Arithmetic Progressions

1

Pathology Lab

In a pathology lab, a culture test has been conducted. In the test, the number of bacteria taken into consideration in various samples is all 3-digit numbers that are divisible by 7, taken in order.

Syllabus

Motivation for studying Arithmetic Progression.
Derivation of the nth term and sum of the first n terms of A.P.

On the basis of above information, answer the following questions.

(i) How many bacteria are considered in the fifth sample?
(a) 126   (b) 140   (c) 133   (d) 149

(ii) How many samples should be taken into consideration?
(a) 129   (b) 128   (c) 130   (d) 127

(iii) Find the total number of bacteria in the first 10 samples.
(a) 1365   (b) 1335   (c) 1302   (d) 1540

(iv) How many bacteria are there in the 7th sample from the last?
(a) 952   (b) 945   (c) 959   (d) 966

(v) The number of bacteria in 50th sample is
(a) 546   (b) 553   (c) 448   (d) 496

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Discussion on A.P.

In a class the teacher asks every student to write an example of A.P. Two friends Geeta and Madhuri writes their progressions as –5, –2, 1, 4, … and 187, 184, 181, …. respectively. Now, the teacher asks various students of the class the following questions on these two progressions. Help students to find the answers of the questions.

(i) Find the 34th term of the progression written by Madhuri.

(a) 286   (b) 88   (c) –99   (d) 190

(ii) Find the sum of common difference of the two progressions.

(a) 6   (b) –6   (c) 1   (d) 0

(iii) Find the 19th term of the progression written by Geeta.

(a) 49   (b) 59   (c) 52   (d) 62

(iv) Find the sum of first 10 terms of the progression written by Geeta.

(a) 85   (b) 95   (c) 110   (d) 200

(v) Which term of the two progressions will have the same value?

(a) 31   (b) 33   (c) 32   (d) 30

3

Shoe Shop

Meena's mother start a new shoe shop. To display the shoes, she put 3 pairs of shoes in 1st row, 5 pairs in 2nd row, 7 pairs in 3rd row and so on.

On the basis of above information, answer the following questions.

(i) If she puts a total of 120 pairs of shoes, then the number of rows required are

(a) 5   (b) 6   (c) 7   (d) 10

(ii) Difference of pairs of shoes in 17th row and 10th row is

(a) 7   (b) 14   (c) 21   (d) 28

(iii) On next day, she arranges x pairs of shoes in 15 rows, then $x =$

(a) 21   (b) 26   (c) 31   (d) 42

(iv) Find the pairs of shoes in 30th row.

    (a) 61   (b) 67   (c) 56   (d) 59

(v) The total number of pairs of shoes in 5th and 8th row is

    (a) 7   (b) 14   (c) 28   (d) 56

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Pocket Money

Anuj gets pocket money from his father everyday. Out of the pocket money, he saves ₹ 2.75 on first day, ₹ 3 on second day, ₹ 3.25 on third day and so on.

On the basis of above information, answer the following questions.

(i) What is the amount saved by Anuj on 14th day?

    (a) ₹ 6.25   (b) ₹ 6   (c) ₹ 6.50   (d) ₹ 6.75

(ii) What is the total amount saved by Anuj in 8 days?

    (a) ₹ 18   (b) ₹ 33   (c) ₹ 24   (d) ₹ 29

(iii) What is the amount saved by Anuj on 30th day?

    (a) ₹ 10   (b) ₹ 12.75   (c) ₹ 10.25   (d) ₹ 9.75

(iv) What is the total amount saved by him in the month of June, if he starts savings from 1st June?

    (a) ₹ 191   (b) ₹ 191.25   (c) ₹ 192   (d) ₹ 192.5

(v) On which day, he save tens times as much as he saved on day-1?

    (a) 9th (b) 99th (c) 10th (d) 100th

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Board Game

In a board game, the number of sea shells in various cells forms an A.P. If the number of sea shells in the 3rd and 11th cell together is 68 and number of shells in 11th cell is 24 more than that of 3rd cell, then answer the following questions based on this data.

(i) What is the difference between the number of sea shells in the 19th and 20th cells?

    (a) 2   (b) 3   (c) 8   (d) 7

(ii) How many sea shells are there in the first cell?

    (a) 52   (b) 18   (c) 16   (d) 54

(iii) How many total sea shells are there in first 13 cells?

    (a) 442   (b) 221   (c) 204   (d) Can't be determined

(iv) Altogether, how many sea shells are there in the first 5 cells?

    (a) 220   (b) 125   (c) 96   (d) 110

(v) What is the sum of number of sea shells in the 7th and 9th cell?

    (a) 42   (b) 32   (c) 74   (d) 80

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Number Cards Game

Amit was playing a number card game. In the game, some number cards (having both +ve or –ve numbers) are arranged in a row such that they are following an arithmetic progression. On his first turn, Amit picks up 6th and 14th card and finds their sum to be –76. On the second turn he picks up 8th and 16th card and finds their sum to be –96.

Based on the above information, answer the following questions.

(i) What is the difference between the numbers on any two consecutive cards?

    (a) 7   (b) –5   (c) 11   (d) –3

(ii) The number on first card is,

    (a) 12   (b) 3   (c) 5   (d) 7

(iii) What is the number on the 19th card?

    (a) –88   (b) –83   (c) –92   (d) –102

(iv) What is the number on the 23rd card?

    (a) –103   (b) –122   (c) –108   (d) –117

(v) The sum of numbers on the first 15 cards is

    (a) –840   (b) –945   (c) –427   (b) –420

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Treasure Hunt Game

While playing a treasure hunt game, some clues (numbers) are hidden in various spots collectively forms an A.P. If the number on the nth spot is $20 + 4n$, then answer the following questions to help the player in spotting the clues.

(i) Which number is on the first spot?

    (a) 20   (b) 24   (c) 16   (d) 28

(ii) Which number is on the $(n-2)^{th}$ spot?

    (a) $16 + 4n$ (b) $24 + 4n$ (c) $12 + 4n$ (d) $28 + 4n$

(iii) Which number is on the 34th spot?

    (a) 156   (b) 116   (c) 120   (d) 160

(iv) What is the sum of all the numbers on the first 10 spots?

    (a) 410   (b) 420   (c) 480   (d) 410

(v) Which spot is numbered as 116?

    (a) 5th (b) 8th (c) 9th (d) 24th

8

A sequence is an ordered list of numbers. A sequence of numbers such that the difference between the consecutive terms is constant is said to be an arithmetic progression (A.P.).

On the basis of above information, answer the following questions.

(i) Which of the following sequence is an A.P.?

    (a) 10, 24, 39, 52, ….   (b) 11, 24, 39, 52, …   (c) 10, 24, 38, 52, …   (d) 10, 38, 52, 66, ….

(ii) If x, y and z are in A.P., then

    (a) $x + z = y$ (b) $x - z = y$ (c) $x + z = 2y$ (d) None of these

(iii) If $a_1, a_2, a_3, ….., a_n$ are in A.P., then which of the following is true?

    (a) $a_1 + k, a_2 + k, a_3 + k, ….., a_n + k$ are in A.P., where k is a constant.

    (b) $k - a_1, k - a_2, k - a_3, ….., k - a_n$ are in A.P., where k is a constant.

    (c) $ka_1, ka_2, ka_3, ….., ka_n$ are in A.P., where k is a constant.

    (d) All of these

(iv) If the nth term $(n > 1)$ of an A.P. is smaller than the first term, then nature of its common difference (d) is

    (a) $d > 0$ (b) $d < 0$

    (c) $d = 0$ (d) Can't be determined

(v) Which of the following is incorrect about A.P.?

    (a) All the terms of constant A.P. are same.

    (b) Some terms of an A.P. can be negative.

    (c) All the terms of an A.P. can never be negative.

    (d) None of these.

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Assessment on A.P.

Jack is much worried about his upcoming assessment on A.P. He was vigorously practicing for the exam but unable to solve some questions. One of these questions is as shown below.

If the 3rd and the 9th terms of an A.P. are 4 and -8 respectively, then help Jack in solving the problem.

(i) What is the common difference?
   (a) 2   (b) –1   (c) –2   (d) 4

(ii) What is the first term?
   (a) 6   (b) 2   (c) –2   (d) 8

(iii) Which term of the A.P. is –160?
   (a) 80th (b) 85th (c) 81th (d) 84th

(iv) Which of the following is not a term of the given A.P.?
   (a) –123   (b) –100   (c) 0   (d) –200

(v) What is the 75th term of the A.P.?
   (a) –140   (b) –102   (c) –150   (d) –158

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Application of A.P. in Day to Day Life

Do you know, we can find A.P. in many situations in our day-to-day life. One such example is a tissue paper roll, in which the first term is the diameter of the core of the roll and twice the thickness of the paper is the common difference. If the sum of first n rolls of tissue on a roll is $S_n = 0.1\,n^2 + 7.9n$, then answer the following questions.

(i) Find S_{n-1}.
   (a) $0.1n^2 - 0.2n - 7.8$ (b) $0.1n^2 - 7.9n$
   (c) $0.1n^2 + 7.7n - 7.8$ (d) None of these

(ii) Find the radius of the core.
   (a) 8 cm   (b) 4 cm
   (c) 16 cm   (d) Can't be determined

(iii) $S_2 =$
   (a) 16.2   (b) 8.2   (c) 2.8   (d) 4.8

(iv) What is the diameter of roll when one tissue sheet is rolled over it?
   (a) 7.6 cm   (b) 7.9 cm   (c) 8.1 cm   (d) 8.2 cm

(v) Find the thickness of each tissue sheet.
   (a) 2 cm   (b) 1 cm   (c) 1 mm   (d) 2 mm

1. Here the smallest 3-digit number divisible by 7 is 105. So, the number of bacteria taken into consideration is 105, 112, 119, ..., 994
So, first term $(a) = 105$, $d = 7$ and last term $= 994$

(i) (c): $t_5 = a + 4d = 105 + 28 = 133$

(ii) (b): Let n samples be taken under consideration.
∵ Last term $= 994$
$\Rightarrow a + (n-1)d = 994 \Rightarrow 105 + (n-1)7 = 994 \Rightarrow n = 128$

(iii) (a): Total number of bacteria in first 10 samples
$= S_{10} = \dfrac{10}{2}[2(105)+9(7)] = 1365$

(iv) (a): t_7 from end $= (128 - 7 + 1)^{th}$ term from beginning $= 122^{th}$ term $= 105 + 121(7) = 952$

(v) (c): $t_{50} = 105 + 49 \times 7 = 448$

2. Geeta's A.P. is –5, –2, 1, 4, ...
Here, first term $(a_1) = -5$ and common difference $(d_1) = -2 + 5 = 3$
Similarly, Madhuri's A.P. is 187, 184, 181, ...
Here first term $(a_2) = 187$ and common difference $(d_2) = 184 - 187 = -3$

(i) (b): $t_{34} = a_2 + 33d_2 = 187 + 33(-3) = 88$

(ii) (d): Required sum $= 3 + (-3) = 0$

(iii) (a): $t_{19} = a_1 + 18d_1 = (-5) + 18(3) = 49$

(iv) (a): $S_{10} = \dfrac{n}{2}[2a_1 + (n-1)d_1] = \dfrac{10}{2}[2(-5)+9(3)] = 85$

(v) (b): Let n^{th} terms of the two A.P.'s be equal.
∴ $-5 + (n-1)3 = 187 + (n-1)(-3)$
$\Rightarrow 6(n-1) = 192 \Rightarrow n = 33$

3. Number of pairs of shoes in 1st, 2nd, 3rd row, ... are 3, 5, 7, ...
So, it forms an A.P. with first term $a = 3$, $d = 5 - 3 = 2$

(i) (d): Let n be the number of rows required.
∴ $S_n = 120$
$\Rightarrow \dfrac{n}{2}[2(3)+(n-1)2] = 120$
$\Rightarrow n^2 + 2n - 120 = 0 \Rightarrow n^2 + 12n - 10n - 120 = 0$
$\Rightarrow (n + 12)(n - 10) = 0 \Rightarrow n = 10$
So, 10 rows required to put 120 pairs.

(ii) (b): No. of pairs in 17th row $= t_{17} = 3 + 16(2) = 35$
No. of pairs in 10th row $= t_{10} = 3 + 9(2) = 21$
∴ Required difference $= 35 - 21 = 14$

(iii) (c): Here $n = 15$
∴ $t_{15} = 3 + 14(2) = 3 + 28 = 31$

(iv) (a): No. of pairs in 30th row $= t_{30} = 3 + 29(2) = 61$

(v) (c): No. of pairs in 5th row $= t_5 = 3 + 4(2) = 11$
No. of pairs in 8th row $= t_8 = 3 + 7(2) = 17$
∴ Required sum $= 11 + 17 = 28$

4. Here the savings form an A.P. i.e., ₹ 2.75, ₹ 3, ₹ 3.25, ...
So, $a = 2.75$, $d = 3 - 2.75 = 0.25$

(i) (b): Amount saved by Anuj on 14th day
$= t_{14} = a + 13d = 2.75 + 13(0.25) = ₹ 6$

(ii) (d): Total amount saved by Anuj in 8 days
$= S_8 = \dfrac{8}{2}[2(2.75)+7(0.25)] = ₹ 29$

(iii) (a): Amount saved by Anuj on 30th day
$= t_{30} = a + 29d = 2.75 + 29(0.25) = ₹ 10$

(iv) (b): Number of days in June $= 30$
∴ $S_{30} = \dfrac{30}{2}[2(2.75)+29(0.25)] = ₹ 191.25$

(v) (d): Let on n^{th} day, he saves 10 times as he saves on 1st day.
$t_n = 10(2.75) \Rightarrow a + (n - 1)d = 27.5 \Rightarrow n = 100$

5. Let the number of sea shells in the cells be of the form a, $a + d$, $a + 2d$, ...
According to question, we have
$(a + 2d) + (a + 10d) = 68$
$\Rightarrow 2a + 12d = 68 \Rightarrow a + 6d = 34$ ...(1)
Also, $(a + 10d) - (a + 2d) = 24 \Rightarrow d = 3$
From (1), we get $a + 6(3) = 34 \Rightarrow a = 16$

(i) (b): Required difference, $d = 3$

(ii) (c): Number of sea shells in the first cell $(a) = 16$

(iii) (a): Total number of sea shells in 13 cells $= S_{13}$
$= \dfrac{13}{2}[2(16)+12(3)] = 6.5(68) = 442$

(iv) (d): $S_5 = \dfrac{5}{2}[2(16)+4(3)] = 110$

(v) (c): Required sum $= t_7 + t_9 = (a + 6d) + (a + 8d)$
$= 2(16) + 14(3) = 74$

6. Let the numbers on the cards be a, $a + d$, $a + 2d$, ...
According to question, We have $(a + 5d) + (a + 13d)$
$= -76$

$\Rightarrow 2a + 18d = -76 \Rightarrow a + 9d = -38$ ...(1)
And $(a + 7d) + (a + 15d) = -96$
$\Rightarrow 2a + 22d = -96 \Rightarrow a + 11d = -48$ ...(2)
From (1) and (2), we get
$2d = -10 \Rightarrow d = -5$
From (1), $a + 9(-5) = -38 \Rightarrow a = 7$

(i) (b): The difference between the numbers on any two consecutive cards = common difference of the A.P. = -5

(ii) (d): Number on first card $= a = 7$

(iii) (b): Number on 19th card $= a + 18d = 7 + 18(-5)$
$= -83$

(iv) (a): Number on 23rd card $= a + 22d = 7 + 22(-5)$
$= -103$

(v) (d): $S_{15} = \dfrac{15}{2}[2(7) + 14(-5)] = -420$

7. Number on n^{th} spot $= 20 + 4n$
i.e., $t_n = 20 + 4n$

(i) (b): Number on 1st spot $= t_1 = 20 + 4(1) = 24$

(ii) (c): Number on $(n - 2)^{th}$ spot $= t_{n-2}$
$= 20 + 4(n - 2)$
$= 20 + 4n - 8 = 12 + 4n$

(iii) (a): Number on 34th spot $= t_{34}$
$= 20 + 4(34) = 156$

(iv) (b): Here $a = t_1 = 24$
Now, $t_2 = 20 + 4(2) = 20 + 8 = 28$
$\therefore d = t_2 - t_1 = 4$

So, required sum $= S_{10} = \dfrac{10}{2}[2(24) + 9(4)] = 420$

(v) (d): Let n^{th} spot is numbered as 116.
$\therefore t_n = 116$
$\Rightarrow 20 + 4n = 116 \Rightarrow 4n = 96 \Rightarrow n = 24$

8. (i) (c) (ii) (c) (iii) (d)
(iv) (b) (v) (c)

9. We have, 3rd term = 4 and 9th term = -8
i.e., $a + 2d = 4$ ...(1)
and $a + 8d = -8$ ...(2)
Solving (1) and (2), we get
$d = -2, a = 8$

(i) (c) (ii) (d)

(iii) (b): Let $t_n = -160 \Rightarrow a + (n - 1)d = -160$
$\Rightarrow 8 + (n - 1)(-2) = -160 \Rightarrow (n - 1)(-2) = -168$
$\Rightarrow n - 1 = 84 \Rightarrow n = 85$
So, $t_{85} = -160$

(iv) (a)

(v) (a): $t_{75} = a + 74d = 8 + 74(-2) = -140$

10. Here $S_n = 0.1n^2 + 7.9n$

(i) (c): $S_{n-1} = 0.1(n - 1)^2 + 7.9(n - 1)$
$= 0.1n^2 + 7.7n - 7.8$

(ii) (b): $S_1 = t_1 = a = 0.1(1)^2 + 7.9(1) = 8$ cm
$=$ Diameter of core
So, radius of the core = 4 cm

(iii) (a): $S_2 = 0.1(2)^2 + 7.9(2) = 16.2$

(iv) (d): Required diameter $= t_2 = S_2 - S_1 = 16.2 - 8$
$= 8.2$ cm

(v) (c): As $d = t_2 - t_1 = 8.2 - 8 = 0.2$ cm
So, thickness of tissue $= 0.2 \div 2 = 0.1$ cm = 1 mm