Chapter 9

Areas of Parallelograms and Triangles

Exercise No. 9.1

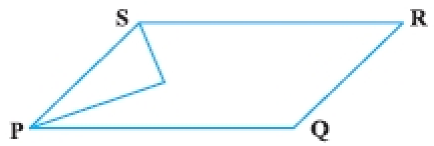
Multiple Choice Questions:

Write the correct answer in each of the following:

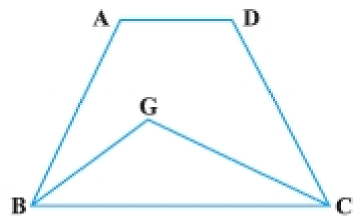
1. The median of a triangle divides it into two
(A) triangles of equal area
(B) congruent triangles
(C) right triangles
(D) isosceles triangles

Solution:
A median of a triangle divides it into two triangles of equal area.
Hence, the correct option is (A).

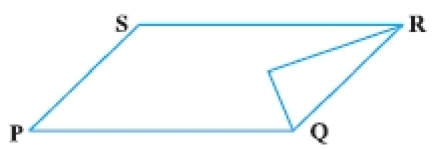
2. In which of the following figures, you find two polygons on the same base and between the same parallels?
(A)



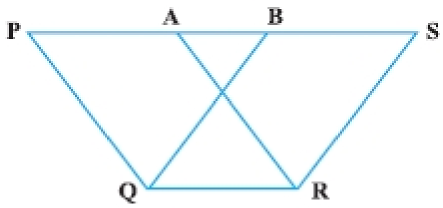
(B)



(C)



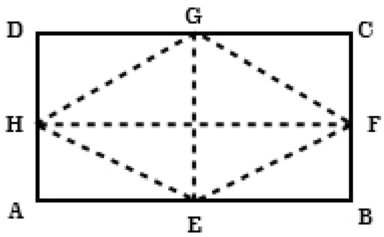
(D)



Solution:
In figure (d), we find two polygons (PQRA and BQRS) on the same base and between the same parallels.
Hence, the correct option is (D).

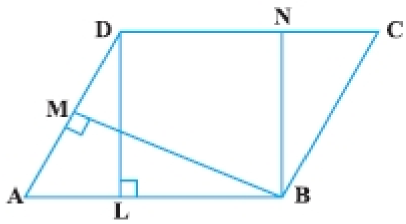
3. The figure obtained by joining the mid-points of the adjacent sides of a rectangle of sides 8 cm and 6 cm is:
(A) a rectangle of area $24\ \text{cm}^2$.
(B) a square of area $25\ \text{cm}^2$.
(C) a trapezium of area $24\ \text{cm}^2$.
(D) a rhombus of area $24\ \text{cm}^2$.

Solution:
According to the question,



ABCD is a rectangle and E, F, G and H are the mid-point of the sides AB, BC, CD and DA respectively. The figure formed is rhombus hose area:
$= \frac{1}{2} \times EG \times FH$
$= \frac{1}{2} \times 6cm \times 8cm$
$= 24cm^2$
Hence, the correct option is (D).

4. In Fig., the area of parallelogram ABCD is:

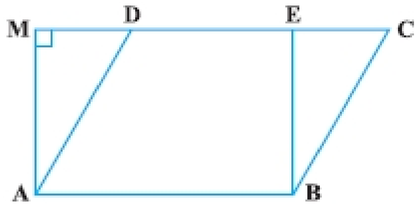


(A) AB × BM

(B) BC × BN
(C) DC × DL
(D) AD × DL

Solution:
Area of parallelogram = Base $\times$Corresponding altitude
$= AB \times DL = DC \times DL$ [Since, AB = DC (opposite side of a parallelogram)]
Hence, the correct option is (C).

5. In Fig., if parallelogram ABCD and rectangle ABEF are of equal area, then:



(A) Perimeter of ABCD = Perimeter of ABEM
(B) Perimeter of ABCD < Perimeter of ABEM
(C) Perimeter of ABCD > Perimeter of ABEM
(D) $\text{Perimeter of ABCD} = \frac{1}{2}(\text{Perimeter of ABEM})$

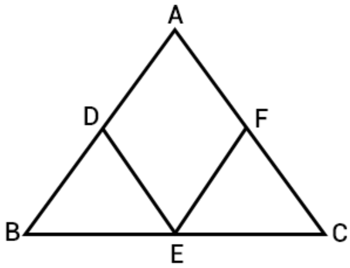
Solution:
If parallelogram ABCD and rectangle ABEF are of equal area then perimeter of ABCD > Perimeter of ABEM because:

As we know that, the perimeter distance between two parallel sides of a parallelogram is always less than the length of the other parallel sides.
BE < BC and AM < AD.
Hence, the correct option is (C).

6. The mid-point of the sides of a triangle along with any of the vertices as the fourth point make a parallelogram of area equal to
(A) $\frac{1}{2}\text{ar}(\text{ABC})$
(B) $\frac{1}{3}\text{ar}(\text{ABC})$
(C) $\frac{1}{4}\text{ar}(\text{ABC})$
(D) $\text{ar}(\text{ABC})$

Solution:

We know that, median of a triangle divides it into two triangle of equal area.



So, $\text{area}(\Delta ADE) = \text{area}(\Delta BDE)$ ...(I)

$\text{area}(\Delta AEF) = \text{area}(\Delta EFC)$ ...(II)

Now, AE is the diagonal of a parallelogram ADEF. That is divides it into two triangles of equal area.

So, $\text{area}(\Delta ADE) = \text{area}(\Delta AFE)$ ...(III)

Now, from equation (I), (II), and (III), get:

$\text{area}(\Delta ADE) = \text{area}(\Delta BDE) = \text{area}(\Delta AEE) = \text{area}(\Delta EFC)$

Hence, $\text{area}(\Delta ADEF) = \frac{1}{2}\text{area}(\Delta ABC)$

Therefore, the correct option is (A).

7. Two parallelograms are on equal bases and between the same parallels. The ratio of their areas is

(A) 1 : 2

(B) 1 : 1

(C) 2 : 1

(D) 3 : 1

Solution:

As we know that parallelogram on the same or equal bases and between the same parallels are equal in area.

So, the ratio of these area is 1:1.

Hence, the correct option is (B).

8. ABCD is a quadrilateral whose diagonal AC divides it into two parts, equal in area, then ABCD

(A) is a rectangle

(B) is always a rhombus

(C) is a parallelogram

(D) need not be any of (A), (B) or (C)

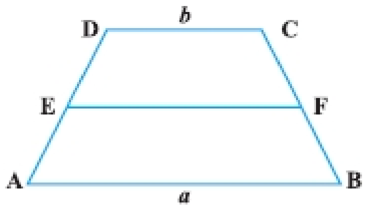
Solution:

The quadrilateral ABCD need not be any of rectangle, rhombus and parallelogram because if quadrilateral ABCD is a square then its diagonal AC also divides it into two parts which are equal in area.
Hence, the correct option is (D).

9. If a triangle and a parallelogram are on the same base and between same parallels, then the ratio of the area of the triangle to the area of parallelogram is
(A) 1 : 3
(B) 1 : 2
(C) 3 : 1
(D) 1 : 4

Solution:
As we know that, if a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram. Therefore, the ratio of the area of the triangle to the area of parallelogram is 1:2.
Hence, the correct option is (B).

10. ABCD is a trapezium with parallel sides AB = *a* cm and DC = *b* cm. E and F are the mid-points of the non-parallel sides. The ratio of ar (ABFE) and ar (EFCD) is



(A) *a* : *b*
(B) (3*a* + *b*) : (*a* + 3*b*)
(C) (*a* + 3*b*) : (3*a* + *b*)
(D) (2*a* + *b*) : (3*a* + *b*)

Solution:
Given:
ABCD is a trapezium with parallel sides such that AB||DC and AB = a cm and DC = b cm. E and F are the mid-points of the non-parallel sides that are AD and BC. So,

$$EF = \frac{1}{2}(a+b)$$

ABEF and EFCD are also trapezium.

$$\text{area}(ABEF) = \frac{1}{2}\left[\frac{1}{2}(a+b)+a\right]\times h = \frac{h}{4}(3a+b)$$

$$\text{area}(EFCD) = \frac{1}{2}\left[b+\frac{1}{2}(a+b)\right]\times h = \frac{h}{4}(a+3b)$$

So,

$$\frac{\text{area}(ABEF)}{\text{area}(EFCD)}=\frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)}=\frac{3a+b}{a+3b}$$

So, the required ratio is (3a+b):(a+3b).
Hence, the correct option is (B).

Exercise No. 9.2

Short Answer Questions with Reasoning:

Write True or False and justify your answer:

1. ABCD is a parallelogram and X is the mid-point of AB. If $\text{ar}(\text{AXCD}) = 24\text{ cm}^2$**, then** $\text{ar}(\text{ABC}) = 24\text{ cm}^2$**.**

Solution:

Given in the question, ABCD is a parallelogram and X is the mid-point of AB.
So, $\text{area}(ABCD) = \text{area}(AXCD) + \text{area}(\Delta XBC)$ … (I)

Now, diagonal AC of a parallelogram divides it into two triangles of equal area.
$\text{area}(ABCD) = 2\text{area}(\Delta ABC)$ … (II)

Similarly, X is the mid-point of AB, So,
$\text{area}(\Delta CXB) = \frac{1}{2}\text{area}(\Delta ABC)$ … (III)[Median divides the triangle in two triangles of equal area]

$2\text{area}(\Delta ABC) = 24 + \frac{1}{2}\text{area}(\Delta ABC)$ [By using equation (I), (II) and (III)]

Now, $2\text{area}(\Delta ABC) - \frac{1}{2}\text{area}(\Delta ABC) = 24$

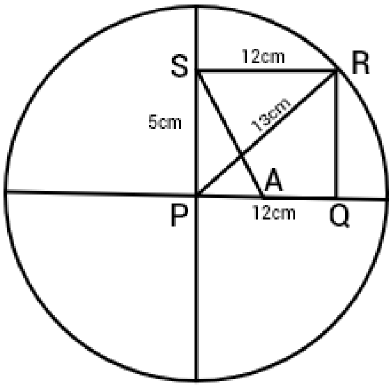
$\frac{3}{2}\text{area}(\Delta ABC) = 24$

Therefore, $\text{area}(\Delta ABC) = \frac{2\times 24}{3} = 16\text{cm}^2$.

2. PQRS is a rectangle inscribed in a quadrant of a circle of radius 13 cm. A is any point on PQ. If PS = 5 cm, then $\text{ar}(\text{PAS}) = 30\text{ cm}^2$

Solution:

Given: A is any point on PQ. Since, PA < PQ



Now, area of triangle PQR is:

$$\text{area}(\Delta PQR)=\frac{1}{2}\times\text{base}\times\text{height}$$

So, $\text{area}(\Delta PQR)=\frac{1}{2}\times PQ\times QR=\frac{1}{2}\times 12\times 5=30\text{cm}^2$ [PQRS is a rectangle, RQ=SP=5 cm]

As PA < PQ

So, $\text{area}(\Delta PAS)<\text{area}(\Delta PQR)$

Or $\text{area}(\Delta PAS)<30\text{cm}^2$ $\quad$ $[\text{area}(\Delta PQR)=30\text{cm}^2]$

Hence, the given statement is false.

3. PQRS is a parallelogram whose area is 180 cm^2 and A is any point on the diagonal QS. The area of $\Delta\text{ASR}=90\text{ cm}^2$.

Solution:

Given: PQRS is a parallelogram.

As we know that diagonal of a parallelogram divides parallelogram into two triangles of equal area.

So,

$$\text{area}(\Delta QRS)=\frac{1}{2}\text{area}(PQRS)$$

$$=\frac{1}{2}\times 180=90\text{cm}^2$$

Now, A is any point on SQ. So,

$\text{area}(\Delta ASR)<\text{area}(\Delta QRS)$

Therefore, $\text{area}(\Delta ASR)<90\text{cm}^2$

Hence, the given statement is false.

4. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then $\text{ar}(\text{BDE})=\frac{1}{4}\text{ar}(\text{ABC})$.

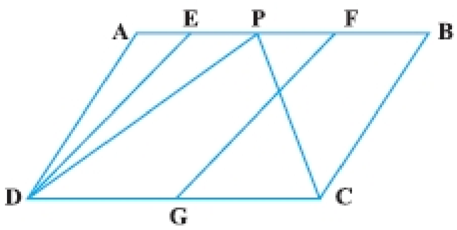
Solution:

Given: ΔABC and ΔBDE are two equilateral triangles.
Suppose that each sides of triangle ABC be x.
Similarly, D is the mid-point of BC. So, each side of triangle BDE is $\frac{x}{2}$.
Now,

$$\frac{\text{area}(\Delta BDE)}{\text{area}(\Delta ABC)}=\frac{\frac{\sqrt{3}}{4}\times\left(\frac{x}{2}\right)^2}{\frac{\sqrt{3}}{4}\times x^2}=\frac{x^2}{4x^2}=\frac{1}{4}$$

Therefore, $\text{area}(\Delta BDE)=\frac{1}{4}\text{area}(\Delta ABC)$.
Hence, the given statement is true.

5. In Fig., ABCD and EFGD are two parallelograms and G is the mid-point of CD. Then $\text{ar}(\text{DPC})=\frac{1}{4}\text{ar}(\text{EFGD})$.



Solution:
As triangle DPC and parallelogram ABCD are on same base DC and between the same parallels AB and DC. So,

$$\text{area}(\Delta DPC)=\frac{1}{2}area(ABCD) \qquad \ldots(\text{I})$$

Now,

$$\frac{area(EFGD)}{area(ABCD)}=\frac{DG\times h}{DC\times h}=\frac{DG}{2DG}=\frac{1}{2}\text{(G is the mid-point of DC)}$$

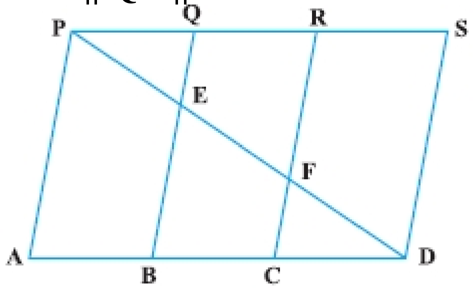
Implies that, $\text{area}(EFGD)=\frac{1}{2}\text{area}(ABCD)$

So, $\text{area}(DPC)=\text{area}(EFGD)$ [From equation (I)]
Hence, the given statement is false.

Exercise No. 9.3

Short Answer Questions:

1. In Fig., PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC. Prove that ar (PQE) = ar (CFD).



Solution:

Given: PSDA is a parallelogram. Points Q and R are taken on PS such that PQ = QR = RS and PA || QB || RC.

To prove that ar (PQE) = ar (CFD).

Proof: PS = AD [opposite angle of a parallelogram]

$\frac{1}{3}PS = \frac{1}{3}AD$

PQ = CD …(I)

Similarly, PS||AD and QB cut them. So,

$\angle PQE = \angle CBE$ [Alternate angles]…(II)

Again, QB||RC and AD cut them,

$\angle QBD = \angle RCD$ [Corresponding angle] …(III)

So, $\angle PQE = \angle FCD$ …(IV) [From (II) and (III), $\angle CBE$ and $\angle QBD$ are same and $\angle RCD$ and $\angle FCD$ are same]

Now, in triangle PQE and triangle CFD,

$\angle PQE = \angle CDF$ [Alternate angle]

PQ = CD [From equation (I)]

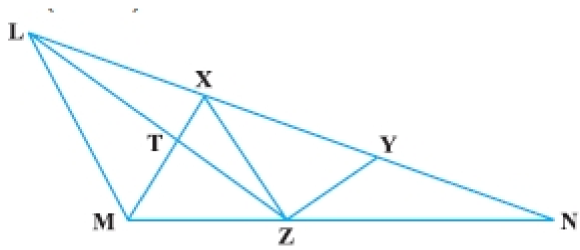
$\angle QPE = \angle FCD$ [From equation (IV)]

$\Delta PQE \cong \Delta CFD$ [By ASA congruence rule]

Hence, $ar(\Delta PQE) = ar(\Delta CFD)$. [Congruence triangle are equal in area]

2. X and Y are points on the side LN of the triangle LMN such that LX = XY = YN. Through X, a line is drawn parallel to LM to meet MN at Z (See Fig.). Prove that

ar (LZY) = ar (MZYX)



Solution:

Prove that ar (LZY) = ar (MZYX)

Proof: As ΔLXZ and ΔXMZ are on the same base and between the same parallels LM and XZ.

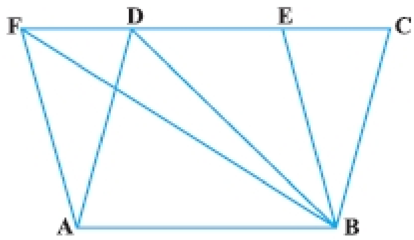
$ar(\Delta LXZ) = ar(\Delta XMZ)$

Now, adding $ar(\Delta XYZ)$ to both sides of (I), get:

$ar(\Delta LXZ) + ar(\Delta XYZ) = ar(\Delta XMZ) + ar(\Delta XYZ)$

$ar(\Delta LZY) = ar(MZYX)$

3. The area of the parallelogram ABCD is 90 cm^2 (see Fig.). Find
(i) ar (ABEF)
(ii) ar (ABD)
(iii) ar (BEF)



Solution:

(i) As we know that parallelogram on the same base and between the same parallels are equal in area.

$ar(ABEF) = ar(ABCD)$

Hence, $ar(ABEF) = ar(ABCD) = 90\text{cm}^2$.

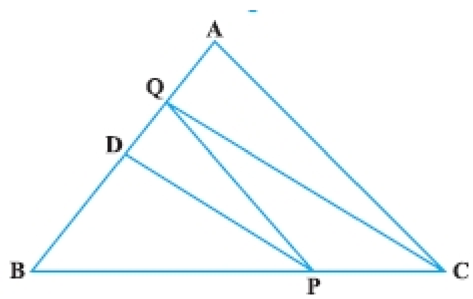
(ii) $ar(\Delta ABD) = \frac{1}{2} ar(ABCD)$ [A diagonal of a parallelogram divides the parallelogram in two triangle of equal area]

$= \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$

(iii) $ar(\Delta BEF) = \frac{1}{2} ar(ABEF) = \frac{1}{2} \times 90\text{cm}^2 = 45\text{cm}^2$

4. In ΔABC, D is the mid-point of AB and P is any point on BC. If CQ || PD meets AB in Q
(Fig.), then prove that

$$\operatorname{ar}(BPQ)=\frac{1}{2}\operatorname{ar}(ABC)$$

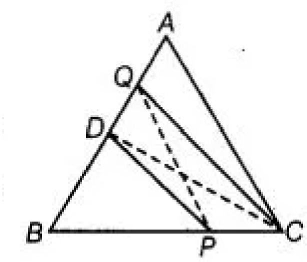


Solution:

Given in triangle ABC, D is the mid-point of AB and P is any point on BC.
CQ||PD means AB in Q.

To prove that $ar(\Delta BPQ)=\frac{1}{2}ar(\Delta ABC)$

Construction: Join PQ and CD.



Proof:
As we know that median of a triangle divides it into two triangles of equal area. So,

$ar(\Delta BCD)=\frac{1}{2}ar(\Delta ABC) \quad \ldots(\mathrm{I})$

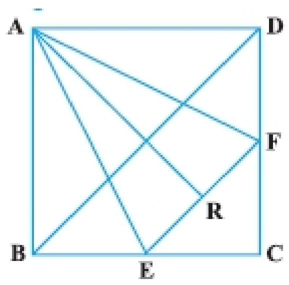
Also, we know that triangles on the same base and between the same parallels are equal in area.
So,

$ar(\Delta DPQ)=ar(\Delta DPC)$ [Triangle DPQ and DPC are on the same base DP and between the same parallels DP and CQ]

$ar(\Delta DPQ)+ar(\Delta DPB)=ar(\Delta DPC)+ar(DPB)$

Hence, $ar(\Delta BPQ)=ar(\Delta BCD)=\frac{1}{2}ar(\Delta ABC)$.

5. ABCD is a square. E and F are respectively the midpoints of BC and CD. If R is the mid-point of EF (Fig.), prove that
ar (AER) = ar (AFR)



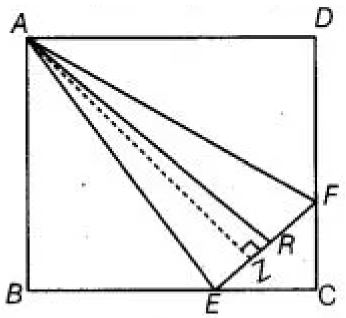
Solution:

Given: ABCD is a square. E and F are respectively the midpoints of BC and CD. Also, R is the mid-point of EF.

To prove that $ar(\Delta AER) = ar(\Delta AFR)$

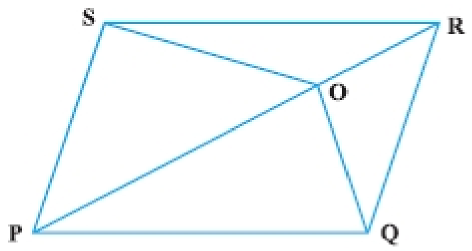
Construction: Draw $AN \perp EF$

Proof:



$$\begin{aligned} ar(\Delta AER) &= \frac{1}{2} \times \text{Base} \times \text{Height} \\ &= \frac{1}{2} \times ER \times AN \\ &= \frac{1}{2} \times FR \times AN \qquad \text{[R is the mid-point of EF so ER = FR]} \\ &= ar(\Delta AFR) \end{aligned}$$

6. O is any point on the diagonal PR of a parallelogram PQRS (Fig.). Prove that ar (PSO) = ar (PQO).

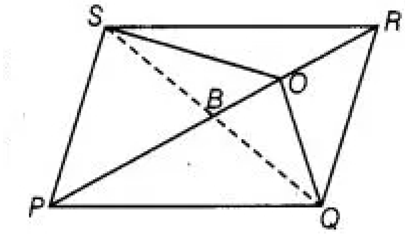


Solution:

Given: O is any point on the diagonal PR of a parallelogram PQRS.

To prove that ar (PSO) = ar (PQO).

Construction: Join SQ which intersects PR at B.

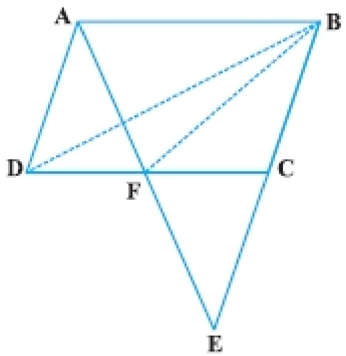


Proof: B is the mid-point of SQ because diagonal of a parallelogram bisect each other.
See the above figure, PB is a median of ΔQPS and as we know that a median of a triangle divides it into two triangle of equal area.
$ar(\Delta BPQ) = ar(\Delta BPS)$ ...(I)

Similarly, OB is the median of ΔOSQ.
$ar(\Delta OBQ) = ar(OBS)$ ...(II)

Now, adding equation (I) and (II), get:
$ar(\Delta BPQ) + ar(\Delta OBQ) = ar(\Delta BPS) + ar(\Delta OBS)$
$ar(\Delta PQO) = ar(\Delta PSO)$

7. ABCD is a parallelogram in which BC is produced to E such that CE = BC (Fig.). AE intersects CD at F.
If $\text{ar}(\text{DFB}) = 3\text{ cm}^2$, find the area of the parallelogram ABCD.



Solution:
Given: ABCD is a parallelogram in which BC is produced to E such that CE = BC. C is the mid-point BE and $ar(\Delta DFB) = 3\text{cm}^2$.

In triangle ADF and triangle EFC,
$\angle DAF = \angle CEF$ [Alternate interior angle]
AD = CE [AD = BC = CE]
$\angle ADF = \angle FCE$ [Alternate interior angle]
So, $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]
Now, $\Delta ADF \cong \Delta ECF$ [By SAS rule of congruence]

$DF = CF$ [CPCT]

As BF is the median of triangle BCD.

$ar(\Delta BDF) = \frac{1}{2} ar(BCD)$ ...(I) [Median divides a triangle into two triangle of equal area]

As we know that a triangle and parallelogram are on the same base and between the same parallels then the area of the triangles is equal to half the area of the parallelogram.

$ar(\Delta BCD) = \frac{1}{2} ar(ABCD)$ ...(II)

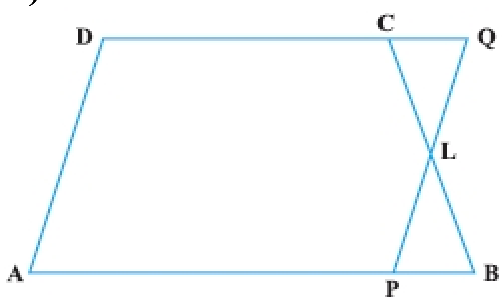
$ar(\Delta BDF) = \frac{1}{2}\left\{\frac{1}{2} ar(ABCD)\right\}$ [By equation (I)]

$3\text{cm}^2 = \frac{1}{4} ar(ABCD)$

$ar(ABCD) = 12\text{cm}^2$

Hence, the area of the parallelogram is $12\,\text{cm}^2$.

8. In trapezium ABCD, AB || DC and L is the mid-point of BC. Through L, a line PQ || AD has been drawn which meets AB in P and DC produced in Q (Fig.). Prove that
ar (ABCD) = ar (APQD)



Solution:

To prove that ar (ABCD) = ar (APQD)
Proof: AS AB||DC and AB||DQ

In triangle CLQ and triangle BLP,

$\angle QCL = \angle LBP$ [Alternate angles]

CL = LP [L is the mid-point og BC]

$\angle CLQ = \angle BLP$ [Vertical opposite angles]

$\Delta CLQ \cong \Delta BLP$ [By ASA congruence rule]

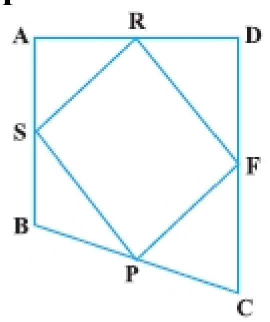
So, $ar(\Delta CLQ) = ar(\Delta BLP)$ ...(I) [Congruent triangles are equal in area]

Now, adding $ar(APLCD)$ both side in above equation, get:

$$ar(\Delta CLQ)+ar(APLCD)=ar(\Delta BLP)+ar(APLCD)$$
$$ar(\Delta APQD)=ar(ABCD)$$

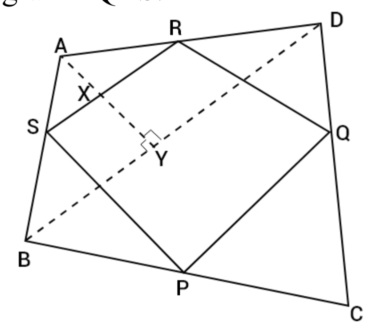
Hence, proved.

9. If the mid-points of the sides of a quadrilateral are joined in order, prove that the area of the parallelogram so formed will be half of the area of the given quadrilateral (Fig.).
[Hint: Join BD and draw perpendicular from A on BD.]



Solution:

According to the question, a quadrilateral ABCD in which the mid-point of the sides of it are joined in order of form parallelogram PQRS.



To Prove that $ar(PQRS)=\frac{1}{2}ar(ABCD)$

Construction: Join BD and draw perpendicular from A and BD which interest SR and BD at X and Y respectively.

Proof: In triangle ABD, S and R are the mid-points of sides AB and AD respectively. So, $SR \parallel BD$

And: $ASX \parallel BY$

See the figure, x is the mid-point of AY. So,
AX = XY

And $SR=\frac{1}{2}BD$ …(II)[mid-point theorem]

Now, $ar(\Delta ABD)=\frac{1}{2}\times BD\times AY$

$$ar(\Delta ASR)=\frac{1}{2}\times SR\times AX$$

$$ar(\Delta ASR)=\frac{1}{2}\times\left(\frac{1}{2}BD\right)\times\left(\frac{1}{2}AY\right)\quad \text{[Using equation (I) and (II)]}$$

$$ar(\Delta ASR)=\frac{1}{4}\times\left(\frac{1}{2}BD\times AY\right)$$

$$ar(\Delta ASR)=\frac{1}{4}\times(\Delta ABD)\qquad \ldots\text{(III)}$$

Again, $ar(\Delta CPQ)=\frac{1}{4}ar(\Delta CBD)\quad \ldots\text{(IV)}$

$$ar(\Delta BPS)=\frac{1}{4}ar(\Delta BAC)\qquad \ldots\text{(V)}$$

$$ar(\Delta DRQ)=\frac{1}{4}ar(DAC)\qquad \ldots\text{(VI)}$$

Now, adding equation (III), (IV), (V) and (VI), get:

$$ar(\Delta ASR)+ar(\Delta CPQ)+ar(BPS)+ar(\Delta DRQ)$$

$$=\frac{1}{4}ar(\Delta ABD)+\frac{1}{4}ar(\Delta CBD)+\frac{1}{4}ar(\Delta ABC)+\frac{1}{4}ar(\Delta DAC)$$

$$=\frac{1}{4}\left[ar(\Delta ABD)+ar(\Delta CBD)+ar(\Delta ABC)+ar(\Delta DAC)\right]$$

$$=\frac{1}{4}\left[ar(ABCD)+ar(ABCD)\right]$$

$$=\frac{1}{4}\times 2ar(ABCD)$$

$$=\frac{1}{2}ar(ABCD)$$

So, $ar(\Delta ASR)+ar(\Delta CPQ)+ar(\Delta BPS)+ar(\Delta DRQ)=\frac{1}{2}ar(ABCD)$

$$ar(ABCD)-ar(PQRS)=\frac{1}{2}ar(ABCD)$$

Now, $ar(PQRS)=ar(ABCD)-\frac{1}{2}ar(ABCD)$

$$ar(PQRS)=\frac{1}{2}ar(ABCD)$$

Hence, proved.

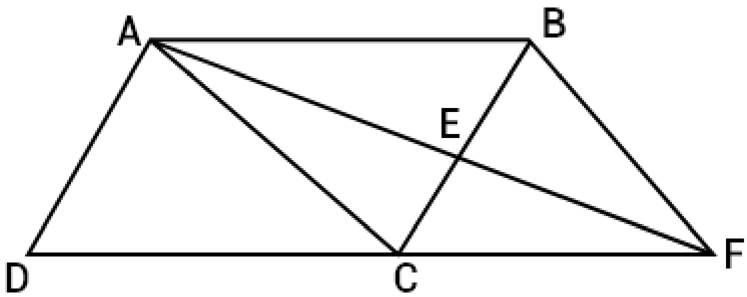
Exercise No. 9.4

Long Answer Questions:

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ADF) = ar (ABFC)

Solution:

Given in the question, A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F.
Prove that ar (ADF) = ar (ABFC)
Proof: ABCD is a parallelogram and AC divides it into two triangle of equal area.



$ar(\Delta ADC) = ar(\Delta ABC)$
So, DC||AB and CF||AB

As we know that triangle on the same base and between the same parallels are equal in area.
So,
$ar(\Delta ACF) = ar(\Delta BCF) \qquad \ldots(\text{II})$

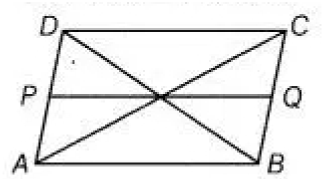
Adding equation (I) and (II), get:
$ar(\Delta ADC) + ar(ACF) = ar(\Delta ABC) + ar(\Delta BCF)$
$ar(\Delta ADF) = ar(ABFC)$
Hence, proved.

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. show that PQ divides the parallelogram into two parts of equal area.

Solution:

Given: ABCD is a parallelogram and diagonal interact at O, and draw a line PQ which intersects AD and BC.

To prove that PQ divides the parallelogram ABCD into two parts of equal area that $ar(ABQP) = ar(CDPQ)$.



Proof: AC is a diagonal of the parallelogram ABCD.

$$ar\left(\Delta \frac{1}{2} ACD\right) = \frac{1}{2} ar(ABCD) \qquad \ldots(I)$$

In triangle AOP and triangle COQ,

AO = CO [Diagonals of a parallelogram bisect each other]

$\angle AOP = \angle COQ$ [Vertical opposite angles]

$\angle OAP = \angle OCQ$ [Alternate angles, AB||CD]

$\Delta AOP = \Delta COQ$ [By ASA congruent rule]

Since, $ar(\Delta AOP) = ar(\Delta COQ)$ [Congruent area axiom] ...(II)

Now, adding $ar(AOQD)$ to both sides of (II), get:

$ar(\Delta ACD) = \frac{1}{2} ar(ABCD)$ [From equation (I)]

Hence, $ar(APQD) = \frac{1}{2} ar(ABCD)$.

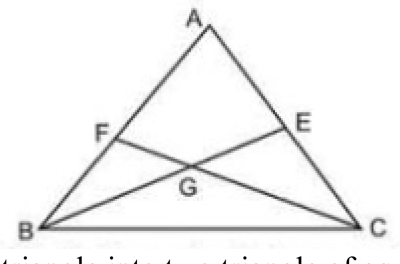
3. The medians BE and CF of a triangle ABC intersect at G. Prove that the

area of ΔGBC = area of the quidrilateral AFGE

Solution:

Given: The medians BE and CF of a triangle ABC intersect at G

To prove that $ar(\Delta GBC) = ar(AFGE)$.



Proof: As median CF divides a triangle into two triangle of equal area. So,

$ar(\Delta BCF) = ar(\Delta ACF)$

$ar(\Delta GBF) + ar(\Delta GBC) = ar(AFGE) + ar(\Delta GCE) \qquad \ldots(I)$

Now, median BE divides a triangle into two triangle of equal area. So,

$ar(\Delta GBF) + ar(AFGE) = ar(\Delta GCE) + ar(\Delta GBC) \qquad \ldots(II)$

Now, subtracting (II) from (I), get:

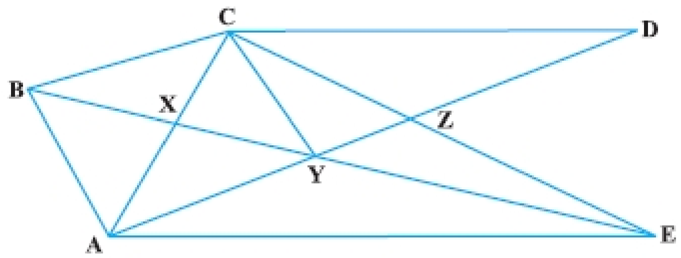
$$ar(\Delta GBC) - ar(AFGE) = ar(\Delta AFGE) - ar(\Delta GBC)$$

$$ar(\Delta GBC) + ar(\Delta GBC) = ar(\Delta AFGE) + ar(\Delta AFGE)$$

$$2ar(\Delta GBC) = 2ar(AFGE)$$

Hence, $ar(\Delta GBC) = ar(AFGE)$.

4. In Fig., CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY)



Solution:

Given: CD || AE and CY || BA

To prove that ar (CBX) = ar (AXY).

Proof: As we know that triangle on the same base and between the same parallels are equal in area. So,

$$ar(\Delta ABC) = ar(\Delta ABY)$$

$$ar(\Delta CBX) + ar(\Delta ABX) = ar(\Delta ABX) + ar(\Delta AXY)$$

Hence, $ar(\Delta CBX) = ar(\Delta AXY)$.

5. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that $\mathrm{ar}(\mathrm{DCYX}) = \frac{7}{9}\mathrm{ar}(\mathrm{XYBA})$

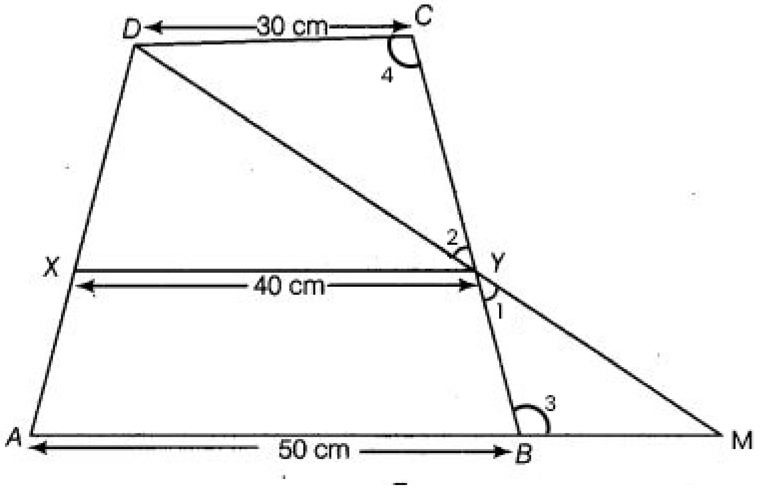
Solution:

To prove that $\mathrm{ar}(\mathrm{DCYX}) = \frac{7}{9}\mathrm{ar}(\mathrm{XYBA})$

Proof: In triangle MBY and triangle DCY,

$\angle 1 = \angle 2$ [Vertically opposite angles]

$\angle 3 = \angle 4$ [AB||DC and alternate angles are equal]



BY = CY [Y is the mid-point of BC]
$\Delta MBY \cong \Delta DCY$ [By ASA congruent angle]
So, MB = DC = 30 cm [CPCT]
Now, AM = AB + BM
=50cm+30cm
=80cm

In triangle ADM,
$XY = \frac{1}{2} AM = \frac{1}{2} \times 80\text{cm} = 40\text{cm}$

Now, AB||XY||DC and X and Y are the mid-points of AD and BC, So, height of trapezium DCXY and XYBA are equal and assume the equal height be h cm.

$$\frac{ar(DCYX)}{ar(XYBA)} = \frac{\frac{1}{2}\times(30+40)\times h}{\frac{1}{2}\times(30+50)\times h} = \frac{70}{90} = \frac{7}{9}$$

Hence, $ar(DCYX) = \frac{7}{9} ar(XYBA)$.

6. In ΔABC, if L and M are the points on AB and AC, respectively such that LM || BC. Prove that ar (LOB) = ar (MOC)

Solution:

Given: in triangle ABC and L and M are the points on AB and AC, respectively such that LM || BC.
Prove that ar (LOB) = ar (MOC)

Proof: As we know that triangle on the same base and between the same parallels are equal in area.
$ar(\Delta LBM) = ar(\Delta LCM)$

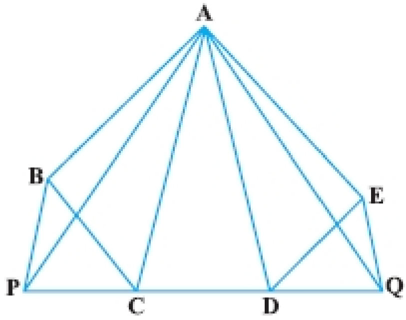
[Triangle LBM and triangle LCM are on the same base LM and between the same parallels LM and BC]

$$ar(\Delta LBM) = ar(\Delta LCM)$$

$$ar(\Delta LOM) + ar(\Delta LOB) = ar(\Delta LOM) + ar(\Delta MOC)$$

Hence, $ar(\Delta LOB) = ar(\Delta MOC)$.

7. In Fig., ABCDE is any pentagon. BP drawn parallel to AC meets DC produced at P and EQ drawn parallel to AD meets CD produced at Q. Prove that
ar (ABCDE) = ar (APQ)



Solution:

Given: ABCDE is any pentagon and BP || AC meets DC produced at P and EQ || AD meets CD produced at Q.
Prove that ar (ABCDE) = ar (APQ)

Proof: As we know that triangle on the same base and between the same parallels are equal in area.

$ar(\Delta ABC) = ar(\Delta APC) \quad \ldots(\text{I})$

$ar(\Delta ADE) = ar(\Delta ADQ) \quad \ldots(\text{II})$

Now, adding equation (I) and (II), get:

$ar(\Delta ABC) + ar(\Delta ADE) = ar(\Delta APC) + ar(\Delta ADQ)$

Now, adding $ar(\Delta ACD)$ to both side, get:

$ar(\Delta ABC) + ar(\Delta ADE) + ar(ACD) = ar(\Delta APC) + ar(\Delta ADQ) + ar(ACD)$

Hence, $ar(ABCDE) = ar(\Delta APQ)$.

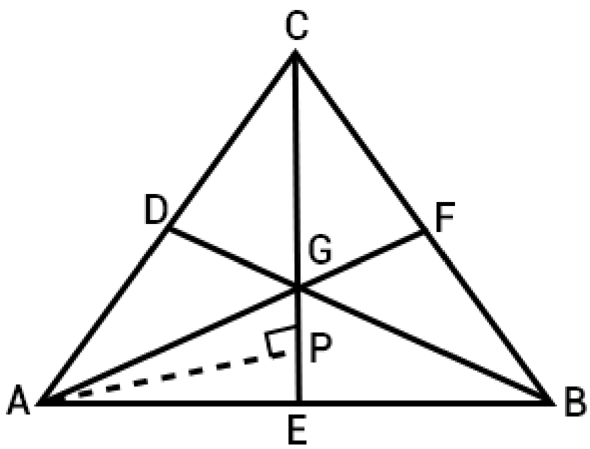
8. If the medians of a ΔABC intersect at G, show that

ar (AGB) = ar (AGC) = ar (BGC)= $\frac{1}{3}$ ar (ABC)

Solution:

Given: The median of a triangle ABC intersect at G.

To prove that ar (AGB) = ar (AGC) = ar (BGC) = $\frac{1}{3}$ ar (ABC)



Construction: Draw BP ⊥ EG

Proof: $AG = \frac{2}{3}AE$ [Centroid divides the median in the ration 2:1]

Now, $ar\left(\Delta AGB\right) = \frac{1}{2} \times AG \times BP$

[Median divides a triangle into two triangles equal in area]

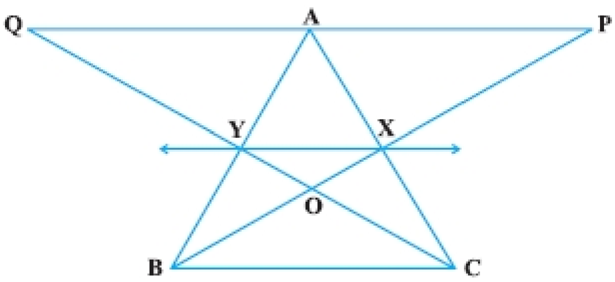
$= \frac{1}{3}ar\left(\Delta ABC\right)$

Again, $ar\left(\Delta AGC\right) = ar\left(\Delta BGC\right) = \frac{1}{3}ar\left(\Delta ABC\right)$

So, $ar\left(\Delta AGB\right) = ar\left(\Delta AGC\right) = ar\left(\Delta BGC\right) = \frac{1}{3}ar\left(\Delta ABC\right)$

Hence, proved.

9. In Fig., X and Y are the mid-points of AC and AB respectively, QP || BC and CYQ and BXP are straight lines. Prove that ar (ABP) = ar (ACQ).



Solution:

Given: In triangle ABC, X and Y are the mid-points of AB and AC.
To prove that ar (ABP) = ar (ACQ).
Proof: Since, XY||BY [BY mid-point theorem]
As we know that triangle on the same base and between the same parallels lines are equal in area. So,

$ar(\Delta BYC) = ar(\Delta BXC) \qquad \ldots(\text{I})$

Now, subtracting $ar(\Delta BOC)$ from both sides in the above, get:

$ar(\Delta BYC) - ar(\Delta BOC) = ar(\Delta BXC) - ar(\Delta BOC)$

$ar(\Delta BOY) = ar(\Delta COX) \ \ldots(\text{II})$

Now, adding $ar(\Delta XOY)$ to both side in equation (II), get:

$ar(\Delta BOY) + ar(\Delta XOY) = ar(\Delta COX) + ar(\Delta XOY) \qquad \ldots(\text{III})$

Again, quadrilaterals XYAP and YXAQ are on the same base XY and between the same parallels XY and PQ. So,

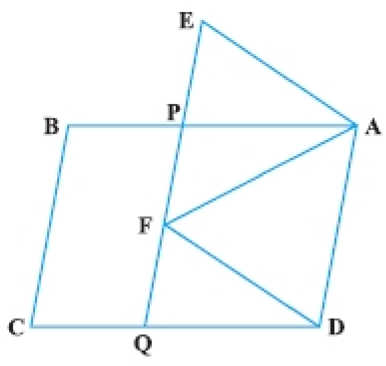
$ar(XYAP) = ar(XYQA) \qquad \ldots(\text{IV})$

Now, adding equation (III) and (IV), get:

$ar(\Delta BXY) + ar(XYAP) = ar(\Delta CXY) + ar(XYAQ)$

Hence, $ar(\Delta ABP) = ar(\Delta ACQ)$.

10. In Fig., ABCD and AEFD are two parallelograms. Prove that ar (PEA) = ar (QFD) [Hint: Join PD].

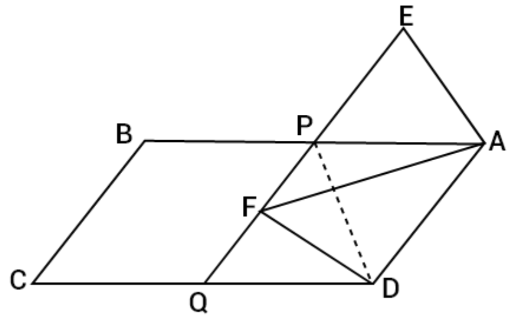


Solution:

Given: ABCD and AEFD are two parallelogram.

To prove that $ar(\Delta PEA) = ar(\Delta QFD)$

Construction: join PD.



Proof: In triangle PEA and triangle QFD,

$\angle APE = \angle DQF$ [Corresponding angles are equal as AB||CD]

$\angle AEP = \angle DEQ$ [Corresponding angles are equal as AE||DF]

AE = DF [Opposite sides of a parallelogram are equal]

So, $\Delta PEA \cong \Delta QFD$ [By AAS congruent rule]

Hence, $ar\left(\Delta PEA\right) = ar\left(\Delta QED\right)$.