

Chapter - 1

Rational Numbers

Exercise

In questions 1 to 25, there are four options out of which one is correct. Choose the correct answer.

1. A number which can be expressed as $\frac{p}{q}$ where p and q are integers and

$q \neq 0$ is

- (a) natural number.
- (b) whole number.
- (c) integer.
- (d) rational number.

Solution:

(d) rational number.

A number which can be expressed as $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is rational number.

2. A number of the form $\frac{p}{q}$ is said to be a rational number if

- (a) p and q are integers.
- (b) p and q are integers and $q \neq 0$.
- (c) p and q are integers and $p \neq 0$.
- (d) p and q are integers and $p \neq 0$ also $q \neq 0$.

Solution:

(b) p and q are integers and $q \neq 0$

3. The numerical expression $\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$ shows that

- (a) rational numbers are closed under addition.
- (b) rational numbers are not closed under addition.
- (c) rational numbers are closed under multiplication.
- (d) addition of rational numbers is not commutative.

Solution:

(a) rational numbers are closed under addition.

As,

$$\frac{3}{8} + \frac{(-5)}{7}$$

We take the LCM of the denominators of the given rational numbers.
LCM of 8 and 7 is 56.

Express each of the given rational numbers with the above LCM as the common denominator.

Then,

$$\frac{3}{8} + \frac{(-5)}{7} = \frac{21-40}{56} \quad \dots \text{ [denominator is same in both the rational numbers]}$$

$$\frac{3}{8} + \frac{(-5)}{7} = \frac{-19}{56}$$

4. Which of the following is not true?

- (a) rational numbers are closed under addition.
- (b) rational numbers are closed under subtraction.
- (c) rational numbers are closed under multiplication.
- (d) rational numbers are closed under division.

Solution:

(d) Rational numbers are closed under division.

As, rational numbers are closed under the operations of addition, subtraction and multiplication.

5. $-\frac{3}{8} + \frac{1}{7} = \frac{1}{7} + \left(\frac{-3}{8}\right)$ is an example to show that

- (a) addition of rational numbers is commutative.
- (b) rational numbers are closed under addition.
- (c) addition of rational number is associative.
- (d) rational numbers are distributive under addition.

Solution:

(a) Addition of rational numbers is commutative.

The arrangement in given rational numbers is in the form of Commutative law of addition.

[a + b = b + a]

6. Which of the following expressions shows that rational numbers are associative under multiplication.

(a) $\frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5}\right) = \left(\frac{2}{3} \times \frac{-6}{7}\right) \times \frac{3}{5}$

$$(b) \frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5} \right) = \frac{2}{3} \times \left(\frac{3}{5} \times \frac{-6}{7} \right)$$

$$(c) \frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5} \right) = \left(\frac{3}{5} \times \frac{2}{3} \right) \times \frac{-6}{7}$$

$$(d) \left(\frac{2}{3} \times \frac{-6}{7} \right) \times \frac{3}{5} = \left(\frac{-6}{7} \times \frac{2}{3} \right) \times \frac{3}{5}$$

Solution:

$$(a) \frac{2}{3} \times \left(\frac{-6}{7} \times \frac{3}{5} \right) = \left(\frac{2}{3} \times \frac{-6}{7} \right) \times \frac{3}{5}$$

As, the arrangement of above rational numbers is in the form of Associative law of Multiplication.

$$[a \times (b \times c)] = [(a \times b) \times c]$$

7. Zero (0) is

- (a) the identity for addition of rational numbers.
- (b) the identity for subtraction of rational numbers.
- (c) the identity for multiplication of rational numbers.
- (d) the identity for division of rational numbers.

Solution:

- (a) the identity for addition of rational numbers.

8. One (1) is

- (a) the identity for addition of rational numbers.
- (b) the identity for subtraction of rational numbers.
- (c) the identity for multiplication of rational numbers.
- (d) the identity for division of rational numbers.

Solution:

- (c) is the identity for multiplication of rational numbers.

9. The additive inverse of $\frac{-7}{19}$ is

(a) $\frac{-7}{19}$

(b) $\frac{7}{19}$

(c) $\frac{19}{7}$

(d) $\frac{-19}{7}$

Solution:

(b) $\frac{7}{19}$

Additive inverse of $\frac{-7}{19}$ is $\frac{7}{19}$.

10. Multiplicative inverse of a negative rational number is

- (a) a positive rational number.
- (b) a negative rational number.
- (c) 0
- (d) 1

Solution:

(b) a negative rational number.

11. If $x + 0 = 0 + x = x$, which is rational number, then 0 is called

- (a) identity for addition of rational numbers.
- (b) additive inverse of x .
- (c) multiplicative inverse of x .
- (d) reciprocal of x .

Solution:

$x + 0 = 0 + x = x$ is

(a) identity for addition of rational numbers.

12. To get the product 1, we should multiply $\frac{8}{21}$ by

- (a) $\frac{8}{21}$
- (b) $\frac{-8}{21}$
- (c) $\frac{21}{8}$
- (d) $\frac{-21}{8}$

Solution:

(c) $\frac{21}{8}$

As,

$$\left(\frac{8}{21}\right) \times \left(\frac{21}{8}\right) = 1$$

13. $-(-x)$ is same as

- (a) $-x$
- (b) x
- (c) $\frac{1}{x}$
- (d) $\frac{-1}{x}$

Solution:

(b) x

We have,

$(- \times - = +)$

14. The multiplicative inverse of $-1\frac{1}{7}$ is

- (a) $\frac{8}{7}$
- (b) $\frac{-8}{7}$
- (c) $\frac{7}{8}$
- (d) $\frac{7}{-8}$

Solution:

(d) $\frac{7}{-8}$

As,

$$\begin{aligned} -1\frac{1}{7} &= \frac{-8}{7} \\ &= \frac{7}{-8} \end{aligned}$$

[reciprocal]

15. If x be any rational number then $x + 0$ is equal to

- (a) x
- (b) **0**
- (c) $-x$
- (d) **Not defined**

Solution:

(a) $x = x + 0$
 $= x$

[Identity for addition of rational numbers]

16. The reciprocal of 1 is

- (a) 1
- (b) -1
- (c) 0
- (d) Not defined

Solution:

(a) 1
Reciprocal of 1 = 1

17. The reciprocal of -1 is

- (a) 1
- (b) -1
- (c) 0
- (d) Not defined

Solution:

(b) -1
Reciprocal of -1 = -1

18. The reciprocal of 0 is

- (a) 1
- (b) -1
- (c) 0
- (d) Not defined

Solution:

(d) Not defined

Reciprocal of 0 = $\frac{1}{0}$
= not defined

19. The reciprocal of any rational number $\frac{p}{q}$, where p and q are integers and $q \neq 0$, is

- (a) $\frac{p}{q}$
- (b) 1
- (c) 0
- (d) $\frac{q}{p}$

Solution:

(d) $\frac{q}{p}$

The reciprocal of $\frac{p}{q} = \frac{q}{p}$

20. If y be the reciprocal of rational number x , then the reciprocal of y will be

(a) x

(b) y

(c) $\frac{x}{y}$

(d) $\frac{y}{x}$

Solution:

(a) x

If y be the reciprocal of rational number x ,

$$y = \frac{1}{x}$$

$$x = \frac{1}{y}$$

Then,

Reciprocal of $y = x$

21. The reciprocal of $-\frac{3}{8} \times \left(\frac{-7}{13}\right)$ is

(a) $\frac{104}{21}$

(b) $\frac{-104}{21}$

(c) $\frac{21}{104}$

(d) $\frac{-21}{104}$

Solution:

(a) $\frac{104}{21}$

$$\frac{-3}{8} \times \left(\frac{-7}{13} \right) = \frac{21}{104}$$

Reciprocal of $\frac{21}{104}$ is $\frac{104}{21}$

22. Which of the following is an example of distributive property of multiplication over addition for rational numbers.

(a) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left[-\frac{1}{4} \times \frac{2}{3} \right] + \left[-\frac{1}{4} \times \left(\frac{-4}{7} \right) \right]$

(b) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left[\frac{1}{4} \times \frac{2}{3} \right] - \left(\frac{-4}{7} \right)$

(c) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \frac{2}{3} + \left(-\frac{1}{4} \right) \times \frac{-4}{7}$

(d) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} - \frac{1}{4}$

Solution:

(a) $-\frac{1}{4} \times \left\{ \frac{2}{3} + \left(\frac{-4}{7} \right) \right\} = \left[-\frac{1}{4} \times \frac{2}{3} \right] + \left[-\frac{1}{4} \times \left(\frac{-4}{7} \right) \right]$

Because, we know the rule of distributive law,
 $a \times (b + c) = [(a \times b) + (a \times c)]$

23. Between two given rational numbers, we can find

(a) one and only one rational number.

(b) only two rational numbers.

(c) only ten rational numbers.

(d) infinitely many rational numbers.

Solution:

(d) Infinitely many rational numbers.

24. $\frac{x+y}{2}$ is a rational number.

(a) Between x and y

(b) Less than x and y both.

(c) Greater than x and y both.

(d) Less than x but greater than y .

Solution:

(a) Between x and y

Let us assume the value of x and y is 4 and 8 respectively

Then,

$$\frac{4+8}{2} = \frac{12}{2}$$
$$= 6$$

So, the value 6 is lies between 4 and 8.

25. Which of the following statements is always true?

- (a) $\frac{x-y}{2}$ is a rational number between x and y.
- (b) $\frac{x+y}{2}$ is a rational number between x and y.
- (c) $\frac{x \times y}{2}$ is a rational number between x and y.
- (d) $\frac{x \div y}{2}$ is a rational number between x and y.

Solution:

- (b) $\frac{x+y}{2}$ is a rational number between x and y

Let us assume the value of x and y is 6 and 9 respectively.

Then,

$$\frac{6+9}{2} = \frac{14}{2}$$
$$= 7$$

So, the value 7 is lies between 6 and 9.

In questions 26 to 47, fill in the blanks to make the statements true.

26. The equivalent of $\frac{5}{7}$, whose numerator is 45 is _____.

Solution:

According to question it is given that equivalent of $\frac{5}{7} = \frac{45}{\text{denominator}}$

To get 45 in the numerator multiply both numerator and denominator by 9.

So,

$$\frac{5 \times 9}{7 \times 9} = \frac{45}{63}$$

So, the equivalent of $\frac{5}{7}$, whose numerator is 45 is ($\frac{45}{63}$).

27. The equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is _____.

Solution:

Form the question it is given that equivalent of $\frac{7}{9} = \frac{\text{Numerator}}{45}$

To get 45 in the denominator multiply both numerator and denominator by 5
Then,

$$\frac{7 \times 5}{9 \times 5} = \frac{35}{45}$$

So, the equivalent rational number of $\frac{7}{9}$, whose denominator is 45 is $(\frac{35}{45})$.

28. Between the numbers $\frac{15}{20}$ and $\frac{35}{40}$, the greater number is _____.

Solution:

The LCM of the denominators 20 and 40 is 40.

$$\begin{aligned} \left(\frac{15}{20}\right) &= \left[\left(\frac{15 \times 2}{20 \times 2}\right)\right] \\ &= \left(\frac{30}{40}\right) \end{aligned}$$

and

$$\begin{aligned} \left(\frac{35}{40}\right) &= \left[\left(\frac{35 \times 1}{40 \times 1}\right)\right] \\ &= \frac{35}{40} \end{aligned}$$

Now, $30 < 35$

$$\left(\frac{30}{40}\right) < \left(\frac{35}{40}\right)$$

Hence, $\left(\frac{15}{20}\right) < \left(\frac{35}{40}\right)$

So, $\frac{35}{40}$ is greater.

29. The reciprocal of a positive rational number is _____.

Solution:

The reciprocal of a positive rational number is positive rational number.

30. The reciprocal of a negative rational number is _____.

Solution:

The reciprocal of a negative rational number is negative rational number.

31. Zero has _____ reciprocal.

Solution:

Zero has no reciprocal.

The reciprocal of 0 = Not defined.

32. The numbers _____ and _____ are their own reciprocal.

Solution:

The numbers 1 and -1 are their own reciprocal.

33. If y be the reciprocal of x, then the reciprocal of y^2 in terms of x will be _____.

Solution:

If y be the reciprocal of x, then the reciprocal of y^2 in terms of x will be $\frac{1}{y^2}$.

According to question,

$$\left(\frac{1}{x}\right) = y$$

Then,

$$\text{Reciprocal of } y^2 = \frac{1}{y^2}$$

Putting $\left(\frac{1}{x}\right)$ in the place of y,

$$\frac{1}{\left(\frac{1}{x}\right)^2} = x^2$$

34. The reciprocal of $\frac{2}{5} \times \left(\frac{-4}{9}\right)$ is _____.

Solution:

$$\frac{2}{5} \times \left(\frac{-4}{9}\right) = \frac{-8}{45}$$

$$\text{Reciprocal} = \frac{-45}{8}$$

35. $(213 \times 657)^{-1} = 213^{-1} \times$ _____.

Solution:

Let us take the missing number be x.

$$(213 \times 657)^{-1} = 213^{-1} \times x$$

$$x = 657^{-1}$$

36. The negative of 1 is _____.

Solution:

The negative of 1 is -1.

37. For rational numbers $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f}$, we have $\frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right)$ _____ + _____.

Solution:

$$\text{For rational numbers } \frac{a}{b}, \frac{c}{d} \text{ and } \frac{e}{f}, \text{ we have } \frac{a}{b} \times \left(\frac{c}{d} + \frac{e}{f}\right) = \underline{\left(\frac{a}{b} \times \frac{c}{d}\right) + \left(\frac{a}{b} \times \frac{e}{f}\right)}$$

38. $\frac{-5}{7}$ is _____ than -3.

Solution:

$\frac{-5}{7}$ is more than -3.

Solution:

39. There are _____ rational numbers between any two rational numbers.

Solution:

There are infinitely many rational numbers between any two rational numbers.

40. The rational numbers $\frac{1}{3}$ and $\frac{-1}{3}$ are on the _____ sides of zero on the number line.

Solution:

The rational numbers $\frac{1}{3}$ and $\frac{-1}{3}$ are on the opposite sides of zero on the number line.

41. The negative of a negative rational number is always a _____ rational number.

Solution:

Positive

Let x be a positive rational number.
Then, $-x$ be a negative rational number.

Now, negative of a negative rational number = $-(-x) = x$
= positive rational number.

42. Rational numbers can be added or multiplied in any _____.

Solution:

Order

Rational numbers can be added or multiplied in any order and this concept is known as commutative property.

43. The reciprocal of $\frac{-5}{7}$ is _____.

Solution:

The reciprocal of $\frac{-5}{7}$ is $\frac{-7}{5}$.

44. The multiplicative inverse of $\frac{4}{3}$ is _____.

Solution:

The multiplicative inverse of $\frac{4}{3}$ is $\frac{3}{4}$.

45. The rational number 10.11 in the form $\frac{p}{q}$ is _____.

Solution:

The rational number 10.11 in the form $\frac{p}{q}$ is $\frac{1011}{100}$.

46. $\frac{1}{5} \times \left[\frac{2}{7} + \frac{3}{8} \right] = \left[\frac{1}{5} \times \frac{2}{7} \right] +$ _____.

Solution:

$$\frac{1}{5} \times \left[\frac{2}{7} + \frac{3}{8} \right] = \left[\frac{1}{5} \times \frac{2}{7} \right] + \left[\frac{1}{5} \times \frac{3}{8} \right]$$

From the rule of distributive law of multiplication,
[$a \times (b + c) = (a \times b) + (a \times c)$]

47. The two rational numbers lying between -2 and -5 with denominator as 1 are _____ and _____.

Solution:

The two rational numbers lying between -2 and -5 with denominator as 1 are $\underline{-3}$ and $\underline{-4}$.

In each of the following, state whether the statements are true (T) or false (F).

48. If $\frac{x}{y}$ is a rational number, then y is always a whole number.

Solution:

False.

If $\frac{x}{y}$ is a rational number, then y is not equal to 0.

But 0 is a whole number.

49. If $\frac{p}{q}$ is a rational number, then p cannot be equal to zero.

Solution:

False.

If $\frac{p}{q}$ is a rational number, p can be equal to zero (0) or any integer.

50. If $\frac{r}{s}$ is a rational number, then s cannot be equal to zero.

Solution:

True.

51. $\frac{5}{6}$ lies between $\frac{2}{3}$ and 1.

Solution:

True.

52. $\frac{5}{10}$ lies between $\frac{1}{2}$ and 1.

Solution:

False.

53. $\frac{-7}{2}$ lies between -3 and -4 .

Solution:

True.

54. $\frac{9}{6}$ lies between 1 and 2.

Solution:

True.

As $\frac{9}{6} = 1.5$

55. If $a \neq 0$, the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$.

Solution:

True.

56. The multiplicative inverse of $\frac{-3}{5}$ is $\frac{5}{3}$.

Solution:

False.

Because, the correct answer is reciprocal of a negative rational number is negative rational number. i.e. reciprocal of $\frac{-3}{5}$ is $-\frac{5}{3}$.

57. The additive inverse of $\frac{1}{2}$ is -2 .

Solution:

False.

The additive inverse of $\frac{1}{2}$ is $-\frac{1}{2}$.

58. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y} + \frac{c}{d} = 0$.

Solution:

True.

Let $x/y = \frac{1}{2}$ and its additive inverse $c/d = -\frac{1}{2}$

So,

$$\begin{aligned}\left(\frac{x}{y}\right) + \left(\frac{c}{d}\right) &= \frac{1}{2} + \left(-\frac{1}{2}\right) \\ &= \frac{1}{2} - \frac{1}{2} \\ &= 0\end{aligned}$$

59. For every rational number x , $x + 1 = x$.

Solution:

False.

Let $x = 3$

So,

$$3 + 1 = 4$$

$$3 \neq 4$$

Hence, it is clear that $x + 1 \neq x$

60. If $\frac{x}{y}$ is the additive inverse of $\frac{c}{d}$, then $\frac{x}{y} - \frac{c}{d} = 0$.

Solution:

False.

Let $\frac{x}{y} = 2/3$ and its additive inverse $\frac{c}{d} = -2/3$

So,

$$\begin{aligned}\left(\frac{x}{y}\right) - \left(\frac{c}{d}\right) &= (2/3) - (-2/3) \\ &= (2/3) + (2/3) \\ &= 4/3\end{aligned}$$

61. The reciprocal of a non-zero rational number $\frac{q}{p}$ is the rational number $\frac{q}{p}$.

Solution:

False.

Reciprocal of non-zero rational number $\frac{q}{p}$ is $\frac{p}{q}$.

62. If $x + y = 0$, then $-y$ is known as the negative of x , where x and y are rational numbers.

Solution:

False.

If x and y are rational numbers, then y is known as the negative of x

63. The negative of the negative of any rational number is the number itself.

Solution:

True.

Let y be a positive rational number.

So,

The negative of the negative of y is $= -(-y)$
 $= y$

64. The negative of 0 does not exist.

Solution:

True.

65. The negative of 1 is 1 itself.

Solution:

False.

The negative of 1 = -1

66. For all rational numbers x and y , $x - y = y - x$.

Solution:

False.

Let $x = 2$,
 $y = 3$

So,

$$\begin{aligned}\text{LHS} &= x - y \\ &= 2 - 3 \\ &= -1\end{aligned}$$

$$\begin{aligned}\text{RHS} &= y - x \\ &= 3 - 2 \\ &= 1\end{aligned}$$

By comparing LHS and RHS

$$-1 \neq 1$$

LHS \neq RHS

67. For all rational numbers x and y , $x \times y = y \times x$.

Solution:

True

Let $x = 2$,
 $y = 3$

Now,

$$\begin{aligned}\text{LHS} &= 2 \times 3 \\ &= 6\end{aligned}$$

$$\begin{aligned}\text{RHS} &= 3 \times 2 \\ &= 6\end{aligned}$$

By comparing LHS and RHS

$$6 = 6$$

LHS = RHS

68. For every rational number x , $x \times 0 = x$.

Solution:

False.

Let $x = 2$

So,

For every rational number x

$$(x) \times (0) = 0$$

$$2 \times 0 = 0$$

69. For every rational numbers x, y and z , $x + (y \times z) = (x + y) \times (x + z)$.

Solution:

False.

For every rational numbers a, b and c , $[a \times (b + c) = (a \times b) + (a \times c)]$

70. For all rational numbers a, b and c , $a(b + c) = ab + bc$.

Solution:

False.

Because, for every rational numbers a, b and c , $[a \times (b + c) = (a \times b) + (a \times c)]$

71. 1 is the only number which is its own reciprocal.

Solution:

False.

As, the reciprocal of -1 is -1 and reciprocal of 1 is 1.

72. -1 is not the reciprocal of any rational number.

Solution:

False.

The reciprocal of -1 is -1.

73. For any rational number x , $x + (-1) = -x$.

Solution:

False.

The correct form is for any rational number x ,

$$(x) \times (-1) = -x.$$

74. For rational numbers x and y , if $x < y$ then $x - y$ is a positive rational number.

Solution:

False.

As, for rational numbers x and y , if $x < y$ then $x - y$ is a negative rational number.

75. If x and y are negative rational numbers, then so is $x + y$.

Solution:

True

For example,

Let $x = -1/3$ and $y = -2/3$

So,

$$\begin{aligned}x + y &= (-1/3) + (-2/3) \\ &= -1/3 - 2/3 \\ &= -3/3 \\ &= -1\end{aligned}$$

76. Between any two rational numbers there are exactly ten rational numbers.

Solution:

False

As between any two rational numbers there are infinite rational numbers.

77. Rational numbers are closed under addition and multiplication but not under subtraction.

Solution:

False

Rational numbers are closed under addition, subtraction and multiplication.

78. Subtraction of rational number is commutative.

Solution:

False

Subtraction of rational number is not commutative.

Let x and y are any two rational number,

So,

$$x - y \neq y - x$$

79. $-\frac{3}{4}$ is smaller than -2 .

Solution:

False

$$\text{As, } -\frac{3}{4} = -0.75$$

80. 0 is a rational number.

Solution:

True

Because, $\frac{0}{1}$ is a rational number.

81. All positive rational numbers lie between 0 and 1000.

Solution:

False

There are infinite positive rational number on the right side of 0 on the number line.

82. The population of India in 2004 - 05 is a rational number.

Solution:

False

Because rational numbers are expressed in the form of p/q.

83. There are countless rational numbers between $\frac{5}{6}$ and $\frac{8}{9}$.

Solution:

True

84. The reciprocal of x^{-1} is $\frac{1}{x}$.

Solution:

False.

$$x^{-1} = \frac{1}{x}$$

So, reciprocal of $\frac{1}{x} = x$

85. The rational number $\frac{57}{23}$ lies to the left of zero on the number line.

Solution:

False

As $\frac{57}{23}$ is positive so it lies to the right side of 0 on the number line.

86. The rational number $\frac{7}{-4}$ lies to the right of zero on the number line.

Solution:

False

As $\frac{7}{-4}$ is negative so it lies to the left side of 0 on the number line.

87. The rational number $\frac{-8}{-3}$ lies neither to the right nor to the left of zero on the number line.

Solution:

False

$\frac{-8}{-3}$ is a positive rational number. So it lies to the right side of 0 on the number line.

88. The rational numbers $\frac{1}{2}$ and -1 are on the opposite sides of zero on the number line.

Solution:

True

$\frac{1}{2}$ is positive rational number so it lies to the right side of 0 on the number line.

-1 is negative rational number so it lies to the left side of 0 on the number line.

89. Every fraction is a rational number.

Solution:

True

As rational numbers can be expressed in the p/q form and fraction is also a part of whole which can be expressed in the form of p/q .

90. Every integer is a rational number.

Solution:

True

In integer denominator = 1. Hence, every integer is a rational number.

91. The rational numbers can be represented on the number line.

Solution:

True

92. The negative of a negative rational number is a positive rational number.

Solution:

True

Let us take $-\frac{1}{2}$ is a negative rational number.

$$\begin{aligned} \text{Then negative of negative rational number} &= -\left(-\frac{1}{2}\right) \\ &= \frac{1}{2} \quad (\text{positive rational number}) \end{aligned}$$

93. If x and y are two rational numbers such that $x > y$, then $x - y$ is always a positive rational number.

Solution:

True

Let

$$x = 4,$$

$$y = 2$$

So,

$$\begin{aligned} x - y &= 4 - 2 \\ &= 2 \end{aligned}$$

94. 0 is the smallest rational number.

Solution:

False.

Negative rational number below 0 is infinite. So, the smallest rational number does not exist.

95. Every whole number is an integer.

Solution:

True

Every whole number is an integer but, every integer is not whole number.

96. Every whole number is a rational number.

Solution:

True

97. 0 is whole number but it is not a rational number.

Solution:

False

0 is whole number and also a rational number.

98. The rational numbers $\frac{1}{2}$ and $-\frac{5}{2}$ are on the opposite sides of 0 on the number line.

Solution:

True

$\frac{1}{2}$ is positive rational number so it lies to the right side of 0 on the number line.

$-\frac{5}{2}$ is negative rational number so it lies to the left side of 0 on the number line.

99. Rational numbers can be added (or multiplied) in any order

$$\frac{-4}{5} \times \frac{-6}{5} = \frac{-6}{5} \times \frac{-4}{5}$$

Solution:

True

The arrangements of given rational number is as per the commutative law under multiplication.

$$a \times b = b \times c$$

100. Solve the following: Select the rational numbers from the list which are also the integers.

$$\frac{9}{4}, \frac{8}{4}, \frac{7}{4}, \frac{6}{4}, \frac{9}{3}, \frac{8}{3}, \frac{7}{3}, \frac{6}{3}, \frac{5}{2}, \frac{4}{2}, \frac{3}{1}, \frac{3}{2}, \frac{1}{1}, \frac{1}{1}, \frac{0}{1}, \frac{-1}{1}, \frac{-2}{1}, \frac{-3}{1}, \frac{-4}{2}, \frac{-5}{2}, \frac{-6}{2},$$

Solution:

The rational number from the given list which also the integers are,

$$\frac{8}{4} = 2, \frac{9}{3} = 3, \frac{6}{3} = 2, \frac{4}{2} = 2, \frac{3}{1} = 3, \frac{1}{1} = 1, \frac{0}{1} = 0, \frac{-1}{1} = -1, -\frac{-2}{1} = -2, \frac{-4}{2} = -2, \frac{-6}{2} = -3$$

101. Select those which can be written as a rational number with denominator 4 in their lowest form:

$$\frac{7}{8}, \frac{64}{16}, \frac{36}{-12}, \frac{-16}{17}, \frac{5}{-4}, \frac{140}{28}$$

Solution:

Rational number with denominator 4 in their lowest form are,

$$\frac{64}{16} = \frac{16}{4},$$

$$\frac{36}{-12} = \frac{12}{-4},$$

$$\frac{5}{-4},$$

$$\frac{140}{28} = \frac{20}{4}$$

102. Using suitable rearrangement and find the sum:

$$(a) \frac{4}{7} + \left(\frac{-4}{9}\right) + \frac{3}{7} + \left(\frac{-13}{9}\right)$$

$$(b) -5 + \frac{7}{10} + \frac{3}{7} + (-3) + \frac{5}{14} + \frac{-4}{5}$$

Solution:

(a)

$$\begin{aligned} \frac{4}{7} + \left(\frac{-4}{9}\right) + \frac{3}{7} + \left(\frac{-13}{9}\right) &= \frac{4+3}{7} + \left(\frac{-4-13}{9}\right) \\ &= \frac{7}{7} + \left(\frac{-17}{9}\right) \\ &= \left(\frac{-8}{9}\right) \end{aligned}$$

(b)

$$\begin{aligned}
 -5 + \frac{7}{10} + \frac{3}{7} + (-3) + \frac{5}{14} + \frac{-4}{5} &= -8 + \left(\frac{7-8}{10}\right) + \left(\frac{6+5}{14}\right) \\
 &= -8 - \frac{1}{10} + \frac{11}{14}
 \end{aligned}$$

LCM of 1, 10 and 14 is 70

$$\begin{aligned}
 -5 + \frac{7}{10} + \frac{3}{7} + (-3) + \frac{5}{14} + \frac{-4}{5} &= \frac{-512}{70} \\
 &= \frac{-256}{35}
 \end{aligned}$$

103. Verify $-(-x) = x$ for

(i) $x = \frac{3}{5}$

(ii) $x = \frac{-7}{9}$

(iii) $x = \frac{13}{-15}$

Solution:

(i) $x = \frac{3}{5}$

$$-x = -\frac{3}{5}$$

$$-(-x) = -\left(-\frac{3}{5}\right)$$

$$x = \frac{3}{5}$$

(ii) $x = \frac{-7}{9}$

$$-x = -\left(\frac{-7}{9}\right)$$

$$-x = \frac{7}{9}$$

$$-(-x) = \frac{-7}{9}$$

$$x = \frac{-7}{9}$$

(iii) $x = \frac{13}{-15}$

$$-x = -\left(\frac{13}{-15}\right)$$

$$-x = \frac{13}{15}$$

$$-(-x) = -\left(\frac{13}{15}\right)$$

$$x = \frac{13}{-15}$$

104. Give one example each to show that the rational numbers are closed under addition, subtraction and multiplication. Are rational numbers closed under division? Give two examples in support of your answer.

Solution:

Rational numbers are closed under addition:-

Example:

$$\begin{aligned} 5/4 + 1/2 &= \frac{5+1}{4} \\ &= \frac{6}{4} \\ &= \frac{3}{2} \text{ is a rational number} \end{aligned}$$

Rational numbers are closed under subtraction:-

Example:

$$\begin{aligned} \frac{5}{4} - \frac{1}{2} &= \frac{5-1}{4} \\ &= \frac{4}{4} \\ &= 1 \text{ is a rational number} \end{aligned}$$

For any rational number x,

$x \div 0$ is not defined,

Therefore not all rational numbers are closed under division. We can say that except zero, all rational numbers are closed under division.

Example:

$\frac{3}{4} \div \frac{4}{5} = \frac{15}{16}$ is a rational number.

105. Verify the property $x + y = y + x$ of rational numbers by taking

(a) $x = \frac{1}{2}, y = \frac{1}{2}$

(b) $x = \frac{-2}{3}, y = \frac{-5}{6}$

(c) $x = \frac{-3}{7}, y = \frac{20}{21}$

(d) $x = \frac{-2}{5}, y = \frac{-9}{10}$

Solution:

(a) $x = \frac{1}{2}, y = \frac{1}{2}$

In the question is given to verify the property :
 $x + y = y + x$

We have,

$$x = \frac{1}{2}, y = \frac{1}{2}$$

So,

$$\frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$$

$$\begin{aligned} \text{LHS} &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{1}{2} + \frac{1}{2} \\ &= 1 \end{aligned}$$

On comparing LHS and RHS

$$\begin{aligned} \text{LHS} &= \text{RHS} \\ 1 &= 1 \end{aligned}$$

So, $x + y = y + x$

(b) $x = \frac{-2}{3}, y = \frac{-5}{6}$

In the question is given to verify the property

$$x + y = y + x$$

We have,

$$x = \frac{-2}{3}, y = \frac{-5}{6}$$

So,

$$\frac{-2}{3} + \left(\frac{-5}{6}\right) = \frac{-5}{6} + \left(\frac{-2}{3}\right)$$

$$\text{LHS} = \frac{-2}{3} + \left(\frac{-5}{6}\right)$$

$$= \frac{-4-5}{6}$$

$$= -\frac{9}{6}$$

$$\text{RHS} = \frac{-5}{6} + \left(\frac{-2}{3}\right)$$

$$= \frac{-4-5}{6}$$

$$= -\frac{9}{6}$$

So,

$$\text{LHS} = \text{RHS}$$

Hence $x + y = y + x$

$$(c) x = \frac{-3}{7}, y = \frac{20}{21}$$

According to question is given to verify the property $x + y = y + x$

So,

$$\frac{-3}{7} + \frac{20}{21} = \frac{20}{21} + \left(\frac{-3}{7}\right)$$

$$\text{LHS} = \frac{-3}{7} + \frac{20}{21}$$

$$= \frac{-9+20}{21}$$

$$= \frac{11}{21}$$

$$\text{RHS} = \frac{20}{21} + \left(\frac{-3}{7}\right)$$

$$= \frac{-9+20}{21}$$

$$= \frac{11}{21}$$

So,

$$\text{LHS} = \text{RHS}$$

Hence $x + y = y + x$

$$(d) \ x = \frac{-2}{5}, y = \frac{-9}{10}$$

According to question is given to verify the property $x + y = y + x$

$$\text{So, } -\frac{2}{5} + \left(\frac{-9}{10}\right) = \frac{-9}{10} + \left(\frac{-2}{5}\right)$$

$$\text{LHS} = \frac{-2}{5} + \left(\frac{-9}{10}\right)$$

$$= \frac{-4-9}{10}$$

$$= \frac{-13}{10}$$

$$\text{RHS} = \frac{-9}{10} + \left(\frac{-2}{5}\right)$$

$$= \frac{-4-9}{10}$$

$$= \frac{-13}{10}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

Hence $x + y = y + x$

106. Simplify each of the following by using suitable property. Also name the property.

$$(a) \left[\frac{1}{2} \times \frac{1}{4}\right] + \left[\frac{1}{2} \times 6\right]$$

$$(b) \left[\frac{1}{5} \times \frac{2}{15}\right] - \left[\frac{1}{5} \times \frac{2}{5}\right]$$

$$(c) \frac{-3}{5} \times \left\{ \frac{3}{7} + \left(\frac{-5}{6}\right) \right\}$$

Solution:

$$(a) \left[\frac{1}{2} \times \frac{1}{4} \right] + \left[\frac{1}{2} \times 6 \right]$$

The arrangement of the given rational number is as per the rule of distributive law over addition.

Now take out $\frac{1}{2}$ as common.

$$\begin{aligned} \left[\frac{1}{2} \times \frac{1}{4} \right] + \left[\frac{1}{2} \times 6 \right] &= \frac{1}{2} \left[\frac{1}{4} + 6 \right] \\ &= \frac{1}{2} \left[\frac{1+24}{4} \right] \\ &= \frac{1}{2} \left[\frac{25}{4} \right] \\ &= \frac{25}{8} \end{aligned}$$

$$(b) \left[\frac{1}{5} \times \frac{2}{15} \right] - \left[\frac{1}{5} \times \frac{2}{5} \right]$$

Solution:-

The arrangement of the given rational number is as per the rule of distributive law over subtraction.

Now take out $\frac{1}{5}$ as common.

Then,

$$\begin{aligned} \left[\frac{1}{5} \times \frac{2}{15} \right] - \left[\frac{1}{5} \times \frac{2}{5} \right] &= \frac{1}{5} \left[\left(\frac{2}{15} \right) - \left(\frac{2}{5} \right) \right] \\ &= \frac{1}{5} \left[\frac{2-6}{15} \right] \\ &= \frac{1}{5} \left[\frac{-4}{15} \right] \\ &= \frac{-4}{75} \end{aligned}$$

$$(c) \frac{-3}{5} \times \left\{ \frac{3}{7} + \left(\frac{-5}{6} \right) \right\}$$

Solution:-

The arrangement of the given rational number is as per the rule of distributive law over addition.

$$\begin{aligned} \frac{-3}{5} \times \left\{ \frac{3}{7} + \left(\frac{-5}{6} \right) \right\} &= \left(\frac{-3}{5} \right) \times \left\{ \left(\frac{3}{7} \right) + \frac{-5}{6} \right\} \\ &= \frac{-3}{5} \left[\frac{18-35}{42} \right] \\ &= \frac{-3}{5} \left[\frac{-17}{42} \right] \\ &= \frac{51}{210} \\ &= \frac{17}{30} \end{aligned}$$

107. Tell which property allows you to compute

$$\frac{1}{5} \times \left[\frac{5}{6} \times \frac{7}{9} \right] \text{ as } \left[\frac{1}{5} \times \frac{5}{6} \right] \times \frac{7}{9}$$

Solution:

The arrangement of the given rational number is as per the rule of Associative property for Multiplication.

108. Verify the property $x \times y = y \times x$ of rational numbers by using

(a) $x = 7$ and $y = \frac{1}{2}$

(b) $x = \frac{2}{3}$ and $y = \frac{9}{4}$

(c) $x = \frac{-5}{7}$ and $y = \frac{14}{15}$

(d) $x = \frac{-3}{8}$ and $y = \frac{-4}{9}$

Solution:

(a) $x = 7$ and $y = \frac{1}{2}$

In the question is given to verify the property $x \times y = y \times x$

Where, $x = 7$, $y = \frac{1}{2}$

$$7 \times \frac{1}{2} = \frac{1}{2} \times 7$$

$$\begin{aligned} \text{LHS} &= 7 \times \frac{1}{2} \\ &= \frac{7}{2} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{1}{2} \times 7 \\ &= \frac{7}{2} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times y = y \times x$$

$$(b) \ x = \frac{2}{3} \text{ and } y = \frac{9}{4}$$

According to question is given to verify the property = $x \times y = y \times x$

$$\left(\frac{2}{3}\right) \times \left(\frac{9}{4}\right) = \left(\frac{9}{4}\right) \times \left(\frac{2}{3}\right)$$

$$\begin{aligned} \text{LHS} &= \left(\frac{2}{3}\right) \times \left(\frac{9}{4}\right) \\ &= \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{9}{4}\right) \times \left(\frac{2}{3}\right) \\ &= \frac{3}{2} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times y = y \times x$$

$$(c) \ x = \frac{-5}{7} \text{ and } y = \frac{14}{15}$$

Acc to question is given to verify the property = $x \times y = y \times x$

$$\text{So, } \left(\frac{-5}{7}\right) \times \left(\frac{14}{15}\right) = \left(\frac{14}{15}\right) \times \left(\frac{-5}{7}\right)$$

$$\begin{aligned} \text{LHS} &= \left(\frac{-5}{7}\right) \times \left(\frac{14}{15}\right) \\ &= \frac{-2}{3} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{14}{15}\right) \times \left(\frac{-5}{7}\right) \\ &= \frac{-2}{3} \end{aligned}$$

By comparing LHS and RHS

LHS = RHS

Hence $x \times y = y \times x$

(d) $x = \frac{-3}{8}$ and $y = \frac{-4}{9}$

Acc to question is given to verify the property $x \times y = y \times x$

So, $\left(\frac{-3}{8}\right) \times \left(\frac{-4}{9}\right) = \left(\frac{-4}{9}\right) \times \frac{-3}{8}$

$$\begin{aligned} \text{LHS} &= \left(\frac{-3}{8}\right) \times \left(\frac{-4}{9}\right) \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{-4}{9}\right) \times \left(\frac{-3}{8}\right) \\ &= \frac{1}{6} \end{aligned}$$

By comparing LHS and RHS

LHS = RHS

Hence $x \times y = y \times x$

109. Verify the property $x \times (y \times z) = (x \times y) \times z$ of rational numbers by using

(a) $x = 1, y = \frac{-1}{2}$ and $z = \frac{1}{4}$

(b) $x = \frac{2}{3}, y = \frac{-3}{7}$ and $z = \frac{1}{2}$

(c) $x = \frac{-2}{7}, y = \frac{-5}{6}$ and $z = \frac{1}{4}$

(d) $x = 0, y = \frac{1}{2}$

and what is the name of this property?

Solution:

(a) $x = 1, y = \frac{-1}{2}$ and $z = \frac{1}{4}$

In the question is given to verify the property $x \times (y \times z) = (x \times y) \times z$

The arrangement of the given rational number is as per the rule of associative property for multiplication.

$$1 \times \left(\frac{-1}{2} \times \frac{1}{4} \right) = \left(1 \times \frac{-1}{2} \right) \times \frac{1}{4}$$

$$\begin{aligned} \text{LHS} &= 1 \times \left(\frac{-1}{2} \times \frac{1}{4} \right) \\ &= \frac{-1}{8} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(1 \times \frac{-1}{2} \right) \times \frac{1}{4} \\ &= \left(\frac{-1}{2} \right) \times \frac{1}{4} \\ &= \frac{-1}{8} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times (y \times z) = (x \times y) \times z$$

$$(b) \ x = \frac{2}{3}, y = \frac{-3}{7} \text{ and } z = \frac{1}{2}$$

Acc to question is given to verify the property $x \times (y \times z) = (x \times y) \times z$

The arrangement of the given rational number is as per the rule of associative property for multiplication.

$$\text{So, } \left(\frac{2}{3} \right) \times \left(\frac{-3}{7} \times \frac{1}{2} \right) = \left(\left(\frac{2}{3} \right) \times \frac{-3}{7} \right) \times \frac{1}{2}$$

$$\begin{aligned} \text{LHS} &= \left(\frac{2}{3} \right) \times \left(\frac{-3}{7} \times \frac{1}{2} \right) \\ &= \frac{-6}{42} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\left(\frac{2}{3} \right) \times \frac{-3}{7} \right) \times \frac{1}{2} \\ &= \frac{-6}{42} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times (y \times z) = (x \times y) \times z$$

$$(c) \ x = \frac{-2}{7}, y = \frac{-5}{6} \text{ and } z = \frac{1}{4}$$

Acc to question is given to verify the property $x \times (y \times z) = (x \times y) \times z$

The arrangement of the given rational number is as per the rule of associative property for multiplication.

$$\text{So, } \left(\frac{-2}{7}\right) \times \left(\frac{-5}{6} \times \frac{1}{4}\right) = \left(\left(\frac{-2}{7}\right) \times \left(\frac{-5}{6}\right)\right) \times \frac{1}{4}$$

$$\begin{aligned}\text{LHS} &= \left(\frac{-2}{7}\right) \times \left(\frac{-5}{6} \times \frac{1}{4}\right) \\ &= \frac{10}{168}\end{aligned}$$

$$\begin{aligned}\text{RHS} &= \left(\left(\frac{-2}{7}\right) \times \left(\frac{-5}{6}\right)\right) \times \frac{1}{4} \\ &= \frac{10}{168}\end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times (y \times z) = (x \times y) \times z$$

110. Verify the property $x \times (y + z) = x \times y + x \times z$ of rational numbers by taking.

(a) $x = \frac{-1}{2}, y = \frac{3}{4}$ and $z = \frac{1}{4}$

(b) $x = \frac{-1}{2}, y = \frac{2}{3}$ and $z = \frac{3}{4}$

(c) $x = \frac{-2}{3}, y = \frac{-4}{6}$ and $z = \frac{-7}{9}$

(d) $x = \frac{-1}{5}, y = \frac{2}{15}, z = \frac{-3}{10}$

Solution:

(a) $x = \frac{-1}{2}, y = \frac{3}{4}$ and $z = \frac{1}{4}$

Acc to question verify the property $x \times (y + z) = x \times y + x \times z$

The arrangement of the given rational number is as per the rule of distributive property of multiplication over addition.

So,

$$\left(\frac{-1}{2}\right) \times \left(\frac{3}{4} + \frac{1}{4}\right) = \left(\frac{-1}{2} \times \frac{3}{4}\right) + \left(\frac{-1}{2} \times \frac{1}{4}\right)$$

$$\begin{aligned} \text{LHS} &= \left(\frac{-1}{2}\right) \times \left(\frac{3}{4} + \frac{1}{4}\right) \\ &= \left(\frac{-1}{2}\right) \times \left(\frac{3+1}{4}\right) \\ &= \frac{-1}{2} \times \left(\frac{4}{4}\right) \\ &= \frac{-1}{2} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{-1}{2} \times \frac{3}{4}\right) + \left(\frac{-1}{2} \times \frac{1}{4}\right) \\ &= \frac{-3-1}{8} \\ &= \frac{-1}{2} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times (y + z) = x \times y + x \times z$$

$$(b) \ x = \frac{-1}{2}, y = \frac{2}{3} \text{ and } z = \frac{3}{4}$$

Acc to question verify the property $x \times (y + z) = x \times y + x \times z$

The arrangement of the given rational number is as per the rule of distributive property of multiplication over addition.

So,

$$\left(\frac{-1}{2}\right) \times \left(\left(\frac{2}{3}\right) + \frac{3}{4}\right) = \left(\frac{-1}{2} \times \left(\frac{2}{3}\right)\right) + \left(\frac{-1}{2} \times \frac{3}{4}\right)$$

$$\begin{aligned} \text{LHS} &= \left(\frac{-1}{2}\right) \times \left(\left(\frac{2}{3}\right) + \frac{3}{4}\right) \\ &= -\frac{17}{24} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\frac{-1}{2} \times \left(\frac{2}{3}\right)\right) + \left(-\frac{1}{2} \times \frac{3}{4}\right) \\ &= -\frac{17}{24} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

$$\text{Hence } x \times (y + z) = x \times y + x \times z$$

$$(c) \quad x = \frac{-2}{3}, y = \frac{-4}{6} \text{ and } z = \frac{-7}{9}$$

Acc to question verify the property $x \times (y + z) = x \times y + x \times z$

The arrangement of the given rational number is as per the rule of distributive property of multiplication over addition.

So,

$$\frac{-2}{3} \times \left(\left(\frac{-4}{6} \right) + \left(\frac{-7}{9} \right) \right) = \left(\frac{-2}{3} \times \frac{-4}{6} \right) + \left(\left(\frac{-2}{3} \right) \times \left(\frac{-7}{9} \right) \right)$$

$$\begin{aligned} \text{LHS} &= \left(\frac{-2}{3} \right) \times \left(\left(\frac{-4}{6} \right) + \left(\frac{-7}{9} \right) \right) \\ &= \frac{26}{27} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \frac{-2}{3} \times \left(\frac{-4}{6} \right) + \frac{-2}{3} \times \left(\frac{-7}{9} \right) \\ &= \frac{26}{27} \end{aligned}$$

By comparing LHS and RHS

$$\text{LHS} = \text{RHS}$$

Hence $x \times (y + z) = x \times y + x \times z$

$$(d) \quad x = \frac{-1}{5}, y = \frac{2}{15}, z = \frac{-3}{10}$$

Acc to question verify the property $x \times (y + z) = x \times y + x \times z$

The arrangement of the given rational number is as per the rule of distributive property of multiplication over addition.

So,

$$\frac{-1}{5} \times \left(\left(\frac{2}{15} \right) + \left(\frac{-3}{10} \right) \right) = \left(\left(\frac{-1}{5} \right) \times \left(\frac{2}{15} \right) \right) + \left(\left(\frac{-1}{5} \right) \times \left(\frac{-3}{10} \right) \right)$$

$$\begin{aligned} \text{LHS} &= \frac{-1}{5} \times \left(\left(\frac{2}{15} \right) + \left(\frac{-3}{10} \right) \right) \\ &= \frac{1}{30} \end{aligned}$$

$$\begin{aligned} \text{RHS} &= \left(\left(\frac{-1}{5} \right) \times \left(\frac{2}{15} \right) \right) + \left(\left(\frac{-1}{5} \right) \times \left(\frac{-3}{10} \right) \right) \\ &= \frac{1}{30} \end{aligned}$$

LHS = RHS

Hence $x \times (y + z) = x \times y + x \times z$

111. Use the distributivity of multiplication of rational numbers over addition to simplify

(a) $\frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1} \right]$

(b) $\frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15} \right]$

(c) $\frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4} \right]$

(d) $\frac{3}{4} \times \left[\frac{8}{9} - 40 \right]$

Solution:

(a)

We know that the distributivity of multiplication of rational numbers over addition, $a \times (b + c) = a \times b + a \times c$

$$\frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1} \right]$$

So,

$$\begin{aligned} \frac{3}{5} \times \left[\frac{35}{24} + \frac{10}{1} \right] &= ((3/5) \times (35/24)) + ((3/5) \times (10/1)) \\ &= ((1/1) \times (7/8)) + ((3/1) \times (2/1)) \\ &= \frac{55}{8} \end{aligned}$$

(b) $\frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15} \right]$

We know that the distributivity of multiplication of rational numbers over addition, $a \times (b + c) = a \times b + a \times c$

So,

$$\begin{aligned} \frac{-5}{4} \times \left[\frac{8}{5} + \frac{16}{15} \right] &= ((-5/4) \times (8/5)) + ((-5/4) \times (16/15)) \\ &= ((-1/1) \times (2/1)) + ((-1/1) \times (4/3)) \\ &= \frac{-10}{3} \end{aligned}$$

$$(c) \frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4} \right]$$

We know that the distributivity of multiplication of rational numbers over subtraction, $a \times (b - c) = a \times b - a \times c$

So,

$$\begin{aligned} \frac{2}{7} \times \left[\frac{7}{16} - \frac{21}{4} \right] &= \left(\left(\frac{2}{7} \right) \times \left(\frac{7}{16} \right) \right) - \left(\left(\frac{2}{7} \right) \times \left(\frac{21}{4} \right) \right) \\ &= \frac{-11}{8} \end{aligned}$$

$$(d) \frac{3}{4} \times \left[\frac{8}{9} - 40 \right]$$

We know that the distributivity of multiplication of rational numbers over subtraction, $a \times (b - c) = a \times b - a \times c$

So,

$$\begin{aligned} \frac{3}{4} \times \left[\frac{8}{9} - 40 \right] &= \left(\left(\frac{3}{4} \right) \times \left(\frac{8}{9} \right) \right) - \left(\left(\frac{3}{4} \right) \times (40) \right) \\ &= \frac{-88}{3} \end{aligned}$$

112. Simplify

$$(a) \frac{32}{5} + \frac{23}{11} \times \frac{22}{15}$$

$$(b) \frac{3}{7} \times \frac{28}{15} \div \frac{14}{5}$$

$$(c) \frac{3}{7} + \frac{-2}{21} \times \frac{-5}{6}$$

$$(d) \frac{7}{8} + \frac{1}{16} - \frac{1}{12}$$

Solution:

$$(a) \frac{32}{5} + \frac{23}{11} \times \frac{22}{15}$$

$$\begin{aligned} \frac{32}{5} + \frac{23}{11} \times \frac{22}{15} &= (32/5) + (46/15) \\ &= (96 + 46)/15 \\ &= \frac{142}{15} \end{aligned}$$

$$(b) \frac{3}{7} \times \frac{28}{15} \div \frac{14}{5}$$

$$\frac{3}{7} \times \frac{28}{15} \div \frac{14}{5} = (1/1) \times (4/5) \div (14/5)$$

$$= \frac{4}{5} \div \frac{14}{5}$$

$$= \frac{2}{7}$$

$$(c) \frac{3}{7} + \frac{-2}{21} \times \frac{-5}{6}$$

$$\frac{3}{7} + \frac{-2}{21} \times \frac{-5}{6} = \frac{32}{63}$$

$$(d) \frac{7}{8} + \frac{1}{16} - \frac{1}{12}$$

$$\frac{7}{8} + \frac{1}{16} - \frac{1}{12} = \frac{41}{48}$$

113. Identify the rational number that does not belong with the other three.

Explain your reasoning

$$\frac{-5}{11}, \frac{-1}{2}, \frac{-4}{9}, \frac{-7}{3}$$

Solution:

The rational number that does not belong with the other three is $-7/3$ as it is smaller than -1 whereas rest of the numbers are greater than -1 .

114. The cost of $\frac{19}{4}$ meters of wire is Rs. $\frac{171}{2}$. Find the cost of one metre of the wire.

Solution:

$$\text{The cost of } \frac{19}{4} \text{ meters of wire is } = ₹ \frac{171}{2}$$

$$\text{Then, cost of one meter of wire } = \left(\frac{171}{2}\right) \div \left(\frac{19}{4}\right)$$

$$= \left(\frac{171}{2}\right) \times \left(\frac{4}{19}\right)$$

$$= ₹ 18$$

The cost of one meter of wire is ₹ 18.

115. A train travels $\frac{1445}{2}$ km in $\frac{17}{2}$ hours . Find the speed of the train in km/h.

Solution:

Distance travelled by train = $\frac{1445}{2}$ km

Time taken by the train to cover distance $\frac{1445}{2} = \frac{17}{2}$ hours

$$\begin{aligned}\text{The speed of the train} &= \left(\frac{1445}{2}\right) \div \left(\frac{17}{2}\right) \\ &= \left(\frac{1445}{2}\right) \times \left(\frac{2}{17}\right) \\ &= 85 \text{ km/h}\end{aligned}$$

The speed of the train is 85 km/h.

116. If 16 shirts of equal size can be made out of 24 m of cloth, how much cloth is needed for making one shirt?

Solution:

The 16 shirts are made out of= 24m of cloth.

$$\begin{aligned}\text{Cloth needed for making one shirt} &= \frac{24}{16} \text{ m of cloth} \\ &= \frac{3}{2} \text{ m of cloth} \\ &= 1.5\text{m}\end{aligned}$$

So, Cloth is needed for making one shirt is 1.5m.

117. $\frac{7}{11}$ of all the money in Hamid's bank account is Rs. 77,000. How much money does Hamid have in his bank account?

Solution:

$\frac{7}{11}$ of all the money in Hamid's bank account = ₹ 77,000

Now, let the money in Hamid's bank account be ₹ y.

Then,

$$\left(\frac{7}{11}\right) \times (x) = 77,000$$

$$x = 77,000 / \left(\frac{7}{11}\right)$$

$$x = 77000 \times \left(\frac{11}{7}\right)$$

$$x = 121000$$

The total money in Hamid's bank account is ₹ 121000.

118. A $117\frac{1}{3}$ m long rope is cut into equal pieces measuring $7\frac{1}{3}$ m each. How many such small pieces are these?

Solution:

$$\text{The length of the rope } 117\frac{1}{3} \text{ m} = \frac{352}{3} \text{ m}$$

Then length of each piece measures,

$$7\frac{1}{3} \text{ m} = \frac{22}{3} \text{ m}$$

So, the number of pieces of the rope = total length of the rope / length of each piece

$$\begin{aligned} &= \frac{352}{3} \div \left(\frac{22}{3}\right) \\ &= 16 \end{aligned}$$

Hence, $117\frac{1}{3}$ m long rope is 16.

119. $\frac{1}{6}$ of the class students are above average, $\frac{1}{4}$ are average and rest are below average. If there are 48 students in all, how many students are below average in the class?

Solution:

$$\text{Number of students in the class are above average} = \frac{1}{6}$$

$$\text{Number of students in the class are average} = \frac{1}{4}$$

$$\begin{aligned} \text{Number of students in the class are below average} &= 1 - \left(\left(\frac{1}{6}\right) + \left(\frac{1}{4}\right)\right) \\ &= \frac{7}{12} \text{ students.} \end{aligned}$$

So, the number of students in the class = 48

$$\begin{aligned}\text{Number of students below average} &= \left(\frac{7}{12}\right) \times 48 \\ &= 28 \text{ students}\end{aligned}$$

The number of students in the class are below average are 28.

120. $\frac{2}{5}$ of total number of students of a school come by car while $\frac{1}{4}$ of students come by bus to school. All the other students walk to school of which $\frac{1}{3}$ walk on their own and the rest are escorted by their parents. If 224 students come to school walking on their own, how many students study in that school?

Solution:

Let us assume total number of students in the school be y .

$$\text{The number of students come by car} = \left(\frac{2}{5}\right) \times x$$

$$\text{The number of students come by bus} = \left(\frac{1}{4}\right) \times x$$

$$\begin{aligned}\text{Remaining students walk to school} &= x - \left(\frac{2}{5}x + \left(\frac{1}{4}x\right)\right) \\ &= \frac{7x}{20}\end{aligned}$$

$$\begin{aligned}\text{Then, number of students walk to school on their own} &= \left(\frac{1}{3}\right) \text{ of } \left(\frac{7x}{20}\right) \\ &= \frac{7x}{60}\end{aligned}$$

Since, 224 students come to school on their own.

As per the data given in the question,

$$\frac{7x}{60} = 224$$

$$x = 1920$$

The total number of students in that school is 1920.

121. Huma, Hubna and Seema received a total of Rs. 2,016 as monthly allowance from their mother such that Seema gets $\frac{1}{2}$ of what Huma gets and

Hubna gets $1\frac{2}{3}$ times Seema's share. How much money do the three sisters get individually?

Solution:

Total monthly allowance received by Huma, Hubna and Seema = ₹ 2,016

from their mother.

Seema gets allowance = $\frac{1}{2}$ of Huma's share

Hubna gets allowance = $1\frac{2}{3}$ of Seema's share
= $\frac{5}{3}$ of Seema's share
= $\frac{5}{3}$ of $\frac{1}{2}$ of Huma's share [given]
= $\frac{5}{6}$ of Huma's share

So,

Huma's share + Hubna's share + Seema's share = ₹ 2,016

Let Huma's share be 1,

$1 + (\frac{5}{6})$ Huma's share + $\frac{1}{2}$ Huma's share = ₹ 2,016

$(\frac{14}{6})$ Huma's share = ₹ 2,016

So,

Huma's share = ₹ 2,016 \div $(\frac{14}{6})$
= 144×6

Huma's Share is = ₹ 864

Seema's share = $\frac{1}{2}$ Huma's share
= $\frac{1}{2} \times 864$
= ₹ 432

Hubna's share = $\frac{5}{6}$ of Huma's share
= $\frac{5}{6} \times 864$

$$= 5 \times 144$$

$$= ₹ 720$$

122. A mother and her two daughters got a room constructed for Rs. 62,000. The elder daughter contributes $\frac{3}{8}$ of her mother's contribution while the younger daughter contributes $\frac{1}{2}$ of her mother's share. How much do the three contribute individually?

Solution:

A mother and her two daughters got a room constructed for = ₹ 62,000
Let us assume mother's share be x,

$$\text{The elder daughter's contribute} = \frac{3}{8} \text{ of her mother's share}$$

$$= \frac{3}{8} x$$

$$\text{The younger daughter's contribute} = \frac{1}{2} \text{ of her mother's share}$$

$$= \frac{1}{2} x$$

So, mother's share + elder daughter's share + younger daughter's share = ₹ 62,000

$$x + \frac{3}{8}x + \frac{1}{2}x = ₹ 62,000$$

$$15x = 62,000 \times 8$$

$$x = \frac{496000}{16}$$

$$x = ₹ 33,066.6$$

Mother's share = ₹ 33,066.6

Elder daughter's share = $\frac{3}{8}$ of her mother's share

$$= \frac{3}{8}x$$

$$= \frac{3}{8} \times 33066.6$$

$$= ₹ 12,400$$

Younger daughter's share = $\frac{1}{2}$ of her mother's share

$$\begin{aligned}
&= \frac{1}{2}x \\
&= \frac{1}{2} \times 33066.6 \\
&= ₹ 16,533.3
\end{aligned}$$

123. Tell which property allows you to compare

$$\frac{2}{3} \times \left[\frac{3}{4} \times \frac{5}{7} \right] \text{ and } \left[\frac{2}{3} \times \frac{5}{7} \right] \times \frac{3}{4}$$

Solution:

$\frac{2}{3} \times \left[\frac{3}{4} \times \frac{5}{7} \right]$ and $\left[\frac{2}{3} \times \frac{5}{7} \right] \times \frac{3}{4}$ this can be compared with associative property and commutative property.

124. Name the property used in each of the following.

(i) $-\frac{7}{11} \times \frac{-3}{5} = \frac{-3}{5} \times \frac{-7}{11}$

(ii) $-\frac{2}{3} \times \left[\frac{3}{4} + \frac{-1}{2} \right] = \left[\frac{-2}{3} \times \frac{3}{4} \right] + \left[\frac{-2}{3} \times \frac{-1}{2} \right]$

(iii) $\frac{1}{3} + \left[\frac{4}{9} + \left(\frac{-4}{3} \right) \right] = \left[\frac{1}{3} + \frac{4}{9} \right] + \left[\frac{-4}{3} \right]$

(iv) $\frac{-2}{7} + 0 = 0 + \frac{-2}{7} = -\frac{2}{7}$

(v) $\frac{3}{8} \times 1 = 1 \times \frac{3}{8} = \frac{3}{8}$

Solution:

(i) $-\frac{7}{11} \times \frac{-3}{5} = \frac{-3}{5} \times \frac{-7}{11}$

The above rational number is in the form of Commutative property over multiplication.

(ii) $-\frac{2}{3} \times \left[\frac{3}{4} + \frac{-1}{2} \right] = \left[\frac{-2}{3} \times \frac{3}{4} \right] + \left[\frac{-2}{3} \times \frac{-1}{2} \right]$

The above rational number is in the form of Distributive property over addition.

(iii) $\frac{1}{3} + \left[\frac{4}{9} + \left(\frac{-4}{3} \right) \right] = \left[\frac{1}{3} + \frac{4}{9} \right] + \left[\frac{-4}{3} \right]$

The above rational number is in the form of Associative property over addition.

(iv) $\frac{-2}{7} + 0 = 0 + \frac{-2}{7} = -\frac{2}{7}$

The above rational number is in the form of Additive identity of rational number.

$$(v) \frac{3}{8} \times 1 = 1 \times \frac{3}{8} = \frac{3}{8}$$

The above rational number is in the form of Multiplicative identity of rational number.

125. Find the multiplicative inverse of

(i) $-1\frac{1}{8}$

(ii) $3\frac{1}{3}$

Solution:

(i)

The given number $-1\frac{1}{8} = \frac{-9}{8}$

The multiplicative inverse = $\frac{-8}{9}$

(ii) $3\frac{1}{3}$

The given number

$$3\frac{1}{3} = \frac{10}{3}$$

The multiplicative inverse = $\frac{3}{10}$

126. Arrange the numbers $\frac{1}{4}, \frac{13}{16}, \frac{5}{8}$ in the descending order.

Solution:

The LCM of the denominators 4, 16 and 8 is 16.

$$\begin{aligned} \frac{1}{4} &= [(1 \times 4) / (4 \times 4)] \\ &= \left(\frac{4}{16}\right) \end{aligned}$$

$$\begin{aligned} \left(\frac{13}{16}\right) &= [(13 \times 1) / (16 \times 1)] \\ &= \frac{13}{16} \end{aligned}$$

$$\left(\frac{5}{8}\right) = [(5 \times 2) / (8 \times 2)]$$

$$= \frac{10}{16}$$

So, $13 < 10 < 4$

$$\frac{13}{16} > \frac{5}{8} > \frac{4}{16}$$

Hence, $\frac{13}{16} > \left(\frac{5}{8}\right) > \frac{1}{4}$

127. The product of two rational numbers is $\frac{-14}{27}$. If one of the numbers be $\frac{7}{9}$, find the other.

Solution:

Let us take the other number be y.

$$\text{Product of two rational number} = \frac{-14}{27}$$

$$\text{One number} = \frac{7}{9}$$

Then,

$$y \times \left(\frac{7}{9}\right) = \frac{-14}{27}$$

$$y = \frac{-14}{27} \div \left(\frac{7}{9}\right)$$

$$y = \frac{-14}{27} \times \left(\frac{9}{7}\right)$$

$$y = \frac{-2}{3}$$

Therefore, the other number is $\frac{-2}{3}$.

128. By what numbers should we multiply $\frac{-15}{20}$ so that the product maybe $\frac{-5}{7}$.

Solution:

Let us assume the other number be y.

$$\text{Given, product of two rational number} = \frac{-5}{7}$$

$$\text{One number} = \frac{-15}{20}$$

$$y \times \left(\frac{-15}{20}\right) = \frac{-5}{7}$$

$$y = \frac{-5}{7} \div \frac{-15}{20}$$

$$y = \frac{-20}{21}$$

Therefore, the other number is $\frac{-20}{21}$.

129. By what number should we multiply $\frac{-8}{13}$ so that the product may be 24?

Solution:

Let us take the other number be y.

Product of two rational number = 24

$$\text{One number} = \frac{-8}{13}$$

$$y \times \left(\frac{-8}{13}\right) = 24$$

$$y = 24 \div \frac{-8}{13}$$

$$y = -39$$

So, the other number is -39.

130. The product of two rational numbers is -7. If one of the number is -5, find the other?

Solution:

Let us assume the other number be y.

Given, product of two rational number = -7

One number = -5

$$y \times (-5) = -7$$

$$y = \frac{7}{5}$$

So, the other number is $\frac{7}{5}$.

131. Can you find a rational number whose multiplicative inverse is -1?

Solution:

No, we cannot find a rational number whose multiplicative inverse is -1 .

132. Find five rational numbers between 0 and 1.**Solution:**

The five rational numbers between 0 and 1 are $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{4}{6}, \frac{5}{6}$.

133. Find two rational numbers whose absolute value is $\frac{1}{5}$.**Solution:**

$\frac{1}{5}$ and $-\frac{1}{5}$ are the rational number whose absolute value is $\frac{1}{5}$.

134. From a rope 40 metres long, pieces of equal size are cut. If the length of one piece is $\frac{10}{3}$ metre, find the number of such pieces.**Solution:**

The length of rope = 40 m

The length of one piece of rope = $\frac{10}{3}$ metre

Let us take the total number of pieces be y .

$$\left(\frac{10}{3}\right) y = 40$$

$$y = 12 \text{ pieces}$$

The number of pieces cut from the rope are 12.

135. $5\frac{1}{2}$ metres long rope is cut into 12 equal pieces. What is the length of each piece?**Solution:**

The length of rope = $5\frac{1}{2}$ metres

$$= \frac{11}{2} \text{ metres}$$

The total number of pieces = 12

Let us assume the length of one piece of rope be y .

$$12y = \frac{11}{2} \text{ m}$$

$$y = \left(\frac{11}{2}\right) \times (1/12)$$

$$y = \frac{11}{24}$$

The length of one piece of rope $\frac{11}{24}$.

136. Write the following rational numbers in the descending order.

$$\frac{8}{7}, \frac{-9}{8}, \frac{-3}{2}, 0, \frac{2}{5}$$

Solution:

The LCM of the denominators 7, 8, 2 and 5 is 280.

$$\begin{aligned}\frac{8}{7} &= [(8 \times 40) / (7 \times 40)] \\ &= (320/280)\end{aligned}$$

$$\begin{aligned}\frac{-9}{8} &= [(-9 \times 35) / (8 \times 35)] \\ &= (-315 / 280)\end{aligned}$$

$$\begin{aligned}\frac{-3}{2} &= [(-3 \times 140) / (2 \times 140)] \\ &= (-420 / 280)\end{aligned}$$

$$\begin{aligned}\left(\frac{2}{5}\right) &= [(2 \times 56) / (56 \times 56)] \\ &= (112/280)\end{aligned}$$

So, $320 > 112 > 0 > -315 > -420$.

$$\text{Hence, } \frac{8}{7} > \frac{2}{5} > 0 > \frac{-9}{8} > \frac{-3}{2}$$

137. Find

(i) $0 \div \frac{2}{3}$

(ii) $\frac{1}{3} \times \frac{-5}{7} \times \frac{-21}{10}$

Solution:

(i)

$$0 \div \frac{2}{3} = 0$$

(ii)

$$\frac{1}{3} \times \frac{-5}{7} \times \frac{-21}{10} = \frac{1}{2}$$

138. On a winter day the temperature at a place in Himachal Pradesh was -16°C . Convert it in degree Fahrenheit ($^{\circ}\text{F}$) by using the formula.

$$\frac{C}{5} = \frac{F-32}{9}$$

Solution:

We have, a winter day the temperature at a place in Himachal Pradesh was -16°C .

$$\frac{C}{5} = \frac{F-32}{9}$$

$$C = -16^{\circ}$$

$$(-16^{\circ}/5) = (F - 32) / 9$$

$$(-16^{\circ}/5) \times 9 = F - 32$$

$$(-144/5) = F - 32$$

$$F = 32 - (144/5)$$

$$F = (160 - 144) / 5$$

$$F = 16/5$$

$$F = 3.2^{\circ}\text{F}$$

139. Find the sum of additive inverse and multiplicative inverse of 7.

Solution:

Additive inverse of 7 = -7

Multiplicative inverse of 7 = $\frac{1}{7}$

$$\begin{aligned} \text{Sum of additive inverse and multiplicative inverse of 7} &= -7 + \left(\frac{1}{7}\right) \\ &= \frac{-48}{7} \end{aligned}$$

140. Find the product of additive inverse and multiplicative inverse of $-\frac{1}{3}$.

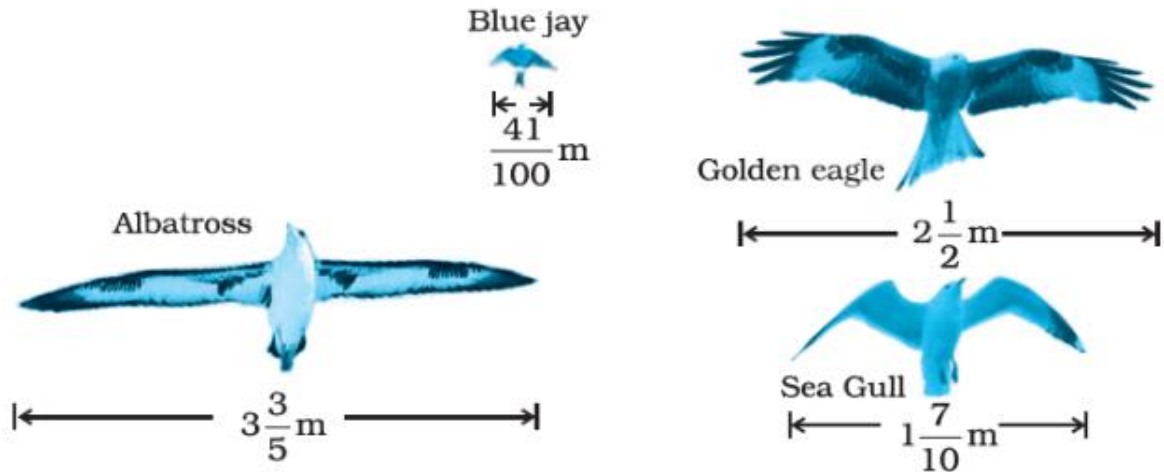
Solution:

Additive inverse of $-\frac{1}{3} = \frac{1}{3}$

Multiplicative inverse of $-\frac{1}{3} = -\frac{3}{1}$

The product of additive inverse and multiplicative inverse of $1/3 = 1/3 \times (-3)$

141. The diagram shows the wingspans of different species of birds. Use the diagram to answer the question given below:



(a) How much longer is the wingspan of an Albatross than the wingspan of a Sea gull?

(b) How much longer is the wingspan of a Golden eagle than the wingspan of a Blue jay?

Solution:

(a)

We have to find out the difference of wingspan of an Albatross and wingspan of a Sea gull.

$$\text{Length of wingspan of an Albatross} = 3\frac{3}{5} = \frac{18}{5} \text{ m}$$

$$\text{Length of wingspan of a Sea gull} = 1\frac{7}{10} = \frac{17}{10} \text{ m}$$

$$\begin{aligned} \text{Difference of both} &= \left(\frac{18}{5}\right) - \left(\frac{17}{10}\right) \\ &= \frac{(36 - 17)}{10} \\ &= \frac{19}{10} \text{ m} \end{aligned}$$

The wingspan of an Albatross is $\frac{19}{10}$ m longer than the wingspan of a Sea gull.

(b)

We have to find out the difference of wingspan of a Golden eagle and wingspan of a Blue jay.

$$\text{Length of wingspan of a Golden eagle} = 2\frac{1}{2}$$

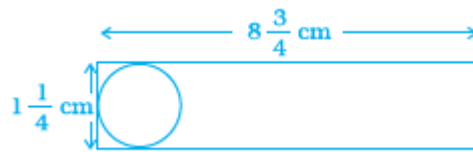
$$= \frac{5}{2} \text{ m}$$

Length of wingspan of a Blue jay = $\frac{41}{100}$ m

$$\begin{aligned} \text{Difference of both} &= \frac{5}{2} - \left(\frac{41}{100}\right) \\ &= \frac{209}{100} \text{ m} \end{aligned}$$

The wingspan of a Golden eagle is $\frac{209}{100}$ m longer than the wingspan of a Blue jay.

142. Shalini has to cut out circles of diameter $1\frac{1}{4}$ cm from an aluminum strip of dimensions $8\frac{3}{4}$ cm by $1\frac{1}{4}$ cm. How many full circles can Shalini cut? Also calculate the wastage of the aluminum strip.



Solution:

Diameter of the circle = Breadth of the aluminium strip $1\frac{1}{4}$ cm = $\frac{5}{4}$ cm

$$\begin{aligned} \text{Length of aluminium strip} &= 8\frac{3}{4} \text{ cm} \\ &= \frac{35}{4} \text{ cm} \end{aligned}$$

The number of full circles cut from the aluminum strip = $\left(\frac{35}{4}\right) \div \left(\frac{5}{4}\right)$
 $= 7$ circles

Radius of circle ,

$$\begin{aligned} &= \frac{5}{4 \times 2} \\ &= \frac{5}{8} \end{aligned}$$

Area to be cut by one circle = πr^2

$$\begin{aligned} &= (22/7) \times \left(\frac{5}{8}\right)^2 \\ &= (22/7) \times \left(\frac{25}{64}\right) \text{ cm}^2 \end{aligned}$$

$$\begin{aligned}
 \text{Now, area to be cut by 7 full circles} &= 7 \times (22/7) \times \left(\frac{25}{64}\right) \\
 &= (22 \times 25)/64 \\
 &= \frac{550}{64} \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of the aluminum strip} &= \text{length} \times \text{breadth} \\
 &= \left(\frac{35}{4}\right) \times \left(\frac{5}{4}\right) \text{ cm}^2 \\
 &= \left(\frac{175}{16}\right) \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{The wastage of aluminum strip} &= \left(\frac{175}{16}\right) - \left(\frac{550}{64}\right) \\
 &= \frac{75}{32} \text{ cm}^2
 \end{aligned}$$

143. One fruit salad recipe requires $\frac{1}{2}$ cup of sugar. Another recipe for the same fruit salad requires 2 tablespoons of sugar. If 1 tablespoon is equivalent to $\frac{1}{16}$ cup, how much more sugar does the first recipe require?

Solution:

$$\text{One fruit salad recipe requires} = \frac{1}{2} \text{ cup of sugar}$$

$$\begin{aligned}
 \text{Sugar required for another salad} &= 2 \times \left(\frac{1}{16}\right) \\
 &= \frac{2}{16} \text{ cup}
 \end{aligned}$$

$$\begin{aligned}
 \text{Hence, the required sugar} &= \frac{1}{2} - \left(\frac{2}{16}\right) \\
 &= \frac{3}{8} \text{ cup of sugar.}
 \end{aligned}$$

144. Four friends had a competition to see how far could they hop on one foot. The table given shows the distance covered by each.

Name	Distance covered (km)
Seema	$\frac{1}{25}$
Nancy	$\frac{1}{32}$

Megha	$\frac{1}{40}$
Soni	$\frac{1}{20}$

- (a) How farther did Soni hop than Nancy?
 (b) What is the total distance covered by Seema and Megha?
 (c) Who walked farther, Nancy or Megha?

Solution:

The LCM of the denominators 25, 32, 40 and 20 is 800

$$\begin{aligned}\frac{1}{25} &= [(1 \times 32) / (25 \times 32)] \\ &= \left(\frac{32}{800}\right)\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{32}\right) &= [(1 \times 25) / (32 \times 25)] \\ &= \left(\frac{25}{800}\right)\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{40}\right) &= [(1 \times 20) / (40 \times 20)] \\ &= \left(\frac{20}{800}\right)\end{aligned}$$

$$\begin{aligned}\left(\frac{1}{20}\right) &= [(1 \times 40) / (20 \times 40)] \\ &= \left(\frac{40}{800}\right)\end{aligned}$$

$$\begin{aligned}\text{(a) Soni hop more than Nancy} &= \left(\frac{40}{800}\right) - \left(\frac{25}{800}\right) \\ &= \frac{3}{160} \text{ km}\end{aligned}$$

$$\begin{aligned}\text{(b) The total distance covered by Seema and Megha} &= \left(\frac{32}{800}\right) + \left(\frac{20}{800}\right) \\ &= \frac{13}{200} \text{ km}\end{aligned}$$

(c) Nancy walked farther.

145. The table given below shows the distances, in kilometres, between four villages of a state. To find the distance between two villages, locate the

square where the row for one village and the column for the other village intersect.

	Sonapur	Ramgarh	Himgaon	Rawalpur
Sonapur		$40\frac{2}{3}$	$100\frac{5}{6}$	$16\frac{1}{2}$
Ramgarh	$40\frac{2}{3}$		$210\frac{3}{8}$	$16\frac{2}{3}$
Himgaon	$100\frac{5}{6}$	$210\frac{3}{8}$		$98\frac{3}{4}$
Rawalpur	$16\frac{1}{2}$	$30\frac{2}{3}$	$98\frac{3}{4}$	

(a) Compare the distance between Himgaon and Rawalpur to Sonapur and Ramgarh?

(b) If you drove from Himgaon to Sonapur and then from Sonapur to Rawalpur, how far would you drive?

Solution:

(a)

$$\begin{aligned} \text{From the table the distance between Himgaon and Rawalpur} &= 98\frac{3}{4} \text{ km} \\ &= \frac{395}{4} \text{ km} \end{aligned}$$

$$\text{The distance between Sonapur and Ramgarh } 40\frac{2}{3} = \frac{122}{3} \text{ km}$$

Difference of the distance between Himgaon and Rawalpur to Sonapur and Ramgarh,

$$\begin{aligned} \left(\frac{395}{4}\right) - \left(\frac{122}{3}\right) &= \frac{697}{12} \\ &= 58\frac{1}{12} \end{aligned}$$

(b)

Distance between Himgaon and Sonapur,

$$100\frac{5}{6} = \frac{605}{6} \text{ km}$$

$$\text{Distance between Sonapur and Rawalpur} = 16\frac{1}{2} \text{ km}$$

$$= \frac{33}{2}$$

$$\begin{aligned}
\text{Total distance that he would drive} &= \frac{605}{6} + \frac{33}{2} \\
&= (605 + 99)/6 \\
&= \frac{352}{3} \\
&= 117\frac{1}{3}
\end{aligned}$$

146. The table shows the portion of some common materials that are recycled.

Material	Recycled
Paper	$\frac{5}{11}$
Aluminium cans	$\frac{5}{8}$
Glass	$\frac{2}{5}$
Scrap	$\frac{3}{4}$

- (a) Is the rational number expressing the amount of paper recycled more than $\frac{1}{2}$ or less than $\frac{1}{2}$?
- (b) Which items have a recycled amount less than $\frac{1}{2}$?
- (c) Is the quantity of aluminium cans recycled more (or less) than half of the quantity of aluminium cans?
- (d) Arrange the rate of recycling the materials from the greatest to the smallest.

Solution:

(a)

The rational number expressing the amount of paper recycled is less than $\frac{1}{2}$.

(b)

Paper and Glass have a recycled amount less than $\frac{1}{2}$.

(c)

The quantity of aluminium cans recycled is more than half of the quantity of aluminium cans.

(d)

The LCM of the denominators 11, 8, 5 and 4 is 440.

$$\frac{5}{11} = [(5 \times 40) / (11 \times 40)]$$

$$= (200/440)$$

$$\left(\frac{5}{8}\right) = [(5 \times 55) / (8 \times 55)]$$

$$= (275/440)$$

$$\left(\frac{2}{5}\right) = [(2 \times 88) / (5 \times 88)]$$

$$= (176/440)$$

$$\left(\frac{3}{4}\right) = [(3 \times 110) / (4 \times 110)]$$

$$= (330/440)$$

Now,

$$330 > 275 > 200 > 176$$

So,

$$\frac{3}{4} > \frac{5}{8} > \frac{5}{11} > \frac{2}{5}$$

Therefore, Scrap > Aluminium > Cans > Paper > Glass.

147. The overall width in cm of several wide-screen televisions are 97.28 cm, $98\frac{4}{9}$ cm, $98\frac{1}{25}$ cm, and 97.94 cm. Express these numbers as rational numbers in the form $\frac{p}{q}$ and arrange the widths in ascending order.

Solution:

The overall width in cm of several wide screen television are,

$$97.28 \text{ cm} = \frac{9728}{100}$$

By dividing both numerator and denominator by 4.

$$= \frac{2432}{25} \text{ cm}$$

By converting mixed fraction, we get,

$$98\frac{4}{9} \text{ cm} = \frac{886}{9} \text{ cm}$$

$$98\frac{1}{25} \text{ cm} = \frac{2421}{25} \text{ cm}$$

Now,

$$97.94 \text{ cm} = \frac{9794}{100}$$

By dividing both numerator and denominator by 2 we get,

$$= \frac{4897}{50} \text{ cm}$$

Therefore,

The LCM of the denominators 25, 9, 25 and 50 is 450.

$$\begin{aligned} \frac{2432}{25} &= [(2432 \times 18) / (25 \times 18)] \\ &= (43776/450) \end{aligned}$$

$$\begin{aligned} \left(\frac{886}{9}\right) &= [(886 \times 50) / (9 \times 50)] \\ &= (44300/450) \end{aligned}$$

$$\begin{aligned} \left(\frac{2421}{25}\right) &= [(2421 \times 18) / (25 \times 18)] \\ &= (44118/450) \end{aligned}$$

$$\begin{aligned} \left(\frac{4897}{50}\right) &= [(4897 \times 9) / (50 \times 9)] \\ &= (44073/450) \end{aligned}$$

Then,

Now, $43776 < 44073 < 44118 < 44300$

Therefore, in ascending order $= \left(\frac{2432}{25}\right) < \left(\frac{4897}{50}\right) < \left(\frac{2421}{25}\right) < \left(\frac{886}{9}\right)$.

So, $97.28 < 97.94 < 98\frac{1}{25} \text{ cm} < 98\frac{4}{9} \text{ cm}$.

148. Roller Coaster at an amusement park is $\frac{2}{3}$ m high. If a new rollercoaster is built that is $\frac{3}{5}$ times the height of the existing coaster, what will be the height of the new roller coaster?

Solution:

Height of the roller coaster at an amusement park = $\frac{2}{3}$ m

Height of the new roller coaster is about to build = $\frac{3}{5}$ times the height of the existing, coaster

$$= \left(\frac{2}{3} \text{ m}\right) \times \left(\frac{3}{5}\right)$$

$$= \left(\frac{2}{5}\right) \text{ m}$$

149. Here is a table which gives the information about the total rainfall for several months compared to the average monthly rains of a town. Write each decimal in the form of rational number $\frac{p}{q}$.

Month	Above / Below normal (in cm)
May	2.6924
June	0.6096
July	-6.9088
August	-8.636

Solution:

(i) May

$$2.6924 \text{ cm} = \frac{26294}{10000} \quad \text{[by the decimal removing method]}$$

On dividing both numerator and denominator by 4 we get,

$$\frac{26294}{10000} = \frac{6731}{2500} \text{ cm}$$

(ii) June

$$0.6096 \text{ cm} = \frac{6096}{10000} \quad \text{[by the decimal removing method]}$$

By dividing both numerator and denominator by 16 we get,

$$= \frac{381}{625} \text{ cm}$$

(iii) July

$$-6.9088 \text{ cm} = -\frac{69088}{10000} \quad \text{[by the decimal removing method]}$$

By dividing both numerator and denominator by 4 we get,

$$= -\frac{4318}{625} \text{ cm}$$

(iv) August

$$-8.636 \text{ cm} = -\frac{8636}{1000}$$

[by the decimal removing method]

By dividing both numerator and denominator by 4 we get,

$$= -\frac{2159}{250} \text{ cm}$$

150. The average life expectancies of males for several states are shown in the table. Express each decimal in the form $\frac{p}{q}$ and arrange the states from

the least to the greatest male life expectancy.

State-wise data are included below; more indicators can be found in the “FACTFILE” section on the homepage for each state.

State	Male	$\frac{p}{q}$ form	Lowest term
Andhra Pradesh	61.6		
Assam	57.1		
Bihar	60.7		
Gujarat	61.9		
Haryana	64.1		
Himachal Pradesh	65.1		
Karnataka	62.4		
Kerala	70.6		
Madhya Pradesh	56.5		
Maharashtra	64.5		
Orissa	57.6		
Punjab	66.9		
Rajasthan	59.8		
Tamil Nadu	63.7		
Uttar Pradesh	58.9		
West Bengal	62.8		
India	60.8		

Source: Registrar General of India (2003) SRS Based Abridged Life Tables. SRS Analytical Studies, Report No. 3 of 2003, New Delhi: Registrar General of India. The data are for the 1995-99 period; states subsequently

divided are therefore included in their pre-partition states (Chhatisgarh in MP, Uttaranchal in UP and Jharkhand in Bihar)

Solution:

State	Male	$\frac{p}{q}$ form	Lowest term
Andhra Pradesh	61.6	616/10	308/5
Assam	57.1	571/10	571/10
Bihar	60.7	607/10	607/10
Gujarat	61.9	619/10	619/10
Haryana	64.1	641/10	641/10
Himachal Pradesh	65.1	651/10	651/10
Karnataka	62.4	624/10	312/5
Kerala	70.6	706/10	353/5
Madhya Pradesh	56.5	565/10	113/2
Maharashtra	64.5	645/10	129/2
Orissa	57.6	576/10	288/5
Punjab	66.9	669/10	669/10
Rajasthan	59.8	598/10	299/5
Tamil Nadu	63.7	637/10	637/10
Uttar Pradesh	58.9	589/10	589/10
West Bengal	62.8	628/10	314/5
India	60.8	608/10	304/5

Kerala; Punjab; HP; Maharashtra; Haryana; Tamil Nadu; West Bengal; Karnataka; Gujarat; Andhra Pradesh; Bihar; Rajasthan; UP; Orissa; Assam; MP.

151. A skirt that is $35\frac{7}{8}$ cm long has a hem of $3\frac{1}{8}$. How long will the skirt be if the hem is let down?

Solution:

$$\begin{aligned} \text{Length of the skirt} &= 35\frac{7}{8} \text{ cm} \\ &= \frac{287}{8} \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Dimension of hem} &= 3\frac{1}{8} \\ &= \frac{25}{8} \text{ cm} \end{aligned}$$

$$\text{Length of skirt, if hem is let down} = \left(\left(\frac{287}{8} \right) + \left(\frac{25}{8} \right) \right) \text{ cm}$$

$$= \frac{312}{8} \text{ cm}$$

$$= 39 \text{ cm}$$

152. Manavi and Kuber each receives an equal allowance. The table shows the fraction of their allowance each deposits into his/her saving account and the fraction each spends at the mall. If allowance of each is Rs. 1260 find the amount left with each.

Where money goes	Fraction of allowance	
	Manvi	Kuber
Saving account	$\frac{1}{2}$	$\frac{1}{3}$
Spend at mall	$\frac{1}{4}$	$\frac{3}{5}$
Left over	?	?

Solution:

Manavi and Kuber each receives and equal allowance = ₹ 1260

Let us assume total cost be ₹ 1.

For Manavi, left over = Total cost – Total spends

$$1 - \left(\frac{1}{2} + \frac{1}{4} \right) = 1 - \frac{(2+1)}{4}$$

$$= \frac{1}{4}$$

$$\text{Amount} = 1260 \times \frac{1}{4}$$

$$= ₹ 315$$

For Kuber, left over = Total cost – Total spends

$$= 1 - \left(\frac{1}{3} + \frac{3}{5} \right)$$

$$\text{So, Amount} = 1260 \times \left(\frac{1}{15} \right) = ₹ 84$$

$$= \frac{1}{15}$$