

Ex 3.1

Binary Operations Ex 3.1 Q1(i)

We have,

$$a * b = a^b \text{ for all } a, b \in \mathbb{N}$$

Let $a \in \mathbb{N}$ and $b \in \mathbb{N}$

$$\Rightarrow a^b \in \mathbb{N}$$

$$\Rightarrow a * b \in \mathbb{N}$$

The operation $*$ defines a binary operation on $\mathbb{N}$

Binary Operations Ex 3.1 Q1(ii)

We have,

$$a \circ b = a^b \text{ for all } a, b \in \mathbb{Z}$$

Let $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$

$$\Rightarrow a^b \notin \mathbb{Z} \quad \Rightarrow a \circ b \notin \mathbb{Z}$$

For example, if $a = 2, b = -2$

$$\Rightarrow a^b = 2^{-2} = \frac{1}{4} \notin \mathbb{Z}$$

$\therefore$ The operation ' $\circ$ ' does not define a binary operation on $\mathbb{Z}$.

Binary Operations Ex 3.1 Q1(iii)

We have,

$$a * b = a + b - 2 \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

Then, $a + b - 2 \notin N$ for all $a, b \in N$

$$\Rightarrow a * b \notin N$$

For example $a = 1, b = 1$

$$\Rightarrow a + b - 2 = 0 \notin N$$

$\therefore$ The operation $*$ does not define a binary operation on N

Binary Operations Ex 3.1 Q1(iv)

We have,

$$S = \{1, 2, 3, 4, 5\}$$

and, $a \times_6 b = \text{Remainder when } ab \text{ is divided by } 6$

Let $a \in S$ and $b \in S$

$$\Rightarrow a \times_6 b \notin S \text{ for all } a, b \in S$$

For example, $a = 2, b = 3$

$$\Rightarrow 2 \times_6 3 = \text{Remainder when } 6 \text{ is divided by } 6 = 0 \notin S$$

$\therefore \times_6$ does not define a binary operation on S

Binary Operations Ex 3.1 Q1(v)

We have,

$$S = \{0, 1, 2, 3, 4, 5\}$$

$$\text{and, } a +_6 b = \begin{cases} a + b; & \text{if } a + b < 6 \\ a + b - 6; & \text{if } a + b \geq 6 \end{cases}$$

Let $a \in S$ and $b \in S$ such that $a + b < 6$

$$\text{Then } a +_6 b = a + b \in S \quad [\because a + b < 6 = 0, 1, 2, 3, 4, 5]$$

Let $a \in S$ and $b \in S$ such that $a + b \geq 6$

$$\text{Then } a +_6 b = a + b - 6 \in S \quad [\because \text{if } a + b \geq 6 \text{ then } a + b - 6 \geq 0 = 0, 1, 2, 3, 4, 5]$$

$$\therefore a +_6 b \in S \text{ for } a, b \in S$$

$\therefore +_6$ defines a binary operation on S

Binary Operations Ex 3.1 Q1(vi)

We have,

$$a \circ b = a^b + b^a \text{ for all } a, b \in N$$

Let $a \in N$ and $b \in N$

$$\Rightarrow a^b \in N \text{ and } b^a \in N$$

$$\Rightarrow a^b + b^a \in N$$

$$\Rightarrow a \circ b \in N$$

Thus, the operation ' $\circ$ ' defines a binary relation on N

Binary Operations Ex 3.1 Q1(vii)

We have,

$$a * b = \frac{a-1}{b+1} \text{ for all } a, b \in \mathbb{Q}$$

Let $a \in \mathbb{Q}$ and $b \in \mathbb{Q}$

Then $\frac{a-1}{b+1} \notin \mathbb{Q}$ for $b = -1$

$\Rightarrow a * b \notin \mathbb{Q}$ for all $a, b \in \mathbb{Q}$

Thus, the operation $*$ does not define a binary operation on $\mathbb{Q}$

Binary Operations Ex 3.1 Q2

(i) On $\mathbb{Z}^+$, $*$ is defined by $a * b = a - b$.

It is not a binary operation as the image of $(1, 2)$ under $*$ is $1 * 2 = 1 - 2 = -1 \notin \mathbb{Z}^+$.

(ii) On $\mathbb{Z}^+$, $*$ is defined by $a * b = ab$.

It is seen that for each $a, b \in \mathbb{Z}^+$, there is a unique element ab in $\mathbb{Z}^+$.

This means that $*$ carries each pair (a, b) to a unique element $a * b = ab$ in $\mathbb{Z}^+$. Therefore, $*$ is a binary operation.

(iii) On $\mathbb{R}$, $*$ is defined by $a * b = ab^2$.

It is seen that for each $a, b \in \mathbb{R}$, there is a unique element ab^2 in $\mathbb{R}$.

This means that $*$ carries each pair (a, b) to a unique element $a * b = ab^2$ in $\mathbb{R}$. Therefore, $*$ is a binary operation.

(iv) On $\mathbb{Z}^+$, $*$ is defined by $a * b = |a - b|$.

It is seen that for each $a, b \in \mathbb{Z}^+$, there is a unique element $|a - b|$ in $\mathbb{Z}^+$.

This means that $*$ carries each pair (a, b) to a unique element $a * b = |a - b|$ in $\mathbb{Z}^+$.

Therefore, $*$ is a binary operation.

(v) On $\mathbb{Z}^+$, $*$ is defined by $a * b = a$.

$*$ carries each pair (a, b) to a unique element $a * b = a$ in $\mathbb{Z}^+$.

Therefore, $*$ is a binary operation.

(vi) on $\mathbb{R}$, $*$ is defined by $a * b = a + 4b^2$

it is seen that for each element $a, b \in \mathbb{R}$, there is unique element $a + 4b^2$ in $\mathbb{R}$

This means that $*$ carries each pair (a, b) to a unique element $a * b =$

$a + 4b^2$ in $\mathbb{R}$.

Therefore, $*$ is a binary operation.

Binary Operations Ex 3.1 Q3

It is given that, $a * b = 2a + b - 3$

Now

$$\begin{aligned} 3 * 4 &= 2 \times 3 + 4 - 3 \\ &= 10 - 3 \\ &= 7 \end{aligned}$$

Binary Operations Ex 3.1 Q4

The operation $*$ on the set $A = \{1, 2, 3, 4, 5\}$ is defined as

$a * b = \text{L.C.M. of } a \text{ and } b$.

$2 * 3 = \text{L.C.M of } 2 \text{ and } 3 = 6$. But 6 does not belong to the given set.

Hence, the given operation $*$ is not a binary operation.

Binary Operations Ex 3.1 Q5

We have,

$$S = \{a, b, c\}$$

We know that the total number of binary operation on a set S with n element is n^{n^2}

$\Rightarrow$ Total number of binary operation on $S = \{a, b, c\} = 3^{3^2} = 3^9$

Binary Operations Ex 3.1 Q6

We have,

$$S = \{a, b\}$$

The total number of binary operation on $S = \{a, b\}$ is $2^{2^2} = 2^4 = 16$

Binary Operations Ex 3.1 Q7

We have,

$$M = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in R - \{0\} \right\} \text{ and} \\ A * B = AB \text{ for all } A, B \in M$$

$$\text{Let } A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \in M \text{ and } B = \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} \in M$$

$$\text{Now, } AB = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} c & 0 \\ 0 & d \end{bmatrix} = \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix}$$

$$\therefore a \in R, b \in R, c \in R, \& d \in R$$

$$\Rightarrow ac \in R \text{ and } bd \in R$$

$$\Rightarrow \begin{bmatrix} ac & 0 \\ 0 & bd \end{bmatrix} \in M$$

$$\Rightarrow A * B \in M$$

Thus, the operator $*$ defines a binary operation on M

Binary Operations Ex 3.1 Q8

S = set of rational numbers of the form $\frac{m}{n}$ where $m \in Z$ and $n = 1, 2, 3$

$$\text{Also, } a * b = ab$$

Let $a \in S$ and $b \in S$

$$\Rightarrow ab \notin S$$

For example $a = \frac{7}{3}$ and $b = \frac{5}{2}$

$$\Rightarrow ab = \frac{35}{6} \notin S$$

$$\therefore a * b \notin S$$

Hence, the operator $*$ does not define a binary operation on S

Binary Operations Ex 3.1 Q9

It is given that, $a * b = 2a + b$

Now

$$\begin{aligned} (2 * 3) &= 2 \times 2 + 3 \\ &= 4 + 3 \\ &= 7 \end{aligned}$$

$$\begin{aligned} (2 * 3) * 4 &= 7 * 4 = 2 \times 7 + 4 \\ &= 14 + 4 \\ &= 18 \end{aligned}$$

Binary Operations Ex 3.1 Q10

It is given that, $a * b = \text{LCM}(a, b)$

Now

$$\begin{aligned} 5 * 7 &= \text{LCM}(5, 7) \\ &= 35 \end{aligned}$$

Ex 3.2

Binary Operations Ex 3.2 Q1

We have,

$$a * b = \text{l.c.m.}(a, b) \text{ for all } a, b \in \mathbb{N}$$

(1)

Now,

$$2 * 4 = \text{l.c.m.}(2, 4) = 4$$

$$3 * 5 = \text{l.c.m.}(3, 5) = 15$$

$$1 * 6 = \text{l.c.m.}(1, 6) = 6$$

(ii)

Commutativity:

Let $a, b \in \mathbb{N}$ then,

$$\begin{aligned} a * b &= \text{l.c.m.}(a, b) \\ &= \text{l.c.m.}(b, a) \\ &= b * a \end{aligned}$$

$$\Rightarrow a * b = b * a$$

$\therefore *$ is commutative on $\mathbb{N}$.

Associativity:

Let $a, b, c \in \mathbb{N}$ then,

$$\begin{aligned} (a * b) * c &= \text{l.c.m.}(a, b) * c \\ &= \text{l.c.m.}(a, b, c) \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \text{l.c.m.}(b, c) \\ &= \text{l.c.m.}(a, b, c) \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore *$ is associative on $\mathbb{N}$.

Binary Operations Ex 3.2 Q2

(i) Clearly, by definition $a * b = 1 = b * a$, $\forall a, b \in \mathbb{N}$

$$\text{Also, } (a * b) * c = (1 * c) = 1$$

$$\text{and } a * (b * c) = (a * 1) = 1 \quad \forall a, b, c \in \mathbb{N}$$

Hence, $\mathbb{N}$ is both associative and commutative.

$$(ii) \quad a * b = \frac{a+b}{2} = \frac{b+a}{2} = b * a,$$

which shows $*$ is commutative.

$$\text{Further, } (a * b) * c = \left(\frac{a+b}{2} \right) * c = \frac{\left(\frac{a+b}{2} \right) + c}{2} = \frac{a+b+2c}{4}$$

$$a * (b * c) = a * \left(\frac{b+c}{2} \right) = \frac{a + \left(\frac{b+c}{2} \right)}{2} = \frac{2a+b+c}{2} \neq \frac{a+b+2c}{4}$$

Hence, $*$ is not associative.

Binary Operations Ex 3.2 Q3

We have, binary operator $*$ defined on A and is given by

$$a * b = b \text{ for all } a, b \in A$$

Commutativity: Let $a, b \in A$, then

$$a * b = b \neq a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$\therefore$ $'*'$ is not commutative on A .

Assodativity: Let $a, b, c \in A$, then

$$(a * b) * c = b * c = c \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * c = c \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $'*'$ is assodative on A .

Binary Operations Ex 3.2 Q4(i)

$'*'$ is a binary operator on Z defined by $a * b = a + b + ab$ for all $a, b \in Z$.

Commutativity of $'*'$:

Let $a, b \in Z$, then

$$a * b = a + b + ab = b + a + ba = b * a$$

$$\therefore a * b = b * a$$

Assodative of $'*'$:

Let $a, b \in Z$, then

$$\begin{aligned} (a * b) * c &= (a + b + ab) * c = a + b + ab + c + ac + bc + abc \\ &= a + b + c + ab + bc + ac + abc \end{aligned} \quad \text{--- (i)}$$

Again, $a * (b * c) = a * (b + c + bc)$

$$= a + b + c + bc + ab + ac + abc \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c = a * (b * c)$$

$\therefore$ $*$ is commutative and assodative on Z

Binary Operations Ex 3.2 Q4(ii)

Commutative:

Let $a, b \in N$, then

$$a * b = 2^{ab} = 2^{ba} = b * a$$

$$\therefore a * b = b * a$$

$\therefore *$ is commutative on N

Associative:

Let $a, b, c \in N$, then

$$(a * b) * c = 2^{ab} * c = 2^{2^{ab}c} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * 2^{bc} = 2^{a \cdot 2^{bc}} \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore *$ is not associative on N

Binary Operations Ex 3.2 Q4(iii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a - b \neq b - a = b * a$$

$$\therefore a * b \neq b * a$$

$\Rightarrow *$ is not commutative on Q

Associative:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a - b) * c = a - b - c \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b - c) = a - b + c \quad \text{--- (ii)}$$

From (i) & (ii), we get

$$(a * b) * c \neq a * (b * c)$$

$\therefore *$ is not associative on Q

Binary Operations Ex 3.2 Q4(iv)

Commutative:

Let $a, b \in Q$, then

$$a \circ b = a^2 + b^2 = b^2 + a^2 = b \circ a$$

$$\Rightarrow a \circ b = b \circ a$$

$\therefore \circ$ is commutative on Q .

Associative:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = (a^2 + b^2) \circ c = (a^2 + b^2)^2 + c^2 \quad \text{--- (i)}$$

$$\text{and, } a \circ (b \circ c) = a \circ (b^2 + c^2) = a^2 + (b^2 + c^2)^2 \quad \text{--- (ii)}$$

From (i) & (ii),

$$(a \circ b) \circ c \neq a \circ (b \circ c)$$

$\therefore \circ$ is not associative on Q .

Binary Operations Ex 3.2 Q4(v)

Binary operation ' $\circ$ ' defined on Q , given by $a \circ b = \frac{ab}{2}$ for all $a, b \in Q$

Commutative:

Let $a, b \in Q$, then

$$a \circ b = \frac{ab}{2} = \frac{ba}{2} = b \circ a$$

$$\Rightarrow a \circ b = b \circ a$$

$\therefore \circ$ is commutative on Q .

Associativity:

Let $a, b, c \in Q$, then

$$(a \circ b) \circ c = \left(\frac{ab}{2}\right) \circ c = \frac{abc}{4} \quad \text{--- (i)}$$

$$a \circ (b \circ c) = a \circ \left(\frac{bc}{2}\right) = \frac{abc}{4} \quad \text{--- (ii)}$$

From (i) & (ii) we get

$$(a \circ b) \circ c = a \circ (b \circ c)$$

$\therefore \circ$ is associative on Q .

Binary Operations Ex 3.2 Q4(vi)

Commutative:

Let $a, b \in Q$, then

$$a * b = ab^2 \neq ba^2 = b * a$$

$$\Rightarrow a * b \neq b * a$$

$\therefore *$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = ab^2 * c = ab^2c^2 \quad \text{--- (i)}$$

$$\& \quad a * (b * c) = a * bc^2 = a(bc^2)^2 \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\therefore *$ is not associative on Q

Binary Operations Ex 3.2 Q4(vii)

Commutativity:

Let $a, b \in Q$, then

$$a * b = a + ab \quad \text{--- (i)}$$

$$b * a = b + ab \quad \text{--- (ii)}$$

From (i) & (ii)

$$a * b \neq b * a$$

$\Rightarrow$ $*$ is not commutative on Q

Associativity:

Let $a, b, c \in Q$, then

$$(a * b) * c = (a + ab) * c = a + ab + ac + abc \quad \text{--- (i)}$$

$$\begin{aligned} a * (b * c) &= a * (b + bc) \\ &= a + ab + abc \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$\Rightarrow$ $*$ is not associative on Q

Binary Operations Ex 3.2 Q4(viii)

Commutativity: Let $a, b \in R$, then

$$\begin{aligned} a * b &= a + b - 7 \\ &= b + a - 7 \\ &= b * a \end{aligned}$$

$$\Rightarrow a * b = b * a$$

$\Rightarrow$ $*$ is commutative on R

Associativity: Let $a, b, c \in Q$, then

$$\begin{aligned} (a * b) * c &= (a + b - 7) * c \\ &= a + b - 7 + c - 7 \\ &= a + b + c - 17 \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (b + c - 7) \\ &= a + b + c - 7 - 7 \\ &= a + b + c - 17 \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on R

Binary Operations Ex 3.2 Q4(ix)

Commutativity:

Let $a, b \in R - \{-1\}$, then

$$a * b = \frac{a}{b+1} \neq \frac{b}{a+1} = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow * \text{ is not commutative on } R - \{-1\}$$

Associativity:

Let $a, b, c \in R - \{-1\}$, then

$$\begin{aligned} (a * b) * c &= \left(\frac{a}{b+1} \right) * c \\ &= \frac{\frac{a}{b+1}}{c+1} = \frac{a}{(b+1)(c+1)} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \& \quad a * (b * c) &= a * \left(\frac{b}{c+1} \right) \\ &= \frac{a}{\frac{b}{c+1} + 1} = \frac{a(c+1)}{b+c+1} \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow * \text{ is not associative on } R - \{-1\}$$

Binary Operations Ex 3.2 Q4(x)

Commutativity:

Let $a, b \in Q$, then

$$a * b = ab + 1 = ba + 1 = b * a$$

$$\Rightarrow a * b = b * a$$

$$\Rightarrow * \text{ is commutative on } Q$$

Associativity:

Let $a, b, c \in Q$, then

$$\begin{aligned} (a * b) * c &= (ab + 1) * c \\ &= abc + c + 1 \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} a * (b * c) &= a * (bc + 1) \\ &= abc + a + 1 \end{aligned} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow * \text{ is not associative on } Q.$$

Binary Operations Ex 3.2 Q4(xi)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow '*' \text{ is not commutative on } N$$

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = (a^b)^c = a^{bc} \quad \text{--- (i)}$$

$$a * (b * c) = a * b^c = (a)^{b^c} \quad \text{--- (ii)}$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } N.$$

Binary Operations Ex 3.2 Q4(xii)

Commutativity:

Let $a, b \in N$, then

$$a * b = a^b \neq b^a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow '*' \text{ is not commutative on } N$$

Associativity:

Let $a, b, c \in N$, then

$$(a * b) * c = a^b * c = (a^b)^c = a^{bc} \quad \text{--- (i)}$$

$$a * (b * c) = a * b^c = (a)^{b^c} \quad \text{--- (ii)}$$

From (i) and (ii)

$$a^{bc} \neq (a)^{b^c}$$

$$\Rightarrow (a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } N.$$

Binary Operations Ex 3.2 Q4(xiii)

Commutativity:

Let $a, b \in \mathbb{Z}$ then,

$$a * b = a - b \neq b - a = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow * \text{ is not commutative on } \mathbb{Z}$$

Associativity:

Let $a, b, c \in \mathbb{Z}$, then

$$(a * b) * c = (a - b) * c = (a - b - c) \quad \text{--- (i)}$$

$$\& \quad a * (b * c) = a * (b - c) = (a - b + c) \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\Rightarrow '*' \text{ is not associative on } \mathbb{Z}.$$

Binary Operations Ex 3.2 Q4(xiv)

Commutativity:

Let $a, b \in \mathbb{Q}$ then,

$$a * b = \frac{ab}{4} = \frac{ba}{4} = b * a$$

$$\Rightarrow a * b = b * a$$

$$\therefore * \text{ is commutative on } \mathbb{Q}$$

Associativity:

Let $a, b, c \in \mathbb{Q}$ then,

$$(a * b) * c = \frac{ab}{4} * c = \frac{abc}{16} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{4} = \frac{abc}{16} \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$$\therefore '*' \text{ is associative on } \mathbb{Q}.$$

Binary Operations Ex 3.2 Q4(xv)

Commutativity:

Let $a, b \in \mathbb{Q}$ then,

$$a * b = (a - b)^2 = (b - a)^2 = b * a$$

$$\Rightarrow a * b = b * a$$

$$\therefore '*' \text{ is commutative on } \mathbb{Q}.$$

Associativity:

Let $a, b, c \in \mathbb{Q}$ then,

$$(a * b) * c = (a - b)^2 * c = [(a - b)^2 - c]^2 \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b - c)^2 = [a - (b - c)^2]^2 \quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c \neq a * (b * c)$$

$$\therefore * \text{ is not associative on } \mathbb{Q}.$$

Binary Operations Ex 3.2 Q5

The binary operator $\circ$ defined on $Q - \{-1\}$ is given by

$$a \circ b = a + b - ab \text{ for all } a, b \in Q - \{-1\}$$

Commutativity:

Let $a, b \in Q - \{-1\}$, then

$$a \circ b = a + b - ab = b + a - ba = b \circ a$$

$$\Rightarrow a \circ b = b \circ a$$

$$\Rightarrow '\circ' \text{ is commutative on } Q - \{-1\}.$$

Binary Operations Ex 3.2 Q6

The binary operator $*$ defined on Z and is given by

$$a * b = 3a + 7b$$

Commutativity: Let $a, b \in Z$, then

$$a * b = 1a + 7b \text{ and}$$

$$b * a = 3b + 7a$$

$$\therefore a * b \neq b * a$$

Hence, ' $*$ ' is not commutative on Z .

Binary Operations Ex 3.2 Q7

We have, $*$ is a binary operator defined on Z is given by

$$a * b = ab + 1 \text{ for all } a, b \in Z$$

Associativity: Let $a, b, c \in Z$, then

$$\begin{aligned} (a * b) * c &= (ab + 1) * c \\ &= abc + c + 1 \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (bc + 1) \\ &= abc + a + 1 \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$\therefore (a * b) * c \neq a * (b * c)$$

Hence, ' $*$ ' is not associative on Z .

Binary Operations Ex 3.2 Q8

We have, set of real numbers except -1 and $*$ is an operator given by

$$a * b = a + b + ab \text{ for all } a, b \in S = R - \{-1\}$$

Now, $\forall a, b \in S$

$$a * b = a + b + ab \in S$$

$$\therefore \text{ if } a + b + ab = -1$$

$$\Rightarrow a + b(1 + a) + 1 = 0$$

$$\Rightarrow (a + 1)(b + 1) = 0$$

$$\Rightarrow a = -1 \text{ or } b = -1$$

but $a \neq -1$ and $b \neq -1$ (given)

$$\therefore a + b + ab \neq -1$$

$$\Rightarrow a * b \in S \text{ for } a, b \in S$$

$$\Rightarrow '\ast' \text{ is a binary operator on } S$$

Commutativity: Let $a, b \in S$

$$\Rightarrow a * b = a + b + ab = b + a + ba = b * a$$

$$\Rightarrow a * b = b * a$$

$$\begin{aligned}\text{and, } a * (b * c) &= a * (b + c + bc) \\ &= a + b + c + bc + ab + ac + abc\end{aligned}\quad \text{--- (ii)}$$

From (i) and (ii)

$$(a * b) * c = a * (b * c)$$

$\therefore$ '*' is associative on S .

$$\begin{aligned}\text{Now, } (2 * x) * 3 &= 7 \\ \Rightarrow (2 + x + 2x) * 3 &= 7 \\ \Rightarrow 2 + x + 2x + 3 + 6 + 3x + 6x &= 7 \\ \Rightarrow 11 + 12x &= 7 \\ \Rightarrow 12x &= -4 \\ \Rightarrow x &= \frac{-4}{12} \quad \Rightarrow x = \frac{-1}{3}\end{aligned}$$

Binary Operations Ex 3.2 Q9

The binary operator $*$ defined as

$$a * b = \frac{a - b}{2} \text{ for all } a, b \in \mathbb{Q}.$$

Now,

Associativity: Let $a, b, c \in \mathbb{Q}$, then

$$\begin{aligned}(a * b) * c &= \frac{a - b}{2} * c = \frac{\frac{a - b}{2} - c}{2} \\ &= \frac{a - b - 2c}{4}\end{aligned}\quad \text{--- (i)}$$

$$\begin{aligned}\text{and, } a * (b * c) &= a * \frac{b - c}{2} = \frac{a - \frac{b - c}{2}}{2} \\ &= \frac{2a - b + c}{4}\end{aligned}\quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, '*' is not associative on $\mathbb{Q}$.

Binary Operations Ex 3.2 Q10

The binary operator $*$ defined as

$$a * b = a + 3b - 4 \text{ for all } a, b \in \mathbb{Z}$$

Now,

Commutativity: Let $a, b \in \mathbb{Z}$, then

$$a * b = a + 3b - 4 \neq b + 3a - 4 = b * a$$

$$\Rightarrow a * b \neq b * a$$

$$\Rightarrow \text{'*'} \text{ is not commutative on } \mathbb{Z}.$$

Associativity: Let $a, b, c \in \mathbb{Z}$, then

$$\begin{aligned}(a * b) * c &= (a + 3b - 4) * c = a + 3b - 4 + 3c - 4 \\ &= a + 3b + 3c - 8\end{aligned}\quad \text{--- (i)}$$

$$\begin{aligned}\text{and, } a * (b * c) &= a * (b + 3c - 4) = a + 3(b + 3c - 4) - 4 \\ &= a + 3b + 9c - 16\end{aligned}\quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, '*' is not associative on $\mathbb{Z}$.

Binary Operations Ex 3.2 Q11

Q be the set of rational numbers and $*$ be a binary operation defined as

$$a * b = \frac{ab}{5} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{5} * c = \frac{abc}{25} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{5} = \frac{abc}{25} \quad \text{--- (ii)}$$

From (i) & (ii)

$$\therefore (a * b) * c = a * (b * c)$$

$\Rightarrow$ $*$ is associative on Q .

Binary Operations Ex 3.2 Q12

The binary operator $*$ is defined as

$$a * b = \frac{ab}{7} \text{ for all } a, b \in Q$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$(a * b) * c = \frac{ab}{7} * c = \frac{abc}{49} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{7} = \frac{abc}{49} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$\Rightarrow$ $'*$ is associative on Q .

Binary Operations Ex 3.2 Q13

The binary operator $*$ defined as

$$a * b = \frac{a+b}{2} \text{ for all } a, b \in Q.$$

Now,

Associativity: Let $a, b, c \in Q$, then

$$\begin{aligned} (a * b) * c &= \frac{a+b}{2} * c = \frac{\frac{a+b}{2} + c}{2} \\ &= \frac{a+b+2c}{4} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \frac{b+c}{2} \\ &= \frac{a + \frac{b+c}{2}}{2} \\ &= \frac{2a+b+c}{4} \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c \neq a * (b * c)$$

Hence, $'*$ is not associative on Q .

Ex 3.3

Binary Operations Ex 3.3 Q1

The binary operator $*$ is defined on I^+ and is given by,

$$a * b = a + b \text{ for all } a, b \in I^+$$

Let $a \in I^+$ and $e \in I^+$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$$\Rightarrow a + e = a$$

$$\Rightarrow e = 0$$

Thus the required identity element is 0.

Binary Operations Ex 3.3 Q2

Let $R - \{-1\}$ be the set and $*$ be a binary operator, given by

$$a * b = a + b + ab \text{ for all } a, b \in R - \{-1\}$$

Now,

Let $a \in R - \{-1\}$ and $e \in R - \{-1\}$ be the identity element with respect to $*$.
by identity property, we have,

$$a * e = e * a = a$$

$$\Rightarrow a + e + ae = a$$

$$\Rightarrow e(1 + a) = 0$$

$$\Rightarrow e = 0 \quad [\because 1 + a \neq 0 \text{ as } a \neq -1]$$

$\therefore$ The required identity element is 0.

Binary Operations Ex 3.3 Q3

We are given the binary operator $*$ defined on Z as

$$a * b = a + b - 5 \text{ for all } a, b \in Q.$$

Let e be the identity element with respect to $*$

$$\text{Then, } a * e = e * a = a \quad [\text{By identity property}]$$

$$\Rightarrow a + e - 5 = a$$

$$\Rightarrow e = 5$$

Hence, the required identity element with respect to $*$ is 5.

Binary Operations Ex 3.3 Q4

The binary operator $*$ is defined on Z , and is given by

$$a * b = a + b + 2 \text{ for all } a, b \in Z.$$

Let $a \in Z$ and $e \in Z$ be the identity element with respect to $*$, then

$$a * e = e * a = a \quad [\text{By identity property}]$$

$$\Rightarrow a + e + 2 = a$$

$$\Rightarrow e = -2 \in Z$$

Hence, the identity element with respect to $*$ is -2 .

Ex 3.4

Binary Operations Ex 3.4 Q1

Given,

$$a * b = a + b - 4 \text{ for all } a, b \in \mathbb{Z}$$

(i)

Commutative: Let $a, b \in \mathbb{Z}$, then

$$\Rightarrow a * b = a + b - 4 = b + a - 4 = b * a$$

$$\Rightarrow a * b = b * a$$

So, '*' is commutative on $\mathbb{Z}$.

Associativity: Let $a, b, c \in \mathbb{Z}$, then

$$\begin{aligned} (a * b) * c &= (a + b - 4) * c = a + b - 4 + c - 4 \\ &= a + b + c - 8 \end{aligned} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * (b + c - 4) = a + b + c - 8 \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on $\mathbb{Z}$.

(ii)

Let $e \in \mathbb{Z}$ be the identity element with respect to *.

By identity property, we have

$$a * e = e * a = a \text{ for all } a \in \mathbb{Z}$$

$$\Rightarrow a + e - 4 = a$$

$$\Rightarrow e = 4$$

So, $e = 4$ will be the identity element with respect to *

(iii)

Let $b \in \mathbb{Z}$ be the inverse element of $a \in \mathbb{Z}$

$$\text{Then, } a * b = b * a = e$$

$$\Rightarrow a + b - 4 = e$$

$$\Rightarrow a + b - 4 = 4 \quad [\because e = 4]$$

$$\Rightarrow b = 8 - a$$

Thus, $b = 8 - a$ will be the inverse element of $a \in \mathbb{Z}$.

Binary Operations Ex 3.4 Q2

We have,

$$a * b = \frac{3ab}{5} \text{ for all } a, b \in Q_0$$

(i)

Commutative: Let $a, b \in Q_0$, then

$$a * b = \frac{3ab}{5} = \frac{3ba}{5} = b * a$$

$$\Rightarrow a * b = b * a$$

So, '*' is commutative on Q_0

Associativity: Let $a, b, c \in Q_0$, then

$$\begin{aligned} (a * b) * c &= \frac{3ab}{5} * c \\ &= \frac{9abc}{25} \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * \frac{3bc}{5} \\ &= \frac{9abc}{25} \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on Q_0

(ii)

Let $e \in Q_0$ be the identity element with respect to *, then

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \frac{3ae}{5} = a$$

$$\begin{aligned} \Rightarrow e &= \frac{5}{3} \\ &\text{will be the identity element with respect to } *. \end{aligned}$$

(iii)

Let $b \in Q_0$ be the inverse element of $a \in Q_0$, then

$$a * b = b * a = e$$

$$\begin{aligned} \Rightarrow \frac{3}{5}ab &= e \\ \Rightarrow \frac{3}{5}ab &= \frac{5}{3} & \left[\because e = \frac{5}{3} \right] \\ \Rightarrow b &= \frac{25}{9a} \end{aligned}$$

$$\therefore b = \frac{25}{9a} \text{ is the inverse of } a \in Q_0.$$

Binary Operations Ex 3.4 Q3

We have,

$$a * b = a + b + ab \text{ for all } a, b \in Q - \{-1\}$$

(i)

Commutativity: Let $a, b \in Q - \{-1\}$

$$\Rightarrow a * b = a + b + ab = b + a + ba = b * a$$

$$\Rightarrow a * b = b * a$$

$$\Rightarrow '*' \text{ is commutative on } Q - \{-1\}$$

Associativity: Let $a, b, c \in Q - \{-1\}$, then

$$\begin{aligned} \Rightarrow (a * b) * c &= (a + b + ab) * c \\ &= a + b + ab + c + ac + bc + abc \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (b + c + bc) \\ &= a + b + c + bc + ab + ac + abc \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$$\Rightarrow * \text{ is associative on } Q - \{-1\}$$

(ii)

Let e be identity element with respect to $*$.

By identity property,

$$a * e = a = e * a \text{ for all } a \in Q - \{-1\}$$

$$\Rightarrow a + e + ae = a$$

$$\Rightarrow e(1 + a) = 0 \Rightarrow e = 0 \quad \left[\because 1 + a \neq 0 \text{ as } a \neq -1 \right]$$

$$\therefore e = 0 \text{ is the identity element with respect to } *$$

(iii)

Let b be the inverse of $a \in Q - \{-1\}$

$$\text{Then, } a * b = b * a = e \quad [e \text{ is the identity element}]$$

$$\Rightarrow a + b + ab = e$$

$$\Rightarrow a + b + ab = 0$$

$$\Rightarrow b(1 + a) = -a$$

$$\Rightarrow b = \frac{-a}{1+a} \quad \left[\begin{array}{l} \because \frac{-a}{1+a} \neq -1, \text{ because if } \frac{-a}{1+a} = -1 \\ \Rightarrow a = 1+a \Rightarrow 1 = 0 \text{ Not possible} \end{array} \right]$$

$$\therefore b = \frac{-a}{1+a} \text{ is the inverse of } a \text{ with respect to } *$$

Binary Operations Ex 3.4 Q4

We have,

$$(a, b) \odot (c, d) = (ac, bc + d) \text{ for all } (a, b), (c, d) \in R_0 \times R$$

(i)

Commutativity: Let $(a, b), (c, d) \in R_0 \times R$, then

$$\Rightarrow (a, b) \odot (c, d) = (ac, bc + d) \quad \text{--- (i)}$$

$$\text{and, } (c, d) \odot (a, b) = (ca, da + b) \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a, b) \odot (c, d) \neq (c, d) \odot (a, b)$$

$$\Rightarrow \text{'}\odot\text{' is not commutative on } R_0 \times R.$$

Associativity: Let $(a, b), (c, d), (e, f) \in R_0 \times R$, then

$$\begin{aligned} \Rightarrow ((a, b) \odot (c, d)) \odot (e, f) &= (ac, bc + d) \odot (e, f) \\ &= (ace, bce + de + f) \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } (a, b) \odot (c, d \odot (e, f)) &= (a, b) \odot (ce, de + f) \\ &= (ace, bce + de + f) \end{aligned} \quad \text{--- (ii)}$$

$$\Rightarrow ((a, b) \odot (c, d)) \odot (e, f) = (a, b) \odot ((c, d) \odot (e, f))$$

$$\Rightarrow \text{'}\odot\text{' is associative on } R_0 \times R.$$

(ii)

Let $(x, y) \in R_0 \times R$ be the identity element with respect to $\odot$, then

$$(a, b) \odot (x, y) = (x, y) \odot (a, b) = (a, b) \text{ for all } (a, b) \in R_0 \times R$$

$$\Rightarrow (ax, bx + y) = (a, b)$$

$$\Rightarrow ax = a \text{ and } bx + y = b$$

$$\Rightarrow x = 1, \text{ and } y = 0$$

$$\therefore (1, 0) \text{ will be the identity element with respect to } \odot.$$

(iii)

Let $(c, d) \in R_0 \times R$ be the inverse of $(a, b) \in R_0 \times R$, then

$$(a, b) \odot (c, d) = (c, d) \odot (a, b) = e$$

$$\Rightarrow (ac, bc + d) = (1, 0) \quad [\because e = (1, 0)]$$

$$\Rightarrow ac = 1 \text{ and } bc + d = 0$$

$$\Rightarrow c = \frac{1}{a} \text{ and } d = -\frac{b}{a}$$

$$\therefore \left(\frac{1}{a}, -\frac{b}{a} \right) \text{ will be the inverse of } (a, b).$$

Binary Operations Ex 3.4 Q5

We have,

$$a * b = \frac{ab}{2} \text{ for all } a, b \in Q_0$$

(i)

Commutativity: Let $a, b \in Q_0$, then

$$\Rightarrow a * b = \frac{ab}{2} = \frac{ba}{2} = b * a$$

$$\Rightarrow a * b = b * a$$

Hence, '*' is commutative on Q_0 .

Associativity: Let $a, b, c \in Q_0$, then

$$\Rightarrow (a * b) * c = \frac{ab}{2} * c = \frac{abc}{4} \quad \text{--- (i)}$$

$$\text{and, } a * (b * c) = a * \frac{bc}{2} = \frac{abc}{4} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

$$\Rightarrow * \text{ is associative on } Q_0.$$

(ii)

Let $e \in Q_0$ be the identity element with respect to *.

By identity property, we have,

$$a * e = e * a = a \text{ for all } a \in Q_0$$

$$\Rightarrow \frac{ae}{2} = a \quad \Rightarrow e = 2$$

Thus, the required identity element is 2.

(iii)

Let $b \in Q_0$ be the inverse of $a \in Q_0$ with respect to *, then,

$$a * b = b * a = e \text{ for all } a \in Q_0$$

$$\begin{aligned} \Rightarrow \frac{ab}{2} &= e & \Rightarrow \frac{ab}{2} &= 2 \\ & & \Rightarrow b &= \frac{4}{a} \end{aligned}$$

Thus, $b = \frac{4}{a}$ is the inverse of a with respect to *.

Binary Operations Ex 3.4 Q6

We have,

$$a * b = a + b - ab \text{ for all } a, b \in R - \{+1\}$$

(i)

Commutative: Let $a, b \in R - \{+1\}$, then,

$$\Rightarrow a * b = a + b - ab = b + a - ba = b * a$$

$$\Rightarrow a * b = b * a$$

So, '*' is commutative on $R - \{+1\}$.

Associativity: Let $a, b, c \in R - \{+1\}$, then

$$\begin{aligned} (a * b) * c &= (a + b - ab) * c \\ &= a + b - ab + c - ac - bc + abc \\ &= a + b + c - ab - ac - bc + abc \end{aligned} \quad \text{--- (i)}$$

$$\begin{aligned} \text{and, } a * (b * c) &= a * (b + c - bc) \\ &= a + b + c - bc - ab - ac + abc \end{aligned} \quad \text{--- (ii)}$$

From (i) & (ii)

$$(a * b) * c = a * (b * c)$$

So, '*' is associative on $R - \{+1\}$.

(ii)

Let $e \in R - \{+1\}$ be the identity element with respect to *, then

$$a * e = e * a = a \text{ for all } a \in R - \{+1\}$$

$$\Rightarrow a + e - ae = a$$

$$\Rightarrow e(1 - a) = 0$$

$$\Rightarrow e = 0 \quad [\because a \neq 1 \Rightarrow 1 - a \neq 0]$$

$\therefore e = 0$ will be the identity element with respect to *.

(iii)

Let $b \in R - \{1\}$ be the inverse element of $a \in R - \{1\}$, then

$$a * b = b * a = e$$

$$\Rightarrow a + b - ab = 0 \quad [\because e = 0]$$

$$\Rightarrow b(1 - a) = -a$$

$$\Rightarrow b = \frac{-a}{1 - a} \neq 1 \quad \left[\begin{array}{l} \because \text{if } \frac{-a}{1 - a} = 1 \\ \Rightarrow -a = 1 - a \Rightarrow 1 = 0 \\ \text{Not possible} \end{array} \right]$$

$\therefore b = \frac{-a}{1 - a}$ is the inverse of $a \in R - \{1\}$ with respect to *.

Binary Operations Ex 3.4 Q7

We have,

$$(a,b) * (c,d) = (ac, bd) \text{ for all } (a,b), (c,d) \in A$$

(i)

Let $(a,b), (c,d) \in A$, then

$$\begin{aligned} (a,b) * (c,d) &= (ac, bd) \\ &= (ca, db) \quad [\because ac = ca \text{ and } bd = db] \\ &= (c,d) * (a,b) \end{aligned}$$

$$\Rightarrow (a,b) * (c,d) = (c,d) * (a,b)$$

So, '*' is commutative on A

Associativity: Let $(a,b), (c,d), (e,f) \in A$, then

$$\begin{aligned} \Rightarrow ((a,b) * (c,d)) * (e,f) &= (ac, bd) * (e,f) \\ &= (ace, bdf) \quad \text{--- (i)} \end{aligned}$$

$$\begin{aligned} \text{and, } (a,b) * ((c,d) * (e,f)) &= (a,b) * (ce, df) \\ &= (ace, bdf) \quad \text{--- (ii)} \end{aligned}$$

From (i) & (ii)

$$\Rightarrow ((a,b) * (c,d)) * (e,f) = (a,b) * ((c,d) * (e,f))$$

So, '*' is associative on A.

(ii)

Let $(x,y) \in A$ be the identity element with respect to *.

$$(a,b) * (x,y) = (x,y) * (a,b) = (a,b) \text{ for all } (a,b) \in A$$

$$\begin{aligned} \Rightarrow (ax, by) &= (a,b) \\ \Rightarrow ax &= a \text{ and } by = b \\ \Rightarrow x &= 1, \text{ and } y = 1 \end{aligned}$$

$\therefore (1,1)$ will be the identity element

(iii)

Let $(c,d) \in A$ be the inverse of $(a,b) \in A$, then

$$(a,b) * (c,d) = (c,d) * (a,b) = e$$

$$\begin{aligned} \Rightarrow (ac, bd) &= (1,1) \quad [\because e = (1,1)] \\ \Rightarrow ac &= 1 \text{ and } bd = 1 \\ \Rightarrow c &= \frac{1}{a} \text{ and } d = \frac{1}{b} \end{aligned}$$

$\therefore \left(\frac{1}{a}, \frac{1}{b}\right)$ will be the inverse of (a,b) with respect to *.

Binary Operations Ex 3.4 Q8

The binary operation $*$ on $\mathbf{N}$ is defined as:

$$a * b = \text{H.C.F. of } a \text{ and } b$$

It is known that:

$$\text{H.C.F. of } a \text{ and } b = \text{H.C.F. of } b \text{ and } a, \quad a, b \in \mathbf{N}.$$

$$\text{Therefore, } a * b = b * a$$

Thus, the operation $*$ is commutative.

For $a, b, c \in \mathbf{N}$, we have:

$$(a * b) * c = (\text{H.C.F. of } a \text{ and } b) * c = \text{H.C.F. of } a, b, \text{ and } c$$

$$a * (b * c) = a * (\text{H.C.F. of } b \text{ and } c) = \text{H.C.F. of } a, b, \text{ and } c$$

$$\text{Therefore, } (a * b) * c = a * (b * c)$$

Thus, the operation $*$ is associative.

Now, an element $e \in \mathbf{N}$ will be the identity for the operation

$$* \text{ if } a * e = a = e * a, \quad \forall a \in \mathbf{N}.$$

But this relation is not true for any $a \in \mathbf{N}$.

Thus, the operation $*$ does not have any identity in $\mathbf{N}$.

Ex 3.5

Binary Operations Ex 3.5 Q1

$a \times_4 b$ = the remainder when ab is divided by 4.

eg. (i) $2 \times 3 = 6 \Rightarrow 2 \times_4 3 = 2$

[When 6 is divided by 4 we get 2 as remainder]

(ii) $2 \times 3 = 4 \Rightarrow 2 \times_4 2 = 0$

[When 4 is divided by 4 we get 0 as remainder]

The composition table for $\times_4$ on set $S = \{0, 1, 2, 3\}$ is :

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Binary Operations Ex 3.5 Q 2

$a +_5 b =$ the remainder when $a + b$ is divided by 5.

eg. $2 + 4 = 6 \Rightarrow 2 +_5 4 = 1 \quad \therefore$ [we get 1 as remainder when 6 is divided by 5]

$2 + 4 = 7 \Rightarrow 3 +_5 4 = 2 \quad \therefore$ [we get 2 as remainder when 7 is divided by 5]

The composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Binary Operations Ex 3.5 Q3

$a \times_6 b =$ the remainder when the product of ab is divided by 6.

The composition table for $\times_6$ on set $S = \{0, 1, 2, 3, 4, 5\}$.

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Binary Operations Ex 3.5 Q4

$a \times_5 b =$ the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$.

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q5

$a \times_{10} b$ = the remainder when the product of ab is divided by 10.

The composition table for $\times_{10}$ on set $S = \{1, 3, 7, 9\}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

We know that an element $b \in S$ will be the inverse of $a \in S$

$$\text{if } a \times_{10} b = 1 \quad \left[\because 1 \text{ is the identity element with respect to multiplication} \right]$$

$$\Rightarrow 3 \times_{10} b = 1$$

From the above table $b = 7$

$\therefore$ Inverse of 3 is 7.

Binary Operations Ex 3.5 Q6

$a \times_7 b$ = the remainder when the product of ab is divided by 7.

The composition table for $\times_7$ on $S = \{1, 2, 3, 4, 5, 6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also, b will be the inverse of a
if, $a \times_7 b = e = 1$

$$\Rightarrow 3 \times_7 b = 1$$

From the above table $3 \times_7 5 = 1$

$$\therefore b = 3^{-1} = 5$$

$$\text{Now, } 3^{-1} \times_7 4 = 5 \times_7 4 = 6$$

Binary Operations Ex 3.5 Q7

$a \times_{11} b$ = the remainder when the product of ab is divided by 11.

The composition table for $\times_{11}$ on Z_{11}

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$$5 \times_{11} 9 = 1 \quad [\because 1 \text{ is the identity element}]$$

$\therefore$ Inverse of 5 is 9.

Binary Operations Ex 3.5 Q8

$$Z_5 = \{0, 1, 2, 3, 4\}$$

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$$a * b = b * a = b$$

$$a * c = c * a = c$$

$$a * d = d * a = d$$

$$b * c = c * b = d$$

$$b * d = d * b = c$$

$$c * d = d * c = b$$

$\therefore$ $*$ is commutative

Again, $a, b, c \in S$

$$\Rightarrow (a * b) * c = b * c = d \text{ and} \\ a * (b * c) = a * d = d$$

$$\therefore (a * b) * c = a * (b * c)$$

$\therefore$ $*$ is associative

We know that e will be identity element with respect to $*$ if

$$a * e = e * a = a \text{ for all } a \in S$$

$$\Rightarrow a * a = a, a * b = b, a * c = c, a * d = d$$

$\therefore$ a will be the identity element

Again,

b will be the inverse of a if

$$b * a = a * b = e$$

From the above table

$$a * a = a, \quad b * b = b, c * c = c \text{ and } d * d = d$$

$\therefore$ Inverse of $a = a$

$$b = b$$

$$c = c$$

$$d = d$$

(ii)

From the above table, we can observe

$$\begin{aligned} aob &= boa, & boc &= cob \\ aoc &= coa, & bod &= dob \\ aod &= doo, & cod &= doc \end{aligned}$$

$\therefore$ 'o' is commutative on S

Again, for $a, b, c \in S$

$$(aob)oc = aoc = a \quad \text{---(i)}$$

$$ao(boc) = aoc = a \quad \text{---(ii)}$$

From (i) & (ii)

$$(aob)oc = ao(boc)$$

So, 'o' is associative on S

Now, we have,

$$aob = a$$

$$bob = b$$

$$cob = c$$

$$dob = d$$

$\Rightarrow$ b is the identity element with respect to 'o'

We know that x will be inverse of y

$$\text{if } xoy = yox = e$$

$$\Rightarrow xoy = yox = b \quad [\because e = b]$$

Now, from the above table we find that

$$bob = b$$

$$cod = b$$

$$doc = b$$

$$\therefore b^{-1} = b, c^{-1} = d, \text{ and } d^{-1} = c$$

Not: a^{-1} does not exist.

Binary Operations Ex 3.5 Q10

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation $*$ on X is defined as:

$$a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$$

An element $e \in X$ is the identity element for the operation $*$, if

$$a * e = a = e * a \quad \forall a \in X.$$

For $a \in X$, we observed that:

$$a * 0 = a + 0 = a \quad [a \in X \Rightarrow a + 0 < 6]$$

$$0 * a = 0 + a = a \quad [a \in X \Rightarrow 0 + a < 6]$$

$$\therefore a * 0 = a = 0 * a \quad \forall a \in X$$

Thus, 0 is the identity element for the given operation $*$.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e., } \begin{cases} a + b = 0 = b + a, & \text{if } a + b < 6 \\ a + b - 6 = 0 = b + a - 6, & \text{if } a + b \geq 6 \end{cases}$$

i.e.,

$$a = -b \text{ or } b = 6 - a$$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.