

4. Inverse Trigonometric Functions

Exercise 4.1

1 A. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

$$\text{Let } \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$$

$$\text{Then } \sin y = \left(-\frac{\sqrt{3}}{2}\right) = -\sin\left(\frac{\pi}{3}\right) = \sin\left(-\frac{\pi}{3}\right)$$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Therefore the principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is $-\frac{\pi}{3}$

1 B. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

$$\text{Let } \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$$

$$\cos y = -\frac{\sqrt{3}}{2}$$

We need to find the value of y .

We know that the value of $\cos$ is negative for the second quadrant and hence the value lies in $[0, \pi]$.

$$\cos y = -\cos\left(\frac{\pi}{6}\right)$$

$$\cos y = \pi - \frac{\pi}{6}$$

$$y = \frac{5\pi}{6}$$

1 C. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)$$

Answer

$$\begin{aligned}
\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) &= \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \\
&= \frac{\pi}{3} - \frac{\pi}{4} \\
&= \frac{\pi}{12}
\end{aligned}$$

1 D. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)$$

Answer

$$\begin{aligned}
\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) &= \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right) \\
&= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) \\
&= \frac{\pi}{3} + \frac{\pi}{4} \\
&= \frac{7\pi}{12}
\end{aligned}$$

1 E. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$$

Answer

$$\text{Let } \sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$$

$$\text{Then } \sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right)$$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$.

1 F. Question

Find the principal value of each of the following:

$$\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$$

Answer

$$\text{Let } y = \sin^{-1}\left(\tan\frac{5\pi}{4}\right)$$

$$\text{Therefore, } \sin y = \left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\text{And } \sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2 A. Question

Find the principal value of each of the following: $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

Answer

$$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$$

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

2 B. Question

Find the principal value of each of the following: $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

Answer

$$\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$$

$$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{3}}{2} \right\}$$

$$= \frac{\pi}{6}$$

3 A. Question

Find the domain of each of the following functions:

$$f(x) = \sin^{-1}x^2$$

Answer

Domain of $\sin^{-1}$ lies in the interval $[-1, 1]$.

Therefore domain of $\sin^{-1}x^2$ lies in the interval $[-1, 1]$.

$$-1 \leq x^2 \leq 1$$

But x^2 cannot take negative values,

$$\text{So, } 0 \leq x^2 \leq 1$$

$$-1 \leq x \leq 1$$

Hence domain of $\sin^{-1}x^2$ is $[-1, 1]$.

3 B. Question

Find the domain of each of the following functions:

$$f(x) = \sin^{-1}x + \sin x$$

Answer

Domain of $\sin^{-1}$ lies in the interval $[-1, 1]$.

$$-1 \leq x \leq 1.$$

The domain of $\sin x$ lies in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$-1.57 \leq x \leq 1.57$$

From the above we can see that the domain of $\sin^{-1}x + \sin x$ is the intersection of the domains of $\sin^{-1}x$ and $\sin x$.

So domain of $\sin^{-1}x + \sin x$ is $[-1, 1]$.

3 C. Question

Find the domain of each of the following functions:

$$f(x) = \sin^{-1}\sqrt{x^2 - 1}$$

Answer

Domain of $\sin^{-1}$ lies in the interval $[-1, 1]$.

Therefore, Domain of $\sin^{-1}\sqrt{x^2 - 1}$ lies in the interval $[-1, 1]$.

$$-1 \leq \sqrt{x^2 - 1} \leq 1$$

$$0 \leq x^2 - 1 \leq 1$$

$$1 \leq x^2 \leq 2$$

$$\pm\sqrt{1} \leq x \leq \pm\sqrt{2}$$

$$\sqrt{2} \leq x \leq -1 \text{ and } 1 \leq x \leq \sqrt{2}$$

Domain of $\sin^{-1}\sqrt{x^2-1}$ is $[-\sqrt{2}, 1] \cup [1, \sqrt{2}]$.

3 D. Question

Find the domain of each of the following functions:

$$f(x) = \sin^{-1}x + \sin^{-1}2x$$

Answer

Domain of $\sin^{-1}$ lies in the interval $[-1, 1]$.

$$-1 \leq x \leq 1$$

Therefore, the domain of $\sin^{-1} 2x$ lies in the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$-1 \leq 2x \leq 1$$

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

The domain of $\sin^{-1}x + \sin^{-1}2x$ is the intersection of the domains of $\sin^{-1}x$ and $\sin^{-1}2x$.

So, Domain of $\sin^{-1}x + \sin^{-1}2x$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

4. Question

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z + \sin^{-1}t = 2\pi$, then find the value of

$$x^2 + y^2 + z^2 + t^2.$$

Answer

Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Give that $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z + \sin^{-1}t = 2\pi$

Each of $\sin^{-1}x, \sin^{-1}y, \sin^{-1}z, \sin^{-1}t$ takes value of $\frac{\pi}{2}$.

So,

$$x = 1, y = 1, z = 1 \text{ and } t = 1.$$

Hence,

$$= x^2 + y^2 + z^2 + t^2$$

$$= 1 + 1 + 1 + 1$$

$$= 4$$

5. Question

If $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = 3/4 \pi^2$. Find $x^2 + y^2 + z^2$.

Answer

Range of $\sin^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Given that $(\sin^{-1}x)^2 + (\sin^{-1}y)^2 + (\sin^{-1}z)^2 = \frac{3}{4}\pi^2$

Each of $\sin^{-1}x$, $\sin^{-1}y$ and $\sin^{-1}z$ takes the value of $\frac{\pi}{2}$.

$x = 1$, $y = 1$, and $z = 1$.

Hence,

$$= x^2 + y^2 + z^2$$

$$= 1 + 1 + 1$$

$$= 3.$$

Exercise 4.2

1. Question

Find the domain of definition of $f(x) = \cos^{-1}(x^2 - 4)$.

Answer

Domain of $\cos^{-1}x$ lies in the interval $[-1, 1]$.

Therefore, the domain of $\cos^{-1}(x^2 - 4)$ lies in the interval $[-1, 1]$.

$$-1 \leq x^2 - 4 \leq 1$$

$$3 \leq x^2 \leq 5$$

$$\pm\sqrt{3} \leq x \leq \pm\sqrt{5}$$

$$-\sqrt{5} \leq x \leq -\sqrt{3} \text{ and } \sqrt{3} \leq x \leq \sqrt{5}$$

Domain of $\cos^{-1}(x^2 - 4)$ is $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$.

2. Question

Find the domain of $f(x) = \cos^{-1}2x + \sin^{-1}x$.

Answer

Domain of $\cos^{-1}x$ lies in the interval $[-1, 1]$.

Therefore, the domain of $\cos^{-1}(2x)$ lies in the interval $[-1, 1]$.

$$-1 \leq 2x \leq 1$$

$$-\frac{1}{2} \leq x \leq \frac{1}{2}$$

Domain of $\cos^{-1}(2x)$ is $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Domain of $\sin^{-1}x$ lies in the interval $[-1, 1]$.

$\therefore$ Domain of $\cos^{-1}(2x) + \sin^{-1}x$ lies in the interval $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

3. Question

Find the domain of $f(x) = \cos^{-1}x + \cos x$.

Answer

Domain of $\cos^{-1}x$ lies in the interval $[-1, 1]$.

Domain of $\cos x$ lies in the interval $[0, \pi] = [0, 3.14]$

$\therefore$ Domain of $\cos^{-1}x + \cos x$ lies in the interval $[-1, 1]$.

4 A. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

We know that for any $x \in [-1, 1]$, $\cos^{-1}$ represents an angle in $[0, \pi]$.

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \text{an angle in } [0, \pi] \text{ whose cosine is } \left(-\frac{\sqrt{3}}{2}\right).$$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

$$\therefore \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

4 B. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

Answer

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y.$$

$$\text{Then, } \cos y = -\frac{1}{\sqrt{2}}$$

$$= -\cos \frac{\pi}{4}$$

$$= \cos\left(\pi - \frac{\pi}{4}\right)$$

$$= \cos\left(\frac{3\pi}{4}\right)$$

We know that the range of the principal value branch of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

4 C. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(\sin \frac{4\pi}{3}\right)$$

Answer

$$\begin{aligned} & \cos^{-1}\left(\sin \frac{4\pi}{3}\right) \\ &= \cos^{-1}\left(\sin\left(\pi + \frac{\pi}{3}\right)\right) \\ &= \cos^{-1}\frac{-\sqrt{3}}{2} \end{aligned}$$

For any $x \in [-1, 1]$, $\cos^{-1}x$ represents an angle in $[0, \pi]$ whose cosine is x .

$$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

$\therefore$ Principal value of $\cos^{-1}\left(\sin \frac{4\pi}{3}\right)$ is $\frac{5\pi}{6}$.

4 D. Question

Find the principal value of each of the following:

$$\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$$

Answer

$$\begin{aligned} & \cos^{-1}\left(\tan \frac{3\pi}{4}\right) \\ &= \cos^{-1}\left(\tan\left(\frac{\pi}{2} + \frac{\pi}{4}\right)\right) \\ &= \cos^{-1}(-1) \end{aligned}$$

For any $x \in [-1, 1]$, $\cos^{-1}x$ represents as an angle in $[0, \pi]$ whose cosine is x .

$$\cos^{-1}(-1) = \pi$$

$\therefore$ Principal value of $\cos^{-1}\left(\tan \frac{3\pi}{4}\right)$ is π .

5 A. Question

For the principal values, evaluate each of the following:

$$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$$

Answer

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x.$$

$$\text{Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(\frac{1}{2}\right) = y.$$

$$\text{Then, } \sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(\frac{1}{2}\right) = \left(\frac{\pi}{6}\right)$$

$$\text{Hence, } \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$$

$$= \frac{\pi}{3} + 2\left(\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

∴ Principal value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

5 B. Question

For the principal values, evaluate each of the following:

$$\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$$

Answer

$$\text{Let } \cos^{-1}\left(\frac{1}{2}\right) = x.$$

$$\text{Then, } \cos x = \frac{1}{2} = \cos\left(\frac{\pi}{3}\right)$$

$$\therefore \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = y.$$

$$\text{Then, } \sin y = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\text{Hence, } \cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{3} - 2\left(-\frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} + \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

∴ Principal value of $\cos^{-1}\left(\frac{1}{2}\right) - 2\sin^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

5 C. Question

For the principal values, evaluate each of the following:

$$\sin^{-1}\left(-\frac{1}{2}\right) + 2\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$

Answer

$$\text{Let } \sin^{-1}\left(-\frac{1}{2}\right) = x$$

$$\text{Then, } \sin x = -\frac{1}{2} = -\sin\left(\frac{\pi}{6}\right) = \sin\left(-\frac{\pi}{6}\right)$$

$$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\text{Let } \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y$$

$$\text{Then, } \cos y = \frac{-\sqrt{3}}{2} = \cos\left(\pi - \frac{\pi}{6}\right)$$

$$\therefore \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

$$\text{Hence, } \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = -\frac{\pi}{6} + 2\left(\frac{5\pi}{6}\right)$$

$$= -\frac{\pi}{6} + \frac{10\pi}{6}$$

$$= \frac{-\pi + 10\pi}{6}$$

$$= \frac{9\pi}{6}$$

$$= \frac{3\pi}{2}$$

$$\therefore \text{Principal value of } \sin^{-1}\left(-\frac{1}{2}\right) + \cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) \text{ is } \frac{3\pi}{2}.$$

5 D. Question

For the principal values, evaluate each of the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Answer

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= -\frac{\pi}{3} + \frac{\pi}{6} \text{ \{Since } \sin^{-1}x = \text{An angle in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ whose sine is } x,$$

Similarly, } \cos^{-1} = \text{An angle in } [0, \pi] \text{ whose cosine is } x\}

$$= -\frac{\pi}{6}$$

Hence,

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$$

$$\therefore \text{Principal value of } \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) \text{ is } -\frac{\pi}{6}$$

Exercise 4.3

1 A. Question

Find the principal value of each of the following:

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

Answer

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x .

So, $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $\frac{1}{\sqrt{3}}$

$$= \frac{\pi}{6}$$

$$\therefore \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Hence, the Principal value of $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

1 B. Question

Find the principal value of each of the following:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

Answer

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x .

So, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ = An angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is $-\frac{1}{\sqrt{3}}$

$$= -\frac{\pi}{6}$$

$$\therefore \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

Hence, Principal value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is $-\frac{\pi}{6}$.

1 C. Question

Find the principal value of each of the following:

$$\tan^{-1}\left(\cos \frac{\pi}{2}\right)$$

Answer

$$\tan^{-1}\left(\cos \frac{\pi}{2}\right) = \tan^{-1}(0) [\because \cos \frac{\pi}{2} = 0]$$

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x .

$$\therefore \tan^{-1}(0) = 0$$

Hence,

Principle value of $\tan^{-1}\left(\cos \frac{\pi}{2}\right)$ is 0.

1 D. Question

Find the principal value of each of the following:

$$\tan^{-1}\left(2\cos\frac{2\pi}{3}\right)$$

Answer

$$\begin{aligned}\tan^{-1}\left(2\cos\frac{2\pi}{3}\right) &= \tan^{-1}\left(2 \times \frac{-1}{2}\right) \\ &= \tan^{-1}(-1)\end{aligned}$$

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x .

$$\therefore \tan^{-1}(-1) = -\frac{\pi}{4}$$

Hence, Principle value of $\tan^{-1}\left(2\cos\frac{2\pi}{3}\right)$ is $-\frac{\pi}{4}$.

2 A. Question

For the principal values, evaluate each of the following:

$$\tan^{-1}(-1) + \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

Answer

$$\text{Let } \tan^{-1}(-1) = x.$$

$$\text{Then } \tan x = -1$$

$$= -\tan\frac{\pi}{4}$$

$$= \tan\left(\pi - \frac{\pi}{4}\right) = \tan\frac{3\pi}{4}$$

$$\therefore \tan^{-1}(-1) = \frac{3\pi}{4}$$

$$\text{Let } \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = y.$$

$$\text{Then } \cos y = \frac{-1}{\sqrt{2}}$$

$$= -\cos\frac{\pi}{4}$$

$$= \cos\left(\pi - \frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$$

$$\therefore \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4}$$

$$\text{Hence, } \tan^{-1}(-1) + \cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \frac{3\pi}{4} + \frac{3\pi}{4} = \frac{6\pi}{4} = \frac{3\pi}{2}$$

2 B. Question

For the principal values, evaluate each of the following:

$$\tan^{-1}\left\{2\sin\left(4\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$$

Answer

$$\text{Let } \cos^{-1} \frac{\sqrt{3}}{2} = x$$

$$\cos x = \cos \left(\frac{\pi}{6} \right)$$

$$x = \left(\frac{\pi}{6} \right)$$

So now,

$$\tan^{-1} \left\{ 2 \sin \left(4 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right\} = \tan^{-1} \left\{ 2 \sin \left(4 \frac{\pi}{6} \right) \right\}$$

$$\tan^{-1} \left\{ 2 \sin \left(4 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right\} = \tan^{-1} \left\{ 2 \sin \left(2 \frac{\pi}{3} \right) \right\}$$

$$\tan^{-1} \left\{ 2 \sin \left(4 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right\} = \tan^{-1} \left(2 \frac{\sqrt{3}}{2} \right)$$

$$\tan^{-1} \left\{ 2 \sin \left(4 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right\} = \tan^{-1} (\sqrt{3})$$

$$\tan^{-1} \left\{ 2 \sin \left(4 \cos^{-1} \frac{\sqrt{3}}{2} \right) \right\} = \frac{\pi}{3}$$

3 A. Question

Evaluate each of the following:

$$\tan^{-1}(1) + \cos^{-1} \left(-\frac{1}{2} \right) + \sin^{-1} \left(-\frac{1}{2} \right)$$

Answer

$$\text{Let } \tan^{-1}(1) = x.$$

$$\text{Then } \tan x = 1 = \frac{\pi}{4}$$

$$\therefore \tan^{-1}(1) = \frac{\pi}{4} \dots\dots(i)$$

$$\text{Let } \cos^{-1} \left(-\frac{1}{2} \right) = y.$$

$$\text{Then } \cos y = -\frac{1}{2} = -\cos \frac{\pi}{3} = \cos \left(\pi - \frac{\pi}{3} \right) = \cos \frac{2\pi}{3}.$$

$$\therefore \cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3} \dots\dots(ii)$$

Again,

$$\text{Let } \sin^{-1} \left(-\frac{1}{2} \right) = z.$$

$$\text{Then } \sin z = -\frac{1}{2} = -\sin \frac{\pi}{6} = \sin \left(-\frac{\pi}{6} \right)$$

$$\therefore \sin^{-1} \left(-\frac{1}{2} \right) = -\frac{\pi}{6} \dots\dots(iii)$$

Now,

$$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} + \left(-\frac{\pi}{6}\right) \text{ [from (i), (ii), (iii)]}$$

$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

$$= \frac{9\pi}{12}$$

$$= \frac{3\pi}{4}$$

3 B. Question

Evaluate each of the following:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

Answer

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x .

$$\therefore \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6},$$

$$\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3} \text{ and,}$$

$$\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = \tan^{-1}(-1) \text{ [}\because \sin\left(-\frac{\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right) = -1\text{]}$$

$$= -\frac{\pi}{4}$$

Now, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ becomes,

$$= \left(-\frac{\pi}{6}\right) + \left(-\frac{\pi}{3}\right) + \left(-\frac{\pi}{4}\right)$$

$$= \frac{-2\pi - 4\pi - 3\pi}{12}$$

$$= \frac{-9\pi}{12}$$

$$= \frac{-3\pi}{4}$$

Therefore the principle value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \tan^{-1}(-\sqrt{3}) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$ is $\frac{-3\pi}{4}$.

3 C. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$$

Answer

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$$

$$\text{Firstly, } \tan\frac{5\pi}{6} = \tan\left(\pi - \frac{\pi}{6}\right) = -\tan\frac{\pi}{6} = -\frac{1}{\sqrt{3}} \dots\dots(i)$$

$$\text{Also, } \cos\left(\frac{13\pi}{6}\right) = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \dots\dots(ii)$$

From (i) and (ii),

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\} \text{ becomes,}$$

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Now,

We know that, for any $x \in \mathbb{R}$, $\tan^{-1}$ represent an angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x.

$$\therefore \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

We know that, for any $x \in [-1, 1]$, $\cos^{-1}$ represent an angle in $(0, \pi)$ whose cosine is x.

$$\therefore \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$\text{Hence, } \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6} + \frac{\pi}{6} = 0$$

Therefore, Principal Value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left\{\cos\left(\frac{13\pi}{6}\right)\right\}$ is 0.

Exercise 4.4

1 A. Question

Find the principal values of each of the following:

$$\sec^{-1}(-\sqrt{2})$$

Answer

$$\text{Let } \sec^{-1}(-\sqrt{2}) = y$$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$= \sec\left(\pi - \frac{\pi}{4}\right)$$

$$= \sec\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$$\text{and } \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$$

$\therefore$ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

1 B. Question

Find the principal values of each of the following:

$$\sec^{-1}(2)$$

Answer

$$\text{Let } \sec^{-1}(2) = y$$

$$\Rightarrow \sec y = 2$$

$$\Rightarrow \sec\left(\frac{\pi}{3}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{And } \sec\left(\frac{\pi}{3}\right) = 2$$

$\therefore$ The principal value of $\sec^{-1}(2)$ is $\frac{\pi}{3}$.

1 C. Question

Find the principal values of each of the following:

$$\sec^{-1}\left(2\sin\frac{3\pi}{4}\right)$$

Answer

$$\text{Let us assume } 2\sin\frac{3\pi}{4} = \theta$$

$$\text{We know } \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\therefore 2\sin\frac{3\pi}{4} = 2\frac{1}{\sqrt{2}}$$

$$\Rightarrow 2\sin\frac{3\pi}{4} = \sqrt{2}$$

$\therefore$ The question becomes $\sec^{-1}(\sqrt{2})$

Now,

$$\text{Let } \sec^{-1}(\sqrt{2}) = y$$

$$\Rightarrow \sec y = \sqrt{2}$$

$$\Rightarrow \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{And } \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$\therefore$ The principal value of $\sec^{-1}(2\sin\frac{3\pi}{4})$ is $\frac{\pi}{4}$.

1 D. Question

Find the principal values of each of the following:

$$\sec^{-1}\left(2\tan\frac{3\pi}{4}\right)$$

Answer

Let us assume $2\tan\frac{3\pi}{4} = \theta$

We know $\tan\frac{3\pi}{4} = -1$

$$\therefore 2\tan\frac{3\pi}{4} = 2(-1)$$

$$\Rightarrow 2\tan\frac{3\pi}{4} = -2$$

$\therefore$ The question converts to $\sec^{-1}(-2)$

Now,

$$\text{Let } \sec^{-1}(-2) = y$$

$$\Rightarrow \sec y = -2$$

$$= -\sec\left(\frac{\pi}{3}\right) = 2$$

$$= \sec\left(\pi - \frac{\pi}{3}\right)$$

$$= \sec\left(\frac{2\pi}{3}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{2\pi}{3}\right) = -2$$

$\therefore$ The principal value of $\sec^{-1}(2\tan\frac{3\pi}{4})$ is $\frac{2\pi}{3}$

2 A. Question

For the principal values, evaluate the following:

$$\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$$

Answer

The Principal value for $\tan^{-1}\sqrt{3}$

$$\text{Let } \tan^{-1}(\sqrt{3}) = y$$

$$\Rightarrow \tan y = \sqrt{3}$$

The range of principal value of $\tan^{-1}$ is $\{-\frac{\pi}{2}, \frac{\pi}{2}\}$

$$\text{And } \tan\left(\frac{\pi}{3}\right) = \sqrt{3}$$

$\therefore$ The principal value of $\tan^{-1}(\sqrt{3})$ is $\frac{\pi}{3}$.

Now,

Principal value for $\sec^{-1}(-2)$

$$\text{Let } \sec^{-1}(-2) = z$$

$$\Rightarrow \sec z = -2$$

$$= -\sec\left(\frac{\pi}{3}\right) = 2$$

$$= \sec\left(\pi - \frac{\pi}{3}\right)$$

$$= \sec\left(\frac{2\pi}{3}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{2\pi}{3}\right) = -2$$

Therefore, the principal value of $\sec^{-1}(-2)$ is $\frac{2\pi}{3}$.

$$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3}$$

$$= \frac{-\pi}{3}$$

$$\therefore \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{-\pi}{3}.$$

2 B. Question

For the principal values, evaluate the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - 2\sec^{-1}\left(2\tan\frac{\pi}{6}\right)$$

Answer

Let,

$$\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = y$$

$$\Rightarrow \sin y = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow -\sin y = \frac{\sqrt{3}}{2}$$

$$\Rightarrow -\sin \frac{\pi}{3}$$

As we know $\sin(-\theta) = -\sin\theta$

$$\therefore -\sin \frac{\pi}{3} = \sin\left(\frac{-\pi}{3}\right)$$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

Therefore, the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ (1)

$$\text{Let us assume } 2\tan\frac{\pi}{6} = \theta$$

$$\text{We know } \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

$$\therefore 2\tan\frac{\pi}{6} = 2\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow 2\tan\frac{\pi}{6} = \frac{2}{\sqrt{3}}$$

$$\therefore \text{The question converts to } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Now,

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = z$$

$$\Rightarrow \sec z = \left(\frac{2}{\sqrt{3}}\right)$$

$$= \sec\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

Therefore, the principal value of $\sec^{-1}(2\tan\frac{\pi}{6})$ is $\frac{\pi}{3}$ (2)

$$\therefore \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - 2\sec^{-1}(2\tan\frac{\pi}{6})$$

$$= \frac{-\pi}{3} - \frac{2\pi}{3} \text{ (from (1) and (2))}$$

$$= \frac{-3\pi}{3}$$

$$= -\pi$$

Therefore, the value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) - 2\sec^{-1}(2\tan\frac{\pi}{6})$ is $-\pi$.

3 A. Question

Find the domain of

$$\sec^{-1}(3x-1)$$

Answer

The range of $\sec x$ is the domain of $\sec^{-1}x$

Now,

The range of $\sec x$ is $(-\infty, -1] \cup [1, \infty)$

$\therefore$ The domain of a given function would be

$$3x-1 \leq -1 \text{ and } 3x-1 \geq 1$$

$$3x \leq 0 \text{ and } 3x \geq 2$$

$$x \leq 0 \text{ and } x \geq \frac{2}{3}$$

$\therefore$ The domain of the given function is $(-\infty, 0] \cup [\frac{2}{3}, \infty)$

3 B. Question

Find the domain of

$$\sec^{-1} x - \tan^{-1} x$$

Answer

Domain of $\sec^{-1}x$ is $(-\infty, -1] \cup [1, \infty)$

Domain of $\tan^{-1}x$ is $\mathbb{R}$

Union of (1) and (2) will be domain of given function

$$(-\infty, -1] \cup [1, \infty) \cup \mathbb{R}$$

$$\Rightarrow (-\infty, -1] \cup [1, \infty)$$

$\therefore$ The domain of given function is $(-\infty, -1] \cup [1, \infty)$.

1 A. Question

Find the principal values of each of the following:

$$\sec^{-1}(-\sqrt{2})$$

Answer

$$\text{Let } \sec^{-1}(-\sqrt{2}) = y$$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$= \sec\left(\pi - \frac{\pi}{4}\right)$$

$$= \sec\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$$

$\therefore$ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

Exercise 4.5

1 A. Question

Find the principal values of each of the following:

$$\operatorname{cosec}^{-1}(-\sqrt{2})$$

Answer

$$\operatorname{cosec}^{-1}(-\sqrt{2}) = y$$

$$\Rightarrow \operatorname{cosec} y = -\sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} y = \sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{4} = \operatorname{cosec} \left(\frac{-\pi}{4}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$$\operatorname{cosec} \left(\frac{-\pi}{4}\right) = -\sqrt{2}$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $\frac{-\pi}{4}$.

1 B. Question

Find the principal values of each of the following:

$$\operatorname{cosec}^{-1}(-2)$$

Answer

$$\operatorname{cosec}^{-1} - 2 = y$$

$$\Rightarrow \operatorname{cosec} y = -2$$

$$\Rightarrow -\operatorname{cosec} y = 2$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{6} = 2$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec} \left(-\frac{\pi}{6} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$ and

$$\operatorname{cosec} \left(-\frac{\pi}{6} \right) = -2$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-2)$ is $-\frac{\pi}{6}$.

1 C. Question

Find the principal values of each of the following:

$$\operatorname{cosec}^{-1} \left(\frac{2}{\sqrt{3}} \right)$$

Answer

$$\text{Let } \operatorname{cosec}^{-1} \left(\frac{2}{\sqrt{3}} \right) = y$$

$$\Rightarrow \operatorname{cosec} y = \left(\frac{2}{\sqrt{3}} \right)$$

$$= \operatorname{cosec} \left(\frac{\pi}{3} \right) = \left(\frac{2}{\sqrt{3}} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$

$$\text{and } \operatorname{cosec} \left(\frac{\pi}{3} \right) = \left(\frac{2}{\sqrt{3}} \right)$$

Therefore, the principal value of $\operatorname{cosec}^{-1} \left(\frac{2}{\sqrt{3}} \right)$ is $\frac{\pi}{3}$.

1 D. Question

Find the principal values of each of the following:

$$\operatorname{cosec}^{-1} \left(2 \cos \frac{2\pi}{3} \right)$$

Answer

$$\operatorname{cosec}^{-1} \left(2 \cos \frac{2\pi}{3} \right)$$

$$\text{Let us assume } 2 \cos \frac{2\pi}{3} = \theta$$

$$\text{We know } \cos \frac{2\pi}{3} = \frac{-1}{2}$$

$$\therefore 2 \cos \frac{2\pi}{3} = 2 \left(\frac{-1}{2} \right)$$

$$\Rightarrow 2 \cos \frac{2\pi}{3} = -1$$

∴ The question converts to $\operatorname{cosec}^{-1}(-1)$

Now,

$$\operatorname{cosec}^{-1}(-1) = y$$

$$\Rightarrow \operatorname{cosec} y = -1$$

$$\Rightarrow -\operatorname{cosec} y = 1$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{4} = 1$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{4} = \operatorname{cosec} \left(\frac{-\pi}{4} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$ and

$$\operatorname{cosec} \left(\frac{-\pi}{4} \right) = -1$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(2\cos\frac{2\pi}{3})$ is $\frac{-\pi}{4}$.

2. Question

Find the set of values of $\operatorname{cosec}^{-1}(\sqrt{3}/2)$.

Answer

$$\text{Let } y = \operatorname{cosec}^{-1}(\sqrt{3}/2)$$

We know that,

Domain of $y = \operatorname{cosec}^{-1} x$ is $(-\infty, -1] \cup [1, \infty)$

$$\text{But } \sqrt{3}/2 < 1$$

Therefore, it can not be a value of y .

Hence, Set of values of $\operatorname{cosec}^{-1}(\sqrt{3}/2)$ is a null set.

3 A. Question

For the principal values, evaluate the following:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$$

Answer

Let,

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = y$$

$$\Rightarrow \sin y = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow -\sin y = \frac{\sqrt{3}}{2}$$

$$\Rightarrow -\sin \frac{\pi}{3}$$

As we know $\sin(-\theta) = -\sin\theta$

$$\therefore -\sin \frac{\pi}{3} = \sin \left(\frac{-\pi}{3} \right)$$

The range of principal value of $\sin^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$

Therefore, the principal value of $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ (1)

Let,

$$\operatorname{cosec}^{-1}\left(\frac{-\sqrt{3}}{2}\right) = z$$

$$\Rightarrow \operatorname{cosec} z = \frac{-\sqrt{3}}{2}$$

$$\Rightarrow -\operatorname{cosec} z = \frac{\sqrt{3}}{2}$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{3}$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{3} = \operatorname{cosec}\left(\frac{-\pi}{3}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$$\operatorname{cosec}\left(\frac{-\pi}{3}\right) = \frac{-\sqrt{3}}{2}$$

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ is $\frac{-\pi}{3}$ (2)

From (1) and (2) we get

$$\Rightarrow \frac{-\pi}{3} + \frac{-\pi}{3}$$

$$= \frac{-2\pi}{3}$$

3 B. Question

For the principal values, evaluate the following:

$$\sec^{-1}(\sqrt{2}) + 2 \operatorname{cosec}^{-1}(-\sqrt{2})$$

Answer

Let $\sec^{-1}(-\sqrt{2}) = y$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$= \sec\left(\pi - \frac{\pi}{4}\right)$$

$$= \sec\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$$\text{and } \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}.$$

Let,

$$\operatorname{cosec}^{-1} - \sqrt{2} = z$$

$$\Rightarrow \operatorname{cosec} z = -\sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} z = \sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{4} = \operatorname{cosec} \left(\frac{-\pi}{4} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$ and

$$\operatorname{cosec} \left(\frac{-\pi}{4} \right) = -\sqrt{2}$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $\frac{-\pi}{4}$.

$$\operatorname{cosec}^{-1} - \sqrt{2} = y$$

$$\Rightarrow \operatorname{cosec} y = -\sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} y = \sqrt{2}$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{4} = \sqrt{2}$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{4} = \operatorname{cosec} \left(\frac{-\pi}{4} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$ and

$$\operatorname{cosec} \left(\frac{-\pi}{4} \right) = -\sqrt{2}$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-\sqrt{2})$ is $\frac{-\pi}{4}$.

From (1) and (2) we get

$$\Rightarrow \frac{3\pi}{4} + 2 \times \frac{-\pi}{4}$$

$$= \frac{3\pi}{4} + \frac{-2\pi}{4}$$

$$= \frac{\pi}{4}$$

3 C. Question

For the principal values, evaluate the following:

$$\sin^{-1} \left[\cos \left\{ \operatorname{cosec}^{-1}(-2) \right\} \right]$$

Answer

First of all we need to find the principal value for $\operatorname{cosec}^{-1}(-2)$

Let,

$$\operatorname{cosec}^{-1} - 2 = y$$

$$\Rightarrow \operatorname{cosec} y = -2$$

$$\Rightarrow -\operatorname{cosec} y = 2$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{6} = 2$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec} \left(\frac{-\pi}{6} \right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2} \right] - \{0\}$ and

$$\operatorname{cosec} \left(\frac{-\pi}{6} \right) = -2$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-2)$ is $\frac{-\pi}{6}$.

$\therefore$ Now, the question changes to

$$\sin^{-1} \left[\cos \frac{-\pi}{6} \right]$$

$$\cos(-\theta) = \cos(\theta)$$

$\therefore$ we can write the above expression as

$$\sin^{-1} \left[\cos \frac{\pi}{6} \right]$$

Let,

$$\sin^{-1} \left(\frac{\sqrt{3}}{2} \right) = y$$

$$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \frac{\pi}{3}$$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2} \right)$ and $\sin \left(\frac{\pi}{3} \right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\sin^{-1} \left(\frac{\sqrt{3}}{2} \right)$ is $\frac{\pi}{3}$.

Hence, the principal value of the given equation is $\frac{\pi}{3}$.

3 D. Question

For the principal values, evaluate the following:

$$\operatorname{cosec}^{-1} \left(2 \tan \frac{11\pi}{6} \right)$$

Answer

We can write,

$$\tan \frac{11\pi}{6} = \tan \left(2\pi - \frac{\pi}{6} \right)$$

$$\tan(2\pi - \theta)$$

$$= \tan(-\theta)$$

$$= -\tan\theta$$

$$\therefore \tan \frac{11\pi}{6} \text{ becomes } -\tan \frac{\pi}{6}$$

$$-\tan \frac{\pi}{6} = -\frac{1}{\sqrt{3}}$$

$$\Rightarrow 2 \tan \frac{11\pi}{6} = -\frac{2}{\sqrt{3}}$$

∴ The question converts to $\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$$\text{Let } \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$$

$$\Rightarrow \operatorname{cosec} y = \left(\frac{2}{\sqrt{3}}\right)$$

$$= \operatorname{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$$\text{and } \operatorname{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3}$.

Exercise 4.6

1 A. Question

Find the principal values of each of the following:

$$\cot^{-1}(-\sqrt{3})$$

Answer

$$\text{Let } \cot^{-1}(-\sqrt{3}) = y$$

$$\Rightarrow \cot y = -\sqrt{3}$$

$$= -\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$= \cot\left(\pi - \frac{\pi}{6}\right)$$

$$= \cot\left(\frac{5\pi}{6}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{5\pi}{6}\right) = -\sqrt{3}$$

∴ The principal value of $\cot^{-1}(-\sqrt{3})$ is $\frac{5\pi}{6}$

1 B. Question

Find the principal values of each of the following:

$$\cot^{-1}(\sqrt{3})$$

Answer

$$\text{Let } \cot^{-1}(\sqrt{3}) = y$$

$$\Rightarrow \cot y = \sqrt{3}$$

$$= \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

∴ The principal value of $\cot^{-1}(\sqrt{3})$ is $\frac{\pi}{6}$

1 C. Question

Find the principal values of each of the following:

$$\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$$

Answer

$$\text{Let } \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\Rightarrow \cot y = \frac{-1}{\sqrt{3}}$$

$$= -\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

$$= \cot\left(\pi - \frac{\pi}{3}\right)$$

$$= \cot\left(\frac{2\pi}{3}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$$

$\therefore$ The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{2\pi}{3}$

1 D. Question

Find the principal values of each of the following:

$$\cot^{-1}\left(\tan \frac{3\pi}{4}\right)$$

Answer

The value of

$$\tan \frac{3\pi}{4} = -1$$

$\therefore$ The question becomes $\cot^{-1}(-1)$

$$\text{Let } \cot^{-1}(-1) = y$$

$$\Rightarrow \cot y = -1$$

$$= -\cot\left(\frac{\pi}{4}\right) = 1$$

$$= \cot\left(\pi - \frac{\pi}{4}\right)$$

$$= \cot\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{3\pi}{4}\right) = -1$$

$\therefore$ The principal value of $\cot^{-1}\left(\tan \frac{3\pi}{4}\right)$ is $\frac{3\pi}{4}$.

2. Question

Find the domain of $f(x) = \cot x + \cot^{-1} x$.

Answer

Now the domain of $\cot x$ is $\mathbb{R}$

While the domain of $\cot^{-1}x$ is $[0, \pi]$

$\therefore$ The union of these two will give the domain of $f(x)$

$$\Rightarrow \mathbb{R} \cup [0, \pi]$$

$$= [0, \pi]$$

$\therefore$ The domain of $f(x)$ is $[0, \pi]$

3 A. Question

Evaluate each of the following:

$$\cot^{-1} \frac{1}{\sqrt{3}} - \operatorname{cosec}^{-1}(-2) + \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

Answer

$$\text{Let } \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\Rightarrow \cot y = \frac{-1}{\sqrt{3}}$$

$$= -\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

$$= \cot\left(\pi - \frac{\pi}{3}\right)$$

$$= \cot\left(\frac{2\pi}{3}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$$

$\therefore$ The principal value of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)$ is $\frac{2\pi}{3} \dots (1)$

Let,

$$\operatorname{cosec}^{-1}(-2) = z$$

$$\Rightarrow \operatorname{cosec} z = -2$$

$$\Rightarrow -\operatorname{cosec} z = 2$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{6} = 2$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec}\left(\frac{-\pi}{6}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$$\operatorname{cosec}\left(\frac{-\pi}{6}\right) = -2$$

Therefore, the principal value of $\operatorname{cosec}^{-1}(-2)$ is $\frac{-\pi}{6} \dots (2)$

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = w$$

$$\Rightarrow \sec w = \left(\frac{2}{\sqrt{3}}\right)$$

$$= \sec\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3} \dots (3)$

From (1), (2) and (3) we can write the above equation as

$$= \frac{2\pi}{3} - \frac{-\pi}{6} + \frac{\pi}{3}$$

$$= \frac{4\pi + \pi + 2\pi}{6}$$

$$= \frac{7\pi}{6}$$

3 B. Question

Evaluate each of the following:

$$\cot^{-1} \left\{ 2 \cos \left(\sin^{-1} \frac{\sqrt{3}}{2} \right) \right\}$$

Answer

For finding the solution we first of need to find the principal value of

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Let,

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$$

$$\Rightarrow \sin y = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin \frac{\pi}{3}$$

The range of principal value of $\sin^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$

Therefore, the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{3}$

$\therefore$ The above equation changes to $\cot^{-1}(2\cos\frac{\pi}{3})$

Now we need to find the value of $2\cos\frac{\pi}{3}$

$$\therefore \cos\frac{\pi}{3} = \frac{1}{2}$$

$$\Rightarrow 2\cos\frac{\pi}{3} = 1 \times \frac{1}{2}$$

$$\Rightarrow 2\cos\frac{\pi}{3} = 1$$

Now the equation simplification to $\cot^{-1}(1)$

$$\text{Let } \cot^{-1}(1) = y$$

$$\Rightarrow \cot y = 1$$

$$= \cot\left(\frac{\pi}{4}\right) = 1$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{\pi}{4}\right) = 1$$

$\therefore$ The principal value of $\cot^{-1}(2\cos(\sin^{-1}(\frac{\sqrt{3}}{2})))$ is $\frac{\pi}{4}$

3 C. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(-\frac{2}{\sqrt{3}}\right) + 2\cot^{-1}(-1)$$

Answer

Now first of the principal value of

$$\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

$$\text{Let } \operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right) = y$$

$$\Rightarrow \operatorname{cosec} y = \left(\frac{2}{\sqrt{3}}\right)$$

$$= \operatorname{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$$\text{and } \operatorname{cosec}\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

Therefore, the principal value of $\operatorname{cosec}^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{3} \dots (1)$

Now, the value of $\cot^{-1}(-1)$

$$\text{Let } \cot^{-1}(-1) = y$$

$$\Rightarrow \cot y = -1$$

$$= -\cot\left(\frac{\pi}{4}\right) = 1$$

$$= \cot\left(\pi - \frac{\pi}{4}\right)$$

$$= \cot\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{3\pi}{4}\right) = -1$$

Therefore, the principal value of $\cot^{-1}(-1)$ is $\frac{3\pi}{4} \dots (2)$

From (1) and (2) we can write the given equation as

$$= \frac{\pi}{3} + 2 \times \frac{3\pi}{4}$$

$$= \frac{\pi}{3} + \frac{3\pi}{2}$$

$$= \frac{11\pi}{6}$$

3 D. Question

Evaluate each of the following:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

Answer

$$\text{Let } \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = y$$

$$\Rightarrow \tan y = \frac{-1}{\sqrt{3}}$$

$$= -\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$$

$$= \tan\left(-\frac{\pi}{6}\right)$$

$$\therefore \text{The principal value of } \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) \text{ is } \frac{-\pi}{6} \dots(1)$$

$$\text{Let } \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) = z$$

$$\Rightarrow \cot z = \frac{-1}{\sqrt{3}}$$

$$= -\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

$$= \cot\left(\pi - \frac{\pi}{3}\right)$$

$$= \cot\left(\frac{2\pi}{3}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{2\pi}{3}\right) = \frac{-1}{\sqrt{3}}$$

$$\therefore \text{The principal value of } \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) \text{ is } \frac{2\pi}{3} \dots(2)$$

$$\sin\frac{-\pi}{2} = -1$$

$$\therefore \tan^{-1}(-1)$$

$$\text{Let } \tan^{-1}(-1) = w$$

$$\Rightarrow \tan w = -1$$

$$= -\tan\left(\frac{\pi}{4}\right) = 1$$

$$= \tan\left(-\frac{\pi}{4}\right)$$

$$\therefore \text{The principal value of } \tan^{-1}(-1) \text{ is } \frac{-\pi}{4} \dots(3)$$

From(1),(2) and (3) we get

$$= \frac{-\pi}{6} + \frac{2\pi}{3} + \frac{-\pi}{4}$$

$$= \frac{\pi}{4}$$

Exercise 4.7

1 A. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin \frac{\pi}{6}\right)$$

Answer

The value of $\sin \frac{\pi}{6}$ is $\frac{1}{2}$

$\therefore$ The question becomes $\sin^{-1}\left(\frac{1}{2}\right)$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$$\Rightarrow \sin y = \frac{1}{2}$$

$$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

The range of principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Therefore, the value of $\sin^{-1}\left(\sin \frac{\pi}{6}\right)$ is $\frac{\pi}{6}$.

Alternate Solution:

$$\sin^{-1}(\sin x) = x$$

$$\text{Provided } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\therefore \text{ we can write } \sin^{-1}\left(\sin \frac{\pi}{6}\right) = \frac{\pi}{6}$$

1 B. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin \frac{7\pi}{6}\right)$$

Answer

The value of $\sin \frac{7\pi}{6}$ is $-\frac{1}{2}$

$\therefore$ The question becomes $\sin^{-1}\left(-\frac{1}{2}\right)$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$

$$\Rightarrow -\sin y = \frac{1}{2}$$

$$= -\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

As, $-\sin(\theta)$ is $\sin(-\theta)$.

$$\Rightarrow -\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{-\pi}{6}\right)$$

The range of principal value of $\sin^{-1}$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\sin\left(\frac{-\pi}{6}\right) = \frac{-1}{2}$

Therefore, the value of $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$ is $\frac{-\pi}{6}$.

1 C. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$$

Answer

The value of $\sin\frac{5\pi}{6}$ is $\frac{1}{2}$

$\therefore$ The question becomes $\sin^{-1}\left(\frac{1}{2}\right)$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$

$$\Rightarrow \sin y = \frac{1}{2}$$

$$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

The range of principal value of $\sin^{-1}$ is $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

Therefore, the value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$ is $\frac{\pi}{6}$.

1 D. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{13\pi}{7}\right)$$

Answer

We can write $\left(\sin\frac{13\pi}{7}\right)$ as $\sin\left(2\pi - \frac{\pi}{7}\right)$

As we know $\sin(2\pi - \theta) = \sin(-\theta)$

So $\sin\left(2\pi - \frac{\pi}{7}\right)$ can be written as $\sin\left(\frac{\pi}{7}\right)$

$\therefore$ The equation becomes $\sin^{-1}\left(\sin\frac{\pi}{7}\right)$

As $\sin^{-1}(\sin x) = x$

Provided $x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ we can write $\sin^{-1}\left(\sin\frac{\pi}{7}\right) = \frac{\pi}{7}$

1 E. Question

Evaluate each of the following:

$$\sin^{-1}\left(\sin\frac{17\pi}{8}\right)$$

Answer

We can write $(\sin\frac{17\pi}{8})$ as $\sin\left(2\pi + \frac{\pi}{8}\right)$

As we know $\sin(2\pi + \theta) = \sin(\theta)$

So $\sin\left(2\pi + \frac{\pi}{8}\right)$ can be written as $\sin\left(\frac{\pi}{8}\right)$

$\therefore$ The equation becomes $\sin^{-1}(\sin\frac{\pi}{8})$

As $\sin^{-1}(\sin x) = x$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ we can write $\sin^{-1}(\sin\frac{\pi}{8}) = \frac{\pi}{8}$

1 F. Question

Evaluate each of the following:

$$\sin^{-1}\left\{\left(\sin-\frac{17\pi}{8}\right)\right\}$$

Answer

As we know $\sin(-\theta)$ is $-\sin(\theta)$

$\therefore$ We can write $(\sin\frac{-17\pi}{8})$ as $-\sin\left(\frac{17\pi}{8}\right)$

Now $-\sin\left(\frac{-17\pi}{8}\right) = -\sin\left(2\pi + \frac{\pi}{8}\right)$

As we know $\sin(2\pi + \theta) = \sin(\theta)$

So $-\sin\left(2\pi + \frac{\pi}{8}\right)$ can be written as $-\sin\left(\frac{\pi}{8}\right)$

And $-\sin\left(\frac{\pi}{8}\right) = \sin\left(\frac{-\pi}{8}\right)$

The equation becomes $\sin^{-1}(\sin\frac{-\pi}{8})$

As $\sin^{-1}(\sin x) = x$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\therefore$ we can write $\sin^{-1}(\sin\frac{-\pi}{8}) = \frac{-\pi}{8}$

1 G. Question

Evaluate each of the following:

$$\sin^{-1}(\sin 3)$$

Answer

$\sin^{-1}(\sin x) = x$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \approx [-1.57, 1.57]$

And in our equation x is 3 which does not lie in the above range.

We know $\sin[\pi - x] = \sin[x]$

$$\therefore \sin(\pi - 3) = \sin(3)$$

Also $\pi - 3$ belongs in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}(\sin 3) = \pi - 3$$

1 H. Question

Evaluate each of the following:

$$\sin^{-1}(\sin 4)$$

Answer

$$\sin^{-1}(\sin x) = x$$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \approx [-1.57, 1.57]$

And in our equation x is 4 which does not lie in the above range.

We know $\sin[\pi - x] = \sin[-x]$

$$\therefore \sin(\pi - 4) = \sin(-4)$$

Also $\pi - 4$ belongs in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}(\sin 4) = \pi - 4$$

1 I. Question

Evaluate each of the following:

$$\sin^{-1}(\sin 12)$$

Answer

$$\sin^{-1}(\sin x) = x$$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \approx [-1.57, 1.57]$

And in our equation x is 12 which does not lie in the above range.

We know $\sin[2n\pi - x] = \sin[-x]$

$$\therefore \sin(2n\pi - 12) = \sin(-12)$$

Here $n = 2$

Also $2\pi - 12$ belongs in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}(\sin 12) = 2\pi - 12$$

1 J. Question

Evaluate each of the following:

$$\sin^{-1}(\sin 2)$$

Answer

$$\sin^{-1}(\sin x) = x$$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \approx [-1.57, 1.57]$

And in our equation x is 3 which does not lie in the above range.

We know $\sin[\pi - x] = \sin[x]$

$$\therefore \sin(\pi - 2) = \sin(2)$$

Also $\pi - 2$ belongs in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\therefore \sin^{-1}(\sin 2) = \pi - 2$$

2 A. Question

Evaluate each of the following:

$$\cos^{-1}\left\{\cos\left(-\frac{\pi}{4}\right)\right\}$$

Answer

As $\cos(-\theta)$ is $\cos(\theta)$

$$\therefore \left(\cos\left(-\frac{\pi}{4}\right)\right) = \left(\cos\left(\frac{\pi}{4}\right)\right)$$

Now,

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\therefore \text{The question becomes } \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$\text{Let } \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = y$$

$$\Rightarrow \cos y = \frac{1}{\sqrt{2}}$$

$$= \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

The range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

Therefore, the value of $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right)$ is $\frac{\pi}{4}$.

2 B. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{5\pi}{4}\right)$$

Answer

The value of $\cos\left(\frac{5\pi}{4}\right)$ is $-\frac{1}{\sqrt{2}}$

Now,

$$\therefore \text{The question becomes } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$$

$$\text{Let } \cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$$

$$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$$

$$= -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

$$= \cos\left(\pi - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

$$\Rightarrow \cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$$

The range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

Therefore, the value of $\cos^{-1}(\cos\left(\frac{5\pi}{4}\right))$ is $\frac{3\pi}{4}$.

2 C. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$$

Answer

The value of $\cos\left(\frac{4\pi}{3}\right)$ is $\frac{-1}{2}$

Now,

$\therefore$ The question becomes $\cos^{-1}\left(\frac{-1}{2}\right)$

$$\text{Let } \cos^{-1}\left(\frac{-1}{2}\right) = y$$

$$\Rightarrow \cos y = \frac{-1}{2}$$

$$= -\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$$

$$= \cos\left(\pi - \frac{\pi}{3}\right) = \frac{-1}{2}$$

$$\Rightarrow \cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$$

The range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = \frac{-1}{2}$

Therefore, the value of $\cos^{-1}(\cos\left(\frac{4\pi}{3}\right))$ is $\frac{2\pi}{3}$.

2 D. Question

Evaluate each of the following:

$$\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$$

Answer

The value of $\cos\left(\frac{13\pi}{6}\right)$ is $\frac{\sqrt{3}}{2}$

Now,

$\therefore$ The question becomes $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\text{Let } \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$$

$$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$$

$$= \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$$

The range of principal value of $\cos^{-1}$ is $[0, \pi]$ and $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, the value of $\cos^{-1}(\cos(\frac{13\pi}{6}))$ is $\frac{\pi}{6}$.

2 E. Question

Evaluate each of the following:

$$\cos^{-1}(\cos 3)$$

Answer

$$\text{As } \cos^{-1}(\cos x) = x$$

Provided $x \in [0, \pi]$

$\therefore$ we can write $\cos^{-1}(\cos 3)$ as 3.

2 F. Question

Evaluate each of the following:

$$\cos^{-1}(\cos 4)$$

Answer

$$\cos^{-1}(\cos x) = x$$

Provided $x \in [0, \pi] \approx [0, 3.14]$

And in our equation x is 4 which does not lie in the above range.

We know $\cos[2\pi - x] = \cos[x]$

$$\therefore \cos(2\pi - 4) = \cos(4)$$

Also $2\pi - 4$ belongs in $[0, \pi]$

$$\therefore \cos^{-1}(\cos 4) = 2\pi - 4$$

2 G. Question

Evaluate each of the following:

$$\cos^{-1}(\cos 5)$$

Answer

$$\cos^{-1}(\cos x) = x$$

Provided $x \in [0, \pi] \approx [0, 3.14]$

And in our equation x is 5 which does not lie in the above range.

We know $\cos[2\pi - x] = \cos[x]$

$$\therefore \cos(2\pi - 5) = \cos(5)$$

Also $2\pi - 5$ belongs in $[0, \pi]$

$$\therefore \cos^{-1}(\cos 5) = 2\pi - 5$$

2 H. Question

Evaluate each of the following:

$$\cos^{-1}(\cos 12)$$

Answer

$$\cos^{-1}(\cos x) = x$$

$$\text{Provided } x \in [0, \pi] \approx [0, 3.14]$$

And in our equation x is 4 which does not lie in the above range.

$$\text{We know } \cos[2n\pi - x] = \cos[x]$$

$$\therefore \cos(2n\pi - 12) = \cos(12)$$

$$\text{Here } n = 2.$$

$$\text{Also } 4\pi - 12 \text{ belongs in } [0, \pi]$$

$$\therefore \cos^{-1}(\cos 12) = 4\pi - 12$$

3 A. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan \frac{\pi}{3}\right)$$

Answer

$$\text{As, } \tan^{-1}(\tan x) = x$$

$$\text{Provided } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\Rightarrow \tan^{-1}\left(\tan \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}$$

3 B. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan \frac{6\pi}{7}\right)$$

Answer

$$\tan \frac{6\pi}{7} \text{ can be written as } \tan\left(\pi - \frac{\pi}{7}\right)$$

$$\tan\left(\pi - \frac{\pi}{7}\right) = -\tan \frac{\pi}{7}$$

$$\therefore \text{As, } \tan^{-1}(\tan x) = x$$

$$\text{Provided } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\tan^{-1}\left(\tan \frac{6\pi}{7}\right) = -\frac{\pi}{7}$$

3 C. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$$

Answer

The value of $\tan \frac{7\pi}{6} = \frac{1}{\sqrt{3}}$

$\therefore$ The question becomes $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Let,

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = y$$

$$\Rightarrow \tan y = \left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \tan\left(\frac{\pi}{6}\right) = \left(\frac{1}{\sqrt{3}}\right)$$

The range of the principal value of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{6}\right) = \left(\frac{1}{\sqrt{3}}\right)$.

$\therefore$ The value of $\tan^{-1}\left(\tan \frac{7\pi}{6}\right)$ is $\frac{\pi}{6}$.

3 D. Question

Evaluate each of the following:

$$\tan^{-1}\left(\tan \frac{9\pi}{4}\right)$$

Answer

The value of $\tan \frac{9\pi}{4} = 1$

$\therefore$ The question becomes $\tan^{-1}1$

Let,

$$\tan^{-1}1 = y$$

$$\Rightarrow \tan y = 1$$

$$\Rightarrow \tan\left(\frac{\pi}{4}\right) = 1$$

The range of the principal value of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(\frac{\pi}{4}\right) = 1$.

$\therefore$ The value of $\tan^{-1}\left(\tan \frac{9\pi}{4}\right)$ is $\frac{\pi}{4}$.

3 E. Question

Evaluate each of the following:

$$\tan^{-1}(\tan 1)$$

Answer

As, $\tan^{-1}(\tan x) = x$

Provided $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\Rightarrow \tan^{-1}(\tan 1)$$

$$= 1$$

3 F. Question

Evaluate each of the following:

$$\tan^{-1}(\tan 2)$$

Answer

$$\text{As, } \tan^{-1}(\tan x) = x$$

$$\text{Provided } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Here our x is 2 which does not belong to our range

$$\text{We know } \tan(\pi - \theta) = -\tan(\theta)$$

$$\therefore \tan(\theta - \pi) = \tan(\theta)$$

$$\therefore \tan(2 - \pi) = \tan(2)$$

Now $2 - \pi$ is in the given range

$$\therefore \tan^{-1}(\tan 2) = 2 - \pi$$

3 G. Question

Evaluate each of the following:

$$\tan^{-1}(\tan 4)$$

Answer

$$\text{As, } \tan^{-1}(\tan x) = x$$

$$\text{Provided } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Here our x is 4 which does not belong to our range

$$\text{We know } \tan(\pi - \theta) = -\tan(\theta)$$

$$\therefore \tan(\theta - \pi) = \tan(\theta)$$

$$\therefore \tan(4 - \pi) = \tan(4)$$

Now $4 - \pi$ is in the given range

$$\therefore \tan^{-1}(\tan 4) = 4 - \pi$$

3 H. Question

Evaluate each of the following:

$$\tan^{-1}(\tan 12)$$

Answer

$$\text{As, } \tan^{-1}(\tan x) = x$$

$$\text{Provided } x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Here our x is 12 which does not belong to our range

$$\text{We know } \tan(n\pi - \theta) = -\tan(\theta)$$

$$\therefore \tan(\theta - 2n\pi) = \tan(\theta)$$

Here $n = 4$

$$\therefore \tan(12 - 4\pi) = \tan(12)$$

Now $12 - 4\pi$ is in the given range

$$\therefore \tan^{-1}(\tan 12) = 12 - 4\pi.$$

4 A. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec \frac{\pi}{3}\right)$$

Answer

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore$ we can write $\sec^{-1}\sec\left(\frac{\pi}{3}\right)$ as $\frac{\pi}{3}$.

4 B. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec \frac{2\pi}{3}\right)$$

Answer

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\therefore$ we can write $\sec^{-1}\sec\left(\frac{2\pi}{3}\right)$ as $\frac{2\pi}{3}$.

4 C. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec \frac{5\pi}{4}\right)$$

Answer

The value of $\sec\left(\frac{5\pi}{4}\right)$ is $-\sqrt{2}$.

$\therefore$ The question becomes $\sec^{-1}(-\sqrt{2})$.

Let $\sec^{-1}(-\sqrt{2}) = y$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$= \sec\left(\pi - \frac{\pi}{4}\right)$$

$$= \sec\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

$$\text{and } \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}$$

$\therefore$ The principal value of $\sec^{-1}(-\sqrt{2})$ is $\frac{3\pi}{4}$.

4 D. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{7\pi}{3}\right)$$

Answer

The value of $\sec\left(\frac{7\pi}{3}\right)$ is 2

Let $\sec^{-1}(2) = y$

$$\Rightarrow \sec y = 2$$

$$\Rightarrow \sec\left(\frac{\pi}{3}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{And } \sec\left(\frac{\pi}{3}\right) = 2$$

$\therefore$ The principal value of $\sec^{-1}(\sec(\frac{7\pi}{3}))$ is $\frac{\pi}{3}$.

4 E. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{9\pi}{5}\right)$$

Answer

$$\sec\left(\frac{9\pi}{5}\right) \text{ can be written as } \sec\left(2\pi - \frac{\pi}{5}\right)$$

Also, we know $\sec(2\pi - \theta) = \sec(\theta)$

$$\therefore \sec\left(2\pi - \frac{\pi}{5}\right) = \sec\left(\frac{\pi}{5}\right)$$

$\therefore$ Now the given equation can be written as $\sec^{-1}\sec\left(\frac{\pi}{5}\right)$

As $\sec^{-1}(\sec x) = x$

Provided $x \in [0, \pi] - \{\frac{\pi}{2}\}$

$\therefore$ we can write $\sec^{-1}\sec\left(\frac{\pi}{5}\right)$ as $\frac{\pi}{5}$.

4 F. Question

Evaluate each of the following:

$$\sec^{-1}\left\{\sec\left(-\frac{7\pi}{3}\right)\right\}$$

Answer

As $\sec(-\theta)$ is $\sec(\theta)$

$$\therefore \sec\left(-\frac{7\pi}{3}\right) = \sec\left(\frac{7\pi}{3}\right)$$

The value of $\sec\left(\frac{7\pi}{3}\right)$ is 2.

$$\text{Let } \sec^{-1}(2) = y$$

$$\Rightarrow \sec y = 2$$

$$\Rightarrow \sec\left(\frac{\pi}{3}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{And } \sec\left(\frac{\pi}{3}\right) = 2$$

$\therefore$ The value of $\sec^{-1}(\sec(\frac{-7\pi}{3}))$ is $\frac{\pi}{3}$.

4 G. Question

Evaluate each of the following:

$$\sec^{-1}\left\{\sec\left(-\frac{13\pi}{4}\right)\right\}$$

Answer

As $\sec(-\theta)$ is $\sec(\theta)$

$$\therefore \sec\left(\frac{-7\pi}{3}\right) = \sec\left(\frac{7\pi}{3}\right)$$

The value of $\sec\left(\frac{-13\pi}{4}\right)$ is $-\sqrt{2}$.

$$\text{Let } \sec^{-1}(-\sqrt{2}) = y$$

$$\Rightarrow \sec y = -\sqrt{2}$$

$$= -\sec\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$= \sec\left(\pi - \frac{\pi}{4}\right)$$

$$= \sec\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{3\pi}{4}\right) = -\sqrt{2}.$$

Therefore, the value of $\sec^{-1}\sec\left(\frac{-13\pi}{4}\right)$ is $\frac{3\pi}{4}$.

4 H. Question

Evaluate each of the following:

$$\sec^{-1}\left(\sec\frac{25\pi}{6}\right)$$

Answer

$$\sec\left(\frac{25\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

$\therefore$ The question converts to $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Now,

$$\text{Let } \sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = z$$

$$\Rightarrow \sec z = \left(\frac{2}{\sqrt{3}}\right)$$

$$= \sec\left(\frac{\pi}{6}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

The range of principal value of $\sec^{-1}$ is $[0, \pi] - \{\frac{\pi}{2}\}$

$$\text{and } \sec\left(\frac{\pi}{3}\right) = \left(\frac{2}{\sqrt{3}}\right)$$

Therefore, the value of $\sec^{-1}\sec\left(\frac{25\pi}{6}\right)$ is $\frac{\pi}{3}$.

5 A. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{\pi}{4}\right)$$

Answer

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$$

$$\text{Provided } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$\therefore \text{ we can write } \operatorname{cosec}^{-1}(\operatorname{cosec}\left(\frac{\pi}{4}\right)) = \frac{\pi}{4}.$$

5 B. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{3\pi}{4}\right)$$

Answer

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$$

$$\text{Provided } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$\therefore \text{ we can write } \operatorname{cosec}^{-1}(\operatorname{cosec}\left(\frac{3\pi}{4}\right)) = \frac{3\pi}{4}.$$

5 C. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{6\pi}{5}\right)$$

Answer

$$\operatorname{cosec}\left(\frac{6\pi}{5}\right) \text{ can be written as } \operatorname{cosec}\left(\pi + \frac{\pi}{5}\right)$$

$$\operatorname{cosec}\left(\pi + \frac{\pi}{5}\right) = -\operatorname{cosec}\left(\frac{\pi}{5}\right)$$

Also,

$$-\operatorname{cosec}(\theta) = \operatorname{cosec}(-\theta)$$

$$\Rightarrow -\operatorname{cosec}\left(\frac{\pi}{5}\right) = \operatorname{cosec}\left(\frac{-\pi}{5}\right)$$

Now the question becomes $\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{-\pi}{5}))$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$$

$$\text{Provided } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$\therefore \text{ we can write } \operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{-\pi}{5})) = \frac{-\pi}{5}.$$

5 D. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{11\pi}{6}\right)$$

Answer

$$\text{The value of } \operatorname{cosec}\left(\frac{11\pi}{6}\right) = -2.$$

Let,

$$\operatorname{cosec}^{-1}(-2) = y$$

$$\Rightarrow \operatorname{cosec} y = -2$$

$$\Rightarrow -\operatorname{cosec} y = 2$$

$$\Rightarrow -\operatorname{cosec} \frac{\pi}{6} = 2$$

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore -\operatorname{cosec} \frac{\pi}{6} = \operatorname{cosec}\left(\frac{-\pi}{6}\right)$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$$\operatorname{cosec}\left(\frac{-\pi}{6}\right) = -2$$

Therefore, the value of $\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{11\pi}{6}))$ is $\frac{-\pi}{6}$.

5 E. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left(\operatorname{cosec}\frac{13\pi}{6}\right)$$

Answer

$$\text{The value of } \operatorname{cosec}\left(\frac{13\pi}{6}\right) \text{ is } 2.$$

$\therefore$ The question becomes $\operatorname{cosec}^{-1}(2)$

Let,

$$\operatorname{cosec}^{-1}(2) = y$$

$$\therefore \operatorname{cosec} y = 2$$

$$\Rightarrow \operatorname{cosec}\left(\frac{\pi}{6}\right) = 2$$

The range of principal value of $\operatorname{cosec}^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ and

$$\operatorname{cosec}\left(\frac{\pi}{6}\right) = 2$$

Therefore, the value of $\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{13\pi}{6}))$ is $\frac{\pi}{6}$.

5 F. Question

Evaluate each of the following:

$$\operatorname{cosec}^{-1}\left\{\operatorname{cosec}\left(-\frac{9\pi}{4}\right)\right\}$$

Answer

As we know $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

$$\therefore \operatorname{cosec}\left(-\frac{9\pi}{4}\right) = -\operatorname{cosec}\left(\frac{9\pi}{4}\right)$$

$$-\operatorname{cosec}\left(\frac{9\pi}{4}\right) \text{ can be written as } -\operatorname{cosec}\left(2\pi + \frac{\pi}{4}\right)$$

Also,

$$\operatorname{cosec}(2\pi + \theta) = \operatorname{cosec}\theta$$

$$\therefore -\operatorname{cosec}\left(2\pi + \frac{\pi}{4}\right) = -\operatorname{cosec}\left(\frac{\pi}{4}\right)$$

As we know $-\operatorname{cosec}(\theta) = \operatorname{cosec}(-\theta)$

$$\therefore -\operatorname{cosec}\left(\frac{\pi}{4}\right) = \operatorname{cosec}\left(-\frac{\pi}{4}\right)$$

Now the question becomes $\operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{-\pi}{4}))$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x$$

Provided $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

$$\therefore \text{we can write } \operatorname{cosec}^{-1}(\operatorname{cosec}(\frac{-\pi}{4})) = \frac{-\pi}{4}.$$

6 A. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot \frac{\pi}{3}\right)$$

Answer

$$\cot^{-1}(\cot x) = x$$

Provided $x \in (0, \pi)$

$$\therefore \cot^{-1}(\cot \frac{\pi}{3}) = \frac{\pi}{3}.$$

6 B. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{4\pi}{3}\right)$$

Answer

$$\cot\frac{4\pi}{3} \text{ can be written as } \cot\left(\pi + \frac{\pi}{3}\right)$$

$$\text{we know } \cot(\pi + \theta) = \cot(\theta)$$

$$\therefore \cot\left(\pi + \frac{\pi}{3}\right) = \cot\left(\frac{\pi}{3}\right)$$

$$\text{Now the question becomes } \cot^{-1}\left(\cot\frac{\pi}{3}\right)$$

$$\cot^{-1}(\cot x) = x$$

$$\text{Provided } x \in (0, \pi)$$

$$\therefore \cot^{-1}\left(\cot\frac{\pi}{3}\right) = \frac{\pi}{3}.$$

6 C. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{9\pi}{4}\right)$$

Answer

$$\text{The value of } \cot\frac{9\pi}{4} \text{ is } 1.$$

$$\therefore \text{The question becomes } \cot^{-1}(1).$$

$$\text{Let } \cot^{-1}(1) = y$$

$$\Rightarrow \cot y = 1$$

$$= \cot\left(\frac{\pi}{4}\right) = 1$$

$$\text{The range of principal value of } \cot^{-1} \text{ is } (0, \pi)$$

$$\text{and } \cot\left(\frac{\pi}{4}\right) = 1$$

$$\therefore \text{The value of } \cot^{-1}\left(\cot\frac{9\pi}{4}\right) \text{ is } \frac{\pi}{4}.$$

6 D. Question

Evaluate each of the following:

$$\cot^{-1}\left(\cot\frac{19\pi}{6}\right)$$

Answer

$$\text{The value of } \cot\frac{19\pi}{6} \text{ is } \sqrt{3}.$$

$$\therefore \text{The question becomes } \cot^{-1}(\sqrt{3}).$$

$$\text{Let } \cot^{-1}(\sqrt{3}) = y$$

$$\Rightarrow \cot y = \sqrt{3}$$

$$= \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$\therefore$ The principal value of $\cot^{-1}(\cot\frac{19\pi}{6})$ is $\frac{\pi}{6}$.

6 E. Question

Evaluate each of the following:

$$\cot^{-1}\left\{\cot\left(-\frac{8\pi}{3}\right)\right\}$$

Answer

$\cot(-\theta)$ is $-\cot(\theta)$

$\therefore$ The equation given above becomes $\cot^{-1}(-\cot\frac{8\pi}{3})$

$$\cot\frac{8\pi}{3} = \frac{-1}{\sqrt{3}}.$$

Therefore

$$\text{Let } \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = y$$

$$\Rightarrow \cot y = \frac{1}{\sqrt{3}}$$

$$= \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

$\therefore$ The value of $\cot^{-1}(\cot\frac{-8\pi}{3})$ is $\frac{\pi}{3}$.

6 F. Question

Evaluate each of the following:

$$\cot^{-1}\left\{\cot\left(-\frac{21\pi}{4}\right)\right\}$$

Answer

$\cot(-\theta)$ is $-\cot(\theta)$

$\therefore$ The equation given above becomes $\cot^{-1}(-\cot\frac{21\pi}{4})$

$$\cot\frac{21\pi}{4} = 1.$$

$$\Rightarrow -\cot\frac{21\pi}{4} = -1.$$

$\therefore$ we get $\cot^{-1}(-1)$

$$\text{Let } \cot^{-1}(-1) = y$$

$$\Rightarrow \cot y = -1$$

$$= -\cot\left(\frac{\pi}{4}\right) = 1$$

$$= \cot\left(\pi - \frac{\pi}{4}\right)$$

$$= \cot\left(\frac{3\pi}{4}\right)$$

The range of principal value of $\cot^{-1}$ is $(0, \pi)$

$$\text{and } \cot\left(\frac{3\pi}{4}\right) = -1$$

$\therefore$ The value of $\cot^{-1}(\cot\frac{-21\pi}{4})$ is $\frac{3\pi}{4}$.

7 A. Question

Write each of the following in the simplest form:

$$\cot^{-1}\left\{\frac{a}{\sqrt{x^2 - a^2}}\right\}, |x| > a$$

Answer

Let us assume $x = a \sec\theta$

$$\theta = \sec^{-1}\frac{x}{a} \dots (1)$$

$\therefore$ we can write

$$\cot^{-1}\left\{\frac{a}{\sqrt{a^2 \sec^2 \theta - a^2}}\right\}$$

$$= \cot^{-1}\left\{\frac{a}{\sqrt{a^2 (\sec^2 \theta - 1)}}\right\}$$

$$= \cot^{-1}\left\{\frac{a}{\sqrt{a^2 \tan^2 \theta}}\right\}$$

$$= \cot^{-1}\left\{\frac{a}{a \tan \theta}\right\}$$

$$= \cot^{-1}\left\{\frac{1}{\tan \theta}\right\}$$

$$= \cot^{-1}(\cot \theta)$$

$$= \theta.$$

From 1 we get the given equation simplification to $\sec^{-1}\frac{x}{a}$.

7 B. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{x + \sqrt{1 + x^2}\right\}, x \in \mathbb{R}$$

Answer

Put $x = \tan \theta$

$$\Rightarrow \theta = \tan^{-1}(x)$$

$$\tan^{-1}\{\tan \theta + \sqrt{1 + \tan^2 \theta}\}$$

$$= \tan^{-1}\{\tan\theta + \sqrt{\sec^2\theta}\}$$

$$= \tan^{-1}\{\tan\theta + \sec\theta\}$$

$$= \tan^{-1}\left\{\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1+\sin\theta}{\cos\theta}\right\}$$

$$\sin\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}, \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$= \tan^{-1}\left\{\frac{2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2} + \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)^2}{\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) \times \left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)}\right\}$$

$$= \tan^{-1}\left\{\frac{\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}{\left(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right)}\right\}$$

Dividing by $\cos\frac{\theta}{2}$ we get,

$$= \tan^{-1}\left\{\frac{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} - \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}\right\}$$

$$= \tan^{-1}\left(\frac{1 + \tan\frac{\theta}{2}}{1 - \tan\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} + \tan\frac{\theta}{2}}{1 - \tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right)$$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{4} + \frac{\theta}{2}$$

From 1 we get

$$= \frac{\pi}{4} + \frac{\tan^{-1}x}{2}$$

Therefore, the simplification of given equation is $\frac{\pi}{4} + \frac{\tan^{-1}x}{2}$.

7 C. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{\sqrt{1+x^2} - x\right\}, x \in \mathbb{R}$$

Answer

Put $x = \tan\theta$

$$\Rightarrow \theta = \tan^{-1}(x)$$

$$\tan^{-1}\{\sqrt{1+\tan^2\theta} - \tan\theta\}$$

$$= \tan^{-1}\{\sqrt{\sec^2\theta} - \tan\theta\}$$

$$= \tan^{-1}\{\sec\theta - \tan\theta\}$$

$$= \tan^{-1}\left\{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right\}$$

$$= \tan^{-1}\left\{\frac{1-\sin\theta}{\cos\theta}\right\}$$

$$\sin\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}, \cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$= \tan^{-1}\left\{\frac{\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} - 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}}\right\}$$

$$= \tan^{-1}\left\{\frac{(\sin\frac{\theta}{2} - \cos\frac{\theta}{2})^2}{(\cos\frac{\theta}{2} - \sin\frac{\theta}{2}) \times (\cos\frac{\theta}{2} + \sin\frac{\theta}{2})}\right\}$$

$$= \tan^{-1}\left\{\frac{(\sin\frac{\theta}{2} - \cos\frac{\theta}{2})}{(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})}\right\}$$

Dividing by $\cos\frac{\theta}{2}$ we get

$$= \tan^{-1}\left\{\frac{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} - \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}{\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)}\right\}$$

$$= \tan^{-1}\left(\frac{1 - \tan\frac{\theta}{2}}{1 + \tan\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\frac{\theta}{2}}{1 + \tan\frac{\pi}{4} \tan\frac{\theta}{2}}\right)$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)\right)$$

$$= \frac{\pi}{4} - \frac{\theta}{2}$$

From 1 we get

$$= \frac{\pi}{4} - \frac{\tan^{-1}x}{2}$$

Therefore, the simplification of given equation is $\frac{\pi}{4} - \frac{\tan^{-1}x}{2}$.

7 D. Question

Write each of the following in the simplest form:

$$\tan^{-1}\left\{\frac{\sqrt{1+x^2}-1}{x}\right\}, x \neq 0$$

Answer

Assume $x = \tan\theta$

$$= \tan^{-1} \left\{ \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sqrt{\sec^2\theta}-1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sec\theta-1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}} \right\}$$

$$= \tan^{-1} \left\{ \frac{1-\cos\theta}{\sin\theta} \right\}$$

$$\cos\theta = 1 - 2\sin^2\frac{\theta}{2} \text{ and } \sin\theta = 2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}$$

$$\Rightarrow 1 - \cos\theta = 2\sin^2\frac{\theta}{2}$$

$$= \tan^{-1} \left\{ \frac{2\sin^2\frac{\theta}{2}}{2 \times \sin\frac{\theta}{2} \times \cos\frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} \right\}$$

$$= \tan^{-1}(\tan\frac{\theta}{2})$$

$$= \frac{\theta}{2}$$

But $\theta = \tan^{-1}x$.

$$\therefore \frac{\theta}{2} = \frac{\tan^{-1}x}{2}$$

Therefore, the simplification of given equation is $\frac{\tan^{-1}x}{2}$

7 E. Question

Write each of the following in the simplest form:

$$\tan^{-1} \left\{ \frac{\sqrt{1+x^2}+1}{x} \right\}, x \neq 0$$

Answer

Assume $x = \tan\theta$

$$= \tan^{-1} \left\{ \frac{\sqrt{1+\tan^2\theta}+1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sqrt{\sec^2\theta}+1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sec\theta+1}{\tan\theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\frac{1}{\cos\theta}+1}{\frac{\sin\theta}{\cos\theta}} \right\}$$

$$= \tan^{-1} \left\{ \frac{1+\cos\theta}{\sin\theta} \right\}$$

$$\cos \theta = 2 \cos^2 \frac{\theta}{2} - 1 \text{ and } \sin \theta = 2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}$$

$$\Rightarrow 1 + \cos \theta = 2 \cos^2 \frac{\theta}{2}$$

$$= \tan^{-1} \left\{ \frac{2 \cos^2 \frac{\theta}{2}}{2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left\{ \frac{\cos \frac{\theta}{2}}{\sin \frac{\theta}{2}} \right\}$$

$$= \tan^{-1}(\cot \frac{\theta}{2})$$

$$\cot \left(\frac{\pi}{2} - \frac{\theta}{2} \right) = \tan \left(-\frac{\theta}{2} \right)$$

$$= \tan^{-1}(\tan \frac{-\theta}{2})$$

$$\text{But } \theta = \tan^{-1} x.$$

$$\therefore \frac{-\theta}{2} = \frac{-\tan^{-1} x}{2}$$

Therefore, the simplification of given equation is $\frac{-\tan^{-1} x}{2}$

7 F. Question

Write each of the following in the simplest form:

$$\tan^{-1} \sqrt{\frac{a-x}{a+x}}, -a < x < a$$

Answer

$$\text{Put } x = a \cos \theta$$

$$\Rightarrow \tan^{-1} \sqrt{\frac{a-a \cos \theta}{a+a \cos \theta}}$$

$$\Rightarrow \tan^{-1} \sqrt{\frac{a(1-\cos \theta)}{a(1+\cos \theta)}}$$

$$\Rightarrow \tan^{-1} \sqrt{\frac{(1-\cos \theta)}{(1+\cos \theta)}}$$

Rationalising it

$$\tan^{-1} \sqrt{\frac{(1-\cos \theta)}{(1+\cos \theta)}} \times \sqrt{\frac{(1-\cos \theta)}{(1-\cos \theta)}}$$

$$\Rightarrow \tan^{-1} \sqrt{\frac{(1-\cos \theta)^2}{(1-\cos^2 \theta)}}$$

$$\Rightarrow \tan^{-1} \sqrt{\frac{(1-\cos \theta)^2}{\sin^2 \theta}}$$

$$\Rightarrow \tan^{-1} \left(\frac{1-\cos \theta}{\sin \theta} \right)$$

$$\cos \theta = 1 - 2 \sin^2 \frac{\theta}{2} \text{ and } \sin \theta = 2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}$$

$$\Rightarrow 1 - \cos \theta = 2 \sin^2 \frac{\theta}{2}$$

$$= \tan^{-1} \left\{ \frac{2 \sin^2 \frac{\theta}{2}}{2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

$$= \frac{\theta}{2}$$

$$\text{But } \theta = \cos^{-1} \frac{x}{a}$$

$\therefore$ The given equation simplification to $\cos^{-1} \frac{x}{a}$.

7 G. Question

Write each of the following in the simplest form:

$$\tan^{-1} \left\{ \frac{x}{a + \sqrt{a^2 - x^2}} \right\}, -a < x < a$$

Answer

Assume $x = a \sin \theta$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 - a^2 \sin^2 \theta}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 (1 - \sin^2 \theta)}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + \sqrt{a^2 (\cos^2 \theta)}} \right\}$$

$$= \tan^{-1} \left\{ \frac{a \sin \theta}{a + a \cos \theta} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sin \theta}{1 + \cos \theta} \right\}$$

$$\cos \theta = \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2} \text{ and } \sin \theta = 2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}, \cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} = 1$$

$$= \tan^{-1} \left\{ \frac{2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}}{\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2} + \cos^2 \frac{\theta}{2} - \sin^2 \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left\{ \frac{2 \times \sin \frac{\theta}{2} \times \cos \frac{\theta}{2}}{2 \cos^2 \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left\{ \frac{\sin \frac{\theta}{2}}{\cos \frac{\theta}{2}} \right\}$$

$$= \tan^{-1} \left(\tan \frac{\theta}{2} \right)$$

$$= \frac{\theta}{2}$$

$$\text{But } \theta = \sin^{-1} \left(\frac{x}{a} \right)$$

$\therefore$ The given equation simplification to $\sin^{-1} \left(\frac{x}{a} \right)$.

7 H. Question

Write each of the following in the simplest form:

$$\sin^{-1} \left\{ \frac{x + \sqrt{1-x^2}}{\sqrt{2}} \right\}, \frac{1}{2} < x < \frac{1}{\sqrt{2}}$$

Answer

Assume $x = \sin \theta$

$$= \sin^{-1} \left\{ \frac{\sin \theta + \sqrt{1-\sin^2 \theta}}{\sqrt{2}} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sin \theta + \cos \theta}{\sqrt{2}} \right\}$$

$$= \sin^{-1} \left\{ \frac{1}{\sqrt{2}} \sin \theta + \frac{1}{\sqrt{2}} \cos \theta \right\}$$

$$= \sin^{-1} \left\{ \cos \frac{\pi}{4} \sin \theta + \sin \frac{\pi}{4} \cos \theta \right\}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$\therefore$ The above expression can be written as

$$= \sin^{-1} \left\{ \sin \frac{\pi}{4} + \theta \right\}$$

$$= \frac{\pi}{4} + \theta$$

$$\text{But } \theta = \sin^{-1} x$$

$\therefore$ the above expression becomes $\frac{\pi}{4} + \sin^{-1} x$.

The given equation simplification to $\frac{\pi}{4} + \sin^{-1} x$.

7 I. Question

Write each of the following in the simplest form:

$$\sin^{-1} \left\{ \frac{\sqrt{1+x} + \sqrt{1-x}}{2} \right\}, 0 < x < 1$$

Answer

Put $x = \sin 2\theta$

And we know $\sin^2 \theta + \cos^2 \theta = 1$

By putting these in above equation, we get

$$= \sin^{-1} \left\{ \frac{\sqrt{\sin^2 \theta + \cos^2 \theta + \sin 2\theta} + \sqrt{\sin^2 \theta + \cos^2 \theta - \sin 2\theta}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sqrt{(\sin \theta + \cos \theta)^2} + \sqrt{(\sin \theta - \cos \theta)^2}}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{\sin \theta + \cos \theta + \sin \theta - \cos \theta}{2} \right\}$$

$$= \sin^{-1} \left\{ \frac{2 \sin \theta}{2} \right\}$$

$$= \sin^{-1}(\sin \theta)$$

$$= \theta$$

$$\text{But } \theta = \frac{1}{2}\sin^{-1}x$$

$\therefore$ The given equation simplification to $\frac{1}{2}\sin^{-1}x$.

7 J. Question

Write each of the following in the simplest form:

$$\sin^{-1}\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$$

Answer

$$\text{Put } x = \cos \theta$$

$$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right)$$

$$1 - \cos\theta = 2 \sin^2\frac{\theta}{2} \text{ and } 1 + \cos\theta = 2 \cos^2\frac{\theta}{2}$$

$$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\frac{2 \sin^2\frac{\theta}{2}}{2 \cos^2\frac{\theta}{2}}}\right)\right)$$

$$= \sin^{-1}\left(2\tan^{-1}\left(\sqrt{\tan^2\frac{\theta}{2}}\right)\right)$$

$$= \sin^{-1}\left(2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right)$$

$$= \sin^{-1}\left(2 \times \frac{\theta}{2}\right)$$

$$= \sin^{-1}(\theta)$$

$$\text{But } \theta = \cos^{-1}x$$

$\therefore$ The above expression becomes $\sin^{-1}(\cos^{-1}x)$

Exercise 4.8

1 A. Question

Evaluate each of the following

$$\sin\left(\sin^{-1}\frac{7}{25}\right)$$

Answer

$$\text{Let } \sin^{-1}\frac{7}{25} = y$$

$$\Rightarrow \sin y = \frac{7}{25}$$

$$\text{Where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin\left(\sin^{-1}\frac{7}{25}\right) = \frac{7}{25} \text{ substituting } y = \sin^{-1}\frac{7}{25}$$

1 B. Question

Evaluate each of the following

$$\sin\left(\cos^{-1}\frac{5}{13}\right)$$

Answer

$$\text{Let } \cos^{-1}\frac{5}{13} = y$$

$$\Rightarrow \cos y = \frac{5}{13} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\text{To find : } \sin\left(\cos^{-1}\frac{5}{13}\right) = \sin y$$

$$\text{As } \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \sin y = \pm\sqrt{1 - \cos^2 y}$$

$$\text{As } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = \sqrt{1 - \cos^2 y}$$

$$\Rightarrow \sin y = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \sin y = \sqrt{1 - \frac{25}{169}}$$

$$\Rightarrow \sin y = \sqrt{\frac{144}{169}}$$

$$\Rightarrow \sin y = \frac{12}{13}$$

$$\Rightarrow \sin\left[\cos^{-1}\left(\frac{5}{13}\right)\right] = \frac{12}{13}$$

1 C. Question

Evaluate each of the following

$$\sin\left(\tan^{-1}\frac{24}{7}\right)$$

Answer

$$\text{Let } \tan^{-1} \frac{24}{7} = y$$

$$\Rightarrow \tan y = \frac{24}{7} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\text{To find : } \sin\left(\tan^{-1} \frac{24}{7}\right) = \sin y$$

$$\text{As } 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$\Rightarrow 1 + \cot^2 y = \operatorname{cosec}^2 y$$

Putting values

$$\Rightarrow 1 + \left(\frac{7}{24}\right)^2 = \operatorname{cosec}^2 y$$

$$\Rightarrow 1 + \frac{49}{576} = \frac{1}{\sin^2 y}$$

$$\Rightarrow \sin^2 y = \frac{576}{625}$$

$$\Rightarrow \sin y = \frac{24}{25} \because y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin\left(\tan^{-1} \frac{24}{7}\right) = \frac{24}{25}$$

1 D. Question

Evaluate each of the following

$$\sin\left(\sec^{-1} \frac{17}{8}\right)$$

Answer

$$\text{Let } \sec^{-1} \frac{17}{8} = y$$

$$\Rightarrow \sec y = \frac{17}{8} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\text{To find : } \sin\left(\sec^{-1} \frac{17}{8}\right) = \sin y$$

$$\text{Now, } \cos y = \frac{1}{\sec y}$$

$$\Rightarrow \cos y = \frac{8}{17}$$

$$\text{Now, } \sin y = \sqrt{1 - \cos^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = \sqrt{1 - \left(\frac{8}{17}\right)^2}$$

$$\Rightarrow \sin y = \sqrt{\frac{225}{289}}$$

$$\Rightarrow \sin y = \frac{15}{17}$$

$$\Rightarrow \sin\left(\sec^{-1}\frac{17}{8}\right) = \frac{15}{17}$$

1 E. Question

Evaluate each of the following

$$\operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right)$$

Answer

$$\text{Let } \cos^{-1}\frac{3}{5} = y$$

$$\Rightarrow \cos y = \frac{3}{5} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\text{To find: } \operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \operatorname{cosec} y$$

$$\text{As } \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \sin y = \sqrt{1 - \cos^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin y = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \sin y = \frac{4}{5}$$

$$\Rightarrow \operatorname{cosec} y = \frac{5}{4}$$

$$\Rightarrow \operatorname{cosec}\left(\cos^{-1}\frac{3}{5}\right) = \frac{5}{4}$$

1 F. Question

Evaluate each of the following

$$\sec\left(\sin^{-1}\frac{12}{13}\right)$$

Answer

$$\text{Let } \sin^{-1}\frac{12}{13} = y \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = \frac{12}{13}$$

$$\Rightarrow \text{To find : } \sec\left(\sin^{-1}\frac{12}{13}\right) = \sec y$$

$$\text{As } \sin^2\theta + \cos^2\theta = 1$$

$$\Rightarrow \cos y = \sqrt{1 - \sin^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \cos y = \sqrt{1 - \left(\frac{12}{13}\right)^2}$$

$$\Rightarrow \cos y = \sqrt{1 - \frac{144}{169}}$$

$$\Rightarrow \cos y = \sqrt{\frac{25}{169}}$$

$$\Rightarrow \cos y = \frac{5}{13}$$

$$\Rightarrow \sec y = \frac{1}{\cos y}$$

$$\Rightarrow \sec y = \frac{13}{5}$$

$$\Rightarrow \sec\left(\sin^{-1}\frac{12}{13}\right) = \frac{13}{5}$$

1 G. Question

Evaluate each of the following

$$\tan\left(\cos^{-1}\frac{8}{17}\right)$$

Answer

$$\text{Let } \cos^{-1}\frac{8}{17} = y \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \cos y = \frac{8}{17}$$

To find: $\tan\left(\cos^{-1}\frac{8}{17}\right) = \tan y$

$\Rightarrow$ As $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \tan y = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$\Rightarrow \tan y = \sqrt{\left(\frac{17}{8}\right)^2 - 1}$

$\Rightarrow \tan y = \sqrt{\frac{289}{64} - 1}$

$\Rightarrow \tan y = \sqrt{\frac{225}{64}}$

$\Rightarrow \tan y = \frac{15}{8}$

$\Rightarrow \tan\left(\cos^{-1}\frac{8}{17}\right) = \frac{15}{8}$

1 H. Question

Evaluate each of the following

$\cot\left(\cos^{-1}\frac{3}{5}\right)$

Answer

Let $\cos^{-1}\frac{3}{5} = y$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \cos y = \frac{3}{5}$

To find: $\cot\left(\cos^{-1}\frac{3}{5}\right) = \cot y$

$\Rightarrow$ As $1 + \tan^2\theta = \sec^2\theta$

$\Rightarrow \tan y = \sqrt{\sec^2 y - 1}$ where $y \in \left[0, \frac{\pi}{2}\right]$

$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{1}{\cos^2 y}\right) - 1}$

$$\Rightarrow \frac{1}{\cot y} = \sqrt{\left(\frac{5}{3}\right)^2 - 1}$$

$$\Rightarrow \frac{1}{\cot y} = \sqrt{\frac{16}{9}}$$

$$\Rightarrow \cot y = \frac{3}{4}$$

$$\Rightarrow \cot\left(\cos^{-1}\frac{3}{5}\right) = \frac{3}{4}$$

1 I. Question

Evaluate each of the following

$$\cos\left(\tan^{-1}\frac{24}{7}\right)$$

Answer

$$\text{Let } \tan^{-1}\frac{24}{7} = y$$

$$\Rightarrow \tan y = \frac{24}{7} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\text{To find: } \cos\left(\tan^{-1}\frac{24}{7}\right) = \cos y$$

$$\text{As } 1 + \tan^2 \theta = \sec^2 \theta$$

$$\Rightarrow 1 + \tan^2 y = \sec^2 y$$

$$\Rightarrow \sec y = \sqrt{1 + \tan^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sec y = \sqrt{1 + \left(\frac{24}{7}\right)^2}$$

$$\Rightarrow \sec y = \sqrt{\frac{625}{49}}$$

$$\Rightarrow \sec y = \frac{25}{7}$$

$$\Rightarrow \cos y = \frac{1}{\sec y}$$

$$\Rightarrow \cos y = \frac{7}{25}$$

$$\Rightarrow \cos\left(\tan^{-1}\frac{24}{7}\right) = \frac{7}{25}$$

2 A. Question

Prove the following results:

$$\tan\left(\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{2}{3}\right) = \frac{17}{6}$$

Answer

$$\text{Let } \cos^{-1}\frac{4}{5} = x \text{ and } \tan^{-1}\frac{2}{3} = y$$

$$\Rightarrow \cos x = \frac{4}{5} \text{ and } \tan y = \frac{2}{3}$$

$$\text{where } x, y \in \left[0, \frac{\pi}{2}\right]$$

Now, LHS is reduced to : $\tan(x+y)$

$$\Rightarrow \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} \text{ ..eq (i)..}$$

$$\text{As } \tan x = \sqrt{\sec^2 x - 1} \text{ where } x \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \tan x = \sqrt{\frac{1}{\cos^2 x} - 1}$$

$$\Rightarrow \tan x = \sqrt{\left(\frac{5}{4}\right)^2 - 1}$$

$$\Rightarrow \tan x = \sqrt{\frac{9}{16}}$$

$$\Rightarrow \tan x = \frac{3}{4}$$

Now putting the values of $\tan x$ and $\tan y$ in eq(i)

$$\Rightarrow \tan(x+y) = \left(\frac{\frac{3}{4} + \frac{2}{3}}{1 - \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)} \right)$$

$$\Rightarrow \tan(x+y) = \frac{17}{6}$$

= RHS

2 B. Question

Prove the following results:

$$\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$$

Answer

$$\text{Let } \sin^{-1} \frac{3}{5} = x \text{ and } \cot^{-1} \frac{3}{2} = y$$

$$\Rightarrow \sin x = \frac{3}{5} \text{ and } \cot y = \frac{3}{2}$$

$$\text{where } x, y \in \left[0, \frac{\pi}{2} \right]$$

Now, LHS is reduced to : $\cos(x+y)$

$$\Rightarrow \cos(x+y) = \cos x \cos y - \sin x \sin y \text{ ..eq(i)}$$

$$\text{As } \cos x = \sqrt{1 - \sin^2 x} \text{ where } x \in \left[0, \frac{\pi}{2} \right]$$

$$\Rightarrow \cos x = \sqrt{1 - \left(\frac{3}{5} \right)^2}$$

$$\Rightarrow \cos x = \sqrt{1 - \frac{9}{25}}$$

$$\Rightarrow \cos x = \frac{4}{5}$$

$$\text{Also, } \operatorname{cosec} y = \sqrt{1 + \cot^2 y} \text{ where } y \in \left[0, \frac{\pi}{2} \right]$$

$$\Rightarrow \operatorname{cosec} y = \sqrt{1 + \left(\frac{3}{2} \right)^2}$$

$$\Rightarrow \operatorname{cosec} y = \frac{\sqrt{13}}{2}$$

$$\Rightarrow \sin y = \frac{1}{\operatorname{cosec} y}$$

$$\Rightarrow \sin y = \frac{2}{\sqrt{13}}$$

$$\Rightarrow \cos y = \sqrt{1 - \sin^2 y} \text{ where } y \in \left[0, \frac{\pi}{2} \right]$$

$$\Rightarrow \cos y = \sqrt{1 - \left(\frac{2}{\sqrt{13}} \right)^2}$$

$$\Rightarrow \cos y = \frac{3}{\sqrt{13}}$$

Putting the values in eq(i),

$$\Rightarrow \cos(x+y) = \left(\frac{4}{5}\right)\left(\frac{3}{\sqrt{13}}\right) - \left(\frac{3}{5}\right)\left(\frac{2}{\sqrt{13}}\right)$$

$$\Rightarrow \cos(x+y) = \frac{6}{5\sqrt{13}}$$

= RHS

2 C. Question

Prove the following results:

$$\tan\left(\sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}\right) = \frac{63}{16}$$

Answer

$$\text{Let } \sin^{-1}\frac{5}{13} = x \text{ and } \cos^{-1}\frac{3}{5} = y$$

$$\Rightarrow \sin x = \frac{5}{13} \text{ and } \cos y = \frac{3}{5}$$

$$\text{where } x, y \in \left[0, \frac{\pi}{2}\right]$$

Now, LHS is reduced to : $\tan(x+y)$

$$\Rightarrow \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \cdot \tan y} \text{ ..eq(i)}$$

$$\text{As } \cos x = \sqrt{1 - \sin^2 x} \text{ where } x \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \cos x = \sqrt{1 - \left(\frac{5}{13}\right)^2}$$

$$\Rightarrow \cos x = \frac{12}{13}$$

Similarly,

$$\sin y = \sqrt{1 - \cos^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = \frac{4}{5}$$

$$\tan x = \frac{\sin x}{\cos x} \text{ and } \tan y = \frac{\sin y}{\cos y}$$

$$\Rightarrow \tan x = \frac{5}{12} \text{ and } \tan y = \frac{4}{3}$$

Putting these values in eq(i)

$$\Rightarrow \tan(x+y) = \frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \cdot \frac{4}{3}}$$

$$\Rightarrow \tan(x+y) = \frac{63}{16}$$

= RHS

2 D. Question

Prove the following results:

$$\sin\left(\cos^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13}\right) = \frac{63}{65}$$

Answer

$$\text{Let } \cos^{-1}\frac{3}{5} = x \text{ and } \sin^{-1}\frac{5}{13} = y$$

$$\Rightarrow \cos x = \frac{3}{5} \text{ and } \sin y = \frac{5}{13}$$

$$\text{where } x, y \in \left[0, \frac{\pi}{2}\right]$$

Now, LHS is reduced to : $\sin(x+y)$

$$\Rightarrow \sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y \text{ ..eq(i)..}$$

$$\text{As } \sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin x = \sqrt{1 - \cos^2 x} \text{ where } x \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin x = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin x = \frac{4}{5}$$

Similarly,

$$\cos y = \sqrt{1 - \sin^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \cos y = \frac{12}{13}$$

Putting these values in eq(i)

$$\Rightarrow \sin(x+y) = \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13}$$

$$\Rightarrow \sin(x+y) = \frac{63}{65}$$

= RHS

3. Question

$$\text{Solve: } \cos(\sin^{-1} x) = \frac{1}{6}$$

Answer

A. Let $\sin^{-1} x = y$

Where $y \in \left[0, \frac{\pi}{2}\right]$ because "cos y" is +ve

$$\Rightarrow \sin y = x$$

where "x" is +ve as $y \in \left[0, \frac{\pi}{2}\right]$

$$\text{As } \sin^2 y + \cos^2 y = 1$$

$$\Rightarrow \cos y = \sqrt{1 - \sin^2 y} \text{ where } y \in \left[0, \frac{\pi}{2}\right]$$

$$\Rightarrow \cos y = \sqrt{1 - x^2}$$

$$\text{According to the question, } \cos(\sin^{-1} x) = \frac{1}{6}$$

$$\Rightarrow \cos y = \frac{1}{6}$$

$$\Rightarrow \sqrt{1 - x^2} = \frac{1}{6}$$

Squaring both sides,

$$\Rightarrow 1 - x^2 = \frac{1}{36}$$

$$\Rightarrow x^2 = \frac{35}{36}$$

As $x > 0$

$$\Rightarrow x = \frac{\sqrt{35}}{6}$$

4. Question

$$\text{Solve: } \cos[2 \sin^{-1}(-x)] = 0$$

Answer

$$\text{A. Let } \sin^{-1}(-x) = y \text{ where } y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\Rightarrow \sin y = -x$$

According to question

$$\Rightarrow \cos 2y = 0$$

$$\Rightarrow 1 - 2 \sin^2 y = 0$$

$$\Rightarrow 1 - 2x^2 = 0$$

$$\Rightarrow x^2 = \frac{1}{2}$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

Exercise 4.9

1 A. Question

Evaluate:

$$\cos \left[\sin^{-1} \left(-\frac{7}{25} \right) \right]$$

Answer

$$\text{Let } \sin^{-1} \left(-\frac{7}{25} \right) = x$$

$$\text{where } x \in \left[-\frac{\pi}{2}, 0 \right]$$

$$\Rightarrow \sin x = -\frac{7}{25}$$

$$\text{To find: } \cos \left[\sin^{-1} \left(-\frac{7}{25} \right) \right] = \cos x$$

$$\text{As } \sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos x = \sqrt{1 - \sin^2 x} \quad \because x \in \left[-\frac{\pi}{2}, 0 \right]$$

$$\Rightarrow \cos x = \sqrt{1 - \frac{49}{625}}$$

$$\Rightarrow \cos x = \sqrt{\frac{576}{625}}$$

$$\Rightarrow \cos x = \frac{24}{25}$$

$$\Rightarrow \cos \left[\sin^{-1} \left(-\frac{7}{25} \right) \right] = \frac{24}{25}$$

1 B. Question

Evaluate:

$$\sec \left[\cot^{-1} \left(-\frac{5}{12} \right) \right]$$

Answer

$$\text{Let } \cot^{-1} \left(-\frac{5}{12} \right) = x$$

$$\text{where } x \in \left(\frac{\pi}{2}, \pi \right)$$

$$\Rightarrow \cot x = -\frac{5}{12}$$

$$\text{To find: } \sec \left[\cot^{-1} \left(-\frac{5}{12} \right) \right] = \sec x$$

$$\text{As } 1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow 1 + \frac{1}{\cot^2 x} = \sec^2 x$$

$$\Rightarrow \sec x = -\sqrt{1 + \frac{1}{\cot^2 x}}$$

$$\text{As } x \in \left(\frac{\pi}{2}, \pi \right)$$

$$\Rightarrow \sec x = -\sqrt{1 + \left(\frac{12}{5} \right)^2}$$

$$\Rightarrow \sec x = -\frac{13}{5}$$

$$\Rightarrow \sec \left[\cot^{-1} \left(-\frac{5}{12} \right) \right] = -\frac{13}{5}$$

1 C. Question

Evaluate:

$$\cot \left[\sec^{-1} \left(-\frac{13}{5} \right) \right]$$

Answer

$$\text{Let } \sec^{-1} \left(-\frac{13}{5} \right) = x \text{ where } x \in \left(\frac{\pi}{2}, \pi \right)$$

$$\Rightarrow \sec x = -\frac{13}{5}$$

$$\text{To find: } \cot \left[\sec^{-1} \left(-\frac{13}{5} \right) \right] = \cot x$$

$$\text{As } 1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow \tan x = -\sqrt{\sec^2 x - 1}$$

$$\Rightarrow \tan x = -\sqrt{\left(-\frac{13}{5} \right)^2 - 1}$$

$$\Rightarrow \tan x = -\frac{12}{5}$$

$$\Rightarrow \cot x = -\frac{5}{12}$$

$$\Rightarrow \cot \left[\sec^{-1} \left(-\frac{13}{5} \right) \right] = -\frac{5}{12}$$

2 A. Question

Evaluate:

$$\tan \left[\cos^{-1} \left(-\frac{7}{25} \right) \right]$$

Answer

$$\text{Let } \cos^{-1} \left(-\frac{7}{25} \right) = x \text{ where } x \in \left(\frac{\pi}{2}, \pi \right)$$

$$\Rightarrow \cos x = -\frac{7}{25}$$

$$\text{To find: } \tan \left[\cos^{-1} \left(-\frac{7}{25} \right) \right] = \tan x$$

$$\text{As } 1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow \tan x = -\sqrt{\sec^2 x - 1} \text{ as } x \in \left(\frac{\pi}{2}, \pi \right)$$

$$\Rightarrow \tan x = -\sqrt{\frac{1}{\cos^2 x} - 1}$$

$$\Rightarrow \tan x = -\sqrt{\left(-\frac{25}{7} \right)^2 - 1}$$

$$\Rightarrow \tan x = -\frac{24}{7}$$

$$\Rightarrow \tan \left[\cos^{-1} \left(-\frac{7}{25} \right) \right] = -\frac{24}{7}$$

2 B. Question

Evaluate:

$$\operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right]$$

Answer

$$\text{Let } \cot^{-1}\left(-\frac{12}{5}\right) = x \text{ where } x \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \cot x = -\frac{12}{5}$$

$$\text{To find: } \operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right] = \operatorname{cosec} x$$

$$\text{As } 1 + \cot^2 x = \operatorname{cosec}^2 x$$

$$\Rightarrow \operatorname{cosec} x = \sqrt{1 + \cot^2 x} \text{ as } x \in \left(\frac{\pi}{2}, \pi\right)$$

$$\Rightarrow \operatorname{cosec} x = \sqrt{1 + \left(-\frac{12}{5}\right)^2}$$

$$\Rightarrow \operatorname{cosec} x = \frac{13}{5}$$

$$\Rightarrow \operatorname{cosec}\left[\cot^{-1}\left(-\frac{12}{5}\right)\right] = \frac{13}{5}$$

2 C. Question

Evaluate:

$$\cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right]$$

Answer

$$\text{Let } \tan^{-1}\left(-\frac{3}{4}\right) = x \text{ where } x \in \left[-\frac{\pi}{2}, 0\right]$$

$$\Rightarrow \tan x = -\frac{3}{4}$$

$$\text{To find : } \cos\left[\tan^{-1}\left(-\frac{3}{4}\right)\right] = \cos x$$

$$\text{As } 1 + \tan^2 x = \sec^2 x$$

$$\Rightarrow \sec x = \sqrt{1 + \tan^2 x} \text{ as } x \in \left[-\frac{\pi}{2}, 0\right]$$

$$\Rightarrow \sec x = \sqrt{1 + \left(-\frac{3}{4}\right)^2}$$

$$\Rightarrow \sec x = \frac{5}{4}$$

$$\Rightarrow \cos x = \frac{1}{\sec x}$$

$$\Rightarrow \cos x = \frac{4}{5}$$

$$\Rightarrow \cos \left[\tan^{-1} \left(-\frac{3}{4} \right) \right] = \frac{4}{5}$$

3. Question

Evaluate: $\sin \left[\cos^{-1} \left(-\frac{3}{5} \right) + \cot^{-1} \left(-\frac{5}{12} \right) \right]$.

Answer

A. Let $\cos^{-1} \left(-\frac{3}{5} \right) = x$ and $\cot^{-1} \left(-\frac{5}{12} \right) = y$

$$\Rightarrow \cos x = -\frac{3}{5} \text{ and } \cot y = -\frac{5}{12}$$

where $x, y \in \left[\frac{\pi}{2}, \pi \right]$

To find: $\sin \left[\cos^{-1} \left(-\frac{3}{5} \right) + \cot^{-1} \left(-\frac{5}{12} \right) \right] = \sin(x + y)$

$$\Rightarrow \sin(x + y) = \sin x \cdot \cos y + \cos x \cdot \sin y \text{ ..eq(i)}$$

As $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow \sin x = \sqrt{1 - \cos^2 x} \text{ as } x \in \left[\frac{\pi}{2}, \pi \right]$$

$$\Rightarrow \sin x = \sqrt{1 - \left(-\frac{3}{5}\right)^2}$$

$$\Rightarrow \sin x = \frac{4}{5}$$

Also, $1 + \cot^2 y = \operatorname{cosec}^2 y$

$$\Rightarrow \operatorname{cosec} y = \sqrt{1 + \cot^2 y}$$

$$\Rightarrow \operatorname{cosec} y = \sqrt{1 + \left(-\frac{5}{12}\right)^2}$$

$$\Rightarrow \operatorname{cosec} y = \frac{13}{12}$$

$$\Rightarrow \sin y = \frac{1}{\operatorname{cosec} y}$$

$$\Rightarrow \sin y = \frac{12}{13}$$

$$\Rightarrow \cos y = \cot y \cdot \sin y$$

$$\Rightarrow \cos y = -\frac{5}{12} \times \frac{12}{13} = -\frac{5}{13}$$

Putting these values in eq(i)

$$\Rightarrow \sin(x+y) = \frac{4}{5} \cdot \left(-\frac{5}{13}\right) + \left(-\frac{3}{5}\right) \cdot \frac{12}{13}$$

$$\Rightarrow \sin(x+y) = -\frac{56}{65}$$

$$\Rightarrow \sin\left[\cos^{-1}\left(-\frac{3}{5}\right) + \cot^{-1}\left(-\frac{5}{12}\right)\right] = -\frac{56}{65}$$

Exercise 4.10

1 A. Question

Evaluate:

$$\cot\left(\sin^{-1}\frac{3}{4} + \sec^{-1}\frac{4}{3}\right)$$

Answer

$$= \cot\left(\sin^{-1}\frac{3}{4} + \cos^{-1}\frac{3}{4}\right)$$

$$\left(\because \sec^{-1}x = \cos^{-1}\frac{1}{x}\right)$$

$$\text{We know, } \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

$$= \cot\frac{\pi}{2}$$

$$= 0$$

1 B. Question

Evaluate:

$$\sin\left(\tan^{-1}x + \tan^{-1}\frac{1}{x}\right) \quad x < 0$$

Answer

$$= \sin(\tan^{-1} x + (\cot^{-1} x - \pi))$$

$$\left(\because \tan^{-1} \theta = \cot^{-1} \frac{1}{\theta} - \pi \quad \text{for } x < 0 \right)$$

$$= \sin\left(\frac{\pi}{2} - \pi\right)$$

$$\left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2} \right)$$

$$= \sin\left(-\frac{\pi}{2}\right)$$

$$= -\sin \frac{\pi}{2} = -1$$

1 C. Question

Evaluate:

$$\sin\left(\tan^{-1} x + \tan^{-1} \frac{1}{x}\right) \quad x > 0$$

Answer

$$= \sin(\tan^{-1} x + \cot^{-1} x)$$

$$\left(\because \tan^{-1} \theta = \cot^{-1} \frac{1}{\theta} \quad \text{for } x > 0 \right)$$

$$= \sin \frac{\pi}{2}$$

$$\left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2} \right)$$

$$= 1$$

1 D. Question

Evaluate:

$$\cot(\tan^{-1} \alpha + \cot^{-1} \alpha)$$

Answer

$$= \cot\left(\frac{\pi}{2}\right)$$

$$\left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2} \right)$$

$$= 0$$

1 E. Question

Evaluate:

$$\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x) \quad |x| \geq 1$$

Answer

$$= \cos\left(\cos^{-1} \frac{1}{x} + \sin^{-1} \frac{1}{x}\right)$$

$$\left(\because \sec^{-1} \theta = \cos^{-1} \frac{1}{\theta} \quad \text{and} \quad \operatorname{cosec}^{-1} \theta = \sin^{-1} \frac{1}{\theta}\right)$$

$$= \cos \frac{\pi}{2}$$

$$\left(\because \sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}\right)$$

$$= 0$$

2. Question

2 If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$, then find the value of $\sin^{-1} x + \sin^{-1} y$.

Answer

$$\text{A. } \cos^{-1} x + \cos^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow \left(\frac{\pi}{2} - \sin^{-1} x\right) + \left(\frac{\pi}{2} - \sin^{-1} y\right) = \frac{\pi}{4}$$

$$\left(\because \sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}\right)$$

$$\Rightarrow \pi - (\sin^{-1} x + \sin^{-1} y) = \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \pi - \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{3\pi}{4}$$

3. Question

If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{3}$ and $\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6}$, then find x and y.

Answer

$$\text{A. } \sin^{-1} x + \sin^{-1} y = \frac{\pi}{3} \quad \dots \text{eq(i)}$$

$$\cos^{-1} x - \cos^{-1} y = \frac{\pi}{6} \quad \dots \text{eq(ii)}$$

Subtracting (ii) from (i)

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + (\sin^{-1} y + \cos^{-1} y) = \frac{\pi}{3} - \frac{\pi}{6}$$

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$

$$\left(\because \sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}\right)$$

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = -\frac{\pi}{3}$$

$$\Rightarrow 2 \cos^{-1} x = \frac{5\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{5\pi}{12}$$

$$\Rightarrow x = \cos\left(\frac{5\pi}{12}\right)$$

$$\Rightarrow x = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

$$\Rightarrow x = \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Now, putting the value of $\cos^{-1} x$ in eq(ii)

$$\Rightarrow \frac{5\pi}{12} - \cos^{-1} y = \frac{\pi}{6}$$

$$\Rightarrow \cos^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow y = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ and } y = \frac{1}{\sqrt{2}}$$

4. Question

If $\cot\left(\cos^{-1} \frac{3}{5} + \sin^{-1} x\right) = 0$, then find the values of x .

Answer

$$A. \cot\left(\cos^{-1}\frac{3}{5} + \sin^{-1}x\right) = 0$$

$$\Rightarrow \cos^{-1}\frac{3}{5} + \sin^{-1}x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}x = \left(n\pi + \frac{\pi}{2}\right) - \cos^{-1}\frac{3}{5}$$

$$\Rightarrow x = \sin\left[\left(n\pi + \frac{\pi}{2}\right) - \cos^{-1}\frac{3}{5}\right]$$

$$\Rightarrow x = \sin\left(n\pi + \frac{\pi}{2}\right)\cos\left(\cos^{-1}\frac{3}{5}\right) - \cos\left(n\pi + \frac{\pi}{2}\right)\sin\left(\cos^{-1}\frac{3}{5}\right)$$

(using $\sin(A-B) = \sin A \cos B - \cos A \sin B$)

$$\Rightarrow x = \pm \frac{3}{5}$$

$$\left(\text{The value of } \sin\left(n\pi + \frac{\pi}{2}\right) \text{ switches between } 1 \text{ and } -1\right)$$

5. Question

$$\left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2 = \frac{17\pi^2}{36}. \text{ Find } x$$

Answer

$$A. \text{ Using } a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x \cos^{-1}x = \frac{17\pi^2}{36}$$

$$\Rightarrow \frac{\pi^2}{4} - 2\sin^{-1}x \left(\frac{\pi}{2} - \sin^{-1}x\right) = \frac{17\pi^2}{36}$$

Substituting $\sin^{-1}x$ with 'a'

$$\Rightarrow 2a^2 - \pi a + \frac{\pi^2}{4} = \frac{17\pi^2}{36}$$

$$\Rightarrow 2a^2 - \pi a - \frac{2\pi^2}{9} = 0$$

$$\Rightarrow 18a^2 - 9\pi a - 2\pi^2 = 0$$

Using quadratic formulae

$$x = \frac{(-b \pm \sqrt{b^2 - 4ac})}{2a}$$

$$\Rightarrow x = \frac{\pi(9 \pm 15)}{36}$$

$$\Rightarrow x = \frac{2\pi}{3}, -\frac{\pi}{6}$$

6. Question

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1. \text{ Find } x$$

Answer

$$A. \sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$$

$$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1}x = n\pi + \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}x = \left(n\pi + \frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}$$

$$\Rightarrow x = \cos\left[\left(n\pi + \frac{\pi}{2}\right) - \sin^{-1}\frac{1}{5}\right]$$

$$\Rightarrow x = \cos\left(n\pi + \frac{\pi}{2}\right)\cos\left(\sin^{-1}\frac{1}{5}\right) + \sin\left(n\pi + \frac{\pi}{2}\right)\sin\left(\sin^{-1}\frac{1}{5}\right)$$

(using $\cos(A-B) = \cos A \cos B + \sin A \sin B$)

$$\Rightarrow x = \pm \frac{1}{5}$$

(The value of $\sin\left(n\pi + \frac{\pi}{2}\right)$ switches between 1 and -1)

7. Question

$$\text{Solve: } \sin^{-1}x = \frac{\pi}{6} + \cos^{-1}x$$

Answer

$$A. \sin^{-1}x = \frac{\pi}{6} + \cos^{-1}x$$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{6} + \left(\frac{\pi}{2} - \sin^{-1}x\right)$$

$$\left(\because \sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}\right)$$

$$\Rightarrow 2\sin^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{3}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

8. Question

Solve: $4 \sin^{-1} x = \pi - \cos^{-1} x$

Answer

A. $4 \sin^{-1} x = \pi - \cos^{-1} x$

$$\Rightarrow 4 \sin^{-1} x = \pi - \left(\frac{\pi}{2} - \sin^{-1} x \right)$$

$$\Rightarrow 3 \sin^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{2}$$

9. Question

Solve: $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$.

Answer

A. $\tan^{-1} x + 2 \cot^{-1} x = \frac{2\pi}{3}$

$$\Rightarrow \tan^{-1} x + 2 \left(\frac{\pi}{2} - \tan^{-1} x \right) = \frac{2\pi}{3}$$

$$\left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2} \right)$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{3}$$

$$\Rightarrow x = \sqrt{3}$$

10. Question

Solve: $5 \tan^{-1} x + 3 \cot^{-1} x = 2\pi$.

Answer

A. $5 \tan^{-1} x + 3 \cot^{-1} x = 2\pi$

$$\Rightarrow 5 \tan^{-1} x + 3 \left(\frac{\pi}{2} - \tan^{-1} x \right) = 2\pi$$

$$\left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2} \right)$$

$$\Rightarrow 2 \tan^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \tan^{-1} x = \frac{\pi}{4}$$

$$\Rightarrow x = 1$$

Exercise 4.11

1 A. Question

Prove the following results:

$$\tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{13} = \tan^{-1} \frac{2}{9}$$

Answer

$$\text{Given:- } \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right) = \tan^{-1}\left(\frac{2}{9}\right)$$

Take

LHS

$$= \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{13}\right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1} \left(\frac{\frac{1}{7} + \frac{1}{13}}{1 - \frac{1}{7} \times \frac{1}{13}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{13+7}{91}}{\frac{91-1}{91}} \right)$$

$$= \tan^{-1} \left(\frac{20}{90} \right)$$

$$= \tan^{-1} \left(\frac{2}{9} \right)$$

= RHS

Hence, Proved.

1 B. Question

Prove the following results:

$$\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$$

Answer

$$\text{Given:- } \sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\frac{4}{5} + \tan^{-1}\left(\frac{63}{16}\right) = \pi$$

Take

LHS

$$\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\frac{4}{5} + \tan^{-1}\left(\frac{63}{16}\right)$$

We know that, Formula

$$\sin^{-1} x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$\cos^{-1} x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{\frac{12}{13}}{\sqrt{1-\left(\frac{12}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \tan^{-1}\left(\frac{12}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \pi + \tan^{-1}\left(\frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{12}{5} \times \frac{3}{4}}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi + \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

We know that, Formula

$$\tan^{-1}(-x) = -\tan^{-1} x$$

$$= \pi - \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)$$

$$= \pi$$

So,

$$\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\frac{4}{5} + \tan^{-1}\left(\frac{63}{16}\right) = \pi$$

Hence, Proved.

1 C. Question

Prove the following results:

$$\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \sin^{-1} \frac{1}{\sqrt{5}}$$

Answer

$$\text{Given:- } \tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Take

LHS

$$\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1} \frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}}$$

$$= \tan^{-1} \frac{\frac{17}{36}}{\frac{36}{36}}$$

$$= \tan^{-1} \frac{17}{36}$$

$$= \tan^{-1} \frac{1}{2}$$

Let,

$$\tan \theta = \frac{1}{2}$$

Therefore,

$$\sin \theta = \frac{1}{\sqrt{5}}$$

So,

$$\theta = \sin^{-1} \frac{1}{\sqrt{5}}$$

$$\Rightarrow \tan^{-1} \left(\frac{1}{2} \right) = \sin^{-1} \left(\frac{1}{\sqrt{5}} \right) = \text{RHS}$$

$$\tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{2}{9} \right) = \sin^{-1} \left(\frac{1}{\sqrt{5}} \right)$$

Hence, Proved.

2. Question

$$\text{Find the value of } \tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

Answer

$$\text{Given: } \tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

Take

$$\tan^{-1} \left(\frac{x}{y} \right) - \tan^{-1} \left(\frac{x-y}{x+y} \right)$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Thus,

$$= \tan^{-1} \frac{\frac{x}{y} - \left(\frac{x-y}{x+y} \right)}{1 + \frac{x}{y} \times \left(\frac{x-y}{x+y} \right)}$$

$$= \tan^{-1} \frac{x(x+y)-y(x-y)}{y(x+y)+x(x-y)}$$

$$= \tan^{-1} \frac{x^2+y^2}{x^2+y^2}$$

$$= \tan^{-1} 1$$

$$= \frac{\pi}{4}$$

So,

$$\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right) = \frac{\pi}{4}$$

3 A. Question

Solve the following equations for x:

$$\tan^{-1} 2x + \tan^{-1} 3x = n\pi + \frac{3\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1}(2x) + \tan^{-1}(3x) = n\pi + \frac{3\pi}{4}$$

Take

LHS

$$\tan^{-1}(2x) + \tan^{-1}(3x) = n\pi + \frac{3\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{2x+3x}{1-2x \times 3x} = n\pi + \frac{3\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{5x}{1-6x^2} = n\pi + \frac{3\pi}{4}$$

$$\Rightarrow \frac{5x}{1-6x^2} = \tan\left(n\pi + \frac{3\pi}{4}\right)$$

$$\Rightarrow \frac{5x}{1-6x^2} = -1$$

$$\Rightarrow 5x = -1 + 6x^2$$

$$\Rightarrow 6x^2 - 5x - 1 = 0$$

$$\Rightarrow 6x^2 - 6x + x - 1 = 0$$

$$\Rightarrow 6x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (6x+1)(x-1) = 0$$

$$\Rightarrow 6x+1 = 0 \text{ or } x-1 = 0$$

$$\Rightarrow x = -\frac{1}{6} \text{ or } x = 1$$

Since,

$$x = -\frac{1}{6} \in \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

So,

$x = -\frac{1}{6}$ is the root of the given equation

Therefore,

$$x = -\frac{1}{6}$$

3 B. Question

Solve the following equations for x:

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$$

Answer

$$\text{Given:- } \tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\left(\frac{8}{31}\right)$$

Take

LHS

$$\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} \frac{8}{31}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{(x+1)+(x-1)}{1-(x+1)(x-1)} = \tan^{-1} \frac{8}{31}$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-(x^2-1)} = \tan^{-1} \frac{8}{31}$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-x^2+1)} = \tan^{-1} \frac{8}{31}$$

$$\Rightarrow \frac{2x}{1-x^2+1)} = \frac{8}{31}$$

$$\Rightarrow 62x = 8 - 8x^2 + 8$$

$$\Rightarrow 4x^2 + 62x - 16 = 0$$

$$\Rightarrow 6x^2 + 31x - 8 = 0$$

$$\Rightarrow 4x(x+8) - 1(x+8) = 0$$

$$\Rightarrow (4x-1)(x+8) = 0$$

$$\Rightarrow 6x+1=0 \text{ or } x-1=0$$

$$\Rightarrow x = \frac{1}{4} \text{ or } x = -8$$

Since,

$$x = \frac{1}{4} \in (-\sqrt{2}, \sqrt{2})$$

So,

$x = \frac{1}{4}$ is the root of the given equation

Therefore,

$$x = \frac{1}{4}$$

3 C. Question

Solve the following equations for x:

$$\tan^{-1}(x-1) + \tan^{-1}x + \tan^{-1}(x+1) = \tan^{-1}3x$$

Answer

$$\text{Given:- } \tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$$

Take

$$\tan^{-1}(x-1) + \tan^{-1}(x) + \tan^{-1}(x+1) = \tan^{-1}3x$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1}\frac{(x+1)+(x-1)}{1-(x+1)(x-1)} + \tan^{-1}(x) = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}\frac{2x}{1-(x^2-1)} + \tan^{-1}(x) = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}\frac{2x}{1-x^2+1} + \tan^{-1}(x) = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}\frac{2x}{2-x^2} + \tan^{-1}(x) = \tan^{-1}3x$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1}\frac{\frac{2x}{2-x^2}+x}{1-x\left(\frac{2x}{2-x^2}\right)} = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}\frac{\frac{2x+2x-x^3}{2-x^2}}{\frac{2-x^2-2x^2}{2-x^2}} = \tan^{-1}3x$$

$$\Rightarrow \tan^{-1}\frac{4x-x^3}{2-3x^2} = \tan^{-1}3x$$

$$\Rightarrow \frac{4x-x^3}{2-3x^2} = 3x$$

$$\Rightarrow 4x - x^3 = 6x - 9x^3$$

$$\Rightarrow 9x^3 - x^3 + 4x - 6x = 0$$

$$\Rightarrow 8x^3 - 2x = 0$$

$$\Rightarrow 2x(4x^2 - 1) = 0$$

$$\Rightarrow x = 0 \text{ or } x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$$

All satisfies x value

So,

$$x = 0 \text{ or } x = \frac{1}{2} \text{ or } x = -\frac{1}{2} \text{ is the root of the given equation}$$

Therefore,

$$x = 0, \pm \frac{1}{2}$$

3 D. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}x = 0, \text{ where } x > 0$$

Answer

$$\text{Given:- } \tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}(x) = 0$$

Take

$$\tan^{-1}\left(\frac{1-x}{1+x}\right) - \frac{1}{2}\tan^{-1}(x) = 0$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Thus,

$$\Rightarrow \tan^{-1}1 - \tan^{-1}x = \frac{1}{2}\tan^{-1}(x)$$

$$\Rightarrow \frac{\pi}{4} = \frac{3}{2}\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(x) = \frac{\pi}{6}$$

$$\Rightarrow x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

3 E. Question

Solve the following equations for x:

$$\cot^{-1}x - \cot^{-1}(x+2) = \frac{\pi}{12}, \text{ where } x > 0$$

Answer

$$\text{Given:- } \cot^{-1}(x) - \cot^{-1}(x+2) = \frac{\pi}{12}$$

Take

$$\cot^{-1}(x) - \cot^{-1}(x+2) = \frac{\pi}{12}$$

We know that, Formula

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{1}{x} - \tan^{-1} \frac{1}{x+2} = \frac{\pi}{12}$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Thus,

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x} \times \frac{1}{x+2}} \right) = \frac{\pi}{12}$$

$$\Rightarrow \frac{\frac{1}{x} - \frac{1}{x+2}}{1 + \frac{1}{x} \times \frac{1}{x+2}} = \tan \frac{\pi}{12}$$

$$\Rightarrow \frac{2}{x^2 + 2x + 1} = \tan \left(\frac{\pi}{3} - \frac{\pi}{4} \right)$$

We know that, Formula

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + (\tan x)(\tan y)}$$

Thus,

$$\Rightarrow \frac{2}{(x+1)^2} = \frac{\tan \frac{\pi}{3} - \tan \frac{\pi}{4}}{1 + (\tan \frac{\pi}{3})(\tan \frac{\pi}{4})}$$

$$\Rightarrow \frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

By rationalisation

$$\Rightarrow \frac{2}{(x+1)^2} = \frac{\sqrt{3}-1}{1+\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$\Rightarrow \frac{2}{(x+1)^2} = \frac{3-1}{(1+\sqrt{3})^2}$$

$$\Rightarrow (x+1)^2 = (1+\sqrt{3})^2$$

$$\Rightarrow x+1 = \pm(1+\sqrt{3})$$

$$\Rightarrow x+1 = 1+\sqrt{3} \text{ or } x+1 = -1-\sqrt{3}$$

$$\Rightarrow x = \sqrt{3} \text{ or } x = -2 - \sqrt{3}$$

as given, $x > 0$

Therefore

$$x = \sqrt{3}$$

3 F. Question

Solve the following equations for x:

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right), x > 0$$

Answer

Given:- $\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\left(\frac{8}{79}\right)$

Take

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1} \frac{8}{79}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{(x+2)+(x-2)}{1-(x+2)(x-2)} = \tan^{-1} \frac{8}{79}$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-(x^2-4)} = \tan^{-1} \frac{8}{79}$$

$$\Rightarrow \tan^{-1} \frac{2x}{1-x^2+4} = \tan^{-1} \frac{8}{79}$$

$$\Rightarrow \frac{2x}{5-x^2} = \frac{8}{79}$$

$$\Rightarrow 40 - 8x^2 = 158x$$

$$\Rightarrow 8x^2 + 158x - 40 = 0$$

$$\Rightarrow 4x^2 + 79x - 20 = 0$$

$$\Rightarrow 4x^2 + 80x - x - 20 = 0$$

$$\Rightarrow 4x(x+20) - 1(x+20) = 0$$

$$\Rightarrow (4x-1)(x+20) = 0$$

$$\Rightarrow 4x-1=0 \text{ or } x+20=0$$

$$\Rightarrow x = \frac{1}{4} \text{ or } x = -20$$

Since,

$$x > 0$$

So,

$$x = \frac{1}{4} \text{ is the root of the given equation}$$

Therefore,

$$x = \frac{1}{4}$$

3 G. Question

Solve the following equations for x:

$$\tan^{-1} \frac{x}{2} + \tan^{-1} \frac{x}{3} = \frac{\pi}{4}, 0 < x < \sqrt{6}$$

Answer

Given:- $\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$

Take

$$\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x}{2} + \frac{x}{3}}{1 - \frac{x}{2} \times \frac{x}{3}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{3x+2x}{6-x^2} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{5x}{6-x^2} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{6-x^2} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{5x}{6-x^2} = 1$$

$$\Rightarrow 5x = 6 - x^2$$

$$\Rightarrow x^2 + 5x - 6 = 0$$

$$\Rightarrow x^2 + 6x - x - 6 = 0$$

$$\Rightarrow x(x+6) - 1(x+6) = 0$$

$$\Rightarrow x = -6, 1$$

as given

$$0 < x < \sqrt{6}$$

Therefore

$$x = 1$$

3 H. Question

Solve the following equations for x:

$$\tan^{-1} \left(\frac{x-2}{x-4} \right) + \tan^{-1} \left(\frac{x+2}{x+4} \right) = \frac{\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1} \left(\frac{x-2}{x-4} \right) + \tan^{-1} \left(\frac{x+2}{x+4} \right) = \frac{\pi}{4}$$

Take

$$\tan^{-1} \left(\frac{x-2}{x-4} \right) + \tan^{-1} \left(\frac{x+2}{x+4} \right) = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x-2}{x-4} + \frac{x+2}{x+4}}{1 - \frac{x-2}{x-4} \times \frac{x+2}{x+4}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{(x-2)(x+4) + (x+2)(x-4)}{(x-4)(x+4)}}{\frac{(x-4)(x+4) - (x-2)(x+2)}{(x-4)(x+4)}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{(x-2)(x+4) + (x+2)(x-4)}{(x-4)(x+4) - (x-2)(x+2)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{x^2 + 2x - 8 + x^2 - 2x - 8}{(x^2 - 16) - (x^2 - 4)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{2x^2 - 16}{-12} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{x^2 - 8}{-6} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{x^2 - 8}{-6} = 1$$

$$\Rightarrow x^2 - 8 = -6$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}$$

3 I. Question

Solve the following equations for x:

$$\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1} \frac{2}{3},$$

$$\text{where } x < -\sqrt{3} \text{ or } x > \sqrt{3}$$

Answer

$$\text{Given: } \tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1} \left(\frac{2}{3} \right)$$

Take

$$\tan^{-1}(2+x) + \tan^{-1}(2-x) = \tan^{-1} \left(\frac{2}{3} \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \frac{(2+x) + (2-x)}{1 - (2+x)(2-x)} = \tan^{-1} \left(\frac{2}{3} \right)$$

$$\Rightarrow \tan^{-1} \frac{4}{1 - (4 - x^2)} = \tan^{-1} \frac{2}{3}$$

$$\Rightarrow \tan^{-1} \frac{4}{1 - 4 + x^2} = \tan^{-1} \frac{2}{3}$$

$$\Rightarrow \frac{4}{1 - 4 + x^2} = \frac{2}{3}$$

$$\Rightarrow 2x^2 - 8 + 2 = 12$$

$$\Rightarrow 2x^2 = 18$$

$$\Rightarrow x = \pm 3$$

Since,

$$x < -\sqrt{3} \text{ or } x > \sqrt{3}$$

So,

$x = +3, -3$ is the root of the given equation

Therefore,

$x = +3, -3$

3 J. Question

Solve the following equations for x :

$$\tan^{-1} \frac{x-2}{x-1} + \tan^{-1} \frac{x+2}{x+1} = \frac{\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1} \left(\frac{x-2}{x-1} \right) + \tan^{-1} \left(\frac{x+2}{x+1} \right) = \frac{\pi}{4}$$

Take

$$\tan^{-1} \left(\frac{x-2}{x-1} \right) + \tan^{-1} \left(\frac{x+2}{x+1} \right) = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{x-2}{x-1} + \frac{x+2}{x+1}}{1 - \frac{x-2}{x-1} \times \frac{x+2}{x+1}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{(x-2)(x+1) + (x+2)(x-1)}{(x-1)(x+1)}}{\frac{(x-1)(x+1) - (x-2)(x+2)}{(x-1)(x+1)}} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{(x-2)(x+1) + (x+2)(x-1)}{(x-1)(x+1) - (x-2)(x+2)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{x^2 + 2x - 2 + x^2 - 2x - 2}{(x^2 - 1) - (x^2 - 4)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{2x^2 - 4}{-5} \right) = \frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2 - 4}{-5} = \tan \frac{\pi}{4}$$

$$\Rightarrow \frac{2x^2 - 4}{-5} = 1$$

$$\Rightarrow 2x^2 - 4 = -5$$

$$\Rightarrow 2x^2 = 1$$

$$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

4. Question

Sum the following series:

$$\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{2}{9} + \tan^{-1} \frac{4}{33} + \dots + \tan^{-1} \frac{2^{n-1}}{1 + 2^{2n-1}}$$

Answer

$$\text{Given:- } \tan^{-1} \left(\frac{1}{3} \right) + \tan^{-1} \left(\frac{2}{9} \right) + \tan^{-1} \left(\frac{4}{33} \right) + \dots + \tan^{-1} \left(\frac{2^{n-1}}{1 + 2^{2n-1}} \right)$$

Take

$$\tan^{-1}\left(\frac{2^{n-1}}{1+2^{2n-1}}\right) = T_n \text{ (Let)}$$

$$\Rightarrow T_n = \tan^{-1}\left(\frac{2^n - 2^{n-1}}{1+2^n 2^{n-1}}\right)$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

$$\Rightarrow T_n = \tan^{-1} 2^n - \tan^{-1} 2^{n-1}$$

So,

$$T_1 = \tan^{-1} 2^1 - \tan^{-1} 2^0$$

$$T_2 = \tan^{-1} 4 - \tan^{-1} 2$$

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$$T_n = \tan^{-1} 2^n - \tan^{-1} 2^{n-1}$$

Adding all the terms, we get

$$= \tan^{-1} 2^n - \tan^{-1} 1$$

$$= \tan^{-1} 2^n - \frac{\pi}{4}$$

Exercise 4.12

1. Question

$$\text{Evaluate : } \cos\left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13}\right)$$

Answer

$$\text{Given:- } \cos\left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13}\right)$$

Take

$$\cos\left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13}\right)$$

We know that, Formula

$$\sin^{-1} x + \sin^{-1} y = \sin^{-1} \left[x\sqrt{1-y^2} + y\sqrt{1-x^2} \right]$$

$$= \cos\left(\sin^{-1} \left[\frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2} + \frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2} \right]\right)$$

$$= \cos\left(\sin^{-1} \left[\frac{3}{5} \times \frac{12}{13} + \frac{5}{13} \times \frac{4}{5} \right]\right)$$

$$= \cos\left(\sin^{-1} \left[\frac{56}{65} \right]\right)$$

We know that, Formula

$$\sin^{-1} x = \cos^{-1} \sqrt{1-x^2}$$

Therefore,

$$= \cos \left(\cos^{-1} \sqrt{1 - \left(\frac{56}{65} \right)^2} \right)$$

$$= \cos \left(\cos^{-1} \sqrt{\frac{33}{65}} \right)$$

$$= \frac{33}{65}$$

So,

$$\cos \left(\sin^{-1} \frac{3}{5} + \sin^{-1} \frac{5}{13} \right) = \frac{33}{65}$$

2 A. Question

Prove the following results:

$$\sin^{-1} \left(\frac{63}{65} \right) = \sin^{-1} \left(\frac{5}{13} \right) + \cos^{-1} \left(\frac{3}{5} \right)$$

Answer

$$\text{Given:- } \sin^{-1} \frac{63}{65} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

Take

RHS

$$\sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

We know that, Formula

$$\cos^{-1} x = \sin^{-1} \sqrt{1-x^2}$$

Thus,

$$= \sin^{-1} \frac{5}{13} + \sin^{-1} \sqrt{1 - \frac{9}{25}}$$

$$= \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{4}{5}$$

By pathagorous theorem

$$= \tan^{-1} \frac{\frac{5}{13}}{\sqrt{1 - \frac{25}{169}}} + \tan^{-1} \frac{\frac{4}{5}}{\sqrt{1 - \frac{16}{25}}}$$

$$= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1} \left(\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \times \frac{4}{3}} \right)$$

$$= \tan^{-1} \left(\frac{63}{16} \right)$$

Now,

LHS

$$= \sin^{-1} \frac{63}{65}$$

$$= \tan^{-1} \frac{\frac{63}{65}}{\sqrt{1 - \left(\frac{25}{169} \right)^2}}$$

$$= \tan^{-1} \left(\frac{63}{16} \right)$$

So,

LHS = RHS

$$\sin^{-1} \frac{63}{65} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

2 B. Question

Prove the following results:

$$\sin^{-1} \left(\frac{5}{13} \right) + \cos^{-1} \frac{3}{5} = \tan^{-1} \frac{63}{16}$$

Answer

$$\text{Given:- } \sin^{-1} \frac{63}{65} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

Take

LHS

$$\sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

We know that, Formula

$$\cos^{-1} x = \sin^{-1} \sqrt{1 - x^2}$$

Thus,

$$= \sin^{-1} \frac{5}{13} + \sin^{-1} \sqrt{1 - \frac{9}{25}}$$

$$= \sin^{-1} \frac{5}{13} + \sin^{-1} \frac{4}{5}$$

By pathagorou theorem

$$= \tan^{-1} \frac{\frac{5}{13}}{\sqrt{1 - \frac{25}{169}}} + \tan^{-1} \frac{\frac{4}{5}}{\sqrt{1 - \frac{16}{25}}}$$

$$= \tan^{-1} \frac{5}{12} + \tan^{-1} \frac{4}{3}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1} \left(\frac{\frac{5}{12} + \frac{4}{3}}{1 - \frac{5}{12} \times \frac{4}{3}} \right)$$

$$= \tan^{-1} \left(\frac{63}{16} \right)$$

Now,

RHS

$$= \sin^{-1} \frac{63}{65}$$

$$= \tan^{-1} \frac{\frac{63}{65}}{\sqrt{1 - \left(\frac{25}{169} \right)^2}}$$

$$= \tan^{-1} \left(\frac{63}{16} \right)$$

So,

LHS = RHS

$$\sin^{-1} \frac{63}{65} = \sin^{-1} \frac{5}{13} + \cos^{-1} \frac{3}{5}$$

2 C. Question

Prove the following results:

$$\frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \left(\frac{1}{3} \right) = \frac{9}{4} \sin^{-1} \left(\frac{2\sqrt{2}}{3} \right)$$

Answer

$$\text{Given:- } \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$$

Take

LHS

$$= \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3}$$

$$= \frac{9}{4} \left(\frac{\pi}{2} - \sin^{-1} \frac{1}{3} \right)$$

We know that, Formula

$$\sin^{-1} x + \cos^{-1} y = \frac{\pi}{2}$$

Thus,

$$= \frac{9}{4} \left(\cos^{-1} \frac{1}{3} \right)$$

Now,

Assume that

$$\cos^{-1} \frac{1}{3} = x$$

Then,

$$\Rightarrow \cos x = \frac{1}{3}$$

$$\text{And } \sin x = \sqrt{1 - \frac{1}{9}}$$

$$\Rightarrow \sin x = \frac{2\sqrt{2}}{3}$$

Therefore,

$$x = \sin^{-1} \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

$$\Rightarrow \frac{9\pi}{8} - \frac{9}{4} \sin^{-1} \frac{1}{3} = \frac{9}{4} \sin^{-1} \frac{2\sqrt{2}}{3}$$

Hence Proved

3 A. Question

Prove the following results: Solve the following:

$$\sin^{-1} x + \sin^{-1} 2x = \pi/3$$

Answer

$$\text{Given:- } \sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$$

Take

$$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} 2x = \sin^{-1} \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin^{-1} 2x = \sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} x$$

We know that, Formula

$$\sin^{-1} x - \sin^{-1} y = \sin^{-1} [x\sqrt{1-y^2} - y\sqrt{1-x^2}]$$

Thus,

$$\Rightarrow \sin^{-1} 2x = \sin^{-1} \left[\frac{\sqrt{3}}{2} \sqrt{1-x^2} - x \sqrt{1 - \frac{3}{4}} \right]$$

$$\Rightarrow \sin^{-1} 2x = \sin^{-1} \left[\frac{\sqrt{3}}{2} \sqrt{1-x^2} - \frac{x}{2} \right]$$

$$\Rightarrow 2x = \left[\frac{\sqrt{3}}{2} \sqrt{1-x^2} - \frac{x}{2} \right]$$

$$\Rightarrow \frac{5x}{2} = \frac{\sqrt{3}}{2} \sqrt{1-x^2}$$

$$\Rightarrow 25x^2 = 3(1-x^2)$$

$$\Rightarrow x^2 = \frac{3}{28}$$

$$\Rightarrow x = \pm \frac{1}{2} \sqrt{\frac{3}{7}}$$

3 B. Question

Prove the following results: Solve the following:

$$\cos^{-1}x + \sin^{-1}x/1 - \pi/6 = 0$$

Answer

$$\text{Given:- } \cos^{-1}x + \sin^{-1}\frac{x}{2} - \frac{\pi}{6} = 0$$

Take

$$\cos^{-1}x + \sin^{-1}\frac{x}{2} - \frac{\pi}{6} = 0$$

$$\Rightarrow \cos^{-1}x + \sin^{-1}\frac{x}{2} = \frac{\pi}{6}$$

We know that, Formula

$$\cos^{-1}x = \sin^{-1}\sqrt{1-x^2}$$

Thus,

$$\Rightarrow \cos^{-1}x + \sin^{-1}\frac{x}{2} = \sin^{-1}\frac{1}{2} - \sin^{-1}\sqrt{1-x^2}$$

We know that, Formula

$$\sin^{-1}x + \sin^{-1}y = \sin^{-1}\left[x\sqrt{1-y^2} + y\sqrt{1-x^2}\right]$$

Thus,

$$\Rightarrow \sin^{-1}\frac{x}{2} = \sin^{-1}\left[\frac{1}{2}\sqrt{1-1+x^2} - \sqrt{1-x^2}\sqrt{1-\frac{1}{4}}\right]$$

$$\Rightarrow \sin^{-1}\frac{x}{2} = \sin^{-1}\left[\frac{x}{2} - \frac{\sqrt{3}\sqrt{1-x^2}}{2}\right]$$

$$\Rightarrow \frac{x}{2} = \frac{x}{2} - \frac{\sqrt{3}\sqrt{1-x^2}}{2}$$

$$\Rightarrow \frac{\sqrt{3}\sqrt{1-x^2}}{2} = 0$$

$$\Rightarrow \sqrt{1-x^2} = 0$$

$$\Rightarrow 1-x^2 = 0$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow x = \pm 1$$

Exercise 4.13

1. Question

If $\cos^{-1}x/2 + \cos^{-1}y/3 = a$, then prove that $9x^2 - 12xy \cos a + 4y^2 = 36 \sin^2 a$.

Answer

$$\text{Given:- } \cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = a$$

Take

$$\cos^{-1}\frac{x}{2} + \cos^{-1}\frac{y}{3} = a$$

We know that, Formula

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$$

Thus,

$$\Rightarrow \cos^{-1}\left[\frac{x}{2} \times \frac{y}{3} - \sqrt{1-\left(\frac{x}{2}\right)^2} \sqrt{1-\left(\frac{y}{3}\right)^2}\right] = a$$

$$\Rightarrow \left[\frac{xy}{6} - \frac{\sqrt{4-x^2}}{2} \times \frac{\sqrt{9-y^2}}{3}\right] = \cos a$$

$$\Rightarrow xy - \sqrt{4-x^2} \times \sqrt{9-y^2} = 6 \cos a$$

$$\Rightarrow xy - 6 \cos a = \sqrt{4-x^2} \sqrt{9-y^2}$$

Now let's take square of both side, we get

$$\Rightarrow (xy - 6 \cos a)^2 = (4-x^2)(9-y^2)$$

$$\Rightarrow x^2y^2 + 36\cos^2a - 12xy \cos a = 36 - 9x^2 - 4y^2 + x^2y^2$$

$$\Rightarrow 9x^2 + 4y^2 - 36 + 36\cos^2a - 12xy \cos a = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos a - 36(1 - \cos^2a) = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos a - 36\sin^2a = 0$$

$$\Rightarrow 9x^2 + 4y^2 - 12xy \cos a = 36\sin^2a$$

Hence Proved

2. Question

Solve the equation: $\cos^{-1} a/x - \cos^{-1} b/x = \cos^{-1} 1/b - \cos^{-1} 1/a$

Answer

$$\text{Given:- } \cos^{-1} \frac{a}{x} - \cos^{-1} \frac{b}{x} = \cos^{-1} \frac{1}{b} - \cos^{-1} \frac{1}{a}$$

Take

$$\cos^{-1} \frac{a}{x} - \cos^{-1} \frac{b}{x} = \cos^{-1} \frac{1}{b} - \cos^{-1} \frac{1}{a}$$

$$\Rightarrow \cos^{-1} \frac{a}{x} + \cos^{-1} \frac{1}{a} = \cos^{-1} \frac{1}{b} + \cos^{-1} \frac{b}{x}$$

We know that, Formula

$$\cos^{-1}x + \cos^{-1}y = \cos^{-1}\left[xy - \sqrt{1-x^2}\sqrt{1-y^2}\right]$$

Thus,

$$\Rightarrow \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2} \sqrt{1-\left(\frac{1}{a}\right)^2}\right] = \cos^{-1}\left[\frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2} \sqrt{1-\left(\frac{1}{b}\right)^2}\right]$$

$$\Rightarrow \frac{1}{x} - \sqrt{1-\left(\frac{a}{x}\right)^2} \sqrt{1-\left(\frac{1}{a}\right)^2} = \frac{1}{x} - \sqrt{1-\left(\frac{b}{x}\right)^2} \sqrt{1-\left(\frac{1}{b}\right)^2}$$

$$\Rightarrow \sqrt{1-\left(\frac{a}{x}\right)^2} \sqrt{1-\left(\frac{1}{a}\right)^2} = \sqrt{1-\left(\frac{b}{x}\right)^2} \sqrt{1-\left(\frac{1}{b}\right)^2}$$

Squaring both side or removing square root, we get

$$\Rightarrow \left(1 - \left(\frac{a}{x}\right)^2\right)\left(1 - \left(\frac{1}{a}\right)^2\right) = \left(1 - \left(\frac{b}{x}\right)^2\right)\left(1 - \left(\frac{1}{b}\right)^2\right)$$

$$\Rightarrow 1 - \left(\frac{a}{x}\right)^2 - \left(\frac{1}{a}\right)^2 + \left(\frac{1}{x}\right)^2 = 1 - \left(\frac{b}{x}\right)^2 - \left(\frac{1}{b}\right)^2 + \left(\frac{1}{x}\right)^2$$

$$\Rightarrow \left(\frac{b}{x}\right)^2 - \left(\frac{a}{x}\right)^2 = \left(\frac{1}{a}\right)^2 - \left(\frac{1}{b}\right)^2$$

$$\Rightarrow (b^2 - a^2)a^2b^2 = x^2(b^2 - a^2)$$

$$\Rightarrow x^2 = a^2b^2$$

$$\Rightarrow x = ab$$

3. Question

Solve: $\cos^{-1} \sqrt{3}x + \cos^{-1} x = \pi/2$

Answer

Given:- $\cos^{-1} \sqrt{3}x + \cos^{-1} x = \frac{\pi}{2}$

Take

$$\cos^{-1} \sqrt{3}x + \cos^{-1} x = \frac{\pi}{2}$$

We know that, Formula

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2}\sqrt{1-y^2}]$$

Thus,

$$\Rightarrow \cos^{-1} [\sqrt{3}x^2 - \sqrt{1-(3x)^2}\sqrt{1-x^2}] = \frac{\pi}{2}$$

$$\Rightarrow \sqrt{3}x^2 - \sqrt{1-(3x)^2}\sqrt{1-x^2} = \cos \frac{\pi}{2}$$

$$\Rightarrow \sqrt{3}x^2 - \sqrt{1-(3x)^2}\sqrt{1-x^2} = 0$$

$$\Rightarrow \sqrt{3}x^2 = \sqrt{1-(3x)^2}\sqrt{1-x^2}$$

Squaring both sides, we get

$$\Rightarrow 3x^4 = 1 - x^2 - 3x^2 + 3x^4$$

$$\Rightarrow 1 - 4x^2 = 0$$

$$\Rightarrow x^2 = \frac{1}{4}$$

$$\Rightarrow x = \pm \frac{1}{2}$$

4. Question

Prove that: $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Answer

Given:- $\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{13} = \cos^{-1} \frac{33}{65}$

Take

LHS

$$\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{3}$$

We know that, Formula

$$\cos^{-1} x + \cos^{-1} y = \cos^{-1} [xy - \sqrt{1-x^2} \sqrt{1-y^2}]$$

Thus,

$$= \cos^{-1} \left[\frac{4}{5} \times \frac{12}{3} - \sqrt{1 - \left(\frac{4}{5}\right)^2} \sqrt{1 - \left(\frac{12}{3}\right)^2} \right]$$

$$= \cos^{-1} \left[\frac{48}{65} - \sqrt{1 - \frac{16}{25}} \sqrt{1 - \frac{144}{169}} \right]$$

$$= \cos^{-1} \left[\frac{48}{65} - \frac{3}{5} \times \frac{5}{13} \right]$$

$$= \cos^{-1} \left[\frac{48}{65} - \frac{15}{65} \right]$$

$$= \cos^{-1} \frac{33}{65}$$

= RHS

So,

$$\cos^{-1} \frac{4}{5} + \cos^{-1} \frac{12}{3} = \cos^{-1} \frac{33}{65}$$

Hence Proved

Exercise 4.14

1 A. Question

Evaluate the following:

$$\tan \left\{ 2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4} \right\}$$

Answer

$$\text{Given:- } \tan \left\{ 2 \tan^{-1} \left(\frac{1}{5} \right) - \frac{\pi}{4} \right\}$$

Now, as we know

$$2 \tan^{-1}(x) = \tan^{-1} \left(\frac{2x}{1-x^2} \right), \text{ if } |x| < 1$$

and $\frac{\pi}{4}$ can be written as $\tan^{-1}(1)$

$$= \tan \left\{ \tan^{-1} \left(\frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}} \right) - \tan^{-1} 1 \right\}$$

$$= \tan \left\{ \tan^{-1} \left(\frac{5}{12} \right) - \tan^{-1} 1 \right\}$$

We know that,

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

$$= \tan \left\{ \tan^{-1} \left(\frac{\frac{5}{12} - 1}{1 + \frac{5}{12}} \right) \right\}$$

$$= \tan \left\{ \tan^{-1} \left(\frac{-7}{17} \right) \right\}$$

$$= -\frac{7}{17}$$

1 B. Question

Evaluate the following:

$$\tan \left(\frac{1}{2} \sin^{-1} \frac{3}{4} \right)$$

Answer

$$\text{Given:- } \tan \left\{ \frac{1}{2} \sin^{-1} \frac{3}{4} \right\}$$

$$\text{Let } \frac{1}{2} \sin^{-1} \frac{3}{4} = t \text{ (say)}$$

Therefore,

$$\Rightarrow \sin^{-1} \frac{3}{4} = 2t$$

$$\Rightarrow \sin 2t = \frac{3}{4}$$

Now, by Pythagoras theorem

$$\Rightarrow \sin 2t = \frac{3}{4} = \frac{\text{perpendicular}}{\text{hypotenuse}}$$

$$\Rightarrow \cos 2t = \frac{\sqrt{4^2 - 3^2}}{4} = \frac{\text{Base}}{\text{hypotenuse}}$$

$$\Rightarrow \cos 2t = \frac{\sqrt{7}}{4}$$

As given, and putting assume value, we get

$$\tan \left\{ \frac{1}{2} \sin^{-1} \frac{3}{4} \right\}$$

$$= \tan(t)$$

We know that: Formula

$$\tan(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$$

$$= \sqrt{\frac{1 - \cos 2t}{1 + \cos 2t}}$$

$$= \sqrt{\frac{1 - \frac{\sqrt{7}}{4}}{1 + \frac{\sqrt{7}}{4}}}$$

$$= \sqrt{\frac{4 - \sqrt{7}}{4 + \sqrt{7}}}$$

$$= \sqrt{\frac{(4 - \sqrt{7})(4 - \sqrt{7})}{(4 + \sqrt{7})(4 - \sqrt{7})}}; \text{ by rationalisation}$$

$$= \sqrt{\frac{(4-\sqrt{7})^2}{9}}$$

$$= \frac{4-\sqrt{7}}{3}$$

Hence

$$\tan\left\{\frac{1}{2}\sin^{-1}\frac{3}{4}\right\} = \frac{4-\sqrt{7}}{3}$$

1 C. Question

Evaluate the following:

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$$

Answer

$$\text{Given:- } \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$$

We know that : Formula

$$\cos^{-1}x = 2\sin^{-1}\left(\pm\sqrt{\frac{1-x}{2}}\right); \text{ choose that formula which actually simplifies function}$$

Thus, given function changes to

$$\sin\left(\frac{1}{2}2\sin^{-1}\left(\pm\sqrt{\frac{1-\frac{4}{5}}{2}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\sqrt{\frac{1}{2 \times 5}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\pm\frac{1}{\sqrt{10}}\right)\right)$$

As we know

$$\sin(\sin^{-1}x) = x \text{ as } x \in [-1, 1]$$

$$= \pm\frac{1}{\sqrt{10}}$$

Hence,

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \pm\frac{1}{\sqrt{10}}$$

1 D. Question

Evaluate the following:

$$\sin\left(2\tan^{-1}\frac{2}{3}\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$$

Answer

$$\text{Given:- } \sin\left(2\tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right)$$

We know that :- Formula (- obtain by Pythagoras theorem)

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}(x)$; Formula of tan in terms of sine, so that it make simplification easier

And

$\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \tan^{-1}(x)$; Formula of tan in terms of cos, so that it make simplification easier

Now given function becomes,

$$= \sin\left(\sin^{-1}\left(\frac{2 \times \frac{2}{3}}{1+\frac{4}{9}}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{\sqrt{1+\frac{4}{9}}}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\frac{12}{13}\right)\right) + \cos\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$$

$$= \frac{12}{13} + \frac{1}{2}$$

$$= \frac{37}{26}$$

Hence,

$$\sin\left(2 \tan^{-1}\left(\frac{2}{3}\right)\right) + \cos\left(\tan^{-1}\sqrt{3}\right) = \frac{37}{26}$$

2 A. Question

Prove the following results:

$$2 \sin^{-1} \frac{3}{5} = \tan^{-1} \frac{24}{7}$$

Answer

$$\text{Given:- } 2 \sin^{-1} \frac{3}{5} = \tan^{-1} \left(\frac{24}{7}\right)$$

Take

LHS

$$= 2 \sin^{-1} \frac{3}{5}$$

We know that, Formula

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Thus,

$$= 2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{9}{25}}}\right)$$

$$= 2 \times \tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right)$$

$$= 2 \times \tan^{-1}\left(\frac{3}{4}\right)$$

Again we know that, Formula

$$2 \tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{ if } |x| < 1$$

Thus,

$$= \tan^{-1} \left(\frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{3}{2}}{\frac{7}{16}} \right)$$

$$= \tan^{-1} \left(\frac{24}{7} \right)$$

$$= \text{RHS}$$

So,

$$2 \sin^{-1} \frac{3}{5} = \tan^{-1} \left(\frac{24}{7} \right)$$

Hence Proved

2 B. Question

Prove the following results:

$$\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \cos^{-1} \frac{3}{5} = \frac{1}{2} \sin^{-1} \frac{4}{5}$$

Answer

$$\text{Given: } \tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{2}{9} \right) = \frac{1}{2} \cos^{-1} \left(\frac{3}{5} \right) = \frac{1}{2} \sin^{-1} \left(\frac{4}{5} \right)$$

Take

LHS

$$= \tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{2}{9} \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$= \tan^{-1} \left(\frac{\frac{1}{4} + \frac{2}{9}}{1 - \frac{1}{4} \times \frac{2}{9}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{9+8}{36}}{\frac{36-2}{36}} \right)$$

$$= \tan^{-1} \left(\frac{17}{34} \right)$$

$$= \tan^{-1} \left(\frac{1}{2} \right)$$

Multiplying and dividing by 2

$$= \frac{1}{2} \left\{ 2 \tan^{-1} \left(\frac{1}{2} \right) \right\}$$

We know that, Formula

$$2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{1-\frac{1}{4}}{1+\frac{1}{4}} \right)$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{\frac{3}{5}}{\frac{4}{5}} \right)$$

$$= \frac{1}{2} \cos^{-1} \left(\frac{3}{4} \right)$$

= RHS

So,

$$\tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{2}{9} \right) = \frac{1}{2} \cos^{-1} \left(\frac{3}{5} \right)$$

Now,

$$= \frac{1}{2} \cos^{-1} \left(\frac{3}{5} \right)$$

We know that, Formula

$$= \cos^{-1} x = \sin^{-1} \sqrt{1 - x^2}$$

Thus,

$$= \frac{1}{2} \sin^{-1} \sqrt{1 - \frac{9}{25}}$$

$$= \frac{1}{2} \sin^{-1} \sqrt{\frac{16}{25}}$$

$$= \frac{1}{2} \sin^{-1} \frac{4}{5}$$

= RHS

So,

$$\tan^{-1} \left(\frac{1}{4} \right) + \tan^{-1} \left(\frac{2}{9} \right) = \frac{1}{2} \cos^{-1} \left(\frac{3}{5} \right) = \frac{1}{2} \sin^{-1} \frac{4}{5}$$

Hence Proved

2 C. Question

Prove the following results:

$$\tan^{-1} \frac{2}{3} = \frac{1}{2} \tan^{-1} \frac{12}{5}$$

Answer

$$\text{Given:- } \tan^{-1} \left(\frac{2}{3} \right) = \frac{1}{2} \tan^{-1} \left(\frac{12}{5} \right)$$

Take

LHS

$$= \tan^{-1} \left(\frac{2}{3} \right)$$

Multiplying and dividing by 2

$$= \frac{1}{2} \left\{ 2 \tan^{-1} \left(\frac{2}{3} \right) \right\}$$

We know that, Formula

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1 - x^2} \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{2 \times \frac{12}{5}}{1 - \frac{144}{25}} \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{4}{\frac{11}{25}} \right)$$

$$= \frac{1}{2} \tan^{-1} \left(\frac{12}{5} \right)$$

= RHS

So,

$$\tan^{-1} \left(\frac{2}{3} \right) = \frac{1}{2} \tan^{-1} \left(\frac{12}{5} \right)$$

Hence Proved

2 D. Question

Prove the following results:

$$\tan^{-1} \left(\frac{1}{7} \right) + 2 \tan^{-1} \left(\frac{1}{3} \right) = \frac{\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1} \left(\frac{1}{7} \right) + 2 \tan^{-1} \left(\frac{1}{3} \right) = \frac{\pi}{4}$$

Take

LHS

$$= \tan^{-1} \left(\frac{1}{7} \right) + 2 \tan^{-1} \left(\frac{1}{3} \right)$$

We know that, Formula

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$= \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{2 \times \frac{1}{3}}{1 - \frac{1}{9}} \right)$$

$$= \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{\frac{2}{3}}{\frac{8}{9}} \right)$$

$$= \tan^{-1} \left(\frac{1}{7} \right) + \tan^{-1} \left(\frac{3}{4} \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$= \tan^{-1} \left(\frac{\frac{1}{7} + \frac{3}{4}}{1 - \frac{1}{7} \times \frac{3}{4}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{25}{28}}{\frac{25}{28}} \right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

= RHS

So,

$$\tan^{-1}\left(\frac{1}{7}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

Hence Proved

2 E. Question

Prove the following results:

$$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$$

Answer

$$\text{Given:- } \sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$$

Take

LHS

$$= \sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right)$$

We know that, Formula

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

And,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{\frac{4}{5}}{\sqrt{1-\frac{16}{25}}}\right) + \tan^{-1}\left(\frac{2 \times \frac{1}{3}}{1-\frac{1}{9}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{5}}{\sqrt{\frac{9}{25}}}\right) + \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{8}{9}}\right)$$

$$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan^{-1}\left(\frac{\frac{4}{3} + \frac{3}{4}}{1 - \frac{4}{3} \times \frac{3}{4}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{25}{12}}{0}\right)$$

$$= \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

= RHS

So,

$$\sin^{-1}\left(\frac{4}{5}\right) + 2\tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{2}$$

Hence Proved

2 F. Question

Prove the following results:

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$$

Answer

$$\text{Given:- } 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

Take

LHS

$$= 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

We know that, Formula

$$\sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

Thus,

$$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\frac{16}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{\frac{9}{25}}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= 2 \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

We know that, Formula

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{2 \times \frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

$$= \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

$$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}}\right)$$

$$= \tan^{-1} \left(\frac{\frac{744-119}{217}}{\frac{217}{217}} \right)$$

$$= \tan^{-1} \left(\frac{625}{625} \right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

$$= \text{RHS}$$

So,

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

Hence Proved

2 G. Question

Prove the following results:

$$2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

Answer

$$\text{Given:- } 2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

Take

LHS

$$= 2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

We know that, Formula

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$$

$$= \tan^{-1}\left(\frac{\frac{5}{12} + \frac{1}{8}}{1 - \frac{5}{12} \times \frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{10+3}{24}}{\frac{96-5}{96}}\right)$$

$$= \tan^{-1}\left(\frac{13}{24} \times \frac{96}{91}\right)$$

$$= \tan^{-1}\left(\frac{4}{7}\right)$$

= RHS

So,

$$2 \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

Hence Proved

2 H. Question

Prove the following results:

$$2 \tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31} = \frac{\pi}{4}$$

Answer

$$\text{Given:- } 2 \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

Take

LHS

$$= 2 \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$$

We know that, Formula

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$= \tan^{-1} \left(\frac{2 \times \frac{3}{4}}{1 - \frac{9}{16}} \right) - \tan^{-1} \left(\frac{17}{31} \right)$$

$$= \tan^{-1} \left(\frac{3}{2} \times \frac{16}{7} \right) - \tan^{-1} \left(\frac{17}{31} \right)$$

$$= \tan^{-1} \left(\frac{24}{7} \right) - \tan^{-1} \left(\frac{17}{31} \right)$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

$$= \tan^{-1} \left(\frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \times \frac{17}{31}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{744-119}{217}}{\frac{247+408}{217}} \right)$$

$$= \tan^{-1} \left(\frac{625}{625} \right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

= RHS

So,

$$2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \frac{\pi}{4}$$

Hence Proved

2 I. Question

Prove the following results:

$$2 \tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{7} = \tan^{-1} \frac{31}{17}$$

Answer

$$\text{Given:- } 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$$

Take

LHS

$$= 2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

We know that, Formula

$$2\tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$= \tan^{-1} \left(\frac{2 \times \frac{1}{2}}{1 - \frac{1}{4}} \right) + \tan^{-1} \left(\frac{1}{7} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2}{2}}{\frac{3}{4}} \right) + \tan^{-1} \left(\frac{1}{7} \right)$$

$$= \tan^{-1} \left(\frac{4}{3} \right) + \tan^{-1} \left(\frac{1}{7} \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$= \tan^{-1} \left(\frac{\frac{4}{3} + \frac{1}{7}}{1 - \frac{4}{3} \times \frac{1}{7}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{31}{21}}{\frac{21}{21}} \right)$$

$$= \tan^{-1} \left(\frac{31}{17} \right)$$

= RHS

So,

$$2 \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \tan^{-1}\left(\frac{31}{17}\right)$$

Hence Proved

2 J. Question

Prove the following results:

$$4 \tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$$

Answer

$$\text{Given:- } 4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$$

Take

LHS

$$= 4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right)$$

We know that, Formula

$$4\tan^{-1}x = \tan^{-1}\left(\frac{4x - 4x^3}{1 - 6x^2 + x^4}\right)$$

Thus,

$$= \tan^{-1}\left(\frac{4 \times \frac{1}{5} - 4\left(\frac{1}{5}\right)^3}{1 - 6\left(\frac{1}{5}\right)^2 + \left(\frac{1}{5}\right)^4}\right) - \tan^{-1}\left(\frac{1}{239}\right)$$

$$= \tan^{-1}\left(\frac{120}{119}\right) - \tan^{-1}\left(\frac{1}{239}\right)$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

$$= \tan^{-1}\left(\frac{\frac{120}{119} - \frac{1}{239}}{1 - \frac{120}{119} \times \frac{1}{239}}\right)$$

$$= \tan^{-1}\left(\frac{120 \times 239 - 119}{119 \times 239 + 120}\right)$$

$$= \tan^{-1}\left(\frac{28561}{28561}\right)$$

$$= \tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

= RHS

So,

$$4\tan^{-1}\left(\frac{1}{5}\right) - \tan^{-1}\left(\frac{1}{239}\right) = \frac{\pi}{4}$$

Hence Proved

3. Question

$$\text{If } \sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2},$$

$$\text{Then prove that } x = \frac{a-b}{1+ab}.$$

Answer

Given:- $\sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

Take

$$\Rightarrow \sin^{-1}\left(\frac{2a}{1+a^2}\right) - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

We know that, Formula

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

And

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$\Rightarrow 2\tan^{-1}(a) - 2\tan^{-1}(b) = 2\tan^{-1}(x)$$

$$\Rightarrow 2(\tan^{-1}(a) - \tan^{-1}(b)) = 2\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(a) - \tan^{-1}(b) = \tan^{-1}(x)$$

We know that, Formula

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$$

Thus,

$$\Rightarrow \tan^{-1}\left(\frac{a-b}{1+ab}\right) = \tan^{-1}(x)$$

On comparing we get,

$$\Rightarrow x = \frac{a-b}{1+ab}$$

Hence Proved

4 A. Question

Prove that:

$$\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{2}$$

Answer

Given:- $\tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x} = \frac{\pi}{2}$

Take

LHS

$$= \tan^{-1}\frac{1-x^2}{2x} + \cot^{-1}\frac{1-x^2}{2x}$$

We know that, Formula

$$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$$

Thus,

$$= \tan^{-1} \left(\frac{1-x^2}{2x} \right) + \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$= \tan^{-1} \left(\frac{\left(\frac{1-x^2}{2x} \right) + \left(\frac{2x}{1-x^2} \right)}{1 - \left(\frac{1-x^2}{2x} \right) \times \left(\frac{2x}{1-x^2} \right)} \right)$$

$$= \tan^{-1} \left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}} \right)$$

$$= \tan^{-1} \left(\frac{1+x^4+2x^2}{0} \right)$$

$$= \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

= RHS

So,

$$\tan^{-1} \frac{1-x^2}{2x} + \cot^{-1} \frac{1-x^2}{2x} = \frac{\pi}{2}$$

Hence Proved

4 B. Question

Prove that:

$$\sin^{-1} \left\{ \tan^{-1} \frac{1-x^2}{2x} + \cos^{-1} \frac{1-x^2}{1+x^2} \right\} = 1$$

Answer

$$\text{Given:- } \sin \left(\tan^{-1} \frac{1-x^2}{2x} + \cos^{-1} \frac{1-x^2}{1+x^2} \right) = 1$$

Take

LHS

$$= \sin \left(\tan^{-1} \frac{1-x^2}{2x} + \cos^{-1} \frac{1-x^2}{1+x^2} \right)$$

We know that, Formula

$$2\tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

Thus,

$$= \sin \left(\tan^{-1} \frac{1-x^2}{2x} + 2\tan^{-1} x \right)$$

Again,

$$2\tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$= \sin \left(\tan^{-1} \frac{1-x^2}{2x} + \tan^{-1} \left(\frac{2x}{1-x^2} \right) \right)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$= \sin \left(\tan^{-1} \left(\frac{\frac{1-x^2}{2x} + \left(\frac{2x}{1-x^2} \right)}{1 - \frac{1-x^2}{2x} \times \left(\frac{2x}{1-x^2} \right)} \right) \right)$$

$$= \sin \left(\tan^{-1} \left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{\frac{2x(1-x^2)-2x(1-x^2)}{2x(1-x^2)}} \right) \right)$$

$$= \sin \left(\tan^{-1} \left(\frac{\frac{1+x^4-2x^2+4x^2}{2x(1-x^2)}}{0} \right) \right)$$

$$= \sin(\tan^{-1}(\infty))$$

$$= \sin \left(\frac{\pi}{2} \right)$$

$$= 1$$

$$= \text{RHS}$$

So,

$$\sin^{-1} \left(\tan^{-1} \frac{1-x^2}{2x} + \cos^{-1} \frac{1-x^2}{1+x^2} \right) = 1$$

Hence Proved

5. Question

$$\text{If } \sin^{-1} \frac{2a}{1+a^2} - \sin^{-1} \frac{2b}{1+b^2} = \tan^{-1} x, \text{ Prove that } x = \frac{ab}{1-ab}$$

Answer

$$\text{Given:- } \sin^{-1} \left(\frac{2a}{1+a^2} \right) + \sin^{-1} \frac{2b}{1+b^2} = 2\tan^{-1}(x)$$

Take

$$\sin^{-1} \left(\frac{2a}{1+a^2} \right) + \sin^{-1} \frac{2b}{1+b^2} = 2\tan^{-1}(x)$$

We know that, Formula

$$2\tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Thus,

$$\Rightarrow 2\tan^{-1}(a) + 2\tan^{-1}(b) = 2\tan^{-1}(x)$$

$$\Rightarrow 2(\tan^{-1}(a) + \tan^{-1}(b)) = 2\tan^{-1}(x)$$

$$\Rightarrow \tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}(x)$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \left(\frac{a+b}{1-ab} \right) = \tan^{-1}(x)$$

On comparing we get,

$$\Rightarrow x = \frac{a+b}{1-ab}$$

Hence Proved

6. Question

Show that $2 \tan^{-1} x + \sin^{-1} \frac{2x}{1+x^2}$ is constant for $x \geq 1$, find the constant.

Answer

$$\text{Given:- } 2 \tan^{-1}(x) + \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Take

$$2 \tan^{-1}(x) + \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

We know that, Formula

$$2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

Thus,

$$= 2 \tan^{-1}(x) + 2 \tan^{-1}(x)$$

$$= 4 \tan^{-1}(x)$$

Now as given,

For, $x \geq 1$

$$= 4 \tan^{-1}(1)$$

$$= 4 \times \frac{\pi}{4}$$

$$= \pi$$

= Constant

So,

$$2 \tan^{-1}(x) + \sin^{-1} \left(\frac{2x}{1+x^2} \right) = \pi$$

7 A. Question

Find the values of each of the following:

$$\tan^{-1} \left\{ 2 \cos \left(2 \sin^{-1} \frac{1}{2} \right) \right\}$$

Answer

Given:- $\tan^{-1}\left(2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right)$

Take

$$\tan^{-1}\left(2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right)$$

We know that, Formula

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Therefore,

$$\cos\left(2 \times \frac{\pi}{6}\right) = \frac{1}{2}$$

Thus,

$$=\tan^{-1}\left(2\cos\left(2 \times \frac{\pi}{6}\right)\right)$$

$$=\tan^{-1}\left(2\cos\left(\frac{\pi}{3}\right)\right)$$

$$=\tan^{-1}\left(2 \times \frac{1}{2}\right)$$

$$=\tan^{-1}(1)$$

$$= \frac{\pi}{4}$$

So,

$$\tan^{-1}\left(2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right) = \frac{\pi}{4}$$

7 B. Question

Find the values of each of the following:

$$\cos\left(\sec^{-1}x - \operatorname{cosec}^{-1}x\right), |x| \geq 1$$

Answer

Given:- $\cos(\sec^{-1}x - \operatorname{cosec}^{-1}x)$

Take

$$\cos(\sec^{-1}x - \operatorname{cosec}^{-1}x)$$

We know that, Formula

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$$

Therefore,

$$=\cos\left(\frac{\pi}{2}\right)$$

$$= 0$$

So,

$$\Rightarrow \cos(\sec^{-1}x - \operatorname{cosec}^{-1}x) = 0$$

8 A. Question

Solve the following equations for x:

$$\tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

Take

$$\Rightarrow \tan^{-1} \frac{1}{4} + 2 \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

We know that, Formula

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$\Rightarrow \tan^{-1} \frac{1}{4} + \tan^{-1} \left(\frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{4} + \tan^{-1} \left(\frac{\frac{2}{5}}{\frac{24}{25}} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \frac{1}{4} + \tan^{-1} \left(\frac{5}{12} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{1}{4} + \frac{5}{12}}{1 - \frac{1}{4} \times \frac{5}{12}} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{8}{48}}{\frac{43}{48}} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{32}{43} \right) + \tan^{-1} \frac{1}{6} + \tan^{-1} \frac{1}{x} = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

$$\text{And } \tan^{-1} 1 = \frac{\pi}{4}$$

Thus,

$$\Rightarrow \tan^{-1} \left(\frac{\frac{32}{43} + \frac{1}{6}}{1 - \frac{32}{43} \times \frac{1}{6}} \right) + \tan^{-1} \frac{1}{x} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1} \left(\frac{\frac{235}{226}}{\frac{258}{226}} \right) + \tan^{-1} \frac{1}{x} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1} \left(\frac{235}{226} \right) + \tan^{-1} \frac{1}{x} = \tan^{-1} 1$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$$

Thus,

$$\Rightarrow \tan^{-1} \left(\frac{\frac{235}{226} + \frac{1}{x}}{1 - \frac{235}{226} \times \frac{1}{x}} \right) = \tan^{-1} 1, \text{ here } \frac{235}{226x} < 1$$

On comparing we get,

$$\Rightarrow \frac{\frac{235}{226} + \frac{1}{x}}{1 - \frac{235}{226} \times \frac{1}{x}} = 1$$

$$\Rightarrow \frac{235x+226}{226x-235} = 1$$

$$\Rightarrow 235x+226 = 226x-235$$

$$\Rightarrow 235x - 226x = 226 - 235$$

$$\Rightarrow x = -\frac{461}{9}$$

8 B. Question

Solve the following equations for x:

$$3 \sin^{-1} \frac{2x}{1+x^2} - 4 \cos^{-1} \frac{1-x^2}{1+x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$$

Answer

$$\text{Given:- } 3 \sin^{-1} \frac{2x}{1-x^2} - 4 \cos^{-1} \frac{1+x^2}{1-x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$$

Take

$$\Rightarrow 3 \sin^{-1} \frac{2x}{1-x^2} - 4 \cos^{-1} \frac{1+x^2}{1-x^2} + 2 \tan^{-1} \frac{2x}{1-x^2} = \frac{\pi}{3}$$

We know that, Formula

$$2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$$

And

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$\Rightarrow 3(2 \tan^{-1}(x)) - 4(2 \tan^{-1}(x)) + 2(2 \tan^{-1}(x)) = \frac{\pi}{3}$$

$$\Rightarrow 6 \tan^{-1}(x) - 8 \tan^{-1}(x) + 4 \tan^{-1}(x) = \frac{\pi}{3}$$

$$\Rightarrow 2 \tan^{-1}(x) = \frac{\pi}{3}$$

$$\Rightarrow \tan^{-1}(x) = \frac{\pi}{6}$$

$$\Rightarrow x = \tan \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

8 C. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{2\pi}{3}, x > 0$$

Answer

$$\text{Given:- } \tan^{-1}\frac{2x}{1-x^2} + \cot^{-1}\frac{1-x^2}{2x} = \frac{2\pi}{3}, x > 0$$

Take

$$\Rightarrow \tan^{-1}\frac{2x}{1-x^2} + \cot^{-1}\frac{1-x^2}{2x} = \frac{2\pi}{3}$$

We know that, Formula

$$\cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$$

Thus,

$$\Rightarrow \tan^{-1}\frac{2x}{1-x^2} + \tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$\Rightarrow 2\tan^{-1}(x) + 2\tan^{-1}(x) = \frac{2\pi}{3}$$

$$\Rightarrow 4\tan^{-1}(x) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}(x) = \frac{\pi}{6}$$

$$\Rightarrow x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

8 D. Question

Solve the following equations for x:

$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$$

Answer

$$\text{Given:- } 2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), x \neq \frac{\pi}{2}$$

Take

$$2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$$

We know that, Formula

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus, here $x = \sin x$

$$\Rightarrow \tan^{-1}\left(\frac{2 \times \sin x}{1 - \sin^2 x}\right) = \tan^{-1}(2 \sec x)$$

We know that, Formula

$$\cos x = 1 - \sin^2 x$$

$$\Rightarrow \frac{2 \sin x}{\cos^2 x} = 2 \sec x$$

$$\Rightarrow \frac{\sin x}{\cos x} = 1$$

$$\Rightarrow \tan x = 1$$

$$\Rightarrow x = \frac{\pi}{4}$$

Thus the solution is $x = n\pi + \frac{\pi}{4}$

8 E. Question

Solve the following equations for x:

$$\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{2\pi}{3}$$

Answer

$$\text{Given:- } \cos^{-1}\frac{x^2-1}{x^2+1} + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$$

Take

$$\Rightarrow \cos^{-1}\frac{x^2-1}{x^2+1} + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$$

$$\Rightarrow \cos^{-1}\left(-\frac{1-x^2}{x^2+1}\right) + \frac{1}{2}\tan^{-1}\frac{2x}{1-x^2} = \frac{2\pi}{3}$$

We know that, Formula

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

And,

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Thus,

$$\Rightarrow \pi - 2\tan^{-1}(x) + \frac{1}{2} \times 2\tan^{-1}(x) = \frac{2\pi}{3}$$

$$\Rightarrow \pi - \tan^{-1}(x) = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}(x) = \pi - \frac{2\pi}{3}$$

$$\Rightarrow x = \tan \frac{\pi}{3}$$

$$\Rightarrow x = \sqrt{3}$$

8 F. Question

Solve the following equations for x:

$$\tan^{-1}\left(\frac{x-2}{x-1}\right) + \tan^{-1}\left(\frac{x+2}{x+1}\right) = \frac{\pi}{4}$$

Answer

$$\text{Given:- } \tan^{-1}\frac{x-2}{x-1} + \tan^{-1}\frac{x+2}{x+1} = \frac{\pi}{4}$$

Take

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} + \tan^{-1}\frac{x+2}{x+1} = \frac{\pi}{4}$$

We know that, Formula

$$\tan^{-1} 1 = \frac{\pi}{4}$$

Thus,

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} + \tan^{-1}\frac{x+2}{x+1} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} = \tan^{-1} 1 - \tan^{-1}\frac{x+2}{x+1}$$

We know that, Formula

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} = \tan^{-1}\left(\frac{1-\frac{x+2}{x+1}}{1+1 \times \frac{x+2}{x+1}}\right)$$

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} = \tan^{-1}\left(\frac{\frac{x+1-x-2}{x+1}}{\frac{x+1+x+2}{x+1}}\right)$$

$$\Rightarrow \tan^{-1}\frac{x-2}{x-1} = \tan^{-1}\left(\frac{-1}{2x+3}\right)$$

On comparing we get,

$$\Rightarrow \frac{x-2}{x-1} = \frac{-1}{2x+3}$$

$$\Rightarrow \frac{(2x+3)(x-2)}{(x-1)} = -1$$

$$\Rightarrow (2x+3)(x-2) = -(x-1)$$

$$\Rightarrow 2x^2 - 4x + 3x - 6 = -x + 1$$

$$\Rightarrow 2x^2 - x - 6 = -x + 1$$

$$\Rightarrow 2x^2 = 7$$

$$\Rightarrow x = \pm \sqrt{\frac{7}{2}}$$

9. Question

Prove that $2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \frac{\theta}{2} \right) = \cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$

Answer

Given:- $2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \left(\frac{\theta}{2} \right) \right) = \cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$

Take

LHS

$$= 2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \left(\frac{\theta}{2} \right) \right)$$

We know that, Formula

$$2 \tan^{-1} x = \cos^{-1} \left(\frac{1 - x^2}{1 + x^2} \right)$$

Thus,

$$= \cos^{-1} \left(\frac{1 - \left(\sqrt{\frac{a-b}{a+b}} \tan \left(\frac{\theta}{2} \right) \right)^2}{1 + \left(\sqrt{\frac{a-b}{a+b}} \tan \left(\frac{\theta}{2} \right) \right)^2} \right)$$

$$= \cos^{-1} \left(\frac{1 - \frac{a-b}{a+b} \tan^2 \left(\frac{\theta}{2} \right)}{1 + \frac{a-b}{a+b} \tan^2 \left(\frac{\theta}{2} \right)} \right)$$

$$= \cos^{-1} \left(\frac{\frac{a+b-(a-b)}{a+b} \tan^2 \left(\frac{\theta}{2} \right)}{\frac{a+b+(a-b)}{a+b} \tan^2 \left(\frac{\theta}{2} \right)} \right)$$

$$= \cos^{-1} \left(\frac{a+b-(a-b) \tan^2 \left(\frac{\theta}{2} \right)}{a+b+(a-b) \tan^2 \left(\frac{\theta}{2} \right)} \right)$$

$$= \cos^{-1} \left(\frac{a \left(1 - \tan^2 \left(\frac{\theta}{2} \right) \right) + b \left(1 + \tan^2 \left(\frac{\theta}{2} \right) \right)}{a \left(1 + \tan^2 \left(\frac{\theta}{2} \right) \right) + b \left(1 - \tan^2 \left(\frac{\theta}{2} \right) \right)} \right)$$

Dividing numerator and denominator by $\left(1 + \tan^2 \left(\frac{\theta}{2} \right) \right)$, we get

$$= \cos^{-1} \left(\frac{a \left(\frac{1 - \tan^2 \left(\frac{\theta}{2} \right)}{1 + \tan^2 \left(\frac{\theta}{2} \right)} \right) + b}{a + b \left(\frac{1 - \tan^2 \left(\frac{\theta}{2} \right)}{1 + \tan^2 \left(\frac{\theta}{2} \right)} \right)} \right)$$

We know that, Formula

$$\cos x = \frac{1 - \tan^2 \left(\frac{x}{2} \right)}{1 + \tan^2 \left(\frac{x}{2} \right)}$$

Thus,

$$= \cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$$

= RHS

So,

$$2 \tan^{-1} \left(\sqrt{\frac{a-b}{a+b}} \tan \left(\frac{\theta}{2} \right) \right) = \cos^{-1} \left(\frac{a \cos \theta + b}{a + b \cos \theta} \right)$$

Hence Proved

10. Question

prove that:

$$\tan^{-1} \frac{2ab}{a^2 - b^2} + \tan^{-1} \frac{2xy}{x^2 - y^2} = \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}, \text{ where } \alpha = ax - by \text{ and } \beta = ay + bx.$$

Answer

$$\text{Given:- } \tan^{-1} \frac{2ab}{a^2 - b^2} + \tan^{-1} \frac{2xy}{x^2 - y^2} = \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

Take

LHS

$$= \tan^{-1} \frac{2ab}{a^2 - b^2} + \tan^{-1} \frac{2xy}{x^2 - y^2}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$$

Thus,

$$\begin{aligned} &= \tan^{-1} \frac{\left(\frac{2ab}{a^2 - b^2} \right) + \left(\frac{2xy}{x^2 - y^2} \right)}{1 - \left(\frac{2ab}{a^2 - b^2} \right) \left(\frac{2xy}{x^2 - y^2} \right)} \\ &= \tan^{-1} \frac{\frac{2abx^2 - 2aby^2 + 2xya^2 - 2xyb^2}{(a^2 - b^2)(x^2 - y^2)}}{\frac{a^2x^2 + b^2y^2 - 2abxy - a^2y^2 - b^2x^2 - 2abxy}{(a^2 - b^2)(x^2 - y^2)}} \\ &= \tan^{-1} \frac{2(abx^2 - aby^2 + xya^2 - xyb^2)}{a^2x^2 + b^2y^2 - 2abxy - a^2y^2 - b^2x^2 - 2abxy} \end{aligned}$$

Formula used:- $a^2 + b^2 + 2ab = (a+b)^2$

$$\begin{aligned} &= \tan^{-1} \frac{2\{(bx+ay)(ax-by)\}}{(ax-by)^2 - (a^2y^2 + b^2x^2 + 2abxy)} \\ &= \tan^{-1} \frac{2\{ax(bx+ay) + by(ay+bx)\}}{(ax-by)^2 - (bx+ay)^2} \end{aligned}$$

As given

$$\alpha = ax - by \text{ and } \beta = ay + bx$$

Thus,

$$\tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

= RHS

So,

$$\tan^{-1} \frac{2ab}{a^2 - b^2} + \tan^{-1} \frac{2xy}{x^2 - y^2} = \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

Hence Proved

11. Question

For any $a, b, x, y > 0$, prove that:

$$\frac{2}{3} \tan^{-1} \left(\frac{3ab^2 - a^3}{b^3 - 3a^2b} \right) + \frac{2}{3} \tan^{-1} \left(\frac{3xy^2 - x^3}{y^3 - 3x^2y} \right) = \tan^{-1} \frac{2a\beta}{a^2 - b^2}, \text{ where } a = -ax \text{ by, } \beta = bx + ay.$$

Answer

$$\text{Given: } -\frac{2}{3} \tan^{-1} \frac{3ab^2 - a^3}{b^3 - 3a^2b} + \frac{2}{3} \tan^{-1} \frac{3xy^2 - x^3}{y^3 - 3x^2y} = \tan^{-1} \frac{2a\beta}{a^2 - b^2}$$

Take

LHS

$$= \frac{2}{3} \tan^{-1} \frac{3ab^2 - a^3}{b^3 - 3a^2b} + \frac{2}{3} \tan^{-1} \frac{3xy^2 - x^3}{y^3 - 3x^2y}$$

Dividing numerator and denominator of 1st term and 2nd term by b^3 and y^3 respectively.

$$\begin{aligned} &= \frac{2}{3} \tan^{-1} \frac{\frac{3ab^2 - a^3}{b^3}}{\frac{b^3 - 3a^2b}{b^3}} + \frac{2}{3} \tan^{-1} \frac{\frac{3xy^2 - x^3}{y^3}}{\frac{y^3 - 3x^2y}{y^3}} \\ &= \frac{2}{3} \tan^{-1} \frac{3\left(\frac{a}{b}\right) - \left(\frac{a}{b}\right)^3}{1 - 3\left(\frac{a}{b}\right)^2} + \frac{2}{3} \tan^{-1} \frac{3\left(\frac{x}{y}\right) - \left(\frac{x}{y}\right)^3}{1 - 3\left(\frac{x}{y}\right)^2} \end{aligned}$$

We know that, Formula

$$3 \tan^{-1} x = \tan^{-1} \left(\frac{3x - x^3}{1 - 3x^2} \right)$$

Thus,

$$\begin{aligned} &= \frac{2}{3} \left\{ 3 \tan^{-1} \left(\frac{a}{b} \right) \right\} + \frac{2}{3} \left\{ 3 \tan^{-1} \left(\frac{x}{y} \right) \right\} \\ &= 2 \tan^{-1} \left(\frac{a}{b} \right) + 2 \tan^{-1} \left(\frac{x}{y} \right) \\ &= 2 \left(\tan^{-1} \left(\frac{a}{b} \right) + \tan^{-1} \left(\frac{x}{y} \right) \right) \end{aligned}$$

We know that, Formula

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x + y}{1 - xy}$$

Thus,

$$\begin{aligned} &= 2 \tan^{-1} \frac{\left(\frac{a}{b}\right) + \left(\frac{x}{y}\right)}{1 - \left(\frac{a}{b}\right)\left(\frac{x}{y}\right)} \\ &= 2 \tan^{-1} \frac{\left(\frac{ay + bx}{by}\right)}{\left(\frac{by - ax}{by}\right)} \\ &= 2 \tan^{-1} \frac{ay + bx}{by - ax} \end{aligned}$$

As given,

$$ay + bx = \beta, -ax + by = \alpha$$

$$= 2 \tan^{-1} \frac{\beta}{\alpha}$$

We know that, Formula

$$2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

Thus,

$$= \tan^{-1} \left(\frac{2 \times \frac{\beta}{\alpha}}{1 - \left(\frac{\beta}{\alpha} \right)^2} \right)$$

$$= \tan^{-1} \left(\frac{2\beta}{\alpha} \times \frac{\alpha^2}{\alpha^2 - \beta^2} \right)$$

$$= \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

= RHS

So,

$$\frac{2}{3} \tan^{-1} \frac{3ab^2 - a^3}{b^3 - 3a^2b} + \frac{2}{3} \tan^{-1} \frac{3xy^2 - x^3}{y^2 - 3x^2y} = \tan^{-1} \frac{2\alpha\beta}{\alpha^2 - \beta^2}$$

Hence Proved

MCQ

1. Question

Choose the correct answer

$$\text{If } \tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha, \text{ then } x^2 =$$

A. $\sin 2\alpha$

B. $\sin \alpha$

C. $\cos 2\alpha$

D. $\cos \alpha$

Answer

We are given that,

$$\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$$

We need to find the value of x^2 .

Take,

$$\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} = \alpha$$

Multiply on both sides by tangent.

$$\Rightarrow \tan \left[\tan^{-1} \left\{ \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right\} \right] = \tan \alpha$$

Since, we know that $\tan(\tan^{-1} x) = x$.

So,

$$\Rightarrow \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} = \tan \alpha$$

Or

$$\tan \alpha = \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}$$

Now, we need to simplify it in order to find x^2 . So, rationalize the denominator by multiplying and dividing by $\sqrt{1+x^2} - \sqrt{1-x^2}$.

$$\begin{aligned}\Rightarrow \tan \alpha &= \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} \right) \times \left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right) \\ &= \frac{(\sqrt{1+x^2} - \sqrt{1-x^2})^2}{(\sqrt{1+x^2} + \sqrt{1-x^2})(\sqrt{1+x^2} - \sqrt{1-x^2})}\end{aligned}$$

Note the denominator is in the form: $(x + y)(x - y)$, where

$$(x + y)(x - y) = x^2 - y^2$$

So,

$$\Rightarrow \tan \alpha = \frac{(\sqrt{1+x^2} - \sqrt{1-x^2})^2}{(\sqrt{1+x^2})^2 - (\sqrt{1-x^2})^2} \dots (i)$$

Numerator:

Applying the algebraic identity in the numerator, $(x - y)^2 = x^2 + y^2 - 2xy$.

We can write as,

$$\begin{aligned}(\sqrt{1+x^2} - \sqrt{1-x^2})^2 &= (\sqrt{1+x^2})^2 + (\sqrt{1-x^2})^2 - 2\sqrt{1+x^2}\sqrt{1-x^2} \\ \Rightarrow (\sqrt{1+x^2} - \sqrt{1-x^2})^2 &= (1+x^2) + (1-x^2) - 2\sqrt{(1+x^2)(1-x^2)}\end{aligned}$$

Again using the identity, $(x + y)(x - y) = x^2 - y^2$.

$$\begin{aligned}\Rightarrow (\sqrt{1+x^2} - \sqrt{1-x^2})^2 &= 1+x^2 + 1-x^2 - 2\sqrt{1-(x^2)^2} \\ \Rightarrow (\sqrt{1+x^2} - \sqrt{1-x^2})^2 &= 2 - 2\sqrt{1-x^4} \dots (ii)\end{aligned}$$

Denominator:

Solving the denominator, we get

$$\begin{aligned}(\sqrt{1+x^2})^2 - (\sqrt{1-x^2})^2 &= (1+x^2) - (1-x^2) \\ \Rightarrow (\sqrt{1+x^2})^2 - (\sqrt{1-x^2})^2 &= 1+x^2 - 1+x^2 \\ \Rightarrow (\sqrt{1+x^2})^2 - (\sqrt{1-x^2})^2 &= 2x^2 \dots (iii)\end{aligned}$$

Substituting values of Numerator and Denominator from (ii) and (iii) in equation (i),

$$\Rightarrow \tan \alpha = \frac{2 - 2\sqrt{1-x^4}}{2x^2}$$

$$\Rightarrow \tan \alpha = \frac{2(1 - \sqrt{1 - x^4})}{2x^2}$$

$$\Rightarrow \tan \alpha = \frac{1 - \sqrt{1 - x^4}}{x^2}$$

By cross-multiplication,

$$\Rightarrow x^2 \tan \alpha = 1 - \sqrt{1 - x^4}$$

$$\Rightarrow \sqrt{1 - x^4} = 1 - x^2 \tan \alpha$$

Squaring on both sides,

$$\Rightarrow [\sqrt{1 - x^4}]^2 = [1 - x^2 \tan \alpha]^2$$

$$\Rightarrow 1 - x^4 = (1)^2 + (x^2 \tan \alpha)^2 - 2x^2 \tan \alpha \quad [\because (x - y)^2 = x^2 + y^2 - 2xy]$$

$$\Rightarrow 1 - x^4 = 1 + x^4 \tan^2 \alpha - 2x^2 \tan \alpha$$

$$\Rightarrow x^4 \tan^2 \alpha - 2x^2 \tan \alpha + x^4 + 1 - 1 = 0$$

$$\Rightarrow x^4 \tan^2 \alpha - 2x^2 \tan \alpha + x^4 = 0$$

Rearranging,

$$\Rightarrow x^4 + x^4 \tan^2 \alpha - 2x^2 \tan \alpha = 0$$

$$\Rightarrow x^4 (1 + \tan^2 \alpha) - 2x^2 \tan \alpha = 0$$

$$\Rightarrow x^4 (\sec^2 \alpha) - 2x^2 \tan \alpha = 0 \quad [\because \sec^2 x - \tan^2 x = 1 \Rightarrow 1 + \tan^2 x = \sec^2 x]$$

Taking x^2 common from both terms,

$$\Rightarrow x^2 (x^2 \sec^2 \alpha - 2 \tan \alpha) = 0$$

$$\Rightarrow x^2 = 0 \text{ or } (x^2 \sec^2 \alpha - 2 \tan \alpha) = 0$$

But $x^2 \neq 0$ as according to the question, we need to find some value of x^2 .

$$\Rightarrow x^2 \sec^2 \alpha - 2 \tan \alpha = 0$$

$$\Rightarrow x^2 \sec^2 \alpha = 2 \tan \alpha$$

In order to find the value of x^2 , shift $\sec^2 \alpha$ to Right Hand Side (RHS).

$$\Rightarrow x^2 = \frac{2 \tan \alpha}{\sec^2 \alpha}$$

Putting $\sec^2 \alpha = \frac{1}{\cos^2 \alpha}$ and $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$,

$$\Rightarrow x^2 = \frac{2 \left(\frac{\sin \alpha}{\cos \alpha} \right)}{\frac{1}{\cos^2 \alpha}}$$

$$\Rightarrow x^2 = 2 \times \frac{\sin \alpha}{\cos \alpha} \times \cos^2 \alpha$$

$$\Rightarrow x^2 = 2 \sin \alpha \cos \alpha$$

Using the trigonometric identity, $2 \sin x \cos x = \sin 2x$.

$$\Rightarrow x^2 = \sin 2\alpha$$

2. Question

Choose the correct answer

The value of $\tan \left\{ \cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right\}$ is

A. $\frac{\sqrt{29}}{3}$

B. $\frac{29}{3}$

C. $\frac{\sqrt{3}}{29}$

D. $\frac{3}{29}$

Answer

We need to find the value of

$$\tan \left\{ \cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right\}$$

Let,

$$\cos^{-1} \frac{1}{5\sqrt{2}} = a \text{ and } \sin^{-1} \frac{4}{\sqrt{17}} = b$$

$$\Rightarrow \cos a = \frac{1}{5\sqrt{2}} \text{ and } \sin b = \frac{4}{\sqrt{17}}$$

Let us find $\sin a$ and $\cos b$.

For $\sin a$,

We know the trigonometric identity, $\sin^2 a + \cos^2 a = 1$

$$\Rightarrow \sin^2 a = 1 - \cos^2 a$$

$$\Rightarrow \sin a = \sqrt{1 - \cos^2 a}$$

Substituting the value of $\cos a$,

$$\Rightarrow \sin a = \sqrt{1 - \left(\frac{1}{5\sqrt{2}} \right)^2}$$

$$= \sqrt{1 - \frac{1}{50}}$$

$$= \sqrt{\frac{50 - 1}{50}}$$

$$= \sqrt{\frac{49}{50}}$$

$$= \frac{7}{5\sqrt{2}}$$

We have $\sin a = \frac{7}{5\sqrt{2}}$ and $\cos a = \frac{1}{5\sqrt{2}}$.

So, we can find $\tan a$.

$$\therefore, \tan a = \frac{\sin a}{\cos a}$$

$$= \frac{\frac{7}{5\sqrt{2}}}{\frac{1}{5\sqrt{2}}}$$

$$= \frac{7}{5\sqrt{2}} \times 5\sqrt{2}$$

$$\Rightarrow \tan a = 7 \dots (i)$$

For $\cos b$,

We know the trigonometric identity,

$$\sin^2 b + \cos^2 b = 1$$

$$\Rightarrow \cos^2 b = 1 - \sin^2 b$$

$$\Rightarrow \cos b = \sqrt{1 - \sin^2 b}$$

Substituting the value of $\sin b$,

$$\Rightarrow \cos b = \sqrt{1 - \left(\frac{4}{\sqrt{17}}\right)^2}$$

$$= \sqrt{1 - \frac{16}{17}}$$

$$= \sqrt{\frac{17 - 16}{17}}$$

$$= \frac{1}{\sqrt{17}}$$

$$\text{We have } \sin b = \frac{4}{\sqrt{17}} \text{ and } \cos b = \frac{1}{\sqrt{17}}.$$

So, we can find $\tan b$.

$$\therefore, \tan b = \frac{\sin b}{\cos b}$$

$$= \frac{\frac{4}{\sqrt{17}}}{\frac{1}{\sqrt{17}}}$$

$$= \frac{4}{\sqrt{17}} \times \sqrt{17}$$

$$\Rightarrow \tan b = 4 \dots (ii)$$

We can write as,

$$\tan \left\{ \cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right\} = \tan \{a - b\}$$

Now, we need to solve Right Hand Side (RHS).

We know the trigonometric identity,

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

Substituting the values of $\tan a$ and $\tan b$ from (i) and (ii),

$$= \frac{7 - 4}{1 + (7)(4)}$$

$$= \frac{3}{1 + 28}$$

$$= \frac{3}{29}$$

So,

$$\tan \left\{ \cos^{-1} \frac{1}{5\sqrt{2}} - \sin^{-1} \frac{4}{\sqrt{17}} \right\} = \frac{3}{29}$$

3. Question

Choose the correct answer

$2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)|$ is equal to

A. $\cot^{-1} x$

B. $\cot^{-1} \frac{1}{x}$

C. $\tan^{-1} x$

D. none of these

Answer

We need to find the value of $2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)|$.

So, take

$$2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)|$$

Using property of inverse trigonometry,

$$\cot^{-1} x = \tan^{-1} \frac{1}{x}$$

$$\Rightarrow 2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)| \\ = 2 \tan^{-1} \left| \operatorname{cosec}(\tan^{-1} x) - \tan\left(\tan^{-1} \frac{1}{x}\right) \right|$$

$$= 2 \tan^{-1} \left| \operatorname{cosec}(\tan^{-1} x) - \frac{1}{x} \right|$$

Now, let $y = \tan^{-1} x$

So, $\tan y = x$

Substituting the value of $\tan^{-1} x$ and x in the equation,

$$\Rightarrow 2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)| = 2 \tan^{-1} \left| \operatorname{cosec} y - \frac{1}{\tan y} \right|$$

Put,

$$\operatorname{cosec} y = \frac{1}{\sin y} \text{ and } \tan y = \frac{\sin y}{\cos y}$$

$$\Rightarrow 2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)| = 2 \tan^{-1} \left| \frac{1}{\sin y} - \frac{1}{\frac{\sin y}{\cos y}} \right|$$

$$= 2 \tan^{-1} \left| \frac{1}{\sin y} - \frac{\cos y}{\sin y} \right|$$

$$= 2 \tan^{-1} \left| \frac{1 - \cos y}{\sin y} \right|$$

Since, we know the trigonometric identity,

$$1 - \cos 2y = 2 \sin^2 y$$

$$\Rightarrow 1 - \cos y = 2 \sin^2 \frac{y}{2}$$

$$\text{Also, } \sin 2y = 2 \sin y \cos y$$

$$\Rightarrow \sin y = 2 \sin \frac{y}{2} \cos \frac{y}{2}$$

We get,

$$= 2 \tan^{-1} \left| \frac{2 \sin^2 \frac{y}{2}}{2 \sin \frac{y}{2} \cos \frac{y}{2}} \right|$$

$$= 2 \tan^{-1} \left| \frac{\sin \frac{y}{2}}{\cos \frac{y}{2}} \right|$$

Since,

$$\tan y = \frac{\sin y}{\cos y}$$

Then,

$$= 2 \tan^{-1} \left| \tan \frac{y}{2} \right|$$

$$= 2 \times \frac{y}{2}$$

$$\Rightarrow 2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)| = y$$

Put $y = \tan^{-1} x$ as let above.

$$\Rightarrow 2 \tan^{-1} |\operatorname{cosec}(\tan^{-1} x) - \tan(\cot^{-1} x)| = \tan^{-1} x$$

4. Question

Choose the correct answer

$$\text{If } \cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha, \text{ then } \frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} =$$

A. $\sin^2 \alpha$

B. $\cos^2 \alpha$

C. $\tan^2 \alpha$

D. $\cot^2 \alpha$

Answer

We are given that,

$$\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$$

We need to find the value of

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2}$$

By property of inverse trigonometry,

$$\cos^{-1} a + \cos^{-1} b = \cos^{-1}(ab - \sqrt{(1-a^2)}\sqrt{(1-b^2)})$$

So,

$$\cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$$

$$\Rightarrow \cos^{-1} \left(\left(\frac{x}{a} \right) \left(\frac{y}{b} \right) - \sqrt{1 - \left(\frac{x}{a} \right)^2} \sqrt{1 - \left(\frac{y}{b} \right)^2} \right) = \alpha$$

Simplifying further,

$$\Rightarrow \cos^{-1} \left(\frac{xy}{ab} - \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}} \right) = \alpha$$

Taking cosine on both sides,

$$\Rightarrow \cos \left[\cos^{-1} \left(\frac{xy}{ab} - \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}} \right) \right] = \cos \alpha$$

Using the property of inverse trigonometric function,

$$\cos(\cos^{-1} x) = x$$

$$\Rightarrow \frac{xy}{ab} - \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}} = \cos \alpha$$

$$\Rightarrow \frac{xy}{ab} - \cos \alpha = \sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}}$$

To simplify it further, take square on both sides.

$$\Rightarrow \left[\frac{xy}{ab} - \cos \alpha \right]^2 = \left[\sqrt{1 - \frac{x^2}{a^2}} \sqrt{1 - \frac{y^2}{b^2}} \right]^2$$

Using algebraic identity,

$$(x - y)^2 = x^2 + y^2 - 2xy$$

$$\Rightarrow \left(\frac{xy}{ab} \right)^2 + \cos^2 \alpha - \frac{2xy}{ab} \cos \alpha = \left(1 - \frac{x^2}{a^2} \right) \left(1 - \frac{y^2}{b^2} \right)$$

Simplifying it further,

$$\Rightarrow \frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - \frac{2xy}{ab} \cos \alpha = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}$$

Shifting all terms at one side,

$$\Rightarrow \frac{x^2 y^2}{a^2 b^2} - \frac{x^2 y^2}{a^2 b^2} + \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab} \cos \alpha = 1 - \cos^2 \alpha$$

Using trigonometric identity,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

We get,

$$\frac{x^2}{a^2} - \frac{2xy}{ab} \cos \alpha + \frac{y^2}{b^2} = \sin^2 \alpha$$

5. Question

Choose the correct answer

The positive integral solution of the equation $\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$ is

- A. $x = 1, y = 2$
- B. $x = 2, y = 1$
- C. $x = 3, y = 2$
- D. $x = -2, y = -1$

Answer

We need to find the positive integral solution of the equation:

$$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

Using property of inverse trigonometry,

$$\cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$$

Also,

$$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$$

Taking,

$$\tan^{-1} x + \cos^{-1} \frac{y}{\sqrt{1+y^2}} = \sin^{-1} \frac{3}{\sqrt{10}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{\sqrt{1 - \left(\frac{y}{\sqrt{1+y^2}}\right)^2}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1} \frac{\frac{3}{\sqrt{10}}}{\sqrt{1 - \left(\frac{3}{\sqrt{10}}\right)^2}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{\sqrt{1 - \frac{y^2}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1} \frac{\frac{3}{\sqrt{10}}}{\sqrt{1 - \frac{9}{10}}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{\sqrt{\frac{1+y^2-y^2}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1} \frac{\frac{3}{\sqrt{10}}}{\sqrt{\frac{10-9}{10}}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{\sqrt{\frac{1}{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}} = \tan^{-1} \frac{\frac{3}{\sqrt{10}}}{\sqrt{\frac{1}{10}}}$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \left(\frac{1}{\sqrt{1+y^2}} \times \frac{\sqrt{1+y^2}}{y} \right) = \tan^{-1} \left(\frac{3}{\sqrt{10}} \times \sqrt{10} \right)$$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \frac{1}{y} = \tan^{-1} 3$$

Using the property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

Similarly,

$$\Rightarrow \tan^{-1} \left(\frac{x + \frac{1}{y}}{1 - (x) \left(\frac{1}{y} \right)} \right) = \tan^{-1} 3$$

Taking tangent on both sides of the equation,

$$\Rightarrow \tan \left[\tan^{-1} \left(\frac{x + \frac{1}{y}}{1 - \frac{x}{y}} \right) \right] = \tan[\tan^{-1} 3]$$

Using property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

Applying this property on both sides of the equation,

$$\Rightarrow \frac{x + \frac{1}{y}}{1 - \frac{x}{y}} = 3$$

Simplifying the equation,

$$\Rightarrow \frac{\frac{xy+1}{y}}{\frac{y-x}{y}} = 3$$

$$\Rightarrow \frac{xy+1}{y} \times \frac{y}{y-x} = 3$$

$$\Rightarrow \frac{xy+1}{y-x} = 3$$

Cross-multiplying in the equation,

$$\Rightarrow xy + 1 = 3(y - x)$$

$$\Rightarrow xy + 1 = 3y - 3x$$

$$\Rightarrow xy + 3x = 3y - 1$$

$$\Rightarrow x(y + 3) = 3y - 1$$

$$\Rightarrow x = \frac{3y - 1}{y + 3}$$

We need to find positive integral solutions using the above result.

That is, we need to find solution which is positive as well as in integer form. A positive integer are all natural numbers.

That is, $x, y > 0$.

So, keep the values of $y = 1, 2, 3, 4, \dots$ and find x .

x	$\frac{3(1) - 1}{1 + 3} = \frac{1}{2}$	$\frac{3(2) - 1}{2 + 3} = 1$	$\frac{3(3) - 1}{3 + 3} = \frac{4}{3}$	$\frac{3(4) - 1}{4 + 3} = \frac{11}{7}$	$\frac{3(5) - 1}{5 + 3} = \frac{7}{4}$
y	1	2	3	4	5

Note that, only at $y = 2$, value is x is positive integer.

Thus, the positive integral solution of the given equation is $x = 1, y = 2$.

6. Question

Choose the correct answer

If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then $x =$

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{2}$

C. $-\frac{1}{2}$

D. none of these

Answer

We are given that,

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6} \dots (i)$$

We need to find the value of x .

By using the property of inverse trigonometry,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

We can find the value of $\sin^{-1} x$ in the terms of $\cos^{-1} x$.

$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$$

Substituting the value of $\sin^{-1} x$ in equation (i),

$$\left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = \frac{\pi}{6}$$

Simplifying it further,

$$\frac{\pi}{2} - \cos^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow \frac{\pi}{2} - 2 \cos^{-1} x = \frac{\pi}{6}$$

$$\Rightarrow 2 \cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{3\pi - \pi}{6}$$

$$= \frac{2\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{6}$$

Multiplying cosine on both sides of the equation,

$$\Rightarrow \cos[\cos^{-1} x] = \cos \frac{\pi}{6}$$

Using property of inverse trigonometry,

$$\cos[\cos^{-1} x] = x$$

$$\Rightarrow x = \cos \frac{\pi}{6}$$

And we know the value,

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

Therefore,

$$x = \frac{\sqrt{3}}{2}$$

7. Question

Choose the correct answer

$\sin \left[\cot^{-1} \left\{ \tan \left(\cos^{-1} x \right) \right\} \right]$ is equal to

A. x

B. $\sqrt{1-x^2}$

C. $\frac{1}{x}$

D. none of these

Answer

We need to find the value of

$$\sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] \dots (i)$$

We can solve such equation by letting the inner most trigonometric function (here, $\cos^{-1} x$) as some variable, and solve systematically following BODMAS rule and other trigonometric identities.

$$\text{Let } \cos^{-1} x = y$$

We can re-write the equation (i),

$$\sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] = \sin [\cot^{-1} \{ \tan y \}]$$

Using trigonometric identity,

$$\tan y = \cot \left(\frac{\pi}{2} - y \right)$$

$\therefore \cot \left(\frac{\pi}{2} - y \right)$ lies in 1st Quadrant and sine, cosine, tangent and cot are positive in 1st Quadrant]

$$\Rightarrow \sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] = \sin \left[\cot^{-1} \left\{ \cot \left(\frac{\pi}{2} - y \right) \right\} \right]$$

Using property of inverse trigonometry,

$$\cot^{-1}(\cot x) = x$$

$$\Rightarrow \sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] = \sin \left[\frac{\pi}{2} - y \right]$$

Using trigonometric identity,

$$\cos y = \sin \left(\frac{\pi}{2} - y \right)$$

Substituting this value of $\sin \left(\frac{\pi}{2} - y \right)$,

$$\Rightarrow \sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] = \cos y$$

We had let above that $\cos^{-1} x = y$.

If,

$$\cos^{-1} x = y$$

$$\Rightarrow x = \cos y$$

Therefore,

$$\sin [\cot^{-1} \{ \tan (\cos^{-1} x) \}] = x$$

8. Question

Choose the correct answer

The number of solutions of the equation $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

A. 2

B. 3

C. 1

D. none of these

Answer

We need to find the number of solutions of the equation,

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

We shall apply the property of inverse trigonometry, that is,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

So,

$$\tan^{-1} \left(\frac{2x+3x}{1-(2x)(3x)} \right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left(\frac{5x}{1-6x^2} \right) = \frac{\pi}{4}$$

Taking tangent on both sides of the equation,

$$\Rightarrow \tan \left[\tan^{-1} \left(\frac{5x}{1-6x^2} \right) \right] = \tan \frac{\pi}{4}$$

Using property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

Also,

$$\tan \frac{\pi}{4} = 1$$

We get,

$$\Rightarrow \frac{5x}{1-6x^2} = 1$$

Simplifying it,

$$\Rightarrow 5x = 1 - 6x^2$$

$$\Rightarrow 6x^2 + 5x - 1 = 0$$

Since, this is a quadratic equation, it is clear that it will have 2 solutions.

Let us check:

We have,

$$6x^2 + 5x - 1 = 0$$

$$\Rightarrow 6x^2 + 6x - x - 1 = 0$$

$$\Rightarrow 6x(x+1) - (x+1) = 0$$

$$\Rightarrow (6x-1)(x+1) = 0$$

$$\Rightarrow (6x-1) = 0 \text{ or } (x+1) = 0$$

$$\Rightarrow 6x = 1 \text{ or } x = -1$$

$$\Rightarrow x = \frac{1}{6} \text{ or } x = -1$$

Hence, there are 2 solutions of the given equation.

9. Question

Choose the correct answer

If $\alpha = \tan^{-1}\left(\tan \frac{5\pi}{4}\right)$ and $\beta = \tan^{-1}\left(-\tan \frac{2\pi}{3}\right)$, then

A. $4\alpha = 3\beta$

B. $3\alpha = 4\beta$

C. $\alpha - \beta = \frac{7\pi}{12}$

D. none of these

Answer

We are given that,

$$\alpha = \tan^{-1}\left(\tan \frac{5\pi}{4}\right) \text{ and } \beta = \tan^{-1}\left(-\tan \frac{2\pi}{3}\right)$$

Take,

$$\alpha = \tan^{-1}\left(\tan \frac{5\pi}{4}\right)$$

We can write $\frac{5\pi}{4}$ as,

$$\frac{5\pi}{4} = \pi + \frac{\pi}{4}$$

Then,

$$\alpha = \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{4}\right)\right)$$

Also, by trigonometric identity

$$\tan\left(\pi + \frac{\pi}{4}\right) = \tan \frac{\pi}{4}$$

[$\therefore \tan\left(\pi + \frac{\pi}{4}\right)$ lies in III Quadrant and tangent is positive in III Quadrant]

$$\Rightarrow \alpha = \tan^{-1}\left(\tan \frac{\pi}{4}\right)$$

Using the property of inverse trigonometry, that is, $\tan^{-1}(\tan A) = A$.

$$\Rightarrow \alpha = \frac{\pi}{4}$$

Now, take

$$\beta = \tan^{-1}\left(-\tan \frac{2\pi}{3}\right)$$

We can write $\frac{2\pi}{3}$ as,

$$\frac{2\pi}{3} = \pi - \frac{\pi}{3}$$

Then,

$$\beta = \tan^{-1}\left(-\tan\left(\pi - \frac{\pi}{3}\right)\right)$$

By trigonometric identity,

$$\tan\left(\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3}$$

$\therefore \tan\left(\pi - \frac{\pi}{3}\right)$ lies in II Quadrant and tangent is negative in II Quadrant]

$$\Rightarrow \beta = \tan^{-1}\left(-\left(-\tan\frac{\pi}{3}\right)\right)$$

$$\Rightarrow \beta = \tan^{-1}\left(\tan\frac{\pi}{3}\right)$$

Using the property of inverse trigonometry, that is, $\tan^{-1}(\tan A) = A$.

$$\Rightarrow \beta = \frac{\pi}{3}$$

We have,

$$\alpha = \frac{\pi}{4} \text{ and } \beta = \frac{\pi}{3}$$

$$\Rightarrow 4\alpha = \pi \text{ and } 3\beta = \pi$$

Since, the values of 4α and 3β are same, that is,

$$4\alpha = 3\beta = \pi$$

Therefore,

$$4\alpha = 3\beta$$

10. Question

Choose the correct answer

The number of real solutions of the equation $\sqrt{1 + \cos 2x} = \sqrt{2} \sin^{-1}(\sin x)$, $-\pi \leq x \leq \pi$ is

- A. 0
- B. 1
- C. 2
- D. infinite

Answer

We are given with equation:

$$\sqrt{1 + \cos 2x} = \sqrt{2} \sin^{-1}(\sin x) \dots(i)$$

Where $-\pi \leq x \leq \pi$

We need to find the number of real solutions of the given equation.

Using trigonometric identity,

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos 2x = \cos^2 x - (1 - \cos^2 x) [\because \sin^2 x + \cos^2 x = 1 \Rightarrow \sin^2 x = 1 - \cos^2 x]$$

$$\Rightarrow \cos 2x = \cos^2 x - 1 + \cos^2 x$$

$$\Rightarrow \cos 2x = 2 \cos^2 x - 1$$

$$\Rightarrow 1 + \cos 2x = 2 \cos^2 x$$

Substituting the value of $(1 + \cos 2x)$ in equation (i),

$$\sqrt{2 \cos^2 x} = \sqrt{2} \sin^{-1}(\sin x)$$

$$\Rightarrow \sqrt{2} |\cos x| = \sqrt{2} \sin^{-1}(\sin x)$$

$\sqrt{2}$ will get cancelled from each sides,

$$\Rightarrow |\cos x| = \sin^{-1}(\sin x)$$

Take interval $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$:

$|\cos x|$ is positive in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, hence $|\cos x| = \cos x$.

And, $\sin x$ is also positive in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, hence $\sin^{-1}(\sin x) = x$.

$$\text{So, } |\cos x| = \sin^{-1}(\sin x)$$

$$\Rightarrow \cos x = x$$

If we draw $y = \cos x$ and $y = x$ on the same graph, we will notice that they intersect at one point, thus giving us 1 solution.

$\therefore$, There is 1 solution of the given equation in interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Take interval $x \in \left[-\pi, -\frac{\pi}{2}\right)$:

$|\cos x|$ is negative in interval $\left[-\pi, -\frac{\pi}{2}\right)$, hence $|\cos x| = -\cos x$.

And, $\sin x$ is also negative in interval $\left[-\pi, -\frac{\pi}{2}\right)$, hence $\sin^{-1}(\sin (\pi + x)) = \pi + x$.

$$\text{So, } |\cos x| = \sin^{-1}(\sin x)$$

$$\Rightarrow -\cos x = \pi + x$$

$$\Rightarrow \cos x = -\pi - x$$

If we draw $y = \cos x$ and $y = -\pi - x$ on the same graph, we will notice that they intersect at one point, thus giving us 1 solution.

$\therefore$, There is 1 solution of the given equation in interval $\left[-\pi, -\frac{\pi}{2}\right)$.

Take interval $x \in \left(\frac{\pi}{2}, \pi\right]$:

$|\cos x|$ is negative in interval $\left(\frac{\pi}{2}, \pi\right]$, hence $|\cos x| = -\cos x$.

And, $\sin x$ is positive in interval $\left(\frac{\pi}{2}, \pi\right]$, hence $\sin^{-1}(\sin (-\pi - x)) = -\pi - x$.

$$\text{So, } |\cos x| = \sin^{-1}(\sin x)$$

$$\Rightarrow -\cos x = -\pi - x$$

$$\Rightarrow -\cos x = -(\pi + x)$$

$$\Rightarrow \cos x = \pi + x$$

If we draw $y = \cos x$ and $y = \pi + x$ on the same graph, we will notice that they doesn't intersect at any point, thus giving us no solution.

$\therefore$, There is 0 solution of the given equation in interval $\left(\frac{\pi}{2}, \pi\right]$.

Hence, we get 2 solutions of the given equation in interval $[-\pi, \pi]$.

11. Question

Choose the correct answer

If $x < 0$, $y < 0$ such that $xy = 1$, then $\tan^{-1}x + \tan^{-1}y$ equals

A. $\frac{\pi}{2}$

B. $-\frac{\pi}{2}$

C. $-\pi$

D. none of these

Answer

We are given that,

$$xy = 1, x < 0 \text{ and } y < 0$$

We need to find the value of $\tan^{-1}x + \tan^{-1}y$.

Using the property of inverse trigonometry,

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

We already know the value of xy , that is, $xy = 1$.

Also, we know that $x, y < 0$.

Substituting $xy = 1$ in denominator,

$$\begin{aligned}\tan^{-1}x + \tan^{-1}y &= \tan^{-1}\left(\frac{x+y}{1-1}\right) \\ &= \tan^{-1}\left(\frac{x+y}{0}\right)\end{aligned}$$

And since $(x+y) = \text{negative value} = \text{integer} = -a$ (say).

$$= \tan^{-1}\left(-\frac{a}{0}\right)$$

$$\Rightarrow \tan^{-1}x + \tan^{-1}y = \tan^{-1} -\infty \dots (i)$$

Using value of inverse trigonometry,

$$\tan^{-1} -\infty = -\frac{\pi}{2}$$

Substituting the value of $\tan^{-1} -\infty$ in the equation (i), we get

$$\tan^{-1}x + \tan^{-1}y = -\frac{\pi}{2}$$

12. Question

Choose the correct answer

If $u = \cot^{-1}\left\{\sqrt{\tan \theta}\right\} - \tan^{-1}\left\{\sqrt{\tan \theta}\right\}$ then, $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right) =$

A. $\sqrt{\tan \theta}$

B. $\sqrt{\cot \theta}$

C. $\tan \theta$

D. $\cot \theta$

Answer

We are given with

$$u = \cot^{-1}\{\sqrt{\tan \theta}\} - \tan^{-1}\{\sqrt{\tan \theta}\}$$

We need to find the value of $\tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$.

$$\text{Let } \sqrt{\tan \theta} = x$$

Then, $u = \cot^{-1}\{\sqrt{\tan \theta}\} - \tan^{-1}\{\sqrt{\tan \theta}\}$ can be written as

$$u = \cot^{-1} x - \tan^{-1} x \dots (i)$$

We know by the property of inverse trigonometry,

$$\cot^{-1} x + \tan^{-1} x = \frac{\pi}{2}$$

Or,

$$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$$

Substituting the value of $\cot^{-1} x$ in equation (i), we get

$$u = (\cot^{-1} x) - \tan^{-1} x$$

$$\Rightarrow u = \left(\frac{\pi}{2} - \tan^{-1} x\right) - \tan^{-1} x$$

$$= \frac{\pi}{2} - \tan^{-1} x - \tan^{-1} x$$

$$= \frac{\pi}{2} - 2 \tan^{-1} x$$

Rearranging the equation,

$$\Rightarrow u + 2 \tan^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow 2 \tan^{-1} x = \frac{\pi}{2} - u$$

Now, divide by 2 on both sides of the equation.

$$\Rightarrow \frac{2 \tan^{-1} x}{2} = \frac{\frac{\pi}{2} - u}{2}$$

$$\Rightarrow \tan^{-1} x = \frac{\frac{\pi}{2}}{2} - \frac{u}{2}$$

$$= \frac{\pi}{2} \times \frac{1}{2} - \frac{u}{2}$$

$$= \frac{\pi}{4} - \frac{u}{2}$$

Taking tangent on both sides, we get

$$\Rightarrow \tan(\tan^{-1} x) = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$$

Using property of inverse trigonometry,

$$\tan(\tan^{-1} x) = x$$

$$\Rightarrow x = \tan\left(\frac{\pi}{4} - \frac{u}{2}\right)$$

Recall the value of x. That is, $x = \sqrt{\tan \theta}$

$$\Rightarrow \tan\left(\frac{\pi}{4} - \frac{u}{2}\right) = \sqrt{\tan \theta}$$

13. Question

Choose the correct answer

If $\cos^{-1} \frac{x}{3} + \cos^{-1} \frac{y}{2} = \frac{\theta}{2}$, then $4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 =$

- A. 36
- B. $36 - 36 \cos \theta$
- C. $18 - 18 \cos \theta$
- D. $18 + 18 \cos \theta$

Answer

We are given with,

$$\cos^{-1} \frac{x}{3} + \cos^{-1} \frac{y}{2} = \frac{\theta}{2} \dots (i)$$

We need to find the value of

$$4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2$$

Take Left Hand Side (LHS) of equation (i),

Using the property of inverse trigonometry,

$$\cos^{-1} A + \cos^{-1} B = \cos^{-1} \left(AB - \sqrt{1 - A^2} \sqrt{1 - B^2} \right)$$

Putting $A = \frac{x}{3}$ and $B = \frac{y}{2}$,

$$\text{LHS} = \cos^{-1} \frac{x}{3} + \cos^{-1} \frac{y}{2}$$

$$\Rightarrow \text{LHS} = \cos^{-1} \left(\left(\frac{x}{3} \right) \left(\frac{y}{2} \right) - \sqrt{1 - \left(\frac{x}{3} \right)^2} \sqrt{1 - \left(\frac{y}{2} \right)^2} \right)$$

$$\Rightarrow \text{LHS} = \cos^{-1} \left(\frac{xy}{6} - \sqrt{1 - \frac{x^2}{9}} \sqrt{1 - \frac{y^2}{4}} \right)$$

$$\Rightarrow \text{LHS} = \cos^{-1} \left(\frac{xy}{6} - \sqrt{\frac{9 - x^2}{9}} \sqrt{\frac{4 - y^2}{4}} \right)$$

Equate LHS to RHS.

$$\cos^{-1} \left(\frac{xy}{6} - \sqrt{\frac{9 - x^2}{9}} \sqrt{\frac{4 - y^2}{4}} \right) = \frac{\theta}{2}$$

Taking cosine on both sides,

$$\Rightarrow \cos \left[\cos^{-1} \left(\frac{xy}{6} - \sqrt{\frac{9-x^2}{9}} \sqrt{\frac{4-y^2}{4}} \right) \right] = \cos \frac{\theta}{2}$$

Using property of inverse trigonometry,

$$\cos(\cos^{-1} A) = A$$

$$\Rightarrow \frac{xy}{6} - \sqrt{\frac{9-x^2}{9}} \sqrt{\frac{4-y^2}{4}} = \cos \frac{\theta}{2}$$

Simplifying the equation,

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{9-x^2}}{3} \frac{\sqrt{4-y^2}}{2} = \cos \frac{\theta}{2}$$

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{9-x^2}\sqrt{4-y^2}}{6} = \cos \frac{\theta}{2}$$

$$\Rightarrow xy - \sqrt{9-x^2}\sqrt{4-y^2} = 6 \cos \frac{\theta}{2}$$

$$\Rightarrow xy - 6 \cos \frac{\theta}{2} = \sqrt{9-x^2}\sqrt{4-y^2}$$

Squaring on both sides,

$$\Rightarrow \left[xy - 6 \cos \frac{\theta}{2} \right]^2 = \left[\sqrt{9-x^2} \sqrt{4-y^2} \right]^2$$

Using algebraic identity,

$$(A - B)^2 = A^2 + B^2 - 2AB$$

$$\Rightarrow (xy)^2 + \left(6 \cos \frac{\theta}{2} \right)^2 - 2(xy) \left(6 \cos \frac{\theta}{2} \right) = (9-x^2)(4-y^2)$$

$$\Rightarrow x^2y^2 + 36 \cos^2 \frac{\theta}{2} - 12xy \cos \frac{\theta}{2} = 36 - 9y^2 - 4x^2 + x^2y^2$$

$$\Rightarrow x^2y^2 - x^2y^2 + 4x^2 + 9y^2 - 12xy \cos \frac{\theta}{2} = 36 - 36 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow 4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 36 - 36 \cos^2 \frac{\theta}{2}$$

Using trigonometric identity,

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta \dots (ii)$$

$$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 - \cos^2 \theta \dots (iii)$$

Putting value of $\sin^2 \theta$ from equation (iii) in equation (ii), we get

$$\cos 2\theta = \cos^2 \theta - (1 - \cos^2 \theta)$$

$$\text{Or, } \cos 2\theta = \cos^2 \theta - 1 + \cos^2 \theta$$

$$\text{Or, } \cos 2\theta = 2 \cos^2 \theta - 1$$

$$\text{Or, } 2 \cos^2 \theta = \cos 2\theta + 1$$

Replace θ by $\theta/2$.

$$2 \cos^2 \frac{\theta}{2} = \cos \frac{2 \times \theta}{2} + 1$$

$$\Rightarrow 2 \cos^2 \frac{\theta}{2} = \cos \theta + 1$$

Substituting the value of $2 \cos^2 \frac{\theta}{2}$ in

$$4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 36 - 36 \cos^2 \frac{\theta}{2}$$

$$\Rightarrow 4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 36 - 18 \left(2 \cos^2 \frac{\theta}{2} \right)$$

$$\Rightarrow 4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 36 - 18(\cos \theta + 1)$$

$$\Rightarrow 4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 36 - 18 \cos \theta - 18$$

$$\Rightarrow 4x^2 - 12xy \cos \frac{\theta}{2} + 9y^2 = 18 - 18 \cos \theta$$

14. Question

Choose the correct answer

If $\alpha = \tan^{-1} \left(\frac{\sqrt{3}x}{2y-x} \right)$, $\beta = \tan^{-1} \left(\frac{2x-y}{\sqrt{3}y} \right)$, then $\alpha - \beta =$

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{3}$

Answer

We are given with,

$$\alpha = \tan^{-1} \left(\frac{\sqrt{3}x}{2y-x} \right)$$

$$\beta = \tan^{-1} \left(\frac{2x-y}{\sqrt{3}y} \right)$$

We need to find the value of $\alpha - \beta$.

So,

$$\alpha - \beta = \tan^{-1} \left(\frac{\sqrt{3}x}{2y-x} \right) - \tan^{-1} \left(\frac{2x-y}{\sqrt{3}y} \right)$$

Using the property of inverse trigonometry,

$$\tan^{-1} A - \tan^{-1} B = \tan^{-1} \left(\frac{A-B}{1+AB} \right)$$

So,

$$\alpha - \beta = \tan^{-1} \left(\frac{\left(\frac{\sqrt{3}x}{2y-x} \right) - \left(\frac{2x-y}{\sqrt{3}y} \right)}{1 + \left(\frac{\sqrt{3}x}{2y-x} \right) \left(\frac{2x-y}{\sqrt{3}y} \right)} \right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{\frac{\sqrt{3}x \times \sqrt{3}y - (2x-y)(2y-x)}{\sqrt{3}y(2y-x)}}{\frac{\sqrt{3}y(2y-x) + \sqrt{3}x(2x-y)}{\sqrt{3}y(2y-x)}} \right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{\sqrt{3}x \times \sqrt{3}y - (2x-y)(2y-x)}{\sqrt{3}y(2y-x)} \times \frac{\sqrt{3}y(2y-x)}{\sqrt{3}y(2y-x) + \sqrt{3}x(2x-y)} \right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{3xy - 4xy + 2x^2 + 2y^2 - xy}{2\sqrt{3}y^2 - \sqrt{3}xy + 2\sqrt{3}x^2 - \sqrt{3}xy} \right)$$

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{2x^2 + 2y^2 - 2xy}{2\sqrt{3}x^2 + 2\sqrt{3}y^2 - 2\sqrt{3}xy} \right)$$

Simplifying it further,

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{2x^2 + 2y^2 - 2xy}{\sqrt{3}(2x^2 + 2y^2 - 2xy)} \right)$$

The term $(2x^2 + 2y^2 - 2xy)$ gets cancelled from numerator and denominator.

$$\Rightarrow \alpha - \beta = \tan^{-1} \left(\frac{1}{\sqrt{3}} \right)$$

Using the value of inverse trigonometry,

$$\tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}$$

$$\therefore \alpha - \beta = \frac{\pi}{6}$$

15. Question

Choose the correct answer

Let $f(x) = e^{\cos^{-1}\{\sin(x+\pi/3)\}}$. Then $f(8\pi/9) =$

A. $e^{5\pi/18}$

B. $e^{13\pi/18}$

C. $e^{-2\pi/18}$

D. none of these

Answer

We are given with,

$$f(x) = e^{\cos^{-1}\{\sin(x+\pi/3)\}}$$

We need to find $f\left(\frac{8\pi}{9}\right)$.

We just need to find put $x = \frac{8\pi}{9}$ in $f(x)$.

So,

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{8\pi}{9} + \frac{\pi}{3}\right)\right\}}$$

Simplify the equation,

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{8\pi+3\pi}{9}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\sin\left(\frac{11\pi}{9}\right)\right\}}$$

Using trigonometric identity,

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{\pi}{2} - \frac{11\pi}{9}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{9\pi-22\pi}{18}\right)\right\}}$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(-\frac{13\pi}{18}\right)\right\}}$$

Using trigonometric identity,

$$\cos(-\theta) = \cos \theta$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\cos^{-1}\left\{\cos\left(\frac{13\pi}{18}\right)\right\}}$$

Using property of inverse trigonometry,

$$\cos^{-1}(\cos \theta) = \theta$$

$$\Rightarrow f\left(\frac{8\pi}{9}\right) = e^{\left(\frac{13\pi}{18}\right)}$$

16. Question

Choose the correct answer

$$\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{2}{11} \text{ is equal to}$$

A. 0

B. $\frac{1}{2}$

C. -1

D. none of these

Answer

We need to find the value of

$$\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{2}{11}$$

Using property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

Replacing the values of A by $\frac{1}{11}$ and B by $\frac{2}{11}$,

$$\tan^{-1} \frac{1}{11} + \tan^{-1} \frac{2}{11} = \tan^{-1} \left(\frac{\frac{1}{11} + \frac{2}{11}}{1 - \left(\frac{1}{11}\right)\left(\frac{2}{11}\right)} \right)$$

Solving it further,

$$= \tan^{-1} \left(\frac{\frac{1+2}{11}}{1 - \frac{2}{121}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{3}{11}}{\frac{121-2}{121}} \right)$$

$$= \tan^{-1} \left(\frac{\frac{3}{11}}{\frac{119}{121}} \right)$$

$$= \tan^{-1} \left(\frac{3}{11} \times \frac{121}{119} \right)$$

$$= \tan^{-1} \left(\frac{3 \times 11}{119} \right)$$

$$= \tan^{-1} \left(\frac{33}{119} \right)$$

$$= 0.27$$

Thus, none of this match the result.

17. Question

Choose the correct answer

If $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta$, then $9x^2 - 12xy \cos \theta + 4y^2$ is equal to

- A. 36
- B. $-36 \sin^2 \theta$
- C. $36 \sin^2 \theta$
- D. $36 \cos^2 \theta$

Answer

We are given with,

$$\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta$$

We need to find the value of $9x^2 - 12xy \cos \theta + 4y^2$.

Using property of inverse trigonometry,

$$\cos^{-1} A + \cos^{-1} B = \cos^{-1} \left(AB - \sqrt{1-A^2} \sqrt{1-B^2} \right)$$

Take Left Hand Side (LHS) of:

$$\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \theta$$

Replace A by $\frac{x}{2}$ and B by $\frac{y}{3}$.

$$\text{LHS} = \cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3}$$

$$\Rightarrow \text{LHS} = \cos^{-1} \left(\left(\frac{x}{2} \right) \left(\frac{y}{3} \right) - \sqrt{1 - \left(\frac{x}{2} \right)^2} \sqrt{1 - \left(\frac{y}{3} \right)^2} \right)$$

$$= \cos^{-1} \left(\frac{xy}{6} - \sqrt{1 - \frac{x^2}{4}} \sqrt{1 - \frac{y^2}{9}} \right)$$

$$= \cos^{-1} \left(\frac{xy}{6} - \sqrt{\frac{4-x^2}{4}} \sqrt{\frac{9-y^2}{9}} \right)$$

Further solving,

$$= \cos^{-1} \left(\frac{xy}{6} - \frac{\sqrt{4-x^2} \sqrt{9-y^2}}{2 \cdot 3} \right)$$

We shall equate LHS to RHS,

$$\cos^{-1} \left(\frac{xy}{6} - \frac{\sqrt{4-x^2} \sqrt{9-y^2}}{2 \cdot 3} \right) = \theta$$

Taking cosine on both sides,

$$\cos \left[\cos^{-1} \left(\frac{xy}{6} - \frac{\sqrt{4-x^2} \sqrt{9-y^2}}{2 \cdot 3} \right) \right] = \cos \theta$$

Using property of inverse trigonometry,

$$\cos(\cos^{-1} A) = A$$

So,

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{4-x^2} \sqrt{9-y^2}}{2 \cdot 3} = \cos \theta$$

$$\Rightarrow \frac{xy}{6} - \frac{\sqrt{4-x^2} \sqrt{9-y^2}}{6} = \cos \theta$$

$$\Rightarrow \frac{xy - \sqrt{4-x^2} \sqrt{9-y^2}}{6} = \cos \theta$$

By cross-multiplying,

$$\Rightarrow xy - \sqrt{4-x^2} \sqrt{9-y^2} = 6 \cos \theta$$

Rearranging it,

$$\Rightarrow xy - 6 \cos \theta = \sqrt{4-x^2} \sqrt{9-y^2}$$

Squaring on both sides,

$$\Rightarrow [xy - 6 \cos \theta]^2 = [\sqrt{4-x^2} \sqrt{9-y^2}]^2$$

Using algebraic identity,

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$\Rightarrow (xy)^2 + (6 \cos \theta)^2 - 2(xy)(6 \cos \theta) = (4 - x^2)(9 - y^2)$$

$$\Rightarrow x^2y^2 + 36 \cos^2 \theta - 12xy \cos \theta = 36 - 9x^2 - 4y^2 + x^2y^2$$

$$\Rightarrow x^2y^2 - x^2y^2 + 9x^2 - 12xy \cos \theta + 4y^2 = 36 - 36 \cos^2 \theta$$

$$\Rightarrow 9x^2 - 12xy \cos \theta + 4y^2 = 36 (1 - \cos^2 \theta)$$

Using trigonometric identity,

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\Rightarrow \sin^2 \theta = 1 - \cos^2 \theta$$

Substituting the value of $(1 - \cos^2 \theta)$, we get

$$\Rightarrow 9x^2 - 12xy \cos \theta + 4y^2 = 36 \sin^2 \theta$$

18. Question

Choose the correct answer

If $\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$, then $x =$

A. 5

B. $\frac{1}{5}$

C. $\frac{5}{14}$

D. $\frac{14}{5}$

Answer

We are given with,

$$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$$

We need to find the value of x .

Using property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A + B}{1 - AB} \right)$$

Let us replace A by 3 and B by x .

$$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} \left(\frac{3 + x}{1 - (3)(x)} \right)$$

$$= \tan^{-1} \left(\frac{3 + x}{1 - 3x} \right)$$

Since, according to the question

$$\tan^{-1} 3 + \tan^{-1} x = \tan^{-1} 8$$

So,

$$\Rightarrow \tan^{-1} \left(\frac{3 + x}{1 - 3x} \right) = \tan^{-1} 8$$

Taking tangent on both sides,

$$\Rightarrow \tan \left[\tan^{-1} \left(\frac{3+x}{1-3x} \right) \right] = \tan[\tan^{-1} 8]$$

Using property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

$$\Rightarrow \frac{3+x}{1-3x} = 8$$

Now, in order to find x, we need to solve the linear equation.

By cross-multiplying,

$$\Rightarrow 3 + x = 8(1 - 3x)$$

$$\Rightarrow 3 + x = 8 - 24x$$

$$\Rightarrow 24x + x = 8 - 3$$

$$\Rightarrow 25x = 5$$

$$\Rightarrow x = \frac{5}{25}$$

$$\Rightarrow x = \frac{1}{5}$$

19. Question

Choose the correct answer

The value of $\sin^{-1} \left(\cos \frac{33\pi}{5} \right)$ is

A. $\frac{3\pi}{5}$

B. $-\frac{\pi}{10}$

C. $\frac{\pi}{10}$

D. $\frac{7\pi}{5}$

Answer

We need to find the value of $\sin^{-1} \left(\cos \frac{33\pi}{5} \right)$.

$$\sin^{-1} \left(\cos \frac{33\pi}{5} \right) = \sin^{-1} \left(\cos \left(6\pi + \frac{3\pi}{5} \right) \right)$$

$$\left[\because \cos \frac{33\pi}{5} = \cos \left(6\pi + \frac{3\pi}{5} \right) \right]$$

Using the trigonometric identity,

$$\cos(6\pi + \theta) = \cos \theta$$

As the function lies in I Quadrant and so it will be positive.

$$\Rightarrow \sin^{-1} \left(\cos \frac{33\pi}{5} \right) = \sin^{-1} \left(\cos \frac{3\pi}{5} \right)$$

Using the trigonometric identity,

$$\cos \theta = \sin\left(\frac{\pi}{2} - \theta\right)$$

$$\Rightarrow \sin^{-1}\left(\cos \frac{33\pi}{5}\right) = \sin^{-1}\left(\sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)\right)$$

Using property of inverse trigonometry,

$$\sin^{-1}(\sin A) = A$$

$$= \frac{\pi}{2} - \frac{3\pi}{5}$$

$$= \frac{5\pi - 6\pi}{10}$$

$$= -\frac{\pi}{10}$$

20. Question

Choose the correct answer

The value of $\cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right)$ is

A. $\frac{\pi}{2}$

B. $\frac{5\pi}{3}$

C. $\frac{10\pi}{3}$

D. 0

Answer

We need to find the value of:

$$\cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right)$$

Let us simplify the trigonometric function.

We can write as:

$$\cos \frac{5\pi}{3} = \cos\left(2\pi - \frac{\pi}{3}\right)$$

Similarly,

$$\sin \frac{5\pi}{3} = \sin\left(2\pi - \frac{\pi}{3}\right)$$

$$\begin{aligned}\Rightarrow \cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right) \\ = \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{3}\right)\right) + \sin^{-1}\left(\sin\left(2\pi - \frac{\pi}{3}\right)\right)\end{aligned}$$

Since, $\cos\left(2\pi - \frac{\pi}{3}\right)$ lies on IV Quadrant and cosine is positive in IV Quadrant.

$$\therefore, \cos\left(2\pi - \frac{\pi}{3}\right) = \cos \frac{\pi}{3}$$

And since, $\sin\left(2\pi - \frac{\pi}{3}\right)$ lies on IV Quadrant and sine is negative in IV Quadrant.

$$\therefore, \sin\left(2\pi - \frac{\pi}{3}\right) = -\sin \frac{\pi}{3}$$

$$\Rightarrow \cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right) = \cos^{-1}\left(\cos \frac{\pi}{3}\right) + \sin^{-1}\left(-\sin \frac{\pi}{3}\right)$$

$$= \cos^{-1}\left(\cos \frac{\pi}{3}\right) - \sin^{-1}\left(\sin \frac{\pi}{3}\right)$$

Using property of inverse trigonometry,

$$\sin^{-1}(\sin A) = A \text{ and } \cos^{-1}(\cos A) = A$$

$$\Rightarrow \cos^{-1}\left(\cos \frac{5\pi}{3}\right) + \sin^{-1}\left(\sin \frac{5\pi}{3}\right) = \frac{\pi}{3} - \frac{\pi}{3}$$

$$= 0$$

21. Question

Choose the correct answer

$$\sin\left\{2 \cos^{-1}\left(\frac{-3}{5}\right)\right\} \text{ is equal to}$$

A. $\frac{6}{25}$

B. $\frac{24}{25}$

C. $\frac{4}{5}$

D. $-\frac{24}{25}$

Answer

We need to find the value of:

$$\sin\left\{2 \cos^{-1}\left(-\frac{3}{5}\right)\right\}$$

$$\text{Let } \cos^{-1}\left(-\frac{3}{5}\right) = x$$

Take cosine on both sides, we get

$$\cos\left[\cos^{-1}\left(-\frac{3}{5}\right)\right] = \cos x$$

Using property of inverse trigonometry,

$$\cos(\cos^{-1} A) = A$$

$$\Rightarrow -\frac{3}{5} = \cos x$$

We have the value of $\cos x$, let us find the value of $\sin x$.

By trigonometric identity,

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \sin^2 x = 1 - \cos^2 x$$

$$\Rightarrow \sin x = \sqrt{1 - \cos^2 x}$$

$$\text{Putting } \cos x = -\frac{3}{5},$$

$$= \sqrt{1 - \left(-\frac{3}{5}\right)^2}$$

$$= \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{25 - 9}{25}}$$

$$= \sqrt{\frac{16}{25}}$$

$$= \frac{4}{5}$$

Now,

$$\sin\left\{2 \cos^{-1}\left(-\frac{3}{5}\right)\right\} = \sin 2x$$

Using the trigonometric identity,

$$\sin 2x = 2 \sin x \cos x$$

$$\Rightarrow \sin\left\{2 \cos^{-1}\left(-\frac{3}{5}\right)\right\} = 2 \sin x \cos x$$

$$\text{Putting the value of } \sin x = \frac{4}{5} \text{ and } \cos x = -\frac{3}{5},$$

$$= 2 \times \frac{4}{5} \times -\frac{3}{5}$$

$$= -\frac{24}{25}$$

22. Question

Choose the correct answer

If $\theta = \sin^{-1}\{\sin(-600^\circ)\}$, then one of the possible values of θ is

A. $\frac{\pi}{3}$

B. $\frac{\pi}{2}$

C. $\frac{2\pi}{3}$

D. $-\frac{2\pi}{3}$

Answer

We are given that,

$$\theta = \sin^{-1} \{\sin (-600^\circ)\}$$

We know that,

$$\sin (2\pi - \theta) = \sin (4\pi - \theta) = \sin (6\pi - \theta) = \sin (8\pi - \theta) = \dots = -\sin \theta$$

As, $\sin (2\pi - \theta)$, $\sin (4\pi - \theta)$, $\sin (6\pi - \theta)$, ... all lie in IV Quadrant where sine function is negative.

So,

If we replace θ by 600° , then we can write as

$$\sin (4\pi - 600^\circ) = -\sin 600^\circ$$

Or,

$$\sin (4\pi - 600^\circ) = \sin (-600^\circ)$$

Or,

$$\sin (720^\circ - 600^\circ) = \sin (-600^\circ) \dots (i)$$

$$[\because, 4\pi = 4 \times 180^\circ = 720^\circ < 600^\circ]$$

Thus, we have

$$\theta = \sin^{-1} \{\sin (-600^\circ)\}$$

$$\Rightarrow \theta = \sin^{-1} \{\sin (720^\circ - 600^\circ)\} \text{ [from equation (i)]}$$

$$\Rightarrow \theta = \sin^{-1} \{\sin 120^\circ\} \dots (ii)$$

We know that,

$$\sin (\pi - \theta) = \sin (3\pi - \theta) = \sin (5\pi - \theta) = \dots = \sin \theta$$

As, $\sin (\pi - \theta)$, $\sin (3\pi - \theta)$, $\sin (5\pi - \theta)$, ... all lie in II Quadrant where sine function is positive.

So,

If we replace θ by 120° , then we can write as

$$\sin (\pi - 120^\circ) = \sin 120^\circ$$

Or,

$$\sin (180^\circ - 120^\circ) = \sin 120^\circ \dots (iii)$$

$$[\because, \pi = 180^\circ < 120^\circ]$$

Thus, from equation (ii),

$$\theta = \sin^{-1} \{\sin 120^\circ\}$$

$$\Rightarrow \theta = \sin^{-1} \{\sin (180^\circ - 120^\circ)\} \text{ [from equation (iii)]}$$

$$\Rightarrow \theta = \sin^{-1} \{\sin 60^\circ\}$$

Using property of inverse trigonometry,

$$\sin^{-1} (\sin A) = A$$

$$\Rightarrow \theta = 60^\circ$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

23. Question

Choose the correct answer

If $3 \sin^{-1} \left(\frac{2x}{1+x^2} \right) - 4 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + 2 \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{\pi}{3}$, then x is equal to

- A. $\frac{1}{\sqrt{3}}$
- B. $-\frac{1}{\sqrt{3}}$
- C. $\sqrt{3}$
- D. $-\frac{\sqrt{3}}{4}$

Answer

We are given that,

$$3 \sin^{-1} \left(\frac{2x}{1+x^2} \right) - 4 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + 2 \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{\pi}{3}$$

We need to find the value of x.

We know that by trigonometric identity, we can represent $\sin \theta$, $\cos \theta$ and $\tan \theta$ in terms of $\tan \theta$.

Note,

$$\sin 2\theta = \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right)$$

$$\cos 2\theta = \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right)$$

$$\tan 2\theta = \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right)$$

So, in the equation given in the question, let $x = \tan \theta$.

Re-writing the equation,

$$3 \sin^{-1} \left(\frac{2x}{1+x^2} \right) - 4 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) + 2 \tan^{-1} \left(\frac{2x}{1-x^2} \right) = \frac{\pi}{3}$$

$$\Rightarrow 3 \sin^{-1} \left(\frac{2 \tan \theta}{1 + \tan^2 \theta} \right) - 4 \cos^{-1} \left(\frac{1 - \tan^2 \theta}{1 + \tan^2 \theta} \right) + 2 \tan^{-1} \left(\frac{2 \tan \theta}{1 - \tan^2 \theta} \right) = \frac{\pi}{3}$$

Substituting the values of trigonometric identities,

$$\Rightarrow 3 \sin^{-1}(\sin 2\theta) - 4 \cos^{-1}(\cos 2\theta) + 2 \tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

Using the property of inverse trigonometry, we have

$$\sin^{-1}(\sin A) = A, \cos^{-1}(\cos A) = A \text{ and } \tan^{-1}(\tan A) = A$$

$$\Rightarrow 3 \times 2\theta - 4 \times 2\theta + 2 \times 2\theta = \frac{\pi}{3}$$

$$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$$

$$\Rightarrow 2\theta = \frac{\pi}{3}$$

$$\Rightarrow \theta = \frac{\pi}{3} \times \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

Now, in order to find the value of x, recall

$$x = \tan \theta$$

Substitute the value of θ derived above,

$$\Rightarrow x = \tan \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

24. Question

Choose the correct answer

If $4\cos^{-1}x + \sin^{-1}x = \pi$, then the value of x is

A. $\frac{3}{2}$

B. $\frac{1}{\sqrt{2}}$

C. $\frac{\sqrt{3}}{2}$

D. $\frac{2}{\sqrt{3}}$

Answer

We are given that,

$$4\cos^{-1}x + \sin^{-1}x = \pi \dots (i)$$

We need to find the value of x.

Using the property of inverse trigonometry,

$$\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}\theta = \frac{\pi}{2} - \cos^{-1}\theta$$

Replacing θ by x, we get

$$\Rightarrow \sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

Substituting the value of $\sin^{-1}x$ in (i),

$$4\cos^{-1}x + \sin^{-1}x = \pi$$

$$\Rightarrow 4 \cos^{-1} x + \left(\frac{\pi}{2} - \cos^{-1} x\right) = \pi$$

$$\Rightarrow 4 \cos^{-1} x + \frac{\pi}{2} - \cos^{-1} x = \pi$$

$$\Rightarrow 3 \cos^{-1} x = \pi - \frac{\pi}{2}$$

$$\Rightarrow 3 \cos^{-1} x = \frac{\pi}{2}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{2} \times \frac{1}{3}$$

$$\Rightarrow \cos^{-1} x = \frac{\pi}{6}$$

Taking cosines on both sides,

$$\Rightarrow \cos[\cos^{-1} x] = \cos \frac{\pi}{6}$$

$$\Rightarrow x = \cos \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\sqrt{3}}{2}$$

25. Question

Choose the correct answer

If $\tan^{-1} \frac{x+1}{x-1} + \tan^{-1} \frac{x-1}{x} = \tan^{-1}(-7)$, then the value of x is

- A. 0
- B. -2
- C. 1
- D. 2

Answer

We are given that,

$$\tan^{-1} \left(\frac{x+1}{x-1}\right) + \tan^{-1} \left(\frac{x-1}{x}\right) = \tan^{-1}(-7) \dots(i)$$

We need to find the value of x.

Using the property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB}\right)$$

Replace A by $\frac{x+1}{x-1}$ and B by $\frac{x-1}{x}$.

$$\tan^{-1} \left(\frac{x+1}{x-1}\right) + \tan^{-1} \left(\frac{x-1}{x}\right) = \tan^{-1} \left(\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)}\right)$$

Putting this value in equation (i),

$$\tan^{-1} \left(\frac{x+1}{x-1}\right) + \tan^{-1} \left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$$

$$\Rightarrow \tan^{-1} \left(\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)} \right) = \tan^{-1}(-7)$$

Taking tangent on both sides,

$$\Rightarrow \tan \left[\tan^{-1} \left(\frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)} \right) \right] = \tan[\tan^{-1}(-7)]$$

Using the property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

$$\Rightarrow \frac{\left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right)}{1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right)} = -7$$

Cross-multiplying, we get

$$\Rightarrow \left(\frac{x+1}{x-1}\right) + \left(\frac{x-1}{x}\right) = -7 \left[1 - \left(\frac{x+1}{x-1}\right)\left(\frac{x-1}{x}\right) \right]$$

Simplifying the equation in order to find the value of x,

$$\Rightarrow \frac{x(x+1) + (x-1)(x-1)}{x(x-1)} = -7 \left[\frac{x(x-1) - (x+1)(x-1)}{x(x-1)} \right]$$

Let us cancel the denominator from both sides of the equation.

$$\Rightarrow x(x+1) + (x-1)(x-1) = -7[x(x-1) - (x+1)(x-1)]$$

$$\Rightarrow x^2 + x + (x-1)^2 = -7[x^2 - x - (x+1)(x-1)]$$

Using the algebraic identity,

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$\text{And, } (a+b)(a-b) = a^2 - b^2$$

$$\Rightarrow x^2 + x + x^2 + 1 - 2x = -7[x^2 - x - (x^2 - 1)]$$

$$\Rightarrow 2x^2 - x + 1 = -7[x^2 - x - x^2 + 1]$$

$$\Rightarrow 2x^2 - x + 1 = -7[1 - x]$$

$$\Rightarrow 2x^2 - x + 1 = -7 + 7x$$

$$\Rightarrow 2x^2 - x - 7x + 1 + 7 = 0$$

$$\Rightarrow 2x^2 - 8x + 8 = 0$$

$$\Rightarrow 2(x^2 - 4x + 4) = 0$$

$$\Rightarrow x^2 - 4x + 4 = 0$$

We need to solve the quadratic equation to find the value of x.

$$\Rightarrow x^2 - 2x - 2x + 4 = 0$$

$$\Rightarrow x(x-2) - 2(x-2) = 0$$

$$\Rightarrow (x-2)(x-2) = 0$$

$$\Rightarrow x = 2 \text{ or } x = 2$$

Hence, $x = 2$.

26. Question

Choose the correct answer

If $\cos^{-1} x > \sin^{-1} x$, then

A. $\frac{1}{\sqrt{2}} < x \leq 1$

B. $0 \leq x < \frac{1}{\sqrt{2}}$

C. $-1 \leq x < \frac{1}{\sqrt{2}}$

D. $x > 0$

Answer

We are given that,

$$\cos^{-1} x > \sin^{-1} x$$

We need to find the range of x.

Using the property of inverse trigonometry,

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

Or,

$$\Rightarrow \sin^{-1} x = \frac{\pi}{2} - \cos^{-1} x$$

So, re-writing the inequality,

$$\cos^{-1} x > \sin^{-1} x$$

$$\Rightarrow \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x$$

Adding $\cos^{-1} x$ on both sides of the inequality,

$$\Rightarrow \cos^{-1} x + \cos^{-1} x > \frac{\pi}{2} - \cos^{-1} x + \cos^{-1} x$$

$$\Rightarrow 2 \cos^{-1} x > \frac{\pi}{2}$$

Dividing both sides of the inequality by 2,

$$\Rightarrow \frac{2 \cos^{-1} x}{2} > \frac{\pi}{2} \times \frac{1}{2}$$

$$\Rightarrow \cos^{-1} x > \frac{\pi}{4}$$

Taking cosine on both sides of the inequality,

$$\Rightarrow \cos[\cos^{-1} x] > \cos \frac{\pi}{4}$$

$$\Rightarrow x > \frac{1}{\sqrt{2}}$$

$\frac{1}{\sqrt{2}}$ is the minimum value of x, while the maximum value of cosine function is 1.

$$\Rightarrow \frac{1}{\sqrt{2}} < x < 1$$

27. Question

Choose the correct answer

In a ΔABC , If C is a right angle, then $\tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) =$

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{5\pi}{2}$

D. $\frac{\pi}{6}$

Answer

We are given that,

ΔABC is a right-angled triangle at C.

Let the sides of the ΔABC be

$$AC = b$$

$$BC = a$$

$$AB = c$$

By Pythagoras theorem, where C is the right angle,

$$(AC)^2 + (BC)^2 = (AB)^2$$

$$\Rightarrow b^2 + a^2 = c^2$$

Or,

$$a^2 + b^2 = c^2 \dots(i)$$

Using the property of inverse trigonometry,

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \left(\frac{A+B}{1-AB} \right)$$

Replacing A by $\left(\frac{a}{b+c}\right)$ and B by $\left(\frac{b}{c+a}\right)$,

$$\Rightarrow \tan^{-1} \left(\frac{a}{b+c} \right) + \tan^{-1} \left(\frac{b}{c+a} \right) = \tan^{-1} \left(\frac{\left(\frac{a}{b+c}\right) + \left(\frac{b}{c+a}\right)}{1 - \left(\frac{a}{b+c}\right)\left(\frac{b}{c+a}\right)} \right)$$

$$= \tan^{-1} \left(\frac{\frac{a(c+a) + b(b+c)}{(b+c)(c+a)}}{\frac{(b+c)(c+a) - ab}{(b+c)(c+a)}} \right)$$

$$= \tan^{-1} \left(\left(\frac{ac + a^2 + b^2 + bc}{(b+c)(c+a)} \right) \times \left(\frac{(b+c)(c+a)}{bc + ab + c^2 + ac - ab} \right) \right)$$

$$\Rightarrow \tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) = \tan^{-1}\left(\frac{a^2 + b^2 + ac + bc}{c^2 + ac + bc}\right)$$

Substituting the value of $a^2 + b^2$ from equation (i),

$$= \tan^{-1}\left(\frac{c^2 + ac + bc}{c^2 + ac + bc}\right)$$

$$= \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\left(\frac{a}{b+c}\right) + \tan^{-1}\left(\frac{b}{c+a}\right) = \frac{\pi}{4}$$

28. Question

Choose the correct answer

The value of $\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

A. $\frac{1}{\sqrt{2}}$

B. $\frac{1}{\sqrt{3}}$

C. $\frac{1}{2\sqrt{2}}$

D. $\frac{1}{3\sqrt{3}}$

Answer

We need to find the value of

$$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$$

$$\text{Let } \sin^{-1}\frac{\sqrt{63}}{8} = x$$

Now, take sine on both sides,

$$\sin\left[\sin^{-1}\frac{\sqrt{63}}{8}\right] = \sin x$$

Using the property of inverse trigonometry,

$$\sin(\sin^{-1} A) = A$$

$$\Rightarrow \sin x = \frac{\sqrt{63}}{8}$$

Let us find the value of $\cos x$.

We know by trigonometric identity, that

$$\sin^2 x + \cos^2 x = 1$$

$$\Rightarrow \cos^2 x = 1 - \sin^2 x$$

$$\Rightarrow \cos x = \sqrt{1 - \sin^2 x}$$

Put the value of sin x,

$$= \sqrt{1 - \left(\frac{\sqrt{63}}{8}\right)^2}$$

$$= \sqrt{1 - \frac{63}{64}}$$

$$= \sqrt{\frac{64 - 63}{64}}$$

$$= \sqrt{\frac{1}{64}}$$

$$= \frac{1}{8}$$

We have,

$$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sin\left(\frac{1}{4}x\right)$$

$$\Rightarrow \sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sin\frac{x}{4} \dots (i)$$

Using the trigonometric identity,

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos 2x = (1 - \sin^2 x) - \sin^2 x \quad [\because \sin^2 x + \cos^2 x = 1]$$

$$\Rightarrow \cos 2x = 1 - \sin^2 x - \sin^2 x$$

$$\Rightarrow \cos 2x = 1 - 2 \sin^2 x$$

Or,

$$2 \sin^2 x = 1 - \cos 2x$$

$$\Rightarrow \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\Rightarrow \sin x = \sqrt{\frac{1 - \cos 2x}{2}}$$

Replacing x by x/4,

$$\Rightarrow \sin\frac{x}{4} = \sqrt{\frac{1 - \cos\left(2 \times \frac{x}{4}\right)}{2}}$$

$$= \sqrt{\frac{1 - \cos\frac{x}{2}}{2}}$$

Substituting the value of $\sin\frac{x}{4}$ in equation (i),

$$\sin\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right) = \sqrt{\frac{1 - \cos\frac{x}{2}}{2}} \dots (ii)$$

Using the trigonometric identity,

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\Rightarrow \cos 2x = \cos^2 x - (1 - \cos^2 x) [\because \sin^2 x + \cos^2 x = 1]$$

$$\Rightarrow \cos 2x = \cos^2 x - 1 + \cos^2 x$$

$$\Rightarrow \cos 2x = 2 \cos^2 x - 1$$

Or,

$$2 \cos^2 x = 1 + \cos 2x$$

$$\Rightarrow \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\Rightarrow \cos x = \sqrt{\frac{1 + \cos 2x}{2}}$$

Replacing x by x/2,

$$\Rightarrow \cos \frac{x}{2} = \sqrt{\frac{1 + \cos \left(2 \times \frac{x}{2}\right)}{2}}$$

$$= \sqrt{\frac{1 + \cos x}{2}}$$

Substituting the value of $\cos \frac{x}{2}$ in equation (ii),

$$\sin \left(\frac{1}{4} \sin^{-1} \frac{\sqrt{63}}{8} \right) = \sqrt{\frac{1 - \sqrt{\frac{1 + \cos x}{2}}}{2}}$$

Put the value of cos x as found above, $\cos x = 1/8$.

$$= \sqrt{\frac{1 - \sqrt{\frac{1 + \frac{1}{8}}{2}}}{2}}$$

$$= \sqrt{\frac{\left(1 - \sqrt{\frac{8+1}{8}}\right)}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{\frac{9}{8}}}{2}}$$

$$= \sqrt{\frac{1 - \sqrt{\frac{9}{16}}}{2}}$$

$$= \sqrt{\frac{\left(1 - \frac{3}{4}\right)}{2}}$$

$$= \sqrt{\frac{\frac{4-3}{4}}{2}}$$

$$= \sqrt{\frac{\frac{1}{4}}{2}}$$

$$= \sqrt{\frac{1}{8}}$$

$$= \frac{1}{2\sqrt{2}}$$

29. Question

Choose the correct answer

$$\cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right) =$$

- A. 4
- B. 6
- C. 5
- D. none of these

Answer

We need to find the value of

$$\cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right)$$

$$\text{Let } 2 \cot^{-1} 3 = y$$

Then,

$$\cot^{-1} 3 = \frac{y}{2}$$

$$\Rightarrow \cot \frac{y}{2} = 3$$

$$\text{Substituting } 2 \cot^{-1} 3 = y,$$

$$\cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right) = \cot\left(\frac{\pi}{4} - y\right)$$

Using the trigonometric identity,

$$\cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

So,

$$\Rightarrow \cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right) = \frac{\cot \frac{\pi}{4} \cot y + 1}{\cot y - \cot \frac{\pi}{4}}$$

We know that,

$$\cot \frac{\pi}{4} = 1$$

$$\Rightarrow \cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right) = \frac{\cot y + 1}{\cot y - 1} \dots (i)$$

We know that, by trigonometric identity,

$$\tan 2y = \frac{2 \tan y}{1 - \tan^2 y}$$

Take reciprocal of both sides,

$$\frac{1}{\tan 2y} = \frac{1 - \tan^2 y}{2 \tan y}$$

$$\Rightarrow \cot 2y = \frac{1 - \tan^2 y}{2 \tan y}$$

$$\left[\because \frac{1}{\tan 2y} = \cot 2y \right]$$

$$\Rightarrow \cot 2y = \frac{1 - \frac{1}{\cot^2 y}}{2 \times \frac{1}{\cot y}}$$

$$= \frac{\frac{\cot^2 y - 1}{\cot^2 y}}{2 \times \frac{1}{\cot y}}$$

$$= \frac{\cot^2 y - 1}{2 \cot y}$$

Put $y = y/2$.

$$\Rightarrow \cot y = \frac{\cot^2 \frac{y}{2} - 1}{2 \cot \frac{y}{2}}$$

Putting the value of $\cot y$ in equation (i),

$$\Rightarrow \cot\left(\frac{\pi}{4} - 2 \cot^{-1} 3\right) = \frac{\frac{\cot^2 \frac{y}{2} - 1}{2 \cot \frac{y}{2}} + 1}{\frac{\cot^2 \frac{y}{2} - 1}{2 \cot \frac{y}{2}} - 1}$$

$$= \frac{\frac{\cot^2 \frac{y}{2} - 1 + 2 \cot \frac{y}{2}}{2 \cot \frac{y}{2}}}{\frac{\cot^2 \frac{y}{2} - 1 - 2 \cot \frac{y}{2}}{2 \cot \frac{y}{2}}}$$

$$= \frac{\cot^2 \frac{y}{2} + 2 \cot \frac{y}{2} - 1}{\cot^2 \frac{y}{2} - 2 \cot \frac{y}{2} - 1}$$

Put the value of $\cot \frac{y}{2} = 3$ derived above and also $\cot^2 \frac{y}{2} = 3^2 = 9$.

$$= \frac{9 + 2 \times 3 - 1}{9 - 2 \times 3 - 1}$$

$$= \frac{9 + 6 - 1}{9 - 6 - 1}$$

$$= \frac{14}{2}$$

$$= 7$$

30. Question

Choose the correct answer

If $\tan^{-1}(\cot \theta) = 2\theta$, then $\theta =$

A. $\pm \frac{\pi}{3}$

B. $\pm \frac{\pi}{4}$

C. $\pm \frac{\pi}{6}$

D. none of these

Answer

We are given that,

$$\tan^{-1}(\cot \theta) = 2\theta$$

We need to find the value of θ .

We have,

$$\tan^{-1}(\cot \theta) = 2\theta$$

Taking tangent on both sides,

$$\Rightarrow \tan[\tan^{-1}(\cot \theta)] = \tan 2\theta$$

Using property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

$$\Rightarrow \cot \theta = \tan 2\theta$$

Or,

$$\Rightarrow \tan 2\theta = \cot \theta$$

Using the trigonometric identity,

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = \cot \theta$$

Using the trigonometric identity,

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\Rightarrow \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{1}{\tan \theta}$$

By cross-multiplying,

$$\Rightarrow \tan \theta \times 2 \tan \theta = 1 - \tan^2 \theta$$

$$\Rightarrow 2 \tan^2 \theta = 1 - \tan^2 \theta$$

$$\Rightarrow 2 \tan^2 \theta + \tan^2 \theta = 1$$

$$\Rightarrow 3 \tan^2 \theta = 1$$

$$\Rightarrow \tan^2 \theta = \frac{1}{3}$$

$$\Rightarrow \tan \theta = \pm \frac{1}{\sqrt{3}}$$

$$\text{And } \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}.$$

$$\Rightarrow \tan \theta = \pm \tan \frac{\pi}{6}$$

Thus,

$$\theta = \pm \frac{\pi}{6}$$

31. Question

Choose the correct answer

If $\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, where $a, x \in (0, 1)$ then, the value of x is

A. 0

B. $\frac{a}{2}$

C. a

D. $\frac{2a}{1-a^2}$

Answer

We are given that,

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Where, $a, x \in (0, 1)$.

We need to find the value of x .

Using property of inverse trigonometry,

$$2 \tan^{-1} a = \sin^{-1}\left(\frac{2a}{1+a^2}\right)$$

$$= \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)$$

$$2 \tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

Then, we can write as

$$\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \cos^{-1}\left(\frac{1-a^2}{1+a^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\Rightarrow 2 \tan^{-1} a + 2 \tan^{-1} a = 2 \tan^{-1} x$$

$$\Rightarrow 4 \tan^{-1} a = 2 \tan^{-1} x$$

Dividing both sides by 2,

$$\Rightarrow \frac{4 \tan^{-1} a}{2} = \frac{2 \tan^{-1} x}{2}$$

$$\Rightarrow 2 \tan^{-1} a = \tan^{-1} x$$

Using property of inverse trigonometry,

$$2 \tan^{-1} a = \tan^{-1}\left(\frac{2a}{1-a^2}\right)$$

Then,

$$\Rightarrow \tan^{-1}\left(\frac{2a}{1-a^2}\right) = \tan^{-1} x$$

Taking tangent on both sides,

$$\Rightarrow \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right] = \tan[\tan^{-1} x]$$

$$\Rightarrow \frac{2a}{1-a^2} = x$$

Or,

$$\Rightarrow x = \frac{2a}{1-a^2}$$

32. Question

Choose the correct answer

The value of $\sin\left(2\left(\tan^{-1} 0.75\right)\right)$ is equal to

A. 0.75

B. 1.5

C. 0.96

D. $\sin^{-1} 1.5$

Answer

We need to find the value of $\sin(2(\tan^{-1} 0.75))$.

We can re-write the equation,

$$\sin(2(\tan^{-1} 0.75)) = \sin(2 \tan^{-1} 0.75)$$

Using the property of inverse trigonometry,

$$2 \tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Replace x by 0.75.

$$2 \tan^{-1} 0.75 = \sin^{-1}\left(\frac{2 \times 0.75}{1+0.75^2}\right)$$

So,

$$\sin(2(\tan^{-1} 0.75)) = \sin(2 \tan^{-1} 0.75)$$

$$\Rightarrow \sin(2(\tan^{-1} 0.75)) = \sin\left(\sin^{-1}\left(\frac{2 \times 0.75}{1 + 0.75^2}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\frac{1.5}{1 + 0.5626}\right)\right)$$

$$= \sin\left(\sin^{-1}\left(\frac{1.5}{1.5626}\right)\right)$$

$$\Rightarrow \sin(2(\tan^{-1} 0.75)) = \sin(\sin^{-1} 0.96)$$

Using the property of inverse trigonometry,

$$\sin(\sin^{-1} A) = A$$

$$\Rightarrow \sin(2(\tan^{-1} 0.75)) = 0.96$$

33. Question

Choose the correct answer

If $x > 1$, then $2 \tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

A. $4 \tan^{-1} x$

B. 0

C. $\frac{\pi}{2}$

D. π

Answer

We are given that, $x > 1$.

We need to find the value of

$$2 \tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Using the property of inverse trigonometry,

$$2 \tan^{-1} x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

We can substitute $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ by $2 \tan^{-1} x$.

So,

$$2 \tan^{-1} x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2 \tan^{-1} x + 2 \tan^{-1} x$$

$$= 4 \tan^{-1} x$$

34. Question

Choose the correct answer

The domain of $\cos^{-1}(x^2 - 4)$ is

A. $[3, 5]$

B. $[-1, 1]$

C. $[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

D. $[-\sqrt{5}, -\sqrt{3}] \cap [-\sqrt{5}, \sqrt{3}]$

Answer

We need to find the domain of $\cos^{-1}(x^2 - 4)$.

We must understand that, the domain of definition of a function is the set of "input" or argument values for which the function is defined.

We know that, domain of an inverse cosine function, $\cos^{-1} x$ is,

$$x \in [-1, 1]$$

Then,

$$(x^2 - 4) \in [-1, 1]$$

Or,

$$-1 \leq x^2 - 4 \leq 1$$

Adding 4 on all sides of the inequality,

$$-1 + 4 \leq x^2 - 4 + 4 \leq 1 + 4$$

$$\Rightarrow 3 \leq x^2 \leq 5$$

Now, since x has a power of 2, so if we take square roots on all sides of the inequality then the result would be

$$\Rightarrow \pm\sqrt{3} \leq x \leq \pm\sqrt{5}$$

But this obviously isn't continuous.

So, we can write as

$$x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$$

35. Question

Choose the correct answer

The value of $\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right)$ is

A. $\frac{19}{8}$

B. $\frac{8}{19}$

C. $\frac{19}{12}$

D. $\frac{3}{4}$

Answer

We need to find the value of

$$\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right)$$

Using the property of inverse trigonometry,

$$\cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$$

Just replace x by 3/5,

$$\cos^{-1}\frac{3}{5} = \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right)$$

So,

$$\tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right) = \tan\left(\tan^{-1}\left(\frac{\sqrt{1-\frac{9}{25}}}{\frac{3}{5}}\right) + \tan^{-1}\frac{1}{4}\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\sqrt{\frac{25-9}{25}}}{\frac{3}{5}}\right) + \tan^{-1}\frac{1}{4}\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\sqrt{\frac{16}{25}}}{\frac{3}{5}}\right) + \tan^{-1}\frac{1}{4}\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\frac{4}{5}}{\frac{3}{5}}\right) + \tan^{-1}\frac{1}{4}\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{4}{5} \times \frac{5}{3}\right) + \tan^{-1}\frac{1}{4}\right)$$

$$= \tan\left(\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{4}\right)$$

Using property of inverse trigonometry,

$$\tan^{-1}A + \tan^{-1}B = \tan^{-1}\left(\frac{A+B}{1-AB}\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\frac{4}{3} + \frac{1}{4}}{1 - \left(\frac{4}{3}\right)\left(\frac{1}{4}\right)}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\frac{16+3}{12}}{\frac{12-4}{12}}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{\frac{19}{12}}{\frac{8}{12}}\right)\right)$$

$$= \tan\left(\tan^{-1}\left(\frac{19}{12} \times \frac{12}{8}\right)\right)$$

$$= \tan\left(\tan^{-1}\frac{19}{8}\right)$$

Using the property of inverse trigonometry,

$$\tan(\tan^{-1} A) = A$$

$$\Rightarrow \tan\left(\cos^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4}\right) = \frac{19}{8}$$

Very short answer

1. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer

Let $\sin^{-1}(-\sqrt{3}/2) = x$ and $\cos^{-1}(-1/2) = y$

$$\Rightarrow \sin x = (-\sqrt{3}/2) \text{ and } \cos y = -1/2$$

We know that the range of the principal value branch of $\sin^{-1}$ is $(-\pi/2, \pi/2)$ and $\cos^{-1}$ is $(0, \pi)$.

We also know that $\sin(-\pi/3) = (-\sqrt{3}/2)$ and $\cos(2\pi/3) = -1/2$

$$\therefore \text{Value of } \sin^{-1}(-\sqrt{3}/2) + \cos^{-1}(-1/2) = -\pi/3 + 2\pi/3$$

$$= \pi/3$$

2. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the difference between maximum and minimum values of $\sin^{-1}x$ for $x \in [-1, 1]$.

Answer

Let $f(x) = \sin^{-1}x$

For x to be defined, $-1 \leq x \leq 1$

For $-1 \leq x \leq 1$, $\sin^{-1}(-1) \leq \sin^{-1}x \leq \sin^{-1}(1)$

$$\Rightarrow -\pi/2 \leq \sin^{-1}x \leq \pi/2$$

$$\Rightarrow -\pi/2 \leq f(x) \leq \pi/2$$

Maximum value = $\pi/2$ and minimum value = $-\pi/2$

$$\therefore \text{The difference between maximum and minimum values of } \sin^{-1}x = \pi/2 - (-\pi/2) = 2\pi/2$$

$$= \pi$$

3. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then write the value of $x + y + z$.

Answer

$$\text{Given } \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = 3\pi/2$$

We know that maximum and minimum values of $\sin^{-1} x$ are $\pi/2$ and $-\pi/2$ respectively.

$$\Rightarrow \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi/2 + \pi/2 + \pi/2$$

$$\Rightarrow \sin^{-1} x = \pi/2, \sin^{-1} y = \pi/2, \sin^{-1} z = \pi/2$$

$$\Rightarrow x = 1, y = 1, z = 1$$

$$\Rightarrow x + y + z = 1 + 1 + 1 = 3$$

$$\therefore x + y + z = 3$$

4. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $x > 1$, then write the value of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ in terms of $\tan^{-1} x$.

Answer

Given $x > 1$

$$\Rightarrow \tan \theta > 1$$

$$\Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2}$$

Multiplying by -2,

$$\Rightarrow -\pi < -2\theta < -\frac{\pi}{2}$$

Subtracting with π ,

$$\Rightarrow 0 < \pi - 2\theta < \frac{\pi}{2}$$

We know that $\sin 2\theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$

Put $\tan \theta = x$

$$\Rightarrow \sin 2\theta = \frac{2x}{1+x^2}$$

For $x > 1$,

$$\Rightarrow \sin(\pi - 2\theta) = \frac{2x}{1+x^2}$$

$$\Rightarrow \pi - 2\theta = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Since $x = \tan \theta$

$$\Rightarrow \theta = \tan^{-1} x$$

$$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2 \tan^{-1} x$$

5. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $x < 0$, then write the value of $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ in terms of $\tan^{-1} x$.

Answer

Given $x < 0$

$$\Rightarrow -\infty < x < 0$$

Let $x = \tan \theta$

$$\Rightarrow -\infty < \tan \theta < 0$$

$$\Rightarrow -\frac{\pi}{2} < \theta < 0$$

Multiplying by -2,

$$\Rightarrow -\pi < -2\theta < 0$$

$$\text{We know that } \cos 2\theta = \frac{1-\tan^2 \theta}{1+\tan^2 \theta}$$

Put $\tan \theta = x$

$$\Rightarrow \cos(-2\theta) = \frac{1-x^2}{1+x^2}$$

$$\Rightarrow -2\theta = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

Since $x = \tan \theta$

$$\Rightarrow \theta = \tan^{-1} x$$

$$\therefore \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2 \tan^{-1} x$$

6. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for $x > 0$.

Answer

Given $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ for $x > 0$

We know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$ if $xy > 1$

$$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}\left[\frac{x + \frac{1}{x}}{1 - x + \frac{1}{x}}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x^2 + 1}{x}}{0}\right]$$

$$= \tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

$$\therefore \tan^{-1} x + \tan^{-1} \left(\frac{1}{x} \right) = \frac{\pi}{2}$$

7. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1} x + \tan^{-1} \left(\frac{1}{x} \right)$ for $x < 0$.

Answer

Given $\tan^{-1} x + \tan^{-1} (1/x)$ for $x < 0$

We know that $\tan^{-1} x + \tan^{-1} y = -\pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$, if $x < 0, y < 0$

$$\Rightarrow \tan^{-1} x + \tan^{-1} \left(\frac{1}{x} \right) = -\pi + \tan^{-1} \left[\frac{x + \frac{1}{x}}{1 - x + \frac{1}{x}} \right]$$

$$= -\pi + \tan^{-1} \left[\frac{\frac{x^2 + 1}{x}}{\frac{x - 1 + 1}{x}} \right]$$

$$= -\pi + \tan^{-1} (\infty)$$

$$= -\pi + \pi/2$$

$$= -\pi/2$$

$$\therefore \tan^{-1} x + \tan^{-1} (1/x) = -\pi/2$$

8. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

What is the value of $\cos^{-1} \left(\cos \frac{2\pi}{3} \right) + \sin^{-1} \left(\sin \frac{2\pi}{3} \right)$?

Answer

We know that $\sin^{-1} (\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$ and $\cos^{-1} (\cos \theta) = \theta$, if $\theta \in [0, \pi]$

Given $\cos^{-1} \left(\cos \frac{2\pi}{3} \right) + \sin^{-1} \left(\sin \frac{2\pi}{3} \right)$

$$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3} \right)$$

$$= \pi$$

$$\therefore \cos^{-1} \left(\cos \frac{2\pi}{3} \right) + \sin^{-1} \left(\sin \frac{2\pi}{3} \right) = \pi$$

9. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $-1 < x < 0$, then write the value of $\sin^{-1} \left(\frac{2x}{1+x^2} \right) + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$.

Answer

Given $-1 < x < 0$

We know that $\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$, if $-1 \leq x \leq 1$ and $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = -2\tan^{-1}x$, if $-\infty < x \leq 0$

$$\text{Given } \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= 2\tan^{-1}x - 2\tan^{-1}x$$

$$= 0$$

10. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin(\cot^{-1}x)$.

Answer

Given $\sin(\cot^{-1}x)$

Let $\cot^{-1}x = \theta$

$$\Rightarrow x = \cot \theta$$

We know that $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$

$$\Rightarrow 1 + x^2 = \operatorname{cosec}^2 \theta$$

We know that $\operatorname{cosec} \theta = 1/\sin \theta$

$$\Rightarrow 1 + x^2 = \frac{1}{\sin^2 \theta}$$

$$\Rightarrow \sin^2 \theta = \frac{1}{1 + x^2}$$

$$\Rightarrow \sin \theta = \frac{1}{\sqrt{1 + x^2}}$$

$$\therefore \sin(\cot^{-1}x) = \frac{1}{\sqrt{1 + x^2}}$$

11. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$.

Answer

Let $\cos^{-1}(1/2) = x$ and $\sin^{-1}(1/2) = y$

$$\Rightarrow \cos x = 1/2 \text{ and } \sin y = 1/2$$

We know that the range of the principal value branch of $\sin^{-1}$ is $(-\pi/2, \pi/2)$ and $\cos^{-1}$ is $(0, \pi)$.

We also know that $\sin(\pi/6) = 1/2$ and $\cos(\pi/3) = 1/2$

$$\Rightarrow \text{Value of } \cos^{-1}(1/2) + 2\sin^{-1}(1/2) = \pi/3 + 2(\pi/6)$$

$$= \pi/3 + \pi/3$$

$$= 2\pi/3$$

$$\therefore \text{Value of } \cos^{-1}(1/2) + 2\sin^{-1}(1/2) = 2\pi/3$$

12. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the range of $\tan^{-1}x$.

Answer

We know that range of $\tan^{-1}x = (-\pi/2, \pi/2)$

13. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}(\cos 1540^\circ)$.

Answer

Given $\cos^{-1}(\cos 1540^\circ)$

$$= \cos^{-1}\{\cos(1440^\circ + 100^\circ)\}$$

$$= \cos^{-1}\{\cos(360^\circ \times 4 + 100^\circ)\}$$

We know that $\cos(2\pi + \theta) = \cos \theta$

$$= \cos^{-1}\{\cos 100^\circ\}$$

We know that $\cos^{-1}(\cos \theta) = \theta$ if $\theta \in [0, \pi]$

$$= 100^\circ$$

$$\therefore \cos^{-1}(\cos 1540^\circ) = 100^\circ$$

14. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}(\sin(-600^\circ))$.

Answer

Given $\sin^{-1}(\sin(-600^\circ))$

$$= \sin^{-1}(\sin(-600 + 360 \times 2))$$

We know that $\sin(2n\pi + \theta) = \sin \theta$

$$= \sin(\sin 120^\circ)$$

We know that $\sin^{-1}(\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$

$$= 180^\circ - 120^\circ$$

$$= 60^\circ$$

$$\therefore \sin^{-1}(\sin(-600^\circ)) = 60^\circ$$

15. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(2\sin^{-1}\frac{1}{3}\right)$.

Answer

Given $\cos(2\sin^{-1} 1/3)$

We know that $\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$

$$= \cos\left(2\tan^{-1} \frac{\frac{1}{3}}{\sqrt{1-\left(\frac{1}{3}\right)^2}}\right)$$

$$= \cos\left(2\tan^{-1} \frac{\frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right)$$

$$= \cos\left(2\tan^{-1} \frac{1}{2\sqrt{2}}\right)$$

We know that $2\tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2}$

$$= \cos\left(\cos^{-1} \frac{1-\left(\frac{1}{2\sqrt{2}}\right)^2}{1+\left(\frac{1}{2\sqrt{2}}\right)^2}\right)$$

$$= \frac{1-\frac{1}{8}}{1+\frac{1}{8}}$$

$$= \frac{7}{9}$$

$$\therefore \cos\left(2\sin^{-1}\frac{1}{3}\right) = \frac{7}{9}$$

16. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}(1550^\circ)$.

Answer

Given $\sin^{-1}(\sin 1550^\circ)$

$$= \sin^{-1}(\sin(1440^\circ + 110^\circ))$$

$$= \sin^{-1}(\sin(360^\circ \times 4 + 110^\circ))$$

We know that $\sin(2n\pi + \theta) = \sin \theta$

$$= \sin^{-1}(\sin 110^\circ)$$

We know that $\sin^{-1}(\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$

$$= 180^\circ - 110^\circ$$

$$= 70^\circ$$

$$\therefore \sin^{-1}(\sin 1550^\circ) = 70^\circ$$

17. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$.

Answer

Given $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$

We know that $\cos^{-1}x = 2\tan^{-1}\sqrt{\frac{1-x}{1+x}}$

$$= \sin\left(\frac{1}{2} \times 2\tan^{-1}\sqrt{\frac{1-\frac{4}{5}}{1+\frac{4}{5}}}\right)$$

$$= \sin\left(\tan^{-1}\frac{1}{3}\right)$$

We know that $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$$= \sin\left(\sin^{-1}\frac{\frac{1}{3}}{\sqrt{1+\left(\frac{1}{3}\right)^2}}\right)$$

$$= \frac{\frac{1}{3}}{\frac{\sqrt{10}}{3}}$$

$$= \frac{1}{\sqrt{10}}$$

$$\therefore \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \frac{1}{\sqrt{10}}$$

18. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin\left(\tan^{-1}\frac{3}{4}\right)$.

Answer

Given $\sin\left(\tan^{-1}\frac{3}{4}\right)$

We know that $\tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}$

$$= \sin\left(\sin^{-1}\frac{\frac{3}{4}}{\sqrt{1+\left(\frac{3}{4}\right)^2}}\right)$$

We know that $\sin(\sin^{-1}\theta) = \theta$

$$= \frac{\frac{3}{4}}{\frac{4}{5}}$$

$$= \frac{3}{5}$$

$$\therefore \sin\left(\tan^{-1}\frac{3}{4}\right) = \frac{3}{5}$$

19. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}\left(\tan\frac{3\pi}{4}\right)$.

Answer

Given $\cos^{-1}(\tan 3\pi/4)$

$$= \cos^{-1}(\tan(\pi - \pi/4))$$

We know that $\tan(\pi - \theta) = -\tan \theta$

$$= \cos^{-1}(-\tan \pi/4)$$

$$= \cos^{-1}(-1)$$

We know that $\cos^{-1} x = \pi$

$$\therefore \cos^{-1}(\tan 3\pi/4) = \pi$$

20. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(2\sin^{-1}\frac{1}{2}\right)$.

Answer

Given $\cos(2\sin^{-1} 1/2)$

$$= \cos(2 \times \pi/6)$$

$$= \cos(\pi/3)$$

$$= 1/2$$

$$\therefore \cos(2\sin^{-1} 1/2) = 1/2$$

21. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}(\cos 350^\circ) - \sin^{-1}(\sin 350^\circ)$.

Answer

Given $\cos^{-1}(\cos 350^\circ) - \sin^{-1}(\sin 350^\circ)$

$$= \cos^{-1}[\cos(360^\circ - 10^\circ)] - \sin^{-1}(\sin(360^\circ - 10^\circ))$$

We know that $\cos(2\pi - \theta) = \cos \theta$ and $\sin(2\pi - \theta) = -\sin \theta$

$$= \cos^{-1} (\cos 10^\circ) - \sin^{-1} (-\sin 10^\circ)$$

We know that $\cos^{-1} (\cos \theta)$, if $\theta \in [0, \pi]$ and $\sin (-\theta) = -\sin \theta$

$$= 10^\circ - \sin^{-1} (\sin (-10^\circ))$$

We know that $\sin^{-1} (\sin \theta) = \theta$, if $\theta \in [-\pi/2, \pi/2]$

$$= 10^\circ - (-10^\circ)$$

$$= 10^\circ + 10^\circ$$

$$= 20^\circ$$

$$\therefore \cos^{-1} (\cos 350^\circ) - \sin^{-1} (\sin 350^\circ) = 20^\circ$$

22. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^2 \left(\frac{1}{2} \cos^{-1} \frac{3}{5} \right)$.

Answer

Given $\cos^2 (1/2 \cos^{-1} 3/5)$

We know that $\cos^{-1} x = 2 \cos^{-1} \sqrt{\frac{1+x}{2}}$

$$= \cos^2 \left(\frac{1}{2} \times 2 \cos^{-1} \sqrt{\frac{1+\frac{3}{5}}{2}} \right)$$

$$= \left(\cos \left(\cos^{-1} \sqrt{\frac{8}{10}} \right) \right)^2$$

$$= \left(\sqrt{\frac{8}{10}} \right)^2$$

$$= \frac{4}{5}$$

$$\therefore \cos^2 \left(\frac{1}{2} \cos^{-1} \frac{3}{5} \right) = \frac{4}{5}$$

23. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then write the value of $x + y + xy$.

Answer

Given $\tan^{-1} x + \tan^{-1} y = \pi/4$

We know that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}(1)$$

$$\Rightarrow \frac{x+y}{1-xy} = 1$$

$$\Rightarrow x + y = 1 - xy$$

$$\Rightarrow x + y + xy = 1$$

$$\therefore x + y + xy = 1$$

24. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1}(\cos 6)$.

Answer

Given $\cos^{-1}(\cos 6)$

We know that $\cos^{-1}(\cos \theta) = 2\pi - \theta$, if $\theta \in [\pi, 2\pi]$

$$= 2\pi - 6$$

$$\therefore \cos^{-1}(\cos 6) = 2\pi - 6$$

25. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(\cos \frac{\pi}{9}\right)$.

Answer

Given $\sin^{-1}(\cos \pi/9)$

We know that $\cos \theta = \sin(\pi/2 - \theta)$

$$= \sin^{-1}(\sin(\pi/2 - \pi/9))$$

$$= \sin^{-1}(\sin 7\pi/18)$$

We know that $\sin^{-1}(\sin \theta) = \theta$

$$= 7\pi/18$$

$$\therefore \sin^{-1}(\cos \pi/9) = 7\pi/18$$

26. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$.

Answer

Given $\sin(\pi/3 - \sin^{-1}(-1/2))$

$$\begin{aligned}
&\text{We know that } \sin^{-1}(-\theta) = -\sin^{-1} \theta \\
&= \sin(\pi/3 + \sin^{-1}(1/2)) \\
&= \sin(\pi/3 + \pi/6) \\
&= \sin(\pi/2) \\
&= 1
\end{aligned}$$

$$\therefore \sin(\pi/3 - \sin^{-1}(-1/2)) = 1$$

27. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1} \left\{ \tan \left(\frac{15\pi}{4} \right) \right\}$.

Answer

$$\text{Given } \tan^{-1} \{ \tan(15\pi/4) \}$$

$$= \tan^{-1} \{ \tan(4\pi - \pi/4) \}$$

$$\text{We know that } \tan(2\pi - \theta) = -\tan \theta$$

$$= \tan^{-1}(-\tan \pi/4)$$

$$= \tan^{-1}(-1)$$

$$= -\pi/4$$

$$\therefore \tan^{-1} \{ \tan(15\pi/4) \} = -\pi/4$$

28. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $2\sin^{-1} \frac{1}{2} + \cos^{-1} \left(-\frac{1}{2} \right)$.

Answer

$$\text{Given } 2\sin^{-1} 1/2 + \cos^{-1}(-1/2)$$

$$= \pi/6 + (\pi - \pi/3)$$

$$= \frac{\pi - 6\pi - 2\pi}{6}$$

$$= \frac{5\pi}{6}$$

$$\therefore 2\sin^{-1} 1/2 + \cos^{-1}(-1/2) = 5\pi/6$$

29. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1} \frac{a}{b} - \tan^{-1} \left(\frac{a-b}{a+b} \right)$.

Answer

$$\text{Given } \tan^{-1} \frac{a}{b} - \tan^{-1} \left(\frac{a-b}{a+b} \right)$$

$$= \tan^{-1} \left[\frac{\frac{a}{b} - \frac{a-b}{a+b}}{1 + \left(\frac{a}{b} \right) \left(\frac{a-b}{a+b} \right)} \right]$$

$$= \tan^{-1} \left[\frac{\frac{a^2 + ab - ab + b^2}{b(a+b)}}{\frac{ba + b^2 + a^2 - ab}{b(a+b)}} \right]$$

$$= \tan^{-1} \left[\frac{a^2 + b^2}{a^2 + b^2} \right]$$

$$= \tan^{-1} (1)$$

$$= \pi/4$$

30. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

$$\text{Write the value of } \cos^{-1} \left(\cos \frac{2\pi}{4} \right).$$

Answer

$$\text{Given } \cos^{-1} (\cos 2\pi/4)$$

$$\text{We know that } \cos^{-1} (\cos \theta) = \theta$$

$$= 2\pi/4$$

$$= \pi/2$$

$$\therefore \cos^{-1} (\cos 2\pi/4) = \pi/2$$

31. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

$$\text{Show that } \sin^{-1} (2x\sqrt{1-x^2}) = 2\sin^{-1} x.$$

Answer

$$\text{Given LHS} = \sin^{-1} (2x - \sqrt{1-x^2})$$

$$\text{Let } x = \sin \theta$$

$$= \sin^{-1} (2\sin \theta \sqrt{1 - \sin^2 \theta})$$

$$\text{We know that } 1 - \sin^2 \theta = \cos^2 \theta$$

$$= \sin^{-1} (2 \sin \theta \cos \theta)$$

$$= \sin^{-1} (\sin^2 \theta)$$

$$= 2\theta$$

$$= 2 \sin^{-1} x$$

$$= \text{RHS}$$

$$\therefore \sin^{-1}(2x - \sqrt{1 - x^2}) = 2 \sin^{-1} x$$

32. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Evaluate: $\sin^{-1}\left(\sin \frac{3\pi}{5}\right)$.

Answer

Given $\sin^{-1}(\sin 3\pi/5)$

We know that $\sin^{-1}(\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$

$$= \pi - 3\pi/5$$

$$= 2\pi/5$$

$$\therefore \sin^{-1}(\sin 3\pi/5) = 2\pi/5$$

33. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\tan^{-1}\left(\sqrt{3}\right) + \cot^{-1} x = \frac{\pi}{2}$, find x.

Answer

Given $\tan^{-1}(\sqrt{3}) + \cot^{-1} x = \pi/2$

$$\Rightarrow \tan^{-1}(\sqrt{3}) = \pi/2 - \cot^{-1} x$$

We know that $\tan^{-1} x + \cot^{-1} x = \pi/2$

$$\Rightarrow \tan^{-1} \sqrt{3} = \tan^{-1} x$$

$$\therefore x = \sqrt{3}$$

34. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1} x = \frac{\pi}{2}$, then find x.

Answer

Given $\sin^{-1}(1/3) + \cos^{-1} x = \pi/2$

$$\Rightarrow \sin^{-1}(1/3) = \pi/2 - \cos^{-1} x$$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$$\Rightarrow \sin^{-1}(1/3) = \sin^{-1} x$$

$$\therefore x = 1/3$$

35. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right)$.

Answer

Given $\sin^{-1}(1/3) - \cos^{-1}(-1/3)$

We know that $\cos^{-1}(-\theta) = \pi - \cos^{-1}\theta$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \left[\pi - \cos^{-1}\left(\frac{1}{3}\right)\right]$$

$$= \sin^{-1}\left(\frac{1}{3}\right) - \pi + \cos^{-1}\left(\frac{1}{3}\right)$$

$$= \sin^{-1}\left(\frac{1}{3}\right) + \cos^{-1}\left(\frac{1}{3}\right) - \pi$$

$$= \frac{\pi}{2} - \pi$$

$$= -\frac{\pi}{2}$$

$$\therefore \sin^{-1}\left(\frac{1}{3}\right) - \cos^{-1}\left(-\frac{1}{3}\right) = -\frac{\pi}{2}$$

36. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $4\sin^{-1}x + \cos^{-1}x = \pi$, then what is the value of x ?

Answer

Given $4\sin^{-1}x + \cos^{-1}x = \pi$

We know that $\sin^{-1}x + \cos^{-1}x = \pi/2$

$$\Rightarrow 4\sin^{-1}x + \frac{\pi}{2} - \sin^{-1}x = \pi$$

$$\Rightarrow 3\sin^{-1}x = \pi - \frac{\pi}{2}$$

$$\Rightarrow 3\sin^{-1}x = \frac{\pi}{2}$$

$$\Rightarrow \sin^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow \sin^{-1}x = \sin^{-1}\frac{1}{2}$$

$$\therefore x = 1/2$$

37. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $x < 0$, $y < 0$ such that $xy = 1$, then write the value of $\tan^{-1}x + \tan^{-1}y$.

Answer

Given if $x < 0$, $y < 0$ such that $xy = 1$

Also given $\tan^{-1}x + \tan^{-1}y$

$$\text{We know that } \tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$= -\pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right)$$

$$= -\pi + \tan^{-1} \left(\frac{x+y}{1-1} \right)$$

$$= -\pi + \tan^{-1} (\infty)$$

$$= -\pi + \frac{\pi}{2}$$

$$= -\frac{\pi}{2}$$

$$\therefore \tan^{-1} x + \tan^{-1} y = -\frac{\pi}{2}$$

38. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

$$\text{What is the principal value of } \sin^{-1} \left(-\frac{\sqrt{3}}{2} \right)?$$

Answer

$$\text{Given } \sin^{-1} (-\sqrt{3}/2)$$

$$\text{We know that } \sin^{-1} (-\theta) = -\sin^{-1} (\theta)$$

$$= -\sin^{-1} (\sqrt{3}/2)$$

$$= -\pi/3$$

$$\therefore \sin^{-1} (-\sqrt{3}/2) = -\pi/3$$

39. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

$$\text{Write the principal value of } \sin^{-1} \left(-\frac{1}{2} \right).$$

Answer

$$\text{Given } \sin^{-1} (-1/2)$$

$$\text{We know that } \sin^{-1} (-\theta) = -\sin^{-1} (\theta)$$

$$= -\sin^{-1} (1/2)$$

$$= \pi/6$$

$$\therefore \sin^{-1} (-1/2) = \pi/6$$

40. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

$$\text{Write the principal value of } \cos^{-1} \left(\cos \frac{2\pi}{3} \right) + \sin^{-1} \left(\sin \frac{2\pi}{3} \right).$$

Answer

We know that $\sin^{-1}(\sin \theta) = \pi - \theta$, if $\theta \in [\pi/2, 3\pi/2]$ and $\cos^{-1}(\cos \theta) = \theta$, if $\theta \in [0, \pi]$

$$\text{Given } \cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right)$$

$$= \frac{2\pi}{3} + \left(\pi - \frac{2\pi}{3}\right)$$

$$= \pi$$

$$\therefore \cos^{-1}\left(\cos \frac{2\pi}{3}\right) + \sin^{-1}\left(\sin \frac{2\pi}{3}\right) = \pi$$

41. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan\left(2 \tan^{-1} \frac{1}{5}\right)$.

Answer

Let $\tan \theta = 1/5$

$$\text{Given } \tan(2 \tan^{-1} 1/5) = \tan 2\theta$$

$$\text{We know that } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2 \times \frac{1}{5}}{1 - \frac{1}{25}}$$

$$= \frac{\frac{2}{5}}{\frac{24}{25}}$$

$$= \frac{5}{12}$$

$$\therefore \tan(2 \tan^{-1} \frac{1}{5}) = \frac{5}{12}$$

42. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$.

Answer

$$\text{Given } \tan^{-1}(1) + \cos^{-1}(-1/2)$$

$$\text{We know that } \cos^{-1}(-\theta) = \pi - \cos^{-1} \theta$$

$$= \frac{\pi}{4} + \left[\pi - \frac{\pi}{3}\right]$$

$$= \frac{\pi}{4} + \frac{2\pi}{3}$$

$$= \frac{3\pi + 8\pi}{12}$$

$$= \frac{11\pi}{12}$$

$$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{11\pi}{12}$$

43. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\left\{2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right\}$.

Answer

$$\text{Given } \tan^{-1}\{2\sin(2\cos^{-1}\sqrt{3}/2)\}$$

$$= \tan^{-1}\{2\sin(2\cos^{-1}\cos\pi/6)\}$$

$$= \tan^{-1}\{2\sin(2 \times \pi/6)\}$$

$$= \tan^{-1}\{2\sin(\pi/3)\}$$

$$= \tan^{-1}\{2 \times \sqrt{3}/2\}$$

$$= \tan^{-1}\{\sqrt{3}\}$$

$$= \pi/3$$

$$\therefore \tan^{-1}\{2\sin(2\cos^{-1}\sqrt{3}/2)\} = \pi/3$$

44. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}$.

Answer

$$\text{Given } \tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3}$$

$$\text{We know that } \tan^{-1}\sqrt{3} = \pi/3 \text{ and } \cot^{-1}\sqrt{3} = \pi/6$$

$$= \frac{\pi}{3} + \frac{\pi}{6}$$

$$= \frac{2\pi + \pi}{6}$$

$$= \frac{3\pi}{6}$$

$$= \pi/2$$

45. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\cos^{-1}(\cos 680^\circ)$.

Answer

$$\begin{aligned}
&\text{Given } \cos^{-1} (\cos 680^\circ) \\
&= \cos^{-1} (\cos (720^\circ - 40^\circ)) \\
&= \cos^{-1} (\cos (2 \times 360^\circ - 40^\circ)) \\
&= \cos^{-1} (\cos 40^\circ) \\
&= 40^\circ \\
&\therefore \cos^{-1} (\cos 680^\circ) = 40^\circ
\end{aligned}$$

46. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sin^{-1} \left(\sin \frac{3\pi}{5} \right)$.

Answer

$$\begin{aligned}
&\text{Given } \sin^{-1} (\sin 3\pi/5) \\
&= \sin^{-1} [\sin (\pi - 2\pi/5)] \\
&= \sin^{-1} (\sin 2\pi/5) \\
&= 2\pi/5 \\
&\therefore \sin^{-1} (\sin 3\pi/5) = 2\pi/5
\end{aligned}$$

47. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\sec^{-1} \left(\frac{1}{2} \right)$.

Answer

We know that the value of $\sec^{-1} (1/2)$ is undefined as it is outside the range i.e. $R - (-1, 1)$.

48. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos^{-1} \left(\cos \frac{14\pi}{3} \right)$.

Answer

$$\begin{aligned}
&\text{Given } \cos^{-1} (\cos 14\pi/3) \\
&= \cos^{-1} [\cos (4\pi + 2\pi/3)] \\
&= \cos^{-1} (\cos 2\pi/3) \\
&= 2\pi/3 \\
&\therefore \cos^{-1} (\cos 14\pi/3) = 2\pi/3
\end{aligned}$$

49. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos(\sin^{-1} x + \cos^{-1} x)$, $|x| \leq 1$.

Answer

Given $|x| \leq 1$

$$\Rightarrow \pm x \leq 1$$

$$\Rightarrow x \leq 1 \text{ or } -x \leq 1$$

$$\Rightarrow x \leq 1 \text{ or } x \geq -1$$

$$\Rightarrow x \in [-1, 1]$$

Now also given $\cos(\sin^{-1} x + \cos^{-1} x)$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$$\therefore \cos(\sin^{-1} x + \cos^{-1} x) = \cos(\pi/2) = 0$$

50. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of the expression $\tan\left(\frac{\sin^{-1} x + \cos^{-1} x}{2}\right)$, when $x = \frac{\sqrt{3}}{2}$.

Answer

Given $\tan\left(\frac{\sin^{-1} x + \cos^{-1} x}{2}\right)$ when $x = \frac{\sqrt{3}}{2}$

$$\Rightarrow \tan\left(\frac{\sin^{-1} x + \cos^{-1} x}{2}\right) = \tan\left(\frac{\sin^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{\sqrt{3}}{2}}{2}\right)$$

We know that $\sin^{-1} x + \cos^{-1} x = \pi/2$

$$= \tan(\pi/4)$$

$$\therefore \tan\left(\frac{\sin^{-1} x + \cos^{-1} x}{2}\right) = 1$$

51. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the principal value of $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{1}{2}\right)\right\}$.

Answer

Given $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{1}{2}\right)\right\}$

$$= \sin^{-1}\left\{\cos\left(\sin^{-1}\left(\sin\frac{\pi}{3}\right)\right)\right\}$$

$$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$$

$$= \sin^{-1}(1/2)$$

$$= \sin^{-1}\left(\sin \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3}$$

52. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

The set of values of $\operatorname{cosec}^{-1}\left(\frac{\sqrt{3}}{2}\right)$.

Answer

We know that the value of $\operatorname{cosec}^{-1}(\sqrt{3}/2)$ is undefined as it is outside the range i.e. $R(-1,1)$.

53. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\tan^{-1}\left(\frac{1}{x}\right)$ for $x < 0$ in terms of $\cot^{-1}(x)$.

Answer

Given $\tan^{-1}(1/x)$

$$= \tan^{-1}\left(-\frac{1}{x}\right) \text{ for } x < 0$$

$$= -\tan^{-1}\left(\frac{1}{x}\right)$$

$$= \cot^{-1} x$$

$$= -(\pi - \cot^{-1} x)$$

$$= -\pi + \cot^{-1} x$$

54. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cot^{-1}(-x)$ for all $x \in R$ in terms of $\cot^{-1}x$.

Answer

We know that $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

$\therefore$ The value of $\cot^{-1}(-x)$ for all $x \in R$ in term of $\cot^{-1} x$ is $\pi - \cot^{-1}(x)$.

55. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Write the value of $\cos\left(\frac{\tan^{-1} x + \cot^{-1} x}{3}\right)$, when $x = -\frac{1}{\sqrt{3}}$.

Answer

Given $\cos\left(\frac{\tan^{-1} x + \cot^{-1} x}{3}\right)$ when $x = -\frac{1}{\sqrt{3}}$

We know that $\tan^{-1} x + \cot^{-1} x = \pi/2$

$$= \cos (\pi/6)$$

$$= \sqrt{3}/2$$

56. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\cos(\tan^{-1} x + \cot^{-1} \sqrt{3}) = 0$, find the value of x .

Answer

$$\text{Given } \cos(\tan^{-1} x + \cot^{-1} \sqrt{3}) = 0$$

$$\Rightarrow \cos(\tan^{-1} x + \cot^{-1} \sqrt{3}) = \cos(\pi/2)$$

$$\Rightarrow \tan^{-1} x + \cot^{-1} \sqrt{3} = \pi/2$$

$$\text{We know that } \tan^{-1} x + \cot^{-1} x = \pi/2$$

$$\therefore x = \sqrt{3}$$

57. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $2 \sec^{-1} 2 + \sin^{-1} \left(\frac{1}{2} \right)$.

Answer

$$\text{Given } 2 \sec^{-1} 2 + \sin^{-1} (1/2)$$

$$= 2 \sec^{-1} (\sec \pi/3) + \sin^{-1} (\sin \pi/6)$$

$$= 2 (\pi/3) + \pi/6$$

$$= 5\pi/6$$

58. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

If $\cos\left(\sin^{-1} \frac{2}{5} + \cos^{-1} x\right) = 0$, find the value of x .

Answer

$$\text{Given } \cos(\sin^{-1} 2/5 + \cos^{-1} x) = 0$$

$$\Rightarrow \cos(\sin^{-1} 2/5 + \cos^{-1} x) = \cos(\pi/2)$$

$$\Rightarrow \sin^{-1} 2/5 + \cos^{-1} x = \pi/2$$

$$\text{We know that } \sin^{-1} x + \cos^{-1} x = \pi/2$$

$$\therefore x = 2/5$$

59. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $\cos^{-1}\left(\cos \frac{13\pi}{6}\right)$.

Answer

$$\text{Given } \cos^{-1}(\cos 13\pi/6)$$

$$= \cos^{-1}[\cos (2\pi + \pi/6)]$$

$$= \cos^{-1}(\cos \pi/6)$$

$$= \pi/6$$

$$\therefore \cos^{-1}(\cos 13\pi/6) = \pi/6$$

60. Question

Answer each of the following questions in one word or one sentence or as per exact requirement of the question:

Find the value of $\tan^{-1}\left(\tan \frac{9\pi}{8}\right)$.

Answer

$$\text{Given } \tan^{-1}(\tan 9\pi/8)$$

$$= \tan^{-1}[\tan (\pi + \pi/8)]$$

$$= \tan^{-1}(\tan \pi/8)$$

$$= \pi/8$$

$$\therefore \tan^{-1}(\tan 9\pi/8) = \pi/8$$