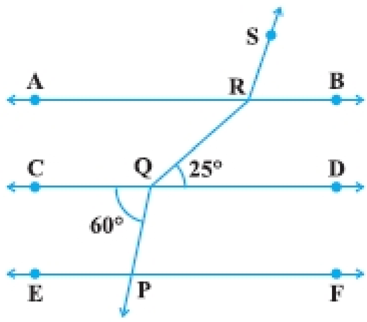
Chapter 6
Lines and Angles

Exercise No. 6.1

Multiple Choice Questions:

Write the correct answer in each of the following:

1. In Fig., if AB || CD || EF, PQ || RS, ∠RQD = 25° and ∠CQP = 60°, then ∠QRS is equal to



(A) 85°
(B) 135°
(C) 145°
(D) 110°

Solution:

As $\angle ARQ = \angle RQD = 25^\circ$ [alt. $\angle s$]

Also, $\angle RQC = 180^\circ - 60^\circ = 120^\circ$ (linear pair)

And, $\angle SRA = 120^\circ$ (Corresponding angle)

Now,

$\angle SRQ = 120^\circ + 25^\circ$

$\angle SRQ = 145^\circ$

Hence, the correct option is (C).

2. If one angle of a triangle is equal to the sum of the other two angles, then the triangle is
(A) an isosceles triangle
(B) an obtuse triangle
(C) an equilateral triangle
(D) a right triangle

Solution:

Given

Let angle of triangle ABC be $\angle A, \angle B$ and $\angle C$
Given that:
$\angle A = \angle B + \angle C$ … (I)

We know that in any triangle $\angle A + \angle B + \angle C = 180^\circ$ … (II)
From equation (I) and (II), get:

$$\angle A + \angle A = 180^\circ$$
$$2\angle A = 180^\circ$$
$$\angle A = \frac{180^\circ}{2}$$
$$\angle A = 90^\circ$$

Hence, the triangle is a right triangle.
Therefore, the correct option is (D).

3. An exterior angle of a triangle is 105° and its two interior opposite angles are equal. Each of these equal angles is

(A) $37\frac{1}{2}^\circ$

(B) $52\frac{1}{2}^\circ$

(C) $72\frac{1}{2}^\circ$

(D) 75°

Solution:

Given: An exterior angle of triangle is 150°.
Let each of the two interior opposite angle be x.

The sum of two interior opposite angle is equal to exterior angle of a triangle. So,

$$105^\circ = x + x$$
$$2x = 105^\circ$$
$$x = 52\frac{1}{2}^\circ$$

Hence, the correct option is (B).

4. The angles of a triangle are in the ratio 5 : 3 : 7. The triangle is
(A) an acute angled triangle
(B) an obtuse angled triangle
(C) a right triangle
(D) an isosceles triangle

Solution:

Let the angle of the triangle are 5x, 3x and 7x. As we know that sum of all angle of triangle is 180°. Now,

$$5x+3x+7x=180^\circ$$
$$15x=180^\circ$$
$$x=\frac{180^\circ}{15}$$
$$x=12^\circ$$

Hence, the angle of the triangle are:

$$5\times12^\circ=60^\circ$$
$$3\times12^\circ=36^\circ$$
$$7\times12^\circ=84^\circ$$

All the angle of this triangle is less than 90 degree.
Hence,, the triangle is an acute angled triangle.

5. If one of the angles of a triangle is 130°, then the angle between the bisectors of the other two angles can be
(A) 50°
(B) 65°
(C) 145°
(D) 155°

Solution:

In triangle ABC, Let $\angle A=130^\circ$.
The bisector of the angle B and C are OB and OC.
Let $\angle OBC=\angle OBA=x$ and $\angle OCB=\angle OCA=y$

In triangle ABC,

$$\angle A+\angle B+\angle C=180^\circ$$
$$130^\circ+2x+2y=180^\circ$$
$$2x+2y=180^\circ-130^\circ$$
$$2x+2y=50^\circ$$
$$x+y=25^\circ$$

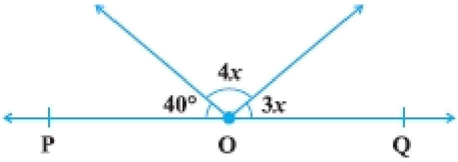
That is $\angle OBC+\angle OCA=25^\circ$

Now, in triangle BOC:

$$\angle BOC=180^\circ-(\angle OBC+\angle OCB)$$
$$=180^\circ-25^\circ$$
$$=155^\circ$$

Hence, the correct option is (D).

6. In Fig., POQ is a line. The value of x is



(A) 20°
(B) 25°
(C) 30°
(D) 35°

Solution:
See the given figure in the question:

$40^\circ + 4x + 3x = 180^\circ$ (Angles on the straight line)

$4x + 3x = 180^\circ - 40^\circ$

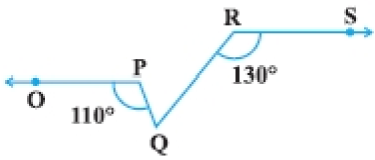
$7x = 140^\circ$

$x = \dfrac{140^\circ}{7}$

$x = 20^\circ$

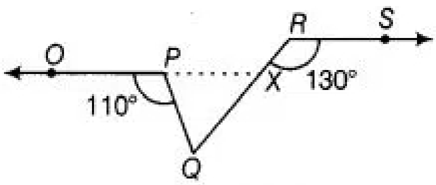
Hence, the correct option is (A).

7. In Fig., if OP||RS, ∠OPQ = 110° and ∠QRS = 130°, then ∠ PQR is equal to



(A) 40°
(B) 50°
(C) 60°
(D) 70°

Solution:
See the given figure, producing OP, to intersect RQ at X.
Given: OP||RS and RX is a transversal.
So, $\angle RXP = \angle XRS$ (alternative angle)



$\angle RXP = 130°$ [Given: $\angle QRS = 130°$]
RQ is a line segment.
So, $\angle PXQ + \angle RXV = 180°$ [linear pair axiom]

$\angle PXQ = 180° - \angle RXP = 180° - 130°$
$\angle PXQ = 50°$

In triangle PQX, $\angle OPQ$ is an exterior angle,
Therefore, $\angle OPQ = \angle PXQ + \angle PQX$ [exterior angle = sum of two opposite interior angles]
$110° = 50° + \angle PQX$
$\angle PQX = 110° - 50°$
$\angle PQR = 60°$

Hence, the correct option is (

8. Angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is
(A) 60°
(B) 40°
(C) 80°
(D) 20°

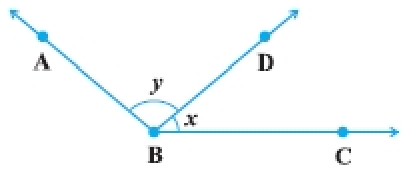
Solution:
Given, the ratio of angles of a triangle is 2 : 4 : 3.
Let the angles of a triangle be ∠A, ∠B and ∠C.
∠A = 2x, ∠B = 4x ∠C = 3x,
∠A+∠B+ ∠C= 180° [sum of all the angles of a triangle is 180°]
2x + 4x + 3x = 180°
9x = 180°
x =180°/9
=20°

∠A=2x =2 x 20° = 40°
∠B = 4x = 4 x 20° = 80°
∠C = 3x = 3 x 20° = 60°
So, the smallest angle of a triangle is 40°.
Hence, the correct option is (B).

Exercise No. 6.2

Short Answer Questions with Reasoning:

1. For what value of $x+y$ in Fig. will ABC be a line? Justify your answer.



Solution:
See the figure, x and y are two adjacent angles.
For ABC to be a straight line, the sum of two adjacent angle must be $180°$.

2. Can a triangle have all angles less than 60°? Give reason for your answer.

Solution:
We know that in a triangle, sum of all the angles is always $180°$. So, a triangle can't have all angles less than $60°$.

3. Can a triangle have two obtuse angles? Give reason for your answer.

Solution:
If an angle whose measure is more than $90°$ but less than $180°$ is called an obtuse angle.
We know that a triangle can't have two obtuse angle because the sum of all the angles of it can't be more than $180°$. It is always equal to $180°$.

4. How many triangles can be drawn having its angles as 45°, 64° and 72°? Give reason for your answer.

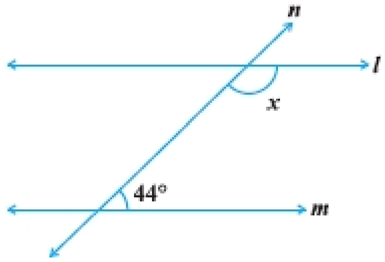
Solution:
We know that sum of all the angles in a triangle is $180°$.
The sum of all the angles is $45°+64°+72°=181°$. So, we can't draw any triangle having sum of all the angle $181°$.

5. How many triangles can be drawn having its angles as 53°, 64° and 63°? Give reason for your answer.

Solution:
We know that sum of all the angles in a triangle is $180°$.

Sum of these angles = $53° + 64° + 63° = 180°$. So, we can draw infinitely many triangles having its angles as 53°, 64° and 63°.

6. In Fig., find the value of *x* for which the lines *l* and *m* are parallel.



Solution:

See the given figure, l || m and if a transversal intersects two parallel lines, then sum of interior angles on the same side of a transversal is supplementary.

$x + 44° = 180°$

$x = 180° - 44°$

$x = 136°$

7. Two adjacent angles are equal. Is it necessary that each of these angles will be a right angle? Justify your answer.

Solution:

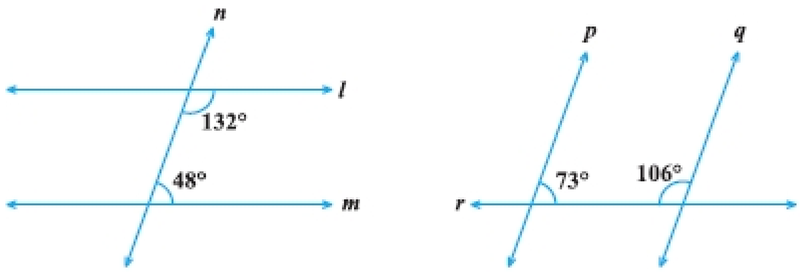
No, because if it will be a right angle only when they form a linear pair.

8. If one of the angles formed by two intersecting lines is a right angle, what can you say about the other three angles? Give reason for your answer.

Solution:

If two intersecting line are formed right then by using linear pair axiom aniom, other three angles will be a right angle.

9. In Fig., which of the two lines are parallel and why?



Solution:

In the first figure, sum of two interior angle is:

$132° + 48° = 180°$ [Equal to 180°]
Hence, we know that, if sum of two interior angle are equal on the same side of n is 180°, then they are the parallel lines.

In the second figure, sum of two interior angle is:
$73° + 106° = 179° \neq 180°$.
Hence, we know that, if sum of two interior angle are equal on the same side of r is not equal to 180°, then they are not the parallel lines.

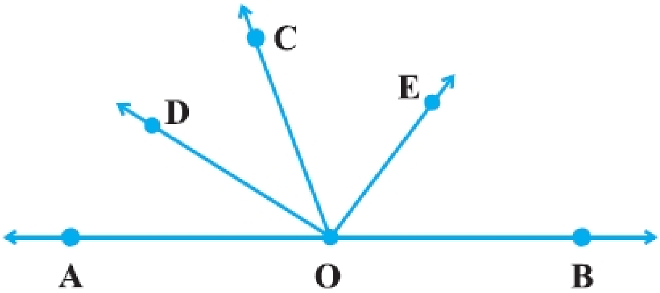
10. Two lines *l* and *m* are perpendicular to the same line *n*. Are *l* and *m* perpendicular to each other? Give reason for your answer.

Solution:
If two lines l and m are perpendicular to the same line n, then each of the two corresponding angles formed by these lines l and m with the line n are equal to $90°$.
Hence the line l and m are not perpendicular but parallel.

Exercise No. 6.3

Short Answer Questions:

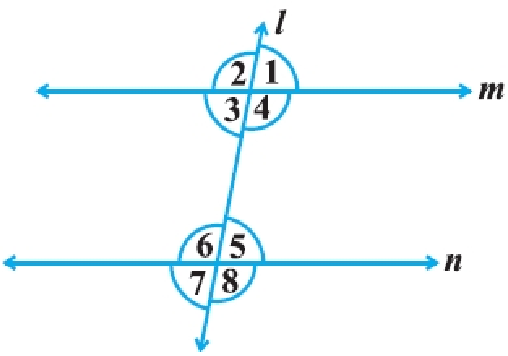
1. In Fig., OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.



Solution:
Given:
OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE
To prove that point A, O and B are collinear that is AOB are straight line.
$\angle AOC = 2\angle DOC$ … (I)
$\angle COB = 2\angle COE$ … (II)

Now, adding equations (I) and (II), get:
$\angle AOC + \angle COB = 2\angle DOC + \angle COE$
$\angle AOC + \angle COB = 2(\angle DOC + \angle COE)$
$\angle AOC + \angle COB = 2\angle DOC$
$\angle AOC + \angle COB = 2 \times 90°$
$\angle AOC + \angle COB = 180°$
$\angle AOC = 180°$
So, $\angle AOC + \angle COB$ are forming linear pair or we can say that AOB is a straight line.
Hence, point A, O and B are collinear.

2. In Fig., ∠1 = 60° and ∠6 = 120°. Show that the lines *m* and *n* are parallel.



Solution:
See the given figure,
$\angle 5+\angle 6=180°$ (Linear pair angle)
$\angle 5+120°=180°$

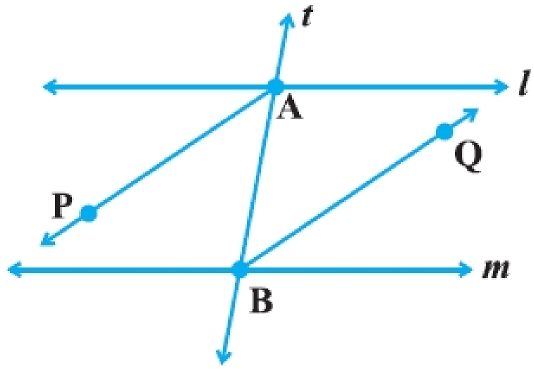
$\angle 5=180°-120°$

$\angle 5=60°$

Then, $\angle 1=\angle 5$ [Each = $60°$]
Since, these are corresponding angles.

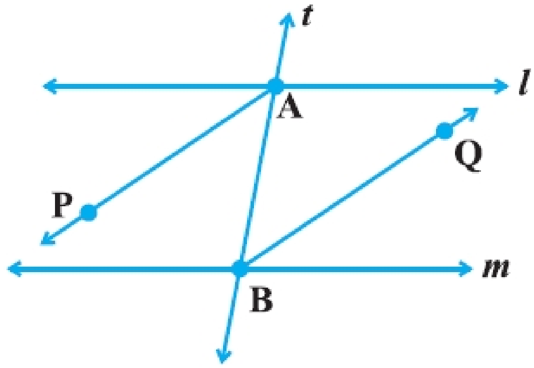
Hence, the line m and n are parallel.

3. AP and BQ are the bisectors of the two alternate interior angles formed by the intersection of a transversal *t* with parallel lines *l* and *m*. Show that AP || BQ.



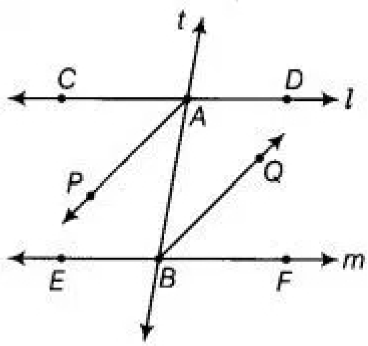
Solution:
According to the question,
Line l || m and t is the transversal.
$\angle MAB=\angle SBA$ [Alt. $\angle s$]
$\frac{1}{2}\angle MAB=\frac{1}{2}\angle SBA$
$\angle PAB=\angle QBA$
But, $\angle PAB$ and $\angle QBA$ are alternate angles.
Hence, AP||BQ.

4. If in Fig., bisectors AP and BQ of the alternate interior angles are parallel, then show that *l* || *m*.



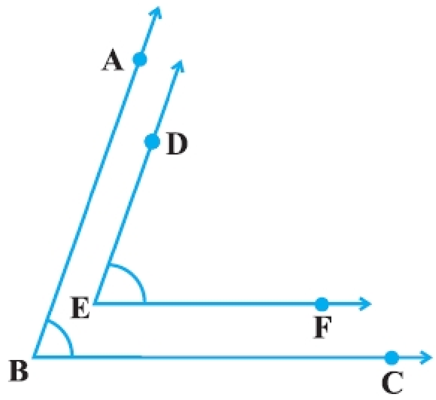
Solution:
See the given figure, AP||BQ, AP and BQ are the bisectors of alternate interior angles $\angle CAB$ and $\angle ABF$.
To show that l||m.

Now, prove that AP||BQ are t is transversal, therefore:
$\angle PAB = \angle ABQ$ [Alternate interior angle] … (I)
$2\angle PAB = 2\angle ABQ$ [Multiplying both sides by 2 in equation (I)]

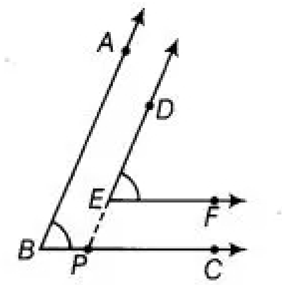


Since, alternate interior angle are equal.
So, if two alternate interior angle are equal then lines are parallel.
Hence, l||m.

5. In Fig., BA || ED and BC || EF. Show that ∠ABC = ∠DEF.
[Hint: Produce DE to intersect BC at P (say)].



Solution:
According to the question:
Given:
Producing DE to intersect BC at P.
EF||BC and DP is the transversal,

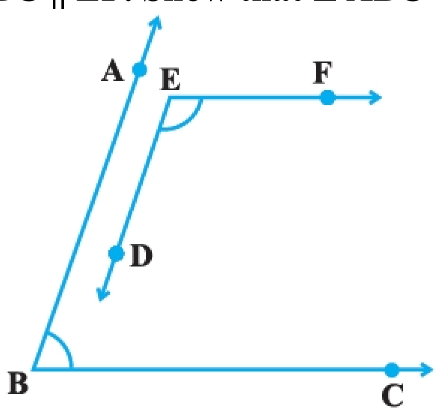


$\angle DEF = \angle DPC$ … (I) [Corresponding $\angle s$]

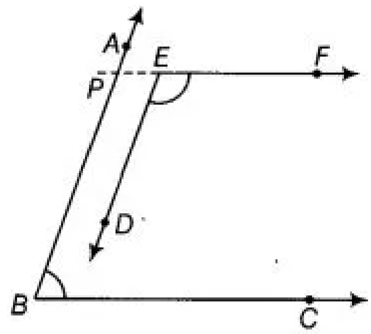
See the above figure, AB||DP and BC is the transversal,
$\angle DPC = \angle ABC$ … (II) [Corresponding $\angle s$]

Now, from equation (I) and (II), get:
$\angle ABC = \angle DEF$
Hence, proved.

6. In Fig., BA || ED and BC || EF. Show that ∠ ABC + ∠ DEF = 180°.



Solution:
See in the figure, BA || ED and BC || EF.
Show that ∠ABC + ∠DEF = 180°.
Produce a ray PE opposite to ray EF.

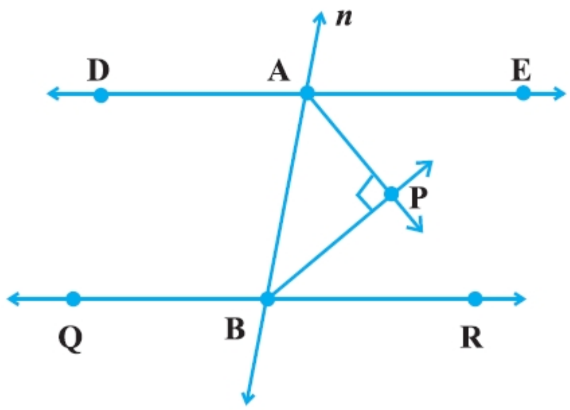


Prove: BC||EF
Now, $\angle EPB + \angle PBC = 180°$ [sum of co interior is $180°$] …(I)

Now, AB||ED and PE is transversal line,
$\angle EPB = \angle DEF$ [Corresponding angles] …(II)

Now, from equation (I) and (II),
$\angle DEF + \angle PBC = 180°$
$\angle ABC + \angle DEF = 180°$ [Because $\angle PBC = \angle ABC$]
Hence, proved.

7. In Fig., DE || QR and AP and BP are bisectors of ∠ EAB and ∠ RBA, respectively. Find ∠APB.



Solution:
See in the given figure, DE||QR and the line n is the transversal line.
$\angle EAB + \angle RBA = 180°$ …(I) [The interior angles on the same side of transversal are supplementary.]

Now, $\angle PAB + \angle PBA = 90°$
Then, from triangle APB, given:

$\angle APB = 180° - (\angle PAB + \angle PBA)$
So, $\angle APB = 180° - 90° = 90°$

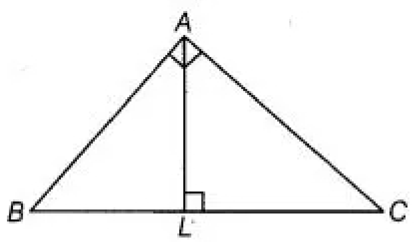
8. The angles of a triangle are in the ratio 2 : 3 : 4. Find the angles of the triangle.

Solution:
Given in the question, ratio of angles is: 2 : 3 : 4.
Let the angles of the triangle be 2x, 3x and 4x.
So,
$2x + 3x + 4x = 180°$ [sum of angles of triangle is $180°$]
$9x = 180°$
$x = \frac{180°}{9}$
$x = 20°$

Therefore, $2x = 2 \times 20° = 40°$
$3x = 2 \times 20° = 60°$
And, $4x = 4 \times 20° = 80°$
Hence, the angle of the triangles are $40°$, $60°$ and $80°$.

9. A triangle ABC is right angled at A. L is a point on BC such that AL ⊥ BC. Prove that ∠ BAL = ∠ ACB.

Solution:



Given:
In triangle ABC,
$\angle A = 90°$ and $AL \perp BC$
To prove: $\angle BAL = \angle ACB$

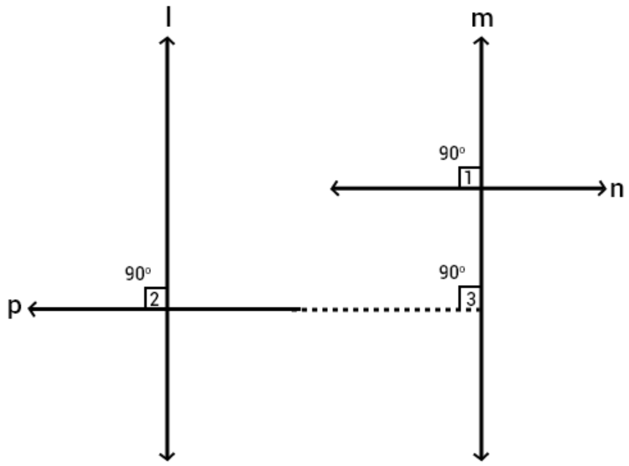
Proof: Let $\angle ABC = x$
$\angle BAL = 90° - x$
As, $\angle A = x$
$\angle CAL = x$
$\angle ABC = \angle CAL$
$\angle ABC = \angle ACB$

Hence, proved.

10. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

Solution:

According to the question:



Two line p and n are respectively perpendicular to two parallel line l and m, that is $P \perp l$ and $n \perp m$.
To prove that p is parallel to n.
Given: $n \perp m$
So, $\angle 1 = 90°$ … (I)
Now, $P \perp l$
So, $\angle 2 = 90°$

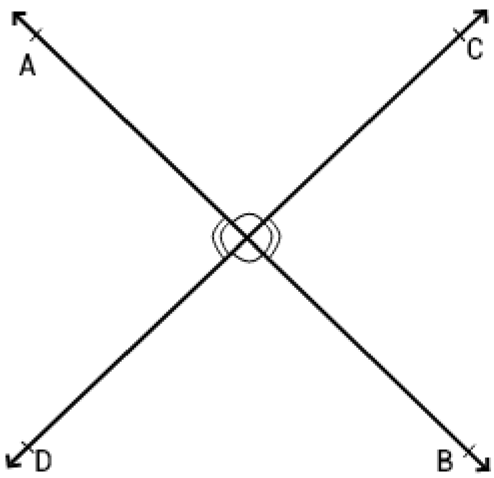
Since, l is parallel to m. So,
$\angle 2 = \angle 3$ [Corresponding $\angle s$]
So,
$\angle 2 = 90°$ … (II)
From equation (I) and (II), get:
$\angle 1 = \angle 3$ [each $90°$]
But these are corresponding angles.
Hence, p||n.

Exercise No. 6.4

Long Answer Questions:

1. If two lines intersect, prove that the vertically opposite angles are equal.

Solution:



Two lines AB and CD intersect at point O.
To prove: (i) $\angle AOC = \angle BOD$
(ii) $\angle AOD = \angle BOC$

Proof: (i)
Ray on stands on line CD. So,
$\angle AOC + \angle AOD = 180°$ …(I) [linear pair axiom]

Similarly, ray OD stands on line AB. So,
$\angle AOD + \angle BOD = 180°$ …(II)
Now, from equation (I) and (II), get:
$\angle AOC + \angle AOD = \angle AOD + \angle BOD$

$$\angle AOC = \angle BOD$$

Hence, proved.

(ii) Ray OD stands on line AB.
$\angle AOD + \angle BOD = 180°$ … (III) [Linear pair axiom]

Similarly, ray OB stands on line CD. So,
$\angle DOB + \angle BOC = 180°$ … (IV)
From equations (III) and (IV), get:
$\angle AOD + \angle BOD = \angle DOB + \angle BOC$

$$\angle AOD = \angle BOC$$

Hence, proved.

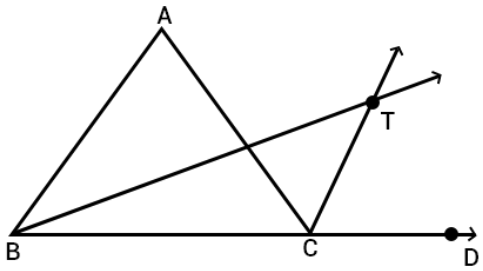
2. Bisectors of interior ∠B and exterior ∠ACD of a Δ ABC intersect at the point T. Prove that

$\angle \mathrm{BTC} = \frac{1}{2}\angle \mathrm{BAC}$

Solution:

Given: in triangle ABC, produce BC to D and the bisectors of $\angle ABC$ and $\angle ACD$ meet at point T.

To prove that $\angle BTC = \frac{1}{2}\angle BAC$



Proof: In triangle ABC, $\angle ACD$ is an exterior angle.

$\angle ACD = \angle ABC + \angle CAB$ [We know that exterior angle of a triangle is equal to the sum of two opposite angle]

$\frac{1}{2}\angle ACD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ [Dividing both sides by 2 in the above equation]

$\angle TCD = \frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC$ …(I) [Since, CT is the bisector of $\angle ACD$ that is $\frac{1}{2}\angle ACD = \angle TCD$]

Now, in triangle BTC,

$\angle TCD = \angle BTC + \angle CBT$ [We know that exterior angle of the triangle is equal to the sum of two opposite angles]

$\angle TCD = \angle BTC + \frac{1}{2}\angle ABC$ …(II) [Since, BT is the bisector of triangle ABC $\angle CBT = \frac{1}{2}\angle ABC$]

Now, from equation (I) and (II), get:

$$\frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC = \angle BTC + \frac{1}{2}\angle ABC$$

$$\frac{1}{2}\angle CAB = \angle BTC$$

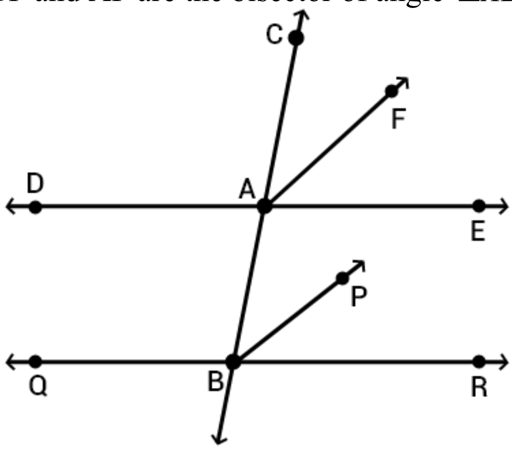
$$\frac{1}{2}\angle BAC = \angle BTC$$

Hence, proved.

3. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Solution:

Given: Lines DE||QR and the line DE intersected by transversal at A and the line QR intersected by transversal at B. Also, BP and AF are the bisector of angle $\angle ABR$ and $\angle CAE$ respectively.



To prove: BP||FA

Proof: DE||QR

$\angle CAE = \angle ABR$ [Corresponding angles]

$\frac{1}{2}\angle CAE = \frac{1}{2}\angle ABR$ [Dividing both side by 2 in the above equation]

$\angle CAF = \angle ABP$ [Since, bisector of angle $\angle ABR$ and $\angle CAE$ are BP and AF respectively]

Because these are the corresponding angles on transversal line n and are equal.

Hence, BP||FA.

4. Prove that through a given point, we can draw only one perpendicular to a given line.
[Hint: Use proof by contradiction].

Solution:

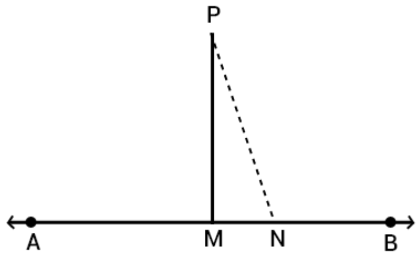
Drawn a perpendicular line from the point p as PM $\perp$ AB. So,

$\angle PMB = 90°$

Let if possible, drown another perpendicular line PN $\perp$ AB. So,

$\angle PMB = 90°$

Since, $\angle PMB = \angle PNB$ it will be possible when PM and PN coincide with each other.



Therefore, at a given point we can draw only one perpendicular to a given line.

5. Prove that two lines that are respectively perpendicular to two intersecting lines intersect each other.
[Hint: Use proof by contradiction].

Solution:

Given:

Let lines *l* and m are two intersecting lines. Again, let $n \perp p$ to the intersecting lines meet at point D.

To prove that two lines n and p intersecting at a point.

Proof:

Let consider that line n and p are intersecting each other it means lines n and p are parallel to each other.

n||p ...(I)

Therefore, lines n and p are perpendicular to m and *l* respectively.

Now, by using equation (I), n||p, it means that *l* and m. it is a contradiction.

Since, our assumption is wrong.

Hence, line *n* and p are intersect at a point.

6. Prove that a triangle must have at least two acute angles.

Solution:

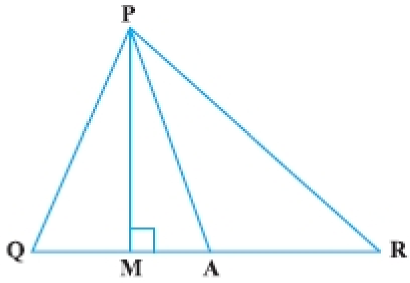
If triangle is an acute triangle then all the angle will be acute angle and sum of the all angle will be $180°$.

If a triangle is a right angle triangle then one angle will be equal to $90°$ and remaining two angle will be acute angles and sum of all the angles will be $180°$.

Hence, a triangle must have at least two acute angles.

7. In Fig., ∠Q > ∠R, PA is the bisector of ∠QPR and PM ⊥ QR. Prove that

$\angle APM = \frac{1}{2}(\angle Q - \angle R)$



Solution:

Given in triangle PQR, $\angle Q > \angle R$, PA is the bisector of $\angle QPR$ and PM $\perp$ QR.

To prove that $\angle APM = \frac{1}{2}(\angle Q - \angle R)$

Proof: PA is the bisector of $\angle QPR$. So,

$\angle QPA = \angle APR$

In angle PQM, $\angle Q + \angle PMQ + \angle QPM = 180°$ … (I) [Angle sum property of a triangle]

$\angle Q + 90° + \angle QPM = 180°$ **[** $\angle PMR = 90°$ **]**

$\angle Q = 90° - \angle QPM$ … (II)

In triangle PMR, $\angle PMR + \angle R + \angle RPM = 180°$ [Angle sum property of a triangle]

$90° + \angle R + \angle RPM = 180°$ **[** $\angle PMR = 90°$ **]**

$\angle R = 180° - 90° - \angle RPM$

$\angle R = 180° - 90° - \angle RPM$

$\angle R = 90° - \angle RPM$ … (III)

Subtracting equation (III) from equation (II), get:

$\angle Q - \angle R = (90° - \angle APM) - (90° - \angle RPM)$

$\angle Q - \angle R = \angle RPM - \angle QPM$

$\angle Q - \angle R = (\angle RPA + \angle APM) - (\angle QPA - \angle APM)$ …(IV)

$\angle Q - \angle R = \angle QPA + \angle APM - \angle QPA + \angle APM$ **[As,** $\angle RPA = \angle QPA$ **]**

$\angle Q - \angle R = 2\angle APM$

$\angle APM = \frac{1}{2}(\angle Q - \angle R)$

Hence, proved.