

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets: –

- (i) All circles are _____. (congruent, similar)
- (ii) All squares are _____. (similar, congruent)
- (iii) All _____ triangles are similar. (isosceles, equilateral)
- (iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are _____ and (b) their corresponding sides are _____. (equal, proportional)

Answer 1:

- (i) Similar
- (ii) Similar
- (iii) Equilateral
- (iv) (a) Equal
(b) Proportional

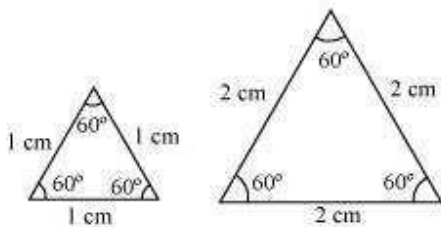
Question 2:

Give two different examples of pair of

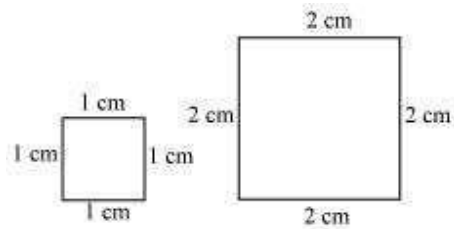
- (i) Similar figures (ii) Non-similar figures

Answer 2:

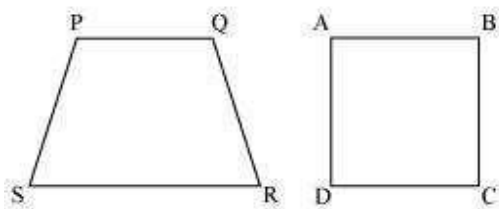
- (i) Two equilateral triangles with sides 1 cm and 2 cm



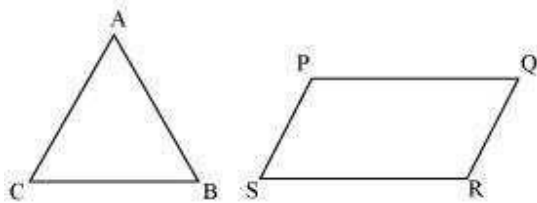
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

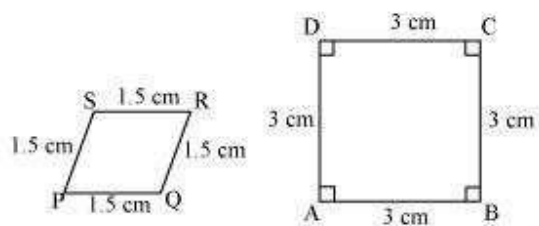


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer 3:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Mathematics

(Chapter – 6) (Triangles)

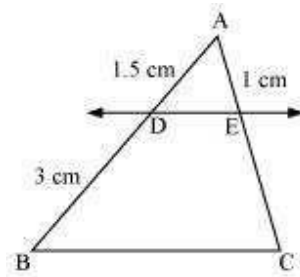
(Class – X)

Exercise 6.2

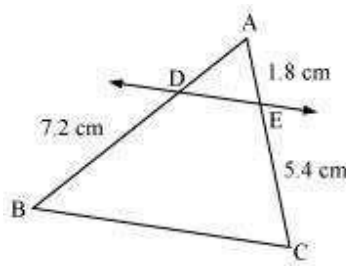
Question 1:

In figure.6.17. (i) and (ii), $DE \parallel BC$. Find EC in (i) and AD in (ii).

(i)

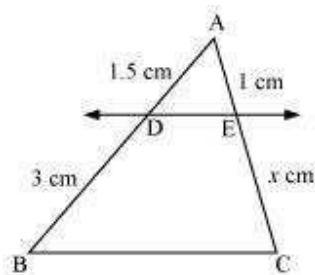


(ii)



Answer 1:

(i)



Let $EC = x$ cm

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

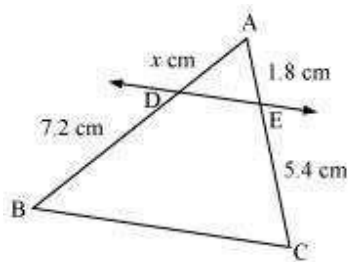
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let $AD = x \text{ cm}$

It is given that $DE \parallel BC$.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

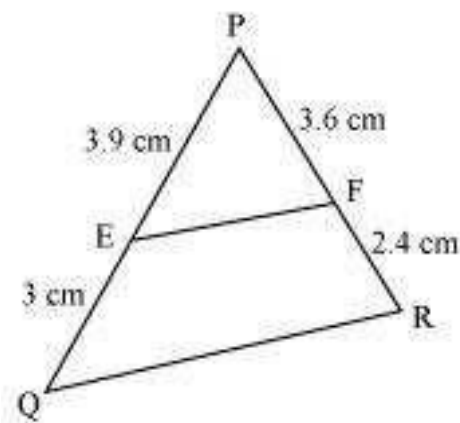
Question 2:

E and F are points on the sides PQ and PR respectively of a ΔPQR . For each of the following cases, state whether $EF \parallel QR$.

- (i) $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm and $FR = 2.4$ cm
(ii) $PE = 4$ cm, $QE = 4.5$ cm, $PF = 8$ cm and $RF = 9$ cm (iii) $PQ = 1.28$ cm, $PR = 2.56$ cm, $PE = 0.18$ cm and $PF = 0.63$ cm

Answer 2:

(i)



Given that, $PE = 3.9$ cm, $EQ = 3$ cm, $PF = 3.6$ cm, $FR = 2.4$ cm

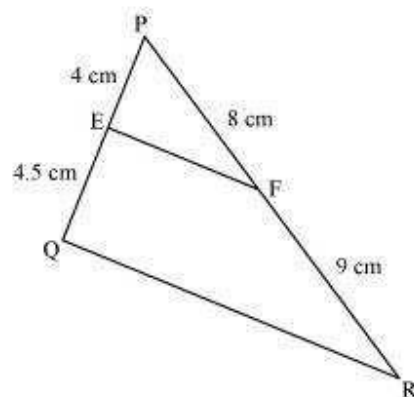
$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$

$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$

$$\text{Hence, } \frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR .

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

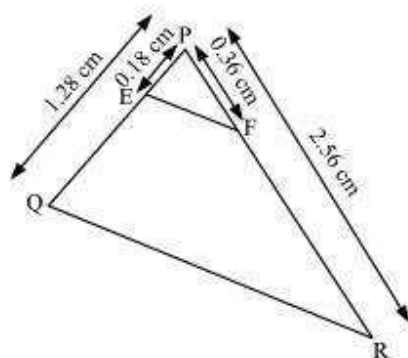
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

$$\text{Hence, } \frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

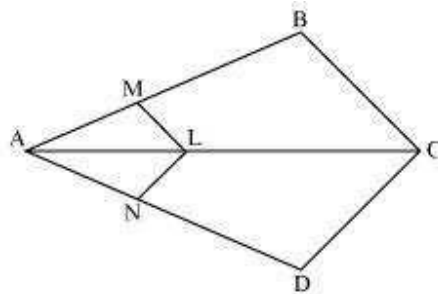
$$\text{Hence, } \frac{PE}{PQ} = \frac{PF}{PR}$$

Therefore, EF is parallel to QR.

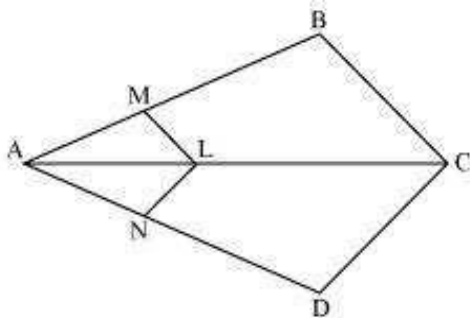
Question 3:

In the following figure, if $LM \parallel CB$ and $LN \parallel CD$, prove that

$$\frac{AM}{AB} = \frac{AN}{AD}$$



Answer 3:



In the given figure, $LM \parallel CB$

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \quad (i)$$

Similarly, $LN \parallel CD$

$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \quad (ii)$$

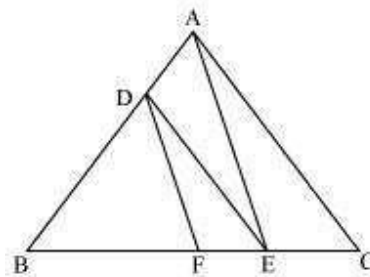
From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

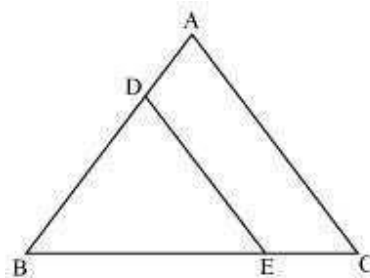
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Question 4:

In the following figure, $DE \parallel AC$ and $DF \parallel AE$. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

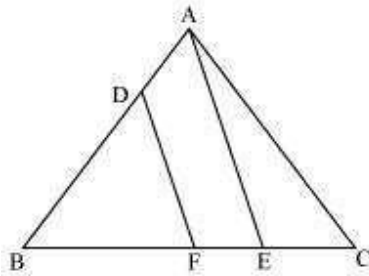


Answer 4:



In $\triangle ABC$, $DE \parallel AC$

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \quad (\text{Basic Proportionality Theorem}) \quad (i)$$



In $\triangle BAE$, $DF \parallel AE$

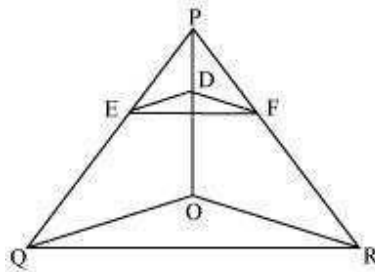
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \quad (\text{Basic Proportionality Theorem}) \quad (ii)$$

From (i) and (ii), we obtain

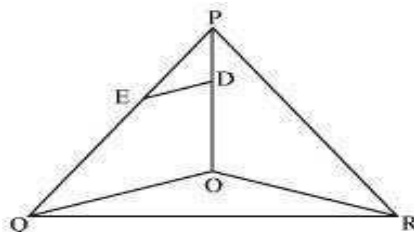
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$.

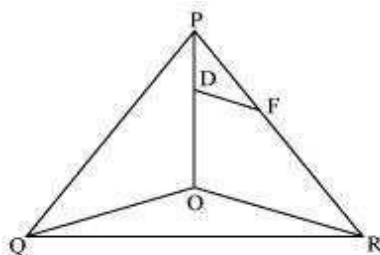


Answer 5:



In ΔPOQ , $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (i)$$



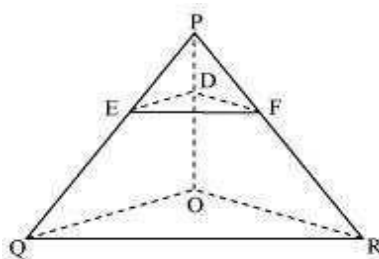
In ΔPOR , $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{Basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

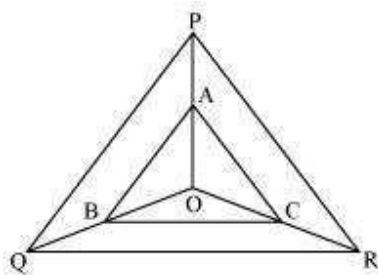
$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$$\therefore EF \parallel QR \quad (\text{Converse of basic proportionality theorem})$$

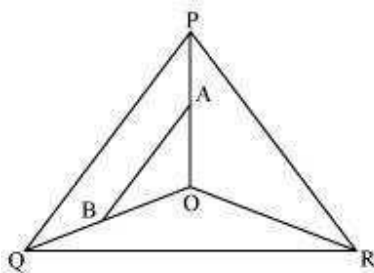


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that $AB \parallel PQ$ and $AC \parallel PR$. Show that $BC \parallel QR$.

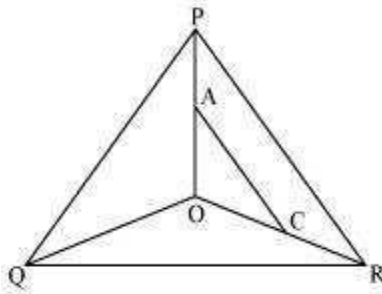


Answer 6:



In $\triangle POQ$, $AB \parallel PQ$

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \quad (\text{Basic proportionality theorem}) \quad (i)$$



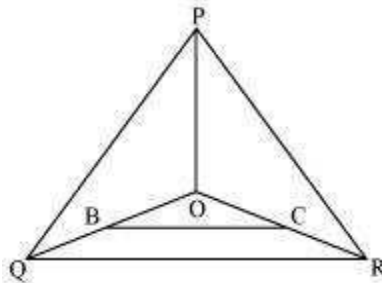
In ΔPOR , $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \quad (\text{By basic proportionality theorem}) \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

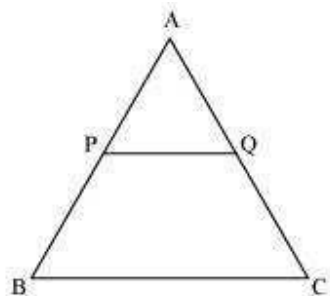
$$\therefore BC \parallel QR \quad (\text{By the converse of basic proportionality theorem})$$



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \quad (\text{P is the mid-point of AB. } \therefore AP = PB)$$

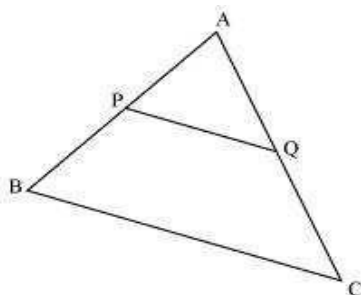
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., $AP = PB$ and $AQ = QC$ It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

$$\text{and } \frac{AQ}{QC} = \frac{1}{1}$$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, by using basic proportionality theorem, we obtain

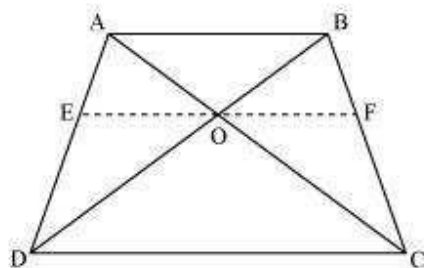
$$PQ \parallel BC$$

Question 9:

ABCD is a trapezium in which $AB \parallel DC$ and its diagonals intersect each other at the

point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer 9:



Draw a line EF through point O, such that $EF \parallel CD$

In $\triangle ADC$, $EO \parallel CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \quad (1)$$

In $\triangle ABD$, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\begin{aligned} \frac{ED}{AE} &= \frac{OD}{BO} \\ \Rightarrow \frac{AE}{ED} &= \frac{BO}{OD} \quad (2) \end{aligned}$$

From equations (1) and (2), we obtain

$$\begin{aligned} \frac{AO}{OC} &= \frac{BO}{OD} \\ \Rightarrow \frac{AO}{BO} &= \frac{OC}{OD} \end{aligned}$$

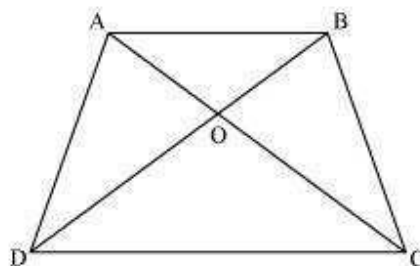
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

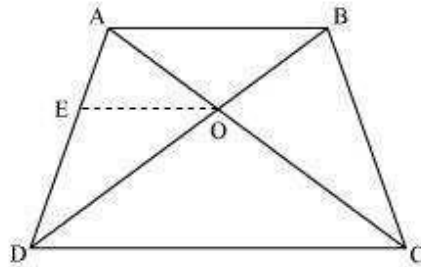
$$\frac{AO}{BO} = \frac{CO}{DO}. \text{ Show that ABCD is a trapezium.}$$

Answer 10:

Let us consider the following figure for the given question.



Draw a line $OE \parallel AB$



In $\triangle ABD$, $OE \parallel AB$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \quad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \quad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

$\Rightarrow EO \parallel DC$ [By the converse of basic proportionality theorem]

$\Rightarrow AB \parallel OE \parallel DC$

$\Rightarrow AB \parallel CD$

$\therefore ABCD$ is a trapezium.

Mathematics

(Chapter – 6) (Triangles)

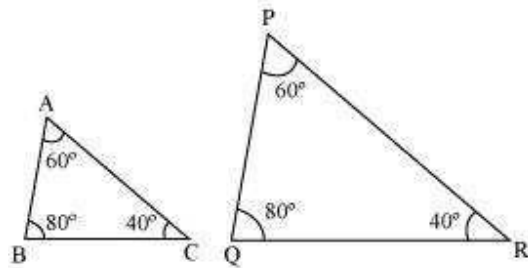
(Class – X)

Exercise 6.3

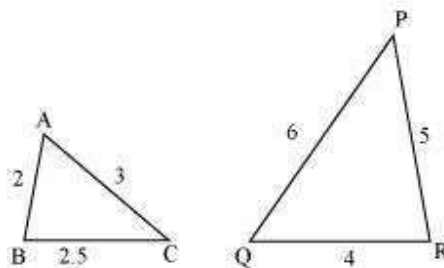
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

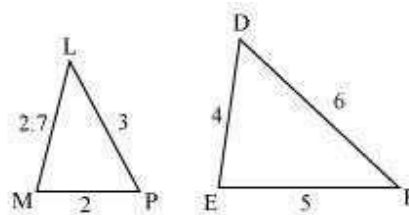
(i)



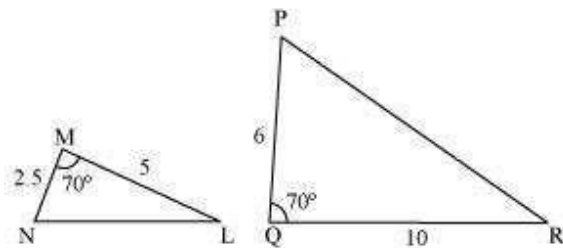
(ii)



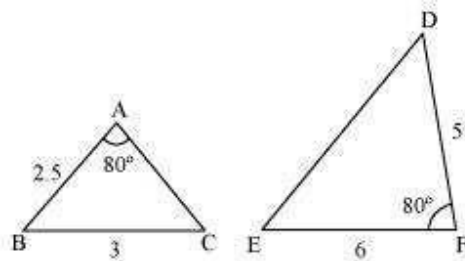
(iii)



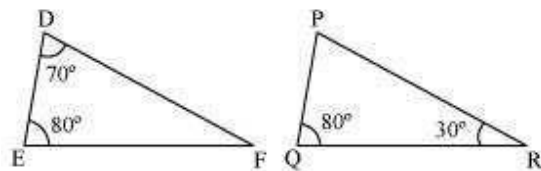
(iv)



(v)



(vi)



Answer 1:

(i) $\angle A = \angle P = 60^\circ$

$\angle B = \angle Q = 80^\circ$

$\angle C = \angle R = 40^\circ$

Therefore, $\triangle ABC \sim \triangle PQR$ [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore \triangle ABC \sim \triangle QRP$ [By SSS similarity criterion]

(iii) The given triangles are not similar as the corresponding sides are not proportional.

(iv) The given triangles are not similar as the corresponding sides are not proportional.

(v) The given triangles are not similar as the corresponding sides are not proportional.

(vi) In $\triangle DEF$,

$$\angle D + \angle E + \angle F = 180^\circ \text{ (Sum of the measures of the angles of a triangle is } 180^\circ\text{.)}$$

$$70^\circ + 80^\circ + \angle F = 180^\circ$$

$$\angle F = 30^\circ$$

Similarly, in $\triangle PQR$,

$$\angle P + \angle Q + \angle R = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\angle P + 80^\circ + 30^\circ = 180^\circ$$

$$\angle P = 70^\circ$$

In $\triangle DEF$ and $\triangle PQR$,

$$\angle D = \angle P \text{ (Each } 70^\circ\text{)}$$

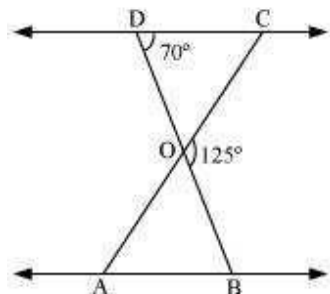
$$\angle E = \angle Q \text{ (Each } 80^\circ\text{)}$$

$$\angle F = \angle R \text{ (Each } 30^\circ\text{)}$$

$\therefore \triangle DEF \sim \triangle PQR$ [By AAA similarity criterion]

Question 2:

In the following figure, $\triangle ODC \sim \triangle OBA$, $\angle BOC = 125^\circ$ and $\angle CDO = 70^\circ$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer 2:

DOB is a straight line.

$$\therefore \angle DOC + \angle COB = 180^\circ$$

$$\Rightarrow \angle DOC = 180^\circ - 125^\circ = 55^\circ$$

In $\triangle DOC$,

$$\angle DCO + \angle CDO + \angle DOC = 180^\circ$$

(Sum of the measures of the angles of a triangle is 180° .)

$$\Rightarrow \angle DCO + 70^\circ + 55^\circ = 180^\circ$$

$$\Rightarrow \angle DCO = 55^\circ$$

It is given that $\triangle ODC \sim \triangle OBA$.

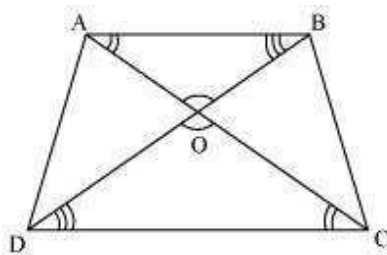
$$\therefore \angle OAB = \angle OCD \text{ [Corresponding angles are equal in similar triangles.]}$$

$$\Rightarrow \angle OAB = 55^\circ$$

**Question 3:**

Diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$ intersect each other at the

point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC} = \frac{OB}{OD}$

Answer 3:

In $\triangle DOC$ and $\triangle BOA$,

$$\angle CDO = \angle ABO \text{ [Alternate interior angles as } AB \parallel DC]$$

$$\angle DCO = \angle BAO \text{ [Alternate interior angles as } AB \parallel DC]$$

$$\angle DOC = \angle BOA \text{ [Vertically opposite angles]}$$

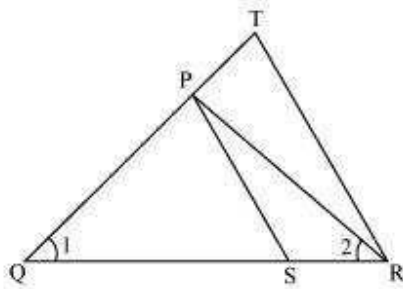
$\therefore \triangle DOC \sim \triangle BOA$ [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \quad \text{[Corresponding sides are proportional]}$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

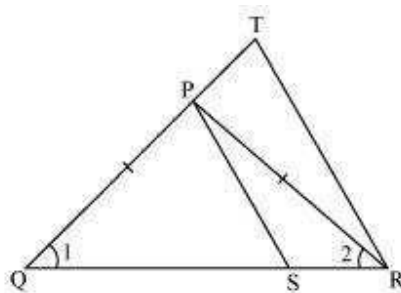
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$.



Show that $\triangle PQS \sim \triangle TQR$

Answer 4:



In $\triangle PQR$, $\angle PQR = \angle PRQ$

$\therefore PQ = PR$ (i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \quad (ii)$$

In ΔPQS and ΔTQR ,

$$\frac{QR}{QS} = \frac{QT}{QP} \quad [\text{Using (ii)}]$$

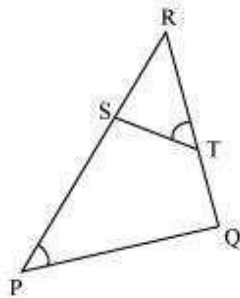
$$\angle Q = \angle Q$$

$$\therefore \Delta PQS \sim \Delta TQR \quad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that $\angle P = \angle RTS$. Show that $\Delta RPQ \sim \Delta RTS$.

Answer 5:



In ΔRPQ and ΔRTS ,

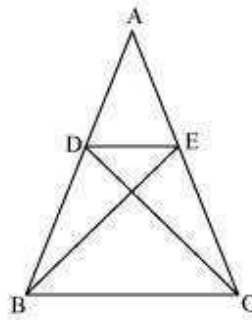
$$\angle RTS = \angle QPS \text{ (Given)}$$

$$\angle R = \angle R \text{ (Common angle)}$$

$$\therefore \Delta RPQ \sim \Delta RTS \text{ (By AA similarity criterion)}$$

Question 6:

In the following figure, if $\triangle ABE \cong \triangle ACD$, show that $\triangle ADE \sim \triangle ABC$.



Answer 6:

It is given that $\triangle ABE \cong \triangle ACD$.

$\therefore AB = AC$ [By CPCT](1)

And, $AD = AE$ [By CPCT](2)

In $\triangle ADE$ and $\triangle ABC$,

$$\frac{AD}{AB} = \frac{AE}{AC} \quad [\text{Dividing equation (2) by (1)}]$$

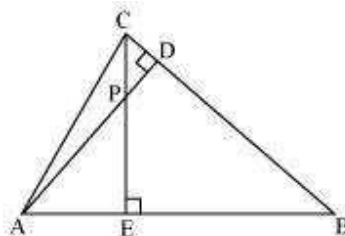
$$\angle A = \angle A \quad [\text{Common angle}]$$

$\therefore \triangle ADE \sim \triangle ABC$ [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of $\triangle ABC$ intersect each other at the point P.

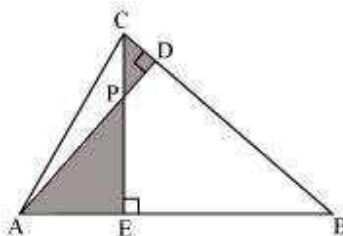
Show that:



- (i) $\triangle AEP \sim \triangle CDP$
- (ii) $\triangle ABD \sim \triangle CBE$
- (iii) $\triangle AEP \sim \triangle ADB$
- (v) $\triangle PDC \sim \triangle BEC$

Answer 7:

(i)



In $\triangle AEP$ and $\triangle CDP$,

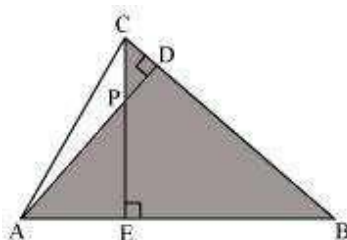
$$\angle AEP = \angle CDP \text{ (Each } 90^\circ\text{)}$$

$$\angle APE = \angle CPD \text{ (Vertically opposite angles)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle CDP$$

(ii)



In $\triangle ABD$ and $\triangle CBE$,

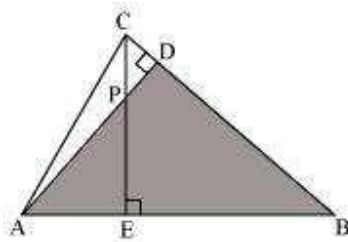
$$\angle ADB = \angle CEB \text{ (Each } 90^\circ\text{)}$$

$$\angle ABD = \angle CBE \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle ABD \sim \triangle CBE$$

(iii)



In $\triangle AEP$ and $\triangle ADB$,

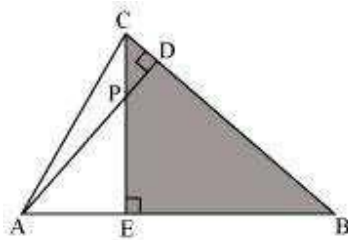
$$\angle AEP = \angle ADB \text{ (Each } 90^\circ\text{)}$$

$$\angle PAE = \angle DAB \text{ (Common)}$$

Hence, by using AA similarity criterion,

$$\triangle AEP \sim \triangle ADB$$

(iv)



In $\triangle PDC$ and $\triangle BEC$,

$$\angle PDC = \angle BEC \text{ (Each } 90^\circ\text{)}$$

$$\angle PCD = \angle BCE \text{ (Common angle)}$$

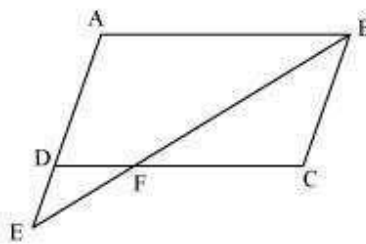
Hence, by using AA similarity criterion,

$$\triangle PDC \sim \triangle BEC$$

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that $\triangle ABE \sim \triangle CFB$

Answer 8:



In $\triangle ABE$ and $\triangle CFB$,

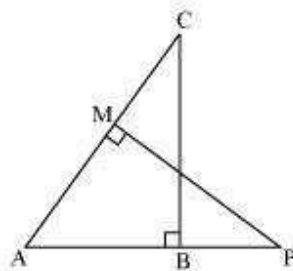
$\angle A = \angle C$ (Opposite angles of a parallelogram)

$\angle AEB = \angle CBF$ (Alternate interior angles as $AE \parallel BC$)

$\therefore \triangle ABE \sim \triangle CFB$ (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) $\triangle ABC \sim \triangle AMP$

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer 9:

In $\triangle ABC$ and $\triangle AMP$,

$$\angle ABC = \angle AMP \text{ (Each } 90^\circ\text{)}$$

$$\angle A = \angle A \text{ (Common)}$$

$\therefore \triangle ABC \sim \triangle AMP$ (By AA similarity criterion)

$$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP} \quad \text{(Corresponding sides of similar triangles are proportional)}$$

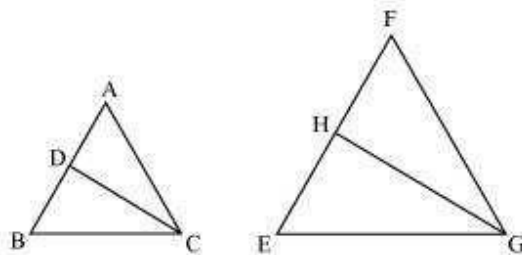
Question 10:

CD and GH are respectively the bisectors of $\angle ACB$ and $\angle EGF$ such that D and H lie on sides AB and FE of $\triangle ABC$ and $\triangle EFG$ respectively. If $\triangle ABC \sim \triangle FEG$, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\triangle DCB \sim \triangle HGE$

(iii) $\triangle DCA \sim \triangle HGF$

Answer 10:

It is given that $\triangle ABC \sim \triangle FEG$.

$$\therefore \angle A = \angle F, \angle B = \angle E, \text{ and } \angle ACB = \angle FGE$$

$$\angle ACB = \angle FGE$$

$$\therefore \angle ACD = \angle FGH \text{ (Angle bisector)}$$

$$\text{And, } \angle DCB = \angle HGE \text{ (Angle bisector)}$$

In $\triangle ACD$ and $\triangle FGH$,

$\angle A = \angle F$ (Proved above)

$\angle ACD = \angle FGH$ (Proved above)

$\therefore \triangle ACD \sim \triangle FGH$ (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In $\triangle DCB$ and $\triangle HGE$,

$\angle DCB = \angle HGE$ (Proved above)

$\angle B = \angle E$ (Proved above)

$\therefore \triangle DCB \sim \triangle HGE$ (By AA similarity criterion)

In $\triangle DCA$ and $\triangle HGF$,

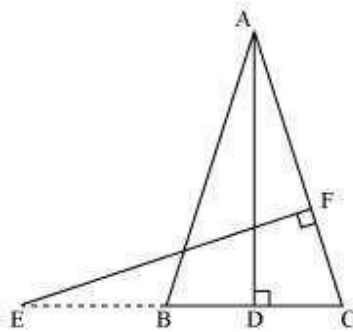
$\angle ACD = \angle FGH$ (Proved above)

$\angle A = \angle F$ (Proved above)

$\therefore \triangle DCA \sim \triangle HGF$ (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$



Answer 11:

It is given that ABC is an isosceles triangle.

$\therefore AB = AC$

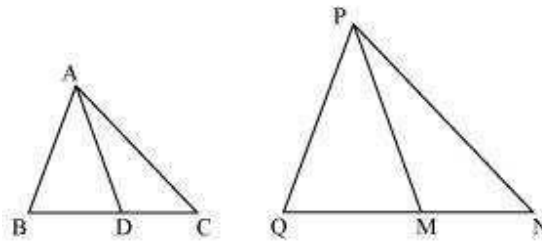
$\Rightarrow \angle ABD = \angle ECF$

In $\triangle ABD$ and $\triangle ECF$,
 $\angle ADB = \angle EFC$ (Each 90°)
 $\angle BAD = \angle CEF$ (Proved above)
 $\therefore \triangle ABD \sim \triangle ECF$ (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of $\triangle PQR$ (see the given figure). Show that $\triangle ABC \sim \triangle PQR$.

Answer 12:



Median divides the opposite side.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2}$$

Given that,

$$\begin{aligned} \frac{AB}{PQ} &= \frac{BC}{QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \\ \Rightarrow \frac{AB}{PQ} &= \frac{BD}{QM} = \frac{AD}{PM} \end{aligned}$$

In $\triangle ABD$ and $\triangle PQM$,

$$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM} \quad (\text{Proved above})$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SSS similarity criterion)

$\Rightarrow \angle ABD = \angle PQM$ (Corresponding angles of similar triangles)

In $\triangle ABC$ and $\triangle PQR$,

$\angle ABD = \angle PQM$ (Proved above)

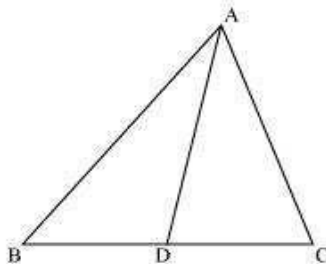
$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$. Show that $CA^2 = CB \cdot CD$.

Answer 13:



In $\triangle ADC$ and $\triangle BAC$,

$\angle ADC = \angle BAC$ (Given)

$\angle ACD = \angle BCA$ (Common angle)

$\therefore \triangle ADC \sim \triangle BAC$ (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

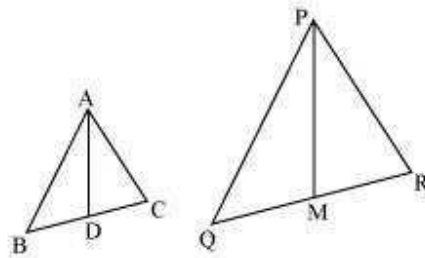
$$\therefore \frac{CA}{CB} = \frac{CD}{CA}$$

$$\Rightarrow CA^2 = CB \times CD$$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\triangle ABC \sim \triangle PQR$

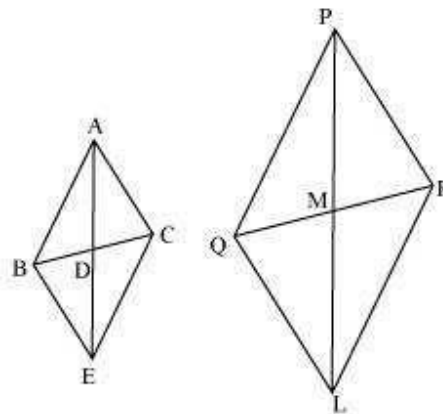
Answer 14:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$\therefore AC = BE$ and $AB = EC$ (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and $PR = QL$,

$PQ = LR$

It was given that

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

$\therefore \triangle ABE \sim \triangle PQL$ (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

$\therefore \angle BAE = \angle QPL \dots (1)$

Similarly, it can be proved that $\triangle AEC \sim \triangle PLR$ and

$\angle CAE = \angle RPL \dots (2)$

Adding equation (1) and (2), we obtain

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \dots (3)$$

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{Given})$$

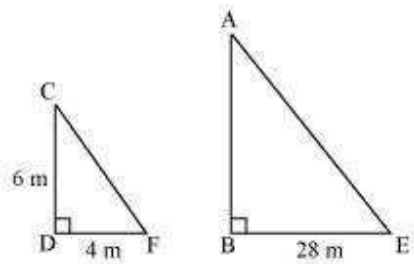
$$\angle CAB = \angle RPQ \quad [\text{Using equation (3)}]$$

$\therefore \triangle ABC \sim \triangle PQR$ (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer 15:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, $\angle DCF = \angle BAE$

And, $\angle DFC = \angle BEA$

$\angle CDF = \angle ABE$ (Tower and pole are vertical to the ground)

$\therefore \triangle ABE \sim \triangle CDF$ (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6 \text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42 \text{ m}$$

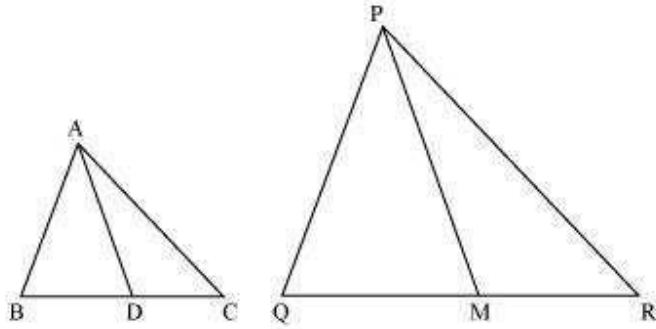
Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where

$\triangle ABC \sim \triangle PQR$ prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer 16:



It is given that $\triangle ABC \sim \triangle PQR$

We know that the corresponding sides of similar triangles are in proportion.

$$\therefore \frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots (1)$$

$$\text{Also, } \angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots (2)$$

Since AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots (3)$$

From equations (1) and (3), we obtain

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots (4)$$

In $\triangle ABD$ and $\triangle PQM$,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ [Using equation (4)]}$$

$\therefore \triangle ABD \sim \triangle PQM$ (By SAS similarity criterion)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.4

Question 1:

Let $\triangle ABC \sim \triangle DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2 . If $EF = 15.4 \text{ cm}$, find BC .

Answer 1:

It is given that $\triangle ABC \sim \triangle DEF$.

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$$EF = 15.4 \text{ cm},$$

$$\text{ar}(\triangle ABC) = 64 \text{ cm}^2,$$

$$\text{ar}(\triangle DEF) = 121 \text{ cm}^2$$

$$\therefore \frac{\text{ar}(ABC)}{\text{ar}(DEF)} = \left(\frac{BC}{EF}\right)^2$$

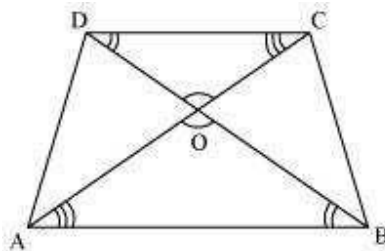
$$\Rightarrow \left(\frac{64 \text{ cm}^2}{121 \text{ cm}^2}\right) = \frac{BC^2}{(15.4 \text{ cm})^2}$$

$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) \text{ cm}$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) \text{ cm} = (8 \times 1.4) \text{ cm} = 11.2 \text{ cm}$$

Question 2:

Diagonals of a trapezium ABCD with $AB \parallel DC$ intersect each other at the point O. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

Answer 2:

Since $AB \parallel CD$,

$\therefore \angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$ (Alternate interior angles)

In $\triangle AOB$ and $\triangle COD$,

$\angle AOB = \angle COD$ (Vertically opposite angles)

$\angle OAB = \angle OCD$ (Alternate interior angles)

$\angle OBA = \angle ODC$ (Alternate interior angles)

$\therefore \triangle AOB \sim \triangle COD$ (By AAA similarity criterion)

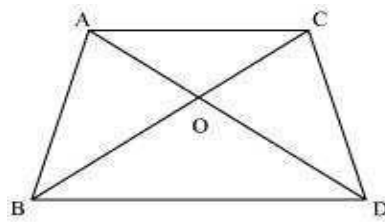
$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{AB}{CD}\right)^2$$

Since $AB = 2CD$,

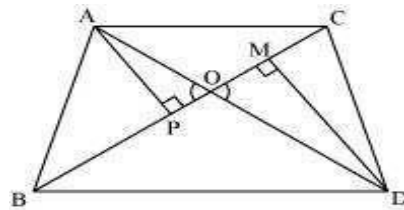
$$\therefore \frac{\text{ar}(\triangle AOB)}{\text{ar}(\triangle COD)} = \left(\frac{2CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\triangle ABC)}{\text{area}(\triangle DBC)} = \frac{AO}{DO}$

**Answer 3:**

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

In $\triangle APO$ and $\triangle DMO$,

$\angle APO = \angle DMO$ (Each = 90°)

$\angle AOP = \angle DOM$ (Vertically opposite angles)

$\therefore \triangle APO \sim \triangle DMO$ (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer 4:

Let us assume two similar triangles as $\Delta ABC \sim \Delta PQR$.

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \quad (1)$$

Given that, $\text{ar}(\Delta ABC) = \text{ar}(\Delta PQR)$

$$\Rightarrow \frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

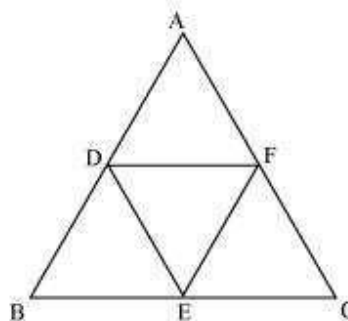
$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

$\Rightarrow AB = PQ, BC = QR, \text{ and } AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC . Find the ratio of the area of ΔDEF and ΔABC .

Answer 5:

D and E are the mid-points of ΔABC .

$$\therefore DE \parallel AC \text{ and } DE = \frac{1}{2} AC$$

In $\triangle BED$ and $\triangle BCA$,

$$\angle BED = \angle BCA \quad (\text{Corresponding angles})$$

$$\angle BDE = \angle BAC \quad (\text{Corresponding angles})$$

$$\angle EBD = \angle CBA \quad (\text{Common angles})$$

$$\therefore \triangle BED \sim \triangle BCA \quad (\text{AAA similarity criterion})$$

$$\frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{\text{ar}(\triangle BED)}{\text{ar}(\triangle BCA)} = \frac{1}{4}$$

$$\Rightarrow \text{ar}(\triangle BED) = \frac{1}{4} \text{ar}(\triangle BCA)$$

$$\text{Similarly, } \text{ar}(\triangle CFE) = \frac{1}{4} \text{ar}(\triangle CBA) \text{ and } \text{ar}(\triangle ADF) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\text{Also, } \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - [\text{ar}(\triangle BED) + \text{ar}(\triangle CFE) + \text{ar}(\triangle ADF)]$$

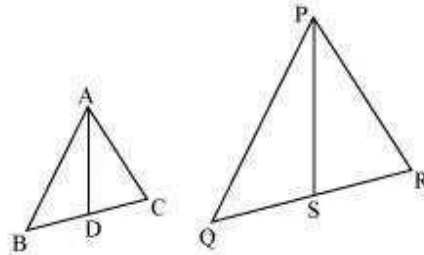
$$\Rightarrow \text{ar}(\triangle DEF) = \text{ar}(\triangle ABC) - \frac{3}{4} \text{ar}(\triangle ABC) = \frac{1}{4} \text{ar}(\triangle ABC)$$

$$\Rightarrow \frac{\text{ar}(\triangle DEF)}{\text{ar}(\triangle ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer 6:



Let us assume two similar triangles as $\triangle ABC \sim \triangle PQR$. Let AD and PS be the medians of these triangles.

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \dots\dots\dots(1)$$

$$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R \dots\dots\dots(2)$$

Since AD and PS are medians,

$$\therefore BD = DC = \frac{BC}{2}$$

$$\text{And, } QS = SR = \frac{QR}{2}$$

Equation (1) becomes

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR} \dots\dots\dots(3)$$

In $\triangle ABD$ and $\triangle PQS$,

$$\angle B = \angle Q \text{ [Using equation (2)]}$$

$$\text{And, } \frac{AB}{PQ} = \frac{BD}{QS} \text{ [Using equation (3)]}$$

$$\therefore \triangle ABD \sim \triangle PQS \text{ (SAS similarity criterion)}$$

Therefore, it can be said that

$$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS} \dots\dots\dots (4)$$

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

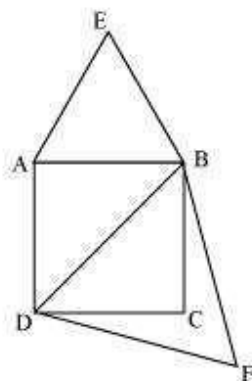
And hence,

$$\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer 7:



Let ABCD be a square of side a .

Therefore, its diagonal $= \sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF .

Side of an equilateral triangle, ΔABE , described on one of its sides $= a$

Side of an equilateral triangle, $\triangle DBF$, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \triangle ABE}{\text{Area of } \triangle DBF} = \left(\frac{a}{\sqrt{2}a} \right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

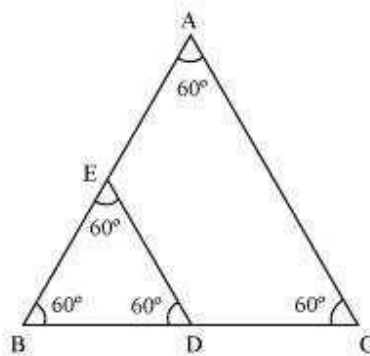
(A) 2 : 1 (B)

1 : 2

(C) 4 : 1

(D) 1 : 4

Answer 8:



We know that equilateral triangles have all its angles as 60° and all its sides of the same length. Therefore, all equilateral triangles are similar to each other.

Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of $\triangle ABC = x$

Therefore, side of $\triangle BDE = \frac{x}{2}$

$$\therefore \frac{\text{area}(\triangle ABC)}{\text{area}(\triangle BDE)} = \left(\frac{x}{\frac{x}{2}} \right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3

(B) 4 : 9

(C) 81 : 16

(D) 16 : 81

Answer 9:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9} \right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Mathematics

(Chapter - 6) (Triangles)

(Class 10)

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm

(ii) 3 cm, 8 cm, 6 cm

(iii) 50 cm, 80 cm, 100 cm

(iv) 13 cm, 12 cm, 5 cm

Answer 1:

(i) Sides of triangle: 7 cm, 24 cm and 25 cm.

Squaring these sides, we get 49, 576 and 625.

$$49 + 576 = 625 \Rightarrow 7^2 + 24^2 = 25^2$$

These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.

We know that the hypotenuses is the longest side in right angled triangle.

Hence, its length is 25 cm.

(ii) Sides of triangle: 3 cm, 6 cm and 8 cm.

Squaring these sides, we get 9, 36 and 64.

$$9 + 36 \neq 64 \Rightarrow 3^2 + 6^2 \neq 8^2$$

These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iii) Sides of triangle: 50 cm, 80 cm and 100 cm.

Squaring these sides, we get 2500, 6400 and 10000.

$$2500 + 6400 \neq 10000 \Rightarrow 50^2 + 80^2 \neq 100^2$$

These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iv) Sides of triangle: 5 cm, 12 cm and 13 cm.

Squaring these sides, we get 25, 144 and 169.

$$25 + 144 = 169 \Rightarrow 5^2 + 12^2 = 13^2$$

These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.

We know that the hypotenuses is the longest side in right angled triangle.

Hence, its length is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that $PM \perp QR$. Show that $PM^2 = QM.MR$.

Answer 2:

Let $\angle MPR = x$

In $\triangle MPR$,

$$\angle MRP = 180^\circ - 90^\circ - x$$

Similarly,

In $\triangle MPQ$,

$$\angle MPQ = 90^\circ - \angle MPR = 90^\circ - x$$

$$\angle MQP = 180^\circ - 90^\circ - (90^\circ - x) = x$$

In $\triangle QMP$ and $\triangle PMR$,

$$\angle MPQ = \angle MRP$$

$$\angle PMQ = \angle RMP$$

$$\angle MQP = \angle MPR$$

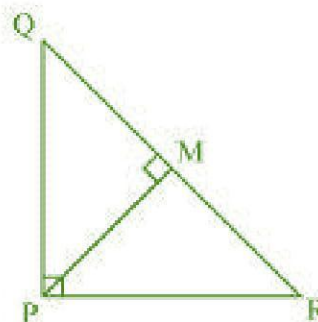
$$\Rightarrow \triangle QMP \sim \triangle PMR \quad [\text{AAA similarity}]$$

We know that the corresponding sides of similar triangles are proportional.

Therefore,

$$\frac{QM}{PM} = \frac{MP}{MR}$$

$$\Rightarrow PM^2 = MQ \times MR$$



Mathematics

(Chapter – 6) (Triangles) (Class 10)

Question 3:

In Figure, ABD is a triangle right angled at A and $AC \perp BD$. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$

Answer 3:

(i) In $\triangle ADB$ and $\triangle CAB$,

$\angle DAB = \angle ACB$

[Each 90°]

$\angle ABD = \angle CBA$

[Common]

$\therefore \triangle ADB \sim \triangle CAB$

[AA similarity]

$\Rightarrow \frac{AB}{CB} = \frac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$

(ii) Let $\angle CAB = x$

In $\triangle CBA$,

$\angle CBA = 180^\circ - 90^\circ - x$

$\Rightarrow \angle CBA = 90^\circ - x$

Similarly, in $\triangle CAD$,

$\angle CAD = 90^\circ - \angle CAB$

$\Rightarrow \angle CAD = 90^\circ - x$

$\angle CDA = 180^\circ - 90^\circ - (90^\circ - x)$

$\Rightarrow \angle CDA = x$

In $\triangle CBA$ and $\triangle CAD$,

$\angle CBA = \angle CAD$

[Proved above]

$\angle CAB = \angle CDA$

[Proved above]

$\angle ACB = \angle DCA$

[Each 90°]

$\therefore \triangle CBA \sim \triangle CAD$

[AAA similarity]

$\Rightarrow \frac{AC}{DC} = \frac{BC}{AC}$

$\Rightarrow AC^2 = BC \times DC$

(iii) In $\triangle DCA$ and $\triangle DAB$,

$\angle DCA = \angle DAB$

[Each 90°]

$\angle CDA = \angle ADB$

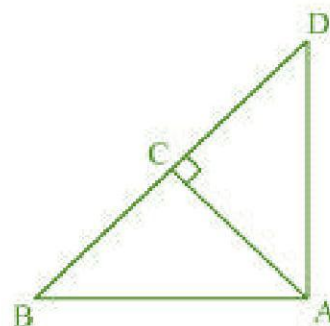
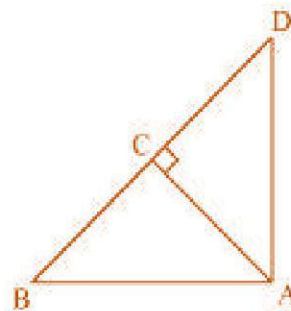
[Common]

$\therefore \triangle DCA \sim \triangle DAB$

[AA similarity]

$\Rightarrow \frac{DC}{DA} = \frac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$



Question 4:

ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer 4:

Given that the triangle ABC is an isosceles triangle such that $AC = BC$ and $\angle C = 90^\circ$,

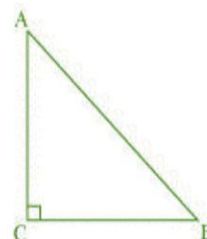
In $\triangle ABC$, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$

[Because $AC = BC$]

$\Rightarrow AB^2 = 2AC^2$



Mathematics

(Chapter – 6) (Triangles) (Class 10)

Question 5:

ABC is an isosceles triangle with $AC = BC$. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer 5:

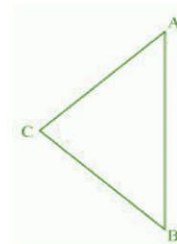
Given that: $AB^2 = 2AC^2$

$$\Rightarrow AB^2 = AC^2 + AC^2$$

$$\Rightarrow AB^2 = AC^2 + BC^2 \quad [\text{Because } AC = BC]$$

These sides satisfy the Pythagoras theorem.

Hence, the triangle ABC is a right angled triangle.



Question 6:

ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer 6:

Let ABC be any equilateral triangle with each sides of length $2a$. Perpendicular AD is drawn from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, $\therefore BD = DC = a$

In $\triangle ADB$, by Pythagoras theorem

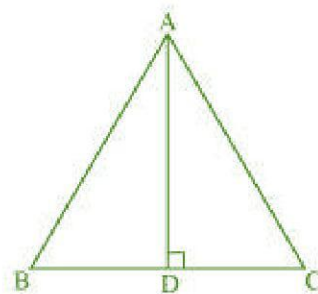
$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow (2a)^2 = AD^2 + a^2 \quad [\text{Because } AB = 2a]$$

$$\Rightarrow 4a^2 = AD^2 + a^2$$

$$\Rightarrow AD^2 = 3a^2 \Rightarrow AD = \sqrt{3}a$$

Hence, the length of each altitude is $\sqrt{3}a$.



Question 7:

Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer 7:

In $\triangle AOB$, by Pythagoras theorem

$$AB^2 = AO^2 + OB^2 \quad \dots (i)$$

In $\triangle BOC$, by Pythagoras theorem

$$BC^2 = BO^2 + OC^2 \quad \dots (ii)$$

In $\triangle COD$, by Pythagoras theorem

$$CD^2 = CO^2 + OD^2 \quad \dots (iii)$$

In $\triangle AOD$, by Pythagoras theorem

$$AD^2 = AO^2 + OD^2 \quad \dots (iv)$$

Adding the equations (i), (ii), (iii) and (iv), we have

$$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$$

$$= 2[OA^2 + OB^2 + OC^2 + OD^2]$$

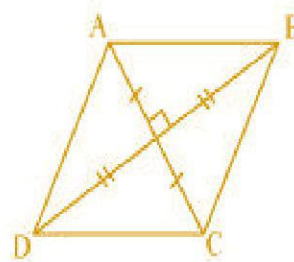
$$= 2[2OA^2 + 2OB^2] \quad [\text{Because } OA = OC, OB = OD]$$

$$= 4[OA^2 + OB^2]$$

$$= 4\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right] \quad [\text{Because } OA = \frac{1}{2} AC, OB = \frac{1}{2} BD]$$

$$= 4\left[\frac{AC^2}{4} + \frac{BD^2}{4}\right]$$

$$= AC^2 + BD^2$$



Mathematics

(Chapter – 6) (Triangles) (Class 10)

Question 8:

In Figure, O is a point in the interior of a triangle ABC, $OD \perp BC$, $OE \perp AC$ and $OF \perp AB$. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$

Answer 8:

Join OA, OB and OC.

(i) In $\triangle AOF$, by Pythagoras theorem

$$OA^2 = OF^2 + AF^2 \quad \dots (i)$$

In $\triangle BOD$, by Pythagoras theorem

$$OB^2 = OD^2 + BD^2 \quad \dots (ii)$$

In $\triangle COE$, by Pythagoras theorem

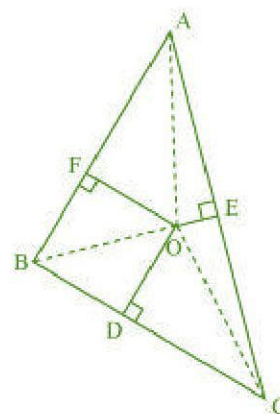
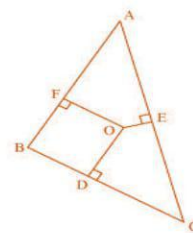
$$OC^2 = OE^2 + EC^2 \quad \dots (iii)$$

Adding equations (i), (ii) and (iii), we have

$$\begin{aligned} OA^2 + OB^2 + OC^2 &= OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2 \\ \Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 &= AF^2 + BD^2 + CE^2 \quad \dots (iv) \end{aligned}$$

(ii) From the equation (iv), we have

$$\begin{aligned} AF^2 + BD^2 + CE^2 &= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 \\ &= (OA^2 - OF^2) + (OC^2 - OD^2) + (OB^2 - OE^2) \\ &= AE^2 + CD^2 + BF^2 \end{aligned}$$



Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.

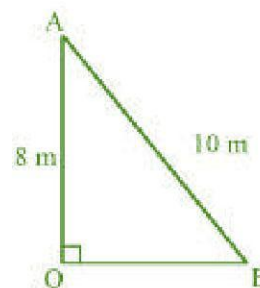
Answer 9:

Let OA is wall and AB is ladder in the figure.

In $\triangle AOB$, by Pythagoras theorem

$$\begin{aligned} AB^2 &= OA^2 + OB^2 \\ \Rightarrow 10^2 &= 8^2 + BO^2 \\ \Rightarrow 100 &= 64 + BO^2 \\ \Rightarrow BO^2 &= 36 \\ \Rightarrow BO &= 6 \text{ m} \end{aligned}$$

Hence, the distance of the foot of the ladder from the base of the wall is 6 m.



Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?

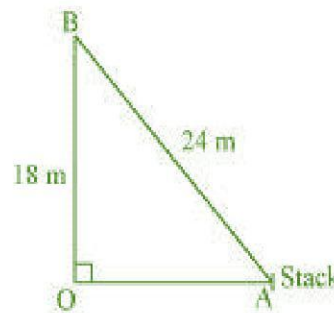
Answer 10:

Let OB is vertical pole in the figure.

In $\triangle AOB$, by Pythagoras theorem

$$\begin{aligned} AB^2 &= OB^2 + OA^2 \\ \Rightarrow 24^2 &= 18^2 + OA^2 \\ \Rightarrow 576 &= 324 + OA^2 \\ \Rightarrow OA^2 &= 252 \\ \Rightarrow OA &= 6\sqrt{7} \text{ m} \end{aligned}$$

Hence, the distance of stake from the base of the pole is $6\sqrt{7}$ m.



Mathematics

(Chapter - 6) (Triangles) (Class 10)

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer 11:

Distance travelled by first aeroplane (due north) in $1\frac{1}{2}$ hours

$$= 1000 \times \frac{3}{2} = 1500 \text{ km}$$

Distance travelled by second aeroplane (due west) in $1\frac{1}{2}$ hours

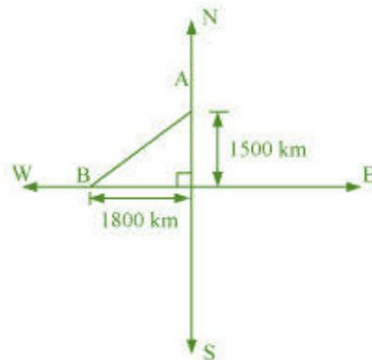
$$= 1200 \times \frac{3}{2} = 1800 \text{ km}$$

Now, OA and OB are the distance travelled.

Now by Pythagoras theorem, the distance between the two planes

$$\begin{aligned} AB &= \sqrt{OA^2 + OB^2} \\ &= \sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000} \\ &= \sqrt{5490000} = 300\sqrt{61} \text{ km} \end{aligned}$$

Hence, $1\frac{1}{2}$ hours, the distance between two planes is $300\sqrt{61}$ km.



Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer 12:

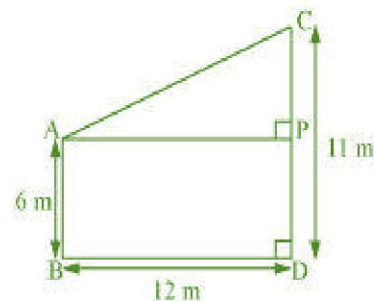
Let AB and CD are the two pole with height 6 m and 11 m respectively.

Therefore, CP = 11 - 6 = 5 m and AP = 12 m

In $\triangle APC$, by Pythagoras theorem

$$\begin{aligned} AP^2 + PC^2 &= AC^2 \\ \Rightarrow 12^2 + 5^2 &= AC^2 \\ \Rightarrow AC^2 &= 144 + 25 = 169 \\ \Rightarrow AC &= 13 \text{ m} \end{aligned}$$

Hence, the distance between the tops of two poles is 13 m.



Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Answer 13:

In $\triangle ACE$, by Pythagoras theorem

$$AC^2 + CE^2 = AE^2 \quad \dots (1)$$

In $\triangle BCD$, by Pythagoras theorem

$$BC^2 + CD^2 = DB^2 \quad \dots (2)$$

From the equation (1) and (2), we have

$$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + DB^2 \quad \dots (3)$$

In $\triangle CDE$, by Pythagoras theorem

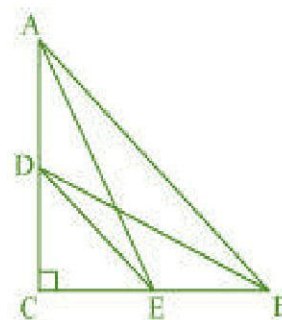
$$DE^2 = CD^2 + CE^2 \quad \dots (4)$$

In $\triangle ABC$, by Pythagoras theorem

$$AB^2 = AC^2 + CB^2 \quad \dots (5)$$

From the equation (3), (4) and (5), we have

$$DE^2 + AB^2 = AE^2 + DB^2$$



Mathematics

(Chapter – 6) (Triangles) (Class 10)

Question 14:

The perpendicular from A on side BC of a ΔABC intersects BC at D such that $DB = 3CD$ (see Figure). Prove that $2AB^2 = 2AC^2 + BC^2$.

Answer 14:

In ΔACD , by Pythagoras theorem

$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow AC^2 - CD^2 = AD^2 \quad \dots (1)$$

In ΔABD , by Pythagoras theorem

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow AB^2 - BD^2 = AD^2 \quad \dots (2)$$

From the equation (1) and (2), we have

$$AC^2 - CD^2 = AB^2 - BD^2 \quad \dots (3)$$

Given that: $3DC = DB$, therefore

$$DC = \frac{BC}{4} \text{ and } BD = \frac{3BC}{4} \quad \dots (4)$$

From the equation (3) and (4), we have

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

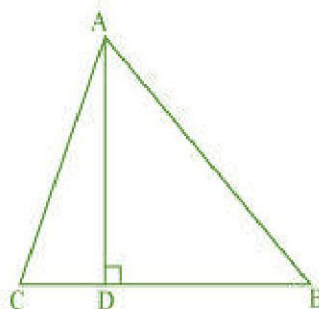
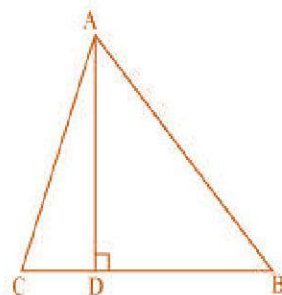
$$\Rightarrow AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$$\Rightarrow 16AC^2 - BC^2 = 16AB^2 - 9BC^2$$

$$\Rightarrow 16AC^2 = 16AB^2 - 8BC^2$$

$$\Rightarrow 2AC^2 = 2AB^2 - BC^2$$

$$\Rightarrow 2AB^2 = 2AC^2 + BC^2$$



Question 15:

In an equilateral triangle ABC, D is a point on side BC such that $BD = \frac{1}{3} BC$. Prove that $9AD^2 = 7AB^2$.

Answer 15:

Triangle ABC is an equilateral triangle with each side a. Draw an altitude AE from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, $BE = EC = \frac{a}{2}$

In ΔAEB , by Pythagoras theorem

$$AB^2 = AE^2 + BE^2$$

$$\Rightarrow (a)^2 = AD^2 + \left(\frac{a}{2}\right)^2 \quad [\text{Because } AB = a]$$

$$\Rightarrow a^2 = AD^2 + \frac{a^2}{4} \quad \Rightarrow AD^2 = \frac{3a^2}{4} \quad \Rightarrow AD = \frac{\sqrt{3}a}{2}$$

Given that: $BD = \frac{1}{3} BC$

$$\therefore BD = \frac{a}{3}$$

$$DE = BE - BD = \frac{a}{2} - \frac{a}{3} = \frac{a}{6}$$

In ΔADE , by Pythagoras theorem,

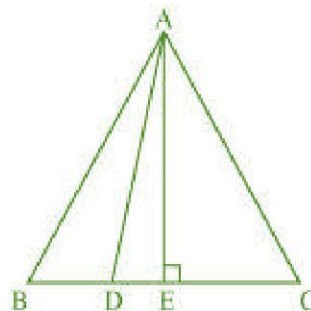
$$AD^2 = AE^2 + DE^2$$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

$$= \frac{3a^2}{4} + \frac{a^2}{36} = \frac{28a^2}{36} = \frac{7}{9}a^2$$

$$\Rightarrow AD^2 = \frac{7}{9}AB^2$$

$$\Rightarrow 9AD^2 = 7AB^2$$



Mathematics

(Chapter - 6) (Triangles) (Class 10)

Question 16:

In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer 16:

Let triangle ABC be an equilateral triangle with side a . Altitude AE is drawn from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

$$\therefore BE = EC = BC/2 = a/2$$

In $\triangle ABE$, by Pythagoras theorem

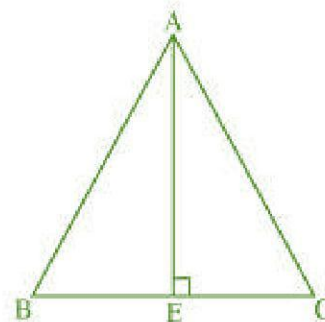
$$AB^2 = AE^2 + BE^2$$

$$\Rightarrow a^2 = AE^2 + \left(\frac{a}{2}\right)^2 = AE^2 + \frac{a^2}{4}$$

$$\Rightarrow AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$4AE^2 = 3a^2$$

$$\Rightarrow 4 \times (\text{Altitude})^2 = 3 \times (\text{Side})^2$$



Question 17:

Tick the correct answer and justify: In $\triangle ABC$, $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm. The angle B is:

(A) 120°

(B) 60°

(C) 90°

(D) 45°

Answer 17:

Given that: $AB = 6\sqrt{3}$ cm, $AC = 12$ cm and $BC = 6$ cm.

Therefore, $AB^2 = 108$, $AC^2 = 144$ and $BC^2 = 36$

Now,

$$AB^2 + BC^2$$

$$= 108 + 36$$

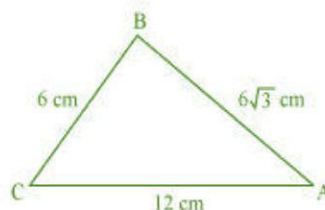
$$= 144$$

$$= AC^2$$

The sides are satisfying the Pythagoras triplet in $\triangle ABC$. Hence, these are the sides of a right angled triangle.

$$\therefore \angle B = 90^\circ$$

Hence, the option (C) is correct.



Mathematics

(Chapter - 6) (Triangles) (Class 10)

Exercise 6.6 (Optional)

Question 1:

In Figure, PS is the bisector of $\angle QPR$ of ΔPQR . Prove that $\frac{QS}{SR} = \frac{PQ}{QR}$.

Answer 1:

A line RT is drawn parallel to SP, which intersects QP produced at T.

Given that, SP bisects angle QPR, therefore

$$\angle QPS = \angle SPR \quad \dots (1)$$

By construction,

$$\angle SPR = \angle PRT \text{ (As } PS \parallel TR) \quad \dots (2)$$

$$\angle QPS = \angle QTR \text{ (As } PS \parallel TR) \quad \dots (3)$$

From the above equations, we have

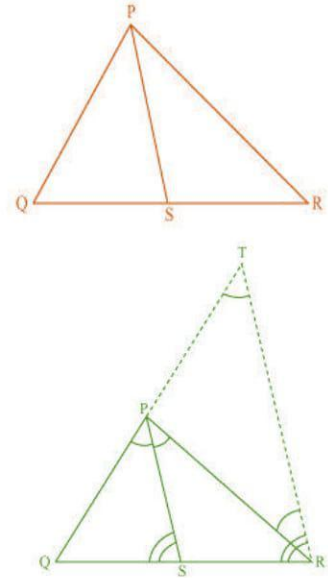
$$\angle PRT = \angle QTR$$

$$\therefore PT = PR$$

By construction, $PS \parallel TR$

In ΔQTR , by Thales theorem

$$\frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{QR} \quad [\because PT = TR]$$



Question 2:

In Figure, D is a point on hypotenuse AC of ΔABC , $DM \perp BC$ and $DN \perp AB$. Prove that:

(i) $DM^2 = DN \cdot MC$

(ii) $DN^2 = DM \cdot AN$

Answer 2:

(i) Join B and D.

Given that, $DN \parallel CB$, $DM \parallel AB$ and $\angle B = 90^\circ$, $\therefore$ DMBN is a rectangle.

$$\therefore DN = MB \text{ and } DM = NB$$

Given that, $BD \perp AC$, $\therefore \angle CDB = 90^\circ$

$$\Rightarrow \angle 2 + \angle 3 = 90^\circ \quad \dots (1)$$

$$\text{In } \Delta CDM, \angle 1 + \angle 2 + \angle DMC = 180^\circ$$

$$\Rightarrow \angle 1 + \angle 2 = 90^\circ \quad \dots (2)$$

$$\text{In } \Delta DMB, \angle 3 + \angle DMB + \angle 4 = 180^\circ$$

$$\Rightarrow \angle 3 + \angle 4 = 90^\circ \quad \dots (3)$$

From the equations (1) and (2), we have, $\angle 1 = \angle 3$

From the equations (1) and (3), we have, $\angle 2 = \angle 4$

In ΔDCM and ΔBDM ,

$$\angle 1 = \angle 3 \quad [\text{Proved above}]$$

$$\angle 2 = \angle 4 \quad [\text{Proved above}]$$

$$\therefore \Delta DCM \sim \Delta BDM \quad [\text{AA similarity}]$$

$$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC} \Rightarrow \frac{DN}{DM} = \frac{DM}{MC} \quad [\because BM = DN]$$

$$\Rightarrow DM^2 = DN \times MC$$

(ii) In ΔDBN , $\angle 5 + \angle 7 = 90^\circ \quad \dots (4)$

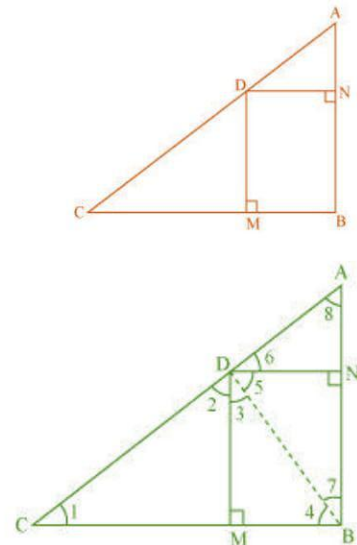
In ΔDAN , $\angle 6 + \angle 8 = 90^\circ \quad \dots (5)$

$$BD \perp AC, \therefore \angle ADB = 90^\circ$$

$$\Rightarrow \angle 5 + \angle 6 = 90^\circ \quad \dots (6)$$

From the equations (4) and (6), we have, $\angle 6 = \angle 7$

From the equations (5) and (6), we have, $\angle 8 = \angle 5$



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(Chapter - 6) (Triangles) (Class 10)

In $\triangle DNA$ and $\triangle BND$,

$$\angle 6 = \angle 7$$

[Proved above]

$$\angle 8 = \angle 5$$

[Proved above]

$$\therefore \triangle DNA \sim \triangle BND$$

[AA similarity]

$$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB} \Rightarrow DN^2 = AN \times NB$$

$$\Rightarrow DN^2 = AN \times DM$$

[$\because NB = DM$]

Question 3:

In Figure, ABC is a triangle in which $\angle ABC > 90^\circ$ and $AD \perp CB$ produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC \cdot BD$.

Answer 3:

In $\triangle ADB$, by Pythagoras theorem

$$AB^2 = AD^2 + DB^2$$

... (1)

In $\triangle ADC$, by Pythagoras theorem

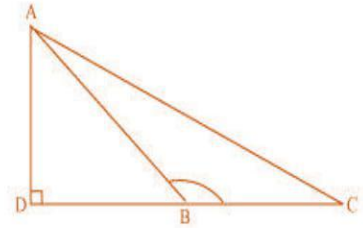
$$AC^2 = AD^2 + DC^2$$

$$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$$

$$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$$

$$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB \times BC$$

[From the equation (1)]



Question 4:

In Figure, ABC is a triangle in which $\angle ABC < 90^\circ$ and $AD \perp BC$. Prove that

$$AC^2 = AB^2 + BC^2 + 2BC \cdot BD.$$

Answer 4:

In $\triangle ADB$, by Pythagoras theorem

$$AD^2 + DB^2 = AB^2$$

$$\Rightarrow AD^2 = AB^2 - DB^2$$

... (1)

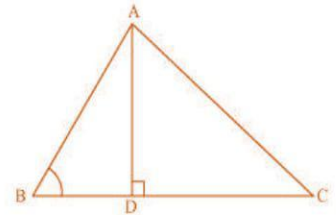
In $\triangle ADC$, by Pythagoras theorem, $AD^2 + DC^2 = AC^2$

$$\Rightarrow AB^2 - DB^2 + DC^2 = AC^2$$

[From the equation (1)]

$$\Rightarrow AB^2 - BD^2 + (BC - BD)^2 = AC^2$$

$$\Rightarrow AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD = AB^2 + BC^2 - 2BC \times BD$$



Question 5:

In Figure, AD is a median of a triangle ABC and $AM \perp BC$. Prove that:

$$(i) AC^2 = AD^2 + BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(ii) AB^2 = AD^2 - BC \cdot DM + \left(\frac{BC}{2}\right)^2$$

$$(iii) AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$$

Answer 5:

(i) In $\triangle AMD$, by Pythagoras theorem

$$AM^2 + MD^2 = AD^2$$

... (1)

In $\triangle AMC$, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$

$$\Rightarrow AM^2 + (MD + DC)^2 = AC^2$$

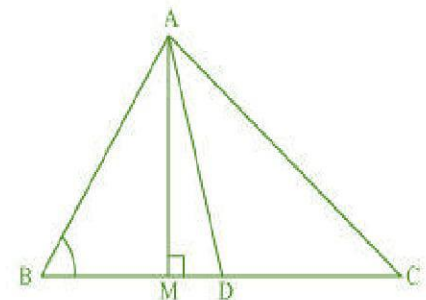
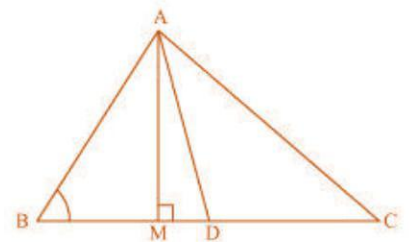
$$\Rightarrow (AM^2 + MD^2) + DC^2 + 2MD \cdot DC = AC^2$$

$$\Rightarrow AD^2 + DC^2 + 2MD \cdot DC = AC^2$$

[From equation (1)]

$$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD \cdot \left(\frac{BC}{2}\right) = AC^2 \quad \left[\because DC = \frac{BC}{2}\right]$$

$$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + MD \cdot BC = AC^2$$



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(ii) In $\triangle ABM$, by Pythagoras theorem

$$\begin{aligned} AB^2 &= AM^2 + MB^2 \\ &= (AD^2 - DM^2) + MB^2 \\ &= (AD^2 - DM^2) + (BD - MD)^2 \\ &= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD \\ &= AD^2 + BD^2 - 2BD \times MD \\ &= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)MD = AC^2 \quad \left[\because BD = \frac{BC}{2}\right] \\ \Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 - BC \cdot MD &= AC^2 \end{aligned}$$

(iii) In $\triangle ABM$, by Pythagoras theorem, $AM^2 + MB^2 = AB^2$... (2)

In $\triangle AMC$, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$... (3)

Adding the equations (2) and (3), we have

$$\begin{aligned} 2AM^2 + MB^2 + MC^2 &= AB^2 + AC^2 \\ \Rightarrow 2AM^2 + (BD - DM)^2 + (MD + DC)^2 &= AB^2 + AC^2 \\ \Rightarrow 2AM^2 + BD^2 + DM^2 - 2BD \cdot DM + MD^2 + DC^2 + 2MD \cdot DC &= AB^2 + AC^2 \\ \Rightarrow 2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD(-BD + DC) &= AB^2 + AC^2 \\ \Rightarrow 2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) &= AB^2 + AC^2 \\ \Rightarrow 2AD^2 + \frac{1}{2}BC^2 &= AB^2 + AC^2 \end{aligned}$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer 6:

In parallelogram ABCD, altitudes AF and DE is drawn on DC and produced BA.

In $\triangle DEA$, by Pythagoras theorem, $DE^2 + EA^2 = DA^2$... (i)

In $\triangle DEB$, by Pythagoras theorem, $DE^2 + EB^2 = DB^2$

$$\begin{aligned} \Rightarrow DE^2 + (EA + AB)^2 &= DB^2 \\ \Rightarrow (DE^2 + EA^2) + AB^2 + 2EA \times AB &= DB^2 \\ \Rightarrow DA^2 + AB^2 + 2EA \times AB &= DB^2 \end{aligned} \quad \dots (ii)$$

In $\triangle ADF$, by Pythagoras theorem, $AD^2 = AF^2 + FD^2$

In $\triangle AFC$, by Pythagoras theorem

$$\begin{aligned} AC^2 &= AF^2 + FC^2 = AF^2 + (DC - FD)^2 = AF^2 + DC^2 + FD^2 - 2DC \times FD \\ &= (AF^2 + FD^2) + DC^2 - 2DC \times FD \\ \Rightarrow AC^2 &= AD^2 + DC^2 - 2DC \times FD \end{aligned} \quad \dots (iii)$$

ABCD is a parallelogram.

Therefore

$$AB = CD \quad \dots (iv)$$

$$\text{and, } BC = AD \quad \dots (v)$$

In $\triangle DEA$ and $\triangle ADF$,

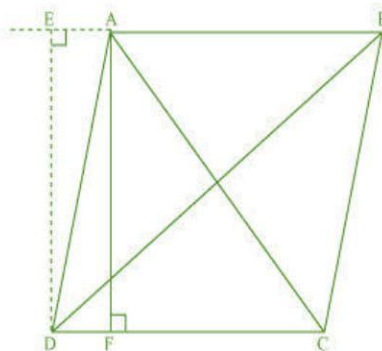
$$\angle DEA = \angle AFD \quad [\text{Each } 90^\circ]$$

$$\angle EAD = \angle ADF \quad [EA \parallel DF]$$

$$AD = AD \quad [\text{Common}]$$

$$\therefore \triangle EAD \cong \triangle FDA \quad [\text{AAS congruency rule}]$$

$$\Rightarrow EA = DF \quad \dots (vi)$$



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Adding equations (ii) and (iii), we have

$$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$$

$$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$$

$$\Rightarrow BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2 \quad [\text{From the equation (iv) and (vi)}]$$

$$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$$

Question 7:

In Figure, two chords AB and CD intersect each other at the point P. Prove that:

(i) $\triangle APC \sim \triangle DPB$

(ii) $AP \cdot BP = CP \cdot DP$

Answer 7:

Join CB.

(i) In $\triangle APC$ and $\triangle DPB$,

$$\angle APC = \angle DPB$$

[Vertically Opposite Angles]

$$\angle CAP = \angle BDP$$

[Angles in the same segment]

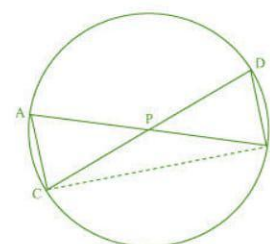
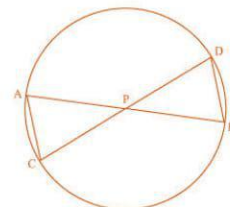
$$\triangle APC \sim \triangle DPB$$

[AA similarity]

(ii) We have already proved that $\triangle APC \sim \triangle DPB$.

We know that the corresponding sides of similar triangles are proportional. So,

$$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD} \Rightarrow \frac{AP}{DP} = \frac{PC}{PB} \Rightarrow AP \cdot PB = PC \cdot DP$$



Question 8:

In Figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\triangle PAC \sim \triangle PDB$

(ii) $PA \cdot PB = PC \cdot PD$

Answer 8:

(i) In $\triangle PAC$ and $\triangle PDB$,

$$\angle P = \angle P$$

[Common]

$$\angle PAC = \angle PDB$$

[The exterior angle of cyclic quadrilateral is equal to opposite interior angle]

$$\therefore \triangle PAC \sim \triangle PDB$$

[AA similarity]

(ii) We know that the corresponding sides of similar triangles are proportional. Therefore,

$$\frac{PA}{PD} = \frac{AC}{DB} = \frac{PC}{PB} \Rightarrow \frac{PA}{PD} = \frac{PC}{PB} \Rightarrow PA \cdot PB = PC \cdot PD$$



Question 9:

In Figure, D is a point on side BC of $\triangle ABC$ such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of $\angle BAC$.

Answer 9:

Produce BA to P, such that $AP = AC$ and join P to C.

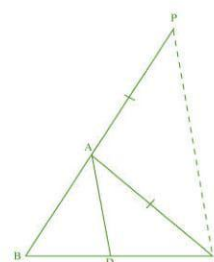
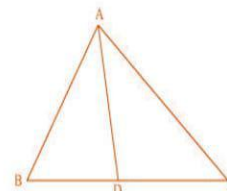
Given that:

$$\frac{BD}{CD} = \frac{AB}{AC} \Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$

By the converse of Thales theorem, we have

$$AD \parallel PC \Rightarrow \angle BAD = \angle APC \quad [\text{Corresponding angle}] \quad \dots (1)$$

$$\text{and, } \angle DAC = \angle ACP \quad [\text{Alternate angle}] \quad \dots (2)$$



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By construction,

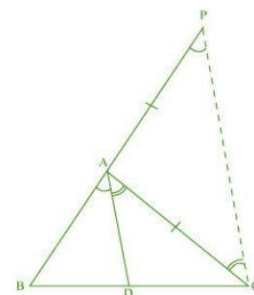
$$AP = AC$$

$$\Rightarrow \angle APC = \angle ACP \quad \dots (3)$$

From the equations (1), (2) and (3), we have

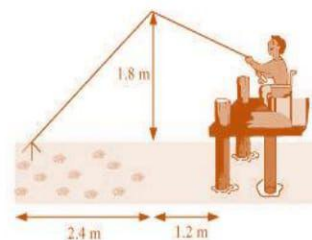
$$\angle BAD = \angle APC$$

$\Rightarrow AD$, bisects angle BAC .



Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



Answer 10:

Let AB be the height of rod tip from the surface of water and BC is the horizontal distance between fly to tip of the rod.

Then, the length of the string is AC .

In $\triangle ABC$, by Pythagoras theorem

$$AC^2 = AB^2 + BC^2$$

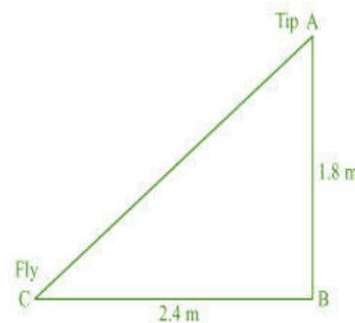
$$\Rightarrow AB^2 = (1.8 \text{ m})^2 + (2.4 \text{ m})^2$$

$$\Rightarrow AB^2 = (3.24 + 5.76) \text{ m}^2$$

$$\Rightarrow AB^2 = 9.00 \text{ m}^2$$

$$\Rightarrow AB = \sqrt{9} = 3 \text{ m}$$

Hence, the length of string, which is out, is 3 m.



If she pulls in the string at the rate of 5 cm/s, then the distance travelled by fly in 12 seconds

$$= 12 \times 5 = 60 \text{ cm} = 0.6 \text{ m}$$

Let, D be the position of fly after 12 seconds.

Hence, AD is the length of string that is out after 12 seconds.

The length of the string pulls in by Nazima = $AD = AC - 12$

$$= (3.00 - 0.6) \text{ m}$$

$$= 2.4 \text{ m}$$

In $\triangle ADB$,

$$AB^2 + BD^2 = AD^2$$

$$\Rightarrow (1.8 \text{ m})^2 + BD^2 = (2.4 \text{ m})^2$$

$$\Rightarrow BD^2 = (5.76 - 3.24) \text{ m}^2 = 2.52 \text{ m}^2$$

$$\Rightarrow BD = 1.587 \text{ m}$$

Horizontal distance travelled by Fly

$$= BD + 1.2 \text{ m}$$

$$= (1.587 + 1.2) \text{ m}$$

$$= 2.787 \text{ m}$$

$$= 2.79 \text{ m}$$

