CHAPTER

1 Real Numbers

CASE STUDY / PASSAGE BASED QUESTIONS

Syllabus

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples. Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$. Decimal representation of rational numbers in terms of terminating/ non-terminating recurring decimals.

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Exponential Laws

Srikanth has made a project on real numbers, where he finely explained the applicability of exponential laws and divisibility conditions on real numbers. He also included some assessment questions at the end of his project as listed below. Answer them.

(i) For what value of n, 4^n ends in 0?

(a) 10 (b) when n is even

(c) when n is odd (d) no value of n

(ii) If a is a positive rational number and n is a positive integer greater than 1, then for what value of n, a^n is a rational number?

(a) when n is any even integer (b) when n is any odd integer

(c) for all $n > 1$ (d) only when $n = 0$

(iii) If x and y are two odd positive integers, then which of the following is true?

(a) $x^2 + y^2$ is even (b) $x^2 + y^2$ is not divisible by 4

(c) $x^2 + y^2$ is odd (d) both (a) and (b)

(iv) The statement 'One of every three consecutive positive integers is divisible by 3' is

(a) always true (b) always false

(c) sometimes true (d) None of these

(v) If n is any odd integer, then $n^2 - 1$ is divisible by

(a) 22 (b) 55 (c) 88 (d) 8

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Daily Life Situations

Real numbers are extremely useful in everyday life. That is probably one of the main reasons we all learn how to count and add and subtract from a very young age. Real numbers help us to count and to measure out quantities of different items in various fields like retail, buying, catering, publishing etc. Every normal person uses real numbers in his daily life. After knowing the importance of real numbers, try and improve your knowledge about them by answering the following questions on real life based situations.

(i) Three people go for a morning walk together from the same place. Their steps measure 80 cm, 85 cm and 90 cm respectively. What is the minimum distance travelled when they meet at first time after starting the walk assuming that their walking speed is same?

(a) 6120 cm (b) 12240 cm (c) 4080 cm (d) None of these

(ii) In a school Independence Day parade, a group of 594 students need to march behind a band of 189 members. The two groups have to march in the same number of columns. What is the maximum number of columns in which they can march?

(a) 9 (b) 6 (c) 27 (d) 29

(iii) Two tankers contain 768 litres and 420 litres of fuel respectively. Find the maximum capacity of the container which can measure the fuel of either tanker exactly.

(a) 4 litres (b) 7 litres (c) 12 litres (d) 18 litres

(iv) The dimensions of a room are 8 m 25 cm, 6 m 75 cm and 4 m 50 cm. Find the length of the largest measuring rod which can measure the dimensions of room exactly.

(a) 1 m 25 cm (b) 75 cm (c) 90 cm (d) 1 m 35 cm

(v) Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads.

(a) 3 and 2 (b) 2 and 5 (c) 3 and 4 (d) 4 and 5

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Activity on Real Numbers

In a classroom activity on real numbers, the students have to pick a number card from a pile and frame question on it if it is not a rational number for the rest of the class. The number cards picked up by first 5 students and their questions on the numbers for the rest of the class are as shown below. Answer them.

(i) Suraj picked up $\sqrt{8}$ and his question was - Which of the following is true about $\sqrt{8}$?

(a) It is a natural number (b) It is an irrational number
(c) It is a rational number (d) None of these

(ii) Shreya picked up 'BONUS' and her question was - Which of the following is not irrational?

(a) $3-4\sqrt{5}$ (b) $\sqrt{7}-6$ (c) $2+2\sqrt{9}$ (d) $4\sqrt{11}-6$

(iii) Ananya picked up $\sqrt{15}-\sqrt{10}$ and her question was - $\sqrt{15}-\sqrt{10}$ is _____ number.

(a) a natural (b) an irrational (c) a whole (d) a rational

(iv) Suman picked up $\frac{1}{\sqrt{5}}$ and her question was - $\frac{1}{\sqrt{5}}$ is _____ number.

(a) a whole (b) a rational (c) an irrational (d) a natural

(v) Preethi picked up $\sqrt{6}$ and her question was - Which of the following is not irrational?

(a) $15+3\sqrt{6}$ (b) $\sqrt{24}-9$ (c) $5\sqrt{150}$ (d) None of these

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Decimal Expansion

Decimal form of rational numbers can be classified into two types.

- Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form

$\frac{p}{q}$, where p and q are co-prime and the prime factorisation of q is of the form $2^n \cdot 5^m$, where n, m are non-negative integers and vice-versa.

- Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form $2^n 5^m$, where n and m are non-negative integers. Then x has a non-terminating repeating decimal expansion.

(i) Which of the following rational numbers have a terminating decimal expansion?

(a) 125/441 (b) 77/210 (c) 15/1600 (d) $129/(2^2 \times 5^2 \times 7^2)$

(ii) $23/(2^3 \times 5^2) =$

(a) 0.575 (b) 0.115 (c) 0.92 (d) 1.15

(iii) $441/(2^2 \times 5^7 \times 7^2)$ is a _____ decimal.

(a) terminating (b) recurring

(c) non-terminating and non-recurring (d) None of these

(iv) For which of the following value(s) of p, $251/(2^3 \times p^2)$ is a non-terminating recurring decimal?

(a) 3 (b) 7 (c) 15 (d) All of these

(v) $241/(2^5 \times 5^3)$ is a _____ decimal.

(a) terminating (b) recurring

(c) non-terminating and non-recurring (d) None of these

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Divisibility Rules

HCF and LCM are widely used in number system especially in real numbers in finding relationship between different numbers and their general forms. Also, product of two positive integers is equal to the product of their HCF and LCM.

Based on the above information answer the following questions.

(i) If two positive integers x and y are expressible in terms of primes as $x = p^2q^3$ and $y = p^3q$, then which of the following is true?

(a) $\text{HCF} = pq^2 \times \text{LCM}$ (b) $\text{LCM} = pq^2 \times \text{HCF}$

(c) $\text{LCM} = p^2q \times \text{HCF}$ (d) $\text{HCF} = p^2q \times \text{LCM}$

(ii) A boy with collection of marbles realizes that if he makes a group of 5 or 6 marbles, there are always two marbles left, then which of the following is correct if the number of marbles is p?

(a) p is odd (b) p is even (c) p is not prime (d) both (b) and (c)

(iii) Find the largest possible positive integer that will divide 398, 436 and 542 leaving remainder 7, 11, 15 respectively.

(a) 3 (b) 1 (c) 34 (d) 17

(iv) Find the least positive integer which on adding 1 is exactly divisible by 126 and 600.

(a) 12600 (b) 12599 (c) 12601 (d) 12500

(v) If A, B and C are three rational numbers such that $85C - 340A = 109$, $425A + 85B = 146$, then the sum of A, B and C is divisible by

(a) 3 (b) 6 (c) 7 (d) 9

1. **(i)** **(d)**: For a number to end in zero it must be divisible by 5, but $4^n = 2^{2n}$ is never divisible by 5.
So, 4^n never ends in zero for any value of n.

(ii) **(c)**: We know that product of two rational numbers is also a rational number.
So, $a^2 = a \times a$ = rational number
$a^3 = a^2 \times a$ = rational number
$a^4 = a^3 \times a$ = rational number
.....
.....
$a^n = a^{n-1} \times a$ = rational number.

(iii) **(d)**: Let $x = 2m + 1$ and $y = 2k + 1$
Then $x^2 + y^2 = (2m + 1)^2 + (2k + 1)^2$
$= 4m^2 + 4m + 1 + 4k^2 + 4k + 1 = 4(m^2 + k^2 + m + k) + 2$
So, it is even but not divisible by 4.

(iv) **(a)**: Let three consecutive positive integers be n, $n + 1$ and $n + 2$.
We know that when a number is divided by 3, the remainder obtained is either 0 or 1 or 2.
So, $n = 3p$ or $3p + 1$ or $3p + 2$, where p is some integer.
If $n = 3p$, then n is divisible by 3.
If $n = 3p + 1$, then $n + 2 = 3p + 1 + 2 = 3p + 3 = 3(p + 1)$ is divisible by 3.
If $n = 3p + 2$, then $n + 1 = 3p + 2 + 1 = 3p + 3 = 3(p + 1)$ is divisible by 3.
So, we can say that one of the numbers among n, $n + 1$ and $n + 2$ is always divisible by 3.

(v) **(d)**: Any odd number is of the form of $(2k + 1)$, where k is any integer.
So, $n^2 - 1 = (2k + 1)^2 - 1 = 4k^2 + 4k$
For $k = 1$, $4k^2 + 4k = 8$, which is divisible by 8.
Similarly, for $k = 2$, $4k^2 + 4k = 24$, which is divisible by 8.
And for $k = 3$, $4k^2 + 4k = 48$, which is also divisible by 8.
So, $4k^2 + 4k$ is divisible by 8 for all integers k, *i.e.*, $n^2 - 1$ is divisible by 8 for all odd values of n.

2. **(i)** **(b)**: Here $80 = 2^4 \times 5$, $85 = 17 \times 5$
and $90 = 2 \times 3^2 \times 5$
L.C.M of 80, 85 and $90 = 2^4 \times 3 \times 3 \times 5 \times 17 = 12240$
Hence, the minimum distance each should walk when they at first time is 12240 cm.

(ii) **(c)**: Here $594 = 2 \times 3^3 \times 11$ and $189 = 3^3 \times 7$
HCF of 594 and $189 = 3^3 = 27$
Hence, the maximum number of columns in which they can march is 27.

(iii) **(c)**: Here $768 = 2^8 \times 3$ and $420 = 2^2 \times 3 \times 5 \times 7$
HCF of 768 and $420 = 2^2 \times 3 = 12$
So, the container which can measure fuel of either tanker exactly must be of 12 litres.

(iv) **(b)**: Here, Length = 825 cm, Breadth = 675 cm and Height = 450 cm
Also, $825 = 5 \times 5 \times 3 \times 11$, $675 = 5 \times 5 \times 3 \times 3 \times 3$ and $450 = 2 \times 3 \times 3 \times 5 \times 5$
HCF $= 5 \times 5 \times 3 = 75$
Therefore, the length of the longest rod which can measure the three dimensions of the room exactly is 75 cm.

(v) **(a)**: LCM of 8 and 12 is 24.
$\therefore$ The least number of pack of pens $= 24/8 = 3$
$\therefore$ The least number of pack of note pads $= 24/12 = 2$

3. **(i)** **(b)**: Here $\sqrt{8} = 2\sqrt{2}$ = product of rational and irrational numbers = irrational number

(ii) **(c)**: Here, $\sqrt{9} = 3$
So, $2 + 2\sqrt{9} = 2 + 6 = 8$, which is not irrational.

(iii) **(b)**: Here $\sqrt{15}$ and $\sqrt{10}$ are both irrational and difference of two irrational numbers is also irrational.

(iv) **(c)**: As $\sqrt{5}$ is irrational, so its reciprocal is also irrational.

(v) **(d)**: We know that $\sqrt{6}$ is irrational.
So, $15 + 3\sqrt{6}$ is irrational.
Similarly, $\sqrt{24} - 9 = 2\sqrt{6} - 9$ is irrational.
And $5\sqrt{150} = 5 \times 5\sqrt{6} = 25\sqrt{6}$ is irrational.

4. **(i)** **(c)**: Here, the simplest form of given options are
$125/441 = 5^3/(3^2 \times 7^2)$, $77/210 = 11/(2 \times 3 \times 5)$,
$15/1600 = 3/(2^6 \times 5)$
Out of all the given options, the denominator of option (c) alone has only 2 and 5 as factors. So, it is a terminating decimal.

(ii) **(b)**: $23/(2^3 \times 5^2) = 23/200 = 0.115$

(iii) (a): $441/(2^2 \times 5^7 \times 7^2) = 9/(2^2 \times 5^7)$, which is a terminating decimal.

(iv) (d): The fraction form of a non-terminating recurring decimal will have at least one prime number other than 2 and 5 as its factors in denominator.
So, p can take either of 3, 7 or 15.

(v) (a): Here denominator has only two prime factors *i.e.*, 2 and 5 and hence it is a terminating decimal.

5. (i) (b): LCM of x and $y = p^3q^3$ and HCF of x and $y = p^2q$
Also, LCM = $pq^2 \times$ HCF.

(ii) (d): Number of marbles = $5m + 2$ or $6n + 2$.
Thus, number of marbles, p = (multiple of 5×6) + 2
$= 30k + 2 = 2(15k + 1)$
= which is an even number but not prime

(iii) (d): Here, required numbers
= HCF (398 – 7, 436 – 11, 542 –15)
= HCF (391, 425, 527) = 17

(iv) (b): LCM of 126 and $600 = 2 \times 3 \times 21 \times 100 = 12600$
The least positive integer which on adding 1 is exactly divisible by 126 and 600 = 12600 – 1 = 12599

(v) (a): Here $85C - 340A = 109$ and $425A + 85B = 146$
On adding them, we get
$85A + 85B + 85C = 255 \Rightarrow A + B + C = 3$, which is divisible by 3.