Theme 5: Introduction to Trigonometry

Prior Knowledge

It is recommended that you revise the following topics before you start working on these questions.

- Trigonometric Ratios
- Determining the unknown sides of a Right Angled Triangle when one side and an Angle is given
- Area of a Rectangle
- Perimeter of a Triangle
- Tangents from an External Point to a Circle
- Volume of a Cylinder

Case Study A - Raising a Flagpole

A simple way of raising a small flagpole for temporary purposes is by using three guylines around the pole, without burying the foot of the pole deep inside the ground. A guyline is a piece of strong rope tied taut between the flagpole and three stakes, which are equidistant from the pole. One end of the guyline will be tied to the pole at a height of about 3/4th of the total height of the pole and the other end to the stake.

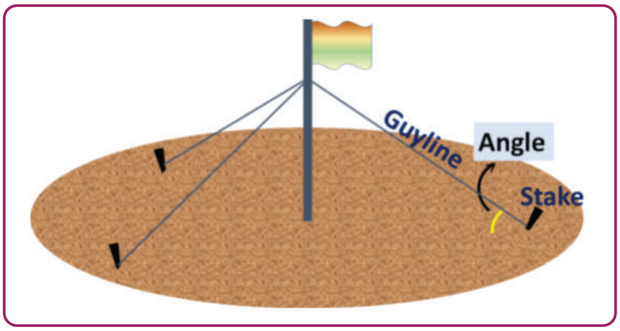


Fig. 5.1, Flagpole using guylines

Sahil wanted to use the 16 ft long pipe, which he had, as the flagpole and wanted to buy rope for making the 3 guylines. As he didn't know the distance from the pole at which he should fix the stake, he tried to figure it out by searching on the internet. He came to know that in order to ensure the stability of the pole, the angle between the ground and guyline (angle in Fig. 5.1, say θ) should be 15 degrees and the maximum angle one can go up to is 25 degrees.

θ	Sin θ	Cos θ	Tan θ
15°	0.26	0.97	0.27
25°	0.42	0.90	0.47

Table 5.1, Trigonometric values

Question 1

Between 15° and 25°, in which case can Sahil get his work done by buying the minimum length of rope?

a. 15°
b. 25°
c. Length of rope does not depend on the angle

Answer

Question 2

What is the minimum possible length of each guyline, such that the angle between the guyline and the ground still falls in the safe range for the maximum stability? Refer to the safe range of the angle in the background information, which is based on Sahil's research.

Answer

Question 3

If the length of the guyline is represented by L and the angle made by the guyline with the ground by A, what would be the distance between the pole and the stakes?

a. L × Sin(A)

b. $\frac{12}{\text{Tan(A)}}$

c. 12 × Tan(A)

d. $\frac{L}{\text{Cos(A)}}$

Answer

Question 4

If the pipe used as the pole was 20 ft long, what would be the total length of rope Sahil would need to buy? Assume that he chooses the angle at which he needs the minimum length of the rope and also consider 10% extra rope for tying knots.

Answer

Case Study B - Styles of Roofs

The steepness of a roof is called the roof pitch. Whenever the roof is not flat, in construction terminology, the roof is said to be pitched. Carpenters will frame the rafter of the roof at an angle with the horizontal to "pitch" a roof. As the pitch of the roof increases, there will be a corresponding increase in the materials needed for framing and sheathing. Among the different types of roofs, gable roofs are the simplest form of roofs. The diagram below shows a simple structure with a gable roof.

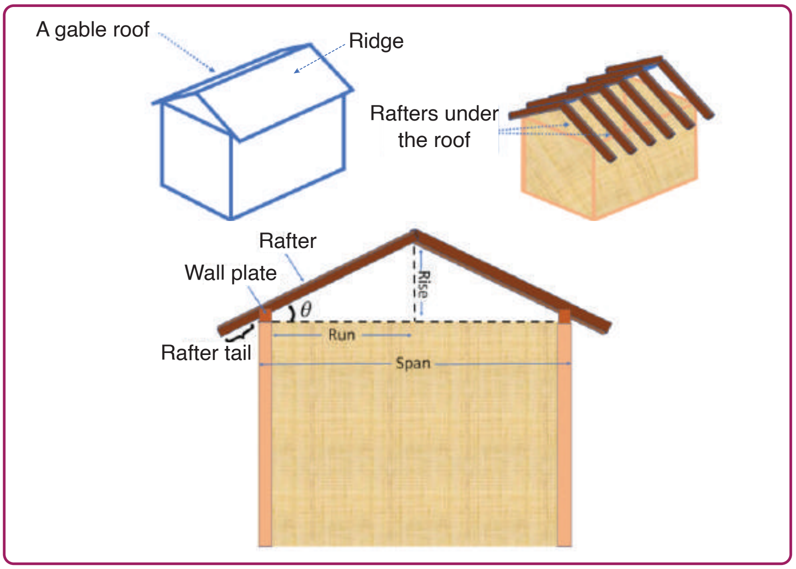


Fig. 5.2, Gable roof, the simplest form of roofs

The line at which the two roof planes meet is called the ridge. The horizontal distance between the two walls is called the span. The horizontal distance between the wall and the point under the ridge is called the run, which is also equal to half the span. The vertical distance between the ridge and the line joining the top of the two walls is called the rise. The rafter is the wooden beam or the metal frame, which runs from the ridge to the wall plate on the wall. The exterior part of the rafter, which extends beyond the wall, is called the raftertail.

Question 5

Speaking in terms of numbers, the pitch represents how much a roof rises as it moves in from the wall. So typically, the pitch is represented by rise over run, i.e. if the rise is 3 units and the run is 4 units, then the pitch is $\frac{3}{4}$. This is also called the slope of the roof. If θ is the angle between the rafter and the line joining the walls, then which of the following correctly represents the pitch/slope?

a. Cot θ	b. Tan θ	Answer
c. Cos θ	d. Sin θ	

Question 6

Thatching is a technique to build a roof with dry plant material, such as reeds and straws (dry stalks of cereal plants, water plants etc), palm branches and leaves, etc. A thatched roof needs to be very steep in order to drain properly. We can also see stone roofs in some old structures. Stone roofs can't be very steep, so as to avoid the stones from falling down.

Fig. 5.3, A thatched roof, stone roof and a metalsheet roof.
Image of thatched roof by Eric Christensen (own work) via Wikimedia Commons, stone roof by Јованвб (own work) via Wikimedia Commons, metal sheet roof via flickr.com

Fig. 5.4 shows three schematic diagrams representing a thatched roof, stone roof and a metal sheet roof.

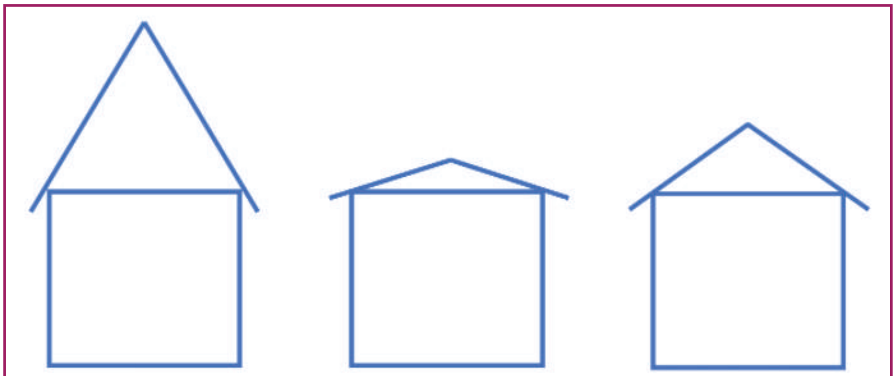


Fig. 5.4, Schematics representing the gable roof made of different materials

Which of the following correctly represents the decreasing order of the value of θ, if the span of all three structures in Fig. 5.4 are the same?

a. Stone roof > Thatched roof > Metal sheet roof
b. Thatched roof > Metal sheet roof > Stone roof
c. Stone roof > Metal sheet roof > Thatched roof
d. Metal sheet roof > Stone roof > Thatched roof

Answer

Question 7

Sahil wanted to make a gable roof for a building in his farm. He thought of using tiles to make the roof. The span of his building is 15.2 feet. In his region, it rains heavily. He decides to have the angle θ, between the rafter with the horizontal, be 40°.

i. What should be the length of the wooden block to make the rafter, assuming a rafter tail of 2 feet? Give your answer to the nearest whole number.

Answer

θ	Sin θ	Cos θ	Tan θ
40°	0.64	0.76	0.83

Table 5.2, Trigonometric values

ii. When Sahil goes to buy the tiles he finds out that the cost of metal sheets is less as compared to tiles. But the disadvantage of having a metal roof is that, there will be a rise in the interior temperature during summer, as metal is a good conductor of heat. So he wanted to experiment by adding a layer of plywood underneath the metal sheet covering. The room which these roof covers has a base of 14 feet x 15.2 feet.

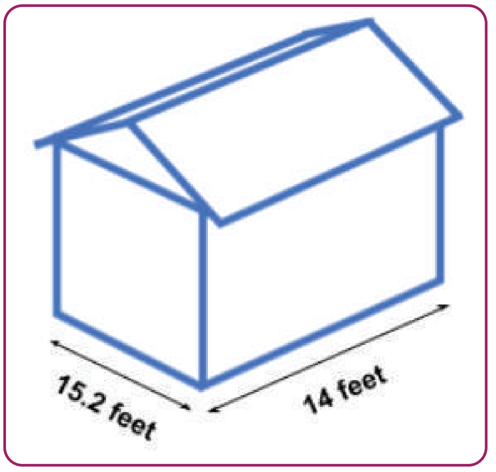


Fig. 5.5, Diagram showing dimensions of the building on Shyam's farm (drawing not to scale)

The region where Sahil lives, plywood is available in the form of sheets of the following standard sizes:

8 feet x 3 feet, 8 feet x 4 feet,
7 feet x 3 feet,
6 feet x 4 feet, 6 feet x 3 feet

Sahil wants to estimate the cost before ordering the raw material. But before that, he wants to calculate the minimum number of plywood sheets that he should buy. What is the minimum number of plywood sheets in total that Sahil will need to make the two sides of the roof, and which sizes should he select to ensure there is no wastage? Assume the false roof made of plywood is also a gable roof and its area is equal to that of the main roof made of metal sheets.

Answer

Case Study C - Optimal Packing

Ria and Avni are designing a triangular storage box for their tennis balls, as shown in Fig. 5.6. The diameter of each ball is 6.8 cm. Ria thought that to calculate the length of the inner side of the box, she could just multiply the diameter of the balls by 5. But Avni pointed out that it is not possible because there is an extended part beyond the last ball on both the sides. Both of them racked their brains and remembered what they have studied about tangents to a circle.

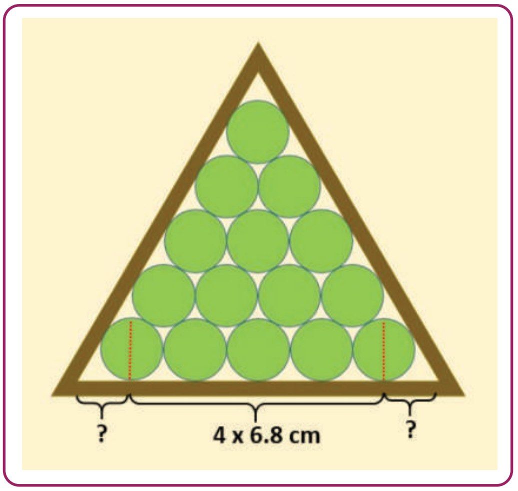


Fig. 5.6, Tennis balls inside a triangular box

They recalled that whenever two tangents are drawn from an external point to a circle, the two triangles formed by the tangents, the radii and the central line together are congruent to each other. In fig. 5.7, $\triangle POA \cong \triangle POB$. Hence $\angle APO = \angle BPO$. In other words, the line PO bisects $\angle APB$.

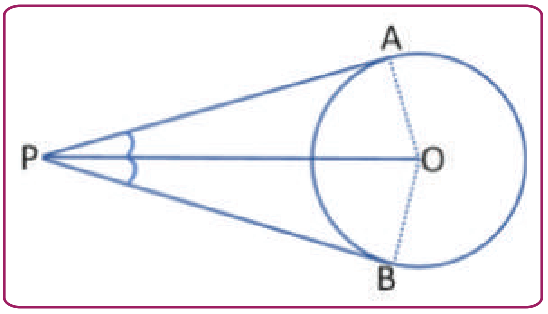


Fig. 5.7, Tangents drawn to a circle from an external point P.

Question 8

Which of the following expressions will give the exact length of the inner side of the triangle?

a. (6.8 x 4) cm + (3.4 x sin 30°) cm	b. (6.8 x 4) cm + (6.8 x cos 30°) cm	Answer
c. (6.8 x 4) cm + (3.4 x tan 30°) cm	d. (6.8 x 4) cm + (6.8 x cot 30°) cm	

Question 9

If Ria decided to make another triangular box, this time to fill 15 balls of diameter 5.7 cm, the inner perimeter of the box will be close to ________.

θ	Sin θ	Cos θ
30°	0.5	0.86

a. 45 cm	b. 75 cm	Answer
c. 84 cm	d. 98 cm	

Sextant

A sextant is an age-old device, used by astronomers, seafarers and engineers for millennia, to measure the angle of elevation of different objects as seen by you. You can make your own simplified version of a Sextant (often called the Clinometer), using a plastic straw, thread, some weight and a protractor template.

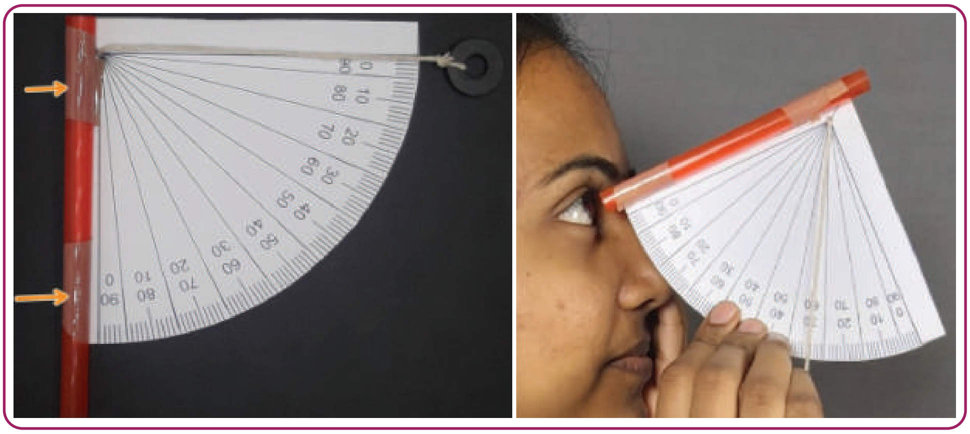


Fig. 5.8, Handmade sextant - a simple device to measure angle of elevation

Case Study D - Estimation

Shyam is standing near a huge overhead water tank at a distance of 35 m from it. He had his own handmade sextant with him. With his sextant he noticed that the angle of elevation of the top and bottom of the tank from his eye level is 30° and 23°, respectively. Based on the distance between opposite pillars under the tank, Shyam estimated the inner diameter of the tank to be 10 metres.

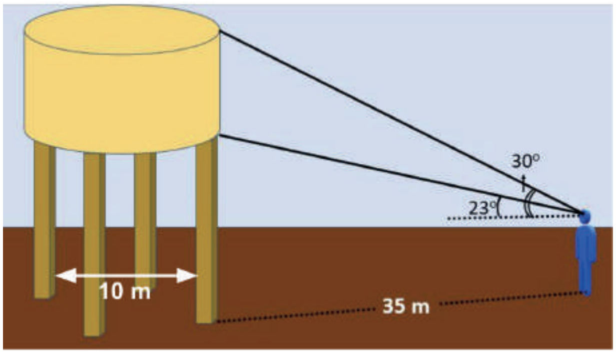


Fig. 5.9, Angle of elevation of the top and bottom of the overhead water tank

θ	Sin θ	Cos θ	Tan θ
23°	0.39	0.92	0.42
30°	0.5	0.86	0.57

Table 5.4, Trigonometric values

Question 10

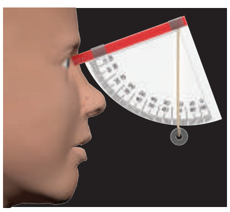
The total amount of water that can be contained in that tank is close to _________

Note: $1 m^3 = 1000$ litres

a. 21000 litres	b. 232000 litres
c. 412000 litres	d. 520000 litres

Answer

Exploration Pathway



Sextant Model

A sextant is an age-old device, used by astronomers, seafarers and engineers for millennia, to measure the angle of elevation to different objects as seen by you.

Here, you make your own Sextant using a plastic straw, thread, some weight and a protractor template.