The Purpose of the Mathematics Laboratory

A mathematics laboratory can foster mathematical awareness, skill building, positive attitudes and learning by doing experiences in different branches of mathematics such as Algebra, Geometry, Mensuration, Trigonometry, Coordinate Geometry, Statistics and Probability etc. It is the place where students can learn certain concepts using concrete objects and verify many mathematical facts and properties using models, measurements and other activities. It will also provide an opportunity to the students to do certain calculations using tables, calculators, etc., and also to listen or view certain audio-video cassettes, remedial instructions, enrichment materials, etc., of his/her own choice on a computer. Thus, it will act as an individualised learning centre for a student. It provides opportunities for discovering, remedial instruction, reinforcement and enrichment. Mathematics laboratory will also provide an opportunity for the teacher to explain and demonstrate many mathematical concepts, facts and properties using concrete materials, models, charts, etc. The teacher may also encourage students to prepare similar models and charts using materials like thermocol, cardboard, etc., in the laboratory. The laboratory will also act as a forum for the teachers to discuss and deliberate on some important mathematical issues and problems of the day. It may also act as a place for teachers and the students to perform a number of mathematical celebrations and recreational activities. Thus, the purpose of a mathematics laboratory is to enable:

- A student to learn mathematics with the help of concrete objects and to exhibit the relatedness of mathematics with everyday life.
- A student to verify or discover some geometric properties using models, measurements, paper cutting, paper folding, etc.
- A student to use different tables and ready reckoners in solving some problems.
- A student to draw graphs and do certain calculations using computers and calculators.
- The students to do some field work like surveying, finding heights, making badminton courts, etc., using instruments kept in the laboratory.

- The students and teachers to organise mathematics club activities including celebration of birthdays of famous mathematicians.
- The students to listen or view certain audio or video cassettes. CDs relating to different mathematical concepts/topics.
- A student to see a certain programme on a computer as a part of remedial instruction or enrichment under the proper guidance of the teacher.
- The students to perform certain experiments, which can be easily evaluated by the teacher.
- The students to do certain projects under the proper guidance of the teacher.
- The students to perform certain recreational activities in mathematics.
- A teacher to visually explain some abstract concepts by using three-dimensional models.
- A teacher to demonstrate certain concepts and patterns using charts and models.
- A teacher to demonstrate and reinforce the truth of certain algebraic identities using different models.
- A teacher to demonstrate the truth of various formulae for areas and volumes of different plane and solid figures using models.
- A teacher to explain certain concepts using computers and calculators.
- The teachers and students to consult good reference mathematics books, journals, etc., kept in the laboratory.
- The teachers to meet and discuss important issues relating to mathematics from time to time.
- A teacher explain certain concepts, data, graphs, etc., using slides.
- A teacher to generate different sets of parallel tests using a computer for testing the achievement of students.
- The budding mathematicians to take inspiration from the lives, works and anecodetes relating to great mathematicians.

Role of Mathematics Laboratory in Teaching-Learning

Mathematics is a compulsory subject at the Secondary Stage. Access to quality mathematics education is the right of every child. Mathematics engages children to use abstractions to establish precise relationships, to see structures, to reason out things, to find truth or falsity of statements (NCF - 2005). Therefore, mathematics teaching in schools must be planned in such away that they should nurture the ability to explore and seek solutions to problems of not only the academic areas but also of daily life. In order to do this, access to laboratory is an essential requirement which our system has not been able to provide so far. It is proposed to fill this gap by providing a mathematics laboratory at the secondary stage to all the schools. This facility will bring about a renewed thrust in our schools so far teaching-learning of mathematics is concerned.

The teaching-learning of mathematics needs to be characterised by focussed emphasis on processes such as activity-based learning, making observations, collection of data, classification, analysis, making hypothesis, drawing inferences and arriving at a conclusion for establishing the objective truth.

The main goal of mathematics education is to 'develop the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstractions' (NCF-2005). To achieve this, a variety of methods and skills have to be adopted in the teaching-learning situation. The basic arithmetical skills offered in the first eight years of schooling will stand in good stead to achieve the higher goals visualised at the secondary stage. A stronger emphasis is to be laid on problem solving and acquisition of analytical skills in order to prepare children to tackle a wide variety of life situations. Abstraction, quantification, analogy, case analysis, guesses and verification exercises are useful in many problem-solving situations (NCF-2005). Another area of concern which teachers will have to address is of the perceived 'stand-alone' status that mathematics has vis-a-vis other subject areas in the school curriculum.

One of the biggest challenges of a mathematics teacher is to create and sustain interest in his students. There is a general feeling that mathematics is all about formulas and mechanical procedures. Under these circumstances, a mathematics laboratory will help teachers to reorient their strategies and make mathematics also an activity-oriented programme in schools.

In the huddled and bundled classroom situations it is indeed difficult to make complex theoretical concepts very clear to all the students. Developing the habit of critical thinking and logical reasoning which are most important in mathematics learning also suffer under such claustrophobic classroom situations. A mathematics corner in the lower classes and a mathematics laboratory with appropriate tools at the secondary stage will enable children to translate abstractions into specific figures, shapes and patterns that will provide opportunities to visualise abstractions with greater ease. To promote interest in the subject, mathematics laboratory has become a reality at many places and is considered as an established strategy for mathematics teaching-learning. Since a practical exercise takes a longer time than a theoretical solution might require, it gives the student additional time for better assimilation leading to stronger retention. For students in whom aptitude for mathematics is limited, practical activities besides overcoming drudgery, boredom and indifference, may help create positive attitude and a new thirst for knowledge.

Management and Maintenance of Laboratory

There is no second opinion that for effective teaching and learning 'Learning by Doing' is of great importance as the experiences gained remains permanently affixed in the mind of the child. Exploring what mathematics is about and arriving at truth provides for pleasure of doing, understanding, developing positive attitude and learning processes of mathematics and above all the great feeling of attachment with the teacher as facilitator. It is said, 'a bad teacher teaches the truth but a good teacher teaches how to arrive at the truth'.

A principle or a concept learnt as a conclusion through activities under the guidance of the teacher stands above all other methods of learning and the theory built upon it, can not be forgotten. On the contrary, a concept stated in the classroom and verified later on in the laboratory doesn't provide for any great experience nor make child's curiosity to know any good nor provides for any sense of achievement.

A laboratory is equipped with instruments, apparatus, equipments, models apart from facilities like water, electricity, etc. Non availability of a single material or facility out of these may hinder the performance of any experiment activity in the laboratory. Therefore, the laboratory must be well managed and well maintained.

A laboratory is managed and maintained by persons and the material required. Therefore, management and maintenance of a laboratory may be classified into two categorise namely the personal management and maintenance and the material management and maintenance.

(A) Personal Management and Maintenance

The persons who manage and maintain laboratories are generally called laboratory assistant and laboratory attendant. Collectively they are known as laboratory staff. Teaching staff also helps in managing and maintenance of the laboratory whenever and wherever it is required.

In personal management and maintenance following points are considered:

1. **Cleanliness**

   A laboratory should always be neat and clean. When students perform experiment activities during the day, it certainly becomes dirty and things are scattered. So, it is the duty of the lab staff to clean the laboratory when the day's work is over and also place the things at their proper places if these are lying scattered.

2. **Checking and arranging materials for the day's work**

   Lab staff should know that what activities are going to be performed on a particular day. The material required for the day's activities must be arranged one day before.

   The materials and instruments should be arranged on tables before the class comes to perform an activity or the teacher brings the class for a demonstration.

3. The facilities like water, electricity, etc. must be checked and made available at the time of experiments.
4. It is better if a list of materials and equipments is pasted on the wall of the laboratory.
5. Many safety measures are required while working in laboratory. A list of such measures may be pasted on a wall of the laboratory.
6. While selecting the laboratory staff, the school authority must see that the persons should have their education with mathematics background.
7. A training of 7 to 10 days may be arranged for the newly selected laboratory staff with the help of mathematics teachers of the school or some resource persons outside the school.
8. A first aid kit may be kept in the laboratory.

(B) Management and Maintenance of Materials

A laboratory requires a variety of materials to run it properly. The quantity of materials however depends upon the number of students in the school.

To manage and maintain materials for a laboratory following points must be considered:

1. A list of instruments, apparatus, activities and material may be prepared according to the experiments included in the syllabus of mathematics.

2. A group of mathematics teachers may visit the agencies or shops to check the quality of the materials and compare the rates. This will help to acquire the material of good quality at appropriate rates.
3. The materials required for the laboratory must be checked from time to time. If some materials or other consumable things are exhausted, orders may be placed for the same.
4. The instruments, equipments and apparatus should also be checked regularly by the laboratory staff. If any repair is required it should be done immediately. If any part is to be replaced, it should be ordered and replaced.
5. All the instruments, equipments, apparatus, etc. must be stored in the almirahs and cupboards in the laboratory or in a separate store room.

Equipments for Mathematics Laboratory at Secondary Stage

As the students will be involved in a lot of model making activities under the guidance of the teacher, the smooth running of the mathematics laboratory will depend upon the supply of oddments such as strings and threads, cellotape, white cardboard, hardboard, needles and pins, drawing pins, sandpaper, pliers, screw-drivers, rubber bands of different colours, gummed papers and labels, squared papers, plywood, scissors, saw, paint, soldering, solder wire, steel wire, cotton wool, tin and plastic sheets, glazed papers, etc. Besides these, some models, charts, slides, etc., made up of a good durable material should also be there for the teacher to demonstrate some mathematical concepts, facts and properties before the students. Different tables, ready reckner should also be there (in the laminated form) so that these can be used by the students for different purposes. Further, for performing activities such as measuring, drawing and calculating, consulting reference books, etc., there should be equipments like mathematical instruments, calculators, computers, books, journals mathematical dictionaries etc., in the laboratory. In view of the above, following is the list of suggested instruments/models for the laboratory:

Equipments

Mathematical instrument set (Wooden Geometry Box for demonstration containing rulers, set-squares, divider, protractor and compasses), some geometry boxes, metre scales of 100 cm, 50 cm and 30 cm, measuring tape, diagonal scale, clinometer, calculators, computers including related software etc.

Models

- Number line
- Geoboards - rectangular, circular and isometric
- Models for verifying the following identities:

(i)	$(a + b)^2 = a^2 + 2ab + b^2$	(ii)	$(a - b)^2 = a^2 - 2ab + b^2$
(iii)	$a^2 - b^2 = (a - b)(a + b)$	(iv)	$k(a + b + c) = ka + kb + kc$
(v)	$(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$	(vi)	$(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
(vii)	$a^3 + b^3 = (a + b)^3 - 3ab(a + b)$	(viii)	$a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

- Concrete models of the following:
  Equilateral triangle, Isosceles triangle, Scalene triangle, Right triangle.

Different types of quadrilaterals such as square, parallelogram, kite, rhombus, rectangle etc., regular pentagon, regular hexagon, regular octagon, circle, sphere, hemisphere, cuboid, cube, right circular cylinder, cone, frustum of a cone, tetrahedron, hexahedron, regular octahedron, dodecahedron, icosahedron.

Model for finding the centre of a circle. Models illustrating the following concepts/properties:

Height and slant height of a cone, various criteria of congruency of triangles (SSS, ASA, SAS, RHS), Angles in a semi-circle. Major and Minor segments of a circle,

Models for verifying Pythagoras theorem by different methods.

- Model for deriving formula for area of a circle sliced into sectors.
- Hinged models for demonstrating the symmetry of a square, rectangle isosceles triangle, equilateral triangle, circle.
- Overhead projector along with slides.
- CDs and films regarding teaching of mathematics, specially on some selected topics.
- Calculators
- Computers
- Reference Books and Journals
- Photographs of mathematicians alongwith their brief life histories and contributions in mathematics.

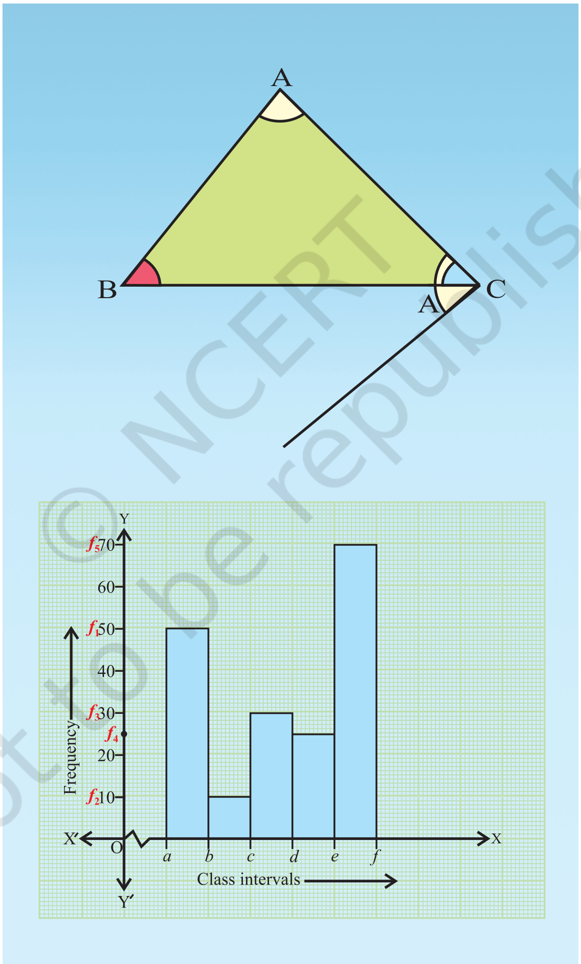
Stationery and oddments

Rubber-bands of different colours, Marbles of different colours, a pack of playing cards, graph paper/squared paper, dotted paper, drawing pins, erasers, pencils, sketch pens, cellotapes, threads of different colours, glazed papers, kite papers, tracing papers, adhesive, pins, scissors and cutters, hammers, saw, thermocol sheets, sand paper, nails and screws of different sizes, screw drivers, drill machine with bit set, and pliers.

The basic principles of learning mathematics are : (a) learning should be related to each child individually (b) the need for mathematics should develop from an intimate acquaintance with the environment (c) the child should be active and interested (d) concrete material and wide variety of illustrations are needed to aid the learning process (e) understanding should be encouraged at each stage of acquiring a particular skill (f) content should be broadly based with adequate appreciation of the links between the various branches of mathematics (g) correct mathematical usage should be encouraged at all stages.

– Ronwill

Activities for
Class IX



Mathematics is one of the most important cultural components of every modern society. Its influence an other cultural elements has been so fundamental and wide-spread as to warrant the statement that her "most modern" ways of life would hardly have been possilbly without mathematics. Appeal to such obvious examples as electronics radio, television, computing machines, and space travel, to substantiate this statement is unnecessary : the elementary art of calculating is evidence enough. Imagine trying to get through three day without using numbers in some fashion or other!

– R.L. Wilder

Activity 1

Objective

To construct a square-root spiral.

Material Required

Coloured threads, adhesive, drawing pins, nails, geometry box, sketch pens, marker, a piece of plywood.

Method of Construction

1. Take a piece of plywood with dimensions 30 cm × 30 cm.
2. Taking 2 cm = 1 unit, draw a line segment AB of length one unit.
3. Construct a perpendicular BX at the line segment AB using set squares (or compasses).
4. From BX, cut off BC = 1 unit. Join AC.
5. Using blue coloured thread (of length equal to AC) and adhesive, fix the thread along AC.
6. With AC as base and using set squares (or compasses), draw CY perpendicular to AC.
7. From CY, cut-off CD = 1 unit and join AD.

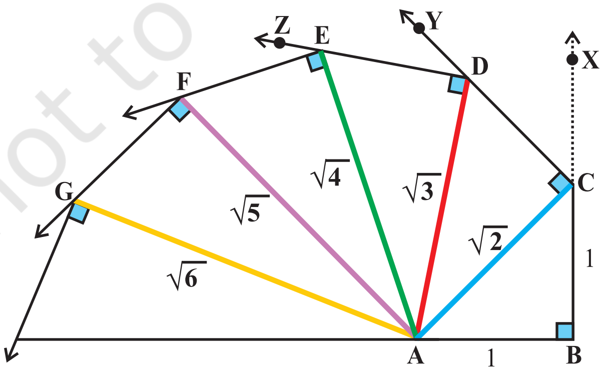


Fig. 1

8. Fix orange coloured thread (of length equal to AD) along AD with adhesive.
9. With AD as base and using set squares (or compasses), draw DZ perpendicular to AD.
10. From DZ, cut off DE = 1 unit and join AE.
11. Fix green coloured thread (of length equal to AE) along AE with adhesive [see Fig. 1].

    Repeat the above process for a sufficient number of times. This is called "a square root spiral".

DEMONSTRATION

1. From the figure, $AC^2 = AB^2 + BC^2 = 12 + 12 = 2$ or $AC = \sqrt{2}$.

   $AD^2 = AC^2 + CD^2 = 2 + 1 = 3$ or $AD = \sqrt{3}$.

2. Similarly, we get the other lengths AE, AF, AG, ... as $\sqrt{4}$ or 2, $\sqrt{5}$, $\sqrt{6}$ ....

OBSERVATION

On actual measurement

AC = ... , AD = ... , AE = ... , AF = ... , AG = ...

$\sqrt{2}$ = AC = ... (approx.),

$\sqrt{3}$ = AD = ... (approx.),

$\sqrt{4}$ = AE = ... (approx.),

$\sqrt{5}$ = AF = ... (approx.)

APPLICATION

Through this activity, existence of irrational numbers can be illustrated.

Activity 2

OBJECTIVE

To represent some irrational numbers on the number line.

MATERIAL REQUIRED

Two cuboidal wooden strips, thread, nails, hammer, two photo copies of a scale, a screw with nut, glue, cutter.

METHOD OF CONSTRUCTION

1. Make a straight slit on the top of one of the wooden strips. Fix another wooden strip on the slit perpendicular to the former strip with a screw at the bottom so that it can move freely along the slit [see Fig.1].
2. Paste one photocopy of the scale on each of these two strips as shown in Fig. 1.
3. Fix nails at a distance of 1 unit each, starting from 0, on both the strips as shown in the figure.
4. Tie a thread at the nail at 0 on the horizontal strip.

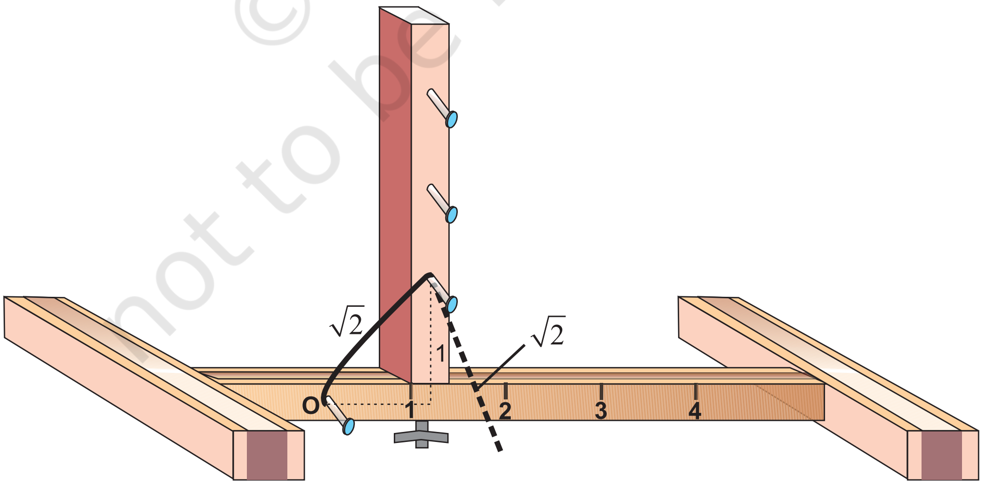


Fig. 1

Demonstration

1. Take 1 unit on the horizontal scale and fix the perpendicular wooden strip at 1 by the screw at the bottom.
2. Tie the other end of the thread to unit '1' on the perpendicular strip.
3. Remove the thread from unit '1' on the perpendicular strip and place it on the horizontal strip to represent $\sqrt{2}$ on the horizontal strip [see Fig. 1].

   Similarly, to represent $\sqrt{3}$, fix the perpendicular wooden strip at $\sqrt{2}$ and repeat the process as above. To represent $\sqrt{a}$, $a > 1$, fix the perpendicular scale at $\sqrt{a-1}$ and proceed as above to get $\sqrt{a}$.

Observation

On actual measurement:

$a - 1 =$ .... $\sqrt{a} =$ ....

Application

The activity may help in representing some irrational numbers such as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, ... on the number line.

Note

You may also find $\sqrt{a}$ such as $\sqrt{13}$ by fixing the perpendicular strip at 3 on the horizontal strip and tying the other end of thread at 2 on the vertical strip.

Activity 3

OBJECTIVE

To verify the algebraic identity :

$(a + b)^2 = a^2 + 2ab + b^2$

MATERIAL REQUIRED

Drawing sheet, cardboard, cello-tape, coloured papers, cutter and ruler.

METHOD OF CONSTRUCTION

1. Cut out a square of side length a units from a drawing sheet/cardboard and name it as square ABCD [see Fig. 1].
2. Cut out another square of length b units from a drawing sheet/cardboard and name it as square CHGF [see Fig. 2].

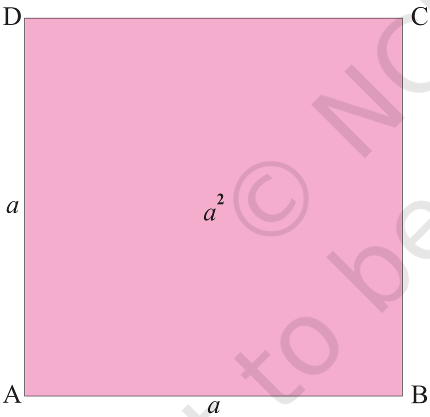


Fig. 1

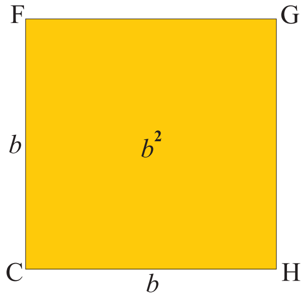


Fig. 2

3. Cut out a rectangle of length a units and breadth b units from a drawing sheet/cardbaord and name it as a rectangle DCFE [see Fig. 3].
4. Cut out another rectangle of length b units and breadth a units from a drawing sheet/cardboard and name it as a rectangle BIHC [see Fig. 4].

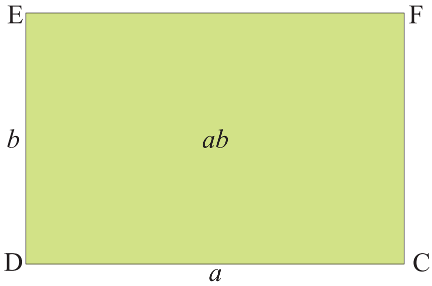


Fig. 3

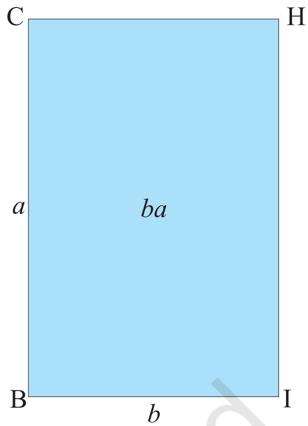


Fig. 4

5. Total area of these four cut-out figures

   = Area of square ABCD + Area of square CHGF + Area of rectangle DCFE + Area of rectangle BIHC

   = $a^2 + b^2 + ab + ba = a^2 + b^2 + 2ab$.

6. Join the four quadrilaterals using cello-tape as shown in Fig. 5.

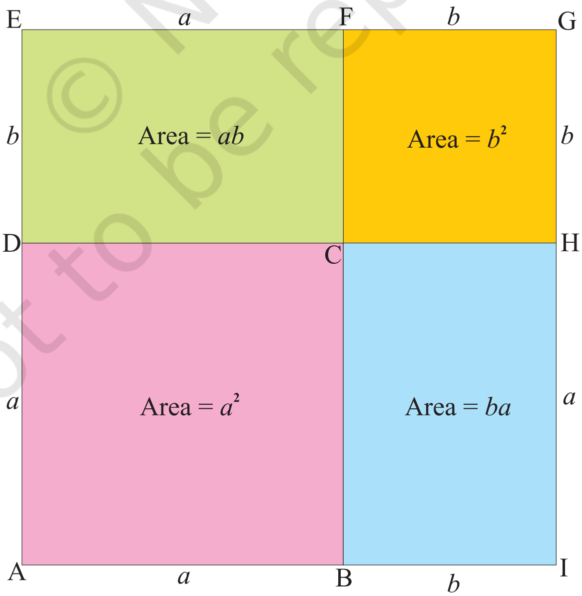


Fig. 5

Clearly, AIGE is a square of side $(a + b)$. Therefore, its area is $(a + b)^2$. The combined area of the constituent units $= a^2 + b^2 + ab + ab = a^2 + b^2 + 2ab$.

Hence, the algebraic identity $(a + b)^2 = a^2 + 2ab + b^2$

Here, area is in square units.

OBSERVATION

On actual measurement:

a = ..., b = ... $(a+b)$ = ...,

So, a^2 = ... b^2 = ..., ab = ...

$(a+b)^2$ = ..., $2ab$ = ...

Therefore, $(a+b)^2 = a^2 + 2ab + b^2$.

The identity may be verified by taking different values of a and b.

APPLICATION

The identity may be used for

1. calculating the square of a number expressed as the sum of two convenient numbers.
2. simplifications/factorisation of some algebraic expressions.

Activity 4

OBJECTIVE

To verify the algebraic identity :

$(a - b)^2 = a^2 - 2ab + b^2$

MATERIAL REQUIRED

Drawing sheets, cardboard, coloured papers, scissors, ruler and adhesive.

METHOD OF CONSTRUCTION

1. Cut out a square ABCD of side a units from a drawing sheet/cardboard [see Fig. 1].
2. Cut out a square EBHI of side b units ($b < a$) from a drawing sheet/cardboard [see Fig. 2].
3. Cut out a rectangle GDCJ of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 3].
4. Cut out a rectangle IFJH of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 4].

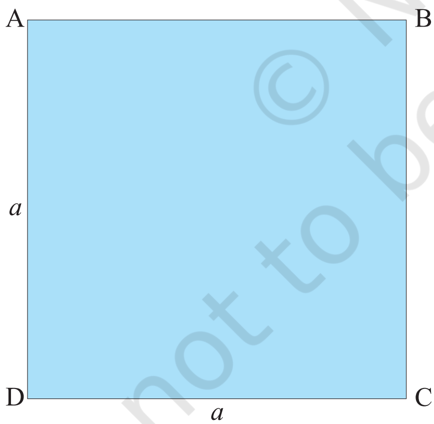


Fig. 1

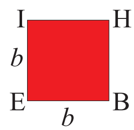


Fig. 2

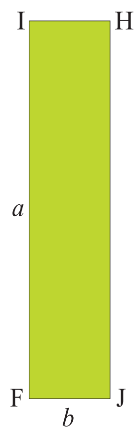


Fig. 4

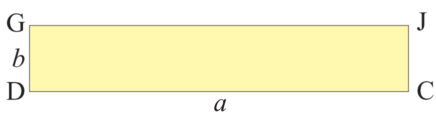


Fig. 3

5. Arrange these cut outs as shown in Fig. 5.

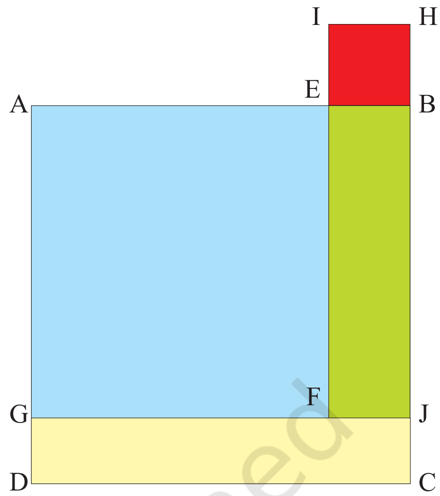


Fig. 5

Demonstration

According to figure 1, 2, 3, and 4, Area of square ABCD = a^2, Area of square EBHI = b^2

Area of rectangle GDCJ = ab, Area of rectangle IFJH = ab

From Fig. 5, area of square AGFE = AG × GF = $(a - b)(a - b) = (a - b)^2$

Now, area of square AGFE = Area of square ABCD + Area of square EBHI

– Area of rectangle IFJH – Area of rectangle GDCJ

$= a^2 + b^2 - ab - ab$

$= a^2 - 2ab + b^2$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ..., $(a - b)$ = ...,

So, a^2 = ..., b^2 = ..., $(a - b)^2$ = ...,

ab = ..., $2ab$ = ...

Therefore, $(a - b)^2 = a^2 - 2ab + b^2$

Application

The identity may be used for

1. calculating the square of a number expressed as a difference of two convenient numbers.
2. simplifying/factorisation of some algebraic expressions.

Activity 5

Objective

To verify the algebraic identity :

$a^2 - b^2 = (a + b)(a - b)$

Material Required

Drawing sheets, cardboard, coloured papers, scissors, sketch pen, ruler, transparent sheet and adhesive.

Method of Construction

1. Take a cardboard of a convenient size and paste a coloured paper on it.
2. Cut out one square ABCD of side a units from a drawing sheet [see Fig. 1].
3. Cut out one square AEFG of side b units ($b < a$) from another drawing sheet [see Fig. 2].

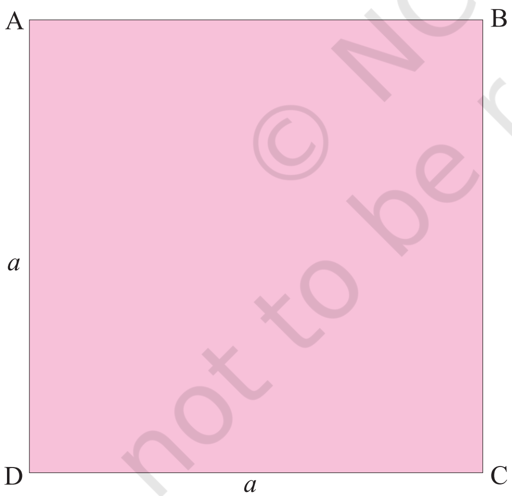


Fig. 1

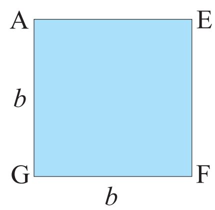


Fig. 2

4. Arrange these squares as shown in Fig. 3.
5. Join F to C using sketch pen. Cut out trapeziums congruent to EBCF and GFCD using a transparent sheet and name them as EBCF and GFCD, respectively [see Fig. 4 and Fig. 5].

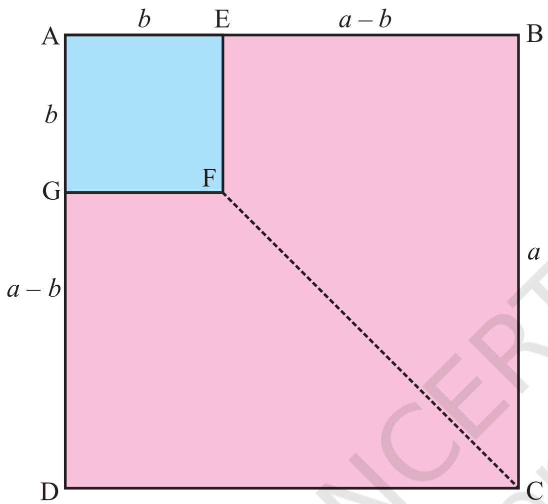


Fig. 3

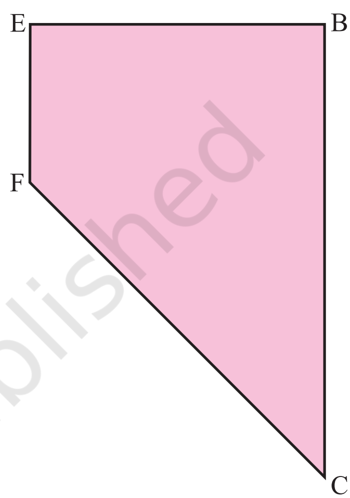


Fig. 4

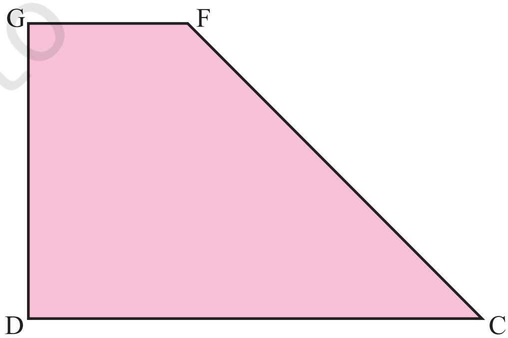


Fig. 5

6. Arrange these trapeziums as shown in Fig. 6.

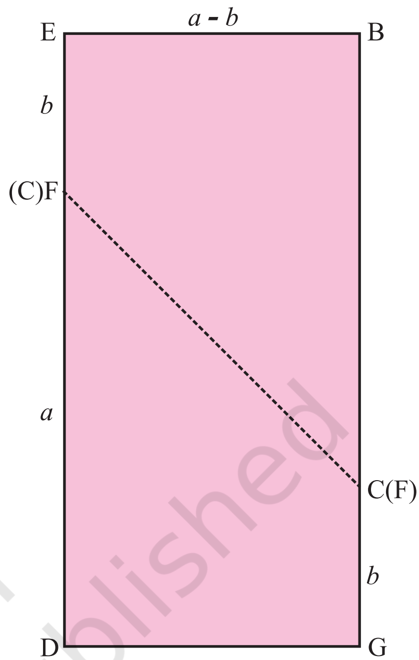


Fig. 6

Demonstration

Area of square ABCD = a^2

Area of square AEFG = b^2

In Fig. 3,

Area of square ABCD – Area of square AEFG

= Area of trapezium EBCF + Area of trapezium GFCD

= Area of rectangle EBGD [Fig. 6].

= ED × DG

Thus, $a^2 - b^2 = (a+b)\ (a-b)$

Here, area is in square units.

Observation

On actual measurement:

a = ..., b = ..., $(a+b)$ = ...,

So, a^2 = ..., b^2 = ..., $(a-b)$ = ...,

a^2-b^2 = ...,$(a+b)\ (a-b)$ = ...,

Therefore, $a^2-b^2 = (a+b)\ (a-b)$

Application

The identity may be used for

1. difference of two squares
2. some products involving two numbers
3. simplification and factorisation of algebraic expressions.

Activity 6

OBJECTIVE

To verify the algebraic identity :

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

MATERIAL REQUIRED

Hardboard, adhesive, coloured papers, white paper.

METHOD OF CONSTRUCTION

1. Take a hardboard of a convenient size and paste a white paper on it.
2. Cut out a square of side a units from a coloured paper [see Fig. 1].
3. Cut out a square of side b units from a coloured paper [see Fig. 2].
4. Cut out a square of side c units from a coloured paper [see Fig. 3].
5. Cut out two rectangles of dimensions $a \times b$, two rectangles of dimensions $b \times c$ and two rectangles of dimensions $c \times a$ square units from a coloured paper [see Fig. 4].

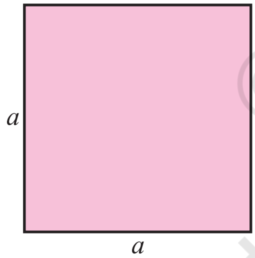


Fig. 1

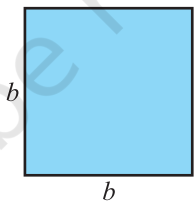


Fig. 2

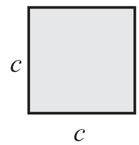


Fig. 3

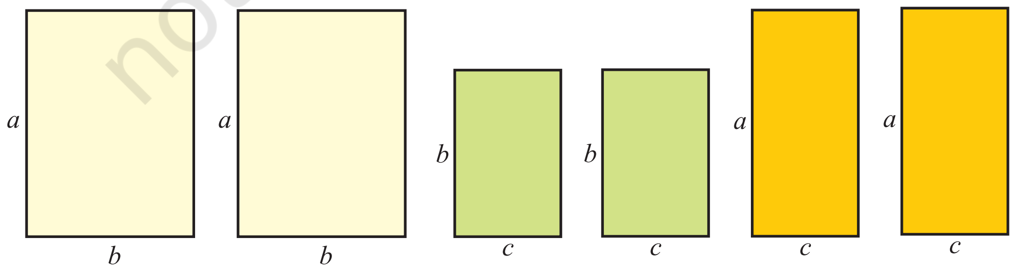


Fig. 4

14/04/18

6. Arrange the squares and rectangles on the hardboard as shown in Fig. 5.

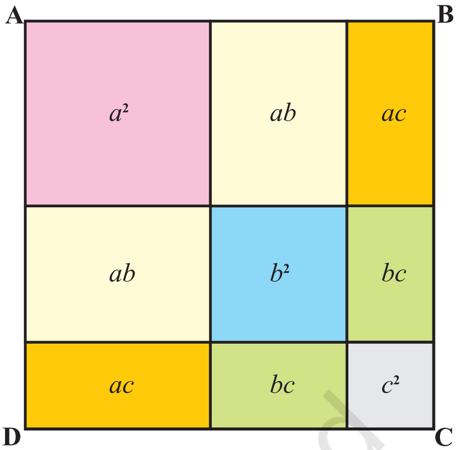


Fig. 5

DEMONSTRATION

From the arrangement of squares and rectangles in Fig. 5, a square ABCD is obtained whose side is $(a+b+c)$ units.

Area of square ABCD = $(a+b+c)^2$.

Therefore, $(a+b+c)^2$ = sum of all the squares and rectangles shown in Fig. 1 to Fig. 4.

$= a^2 + ab + ac + ab + b^2 + bc + ac + bc + c^2$

$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

Here, area is in square units.

OBSERVATION

On actual measurement:

a = ..., b = ..., c = ...,

So, a^2 = ..., b^2 = ..., c^2= ...., ab= ....,

bc= ...., ca = ..., $2ab$ = ..., $2bc$ = ...,

$2ca$= ...., $a+b+c$ = ..., $(a+b+c)^2$ = ...,

Therefore, $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

APPLICATION

The identity may be used for

1. simiplification/factorisation of algebraic expressions
2. calculating the square of a number expressed as a sum of three convenient numbers.

Activity 7

Objective

To verify the algebraic identity :

$(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Material Required

Acrylic sheet, coloured papers, glazed papers, saw, sketch pen, adhesive, Cello-tape.

Method of Construction

1. Make a cube of side a units and one more cube of side b units ($b < a$), using acrylic sheet and cello-tape/adhesive [see Fig. 1 and Fig. 2].
2. Similarly, make three cuboids of dimensions $a \times a \times b$ and three cuboids of dimensions $a \times b \times b$ [see Fig. 3 and Fig. 4].

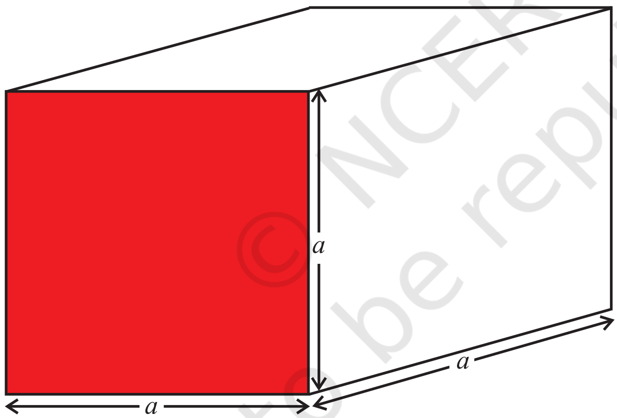


Fig. 1

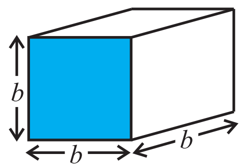


Fig. 2

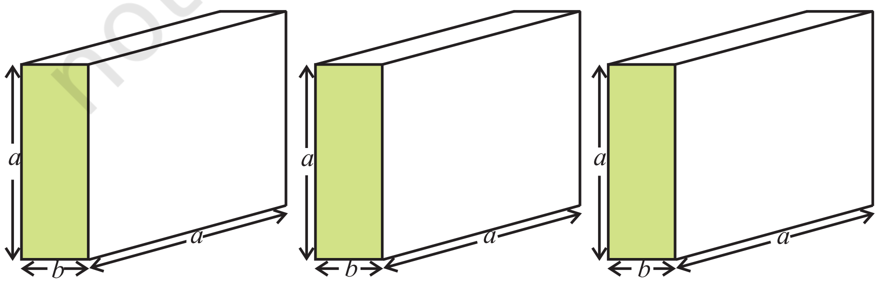


Fig. 3

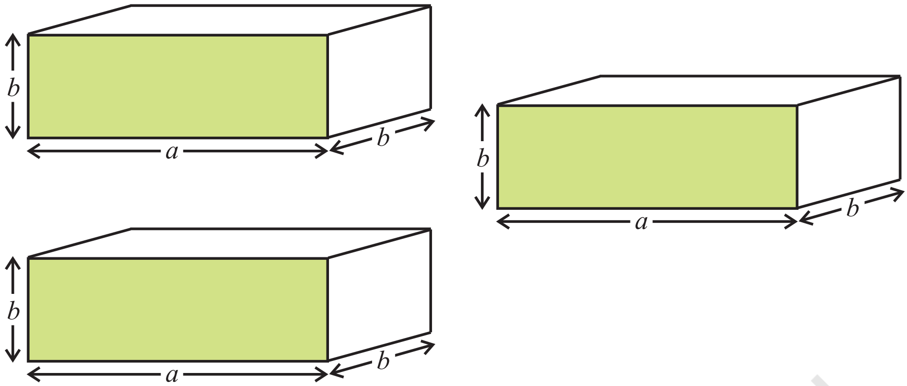


Fig. 4

3. Arrange the cubes and cuboids as shown in Fig. 5.

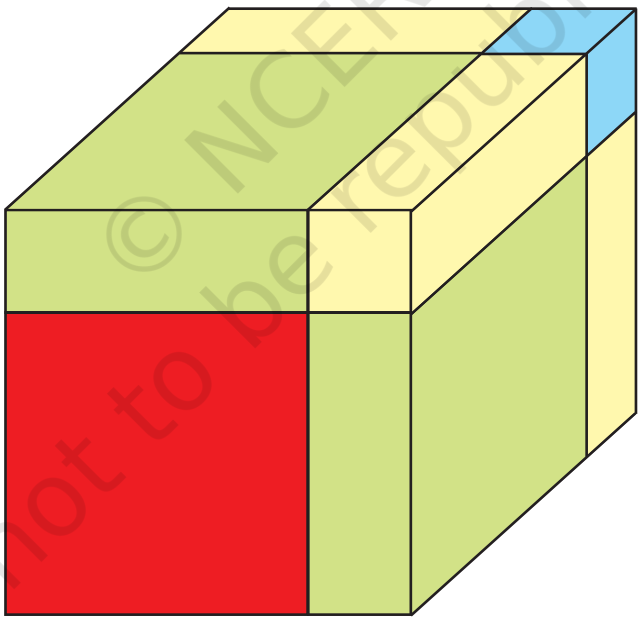


Fig. 5

Demonstration

Volume of the cube of side $a = a \times a \times a = a^3$, volume of the cube of side $b = b^3$

Volume of the cuboid of dimensions $a \times a \times b = a^2b$, volume of three such cuboids $= 3a^2b$

Volume of the cuboid of dimensions $a \times b \times b = ab^2$, volume of three such cuboids $= 3ab^2$

Solid figure obtained in Fig. 5 is a cube of side $(a + b)$

Its volume $= (a + b)^3$

Therefore, $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Here, volume is in cubic units.

Observation

On actual measurement:

a = ..., b = ..., a^3 = ...,

So, a^3 = ..., b^3 = ..., a^2b = ..., $3a^2b$ = ...,

ab^2 = ..., $3ab^2$ = ..., $(a+b)^3$ = ...,

Therefore, $(a+b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$

Application

The identity may be used for

1. calculating cube of a number expressed as the sum of two convenient numbers
2. simplification and factorisation of algebraic expressions.

Activity 8

Objective

To verify the algebraic identity

$(a - b)^3 = a^3 - b^3 - 3(a - b)ab$

Material Required

Acrylic sheet, coloured papers, saw, sketch pens, adhesive, Cellotape.

Method of Construction

1. Make a cube of side $(a - b)$ units $(a > b)$ using acrylic sheet and cellotape/adhesive [see Fig. 1].
2. Make three cuboids each of dimensions $(a-b) \times a \times b$ and one cube of side b units using acrylic sheet and cellotape [see Fig. 2 and Fig. 3].
3. Arrange the cubes and cuboids as shown in Fig. 4.

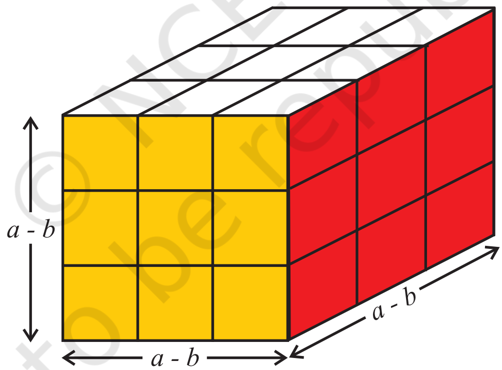


Fig. 1

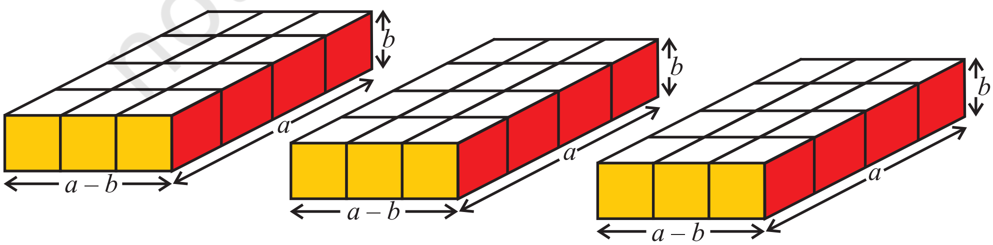


Fig. 2

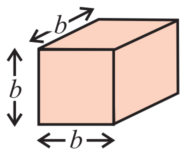


Fig. 3

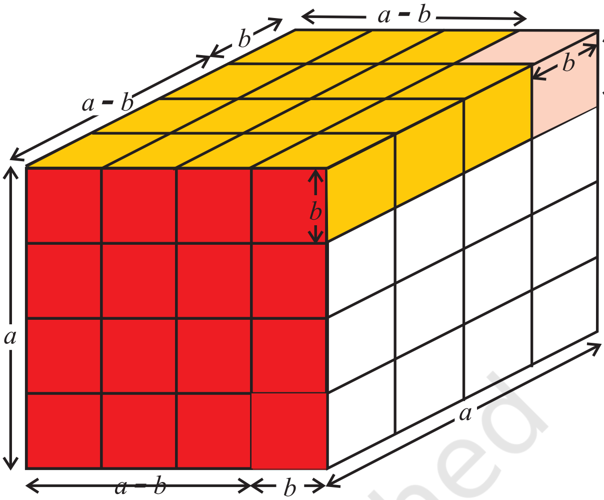


Fig. 4

DEMONSTRATION

Volume of the cube of side $(a - b)$ units in Fig. 1 = $(a- b)^3$

Volume of a cuboid in Fig. 2 = $(a-b)\ ab$

Volume of three cuboids in Fig. 2 = $3\ (a-b)\ ab$

Volume of the cube of side b in Fig. 3 = b^3

Volume of the solid in Fig. 4 = $(a-b)^3 + (a-b)\ ab + (a-b)\ ab + (a - b)\ ab + b^3$

$= (a-b)^3 + 3(a-b)\ ab + b^3$ (1)

Also, the solid obtained in Fig. 4 is a cube of side a

Therefore, its volume = a^3 (2)

From (1) and (2),

$(a-b)^3 + 3(a-b)\ ab + b^3 = a^3$

or $(a-b)^3 = a^3 - b^3 - 3ab\ (a-b)$.

Here, volume is in cubic units.

Observation

On actual measurement:

a = ..., b = ..., $a–b$ = ...,

So, a^3 = ..., ab = ...,

b^3 = ..., $ab(a–b)$ = ...,

$3ab\ (a–b)$ = ..., $(a–b)^3$ = ...,

Therefore, $(a–b)^3 = a^3 - b^3 - 3ab(a–b)$

Application

The identity may be used for

1. calculating cube of a number expressed as a difference of two convenient numbers
2. simplification and factorisation of algebraic expressions.

Note

This identity can also be expressed as :

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$.

Activity 9

Objective

To verify the algebraic identity :

$a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$

Material Required

Acrylic sheet, glazed papers, saw, adhesive, cellotape, coloured papers, sketch pen, etc.

Method of Construction

1. Make a cube of side *a* units and another cube of side *b* units as shown in Fig. 1 and Fig. 2 by using acrylic sheet and cellotape/adhesive.
2. Make a cuboid of dimensions $a \times a \times b$ [see Fig. 3].
3. Make a cuboid of dimensions $a \times b \times b$ [see Fig. 4].
4. Arrange these cubes and cuboids as shown in Fig. 5.

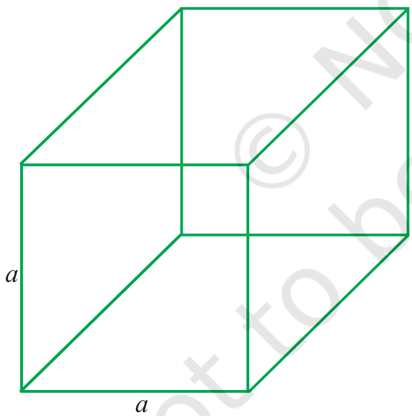


Fig. 1

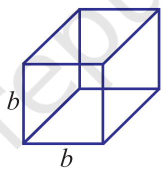


Fig. 2

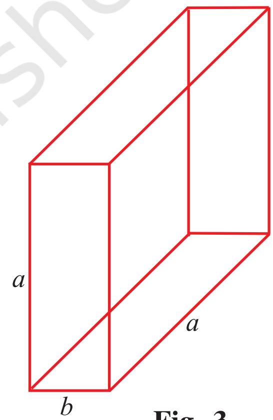


Fig. 3

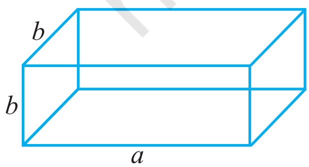


Fig. 4

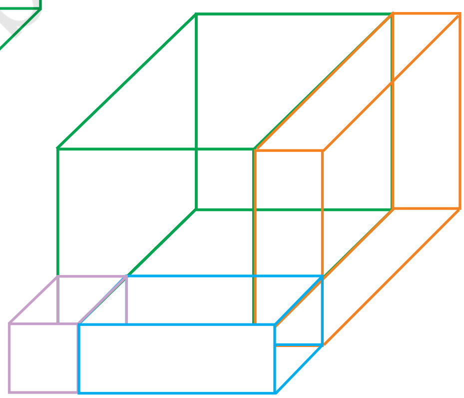


Fig. 5

Demonstration

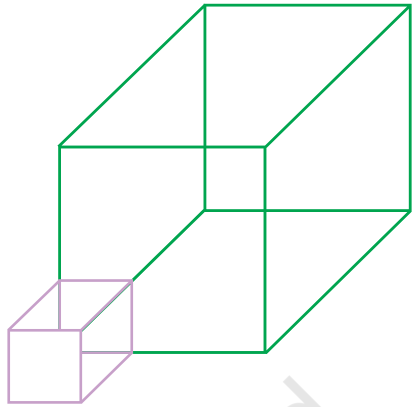


Fig. 6

Volume of cube in Fig. 1 = a^3

Volume of cube in Fig. 2 = b^3

Volume of cuboid in Fig. 3 = a^2b

Volume of cuboid in Fig. 4 = ab^2

Volume of solid in Fig. 5 = $a^3+b^3+a^2b+ab^2$ $= (a+b)(a^2+b^2)$

Removing cuboids of volumes a^2b and ab^2, i.e., $ab\ (a+b)$ from solid obtained in Fig. 5, we get the solid in Fig. 6.

Volume of solid in Fig. 6 = $a^3 + b^3$.

Therefore, $a^3 + b^3 = (a+b)(a^2+b^2) - ab(a+b)$

$= (a+b)(a^2+b^2-ab)$

Here, volumes are in cubic units.

Observation

On actual measurement:

a = ..., b = ...,

So, a^3 = ..., b^3 = ..., $(a+b)$ = ..., $(a+b)a^2$ = ...,

$(a+b)\,b^2$ = ..., a^2b = ..., ab^2 = ...,

$ab\,(a+b)$ = ...,

Therefore, $a^3 + b^3 = (a+b)(a^2+b^2-ab)$.

Application

The identity may be used in simplification and factorisation of algebraic expressions.

Activity 10

OBJECTIVE

To verify the algebraic identity :

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

MATERIAL REQUIRED

Acrylic sheet, sketch pen, glazed papers, scissors, adhesive, cello-tape, coloured papers, cutter.

METHOD OF CONSTRUCTION

1. Make a cuboid of dimensions $(a–b) \times a \times a$ $(b < a)$, using acrylic sheet and cellotape/adhesive as shown in Fig. 1.
2. Make another cuboid of dimensions $(a–b) \times a \times b$, using acrylic sheet and cellotape/adhesive as shown in Fig. 2.
3. Make one more cuboid of dimensions $(a–b) \times b \times b$ as shown in Fig. 3.
4. Make a cube of dimensions $b \times b \times b$ using acrylic sheet as shown in Fig. 4.

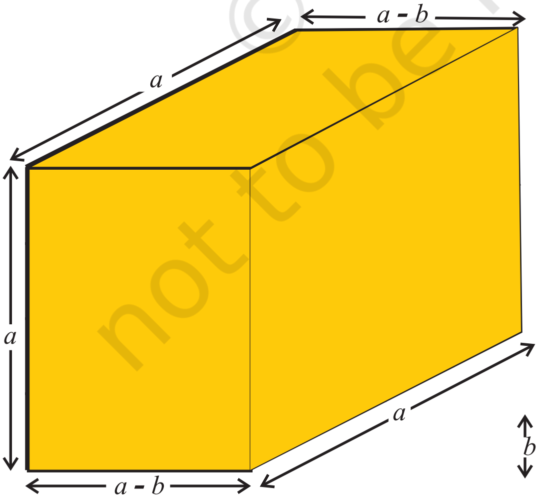


Fig. 1

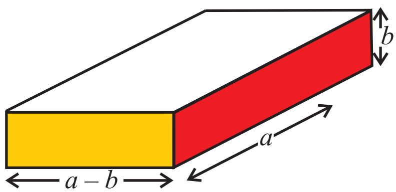


Fig. 2

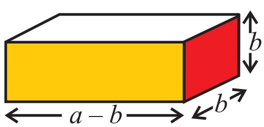


Fig. 3

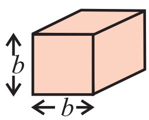


Fig. 4

5. Arrange the cubes and cuboids made above in Steps (1), (2), (3) and (4) to obtain a solid as shown in Fig. 5, which is a cube of volume a^3 cubic units.

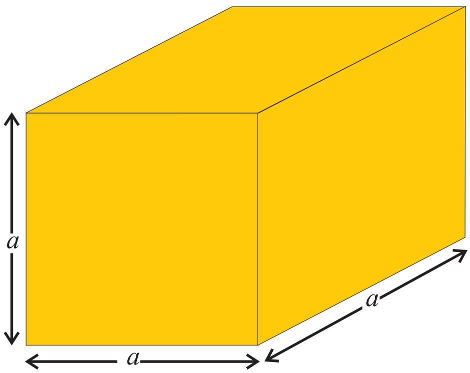


Fig. 5

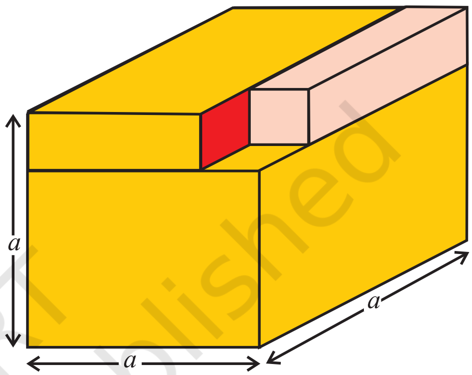


Fig. 6

DEMONSTRATION

Volume of cuboid in Fig. 1 = $(a–b) \times a \times a$ cubic units.

Volume of cuboid in Fig. 2 = $(a–b) \times a \times b$ cubic units.

Volume of cuboid in Fig. 3 = $(a–b) \times b \times b$ cubic units.

Volume of cube in Fig. 4 = b^3 cubic units.

Volume of solid in Fig. 5 = a^3 cubic units.

Removing a cube of size b^3 cubic units from the solid in Fig. 5, we obtain a solid as shown in Fig. 6.

Volume of solid in Fig. 6 = $(a–b)\ a^2 + (a–b)\ ab + (a–b)\ b^2$

$$= (a–b)\ (a^2 + ab + b^2)$$

Therefore, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Observation

On actual measurement:

a = ..., b = ...,

So, a^3 = ..., b^3 = ..., $(a–b)$ = ..., ab = ...,

a^2 = ..., b^2 = ...,

Therefore, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

Application

The identity may be used in simplification/factorisation of algebraic expressions.