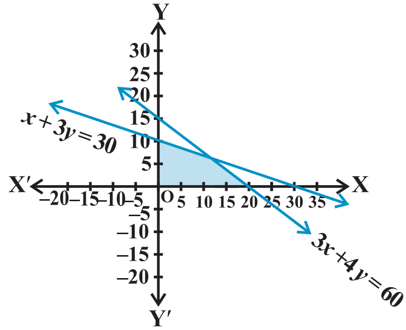
8.


Y
X′
X
O
1 2 3 4 5 6 7 8 9
9 8 7 6 5 4 3 2 1
y = x
x + y = 9

9.



10.



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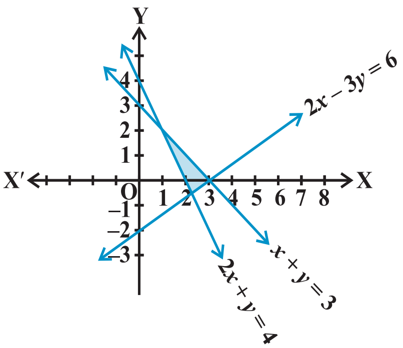
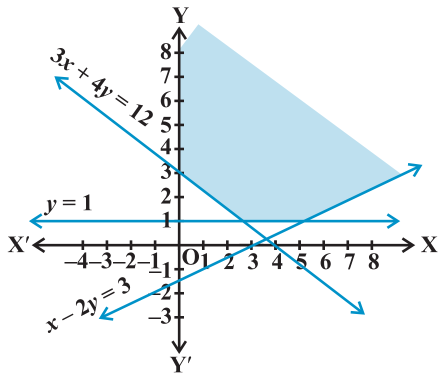
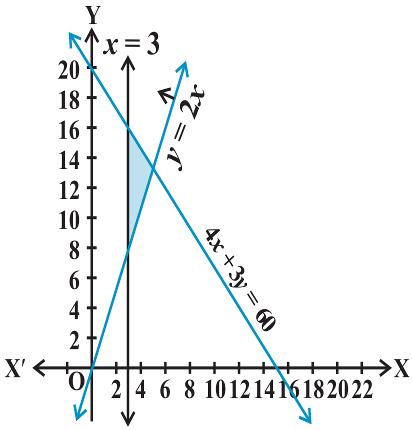


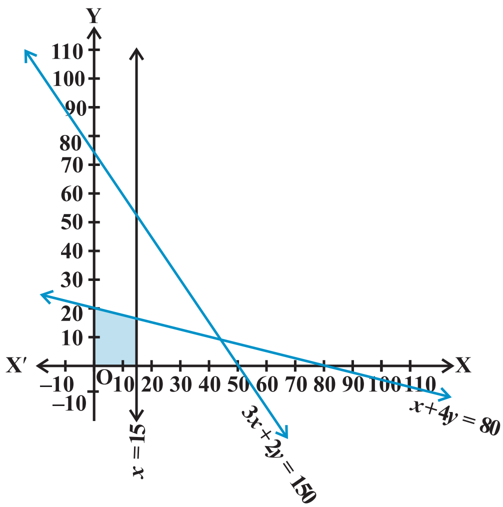
12.



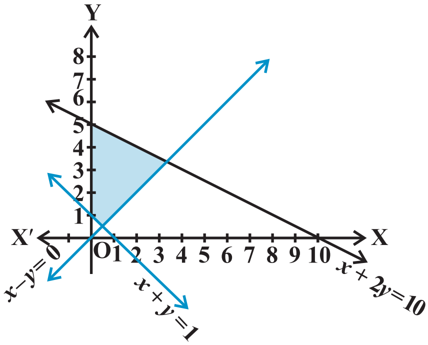
13.



14.



15.



Miscellaneous Exercise on Chapter 6

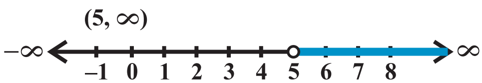
1. [2, 3] **2.** (0, 1] **3.** [– 4, 2]

4. (– 23, 2] **5.** $\left(\frac{-80}{3}, \frac{-10}{3}\right]$ **6.** $\left[1, \frac{11}{3}\right]$

7. (–5, 5)

8. (–1, 7)

9. (5, ∞)



10. [– 7, 11]

11. Between 20°C and 25°C

12. More than 320 litres but less than 1280 litres.

13. More than 562.5 litres but less than 900 litres.

14. $9.6 \le MA \le 16.8$

EXERCISE 7.1

1. (i) 125, (ii) 60. **2.** 108 **3.** 5040 **4.** 336

5. 8 **6.** 20

EXERCISE 7.2

1. (i) 40320, (ii) 18 **2.** 30, No **3.** 28 **4.** 64
5. (i) 30, (ii) 15120

EXERCISE 7.3

1. 504 **2.** 4536 **3.** 60 **4.** 120, 48
5. 56 **6.** 9 **7.** (i) 3, (ii) 4 **8.** 40320
9. (i) 360, (ii) 720, (iii) 240 **10.** 33810
11. (i) 1814400, (ii) 2419200, (iii) 25401600

EXERCISE 7.4

1. 45 **2.** (i) 5, (ii) 6 **3.** 210 **4.** 40
5. 2000 **6.** 778320 **7.** 3960 **8.** 200
9. 35

Miscellaneous Exercise on Chapter 7

1. 3600 **2.** 1440 **3.** (i) 504, (ii) 588, (iii) 1632
4. 907200 **5.** 120 **6.** 50400 **7.** 420
8. ${}^{4}C_1 \times {}^{48}C_4$ **9.** 2880 **10.** ${}^{22}C_7 + {}^{22}C_{10}$ **11.** 151200

EXERCISE 8.1

1. $1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

2. $\frac{32}{x^5} - \frac{40}{x^3} + \frac{20}{x} - 5x + \frac{5}{8}x^3 - \frac{x^5}{32}$

3. $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

4. $\frac{x^5}{243} + \frac{5x^3}{81} + \frac{10}{27}x + \frac{10}{9x} + \frac{5}{3x^3} + \frac{1}{x^5}$

5. $x^6 + 6x^4 + 15x^2 + 20 + \frac{15}{x^2} + \frac{6}{x^4} + \frac{1}{x^6}$

6. 884736 **7.** 11040808032 **8.** 104060401

9. 9509900499 **10.** $(1.1)^{10000} > 1000$ **11.** $8(a^3b + ab^3)$; $40\sqrt{6}$

12. $2(x^6 + 15x^4 + 15x^2 + 1)$, 198

EXERCISE 8.2

1. 1512
2. – 101376
3. $(-1)^r\ {}^6C_r.x^{12-2r}.y^r$
4. $(-1)^r\ {}^{12}C_r.x^{24-r}.y^r$
5. $-1760\ x^9y^3$
6. 18564
7. $\frac{-105}{8}x^9;\ \frac{35}{48}x^{12}$
8. $61236\ x^5y^5$
10. $n = 7;\ r = 3$
12. $m = 4$

Miscellaneous Exercise on Chapter 8

1. $a = 3;\ b = 5;\ n = 6$
2. $a = \frac{9}{7}$
3. 171
5. $396\sqrt{6}$
6. $2a^8 + 12a^6 - 10a^4 - 4a^2 + 2$
7. 0.9510
8. $n = 10$
9. $\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5$
10. $27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$

EXERCISE 9.1

1. 3, 8, 15, 24, 35
2. $\frac{1}{2},\frac{2}{3},\frac{3}{4},\frac{4}{5},\frac{5}{6}$
3. 2, 4, 8, 16 and 32
4. $-\frac{1}{6},\frac{1}{6},\frac{1}{2},\frac{5}{6}$ and $\frac{7}{6}$
5. 25, –125, 625, –3125, 15625
6. $\frac{3}{2},\frac{9}{2},\frac{21}{2}, 21$ and $\frac{75}{2}$
7. 65, 93
8. $\frac{49}{128}$
9. 729
10. $\frac{360}{23}$
11. 3, 11, 35, 107, 323; 3 + 11 + 35 + 107 + 323 + ...
12. $-1,\frac{-1}{2},\frac{-1}{6},\frac{-1}{24},\frac{-1}{120};\ -1+\left(\frac{-1}{2}\right)+\left(\frac{-1}{6}\right)+\left(\frac{-1}{24}\right)+\left(\frac{-1}{120}\right)+...$

13. 2, 2, 1, 0, –1; $2 + 2 + 1 + 0 + (-1) + \ldots$ **14.** $1, 2, \frac{3}{2}, \frac{5}{3}$ and $\frac{8}{5}$

EXERCISE 9.2

1. 1002001 **2.** 98450 **4.** 5 or 20 **6.** 4

7. $\frac{n}{2}(5n+7)$ **8.** $2q$ **9.** $\frac{179}{321}$ **10.** 0

13. 27 **14.** 11, 14, 17, 20 and 23 **15.** 1

16. 14 **17.** Rs 245 **18.** 9

EXERCISE 9.3

1. $\frac{5}{2^{20}}, \frac{5}{2^n}$ **2.** 3072 **4.** -2187

5. (a) 13th, (b) 12th, (c) 9th **6.** ± 1 **7.** $\frac{1}{6}\left[1-(0.1)^{20}\right]$

8. $\frac{\sqrt{7}}{2}\left(\sqrt{3}+1\right)\left(3^{\frac{n}{2}}-1\right)$ **9.** $\frac{\left[1-(-a)^n\right]}{1+a}$ **10.** $\frac{x^3\left(1-x^{2n}\right)}{1-x^2}$

11. $22+\frac{3}{2}\left(3^{11}-1\right)$ **12.** $r=\frac{5}{2}$ or $\frac{2}{5}$; Terms are $\frac{2}{5},1,\frac{5}{2}$ or $\frac{5}{2},1,\frac{2}{5}$

13. 4 **14.** $\frac{16}{7};2;\frac{16}{7}\left(2^n-1\right)$ **15.** 2059 or 463

16. $\frac{-4}{3},\frac{-8}{3},\frac{-16}{3},\ldots$ or $4,-8,16,-32,64,\ldots$ **18.** $\frac{80}{81}\left(10^n-1\right)-\frac{8}{9}n$

19. 496 **20.** rR **21.** 3, –6, 12, –24 **26.** 9 and 27

27. $n=\frac{-1}{2}$ **30.** 120, 480, 30 (2^n) **31.** Rs 500 $(1.1)^{10}$

32. $x^2 - 16x + 25 = 0$

EXERCISE 9.4

1. $\frac{n}{3}(n+1)(n+2)$ **2.** $\frac{n(n+1)(n+2)(n+3)}{4}$

3. $\frac{n}{6}(n+1)(3n^2+5n+1)$ **4.** $\frac{n}{n+1}$ **5.** 2840

6. $3n\ (n+1)\ (n+3)$ **7.** $\frac{n(n+1)^2(n+2)}{12}$

8. $\frac{n(n+1)}{12}(3n^2+23n+34)$

9. $\frac{n}{6}(n+1)(2n+1)+2(2^n-1)$ **10.** $\frac{n}{3}(2n+1)(2n-1)$

Miscellaneous Exercise on Chapter 9

2. 5, 8, 11 **4.** 8729 **5.** 3050 **6.** 1210
7. 4 **8.** 160; 6 **9.** ± 3 **10.** 8, 16, 32
11. 4 **12.** 11

21. (i) $\frac{50}{81}(10^n-1)-\frac{5n}{9}$, (ii) $\frac{2n}{3}-\frac{2}{27}(1-10^{-n})$ **22.** 1680

23. $\frac{n}{3}(n^2+3n+5)$ **25.** $\frac{n}{24}(2n^2+9n+13)$

27. Rs 16680 **28.** Rs 39100 **29.** Rs 43690 **30.** Rs 17000; 20,000
31. Rs 5120 **32.** 25 days

EXERCISE 10.1

1. $\frac{121}{2}$ square unit.

2. (0, a), (0, – a) and $\left(-\sqrt{3}a,0\right)$ or (0, a), (0, – a), and $\left(\sqrt{3}a,0\right)$

3. (i) $|y_2-y_1|$, (ii) $|x_2-x_1|$ **4.** $\left(\frac{15}{2},0\right)$ **5.** $-\frac{1}{2}$

7. $-\sqrt{3}$ **8.** $x=1$ **10.** 135°

11. 1 and 2, or $\frac{1}{2}$ and 1, or – 1 and –2, or $-\frac{1}{2}$ and – 1 **14.** $\frac{1}{2}$, 104.5 Crores

EXERCISE 10.2

1. $y = 0$ and $x = 0$ **2.** $x - 2y + 10 = 0$ **3.** $y = mx$

4. $\left(\sqrt{3}+1\right)x-\left(\sqrt{3}-1\right)y=4\left(\sqrt{3}-1\right)$ **5.** $2x + y + 6 = 0$

6. $x-\sqrt{3}y+2\sqrt{3}=0$ **7.** $5x + 3y + 2 = 0$

8. $\sqrt{3}x+y=10$ **9.** $3x - 4y + 8 = 0$ **10.** $5x - y + 20 = 0$

11. $(1 + n)x + 3(1 + n)y = n + 11$ **12.** $x + y = 5$

13. $x + 2y - 6 = 0$, $2x + y - 6 = 0$

14. $\sqrt{3}x+y-2=0$ and $\sqrt{3}x+y+2=0$ **15.** $2x - 9y + 85 = 0$

16. $L=\frac{.192}{90}(C-20)+124.942$ **17.** 1340 litres. **19.** $2kx + hy = 3kh$.

EXERCISE 10.3

1. (i) $y=-\frac{1}{7}x+0, -\frac{1}{7}, 0$; (ii) $y=-2x+\frac{5}{3}, -2, \frac{5}{3}$; (iii) $y = 0x + 0, 0, 0$

2. (i) $\frac{x}{4}+\frac{y}{6}=1, 4, 6$; (ii) $\frac{x}{\frac{3}{2}}+\frac{y}{-2}=1, \frac{3}{2}, -2$;

(iii) $y=-\frac{2}{3}$, intercept with y-axis $= -\frac{2}{3}$ and no intercept with x-axis.

3. (i) $x \cos 120° + y \sin 120° = 4$, 4, 120° (ii) $x \cos 90° + y \sin 90° = 2$, 2, 90°;

(iii) $x \cos 315° + y \sin 315° = 2\sqrt{2}$, $2\sqrt{2}$, 315° **4.** 5 units

5. $(-2, 0)$ and $(8, 0)$ **6.** (i) $\frac{65}{17}$ units, (ii) $\frac{1}{\sqrt{2}}\left|\frac{p+r}{l}\right|$ units.

7. $3x - 4y + 18 = 0$ **8.** $y + 7x = 21$ **9.** 30° and 150°

10. $\frac{22}{9}$

12. $\left(\sqrt{3}+2\right)x+\left(2\sqrt{3}-1\right)y=8\sqrt{3}+1$ or $\left(\sqrt{3}-2\right)x+\left(1+2\sqrt{3}\right)y=-1+8\sqrt{3}$

13. $2x + y = 5$ **14.** $\left(\frac{68}{25}, -\frac{49}{25}\right)$ **15.** $m = \frac{1}{2}, c = \frac{5}{2}$

17. $y - x = 1, \sqrt{2}$

Miscellaneous Exercise on Chapter 10

1. (a) 3, (b) ± 2, (c) 6 or 1 **2.** $\frac{7\pi}{6}, 1$

3. $2x - 3y = 6, -3x + 2y = 6$ **4.** $\left(0, -\frac{8}{3}\right), \left(0, \frac{32}{3}\right)$

5. $\left|\text{Cos}\left(\frac{\phi - \theta}{2}\right)\right|$ **6.** $x = -\frac{5}{22}$ **7.** $2x - 3y + 18 = 0$

8. k^2 square units **9.** 5 **11.** $3x - y = 7, \quad x + 3y = 9$

12. $13x + 13y = 6$ **14.** 1 : 2 **15.** $\frac{23\sqrt{5}}{18}$ units

16. The line is parallel to x - axis or parallel to y-axis

17. $x = 1, \quad y = 1.$ **18.** $(-1, -4).$ **19.** $\frac{1 \pm 5\sqrt{2}}{7}$

21. $18x + 12y + 11 = 0$ **22.** $\left(\frac{13}{5}, 0\right)$ **24.** $119x + 102y = 125$

EXERCISE 11.1

1. $x^2 + y^2 - 4y = 0$ **2.** $x^2 + y^2 + 4x - 6y - 3 = 0$

3. $36x^2 + 36y^2 - 36x - 18y + 11 = 0$ **4.** $x^2 + y^2 - 2x - 2y = 0$

5. $x^2 + y^2 + 2ax + 2by + 2b^2 = 0$ **6.** $c(-5, 3), r = 6$

7. $c(2, 4), r = \sqrt{65}$ **8.** $c(4, -5), r = \sqrt{53}$ **9.** $c\left(\frac{1}{4}, 0\right); r = \frac{1}{4}$

10. $x^2 + y^2 - 6x - 8y + 15 = 0$ **11.** $x^2 + y^2 - 7x + 5y - 14 = 0$

12. $x^2 + y^2 + 4x - 21 = 0$ & $x^2 + y^2 - 12x + 11 = 0$

13. $x^2 + y^2 - ax - by = 0$ **14.** $x^2 + y^2 - 4x - 4y = 5$

15. Inside the circle; since the distance of the point to the centre of the circle is less than the radius of the circle.

EXERCISE 11.2

1. F (3, 0), axis - x - axis, directrix $x = -3$, length of the Latus rectum = 12

2. F $(0, \frac{3}{2})$, axis - y - axis, directrix $y = -\frac{3}{2}$, length of the Latus rectum = 6

3. F (–2, 0), axis - x - axis, directrix $x = 2$, length of the Latus rectum = 8

4. F (0, –4), axis - y - axis, directrix $y = 4$, length of the Latus rectum = 16

5. F $(\frac{5}{2}, 0)$ axis - x - axis, directrix $x = -\frac{5}{2}$, length of the Latus rectum = 10

6. F $(0, \frac{-9}{4})$, axis - y - axis, directrix $y = \frac{9}{4}$, length of the Latus rectum = 9

7. $y^2 = 24x$ **8.** $x^2 = -12y$ **9.** $y^2 = 12x$

10. $y^2 = -8x$ **11.** $2y^2 = 9x$ **12.** $2x^2 = 25y$

EXERCISE 11.3

1. F $(\pm\sqrt{20}, 0)$; V $(\pm 6, 0)$; Major axis = 12; Minor axis = 8, $e = \frac{\sqrt{20}}{6}$,

Latus rectum = $\frac{16}{3}$

2. F $(0, \pm\sqrt{21})$; V $(0, \pm 5)$; Major axis = 10; Minor axis = 4, $e = \frac{\sqrt{21}}{5}$;

Latus rectum = $\frac{8}{5}$

3. F $(\pm\sqrt{7}, 0)$; V $(\pm 4, 0)$; Major axis = 8; Minor axis = 6, $e = \frac{\sqrt{7}}{4}$;

Latus rectum = $\frac{9}{2}$

4. F $(0, \pm\sqrt{75})$; V $(0, \pm 10)$; Major axis = 20; Minor axis = 10 , $e = \frac{\sqrt{3}}{2}$;
Latus rectum = 5

5. F $(\pm\sqrt{13}, 0)$; V $(\pm 7, 0)$; Major axis =14 ; Minor axis = 12 , $e = \frac{\sqrt{13}}{7}$;
Latus rectum = $\frac{72}{7}$

6. F $(0, \pm 10\sqrt{3})$; V $(0, \pm 20)$; Major axis =40 ; Minor axis = 20 , $e = \frac{\sqrt{3}}{2}$;
Latus rectum = 10

7. F $(0, \pm 4\sqrt{2})$; V $(0, \pm 6)$; Major axis =12 ; Minor axis = 4 , $e = \frac{2\sqrt{2}}{3}$;
Latus rectum = $\frac{4}{3}$

8. F $\left(0, \pm\sqrt{15}\right)$; V $(0, \pm 4)$; Major axis = 8 ; Minor axis = 2 , $e = \frac{\sqrt{15}}{4}$;
Latus rectum = $\frac{1}{2}$

9. F $(\pm\sqrt{5}, 0)$; V $(\pm 3, 0)$; Major axis = 6 ; Minor axis = 4 , $e = \frac{\sqrt{5}}{3}$;
Latus rectum = $\frac{8}{3}$

10. $\frac{x^2}{25}+\frac{y^2}{9}=1$ **11.** $\frac{x^2}{144}+\frac{y^2}{169}=1$ **12.** $\frac{x^2}{36}+\frac{y^2}{20}=1$

13. $\frac{x^2}{9}+\frac{y^2}{4}=1$ **14.** $\frac{x^2}{1}+\frac{y^2}{5}=1$ **15.** $\frac{x^2}{169}+\frac{y^2}{144}=1$

16. $\frac{x^2}{64}+\frac{y^2}{100}=1$ **17.** $\frac{x^2}{16}+\frac{y^2}{7}=1$ **18.** $\frac{x^2}{25}+\frac{y^2}{9}=1$

19. $\frac{x^2}{10}+\frac{y^2}{40}=1$

20. $x^2 + 4y^2 = 52$ or $\frac{x^2}{52}+\frac{y^2}{13}=1$

EXERCISE 11.4

1. Foci $(\pm 5, 0)$, Vertices $(\pm 4, 0)$; $e = \frac{5}{4}$; Latus rectum $= \frac{9}{2}$

2. Foci (0 ± 6), Vertices $(0, \pm 3)$; $e = 2$; Latus rectum $= 18$

3. Foci $(0, \pm\sqrt{13})$, Vertices $(0, \pm 2)$; $e = \frac{\sqrt{13}}{2}$; Latus rectum $= 9$

4. Foci $(\pm 10, 0)$, Vertices $(\pm 6, 0)$; $e = \frac{5}{3}$; Latus rectum $= \frac{64}{3}$

5. Foci $(0, \pm\frac{2\sqrt{14}}{\sqrt{5}})$, Vertices $(0, \pm\frac{6}{\sqrt{5}})$; $e = \frac{\sqrt{14}}{3}$; Latus rectum $= \frac{4\sqrt{5}}{3}$

6. Foci $(0, \pm\sqrt{65})$, Vertices $(0, \pm 4)$; $e = \frac{\sqrt{65}}{4}$; Latus rectum $= \frac{49}{2}$

7. $\frac{x^2}{4}-\frac{y^2}{5}=1$

8. $\frac{y^2}{25}-\frac{x^2}{39}=1$

9. $\frac{y^2}{9}-\frac{x^2}{16}=1$

10. $\frac{x^2}{16}-\frac{y^2}{9}=1$

11. $\frac{y^2}{25}-\frac{x^2}{144}=1$

12. $\frac{x^2}{25}-\frac{y^2}{20}=1$

13. $\frac{x^2}{4}-\frac{y^2}{12}=1$

14. $\frac{x^2}{49}-\frac{9y^2}{343}=1$

15. $\frac{y^2}{5}-\frac{x^2}{5}=1$

Miscellaneous Exercise on Chapter 11

1. Focus is at the mid-point of the given diameter.

2. 2.23 m (approx.)

3. 9.11 m (approx.)

4. 1.56m (approx.)

5. $\frac{x^2}{81}+\frac{y^2}{9}=1$

6. 18 sq units

7. $\frac{x^2}{25}+\frac{y^2}{9}=1$

8. $8\sqrt{3}a$

EXERCISE 12.1

1. y and z - coordinates are zero
2. y - coordinate is zero
3. I, IV, VIII, V, VI, II, III, VII
4. (i) XY - plane (ii) $(x, y, 0)$ (iii) Eight

EXERCISE 12.2

1. (i) $2\sqrt{5}$ (ii) $\sqrt{43}$ (iii) $2\sqrt{26}$ (iv) $2\sqrt{5}$
4. $x - 2z = 0$
5. $9x^2 + 25y^2 + 25z^2 - 225 = 0$

EXERCISE 12.3

1. (i) $\left(\frac{-4}{5}, \frac{1}{5}, \frac{27}{5}\right)$ (ii) $(-8, 17, 3)$
2. $1 : 2$
3. $2 : 3$
5. $(6, -4, -2), (8, -10, 2)$

Miscellaneous Exercise on Chapter 12

1. $(1, -2, 8)$
2. $7, \sqrt{34}, 7$
3. $a = -2, \ b = -\frac{16}{3}, \ c = 2$
4. $(0, 2, 0)$ and $(0, -6, 0)$
5. $(4, -2, 6)$
6. $x^2 + y^2 + z^2 - 2x - 7y + 2z = \frac{k^2 - 109}{2}$

EXERCISE 13.1

1. 6
2. $\left(\pi - \frac{22}{7}\right)$
3. π
4. $\frac{19}{2}$
5. $-\frac{1}{2}$
6. 5
7. $\frac{11}{4}$
8. $\frac{108}{7}$
9. b
10. 2
11. 1
12. $-\frac{1}{4}$
13. $\frac{a}{b}$
14. $\frac{a}{b}$
15. $\frac{1}{\pi}$
16. $\frac{1}{\pi}$

17. 4 **18.** $\dfrac{a+1}{b}$ **19.** 0 **20.** 1

21. 0 **22.** 2 **23.** 3, 6

24. Limit does not exist at $x = 1$

25. Limit does not exist at $x = 0$ **26.** Limit does not exist at $x = 0$

27. 0 **28.** $a = 0, b = 4$

29. $\lim\limits_{x \to a_1} f(x) = 0$ and $\lim\limits_{x \to a} f(x) = (a - a_1)(a - a_2) \ldots (a - a_x)$

30. $\lim\limits_{x \to a} f(x)$ exists for all $a \neq 0$. **31.** 2

32. For $\lim\limits_{x \to 0} f(x)$ to exists, we need $m = n$; $\lim\limits_{x \to 1} f(x)$ exists for any integral value of m and n.

EXERCISE 13.2

1. 20 **2.** 1 **3.** 99

4. (i) $3x^2$ (ii) $2x - 3$ (iii) $\dfrac{-2}{x^3}$ (iv) $\dfrac{-2}{(x-1)^2}$

6. $nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \ldots + a^{n-1}$

7. (i) $2x - a - b$ (ii) $4ax\left(ax^2 + b\right)$ (iii) $\dfrac{a-b}{(x-b)^2}$

8. $\dfrac{nx^n - anx^{n-1} - x^n + a^n}{(x-a)^2}$

9. (i) 2 (ii) $20x^3 - 15x^2 + 6x - 4$ (iii) $\dfrac{-3}{x^4}(5 + 2x)$ (iv) $15x^4 + \dfrac{24}{x^5}$

(v) $\dfrac{-12}{x^5} + \dfrac{36}{x^{10}}$ (vi) $\dfrac{-2}{(x+1)^2} - \dfrac{x(3x-2)}{(3x-1)^2}$ **10.** $-\sin x$

11. (i) $\cos 2x$ (ii) $\sec x \tan x$
(iii) $5\sec x \tan x - 4\sin x$ (iv) $-\operatorname{cosec} x \cot x$
(v) $-3\operatorname{cosec}^2 x - 5 \operatorname{cosec} x \cot x$ (vi) $5\cos x + 6\sin x$
(vii) $2\sec^2 x - 7\sec x \tan x$

Miscellaneous Exercise on Chapter 13

1. (i) – 1 (ii) $\frac{1}{x^2}$ (iii) cos $(x + 1)$ (iv) $-\sin\left(x-\frac{\pi}{8}\right)$ **2.** 1

3. $\frac{-qr}{x^2}+ps$ **4.** $2c\,(ax+b)\,(cx+d)+a\,(cx+d)^2$

5. $\frac{ad-bc}{(cx+d)^2}$ **6.** $\frac{-2}{(x-1)^2},\ x\neq 0,1$ **7.** $\frac{-(2ax+b)}{\left(ax^2+bx+c\right)^2}$

8. $\frac{-apx^2-2bpx+ar-bq}{\left(px^2+qx+r\right)^2}$ **9.** $\frac{apx^2+2bpx+bq-ar}{(ax+b)^2}$ **10.** $\frac{-4a}{x^5}+\frac{2b}{x^3}-\sin x$

11. $\frac{2}{\sqrt{x}}$ **12.** $na(ax+b)^{n-1}$

13. $(ax+b)^{n-1}(cx+d)^{m-1}\left[mc(ax+b)+na(cx+d)\right]$ **14.** $\cos\,(x+a)$

15. $-\text{cosec}^3 x - \text{cosec}\, x\cot^2 x$ **16.** $\frac{-1}{1+\sin x}$

17. $\frac{-2}{(\sin x-\cos x)^2}$ **18.** $\frac{2\sec x\tan x}{(\sec x+1)^2}$ **19.** $n\sin^{n-1}x\cos x$

20. $\frac{bc\cos x+ad\sin x+bd}{(c+d\cos x)^2}$ **21.** $\frac{\cos a}{\cos^2 x}$

22. $x^3\left(5x\cos x+3x\sin x+20\sin x-12\cos x\right)$

23. $-x^2\sin x-\sin x+2x\cos x$

24. $-q\sin x\left(ax^2+\sin x\right)+\left(p+q\cos x\right)\left(2a\,x+\cos x\right)$

25. $-\tan^2 x\left(x+\cos x\right)+\left(x-\tan x\right)\left(1-\sin x\right)$

26. $\frac{35+15x\cos x+28\cos x+28x\sin x-15\sin x}{(3x+7\cos x)^2}$

27. $\dfrac{x\cos\dfrac{\pi}{4}(2\sin x - x\cos x)}{\sqrt{2}\sin^2 x}$

28. $\dfrac{1+\tan x - x\sec^2 x}{(1+\tan x)^2}$

29. $(x+\sec x)(1-\sec^2 x)+(x-\tan x).(1+\sec x\tan x)$

30. $\dfrac{\sin x - n\,x\cos x}{\sin^{n+1} x}$

EXERCISE 14.1

1. (i) This sentence is always false because the maximum number of days in a month is 31. Therefore, it is a statement.

(ii) This is not a statement because for some people mathematics can be easy and for some others it can be difficult.

(iii) This sentence is always true because the sum is 12 and it is greater than 10. Therefore, it is a statement.

(iv) This sentence is sometimes true and sometimes not true. For example the square of 2 is even number and the square of 3 is an odd number. Therefore, it is not a statement.

(v) This sentence is sometimes true and sometimes false. For example, squares and rhombus have equal length whereas rectangles and trapezium have unequal length. Therefore, it is not a statement.

(vi) It is an order and therefore, is not a statement.

(vii) This sentence is false as the product is (–8). Therefore, it is a statement.

(viii) This sentence is always true and therefore, it is a statement.

(ix) It is not clear from the context which day is referred and therefore, it is not a statement.

(x) This is a true statement because all real numbers can be written in the form $a + i \times 0$.

2. The three examples can be:

(i) Everyone in this room is bold. This is not a statement because from the context it is not clear which room is referred here and the term bold is not precisely defined.

(ii) She is an engineering student. This is also not a statement because who 'she' is.

(iii) "$\cos^2\theta$ is always greater than 1/2". Unless, we know what θ is, we cannot say whether the sentence is true or not.

EXERCISES 14.2

1. (i) Chennai is not the capital of Tamil Nadu.
   (ii) $\sqrt{2}$ is a complex number.
   (iii) All triangles are equilateral triangles.
   (iv) The number 2 is not greater than 7.
   (v) Every natural number is not an integer.
2. (i) The negation of the first statement is "the number x is a rational number." which is the same as the second statement" This is because when a number is not irrational, it is a rational. Therefore, the given pairs are negations of each other.
   (ii) The negation of the first statement is "x is an irrational number" which is the same as the second statement. Therefore, the pairs are negations of each other.
3. (i) Number 3 is prime; number 3 is odd (True).
   (ii) All integers are positive; all integers are negative (False).
   (iii) 100 is divisible by 3,100 is divisible by 11 and 100 is divisible by 5 (False).

EXERCISE 14.3

1. (i) "And". The component statements are:
   All rational numbers are real.
   All real numbers are not complex.
   (ii) "Or". The component statements are:
   Square of an integer is positive.
   Square of an integer is negative.
   (iii) "And". the component statements are:
   The sand heats up quickily in the sun.
   The sand does not cool down fast at night.
   (iv) "And". The component statements are:
   $x = 2$ is a root of the equation $3x^2 - x - 10 = 0$
   $x = 3$ is a root of the equation $3x^2 - x - 10 = 0$
2. (i) "There exists". The negation is
   There does not exist a number which is equal to its square.
   (ii) "For every". The negation is
   There exists a real number x such that x is not less than $x + 1$.
   (iii) "There exists". The negation is
   There exists a state in India which does not have a capital.

3. No. The negation of the statement in (i) is "There exists real number x and y for which $x + y \neq y + x$", instead of the statement given in (ii).

4. (i) Exclusive
   (ii) Inclusive
   (iii) Exclusive

EXERCISE 14.4

1. (i) A natural number is odd implies that its square is odd.
   (ii) A natural number is odd only if its square is odd.
   (iii) For a natural number to be odd it is necessary that its square is odd.
   (iv) For the square of a natural number to be odd, it is sufficient that the number is odd
   (v) If the square of a natural number is not odd, then the natural number is not odd.

2. (i) The contrapositive is
   If a number x is not odd, then x is not a prime number.
   The converse is
   If a number x in odd, then it is a prime number.
   (ii) The contrapositive is
   If two lines intersect in the same plane, then they are not parallel
   The converse is
   If two lines do not interesect in the same plane, then they are parallel
   (iii) The contrapositive is
   If something is not at low temperature, then it is not cold
   The converse is
   If something is at low temperature, then it is cold
   (iv) The contrapositive is
   If you know how to reason deductively, then you can comprehend geometry.
   The converse is
   If you do not know how to reason deductively, then you can not comprehend geometry.
   (v) This statement can be written as "If x is an even number, then x is divisible by 4".
   The contrapositive is, If x is not divisible by 4, then x is not an even number.
   The converse is, If x is divisible by 4, then x is an even number.

3. (i) If you get a job, then your credentials are good.
   (ii) If the banana tree stays warm for a month, then it will bloom.

(iii) If diagonals of a quadrilateral bisect each other, then it is a parallelogram.
(iv) If you get A^+ in the class, then you do all the exercises in the book.

4. **a** (i) Contrapositive
(ii) Converse
b (i) Contrapositive
(ii) Converse

EXERCISE 14.5

5. (i) False. By definition of the chord, it should intersect the circle in two points.
(ii) False. This can be shown by giving a counter example. A chord which is not a dimaeter gives the counter example.
(iii) True. In the equation of an ellipse if we put $a = b$, then it is a circle (Direct Method)
(iv) True, by the rule of inequality
(v) False. Since 11 is a prime number, therefore $\sqrt{11}$ is irrational.

Miscellaneous Exercise on Chapter 14

1. (i) There exists a positive real number x such that x–1 is not positive.
(ii) There exists a cat which does not scratch.
(iii) There exists a real number x such that neither $x > 1$ nor $x < 1$.
(iv) There does not exist a number x such that $0 < x < 1$.

2. (i) The statement can be written as “If a positive integer is prime, then it has no divisors other than 1 and itself.
The converse of the statement is
If a positive integer has no divisors other than 1 and itself, then it is a prime.
The contrapositive of the statement is
If positive integer has divisors other than 1 and itself then it is not prime.
(ii) The given statement can be written as “If it is a sunny day, then I go to a beach.
The converse of the statement is
If I go to beach, then it is a sunny day.
The contrapositive is
If I do not go to a beach, then it is not a sunny day.
(iii) The converse is
If you feel thirsty, then it is hot outside.
The contrapositive is
If you do not feel thirsty, then it is not hot outside.

3. (i) If you log on to the server, then you have a password.
(ii) If it rains, then there is traffic jam.
(iii) If you can access the website, then you pay a subscription fee.

4. (i) You watch television if and only if your mind in free.
(ii) You get an A grade if and only if you do all the homework regularly.
(iii) A quadrilateral is equiangular if and only if it is a rectangle.

5. The compound statement with "And" is 25 is a multiple of 5 and 8
This is a false statement.
The compound statement with "Or" is 25 is a multiple of 5 or 8
This is true statement.

7. Same as Q1 in Exercise 14.4

EXERCISE 15.1

1. 3 **2.** 8.4 **3.** 2.33 **4.** 7

5. 6.32 **6.** 16 **7.** 3.23 **8.** 5.1

9. 157.92 **10.** 11.28 **11.** 10.34 **12.** 7.35

EXERCISE 15.2

1. 9, 9.25 **2.** $\frac{n+1}{2}, \frac{n^2-1}{12}$ **3.** 16.5, 74.25 **4.** 19, 43.4

5. 100, 29.09 **6.** 64, 1.69 **7.** 107, 2276 **8.** 27, 132

9. 93, 105.58, 10.27 **10.** 5.55, 43.5

EXERCISE 15.3

1. B **2.** Y **3.** (i) B, (ii) B

4. A **5.** Weight

Miscellaneous Exercise on Chapter 15

1. 4, 8 **2.** 6, 8 **3.** 24, 12

5. (i) 10.1, 1.99 (ii) 10.2, 1.98

6. Highest Chemistry and lowest Mathematics **7.** 20, 3.036

EXERCISE 16.1

1. {HHH, HHT, HTH, THH, TTH, HTT, THT, TTT}

2. $\{(x, y) : x, y = 1,2,3,4,5,6\}$

or {(1,1), (1,2), (1,3), ..., (1,6), (2,1), (2,2), ..., (2,6), ..., (6, 1), (6, 2), ..., (6,6)}

3. {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

4. {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

5. {H1, H2, H3, H4, H5, H6, T}

6. $\{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$

7. {R1, R2, R3, R4, R5, R6, W1, W2, W3, W4, W5, W6, B1, B2, B3, B4, B5, B6}

8. (i) {BB, BG, GB, GG} (ii) {0, 1, 2}

9. {RW, WR, WW}

10. [HH, HT, T1, T2, T3, T4, T5, T6}

11. {DDD, DDN, DND, NDD, DNN, NDN, NND, NNN}

12. {T, H1, H3, H5, H21, H22, H23, H24, H25, H26, H41, H42, H43, H44, H45, H46, H61, H62, H63, H64, H65, H66}

13. {(1,2), (1,3), (1,4), (2,1), (2,3), (2,4), (3,1), (3,2), (3,4), (4,1), (4,2), (4,3)}

14. {1HH, 1HT, 1TH, 1TT, 2H, 2T, 3HH, 3HT, 3TH, 3TT, 4H, 4T, 5HH, 5HT, 5TH, 5TT, 6H, 6T}

15. $\{TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6\}$

16. {6, (1,6), (2,6), (3,6), (4,6), (5,6), (1,1,6), (1,2,6), ..., (1,5,6), (2,1,6). (2,2,6), ..., (2,5,6), ..., (5,1,6), (5,2,6), ... }

EXERCISE 16.2

1. No.

2. (i) {1, 2, 3, 4, 5, 6} (ii) ϕ (iii) {3, 6} (iv) {1, 2, 3} (v) {6}

(vi) {3, 4, 5, 6}, $A\cup B = \{1, 2, 3, 4, 5, 6\}$, $A\cap B = \phi$, $B\cup C = \{3, 6\}$, $E\cap F = \{6\}$, $D\cap E = \phi$,

$A - C = \{1, 2,4,5\}$, $D - E = \{1,2,3\}$, $E\cap F' = \phi$, $F' = \{1, 2\}$

3. A = {(3,6), (4,5), (5, 4), (6,3), (4,6), (5,5), (6,4), (5,6), (6,5), (6,6)}

B = {(1,2), (2,2), (3, 2), (4,2), (5,2), (6,2), (2,1), (2,3), (2,4), (2,5), (2,6)}

C ={(3,6), (6,3), (5, 4), (4,5), (6,6)}

A and B, B and C are mutually exclusive.

4. (i) A and B; A and C; B and C; C and D (ii) A and C (iii) B and D

5. (i) "Getting at least two heads", and "getting at least two tails"

(ii) "Getting no heads", "getting exactly one head" and "getting at least two heads"

(iii) "Getting at most two tails", and "getting exactly two tails"
(iv) "Getting exactly one head" and "getting exactly two heads"
(v) "Getting exactly one tail", "getting exactly two tails", and getting exactly three tails"

Note There may be other events also as answer to the above question.

6. A = {(2, 1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

B = {(1, 1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}

C = {(1, 1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}

(i) $A' = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\} = B$

(ii) $B' = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\} = A$

(iii) $A \cup B = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (2,1), (2,2), (2,3), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\} = S$

(iv) $A \cap B = \phi$

(v) $A - C = \{(2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

(vi) $B \cup C = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\}$

(vii) $B \cap C = \{(1,1), (1,2), (1,3), (1,4), (3,1), (3,2)\}$

(viii) $A \cap B' \cap C' = \{(2,4), (2,5), (2,6), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

7. (i) True (ii) True (iii) True (iv) False (v) False (vi) False

EXERCISE 16.3

1. (a) Yes (b) Yes (c) No (d) No (e) No **2.** $\frac{3}{4}$

3. (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{1}{6}$ (iv) 0 (v) $\frac{5}{6}$ **4.** (a) 52 (b) $\frac{1}{52}$ (c) (i) $\frac{1}{13}$ (ii) $\frac{1}{2}$

5. (i) $\frac{1}{12}$ (ii) $\frac{1}{12}$ **6.** $\frac{3}{5}$

7. Rs 4.00 gain, Rs 1.50 gain, Re 1.00 loss, Rs 3.50 loss, Rs 6.00 loss.

P (Winning Rs 4.00) $=\frac{1}{16}$, P(Winning Rs 1.50) $=\frac{1}{4}$, P (Losing Re. 1.00) $=\frac{3}{8}$

P (Losing Rs 3.50) $=\frac{1}{4}$, P (Losing Rs 6.00) $=\frac{1}{16}$.

8. (i) $\frac{1}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{1}{2}$ (iv) $\frac{7}{8}$ (v) $\frac{1}{8}$ (vi) $\frac{1}{8}$ (vii) $\frac{3}{8}$ (viii) $\frac{1}{8}$ (ix) $\frac{7}{8}$

9. $\frac{9}{11}$

10. (i) $\frac{6}{13}$ (ii) $\frac{7}{13}$

11. $\frac{1}{38760}$

12. (i) No, because P(A∩B) must be less than or equal to P(A) and P(B), (ii) Yes

13. (i) $\frac{7}{15}$ (ii) 0.5 (iii) 0.15

14. $\frac{4}{5}$

15. (i) $\frac{5}{8}$ (ii) $\frac{3}{8}$

16. No

17. (i) 0.58 (ii) 0.52 (iii) 0.74

18. 0.6

19. 0.55

20. 0.65

21. (i) $\frac{19}{30}$ (ii) $\frac{11}{30}$ (iii) $\frac{2}{15}$

Miscellaneous Exercise on Chapter 16

1. (i) $\frac{^{20}C_5}{^{60}C_5}$ (ii) $1-\frac{^{30}C_5}{^{60}C_5}$

2. $\frac{^{13}C_3 \cdot {^{13}C_1}}{^{52}C_4}$

3. (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$ (iii) $\frac{5}{6}$

4. (a) $\frac{999}{1000}$ (b) $\frac{^{9990}C_2}{^{10000}C_2}$ (c) $\frac{^{9990}C_{10}}{^{10000}C_{10}}$

5. (a) $\frac{17}{33}$ (b) $\frac{16}{33}$

6. $\frac{2}{3}$

7. (i) 0.88 (ii) 0.12 (iii) 0.19 (iv) 0.34

8. $\frac{4}{5}$

9. (i) $\frac{33}{83}$ (ii) $\frac{3}{8}$

10. $\frac{1}{5040}$

SUPPLEMENTARY MATERIAL

CHAPTER 3

3.6 Proofs and Simple Applications of Sine and Cosine Formulae

Let ABC be a triangle. By angle A, we mean the angle between the sides AB and AC which lies between 0° and 180°. The angles B and C are similarly defined. The sides AB, BC and CA opposite to the vertices C, A and B will be denoted by *c*, *a* and *b* respectively (see Fig. 3.15).

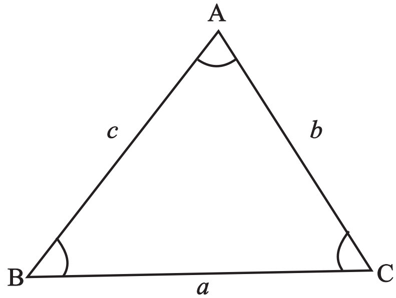


Fig. 3.15

Theorem 1 (Sine formulae) In any triangle, sides are proportional to the sines of the opposite angles. That is, in a triangle ABC

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Proof Let ABC be either of the triangles as shown in Fig. 3.16 (i) and (ii).

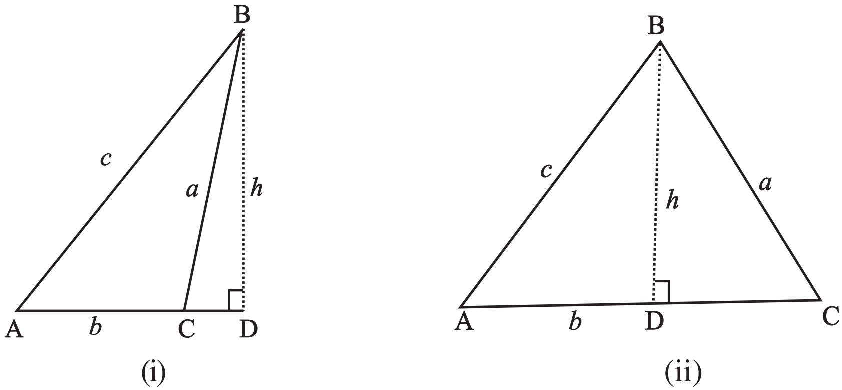


Fig. 3.16

The altitude *h* is drawn from the vertex B to meet the side AC in point D [in (i) AC is produced to meet the altitude in D]. From the right angled triangle ABD in Fig. 3.16(i), we have

$$\sin A = \frac{h}{c}, \text{ i.e., } h = c \sin A \tag{1}$$

and $$\sin (180° - C) = \frac{h}{a} \Rightarrow h = a \sin C \tag{2}$$

From (1) and (2), we get

$$c \sin A = a \sin C, \text{ i.e., } \frac{\sin A}{a} = \frac{\sin C}{c} \tag{3}$$

Similarly, we can prove that

$$\frac{\sin A}{a} = \frac{\sin B}{b} \tag{4}$$

From (3) and (4), we get

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

For triangle ABC in Fig. 3.16 (ii), equations (3) and (4) follow similarly.

Theorem 2 (Cosine formulae) Let A, B and C be angles of a triangle and a, b and c be lengths of sides opposite to angles A, B and C respectively, then

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = c^2 + a^2 - 2ca \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

Proof Let ABC be triangle as given in Fig. 3.17 (i) and (ii)

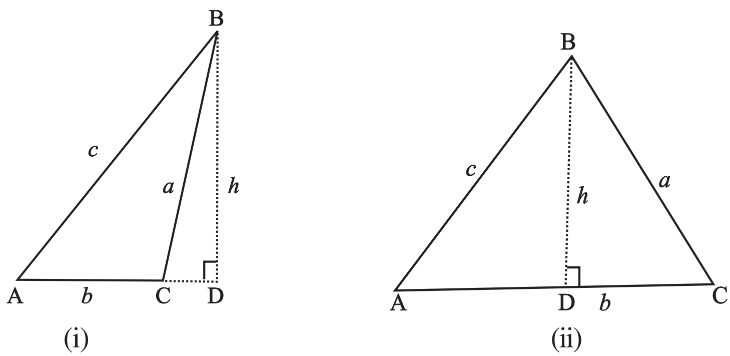


Fig. 3.17

Referring to Fig. 3.17 (ii), we have

$$BC^2 = BD^2 + DC^2 = BD^2 + (AC - AD)^2$$
$$= BD^2 + AD^2 + AC^2 - 2AC.AD$$
$$= AB^2 + AC^2 - 2AC\,AB\cos A$$

or $$a^2 = b^2 + c^2 - 2bc \cos A$$

Similarly, we can obtain

$$b^2 = c^2 + a^2 - 2ca\cos B$$

and $$c^2 = a^2 + b^2 - 2ab\cos C$$

Same equations can be obtained for Fig. 3.17 (i), where C is obtuse.

A convenient form of the cosine formulae, when angles are to be found are as follows:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{c^2 + a^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Example 25 In triangle ABC, prove that

$$\tan\frac{B - C}{2} = \frac{b - c}{b + c}\cot\frac{A}{2}$$
$$\tan\frac{C - A}{2} = \frac{c - a}{c + a}\cot\frac{B}{2}$$
$$\tan\frac{A - B}{2} = \frac{a - b}{a + b}\cot\frac{C}{2}$$

Proof By sine formulae, we have

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k\,(say).$$

Therefore, $\dfrac{b-c}{b+c} = \dfrac{k(\sin B - \sin C)}{k(\sin B + \sin C)}$

$$= \frac{2\cos\frac{B+C}{2}\sin\frac{B-C}{2}}{2\sin\frac{B+C}{2}\cos\frac{B-C}{2}}$$

$$= \cot\frac{(B+C)}{2}\tan\frac{(B-C)}{2}$$

$$= \cot\left(\frac{\pi}{2}-\frac{A}{2}\right)\tan\left(\frac{B-C}{2}\right)$$

$$= \frac{\tan\frac{B-C}{2}}{\cot\frac{A}{2}}$$

Therefore, $\tan\dfrac{B-C}{2} = \dfrac{b-c}{b+c}\cot\dfrac{A}{2}$

Similarly, we can prove other results. These results are well known as Napier's Analogies.

Example 26 In any triangle ABC, prove that

$$a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$$

Solution Consider

$$a\sin(B-C) = a[\sin B\cos C - \cos B\sin C] \qquad (1)$$

Now $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k \text{ (say)}$

Therefore, $\sin A = ak$, $\sin B = bk$, $\sin C = ck$

Substituting the values of sin B and sin C in (1) and using cosine formulae, we get

$$a\sin(\mathrm{B}-\mathrm{C}) = a\left[bk\left(\frac{a^2+b^2-c^2}{2ab}\right) - ck\left(\frac{c^2+a^2-b^2}{2ac}\right)\right]$$

$$= \frac{k}{2}\left(a^2+b^2-c^2-c^2-a^2+b^2\right)$$

$$= k\,(b^2-c^2)$$

Similarly, b sin (C – A) = $k\,(c^2 - a^2)$

and c sin (A – B) = $k\,(a^2 - b^2)$

Hence L.H.S $= k\,(b^2 - c^2 + c^2 - a^2 + a^2 - b^2)$

= 0 = R.H.S.

Example 27 The angle of elevation of the top point P of the vertical tower PQ of height h from a point A is 45° and from a point B, the angle of elevation is 60°, where B is a point at a distance d from the point A measured along the line AB which makes an angle 30° with AQ. Prove that $d = h(\sqrt{3}-1)$

Proof From the Fig. 3.18, we have ∠PAQ = 45°, ∠BAQ = 30°, ∠PBH = 60°

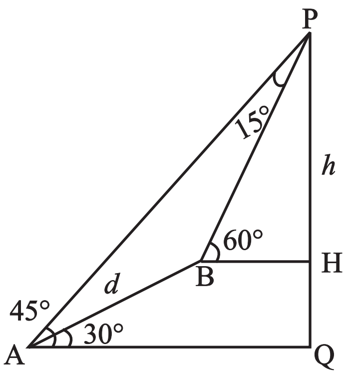


Fig. 3.18

Clearly $\angle \mathrm{APQ} = 45°, \angle \mathrm{BPH} = 30°$, giving $\angle \mathrm{APB} = 15°$

Again $\angle \mathrm{PAB} = 15° \Rightarrow \angle \mathrm{ABP} = 150°$

From triangle APQ, we have $AP^2 = h^2 + h^2 = 2h^2$ (Why?)

or $$AP = \sqrt{2}h$$

Applying sine formulae in Δ ABP, we get

$$\frac{AB}{\sin 15°} = \frac{AP}{\sin 150°} \Rightarrow \frac{d}{\sin 15°} = \frac{\sqrt{2}h}{\sin 150°}$$

i.e., $$d = \frac{\sqrt{2}h \sin 15°}{\sin 30°}$$

$$= h(\sqrt{3} - 1) \qquad \text{(why?)}$$

Example 28 A lamp post is situated at the middle point M of the side AC of a triangular plot ABC with BC = 7 m, CA = 8 m and AB = 9 m. Lamp post subtends an angle 15° at the point B. Determine the height of the lamp post.

Solution From the Fig. 3.19, we have AB = 9 = c, BC = 7 m = a and

AC = 8 m = b.

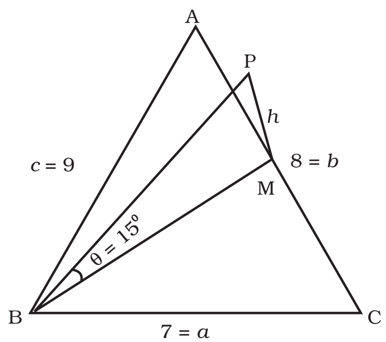


Fig. 3.19

M is the mid-point of the side AC at which lamp post MP of height h (say) is located. Again, it is given that lamp post subtends an angle θ (say) at B which is 15°.

Applying cosine formulae in ΔABC, we have

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} = \frac{49 + 64 - 81}{2 \times 7 \times 8} = \frac{2}{7} \qquad (1)$$

Similarly using cosine formulae in ΔBMC, we get

$$BM^2 = BC^2 + CM^2 - 2\, BC \times CM \cos C.$$

Here $CM = \frac{1}{2} CA = 4$, since M is the mid-point of AC.

Therefore, using (1), we get

$$\begin{aligned} BM^2 &= 49 + 16 - 2 \times 7 \times 4 \times \frac{2}{7} \\ &= 49 \end{aligned}$$

or $\quad BM = 7$

Thus, from ΔBMP right angled at M, we have

$$\tan\theta = \frac{PM}{BM} = \frac{h}{7}$$

or $\quad \frac{h}{7} = \tan(15°) = 2 - \sqrt{3}$ (why?)

or $\quad h = 7(2 - \sqrt{3})\,\text{m}$.

Exercise 3.5

In any triangle ABC, if $a = 18$, $b = 24$, $c = 30$, find

1. cos A, cos B, cos C (Ans. $\frac{4}{5}, \frac{3}{5}, 0$)

2. sin A, sin B, sin C (Ans. $\frac{3}{5}, \frac{4}{5}, 1$)

For any triangle ABC, prove that

3. $\frac{a+b}{c} = \frac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}}$

4. $\dfrac{a-b}{c} = \dfrac{\sin\left(\dfrac{A-B}{2}\right)}{\cos\dfrac{C}{2}}$

5. $\sin\dfrac{B-C}{2} = \dfrac{b-c}{a}\cos\dfrac{A}{2}$

6. $a\,(b\cos C - c\cos B) = b^2 - c^2$

7. $a\,(\cos C - \cos B) = 2\,(b-c)\cos^2\dfrac{A}{2}$

8. $\dfrac{\sin(B-C)}{\sin(B+C)} = \dfrac{b^2-c^2}{a^2}$

9. $(b+c)\cos\dfrac{B+C}{2} = a\cos\dfrac{B-C}{2}$

10. $a\cos A + b\cos B + c\cos C = 2a\sin B\sin C$

11. $\dfrac{\cos A}{a} + \dfrac{\cos B}{b} + \dfrac{\cos C}{c} = \dfrac{a^2+b^2+c^2}{2abc}$

12. $(b^2 - c^2)\cot A + (c^2 - a^2)\cot B + (a^2 - b^2)\cot C = 0$

13. $\dfrac{b^2-c^2}{a^2}\sin 2A + \dfrac{c^2-a^2}{b^2}\sin 2B + \dfrac{a^2-b^2}{c^2}\sin 2C = 0$

14. A tree stands vertically on a hill side which makes an angle of 15° with the horizontal. From a point on the ground 35 m down the hill from the base of the tree, the angle of elevation of the top of the tree is 60°. Find the height of the tree. (Ans. $35\sqrt{2}m$)

15. Two ships leave a port at the same time. One goes 24 km per hour in the direction N45°E and other travels 32 km per hour in the direction S75°E. Find the distance between the ships at the end of 3 hours. (Ans. 86.4 km (approx.))

16. Two trees, A and B are on the same side of a river. From a point C in the river the distance of the trees A and B is 250 m and 300 m, respectively. If the angle C is 45°, find the distance between the trees (use $\sqrt{2} = 1.44$). (Ans. 215.5 m)

CHAPTER 5

5.7 Square-root of a Complex Number

We have discussed solving of quadratic equations involving complex roots on page 108-109 of the textbook. Here we explain the particular procedure for finding square root of a complex number expressed in the standard form. We illustrate the same by an example.

Example 12 Find the square root of $-7 - 24i$

Solution Let $x + iy = \sqrt{-7-24i}$

Then $(x+iy)^2 = -7-24i$

or $x^2 - y^2 + 2xyi = -7 - 24i$

Equating real and imaginary parts, we have

$$x^2 - y^2 = -7 \quad (1)$$

$$2xy = -24$$

We know the identity

$$\left(x^2+y^2\right)^2 = \left(x^2-y^2\right)^2 + (2xy)^2$$

$$= 49 + 576$$

$$= 625$$

Thus, $x^2 + y^2 = 25$ (2)

From (1) and (2), $x^2 = 9$ and $y^2 = 16$

or $x = \pm 3$ and $y = \pm 4$

Since the product xy is negative, we have

$x = 3$, $y = -4$ or, $x = -3$, $y = 4$

Thus, the square roots of $-7 - 24i$ are $3 - 4i$ and $-3 + 4i$.

Exercise 5.4

Find the square roots of the following:

1. $-15 - 8i$ (Ans. $1 - 4i$, $-1 + 4i$)
2. $-8 - 6i$ (Ans. $1 - 3i$, $-1 + 3i$)

3. $1 - i$ (Ans. $\left(\pm\sqrt{\frac{\sqrt{2}+1}{2}} \quad \sqrt{\frac{\sqrt{2}-1}{2}}i\right)$)

4. $-i$ (Ans. $\left(\pm\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}i\right)$)

5. i (Ans. $\left(\pm\frac{1}{\sqrt{2}}\pm\frac{1}{\sqrt{2}}i\right)$)

6. $1 + i$ (Ans. $\left(\pm\sqrt{\frac{\sqrt{2}+1}{2}}\pm\sqrt{\frac{\sqrt{2}-1}{2}}i\right)$)

CHAPTER 9

9.7 Infinite G.P. and its Sum

G.P. of the form a, ar, ar^2, ar^3, ... is called infinite G.P. Now, to find the formulae for finding sum to infinity of a G.P., we begin with an example.

Let us consider the G.P.,

$$1, \frac{2}{3}, \frac{4}{9}, \ldots$$

Here $a = 1$, $r = \frac{2}{3}$. We have

$$S_n = \frac{1-\left(\frac{2}{3}\right)^n}{1-\frac{2}{3}} = 3\left[1-\left(\frac{2}{3}\right)^n\right]$$

Let us study the behaviour of $\left(\frac{2}{3}\right)^n$ as n becomes larger and larger:

n	1	5	10	20
$\left(\frac{2}{3}\right)^n$	0.6667	0.1316872428	0.01734152992	0.00030072866

We observe that as n becomes larger and larger, $\left(\frac{2}{3}\right)^n$ becomes closer and closer to zero. Mathematically, we say that as n becomes sufficiently large, $\left(\frac{2}{3}\right)^n$ becomes sufficiently small. In other words as $n \to \infty, \left(\frac{2}{3}\right)^n \to 0$. Consequently, we find that the sum of infinitely many terms is given by $S_\infty = 3$.

Now, for a geometric progression, a, ar, ar^2, ..., if numerical value of common ratio r is less than 1, then

$$S_n = \frac{a(1-r^n)}{(1-r)} = \frac{a}{1-r} - \frac{ar^n}{1-r}$$

In this case as $n \to \infty$, $r^n \to 0$ since $|r| < 1$. Therefore

$$S_n \to \frac{a}{1-r}$$

Symbolically sum to infinity is denoted by S_∞ or S.

Thus, we have $S = \frac{a}{1-r}$.

For examples,

(i) $$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \ldots = \frac{1}{1-\frac{1}{2}} = 2.$$

(ii) $$1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \ldots = \frac{1}{1-\left(\frac{-1}{2}\right)} = \frac{1}{1+\frac{1}{2}} = \frac{2}{3}$$

Exercise 9.4

Find the sum to infinity in each of the following Geometric Progression.

1. $1, \frac{1}{3}, \frac{1}{9}, \ldots$ (Ans. 1.5)
2. 6, 1.2, .24, ... (Ans. 7.5)
3. $5, \frac{20}{7}, \frac{80}{49}, \ldots$ (Ans. $\frac{35}{3}$)
4. $\frac{-3}{4}, \frac{3}{16}, \frac{-3}{64}, \ldots$ (Ans. $\frac{-3}{5}$)

5. Prove that $3^{\frac{1}{2}} \times 3^{\frac{1}{4}} \times 3^{\frac{1}{8}} \ldots = 3$

6. Let $x = 1 + a + a^2 + \ldots$ and $y = 1 + b + b^2 + \ldots$, where $|a| < 1$ and $|b| < 1$. Prove that

$$1 + ab + a^2b^2 + \ldots = \frac{xy}{x+y-1}$$

CHAPTER 10

10.6 Equation of Family of Lines Passing Through the Point of Intersection of Two Lines

Let the two intersecting lines l_1 and l_2 be given by

$$A_1x + B_1y + C_1 = 0 \quad (1)$$

and $$A_2x + B_2y + C_2 = 0 \quad (2)$$

From the equations (1) and (2), we can form an equation

$$A_1x + B_1y + C_1 + k\left(A_2x + B_2y + C_2\right) = 0 \quad (3)$$

where k is an arbitrary constant called parameter. For any value of k, the equation (3) is of first degree in x and y. Hence it represents a family of lines. A particular member of this family can be obtained for some value of k. This value of k may be obtained from other conditions.

Example 20 Find the equation of line parallel to the y-axis and drawn through the point of intersection of $x - 7y + 5 = 0$ and $3x + y - 7 = 0$

Solution The equation of any line through the point of intersection of the given lines is of the form

$$x - 7y + 5 + k\ (3x + y - 7) = 0$$

$$\text{i.e., } (1 + 3k)\ x + (k - 7)\ y + 5 - 7k = 0 \quad (1)$$

If this line is parallel to y-axis, then the coefficient of y should be zero, i.e., $k - 7 = 0$ which gives $k = 7$.

Substituting this value of k in the equation (1), we get

$22x - 44 = 0$, i.e., $x - 2 = 0$, which is the required equation.

Exercise 10.4

1. Find the equation of the line through the intersection of lines $3x + 4y = 7$ and $x - y + 2 = 0$ and whose slope is 5. (Ans. $35x - 7y + 18 = 0$)
2. Find the equation of the line through the intersection of lines $x + 2y - 3 = 0$ and $4x - y + 7 = 0$ and which is parallel to $5x + 4y - 20 = 0$
   (Ans. $15x + 12y - 7 = 0$)
3. Find the equation of the line through the intersection of the lines $2x + 3y - 4 = 0$ and $x - 5y = 7$ that has its x-intercept equal to -4.
   (Ans. $10x + 93y + 40 = 0$.)
4. Find the equation of the line through the intersection of $5x - 3y = 1$ and $2x + 3y - 23 = 0$ and perpendicular to the line $5x - 3y - 1 = 0$.
   (Ans. $63x + 105y - 781 = 0$)

10.7 Shifting of Origin

An equation corresponding to a set of points with reference to a system of coordinate axes may be simplified by taking the set of points in some other suitable coordinate system such that all geometric properties remain unchanged. One such transformation is that in which the new axes are transformed parallel to the original axes and origin is shifted to a new point. A transformation of this kind is called a *translation of axes*.

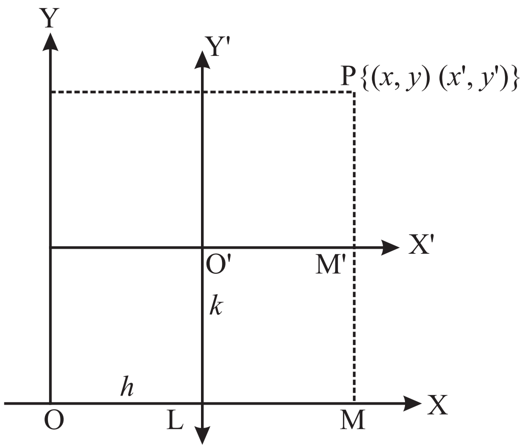


Fig. 10.21

The coordinates of each point of the plane are changed under a translation of axes. By knowing the relationship between the old coordinates and the new coordinates of points, we can study the analytical problem in terms of new system of coordinate axes.

To see how the coordinates of a point of the plane changed under a translation of axes, let us take a point P (x, y) referred to the axes OX and OY. Let $O'X'$ and $O'Y'$ be new axes parallel to OX and OY respectively, where O' is the new origin. Let (h, k)

be the coordinates of O′ referred to the old axes, i.e., OL = h and LO′ = k. Also, OM = x and MP = y (see Fig.10.21)

Let O′ M′ = x' and M′P = y' be respectively, the abscissa and ordinates of a point P referred to the new axes O′ X′ and O′ Y′. From Fig.10.21, it is easily seen that

OM = OL + LM, i.e., $x = h + x'$

and MP = MM′ + M′ P, i.e., $y = k + y'$

Hence, $x = x' + h$, $y = y' + k$

These formulae give the relations between the old and new coordinates.

Example 21 Find the new coordinates of point (3, –4) if the origin is shifted to (1, 2) by a translation.

Solution The coordinates of the new origin are $h = 1$, $k = 2$, and the original coordinates are given to be $x = 3$, $y = -4$.

The transformation relation between the old coordinates (x, y) and the new coordinates (x', y') are given by

$x = x' + h$ i.e., $x' = x - h$

and $y = y' + k$ i.e., $y' = y - k$

Substituting the values, we have

$x' = 3 - 1 = 2$ and $y' = -4 - 2 = -6$

Hence, the coordinates of the point (3, –4) in the new system are (2, –6).

Example 22 Find the transformed equation of the straight line $2x - 3y + 5 = 0$, when the origin is shifted to the point (3, –1) after translation of axes.

Solution Let coordinates of a point P changes from (x, y) to (x', y') in new coordinate axes whose origin has the coordinates $h = 3$, $k = -1$. Therefore, we can write the transformation formulae as $x = x' + 3$ and $y = y' - 1$. Substituting, these values in the given equation of the straight line, we get

$$2(x' + 3) - 3(y' - 1) + 5 = 0$$

or $$2x' - 3y' + 14 = 0$$

Therefore, the equation of the straight line in new system is $2x - 3y + 14 = 0$

Exercise 10.5

1. Find the new coordinates of the points in each of the following cases if the origin is shifted to the point (–3, –2) by a translation of axes.

(i) (1, 1) (Ans (4, 3)) (ii) (0, 1) (Ans. (3, 3))

(iii) (5, 0) (Ans. (8, 2)) (iv) (–1, –2) (Ans. (2, 0))

(v) (3, –5) (Ans. (6, –3))

2. Find what the following equations become when the origin is shifted to the point (1, 1)

(i) $x^2 + xy - 3y^2 - y + 2 = 0$ (Ans. $x^2 - 3y^2 + xy + 3x - 6y + 1 = 0$)

(ii) $xy - y^2 - x + y = 0$ (Ans. $xy - y^2 = 0$)

(iii) $xy - x - y + 1 = 0$ (Ans. $xy = 0$)

CHAPTER 13

13.5 Limits Involving Exponential and Logarithmic Functions

Before discussing evaluation of limits of the expressions involving exponential and logarithmic functions, we introduce these two functions stating their domain, range and also sketch their graphs roughly.

Leonhard Euler (1707–1783), the great Swiss mathematician introduced the number *e* whose value lies between 2 and 3. This number is useful in defining exponential function and is defined as $f(x) = e^x$, $x \in \mathbf{R}$. Its domain is **R**, range is the set of positive real numbers. The graph of exponential function, i.e., $y = e^x$ is as given in Fig.13.11.

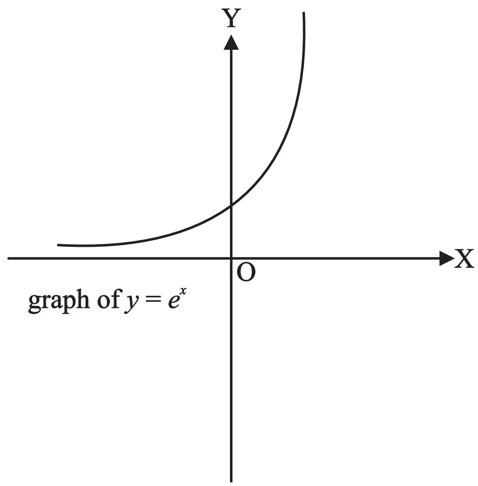


Fig. 13.11

Similarly, the logarithmic function expressed as $\log_e \mathbf{R}^+ \to \mathbf{R}$ is given by $\log_e x = y$, if and only if $e^y = x$. Its domain is $\mathbf{R}^+$ which is the set of all positive real numbers and range is **R**. The graph of logarithmic function $y = \log_e x$ is shown in Fig.13.12.

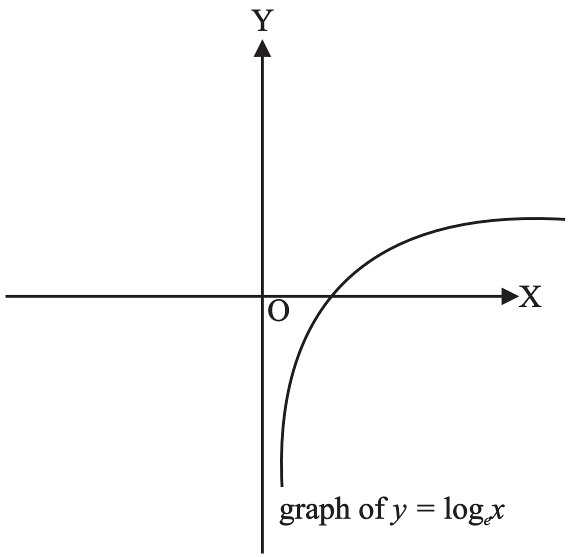


Fig. 13.12

In order to prove the result $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$, we make use of an inequality involving the expression $\frac{e^x - 1}{x}$ which runs as follows:

$\frac{1}{1+|x|} \leq \frac{e^x - 1}{x} \leq 1 + (e-2)|x|$ holds for all x in $[-1, 1] \sim \{0\}$.

Theorem 6 Prove that $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

Proof Using above inequality, we get

$$\frac{1}{1+|x|} \leq \frac{e^x - 1}{x} \leq 1 + |x|(e-2),\ x \hat{I} [-1, 1] \sim \{0\}$$

Also $\quad \lim_{x \to 0} \frac{1}{1+|x|} = \frac{1}{1 + \lim_{x \to 0} |x|} = \frac{1}{1+0} = 1$

and $\lim_{x\to 0}\left[1+(e-2)\left|x\right|\right] = 1+(e-2)\lim_{x\to 0}\left|x\right| = 1+(e-2)0 = 1$

Therefore, by Sandwich theorem, we get

$$\lim_{x\to 0}\frac{e^x-1}{x} = 1$$

Theorem 7 Prove that $\lim_{x\to 0}\frac{\log_e(1+x)}{x} = 1$

Proof Let $\frac{\log_e(1+x)}{x} = y$. Then

$$\log_e(1+x) = xy$$

$$\Rightarrow 1+x = e^{xy}$$

$$\Rightarrow \frac{e^{xy}-1}{x} = 1$$

or $$\frac{e^{xy}-1}{xy}.y = 1$$

$$\Rightarrow \lim_{xy\to 0}\frac{e^{xy}-1}{xy}\lim_{x\to 0} y = 1 \text{ (since } x \to 0 \text{ gives } xy \to 0)$$

$$\Rightarrow \lim_{x\to 0} y = 1\left(\text{as} \lim_{xy\to 0}\frac{e^{xy}-1}{xy} = 1\right)$$

$$\Rightarrow \lim_{x\to 0}\frac{\log_e(1+x)}{x} = 1$$

Example 5 Compute $\lim_{x\to 0}\frac{e^{3x}-1}{x}$

Solution We have

$$\lim_{x\to 0}\frac{e^{3x}-1}{x} = \lim_{3x\to 0}\frac{e^{3x}-1}{3x}\cdot 3$$

$$= 3\left(\lim_{y\to 0}\frac{e^y-1}{y}\right), \quad \text{where } y = 3x$$

$$= 3.1 = 3$$

Example 6 Compute $\lim\limits_{x\to 0}\dfrac{e^x-\sin x-1}{x}$

Solution We have $\lim\limits_{x\to 0}\dfrac{e^x-\sin x-1}{x}=\lim\limits_{x\to 0}\left[\dfrac{e^x-1}{x}-\dfrac{\sin x}{x}\right]$

$$=\lim_{x\to 0}\frac{e^x-1}{x}-\lim_{x\to 0}\frac{\sin x}{x}=1-1=0$$

Example 7 Evaluate $\lim\limits_{x\to 1}\dfrac{\log_e x}{x-1}$

Solution Put $x = 1 + h$, then as $x\to 1\Rightarrow h\to 0$. Therefore,

$$\lim_{x\to 1}\frac{\log_e x}{x-1}=\lim_{h\to 0}\frac{\log_e(1+h)}{h}=1\left(\text{since }\lim_{x\to 0}\frac{\log_e(1+x)}{x}=1\right).$$

Exercise 13.2

Evaluate the following limits, if exist

1. $\lim\limits_{x\to 0}\dfrac{e^{4x}-1}{x}$ (Ans. 4)

2. $\lim\limits_{x\to 0}\dfrac{e^{2+x}-e^2}{x}$ (Ans. e^2)

3. $\lim\limits_{x\to 5}\dfrac{e^x-e^5}{x-5}$ (Ans. e^5)

4. $\lim\limits_{x\to 0}\dfrac{e^{\sin x}-1}{x}$ (Ans. 1)

5. $\lim\limits_{x\to 3}\dfrac{e^x-e^3}{x-3}$ (Ans. e^3)

6. $\lim\limits_{x\to 0}\dfrac{x(e^x-1)}{1-\cos x}$ (Ans. 2)

7. $\lim\limits_{x\to 0}\dfrac{\log_e(1+2x)}{x}$ (Ans. 2)

8. $\lim\limits_{x\to 0}\dfrac{\log(1+x^3)}{\sin^3 x}$ (Ans. 1)

Notes

Notes

BE A STUDENT OF STUDENTS

A teacher who establishes rapport with the taught, becomes one with them, learns more from them than he teaches them. He who learns nothing from his disciples is, in my opinion, worthless. Whenever I talk with someone I learn from him. I take from him more than I give him. In this way, a true teacher regards himself as a student of his students. If you will teach your pupils with this attitude, you will benefit much from them.

Talk to Khadi Vidyalaya Students, Sevagram
Harijan Seva, 15 February 1942 (CW 75, p. 269)

USE ALL RESOURCES TO BE CONSTRUCTIVE AND CREATIVE

What we need is educationists with originality, fired with true zeal, who will think out from day to day what they are going to teach their pupils. The teacher cannot get this knowledge through musty volumes. He has to use his own faculties of observation and thinking and impart his knowledge to the children through his lips, with the help of a craft. This means a revolution in the method of teaching, a revolution in the teachers' outlook. Up till now you have been guided by inspector's reports. You wanted to do what the inspector might like, so that you might get more money yet for your institutions or higher salaries for yourselves. But the new teacher will not care for all that. He will say, '1 have done my duty to my pupil if I have made him a better man and in doing so I have used all my resources. That is enough for me'.

Harijan, 18 February 1939 (CW 68, pp. 374-75)