CHAPTER 2

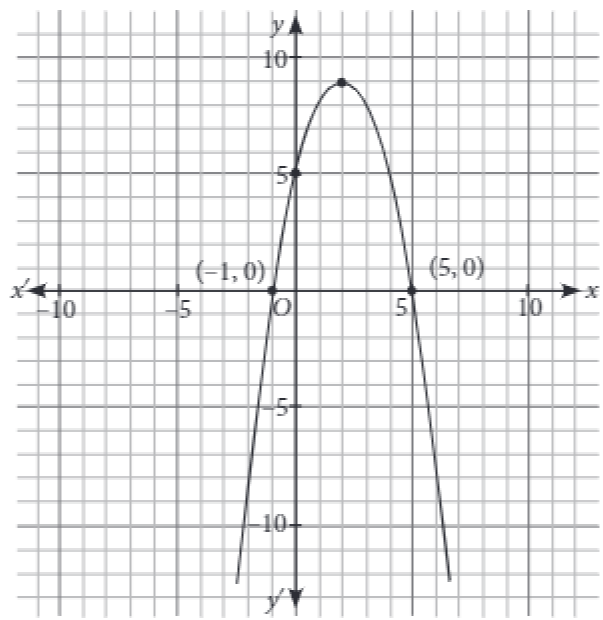
Polynomials

CASE STUDY / PASSAGE BASED QUESTIONS

1

Construction of Humps

ABC construction company got the contract of making speed humps on roads. Speed humps are parabolic in shape and prevents overspeeding, minimise accidents and gives a chance for pedestrians to cross the road. The mathematical representation of a speed hump is shown in the given graph.



Syllabus

Zeroes of a polynomial. Relationship between zeroes and coefficients of quadratic polynomials.

Based on the above information, answer the following questions.

(i) The polynomial represented by the graph can be ______ polynomial.

(a) Linear (b) Quadratic

(c) Cubic (d) Zero

(ii) The zeroes of the polynomial represented by the graph are

(a) 1, 5 (b) 1, –5

(c) –1, 5 (d) –1, –5

(iii) The sum of zeroes of the polynomial represented by the graph are

(a) 4 (b) 5 (c) 6 (d) 7

(iv) If α and β are the zeroes of the polynomial represented by the graph such that $\beta > \alpha$, then $|8\alpha + \beta| =$

(a) 1 (b) 2 (c) 3 (d) 4

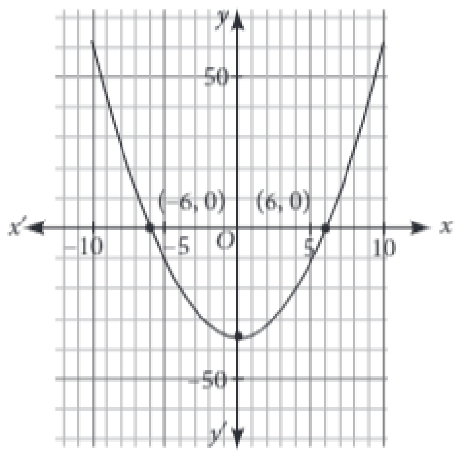
(v) The expression of the polynomial represented by the graph is

(a) $-x^2 - 4x - 5$ (b) $x^2 + 4x + 5$ (c) $x^2 + 4x - 5$ (d) $-x^2 + 4x + 5$

2

Honeycomb

While playing in garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.



Based on the above information, answer the following questions.

(i) Graph of a quadratic polynomial is _______ in shape.

(a) straight line (b) parabolic

(c) circular (d) None of these

(ii) The expression of the polynomial represented by the graph is

(a) $x^2 - 49$ (b) $x^2 - 64$ (c) $x^2 - 36$ (d) $x^2 - 81$

(iii) Find the value of the polynomial represented by the graph when $x = 6$.

(a) −2 (b) −1 (c) 0 (d) 1

(iv) The sum of zeroes of the polynomial $x^2 + 2x - 3$ is

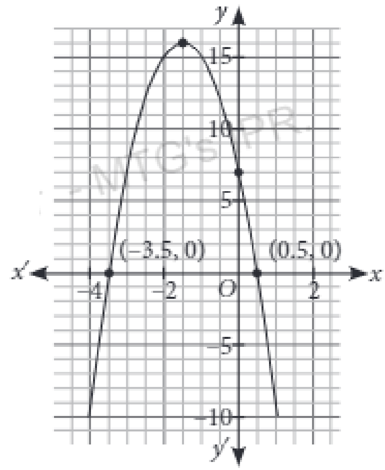
(a) −1 (b) −2 (c) 2 (d) 1

(v) If the sum of zeroes of polynomial $at^2 + 5t + 3a$ is equal to their product, then find the value of a.

(a) −5 (b) −3 (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

3

Just before the morning assembly a teacher of kindergarten school observes some clouds in the sky and so she cancels the assembly. She also observes that the clouds has a shape of the polynomial. The mathematical representation of a cloud is shown in the figure.



(i) Find the zeroes of the polynomial represented by the graph.

(a) −1/2, 7/2 (b) 1/2, −7/2 (c) −1/2, −7/2 (d) 1/2, 7/2

(ii) What will be the expression for the polynomial represented by the graph?

(a) $p(x) = 12x^2 - 4x - 7$ (b) $p(x) = -x^2 - 12x + 3$ (c) $p(x) = 4x^2 + 12x + 7$ (d) $p(x) = -4x^2 - 12x + 7$

(iii) What will be the value of polynomial represented by the graph, when $x = 3$?

(a) 65 (b) −65 (c) 68 (d) −68

(iv) If α and β are the zeroes of the polynomial $f(x) = x^2 + 2x - 8$, then $\alpha^4 + \beta^4 =$

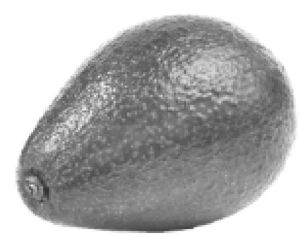
(a) 262 (b) 252 (c) 272 (d) 282

(v) Find a quadratic polynomial where sum and product of its zeroes are $0, \sqrt{7}$ respectively.

(a) $k(x^2 + \sqrt{7})$ (b) $k(x^2 - \sqrt{7})$ (c) $k(x^2 + \sqrt{5})$ (d) none of these

4

Avocado



Pankaj's father gave him some money to buy avocado from the market at the rate of $p(x) = x^2 - 24x + 128$. Let α, β are the zeroes of $p(x)$.

Based on the above information, answer the following questions.

(i) Find the value of α and β, where $\alpha < \beta$.

(a) −8, −16 (b) 8, 16 (c) 8, 15 (d) 4, 9

(ii) Find the value of $\alpha + \beta + \alpha\beta$.

(a) 151 (b) 158 (c) 152 (d) 155

(iii) The value of $p(2)$ is

(a) 80 (b) 81 (c) 83 (d) 84

(iv) If α and β are zeroes of $x^2 + x - 2$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/5

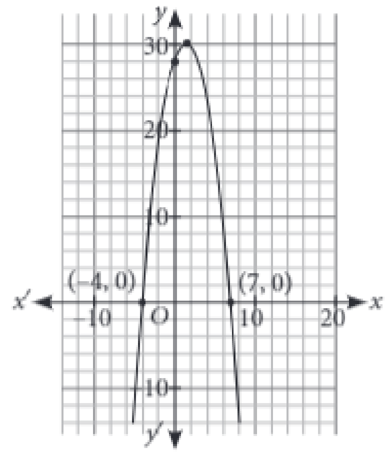
(v) If sum of zeroes of $q(x) = kx^2 + 2x + 3k$ is equal to their product, then $k =$

(a) 2/3 (b) 1/3 (c) −2/3 (d) −1/3

5

Mountain Trekking

Two friends Trisha and Rohan during their summer vacations went to Manali. They decided to go for trekking. While trekking they observes that the trekking path is in the shape of a parabola. The mathematical representation of the track is shown in the graph.



Based on the above information, answer the following questions.

(i) The zeroes of the polynomial whose graph is given, are

(a) 4, 7 (b) –4, 7 (c) 4, 3 (d) 7, 10

(ii) What will be the expression of the given polynomial $p(x)$?

(a) $x^2 - 3x + 28$ (b) $-x^2 + 4x + 28$ (c) $x^2 - 4x + 28$ (d) $-x^2 + 3x + 28$

(iii) Product of zeroes of the given polynomial is

(a) –28 (b) 28 (c) –30 (d) 30

(iv) The zeroes of the polynomial $9x^2 - 5$ are

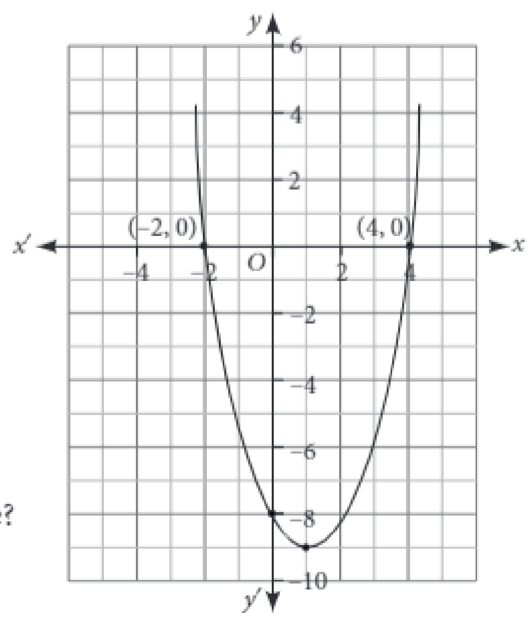
(a) $\frac{3}{\sqrt{5}}, \frac{-3}{\sqrt{5}}$ (b) $\frac{2}{\sqrt{5}}, \frac{-2}{\sqrt{5}}$ (c) $\frac{\sqrt{5}}{3}, \frac{-\sqrt{5}}{3}$ (d) $\frac{\sqrt{5}}{2}, \frac{-\sqrt{5}}{2}$

(v) If $f(x) = x^2 - 13x + 1$, then $f(4) =$

(a) 35 (b) –35 (c) 36 (d) –36

6

Neeru saw a creeper on the boundary of her aunt's house which was in the shape as shown in the figure. Answer the following questions by considering that creeper has same mathematical shape as shown in the figure.



Based on the above information, answer the following questions.

(i) The shape represents a ______ polynomial.

(a) Linear (b) Cubic

(c) Quadratic (d) None of these

(ii) How many zeroes does the polynomial (shape of the creeper) have?

(a) 0 (b) 1

(c) 2 (d) 3

(iii) The zeroes of the polynomial, represented by the graph, are

(a) 4, –2 (b) –4, 2 (c) 4, 2 (d) –5, 6

(iv) The expression of the polynomial, represented by the graph, is

(a) $x^2 + 2x - 8$ (b) $x^2 - 2x - 8$ (c) $x^3 - x + 8$ (d) $x^3 - x^2 + 2x + 8$

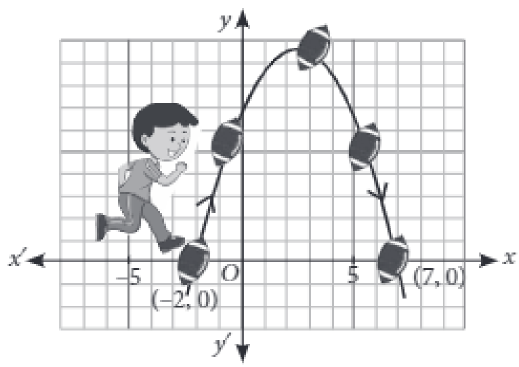
(v) For what value of x, the value of the polynomial, represented by the graph, is –5?

(a) $x = 3$ (b) $x = -1$ (c) Both (a) and (b) (d) Can't be determined

7

Soccer Match

In a soccer match, the path of the soccer ball in a kick is recorded as shown in the following graph.



Based on the above information, answer the following questions.

(i) The shape of path of the soccer ball is a

(a) Circle (b) Parabola (c) Line (d) None of these

(ii) The axis of symmetry of the given parabola is

(a) y-axis (b) x-axis

(c) line parallel to y-axis (d) line parallel to x-axis

(iii) The zeroes of the polynomial, represented in the given graph, are

(a) –1, 7 (b) 5, –2 (c) –2, 7 (d) –3, 8

(iv) Which of the following polynomial has –2 and –3 as its zeroes?

(a) $x^2 - 5x - 5$ (b) $x^2 + 5x - 6$ (c) $x^2 + 6x - 5$ (d) $x^2 + 5x + 6$

(v) For what value of 'x', the value of the polynomial $f(x) = (x - 3)^2 + 9$ is 9?

(a) 1 (b) 2 (c) 3 (d) 4

8

Slinky Spring Dog Toy

Prachi was playing with a slinky spring dog toy and asked her brother Rhythm, what is the shape thus formed called. Rhythm explained her that the shape formed is a parabola. He also explained her that parabola is the graphical representation of a quadratic polynomial.

Based on the above information, answer the following questions.

(i) The general form of polynomial representing the parabolic graph is

(a) $ax^2 + c, a \neq 0$ (b) $ax^2 + bx + c, b \neq 0$

(c) $ax^2 + bx + c$, a, b and $c \neq 0$ (d) $ax^2 + bx + c, a \neq 0$

(ii) Kavita drawn a parabola passing through (–4, 3), (–1, 0), (1, 8), (0, 3), (–3, 0) and (–2, –1) on the graph paper. Then zeroes of the polynomial representing the graph is

(a) 3 and –3 (b) –1 and –2 (c) –3 and –1 (d) 1 and 8

(iii) Which of the following is correct?

(a) A parabola intersects x-axis at maximum 2 points.

(b) A parabola intersects x-axis only at 1 point.

(c) A parabola intersects x-axis exactly at 2 points.

(d) A parabola intersects x-axis at least at 2 points.

(iv) The product of roots of the polynomial $5x(x - 6)$ is

(a) 3/2 (b) 2/3 (c) 3 (d) 0

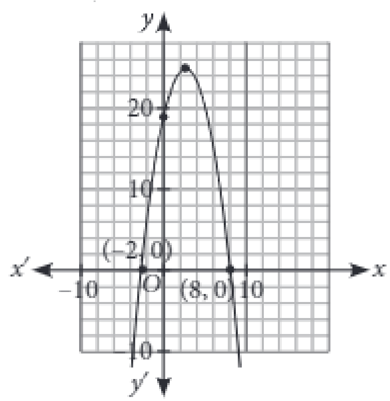
(v) The sum of zeroes of a quadratic polynomial $ax^2 + bx + c, a \neq 0$ is

(a) a/b (b) a/c (c) $-b/a$ (d) $-c/a$

9

Application of Quadratic Polynomial–Highway Tunnel

Shweta and her husband Sunil who is an architect by profession, visited France. They went to see Mont Blanc Tunnel which is a highway tunnel between France and Italy, under the Mont Blanc Mountain in the Alps, and has a parabolic cross-section. The mathematical representation of the tunnel is shown in the graph.



Based on the above information, answer the following questions.

(i) The zeroes of the polynomial whose graph is given, are

(a) –2, 8 (b) –2, –8 (c) 2, 8 (d) –2, 0

(ii) What will be the expression of the polynomial given in diagram?

(a) $x^2 - 6x + 16$ (b) $-x^2 + 6x + 16$ (c) $x^2 + 6x + 16$ (d) $-x^2 - 6x - 16$

(iii) What is the value of the polynomial, represented by the graph, when $x = 4$?

(a) 22 (b) 23 (c) 24 (d) 25

(iv) If the tunnel is represented by $-x^2 + 3x - 2$, then its zeroes are

(a) –1, –2 (b) 1, –2 (c) –1, 2 (d) 1, 2

(v) If one zero is 4 and sum of zeroes is –3, then representation of tunnel as a polynomial is

(a) $x^2 - x + 24$ (b) $-x^2 - 3x + 28$ (c) $x^2 + x + 28$ (d) $x^2 - x + 28$

10

Application of Polynomials–Architectural Structures

Quadratic polynomial can be used to model the shape of many architectural structures in the world. Pershing field of Jersey city in US is one such structure.

Based on the above information, answer the following questions.

(i) If the Arch is represented by $10x^2 - x - 3$, then its zeroes are

(a) $\frac{1}{2}, \frac{-3}{2}$ (b) $\frac{-1}{2}, \frac{3}{5}$ (c) $\frac{-1}{2}, \frac{1}{3}$ (d) $\frac{-1}{3}, \frac{2}{3}$

(ii) The zeroes of the polynomial are the points where its graph

(a) intersect the x-axis (b) intersect the y-axis

(c) intersect either of the axes (d) Can't say

(iii) The quadratic polynomial whose sum of zeroes is 0 and product of zeroes is 1 is given by

(a) $x^2 - x$ (b) $x^2 + x$ (c) $x^2 - 1$ (d) $x^2 + 1$

(iv) Which of the following has $\frac{-1}{2}$ and 2 as their zeroes?

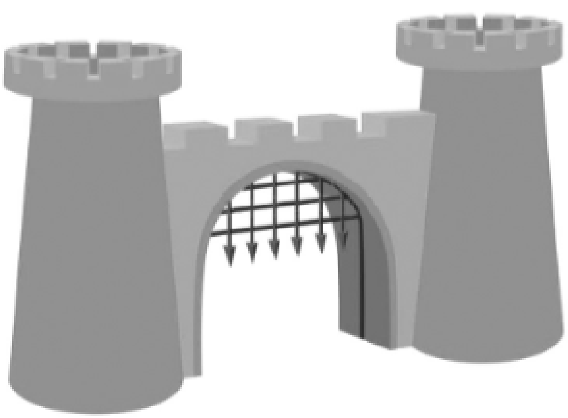
(a) $6x^2 - 4x + 6$ (b) $3x^2 - x + 2$ (c) $2x^2 - 7x + 2$ (d) $2x^2 - 3x - 2$

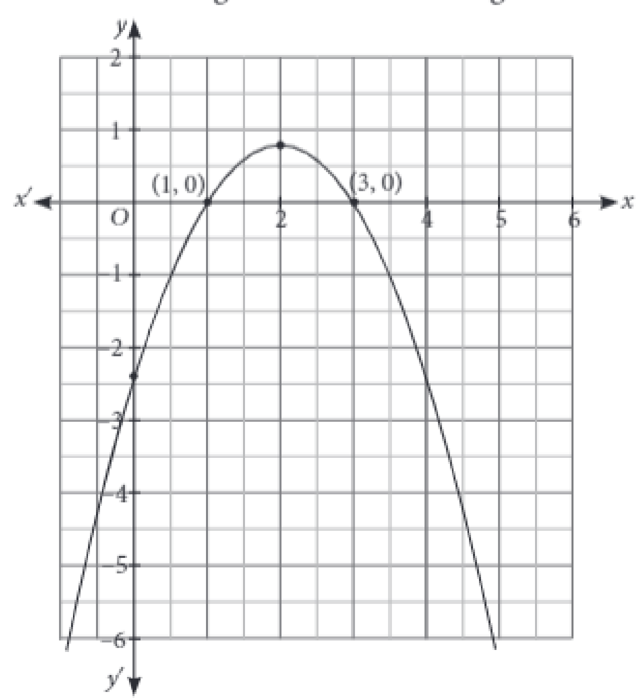
(v) The product of zeroes of the polynomial $\sqrt{3}x^2 - 14x + 8\sqrt{3}$ is

(a) 4 (b) 6 (c) 8 (d) 10

11

Priya visited a temple in Gwalior. On the way she sees the Agra Fort. The entrance gate of the fort has a shape of quadratic polynomial (parabolic). The mathematical representation of the gate is shown in the figure.





Based on the above information, answer the following questions.

(i) Find the zeroes of the polynomial represented by the graph.

(a) $-1, 3$ (b) $1, 3$ (c) $1, -3$ (d) $0, 1$

(ii) What will be the expression for the polynomial represented by the graph?

(a) $x^2 + 4x - 5$ (b) $x^2 - 4x + 5$ (c) $-x^2 + 4x - 3$ (d) $x^2 + 5x - 4$

(iii) What will be the value of polynomial, represented by the graph, when $x = 4$?

(a) -2 (b) 3 (c) -3 (d) 2

(iv) If one zero of a polynomial $p(x)$ is 7 and product of its zeroes is -35, then $p(x) =$

(a) $-x^2 + 2x + 35$ (b) $x^2 + 2x + 35$ (c) $x^2 + 12x - 35$ (d) $x^2 - 12x - 35$

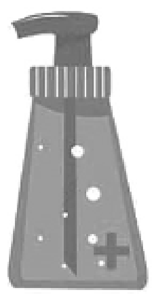
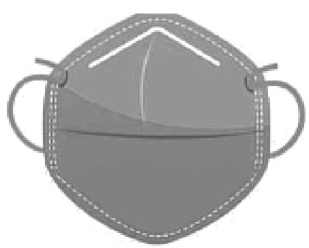
(v) If the gate is represented by the polynomial $-x^2 + 5x - 6$, then its zeroes are

(a) $2, -3$ (b) $2, 3$ (c) $-2, 3$ (d) $-2, -3$

12

Social Service

Shray, who is a social worker, wants to distribute masks, gloves, and hand sanitizer bottles in his block. Number of masks, gloves and sanitizer bottles distributed in 1 day can be represented by the zeroes $\alpha, \beta, \gamma, (\alpha > \beta > \gamma)$ of the polynomial $p(x) = x^3 - 18x^2 + 95x - 150$.

Based on the above information, answer the following questions.

(i) Find the value of α, β, γ.

(a) $-10, -5, -3$ (b) $3, 6, 5$

(c) $10, 5, 3$ (d) $4, 8, 9$

(ii) The sum of product of zeroes taken two at a time is

(a) 91 (b) 92 (c) 94 (d) 95

(iii) Product of zeroes of polynomial $p(x)$ is

(a) 150 (b) 160 (c) 170 (d) 180

(iv) The value of the polynomial $p(x)$, when $x = 4$ is

(a) 5 (b) 6 (c) 7 (d) 8

(v) If α, β, γ are the zeroes of a polynomial $g(x)$ such that $\alpha + \beta + \gamma = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -16$ and $\alpha\beta\gamma = -48$, then $g(x) =$

(a) $x^3 - 2x^2 - 48x + 6$ (b) $x^3 + 3x^2 + +16x - 48$

(c) $x^3 - 48x^2 - 16x + 3$ (d) $x^3 - 3x^2 - 16x + 48$

13

Barrier Chains

While playing badminton Ronit seeing the barrier chains hung between two posts at the edge of the walk way of a street. It is hung in the shape of the parabola. Parabola is the graphical representation of a particular type of polynomial.
Based on the above information, answer the following questions.

(i) Which of the following polynomial is graphically represented by a parabola?
(a) Linear polynomial (b) Quadratic polynomial
(c) Cubic polynomial (d) None of these

(ii) If a polynomial, represented by a parabola, intersects the x-axis at -3, 4 and y-axis at -2, then its zero(es) is/are
(a) -1, 2 and -2 (b) 2 and -2 (c) -1 (d) -3 and 4

(iii) If the barrier chains between two posts is represented by the polynomial $x^2 - x - 12$, then its zeroes are
(a) 4, 3 (b) -2, 5 (c) 4, -3 (d) 4, -5

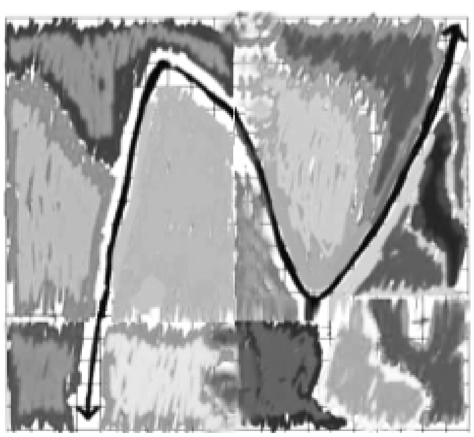
(iv) The sum of zeroes of the polynomial $4x^2 - 9x + 2$ is
(a) 1/4 (b) 9/4 (c) 2/4 (d) $-9/4$

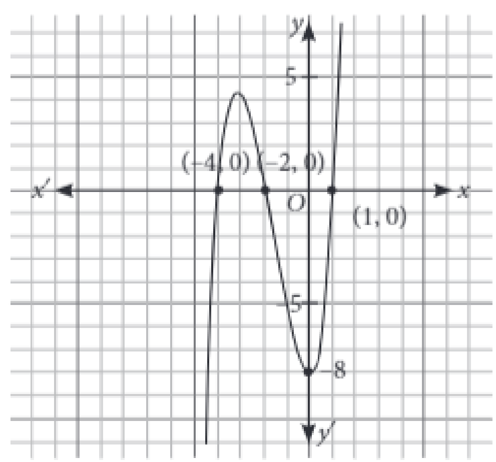
(v) The reciprocal of product of zeroes of the polynomial $x^2 - 9x + 20$ is
(a) 5 (b) 1/8 (c) 1/20 (d) 20

14

Painting Exhibition

Shruti is very good in painting. So she thought of exhibiting her paintings in which she want to display her latest painting which is in the form of a graph of a polynomial as shown below :





Based on the above information, answer the following questions.

(i) The number of zeroes of the polynomial represented by the graph is
(a) 1 (b) 2 (c) 3 (d) can't be determined

(ii) The sum of zeroes of the polynomial represented by the graph is

(a) −4 (b) −3 (c) 2 (d) −5

(iii) Find the value of the polynomial represented by the graph when $x = 0$.

(a) −6 (b) −8 (c) 6 (d) 8

(iv) The polynomial representing the graph drawn in the painting by Shruti is a

(a) quadratic polynomial (b) cubic polynomial

(c) bi-quadratic polynomial (d) linear polynomial

(v) The sum of product of zeroes, taken two at a time, of the polynomial represented by the graph is

(a) 2 (b) 3 (c) −2 (d) −3

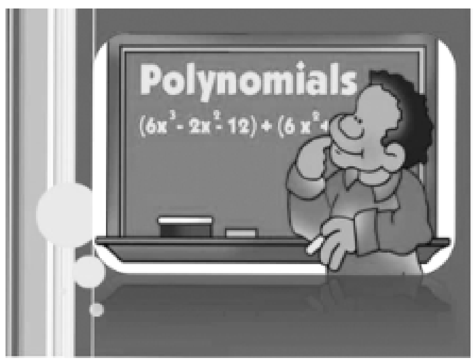
15

The tutor in a coaching centre was explaining the concept of cubic polynomial as - A cubic polynomial is of the form $ax^3 + bx^2 + cx + d$, $a \neq 0$ and it has maximum three real zeroes. The zeroes of a cubic polynomial are namely the x-coordinates of the points where the graph of the polynomial intersects the x-axis. If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then the relation between their zeroes and their coefficients are

$\alpha + \beta + \gamma = -b/a$

$\alpha\beta + \beta\gamma + \alpha\gamma = c/a$

$\alpha\beta\gamma = -d/a$



Based on the above information, answer the following questions.

(i) Which of the following are the zeroes of the polynomial $x^3 - 4x^2 - 7x + 10$?

(a) −3, 1 and 3 (b) −1, 2 and −3

(c) 2, −1 and 5 (d) −2, 1 and 5

(ii) If $-\frac{1}{2}$, −2 and 5 are zeroes of a cubic polynomial, then the sum of product of zeroes taken two at a time is

(a) $\frac{23}{2}$ (b) $-\frac{1}{2}$

(c) −23 (d) $-\frac{23}{2}$

(iii) In which of the following polynomials the sum and product of zeroes are equal ?

(a) $x^3 - x^2 + 5x - 1$ (b) $x^3 - 4x$

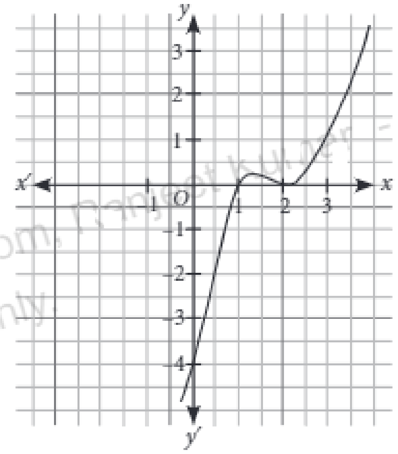
(c) $3x^3 - 5x^2 - 11x - 3$ (d) Both (a) and (b)

(iv) The polynomial whose all the zeroes are same is

(a) $x^3 + x^2 + x - 1$ (b) $x^3 - 3x^2 + 3x - 1$

(c) $x^3 - 5x^2 + 6x - 1$ (d) $3x^3 + x^2 + 2x - 1$

(v) The cubic polynomial, whose graph is as shown below, is



(a) $x^3 - 5x^2 + 8x - 4$ (b) $x^3 - 7x^2 + 11x + 9$
(c) $3x^3 - 4x^2 + x - 5$ (d) $x^3 - 9$

HINTS & EXPLANATIONS

1. **(i) (b):** Since, the given graph is parabolic is shape, therefore it will represent a quadratic polynomial.

[∵ Graph of quadratic polynomial is parabolic in shape]

(ii) (c): Since, the graph cuts the x-axis at -1, 5. So the polynomial has 2 zeroes *i.e.*, -1 and 5.

(iii) (a): Sum of zeroes $= -1 + 5 = 4$

(iv) (c): Since α and β are zeroes of the given polynomial and $\beta > \alpha$

$\therefore \quad \alpha = -1$ and $\beta = 5$.

$\therefore \quad |8\alpha + \beta| = |8(-1) + 5| = |-8 + 5| = |-3| = 3.$

(v) (d): Since the zeroes of the given polynomial are -1 and 5.

$\therefore$ Required polynomial $p(x)$

$= k\{x^2 - (-1 + 5)x + (-1)(5)\} = k(x^2 - 4x - 5)$

For $k = -1$, we get

$p(x) = -x^2 + 4x + 5$, which is the required polynomial.

2. **(i) (b):** Graph of a quadratic polynomial is a parabolic in shape.

(ii) (c): Since the graph of the polynomial cuts the x-axis at $(-6, 0)$ and $(6, 0)$. So, the zeroes of polynomial are -6 and 6.

$\therefore$ Required polynomial is

$p(x) = x^2 - (-6 + 6)x + (-6)(6) = x^2 - 36$

(iii) (c): We have, $p(x) = x^2 - 36$

Now, $p(6) = 6^2 - 36 = 36 - 36 = 0$

(iv) (b): Let $f(x) = x^2 + 2x - 3$. Then,

$$\text{Sum of zeroes} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2} = -\frac{(2)}{1} = -2$$

(v) (d): The given polynomial is $at^2 + 5t + 3a$

Given, sum of zeroes = product of zeroes

$$\Rightarrow \quad \frac{-5}{a} = \frac{3a}{a} \quad \Rightarrow \quad a = \frac{-5}{3}$$

3. **(i) (b):** Since the graph of the polynomial intersect the x-axis at $x = \frac{1}{2}, \frac{-7}{2}$, therefore required zeroes of the polynomial are $\frac{1}{2}$ and $\frac{-7}{2}$.

(ii) (d): $\because \frac{1}{2}$ and $\frac{-7}{2}$ are the zeroes of the polynomial.

So, at $x = \frac{1}{2}, \frac{-7}{2}$, the value of the polynomial will be 0.

From options, required polynomial is

$p(x) = -4x^2 - 12x + 7.$

(iii) (b): We have, $p(x) = -4x^2 - 12x + 7$

$\therefore \quad p(3) = -4(3)^2 - 12(3) + 7 = -36 - 36 + 7 = -65$

(iv) (c): Here $f(x) = x^2 + 2x - 8$ and α, β are its zeroes.

$\therefore \quad \alpha + \beta = -2$ and $\alpha\beta = -8$

$$\begin{aligned}\text{Now, } \alpha^4 + \beta^4 &= (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 \\ &= ((\alpha + \beta)^2 - 2\alpha\beta)^2 - 2(\alpha\beta)^2 \\ &= [(-2)^2 - 2(-8)]^2 - 2(-8)^2 \\ &= [4 + 16]^2 - 2(-8)^2 = (20)^2 - 2(64) \\ &= 400 - 128 = 272\end{aligned}$$

(v) (a): We have sum of zeroes = 0 and product of zeroes = $\sqrt{7}$

So, required polynomial = $k(x^2 - 0\cdot x + \sqrt{7})$
$= k(x^2 + \sqrt{7})$

4. (i) (b): Given, α and β are the zeroes of $p(x) = x^2 - 24x + 128$.
Putting $p(x) = 0$, we get
$x^2 - 8x - 16x + 128 = 0$
$\Rightarrow x(x - 8) - 16(x - 8) = 0$
$\Rightarrow (x - 8)(x - 16) = 0 \Rightarrow x = 8$ or $x = 16$
$\therefore \alpha = 8, \beta = 16$

(ii) (c): $\alpha + \beta + \alpha\beta = 8 + 16 + (8)(16)$
$= 24 + 128 = 152$

(iii) (d): $p(2) = 2^2 - 24(2) + 128 = 4 - 48 + 128 = 84$

(iv) (a): Since α and β are zeroes of $x^2 + x - 2$.
$\therefore \alpha + \beta = -1$ and $\alpha\beta = -2$
Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-1}{-2} = \frac{1}{2}$

(v) (c): Sum of zeroes $= \frac{-2}{k}$
Product of zeroes = $\frac{3k}{k} = 3$
According to question, we have $\frac{-2}{k} = 3$
$\Rightarrow k = \frac{-2}{3}$

5. (i) (b): Since, the graph intersects the x-axis at two points, namely $x = -4, 7$
So, $-4, 7$ are the zeroes of the polynomial.

(ii) (d): $p(x) = -x^2 + 3x + 28$

(iii) (a): Product of zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ Required product of zeroes = $\frac{28}{-1} = -28$

(iv) (c): We have, $9x^2 - 5 = (3x)^2 - (\sqrt{5})^2$
$= (3x - \sqrt{5})(3x + \sqrt{5})$

$\therefore x = \frac{\sqrt{5}}{3}$ or $\frac{-\sqrt{5}}{3}$

(v) (b): Here, $f(x) = x^2 - 13x + 1$
$\therefore f(4) = 4^2 - 13(4) + 1 = 16 - 52 + 1 = -35$

6. (i) (c): The shape represents a quadratic polynomial.

(ii) (c): Since, the graph of polynomial cuts the x-axis at $(-2, 0)$ and $(4, 0)$. So, the polynomial has 2 zeroes.

(iii) (a): The zeroes of the polynomial are -2 and 4.

(iv) (b): Required polynomial is
$p(x) = x^2 - (-2 + 4)x + (-2)(4) = x^2 - 2x - 8$

(v) (c): Consider, $p(x) = -5$
$\Rightarrow x^2 - 2x - 8 = -5 \Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x - 3)(x + 1) = 0 \Rightarrow x = -1$ or $x = 3$
So, at $x = 3$ and at $x = -1$, $p(x) = -5$.

7. (i) (b): The shape of the path of the soccer ball is a parabola.

(ii) (c): The axis of symmetry of the given curve is a line parallel to y-axis.

(iii) (a): The zeroes of the polynomial, represented in the given graph, are -2 and 7, since the curve cuts the x-axis at these points.

(iv) (d): A polynomial having zeroes -2 and -3 is
$p(x) = x^2 - (-2 - 3)x + (-2)(-3) = x^2 + 5x + 6$

(v) (c): We have, $f(x) = (x - 3)^2 + 9$
Now, $9 = (x - 3)^2 + 9$
$\Rightarrow (x - 3)^2 = 0 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

8. (i) (d): The general form of polynomial representing the parabolic graph is $ax^2 + bx + c, a \neq 0$.

(ii) (c): The zeroes of the polynomial are the points at which its graph intersects the x-axis, *i.e.*, whose y coordinate is 0.
$\therefore$ Zeroes are -3 and -1.

(iii) (a): A parabola intersects x-axis at maximum 2 points.

(iv) (d): The product of roots of the polynomial $5x^2 - 30x$ is 0. [$\because$ constant term = 0]

(v) (c): Sum of zeroes of quadratic polynomial $ax^2 + bx + c, a \neq 0$ is $\frac{-b}{a}$.

9. (i) (a): Since, the graph intersects the x-axis at two points, namely $x = 8, -2$.
So, $8, -2$ are the zeroes of the given polynomial.

(ii) (b): The expression of the polynomial given in diagram is $-x^2 + 6x + 16$.

(iii) (c): Let $p(x) = -x^2 + 6x + 16$
When $x = 4$, $p(4) = -4^2 + 6 \times 4 + 16 = 24$

(iv) (d): Let $f(x) = -x^2 + 3x - 2$
Now, consider $f(x) = 0 \Rightarrow -x^2 + 3x - 2 = 0$
$\Rightarrow x^2 - 3x + 2 = 0 \Rightarrow (x - 2)(x - 1) = 0$
$\Rightarrow x = 1, 2$ are its zeroes.

(v) (b): Let α and β are the zeroes of the required polynomial.
Given, $\alpha + \beta = -3$
If $\alpha = 4$, then $\beta = -7$
$\therefore$ Representation of tunnel is $-x^2 - 3x + 28$.

10. **(i)** **(b):** Put $10x^2 - x - 3 = 0$

$\Rightarrow 10x^2 - 6x + 5x - 3 = 0 \Rightarrow (2x+1)(5x-3) = 0$

$\Rightarrow x = \frac{-1}{2}$ or $\frac{3}{5}$

Thus, the zeroes are $\frac{3}{5}$ and $\frac{-1}{2}$.

(ii) **(a):** The zeroes of the polynomial are the points where its graph intersect the x-axis.

(iii) **(d)** **(iv)** **(d)**

(v) **(c):** Product of zeroes $= \frac{8\sqrt{3}}{\sqrt{3}} = 8$

11. **(i)** **(b):** Since, the graph of the polynomial intersect the x-axis at $x = 1, 3$ therefore required zeroes of the polynomial are 1 and 3.

(ii) **(c)**

(iii) **(c):** Let $f(x) = -x^2 + 4x - 3$

Then $f(4) = -4^2 + 4 \times 4 - 3$

$= -16 + 16 - 3 = -3$

(iv) **(a):** Clearly, other zero $= \frac{-35}{7} = -5$

Thus, the zeroes are 7 and -5.

From the options, 7 and -5 satisfies only $-x^2 + 2x + 35$.

So, $p(x) = -x^2 + 2x + 35$

(v) **(b):** Let $p(x) = -x^2 + 5x - 6$

For zeroes, consider $p(x) = 0$

$\Rightarrow -x^2 + 5x - 6 = 0 \Rightarrow x^2 - 5x + 6 = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x-3)(x-2) = 0 \Rightarrow x = 3, 2$

Thus, the required zeroes are 3 and 2.

12. **(i)** **(c):** For finding α, β, γ, consider $p(x) = 0$

$\Rightarrow x^3 - 18x^2 + 95x - 150 = 0$

$\Rightarrow (x-3)(x^2 - 15x + 50) = 0$

$\Rightarrow (x-3)(x-5)(x-10) = 0 \Rightarrow x = 10$ or $x = 5$ or $x = 3$

Thus $\alpha = 10$, $\beta = 5$ and $\gamma = 3$

(ii) **(d):** Here $\alpha = 10$, $\beta = 5$ and $\gamma = 3$

$\therefore$ Sum of product of zeroes taken two at a time

$= \alpha\beta + \beta\gamma + \gamma\alpha = (10)(5) + (5)(3) + (3)(10)$

$= 50 + 15 + 30 = 95$

(iii) **(a):** Product of zeroes of polynomial $p(x) = \alpha\beta\gamma$

$= (10)(5)(3) = 150$

(iv) **(b):** We have $p(x) = x^3 - 18x^2 + 95x - 150$

Now, $p(4) = 4^3 - 18(4)^2 + 95(4) - 150$

$= 64 - 288 + 380 - 150 = 6$

(v) **(d):** $g(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$\Rightarrow g(x) = x^3 - 3x^2 - 16x - (-48) = x^3 - 3x^2 - 16x + 48$

13. **(i)** **(b)**

(ii) **(d):** Since, the parabola intersects the x-axis at -3 and 4. So, zeroes of the polynomial are -3 and 4.

(iii) **(c):** Let $f(x) = x^2 - x - 12$

$= x^2 - 4x + 3x - 12 = (x+3)(x-4)$

Consider $f(x) = 0 \Rightarrow (x+3)(x-4) = 0 \Rightarrow x = 4, -3$

(iv) **(b):** Sum of zeroes $= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= -\frac{(-9)}{4} = \frac{9}{4}$

(v) **(c):** Product of zeroes $= \frac{20}{1} = 20$

$\therefore$ Reciprocal of product of zeroes $= \frac{1}{20}$

14. **(i)** **(c):** Since the graph intersect the x-axis at 3 points, therefore the polynomial has 3 zeroes.

(ii) **(d):** Clearly the graph intersect the x-axis at $x = -4$, $x = -2$ and $x = 1$, therefore the zeroes are -4, -2 and 1.

Now, the sum of zeroes $= -4 - 2 + 1 = -5$

(iii) **(b):** From the graph, it can be seen that

When $x = 0$, then $y = -8$.

(iv) **(b):** Since there are 3 zeroes, therefore the graph represents a cubic polynomial.

(v) **(a):** The sum of product of zeroes taken two at a time $= (-4)(-2) + (-2)(1) + (1)(-4) = 8 - 2 - 4 = 2$

15. **(i)** **(d):** For finding zeroes, check whether $x^3 - 4x^2 - 7x + 10$ is 0 for given zeroes.

Let $p(x) = x^3 - 4x^2 - 7x + 10$. Then,

Clearly $p(-2) = p(1) = p(5) = 0$

So, the zeroes are -2, 1 and 5.

(ii) **(d):** Here $\alpha = \frac{-1}{2}$, $\beta = -2$ and $\gamma = 5$

$\therefore$ Sum of product of zeroes taken two at a time

$= \alpha\beta + \beta\gamma + \gamma\alpha$

$$= \left(\frac{-1}{2}\right)(-2) + (-2)(5) + (5)\left(\frac{-1}{2}\right) = 1 - 10 - \frac{5}{2} = \frac{-23}{2}$$

(iii) **(d):** Consider $x^3 - x^2 + 5x - 1$

Sum of zeroes = 1 = Product of zeroes

Now, consider $x^3 - 4x$

Sum of zeroes = 0 = Product of zeroes.

(iv) **(b):** Let α, α, α, be the zeroes of the cubic polynomial. [$\because$ All zeroes are same]

Then, $\alpha^3 = 1 \Rightarrow \alpha = 1$ [Using given options]

So, the required polynomial is $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

(v) **(a):** Clearly $x = 1$ and $x = 2$ are the zeroes of given polynomial, both of which satisfies $x^3 - 5x^2 + 8x - 4$.