

Chapter - 9
Circles

Exercise No. 9.1

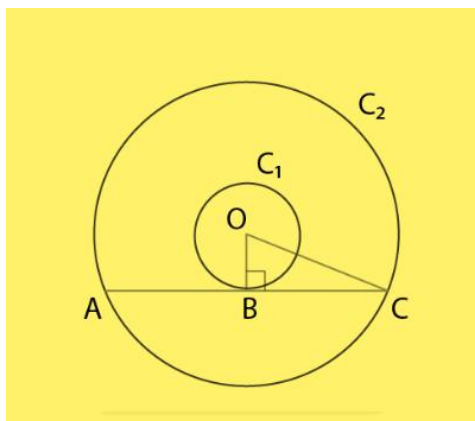
Multiple Choice Questions:

Choose the correct answer from the given four options:

1. If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is

- (A) 3 cm
- (B) 6 cm
- (C) 9 cm
- (D) 1 cm

Solution:



As given in the question,

$OA = 4\text{cm}$,

$OB = 5\text{cm}$

And,

$OA \perp BC$

Therefore,

$$OB^2 = OA^2 + AB^2$$

$$5^2 = 4^2 + AB^2$$

$$AB = 3\text{cm}$$

And,

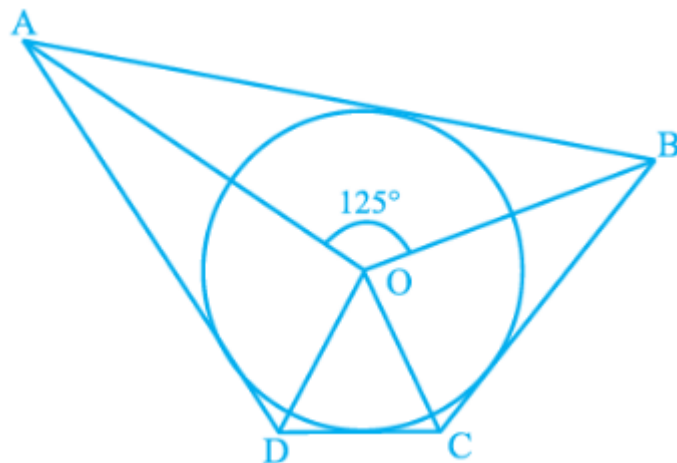
$$BC = 2AB$$

$$= 2 \times 3\text{cm}$$

$$= 6\text{cm}$$

2. In Fig., if $\angle AOB = 125^\circ$, then $\angle COD$ is equal to

- (A) 62.5°
- (B) 45°
- (C) 35°
- (D) 55°



Solution:

ABCD is a quadrilateral circumscribing the circle

And we know that the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the center of the circle.

So,

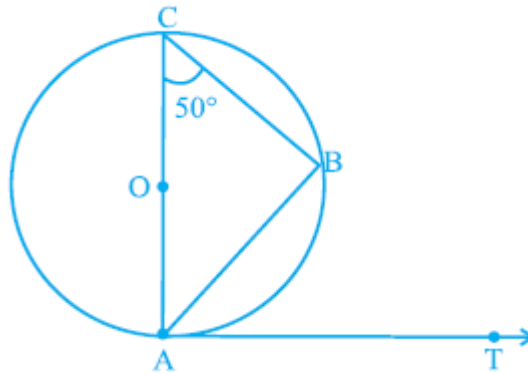
$$\angle AOB + \angle COD = 180^\circ$$

$$125^\circ + \angle COD = 180^\circ$$

$$\angle COD = 55^\circ$$

3. In Fig., AB is a chord of the circle and AOC is its diameter such that $\angle ACB = 50^\circ$. If AT is the tangent to the circle at the point A, then $\angle BAT$ is equal to

- (A) 65°
- (B) 60°
- (C) 50°
- (D) 40°



Solution:

As given in the question,

A circle with centre O, diameter AC and $\angle ACB = 50^\circ$

AT is a tangent to the circle at point A

As, angle in a semicircle is a right angle
 $\angle CBA = 90^\circ$

So using angle sum property of a triangle,

$$\begin{aligned}\angle ACB + \angle CAB + \angle CBA &= 180^\circ \\ 50^\circ + \angle CAB + 90^\circ &= 180^\circ \\ \angle CAB &= 40^\circ\end{aligned}\quad \dots (1)$$

As, tangent to at any point on the circle is perpendicular to the radius through point of contact,

We get,

$$OA \perp AT$$

$$\angle OAT = 90^\circ$$

$$\angle OAT + \angle BAT = 90^\circ$$

$$\angle CAT + \angle BAT = 90^\circ$$

$$40^\circ + \angle BAT = 90^\circ \quad \text{[from equation (1)]}$$

$$\angle BAT = 50^\circ$$

4. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is

(A) 60 cm^2

(B) 65 cm^2

- (C) 30 cm^2
(D) 32.5 cm^2

Solution:

Construction: Draw a circle of radius 5 cm with center O.

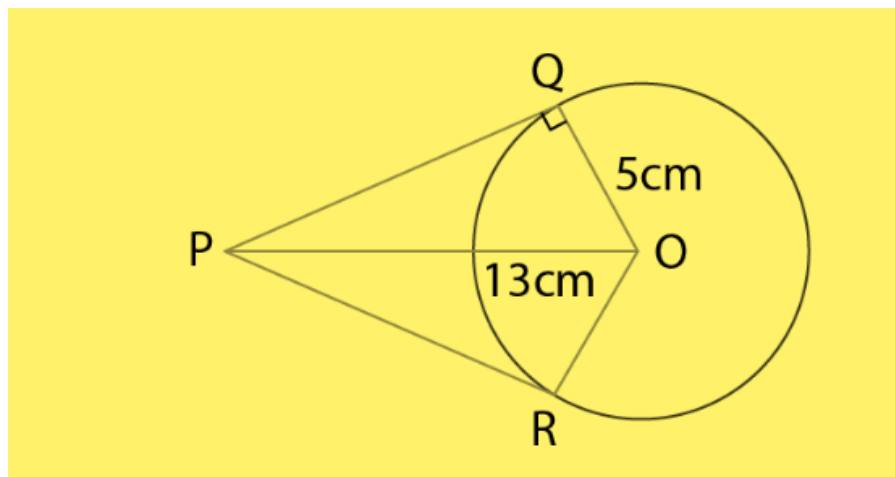
Let P be a point at a distance of 13 cm from O.

Draw a pair of tangents, PQ and PR.

$OQ = OR = \text{radius} = 5 \text{ cm}$

...equation (1)

And $OP = 13 \text{ cm}$



Also, tangent to at any point on the circle is perpendicular to the radius through point of contact,

We get,

$OQ \perp PQ$ and $OR \perp PR$

$\triangle POQ$ and $\triangle POR$ are right-angled triangles.

By using Pythagoras Theorem in $\triangle PQO$,

$$(\text{Base})^2 + (\text{Perpendicular})^2 = (\text{Hypotenuse})^2$$

$$(PQ)^2 + (OQ)^2 = (OP)^2$$

$$(PQ)^2 + (5)^2 = (13)^2$$

$$(PQ)^2 + 25 = 169$$

$$(PQ)^2 = 144$$

$$PQ = 12 \text{ cm}$$

Tangents through an external point to a circle are equal.

So,

$$PQ = PR = 12 \text{ cm} \quad \dots (2)$$

Therefore,

Area of quadrilateral PQRS, $A = \text{area of } \triangle POQ + \text{area of } \triangle POR$

Area of right angled triangle = $\frac{1}{2} \times \text{base} \times \text{height}$

$$A = (\frac{1}{2} \times OQ \times PQ) + (\frac{1}{2} \times OR \times PR)$$

$$A = (\frac{1}{2} \times 5 \times 12) + (\frac{1}{2} \times 5 \times 12)$$

$$A = 30 + 30$$

$$= 60 \text{ cm}^2$$

5. At one end A of a diameter AB of a circle of radius 5 cm, tangent XAY is drawn to the circle. The length of the chord CD parallel to XY and at a distance 8 cm from A is

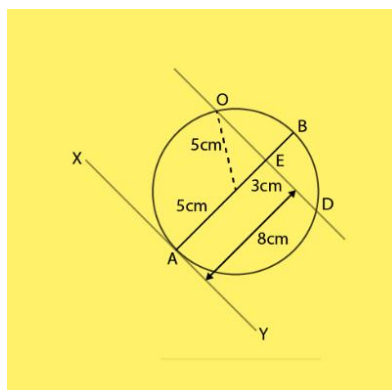
(A) 4 cm

(B) 5 cm

(C) 6 cm

(D) 8 cm

Solution:



As given the question,

Radius of circle,

$$AO = OC = 5 \text{ cm}$$

$$AM = 8 \text{ cm}$$

$$AM = OM + AO$$

$$OM = AM - AO$$

Putting these values in the equation,

$$OM = (8 - 5) \\ = 3 \text{ cm}$$

OM is perpendicular to the chord CD.

In $\triangle OCM$ $\angle OMC = 90^\circ$

By Pythagoras theorem,

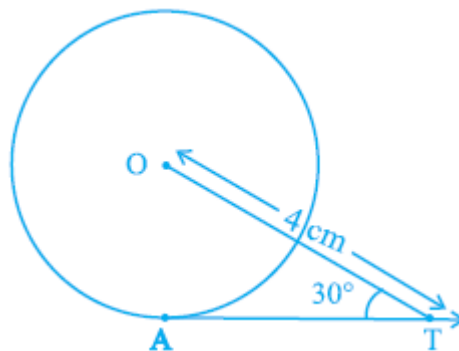
$$OC^2 = OM^2 + MC^2$$

Therefore,

$$CD = 2 \times CM \\ = 8 \text{ cm}$$

6. In Fig., AT is a tangent to the circle with centre O such that OT = 4 cm and $\angle \angle OTA = 30^\circ$. Then AT is equal to

- (A) 4 cm
- (B) 2 cm
- (C) $2\sqrt{3}$ cm
- (D) $4\sqrt{3}$ cm

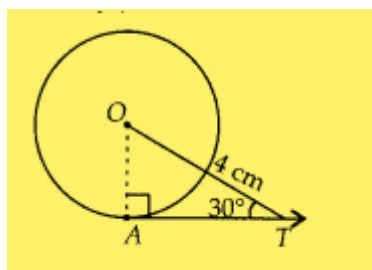


Solution:

(C)

Join OA

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$$\angle OAT = 90^\circ$$

In $\triangle OAT$,

$$\cos 30^\circ = \frac{OA}{OT}$$

$$\frac{\sqrt{3}}{2} = \frac{OA}{4}$$

$$OA = 2\sqrt{3} \text{ cm}$$

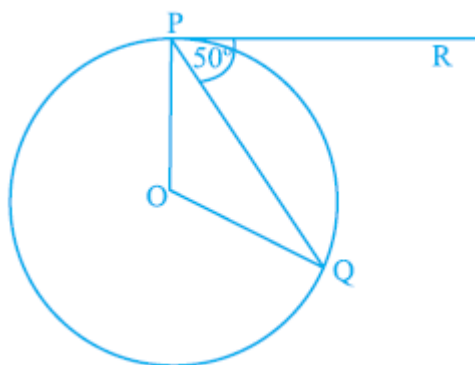
7. In Fig., if O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to

(A) 100°

(B) 80°

(C) 90°

(D) 75°



Solution:

(A)

Given,

$$\angle QPR = 50^\circ$$

We know that, the tangent at any point of a circle is perpendicular to the radius through the point of contact.

$$\angle OPR = 90^\circ$$

$$\angle OPQ + \angle QPR = 90^\circ$$

$$\angle OPQ = 90^\circ - 50^\circ$$

$$= 40^\circ$$

$$[\text{as, } \angle QPR = 50^\circ]$$

Now,

$OP = OQ = \text{radius of circle}$

$$\begin{aligned}\angle OQP &= \angle OPQ \\ &= 40^\circ\end{aligned}$$

[Angles opposite to equal sides are equal]

In $\triangle OPQ$,

$$\angle O + \angle OPQ + \angle Q = 180^\circ$$

[Angle sum property]

$$\begin{aligned}\angle POQ &= 180^\circ - (40 + 40^\circ) \\ &= 180^\circ - 80^\circ\end{aligned}$$

$$[\angle OPQ = 40^\circ = \angle Q]$$

$$\angle POQ = 100^\circ$$

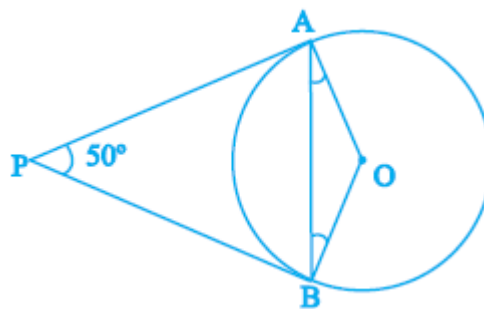
8. In Fig., if PA and PB are tangents to the circle with centre O such that $\angle APB = 50^\circ$, then $\angle OAB$ is equal to

(A) 25°

(B) 30°

(C) 40°

(D) 50°



Solution:

(A)

Given,

PA and PB are tangent lines.

$PA = PB$ [as, Length of tangents drawn from an external point to a circle is equal]

Let,

$$\angle PBA = \angle PAB = \theta$$

In $\triangle PAB$,

$$\angle P + \angle A + \angle B = 180^\circ$$

[Angle sum property]

$$50^\circ + \theta + \theta = 180^\circ$$

$$2\theta = 180^\circ - 50^\circ = 130^\circ$$

$$\theta = 65^\circ$$

Also,

$OA \perp PA$

[as, tangent at any point of a circle is perpendicular to the radius through the point of contact]

So,

$$\angle PAO = 90^\circ$$

$$\angle PAB + \angle BAO = 90^\circ$$

$$65^\circ + \angle BAO = 90^\circ$$

$$\angle BAO = 90^\circ - 65^\circ$$

$$= 25^\circ$$

$$\angle OAB = 25^\circ$$

9. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to

(A) $\frac{3}{2}\sqrt{3}$ cm

(B) 6 cm

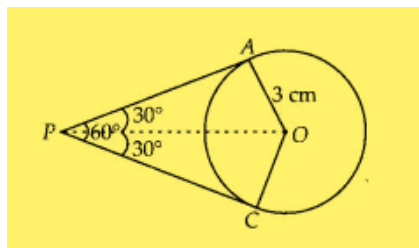
(C) 3 cm

(D) $3\sqrt{3}$ cm

Solution:

(D)

Let P be an external point and a pair of tangents is drawn from point P such that the angle between two tangents is 60° .



Join OA and OP.

Also,

OP is a bisector line of $\angle APC$.

$$\angle APO = \angle CPO = 30^\circ$$

And,

$$OA \perp AP$$

[Tangent at any point of a circle is perpendicular to the radius through the point of contact.]

$$\angle OAP = 90^\circ$$

In right angled $\triangle OAP$,

$$\tan 30^\circ = \frac{OA}{AP}$$

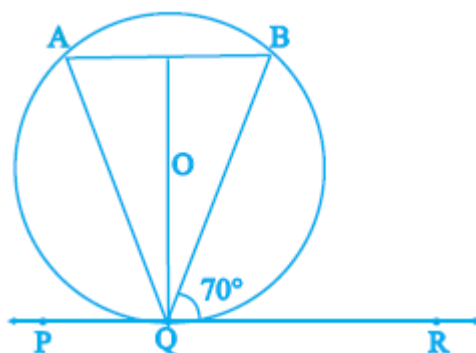
$$\frac{1}{\sqrt{3}} = \frac{3}{AP}$$

$$AP = 3\sqrt{3} \text{ cm}$$

So, the length of each tangent is $3\sqrt{3}$ cm.

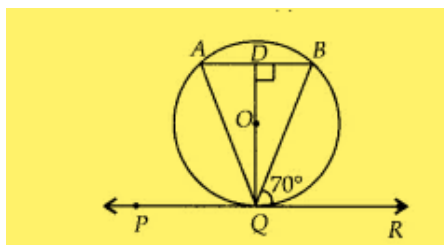
10. In Fig., if PQR is the tangent to a circle at Q whose centre is O, AB is a chord parallel to PR and $\angle BQR = 70^\circ$, then $\angle AQB$ is equal to

- (A) 20°
- (B) 40°
- (C) 35°
- (D) 45°



Solution:

- (B) Given,
 $AB \parallel PR$



Therefore,
 $\angle ABQ = \angle BQR = 70^\circ$ [Alternate angles]
Also,
QD is perpendicular to AB and QD bisects AB.

In $\triangle QDA$ and $\triangle QDB$,
 $\angle QDA = \angle QDB$ [90° each]
 $AD = BD$ [Common side]
 $QD = QD$
So,
 $\triangle ADQ \cong \triangle BDQ$ [by SAS congruency]

Therefore,
 $\angle QAD = \angle QBD$ [CPCT] ... (i)
But,

$$\angle QBD = \angle ABQ = 70^\circ$$

$$\angle QAD = 70^\circ$$

[From (i)]

Now, in $\triangle ABQ$,

$$\angle A + \angle B + \angle AQB = 180^\circ.$$

[Angle sum property]

$$\begin{aligned}\angle AQB &= 180^\circ - (70^\circ + 70^\circ) \\ &= 40^\circ\end{aligned}$$

Exercise No. 9.2

Short Answer Questions with Reasoning:

Write 'True' or 'False' and justify your answer in each of the following:

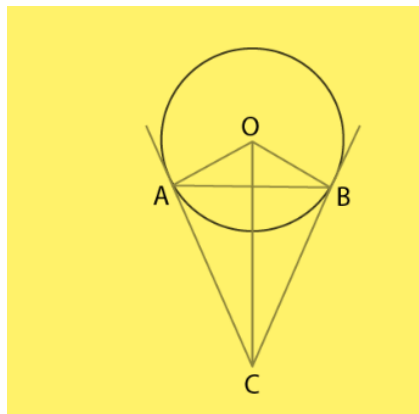
1. If a chord AB subtends an angle of 60° at the centre of a circle, then angle between the tangents at A and B is also 60° .

Solution:

False

Explanation:

Let us consider the given figure. In which we have a circle with centre O and AB a chord with $\angle AOB = 60^\circ$



As, tangent to any point on the circle is perpendicular to the radius through point of contact,

We get,

$$OA \perp AC \text{ and } OB \perp CB$$

$$\angle OBC = \angle OAC = 90^\circ$$

... eq(i)

Using angle sum property of quadrilateral in Quadrilateral AOBC,

We get,

$$\begin{aligned}\angle OBC + \angle OAC + \angle AOB + \angle ACB &= 360^\circ \\ 90^\circ + 90^\circ + 60^\circ + \angle ACB &= 360^\circ \\ \angle ACB &= 120^\circ\end{aligned}$$

Therefore, the angle between two tangents is 120° .

And, we can conclude that, the given statement is false.

2. The length of tangent from an external point on a circle is always greater than the radius of the circle.

Solution:

False

Explanation:

Length of tangent from an external point P on a circle may or may not be greater than the radius of the circle.

3. The length of tangent from an external point P on a circle with centre O is always less than OP.

Solution:

True

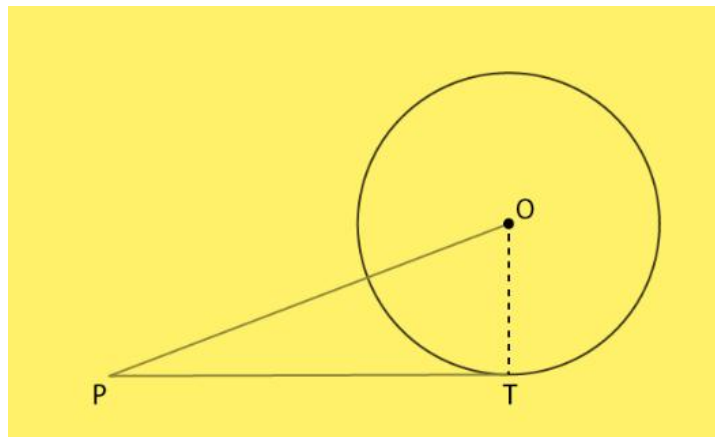
Explanation:

Consider the figure of a circle with centre O.

Let PT be a tangent drawn from external point P.

Now, Join OT.

$OT \perp PT$



We know that,

Tangent at any point on the circle is perpendicular to the radius through point of contact

Therefore, OPT is a right-angled triangle formed.

We also know that,

In a right angled triangle, hypotenuse is always greater than any of the two sides of the triangle.

So,

$OP > PT$ or $PT < OP$

Hence, length of tangent from an external point P on a circle with center O is always less than OP.

4. The angle between two tangents to a circle may be 0° .

Solution:

True

Explanation:

The angle between two tangents to a circle may be 0° only when both tangent lines coincide or are parallel to each other.

5. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 90° , then $OP = a\sqrt{2}$.

Solution:

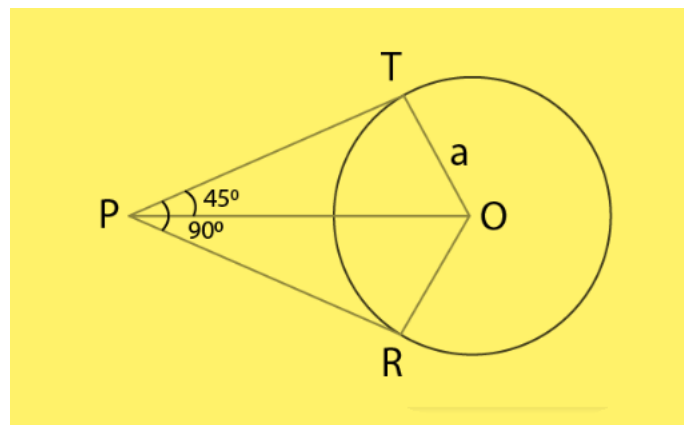
True.

Tangent is always perpendicular to the radius at the point of contact.

So, $\angle OTP = 90^\circ$

If 2 tangents are drawn from an external point, then they are equally inclined to the line segment joining the centre to that point.

Let us consider the following figure,



From point P, two tangents are drawn.

Given,

$OT = a$

Also, line OP bisects the $\angle RPT$

$$\angle TPO = \angle RPO = 45^\circ$$

Also,

$$OT \perp TP$$

$$\angle OTP = 90^\circ$$

In right angled $\triangle OTP$,

$$\sin 45^\circ = \frac{OT}{OP}$$

$$\frac{1}{\sqrt{2}} = \frac{a}{OP}$$

$$OP = a\sqrt{2}$$

6. If angle between two tangents drawn from a point P to a circle of radius a and centre O is 60° , then $OP = a\sqrt{3}$.

Solution:

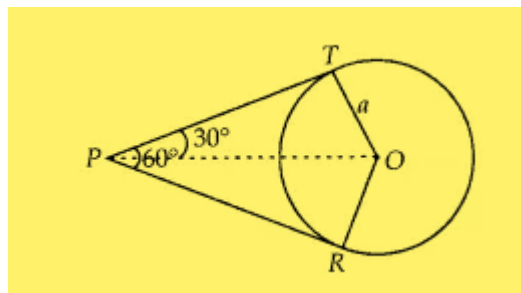
False

Explanation:

From point P, two tangents are drawn.

Given,

$$OT = a$$



Also, line OP bisects the $\angle RPT$.

$$\angle TPO = \angle RPO = 30^\circ$$

Also,

$$OT \perp PT$$

$$\angle OTP = 90^\circ$$

In right angled $\triangle OTP$,

$$\sin 30^\circ = \frac{OT}{OP}$$

$$\frac{1}{2} = \frac{a}{OP}$$

$$OP = 2a$$

$$OP = 2a$$

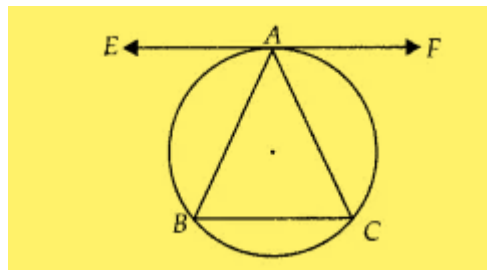
7. The tangent to the circumcircle of an isosceles triangle ABC at A, in which $AB = AC$, is parallel to BC.

Solution:

True

Explanation:

Let EAF be tangent to the circumcircle of $\triangle ABC$.



To prove: $EAF \parallel BC$

We have, $\angle EAB = \angle ACB$

...(i)

[Angle between tangent and chord is equal to angle made by chord in the alternate segment]

Here, $AB = AC$

$\angle ABC = \angle ACB$

...(ii)

From equation (i) and (ii), we get

$\angle EAB = \angle ABC$

Alternate angles are equal.

$EAF \parallel BC$

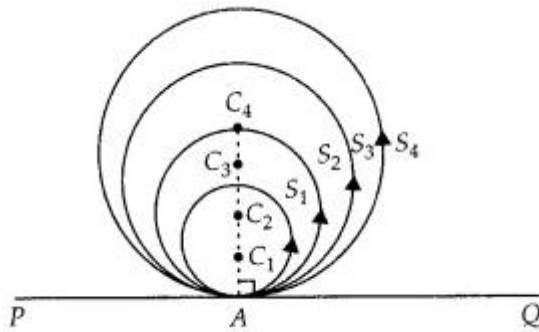
8. If a number of circles touch a given line segment PQ at a point A, then their centres lie on the perpendicular bisector of PQ.

Solution:

False

Explanation:

Given that PQ is any line segment and $S_1, S_2, S_3, S_4, \dots$ circles touch the line segment PQ at a point A. Let the centres of the circles $S_1, S_2, S_3, S_4, \dots$ be $C_1, C_2, C_3, C_4, \dots$ respectively.



To prove: Centres of the circles lie on the perpendicular bisector of PQ.
 Joining each centre of the circles to the point A on the line segment PQ by line segment i.e., C_1A , C_2A , C_3A , C_4A , ... and so on.

We know that, if we draw a line from the centre of a circle to its tangent line, then the line is always perpendicular to the tangent line. But it does not bisect the line segment PQ.

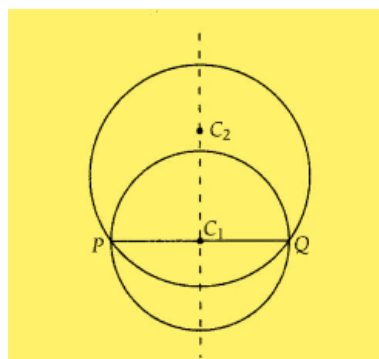
9. If a number of circles pass through the end points P and Q of a line segment PQ, then their centres lie on the perpendicular bisector of PQ.

Solution:

True

Explanation:

We draw two circles with centre C_1 and C_2 passing through the end points P and Q of a line segment PQ. We know that the perpendicular bisector of a chord of circle always passes through the centre of the circle.



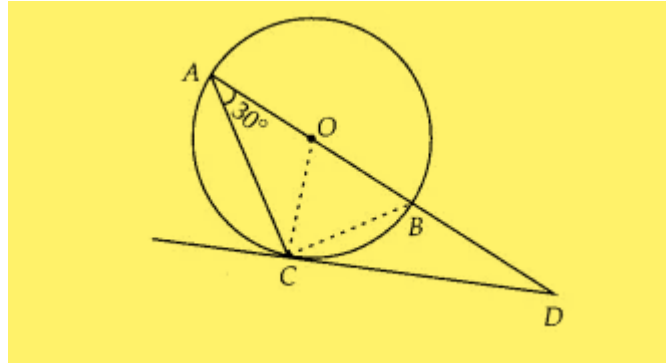
Thus, perpendicular bisector of PQ passes through C_1 and C_2 . Similarly, all the circle passing through the end points of line segment PQ, will have their centres on the perpendicular bisector of PQ.

10. AB is a diameter of a circle and AC is its chord such that $\angle BAC = 30^\circ$. If the tangent at C intersects AB extended at D, then $BC = BD$.

Solution:

True

To prove: $BC = BD$



Join BC and OC.

Given,

$$\angle BAC = 30^\circ$$

$$\angle BCD = 30^\circ$$

[Angle between tangent and chord is equal to angle made by chord in the alternate segment]

$OC \perp CD$ and

$OA = OC = \text{radius}$

$$\angle OAC = \angle OCA = 30^\circ$$

So,

$$\angle ACD = \angle ACO + \angle OCD$$

$$= 30^\circ + 90^\circ$$

$$= 120^\circ$$

Now,

In $\triangle ACD$,

$$\angle DAC + \angle ACD + \angle CDA = 180^\circ \text{ [Angle sum property]}$$

$$30^\circ + 120^\circ + \angle CDA = 180^\circ$$

$$\begin{aligned} \angle CDA &= 180^\circ - (30^\circ + 120^\circ) \\ &= 30^\circ \end{aligned}$$

$$\angle CDA = \angle BCD$$

$$BC = BD$$

[as, Sides opposite to equal angles are equal]

Exercise No. 9.3

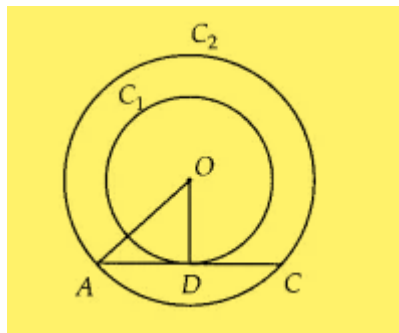
Short Answer Questions:

Question:

1. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circle.

Solution:

Let C_1 and C_2 be the two circles having same centre O. AC is a chord which touches C_1 at point D.



Join OD.

Also, $OD \perp AC$

$AD = DC = 4$ cm

[Perpendicular line OD bisects the chord]

In right angled $\triangle AOD$,

$OA^2 = AD^2 + OD^2$ [By Pythagoras theorem]

$OD^2 = 5^2 - 4^2$

$OD^2 = 25 - 16$

$= 9$

$OD = 3$ cm

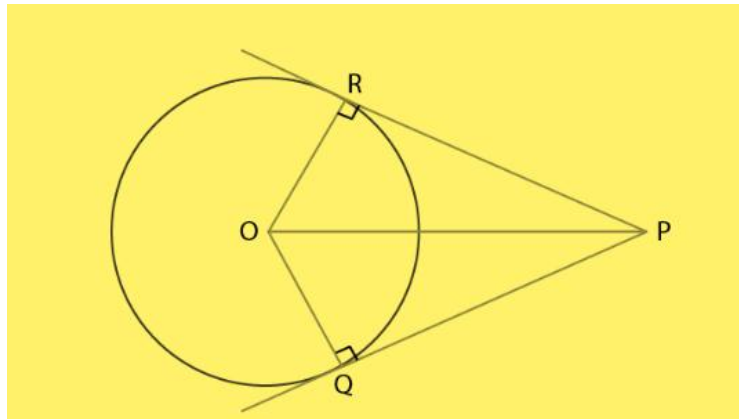
Radius of the inner circle is $OD = 3$ cm

2. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral.

Solution:

We know that,

Radius $\perp$ Tangent = $OR \perp PR$



$$\angle ORP = 90^\circ$$

Similarly,

Radius $\perp$ Tangent = $OQ \perp PQ$

$$\angle OQP = 90^\circ$$

In quadrilateral ORPQ,

Sum of all interior angles = 360°

$$\angle ORP + \angle RPQ + \angle PQO + \angle QOR = 360^\circ$$

$$90^\circ + \angle RPQ + 90^\circ + \angle QOR = 360^\circ$$

So,

$$\angle O + \angle P = 180^\circ$$

PROQ is a cyclic quadrilateral.

3. If from an external point B of a circle with centre O, two tangents BC and BD are drawn such that $\angle DBC = 120^\circ$, prove that $BC + BD = BO$, i.e., $BO = 2BC$.

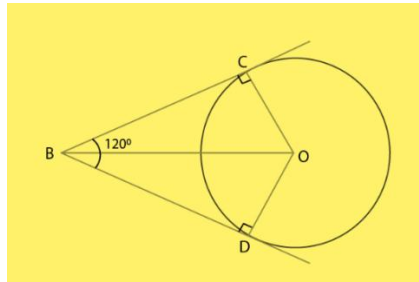
Solution:

As given in the question,

By RHS rule,

$\triangle OBC$ and $\triangle OBD$ are congruent

{By CPCT}



$\angle OBC$ and $\angle OBD$ are equal

Therefore,

$$\angle OBC = \angle OBD = 60^\circ$$

In triangle OBC,

$$\cos 60^\circ = \frac{BC}{OB}$$

$$\frac{1}{2} = \frac{BC}{OB}$$

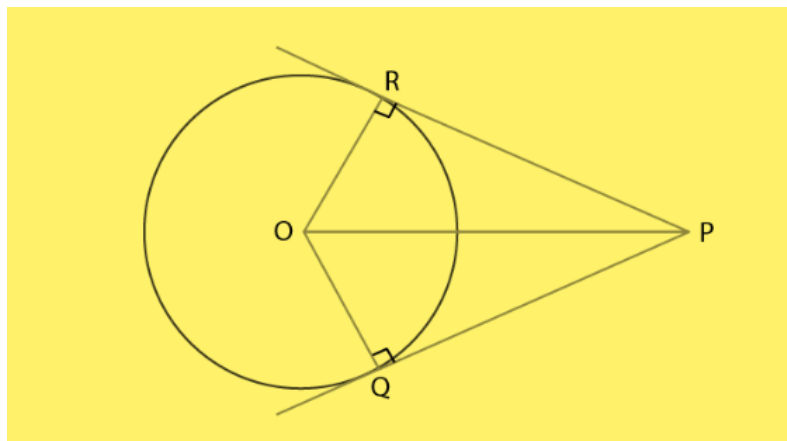
$$OB = 2BC$$

Hence proved.

4. Prove that the centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

Solution:

Let us take the lines be l_1 and l_2 .



Assume that O touches l_1 and l_2 at M and N,

So,

$$OM = ON$$

(Radius of the circle)

Therefore,

From the centre "O" of the circle, it has equal distance from l_1 & l_2 .

In $\triangle OPM$ & OPN ,

$$\begin{aligned} OM &= ON \\ \angle OMP &= \angle ONP \\ OP &= OP \end{aligned}$$

(Radius of the circle)
(As, Radius is perpendicular to its tangent)
(Common sides)

Therefore,

$$\begin{aligned} \triangle OPM &= \triangle OPN \\ \text{By C.P.C.T,} \\ \angle MPO &= \angle NPO \end{aligned}$$

(SSS congruence rule)

So, l bisects $\angle MPN$.

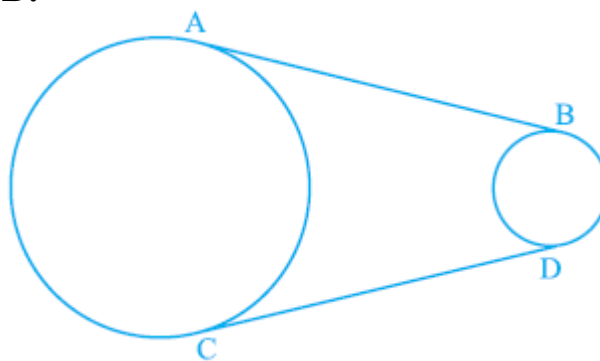
Hence, O lies on the bisector of the angle between l_1 & l_2 .

Also,

Centre of a circle touching two intersecting lines lies on the angle bisector of the lines.

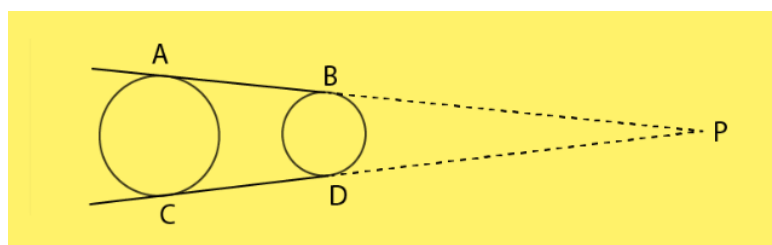
5. In Fig. 9.13, AB and CD are common tangents to two circles of unequal radii.

Prove that $AB = CD$.



Solution:

As given in the question,
 $AB = CD$



Construction: Produce AB and CD , to intersect at P .

Proof:

Consider the circle with greater radius.

Tangents drawn from an external point to a circle are equal

$$AP = CP \quad \dots(i)$$

Also,

Consider the circle with smaller radius.

Tangents drawn from an external point to a circle are equal

$$BP = BD \quad \dots(ii)$$

Subtract Equation (ii) from (i),

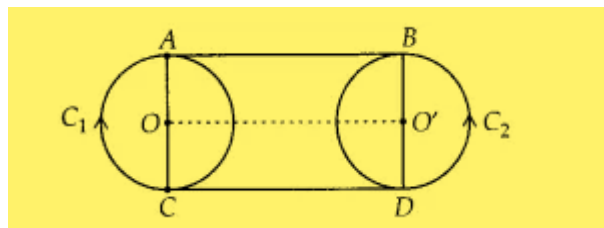
$$AP - BP = CP - BD$$

$$AB = CD$$

Hence Proved.

6. In Question 5 above, if radii of the two circles are equal, prove that $AB = CD$.

Solution:



Join OO'

Since, $OA = O'B$

[Given]

And,

$$\angle OAB = \angle O'BA = 90^\circ$$

[Tangent at any point of a circle is perpendicular to the radius at the point of contact]

Since, perpendicular distance between two straight lines at two different points is same.

AB is parallel to OO'

Also,

CD is parallel to OO'

$AB \parallel CD$

Now,

$$\angle OAB = \angle OCD = \angle O'BA = \angle O'DC = 90^\circ$$

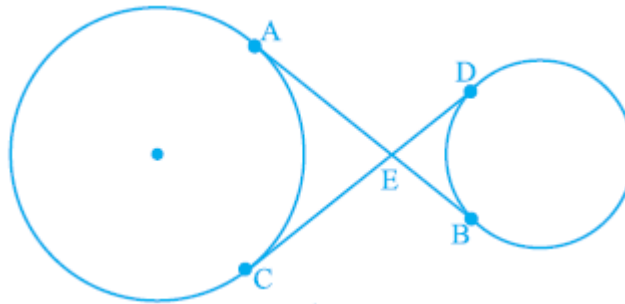
$ABCD$ is a rectangle.

Hence,

$AB = CD$.

7. In Fig., common tangents AB and CD to two circles intersect at E .

Prove that $AB = CD$.



Solution:

Given, common tangents AB and CD to two circles intersecting at E.

To prove:

$AB = CD$

We have,

The length of tangents drawn from an external point to a circle is equal

$EB = ED$ and

$EA = EC$

On adding, we get

$EA + EB = EC + ED$

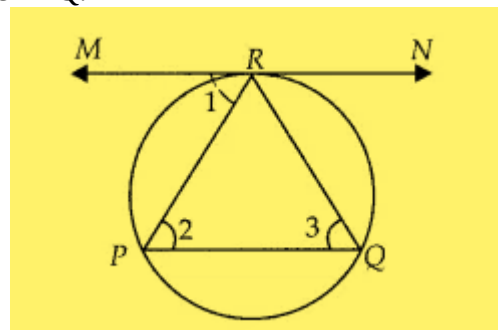
$AB = CD$

8. A chord PQ of a circle is parallel to the tangent drawn at a point R of the circle. Prove that R bisects the arc PRQ.

Solution:

Given, chord PQ is parallel to tangent at R.

To prove : R bisects the arc PRQ.



$\angle 1 = \angle 2$

[Alternate interior angles]

$\angle 1 = \angle 3$

[Angle between tangent and chord is equal to angle made by chord in alternate segment]

$\angle 2 = \angle 3$

$$PR = QR$$

[Sides opposite to equal angles are equal]

As,

$$\text{arc } PR = \text{arc } QR$$

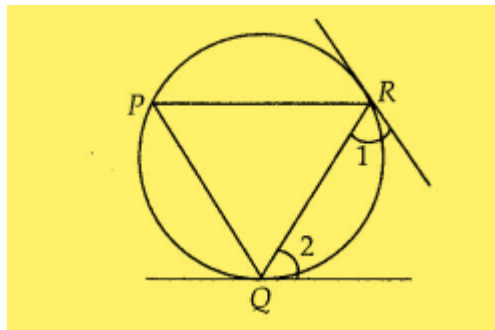
Therefore, R bisects PQ.

9. Prove that the tangents drawn at the ends of a chord of a circle make equal angles with the chord.

Solution:

To prove : $\angle 1 = \angle 2$

Let RQ be a chord of the circle. Tangents are drawn at the points R and Q.



Let P be another point on the circle, then join PQ and PR.

As, at point Q, there is a tangent.

$$\angle 2 = \angle P$$

[Angles in alternate segments are equal]

Also, at point R, there is a tangent.

$$\angle 1 = \angle P$$

[Angles in alternate segments are equal]

$$\angle 1 = \angle 2 = \angle P$$

$$\angle 1 = \angle 2$$

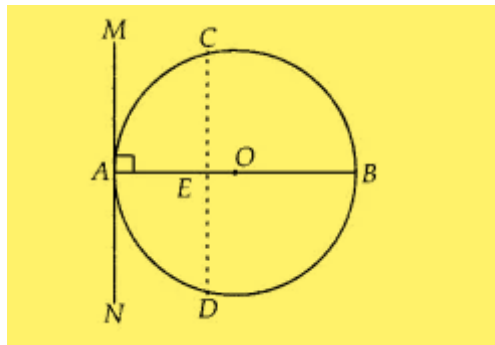
10. Prove that a diameter AB of a circle bisects all those chords which are parallel to the tangent at the point A.

Solution:

Given, AB is a diameter of the circle.

A tangent is drawn at point A.

Draw a chord CD parallel to tangent MAN.



Therefore, CD is a chord of the circle and OA is radius of the circle.

$$\angle MAO = 90^\circ$$

[tangent at any point of a circle is perpendicular to the radius through the point of contact]

$$\angle CEO = \angle MAO$$

[Corresponding angles]

$$\angle CEO = 90^\circ$$

So,

OE bisects CD,

[Perpendicular from centre of circle to chord bisects the chord]

Similarly,

Diameter AB bisects all chords which are parallel to the tangent at the point A.

Exercise No. 9.4

Long Answer Questions:

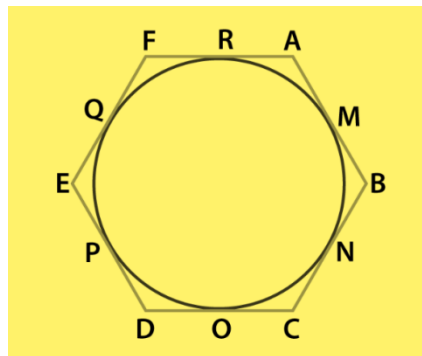
Question:

1. If a hexagon ABCDEF circumscribe a circle, prove that $AB + CD + EF = BC + DE + FA$.

Solution:

As given in the question,

A Hexagon ABCDEF circumscribe a circle.



To prove:

$$AB + CD + EF = BC + DE + FA$$

We know that,

Tangents drawn from an external point to a circle are equal.

So,

$$AM = RA$$

$$BM = BN$$

$$CO = NC$$

$$OD = DP$$

$$EQ = PE$$

$$QF = FR$$

... i [tangents from point A]

... ii [tangents from point B]

... iii [tangents from point C]

... iv [tangents from point D]

... v [tangents from point E]

... vi [tangents from point F]

Now, adding,

$$[eq\ i] + [eq\ ii] + [eq\ iii] + [eq\ iv] + [eq\ v] + [eq\ vi]$$

$$AM + BM + CO + OD + EQ + QF = RA + BN + NC + DP + PE + FR$$

On solving, we get,

$$(AM + BM) + (CO + OD) + (EQ + QF) = (BN + NC) + (DP + PE) + (FR + RA)$$

$$AB + CD + EF = BC + DE + FA$$

Hence Proved!

2. Let s denote the semi-perimeter of a triangle ABC in which $BC = a$, $CA = b$, $AB = c$. If a circle touches the sides BC, CA, AB at D, E, F, respectively, prove that $BD = s - b$.

Solution:

As given in the question,

A triangle ABC with $BC = a$, $CA = b$ and $AB = c$. Also, a circle is inscribed which touches the sides BC, CA and AB at D, E and F respectively and s is semi-perimeter of the triangle

To Prove: $BD = s - b$

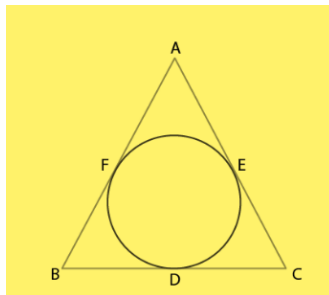
We have,

Semi Perimeter = s

Perimeter = $2s$

$$2s = AB + BC + AC$$

[i]



We know that,

Tangents drawn from an external point to a circle are equal

So,

$$AF = AE$$

$$BF = BD$$

$$CD = CE$$

[ii] [Tangents from point A]

[iii] [Tangents From point B]

[iv] [Tangents From point C]

Adding [ii] [iii] and [iv],

$$\begin{aligned}AF + BF + CD &= AE + BD + CE \\AB + CD &= AC + BD\end{aligned}$$

Adding BD both side,

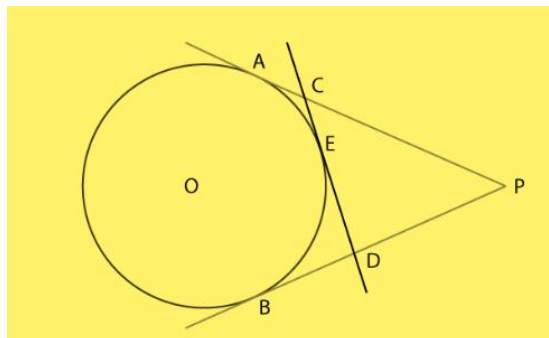
$$\begin{aligned}AB + CD + BD &= AC + BD + BD \\AB + BC - AC &= 2BD \\AB + BC + AC - AC - AC &= 2BD \\2s - 2AC &= 2BD \\2BD &= 2s - 2b \\BD &= s - b\end{aligned}$$

[From i]
[as AC = b]

Hence Proved.

3. From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At one point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of the triangle PCD.

Solution:



As given in the question,

From an external point P, two tangents, PA and PB are drawn to a circle with center O. At a point E on the circle tangent is drawn which intersects PA and PB at C and D, respectively. And PA = 10 cm

To Find : Perimeter of $\triangle PCD$

As we know that, Tangents drawn from an external point to a circle are equal.
So, we have,

$$\begin{aligned}AC &= CE && \text{[i] [Tangents from point C]} \\ED &= DB && \text{[ii] [Tangents from point D]}\end{aligned}$$

Now,

$$\begin{aligned}\text{Perimeter of Triangle PCD} &= PC + CD + DP \\&= PC + CE + ED + DP\end{aligned}$$

$$= PC + AC + DB + DP \quad [\text{From i and ii}]$$

$$= PA + PB$$

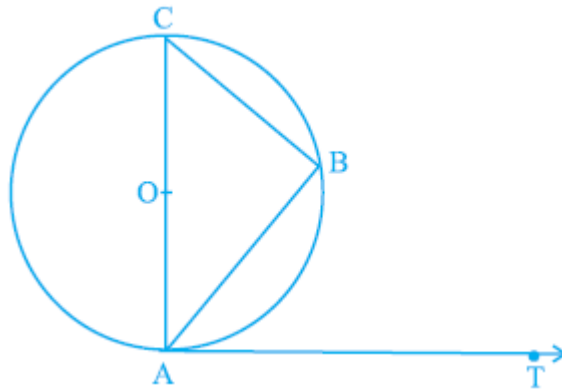
Now,

$PA = PB = 10$ cm as tangents drawn from an external point to a circle are equal

So ,

$$\begin{aligned} \text{Perimeter} &= PA + PB \\ &= 10 + 10 \\ &= 20 \text{ cm} \end{aligned}$$

4. If AB is a chord of a circle with centre O, AOC is a diameter and AT is the tangent at A as shown in Fig.. Prove that $\angle BAT = \angle ACB$.



Solution:

As given in the question,

A circle with center O and AC as a diameter and AB and BC as two chords also AT is a tangent at point A

To Prove : $\angle BAT = \angle ACB$

Proof :

$$\angle ABC = 90^\circ \quad [\text{Angle in a semicircle is a right angle}]$$

In $\triangle ABC$ By angle sum property of triangle

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$\angle ACB + 90^\circ = 180^\circ - \angle BAC$$

$$\angle ACB = 90 - \angle BAC$$

[i]

Now,

$$OA \perp AT$$

[Tangent at a point on the circle is perpendicular to the radius through point of contact]

$$\begin{aligned}\angle OAT &= \angle CAT = 90^\circ \\ \angle BAC + \angle BAT &= 90^\circ \\ \angle BAT &= 90^\circ - \angle BAC\end{aligned}$$

[ii]

From [i] and [ii],
 $\angle BAT = \angle ACB$

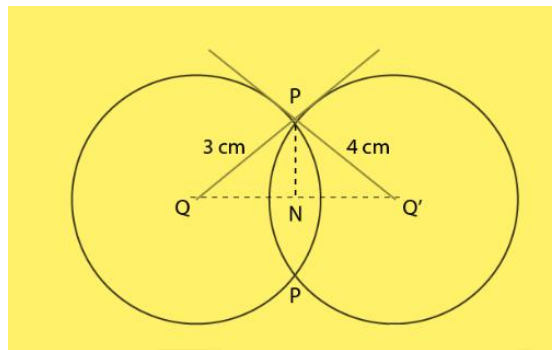
[Proved]

5. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ.

Solution:

We have,

Two circles with centers O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q, such that OP and O'P are tangents to the two circles and PQ is a common chord.



To Find: Length of common chord PQ

$$\angle OPO' = 90^\circ$$

[Tangent at a point on the circle is perpendicular to the radius through point of contact]

Therefore,

OPO is a right-angled triangle at P

By Pythagoras in $\Delta OPO'$, we have

$$\begin{aligned}(OO')^2 &= (O'P)^2 + (OP)^2 \\ (OO')^2 &= (4)^2 + (3)^2 \\ (OO')^2 &= 25 \\ OO' &= 5 \text{ cm}\end{aligned}$$

Let ON = x cm and

$$NO' = 5 - x \text{ cm}$$

In right angled triangle ONP

$$\begin{aligned}
(ON)^2 + (PN)^2 &= (OP)^2 \\
x^2 + (PN)^2 &= (3)^2 \\
(PN)^2 &= 9 - x^2
\end{aligned}
\tag{i}$$

In right angled triangle O'NP

$$\begin{aligned}
(O'N)^2 + (PN)^2 &= (O'P)^2 \\
(5 - x)^2 + (PN)^2 &= (4)^2 \\
25 - 10x + x^2 + (PN)^2 &= 16 \\
(PN)^2 &= -x^2 + 10x - 9
\end{aligned}
\tag{ii}$$

From [i] and [ii]

$$\begin{aligned}
9 - x^2 &= -x^2 + 10x - 9 \\
10x &= 18 \\
x &= 1.8
\end{aligned}$$

From (1) we have

$$\begin{aligned}
(PN)^2 &= 9 - (1.8)^2 \\
&= 9 - 3.24 \\
&= 5.76 \\
PN &= 2.4 \text{ cm}
\end{aligned}$$

$$\begin{aligned}
PQ &= 2PN \\
&= 2(2.4) \\
&= 4.8 \text{ cm}
\end{aligned}$$

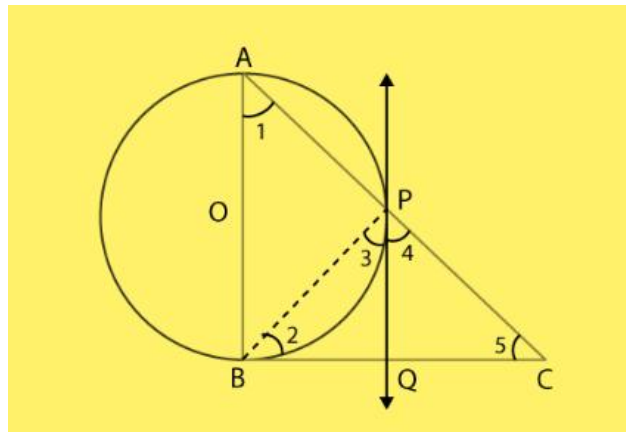
6. In a right triangle ABC in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Prove that the tangent to the circle at P bisects BC.

Solution:

As given in the question,

In a right angle $\triangle ABC$ in which $\angle B = 90^\circ$, a circle is drawn with AB as diameter intersecting the hypotenuse AC at P. Also PQ is a tangent at P

To Prove: PQ bisects BC or, BQ = QC



We have,

$$\angle APB = 90^\circ$$

[Angle in a semicircle is a right-angle]

$$\angle BPC = 90^\circ$$

[Linear Pair]

$$\angle 3 + \angle 4 = 90$$

[i]

Now,

$$\angle ABC = 90^\circ$$

In $\triangle ABC$,

$$\angle ABC + \angle BAC + \angle ACB = 180^\circ$$

$$90 + \angle 1 + \angle 5 = 180$$

$$\angle 1 + \angle 5 = 90$$

[ii]

Now,

$$\angle 1 = \angle 3$$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

Using this in [ii] we have,

$$\angle 3 + \angle 5 = 90$$

[iii]

From [i] and [iii] we have

$$\angle 3 + \angle 4 = \angle 3 + \angle 5$$

$$\angle 4 = \angle 5$$

$$QC = PQ$$

[Sides opposite to equal angles are equal]

Also

$$PQ = BQ$$

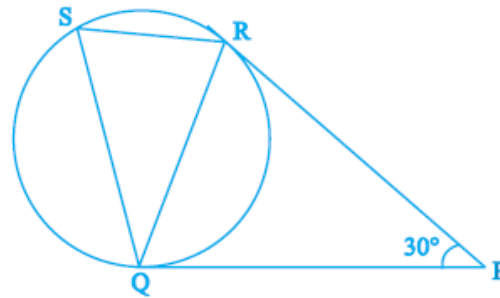
[Tangents drawn from an external point to a circle are equal]

So,

$$BQ = QC$$

Therefore, PQ bisects BC .

7. In Fig., tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ. Find the $\angle RQS$.
[Hint: Draw a line through Q and perpendicular to QP.]



Solution:

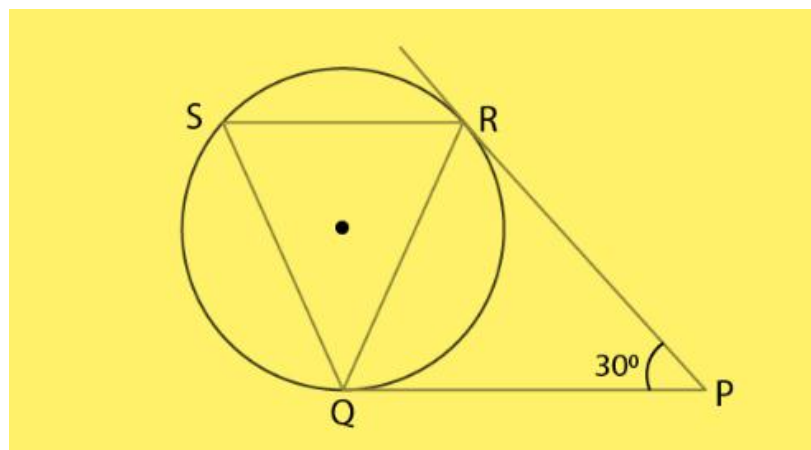
As given in the question,

Tangents PQ and PR are drawn to a circle such that $\angle RPQ = 30^\circ$. A chord RS is drawn parallel to the tangent PQ.

To Find : $\angle RQS$

$PQ = PR$ [Tangents drawn from an external point to a circle are equal]

$\angle PRQ = \angle PQR$ [Angles opposite to equal sides are equal] ..[i]



In $\triangle PQR$

$$\angle PRQ + \angle PQR + \angle QPR = 180^\circ$$

$$\angle PQR + \angle PQR + \angle QPR = 180^\circ \quad \text{[Using 1]}$$

$$2\angle PQR + \angle RPQ = 180^\circ$$

$$2\angle PQR + 30 = 180$$

$$2\angle PQR = 150$$

$$\angle PQR = 75^\circ$$

$$\angle QRS = \angle PQR = 75^\circ$$

[Alternate interior angles]

$$\angle QSR = \angle PQR = 75^\circ$$

[angle between tangent and the chord equals angle made by the chord in alternate segment]

Now

In $\triangle RQS$

$$\angle RQS + \angle QRS + \angle QSR = 180$$

$$\angle RQS + 75 + 75 = 180$$

$$\angle RQS = 30^\circ$$

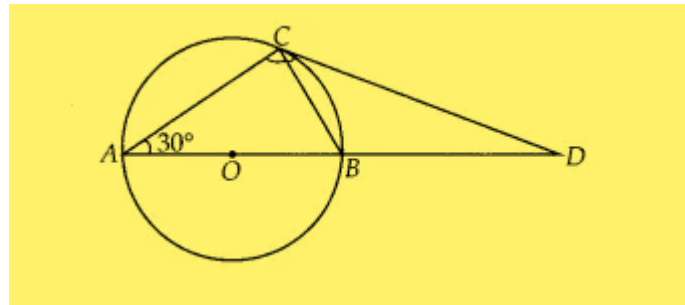
8. AB is a diameter and AC is a chord of a circle with centre O such that $\angle BAC = 30^\circ$. The tangent at C intersects extended AB at a point D. Prove that $BC = BD$.

Solution:

Given,

AB is a diameter and AC is a chord of circle with centre O, $\angle BAC = 30^\circ$

To prove : $BC = BD$



Construction: Join BC

$$\angle BCD = \angle CAB$$

[Angles in alternate segment]

$$\angle CAB = 30^\circ$$

[Given]

$$\angle BCD = 30^\circ$$

...(i)

$$\angle ACB = 90^\circ$$

[Angle in semi-circle]

In $\triangle ABC$,

$$\angle A + \angle B + \angle C = 180^\circ$$

[Angle sum property]

$$30^\circ + \angle CBA + 90^\circ = 180^\circ$$

$$\angle CBA = 60^\circ$$

Also,

$$\angle CBA + \angle CBD = 180^\circ$$

[Linear pair]

$$\angle CBD = 180^\circ - 60^\circ$$

$$= 120^\circ$$

$$[\text{as, } \angle CBA = 60^\circ]$$

Now,

In ACBD,

$$\angle CBD + \angle BDC + \angle DCB = 180^\circ$$

$$120^\circ + \angle BDC + 30^\circ = 180^\circ$$

$$\angle BDC = 30^\circ$$

...(ii)

From (i) and (ii),

$$\angle BCD = \angle BDC$$

$$BC = BD$$

[Sides opposite to equal angles are equal]

9. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

Solution:

Let us take the mid-point of an arc AMB be M and TMT' be the tangent to the circle.
Join AB, AM and MB.

Since,

$$\text{arc AM} = \text{arc MB}$$

$$\text{Chord AM} = \text{Chord MB}$$

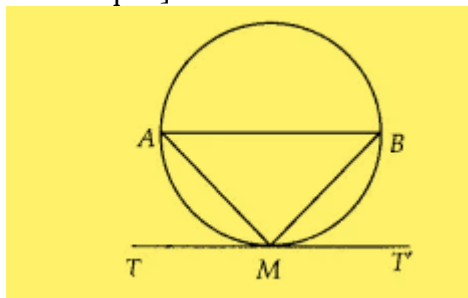
In $\triangle AMB$,

$$AM = MB$$

$$\angle MAB = \angle MBA$$

...(i)

[Sides opposite to equal angles are equal]



Since, TMT' is a tangent line.

Therefore,

$$\angle AMT = \angle MBA$$

[Angles in alternate segments are equal]

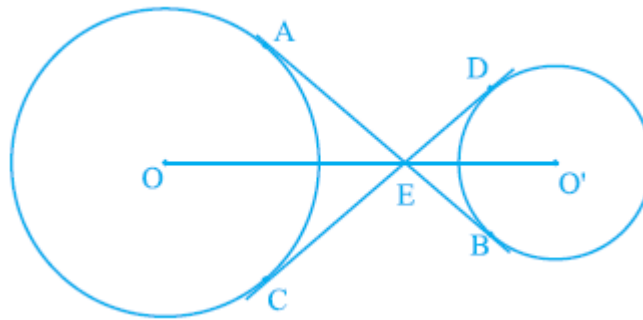
$$= \angle MAB$$

[from equation (i)]

But, $\angle AMT$ and $\angle MAB$ are alternate angles, which is possible only when AB is parallel to TMT'

Hence, the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

10. In Fig., the common tangent, AB and CD to two circles with centres O and O' intersect at E. Prove that the points O, E, O' are collinear.



Solution:

Join AO, OC and O'D, O'B.

Now,

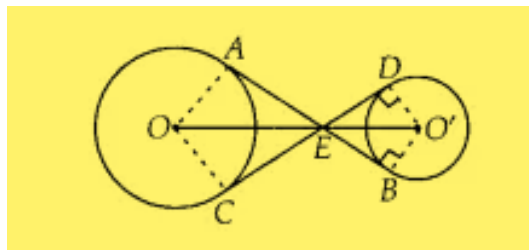
In $\triangle O'DE$ and $\triangle O'BE$,

$O'D = O'B$

$O'E = O'E$

$ED = EB$

[Tangents drawn from an external point to the circle are equal in length]



$\triangle O'DE \cong \triangle O'BE$

[By SSS congruence criterion]

$\angle O'ED = \angle O'EB$

...(i)

Therefore,

O'E is the angle bisector of $\angle DEB$.

Similarly,

OE is the angle bisector of $\angle AEC$.

Now, in quadrilateral DEBO'.

$\angle O'DE = \angle O'BE = 90^\circ$

[CED is a tangent to the circle and O'D is the radius, i.e., $O'D \perp CED$]

$$\begin{aligned}\angle O'DE + \angle O'BE &= 180^\circ \\ \angle DEB + \angle DO'B &= 180^\circ\end{aligned}$$

[as, DEBO' is cyclic quadrilateral] ...(ii)

Since,
AB is a straight line.

$$\begin{aligned}\angle AED + \angle DEB &= 180^\circ \\ \angle AED + 180^\circ - \angle DO'B &= 180^\circ & \text{[from (ii)]} \\ \angle AED &= \angle DO'B & \text{...(iii)}\end{aligned}$$

Similarly,
 $\angle AED = \angle AOC$...(iv)

Again from eq. (ii),
 $\angle DEB = 180^\circ - \angle DO'B$
Dividing by 2 on both sides, we get,

$$\begin{aligned}\frac{1}{2} \angle DEB &= 90^\circ - \frac{1}{2} \angle DO'B \\ \angle DEO' &= 90^\circ - \frac{1}{2} \angle DO'B & \text{...(v)}\end{aligned}$$

Similarly,

$$\angle AEC = 180^\circ - \angle AOC$$

Dividing 2 on both sides,

$$\begin{aligned}\frac{1}{2} \angle AEC &= 90^\circ - \frac{1}{2} \angle AOC \\ \angle AEO &= 90^\circ - \frac{1}{2} \angle AOC & \text{....(vi)}\end{aligned}$$

Now,

$$\begin{aligned}\angle AED + \angle AEO + \angle DEO' &= \angle AED + 90^\circ - \frac{1}{2} \angle DO'B + 90^\circ - \frac{1}{2} \angle AOC \\ &= \angle AED + 180^\circ - \frac{1}{2} (\angle DO'B + \angle AOC) \\ &= \angle AED + 180^\circ - \frac{1}{2} (\angle AED + \angle AED) & \text{(from iii and iv)} \\ &= \angle AED + 180^\circ - \angle AED \\ &= 180^\circ\end{aligned}$$

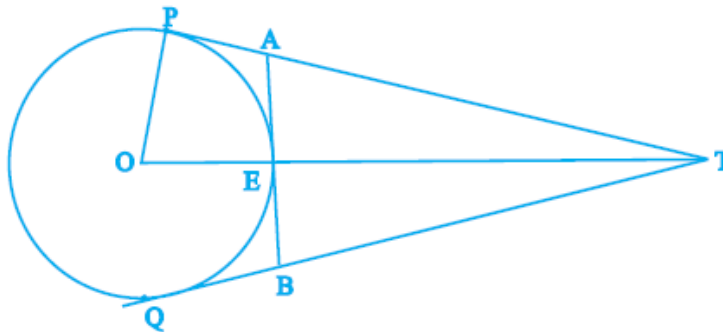
So,

$$\angle AED + \angle AEO + \angle DEO' = 180^\circ$$

So,
OEO' is straight line.

Hence, O, E and O' are collinear.

11. In Fig. 9.20. O is the centre of a circle of radius 5 cm, T is a point such that OT = 13 cm and OT intersects the circle at E. If AB is the tangent to the circle at E, find the length of AB.



Solution:

OP is perpendicular to PT.

In $\triangle OPT$,

$$OT^2 = OP^2 + PT^2$$

$$PT^2 = OT^2 - OP^2$$

$$PT^2 = (13)^2 - (5)^2$$

$$= 169 - 25$$

$$= 144$$

$$PT = 12 \text{ cm}$$

Since, the length of pair of tangents from an external point T is equal.

So,

$$QT = 12 \text{ cm}$$

Now,

$$TA = PT - PA$$

$$TA = 12 - PA$$

...(i)

and

$$TB = QT - QB$$

$$TB = 12 - QB$$

...(ii)

Also,

$$PA = AE \text{ and } QB = EB$$

...(iii) [Pair of tangents]

$$ET = OT - OE$$

[as, OE = 5 cm = radius]

$$ET = 13 - 5$$

$$ET = 8 \text{ cm}$$

Since, AB is a tangent and OE is the radius.

$OE \perp AB$,

$$\angle OEA = 90^\circ$$

$$\angle AET = 180^\circ - \angle OEA$$

[Linear pair]

$$\angle AET = 90^\circ$$

Now, in right angled $\triangle AET$,

$$(AT)^2 = (AE)^2 + (ET)^2$$

[by Pythagoras theorem]

$$(12 - PA)^2 = (PA)^2 + (8)^2$$

On solving,

$$144 + (PA)^2 - 24 PA = (PA)^2 + 64$$

$$24 PA = 80$$

$$PA = \frac{10}{3}$$

So,

$$AE = \frac{10}{3}$$

We join OQ,

Similarly,

$$BE = \frac{10}{3}$$

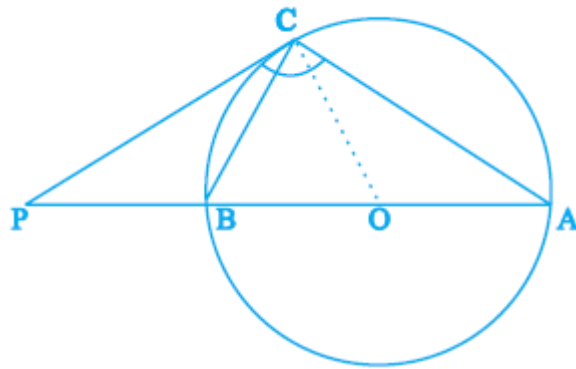
Also,

$$AB = AE + BE$$

$$= \frac{10}{3} + \frac{10}{3}$$

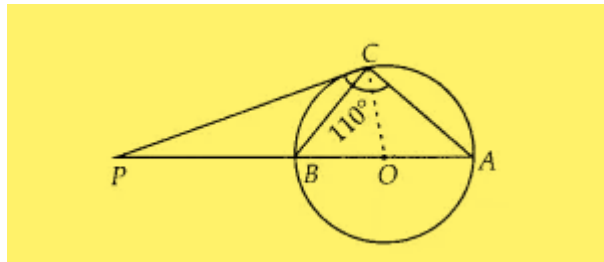
$$= \frac{20}{3}$$

**12. The tangent at a point C of a circle and a diameter AB when extended intersect at P. If $\angle PCA = 110^\circ$, find CBA.
[Hint: Join C with centre O.]**



Solution:

Join OC. In this, OC is the radius.



We know that, tangent at any point of a circle is – perpendicular to the radius through the point of contact.

Therefore,
 $OC \perp PC$

Now,

$$\begin{aligned}\angle PCA &= 110^\circ && \text{[Given]} \\ \angle PCO + \angle OCA &= 110^\circ \\ 90^\circ + \angle OCA &= 110^\circ \\ \angle OCA &= 20^\circ\end{aligned}$$

Also,

$OC = OA = \text{radius of circle}$
 $\angle OCA = \angle OAC = 20^\circ$

[Sides opposite to equal angles are equal]

Since, PC is a tangent,
 $\angle BCP = \angle CAB = 20^\circ$

[Angles in alternate segment]

In $\triangle PAC$,

$$\angle P + \angle C + \angle A = 180^\circ$$

So,

$$\angle P = 180^\circ - (\angle C + \angle A)$$

$$\begin{aligned}
 &= 180^\circ - (110^\circ + 20^\circ) \\
 &= 180^\circ - 130^\circ = 50^\circ
 \end{aligned}$$

In $\triangle PBC$,

$$\begin{aligned}
 \angle BPC + \angle PCB + \angle CBP &= 180^\circ \\
 50^\circ + 20^\circ + \angle PBC &= 180^\circ \\
 \angle PBC &= 180^\circ - 70^\circ \\
 \angle PBC &= 110^\circ
 \end{aligned}$$

Since, ABP is a straight line.

Therefore,

$$\begin{aligned}
 \angle PBC + \angle CBA &= 180^\circ \\
 \angle CBA &= 180^\circ - 110^\circ \\
 &= 70^\circ
 \end{aligned}$$

13. If an isosceles triangle ABC, in which $AB = AC = 6$ cm, is inscribed in a circle of radius 9 cm, find the area of the triangle.

Solution:

Join OB, OC and OA.

In $\triangle ABO$ and $\triangle ACO$,

$$AB = AC$$

[Given]

$$BO = CO$$

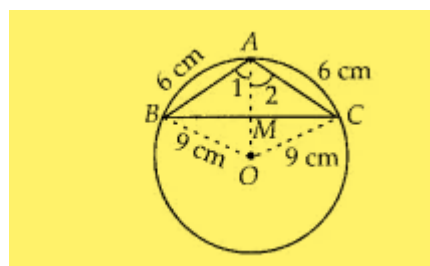
[Radii of same circle]

$$AO = AO$$

[Common side]

$$\triangle ABO \cong \triangle ACO$$

[By SSS congruence criterion]



$$\angle 1 = \angle 2$$

[CPCT]

Now,

In $\triangle ABM$ and $\triangle ACM$,

$$AB = AC$$

[Given]

$$\angle 1 = \angle 2$$

[proved above]

$$AM = AM$$

[Common side]

So,

$$\begin{array}{ll} \triangle AMB \cong \triangle AMC & [\text{By SAS congruence criterion}] \\ \angle AMB = \angle AMC & [\text{CPCT}] \end{array}$$

Also,

$$\begin{array}{ll} \angle AMB + \angle AMC = 180^\circ & [\text{Linear pair}] \\ \angle AMB = 90^\circ & \end{array}$$

We know that a perpendicular from the centre of circle bisects the chord.

So, OA is a perpendicular bisector of BC.

$$\text{Let } AM = x, \text{ then } OM = 9 - x \quad [\text{as, } OA = \text{radius} = 9 \text{ cm}]$$

$$\begin{array}{ll} \text{In right angled } \triangle AMC, & \\ AC^2 = AM^2 + MC^2 & [\text{By Pythagoras theorem}] \\ MC^2 = 6^2 - x^2 & \dots(i) \end{array}$$

$$\begin{array}{ll} \text{In right angle } \triangle OMC, & \\ OC^2 = OM^2 + MC^2 & [\text{By Pythagoras theorem}] \\ MC^2 = 9^2 - (9 - x)^2 & \end{array}$$

$$\begin{array}{l} \text{From equation (i) and (ii),} \\ 6^2 - x^2 = 9^2 - (9 - x)^2 \\ 36 - x^2 = 81 - (81 + x^2 - 18x) \\ 36 = 18x \\ x = 2 \end{array}$$

$$\begin{array}{l} \text{So,} \\ AM = 2 \text{ cm} \\ \text{From equation (ii),} \\ MC^2 = 9^2 - (9 - 2)^2 \\ MC^2 = 81 - 49 \\ = 32 \\ MC = 4\sqrt{2} \text{ cm} \end{array}$$

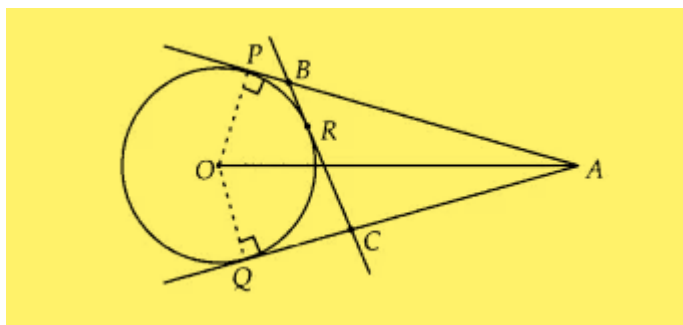
$$\begin{array}{l} \text{So,} \\ BC = 2 MC \\ = 8\sqrt{2} \text{ cm} \end{array}$$

$$\begin{array}{l} \text{Area of } \triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} \\ = \frac{1}{2} \times BC \times AM \\ = \frac{1}{2} \times 8\sqrt{2} \times 2 \\ = 8\sqrt{2} \text{ cm}^2 \end{array}$$

The required area of $\triangle ABC$ is $8\sqrt{2}$ cm².

14. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the perimeter of the $\triangle ABC$.

Solution:



We have,
 $\angle OPA = 90^\circ$

[Tangent at any point of a circle is perpendicular to the radius through the point of contact]

In $\triangle OAP$,
 $OA^2 = OP^2 + PA^2$
 $13^2 = 5^2 + PA^2$
 $PA = 12$ cm

Now,

Perimeter of $\triangle ABC = AB + BC + CA$
 $= AB + BR + RC + CA$
 $= (AB + BR) + (RC + CA)$
 $= (AB + BP) + (CQ + CA)$

[As, $BR = BP$, $RC = CQ$ i.e., tangents from external point to a circle are equal]

Perimeter of $\triangle ABC = AP + AQ$
 $= 2AP$ [as, $AP = AQ$]
 $= 2 \times 12$
 $= 24$ cm

Hence, the perimeter of $\triangle ABC = 24$ cm.