

Ex 7.1

Adjoint and Inverse of Matrix Ex 7.1 Q1(i)

Here, $A = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix}$

Cofactors of A are:

$$C_{11} = 4$$

$$C_{12} = -2$$

$$C_{21} = -5$$

$$C_{22} = -3$$

$$\therefore \text{adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \therefore (\text{adj } A) &= \begin{bmatrix} 4 & -2 \\ -5 & -3 \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} \end{aligned}$$

$$\text{Now, } (\text{adj } A)A = \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

$$\text{And, } |A|.I = \begin{vmatrix} -3 & 5 \\ 2 & 4 \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (-22) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

Also,

$$A(\text{adj } A) = \begin{bmatrix} -3 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 4 & -5 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -22 & 0 \\ 0 & -22 \end{bmatrix}$$

$$\text{Therefore, } (\text{adj } A)A = |A|.I = A(\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(ii)

Here, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Cofactors of A are:

$$\begin{aligned} C_{11} &= d \\ C_{12} &= -c \\ C_{21} &= -b \\ C_{22} &= a \end{aligned}$$

$$\therefore \text{adj}A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \therefore \text{adj}A &= \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}^T \\ &= \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \end{aligned}$$

$$\text{Now, } (\text{adj}A)(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & ad-bc \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\text{And, } |A|.I = \begin{vmatrix} a & b \\ c & d \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (ad-bc) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

Also,

$$A(\text{adj}A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$\therefore (\text{adj}A)(A) = |A|.I = A(\text{adj}A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iii)

Here, $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$

Cofactors of A are:

$$C_{11} = \cos \alpha$$

$$C_{12} = -\sin \alpha$$

$$C_{21} = -\sin \alpha$$

$$C_{22} = \cos \alpha$$

$$\therefore \text{Adj } A = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T$$

$$\begin{aligned} \text{Adj } A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}^T \\ &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, } (\text{adj } A) \cdot (A) &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & \cos \alpha \sin \alpha - \sin \alpha \cos \alpha \\ -\cos \alpha \sin \alpha + \sin \alpha \cos \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{And, } A (\text{adj } A) &= \begin{bmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & -\sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & -\cos \alpha \sin \alpha + \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \cos \alpha \sin \alpha & -\sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

Also,

$$\begin{aligned} |A| \cdot I &= \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & \cos \alpha \end{vmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = (\cos^2 \alpha - \sin^2 \alpha) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha - \sin^2 \alpha & 0 \\ 0 & \cos^2 \alpha - \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos 2\alpha & 0 \\ 0 & \cos 2\alpha \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q1(iv)

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$$c_{11} = 1, \quad c_{12} = -(-\tan \alpha/2) = \tan \alpha/2$$

$$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

We have,

$$A = \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}$$

Cofactors of A are:

$$c_{11} = 1, \quad c_{12} = -(-\tan \alpha/2) = \tan \alpha/2$$

$$c_{21} = -\tan \alpha/2, \quad c_{22} = 1$$

$$\therefore \text{adj } A = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & -\tan \alpha/2 \\ \tan \alpha/2 & 1 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & \tan \alpha/2 \\ -\tan \alpha/2 & 1 \end{vmatrix}$$

$$= 1 + \tan^2 \alpha/2$$

$$= \sec^2 \alpha/2$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(i)

Here $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -3 & C_{21} = +2 & C_{31} = 2 \\ C_{12} = +2 & C_{22} = -3 & C_{32} = 2 \\ C_{13} = 2 & C_{23} = 2 & C_{33} = -3 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Now,

$$(\text{adj } A) \cdot A = \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\begin{aligned} |A| \cdot I &= \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= (-3 + 4 + 4) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \end{aligned}$$

$$A (\text{adj } A) = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\therefore (\text{adj } A) \cdot A = |A| \cdot I = A (\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 2 & C_{21} = 3 & C_{31} = -13 \\ C_{12} = -3 & C_{22} = 6 & C_{32} = 9 \\ C_{13} = 5 & C_{23} = -3 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 2 & -3 & 5 \\ 3 & 6 & -3 \\ -13 & 9 & -1 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj } A = \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix}$$

Now,

$$\begin{aligned} (\text{adj } A) \cdot A &= \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix} \end{aligned}$$

$$|A| \cdot I = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$A \cdot (\text{adj } A) = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -13 \\ -3 & 6 & 9 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 21 \end{bmatrix}$$

$$\therefore (\text{adj } A) \cdot A = |A| \cdot I = A \cdot (\text{adj } A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -22 & C_{21} = 11 & C_{31} = -11 \\ C_{12} = 4 & C_{22} = -2 & C_{32} = 2 \\ C_{13} = 16 & C_{23} = -8 & C_{33} = 8 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -22 & 4 & 16 \\ 11 & -2 & -8 \\ -11 & 2 & 8 \end{bmatrix} \end{aligned}$$

Therefore,

$$\text{adj} A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix}$$

Now,

$$(\text{adj} A) \cdot A = \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} |A| \cdot I &= \begin{vmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{vmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= (44 - 4 + 48) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0 \end{aligned}$$

$$A (\text{adj} A) = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 2 & 5 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} -22 & 11 & -11 \\ 4 & -2 & 2 \\ 16 & -8 & 8 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore (\text{adj} A) \cdot A = |A| I = A (\text{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q2(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 7 & C_{32} = -5 \\ C_{13} = 4 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -15 & 4 \\ -1 & 7 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj} A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix}$$

Now,

$$(\text{adj} A)A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{aligned} |A|.I &= \begin{vmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{vmatrix} I_3 \\ &= (6-4)I_3 = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$A(\text{adj} A) = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 7 & -5 \\ 4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore (\text{adj} A)A = |A|.I = A(\text{adj} A)$$

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 15 & C_{21} = 6 & C_{31} = -15 \\ C_{12} = 0 & C_{22} = -3 & C_{32} = 0 \\ C_{13} = -10 & C_{23} = 0 & C_{33} = 5 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 15 & 0 & -10 \\ 6 & -3 & 0 \\ -15 & 0 & 5 \end{bmatrix} \end{aligned}$$

Therefore,

$$\text{adj} A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix}$$

Now,

$$(\text{adj} A) A = \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\begin{aligned} |A|.I &= \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{vmatrix} I_3 \\ &= (-15)I_3 = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix} \end{aligned}$$

$$A(\text{adj} A) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 5 & 0 \\ 2 & 4 & 3 \end{bmatrix} \begin{bmatrix} 15 & 6 & -15 \\ 0 & -3 & 0 \\ -10 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$$

$$\therefore (\text{adj} A) A = |A|.I = A(\text{adj} A)$$

Adjoint and Inverse of Matrix Ex 7.1 Q3

Here

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$$

Cofactors of A are

$$\begin{array}{lll} C_{11} = 30 & C_{21} = 12 & C_{31} = -3 \\ C_{12} = -20 & C_{22} = -8 & C_{32} = 2 \\ C_{13} = -50 & C_{23} = -20 & C_{33} = 5 \end{array}$$

Therefore,

$$\begin{aligned} \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 30 & -20 & -50 \\ 12 & -8 & -20 \\ -3 & 2 & 5 \end{bmatrix} \end{aligned}$$

So,

$$\text{adj } A = \begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix}$$

Now,

$$\begin{aligned} A(\text{adj } A) &= \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix} \begin{bmatrix} 30 & 12 & -3 \\ -20 & -8 & 2 \\ -50 & -20 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix} (0) \\ &= 0 \end{aligned}$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q4

Here, $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -4 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = 1 & C_{22} = 0 & C_{32} = 1 \\ C_{13} = 4 & C_{23} = 4 & C_{33} = 3 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix} \end{aligned}$$

Therefore, $\text{adj } A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

So, $\text{adj } A = A$

Adjoint and Inverse of Matrix Ex 7.1 Q5

Here $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -3 & C_{21} = 6 & C_{31} = 6 \\ C_{12} = -6 & C_{22} = 3 & C_{32} = -6 \\ C_{13} = -6 & C_{23} = -6 & C_{33} = 3 \end{array}$$

$$\begin{aligned} \therefore \text{adj}A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T \end{aligned}$$

Therefore, $\text{adj}A = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$ --- (i)

Now, $3.A^T = 3 \begin{bmatrix} -1 & 2 & 2 \\ -2 & 1 & -2 \\ -2 & -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$ --- (ii)

$\therefore \text{adj}A = 3.A^T$

Adjoint and Inverse of Matrix Ex 7.1 Q6

Here, $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 9 & C_{21} = 19 & C_{31} = -4 \\ C_{12} = 4 & C_{22} = 14 & C_{32} = 1 \\ C_{13} = 8 & C_{23} = 3 & C_{33} = 2 \end{array}$$

$$\begin{aligned} \therefore \text{adj}A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}^T \end{aligned}$$

Therefore,

$$\text{adj}A = \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } A \text{adj}A &= \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix} \begin{bmatrix} 9 & 19 & -4 \\ 4 & 14 & 1 \\ 8 & 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 25 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 25 \end{bmatrix} \\ &= 25 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= 25I_3 \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (i)

$$A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

Now, $|A| = 1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$\begin{aligned} C_{11} &= \cos \theta & C_{21} &= -\sin \theta \\ C_{12} &= \sin \theta & C_{22} &= \cos \theta \end{aligned}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \end{aligned}$$

$$\therefore \text{adj} A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (ii)

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $|A| = -1 \neq 0$

Hence A^{-1} exists.

Cofactors of A are:

$$\begin{aligned} C_{11} &= 0 & C_{12} &= -1 \\ C_{21} &= -1 & C_{22} &= 0 \end{aligned}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \end{aligned}$$

$$\text{Also, } A^{-1} = \frac{1}{|A|} (\text{adj} A)$$

$$A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iii)

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

$$\text{Now, } |A| = \frac{a+abc}{a} - bc = \frac{a+abc-abc}{a} = 1 \neq 0$$

Hence A^{-1} exists.

Now, cofactors of A are:

$$\begin{aligned} C_{11} &= \frac{1+bc}{a} & C_{12} &= -c \\ C_{21} &= -b & C_{22} &= a \end{aligned}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} \frac{1+bc}{a} & -c \\ -b & a \end{bmatrix}^T \end{aligned}$$

$$\therefore \text{adj } A = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

$$\begin{aligned} \text{Also, } A^{-1} &= \frac{1}{|A|} \cdot \text{adj } A \\ A^{-1} &= \frac{1}{1} \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} \end{aligned}$$

$$\text{or } A^{-1} = \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q7 (iv)

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$$

$$\text{Now, } |A| = 2 + 15 = 17 \neq 0$$

Hence A^{-1} exists.

Now, cofactors of A are:

$$\begin{aligned} C_{11} &= 1 & C_{12} &= 3 \\ C_{21} &= -5 & C_{22} &= 2 \end{aligned}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix} \end{aligned}$$

$$\therefore \text{adj } A = \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

$$\begin{aligned} A^{-1} &= \frac{1}{|A|} \cdot (\text{adj } A) \\ A^{-1} &= \frac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } A^{-1} = \frac{1}{17} \begin{bmatrix} 1 & -5 \\ 3 & 2 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(i)

Here, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

Expanding using 1st row, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} \\ &= 1(6-1) - 2(4-3) + 3(2-9) \\ &= 5 - 2 \times 1 + 3 \times (-7) \\ &= 5 - 2 - 21 = -18 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 5 & C_{21} = -1 & C_{31} = -7 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 5 \\ C_{13} = -7 & C_{23} = 5 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix}^T \\ &= \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} \end{aligned}$$

Now,

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$\text{Hence, } A^{-1} = \frac{1}{(-18)} \begin{bmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & -1 \end{bmatrix} = \begin{bmatrix} \frac{-5}{18} & \frac{1}{18} & \frac{7}{18} \\ \frac{1}{18} & \frac{7}{18} & \frac{-5}{18} \\ \frac{7}{18} & \frac{-5}{18} & \frac{1}{18} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(ii)

Here, $A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$

Expanding using first column, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} -1 & -1 \\ 3 & -1 \end{vmatrix} - 2 \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} + 5 \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} \\ &= 1 \times (1 + 3) - 2(-1 + 2) + 5(3 + 2) \\ &= 4 - 2(1) + 5(5) = 27 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists.

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 4 & C_{21} = +17 & C_{31} = 3 \\ C_{12} = -1 & C_{22} = -11 & C_{32} = +6 \\ C_{13} = 5 & C_{23} = +1 & C_{33} = -3 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & -1 & 5 \\ 17 & -11 & 1 \\ 3 & 6 & -3 \end{bmatrix}^T \\ &= \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \end{aligned}$$

Now, $A^{-1} = \frac{1}{|A|} \cdot \text{adj} A$

$$\therefore A^{-1} = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{3}{27} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{6}{27} \\ \frac{5}{27} & \frac{1}{27} & \frac{-3}{27} \end{bmatrix} = \begin{bmatrix} \frac{4}{27} & \frac{17}{27} & \frac{1}{9} \\ \frac{-1}{27} & \frac{-11}{27} & \frac{2}{9} \\ \frac{5}{27} & \frac{1}{27} & \frac{-1}{9} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iii)

Here, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Expanding using first column, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 \\ 1 & 2 \end{vmatrix} + 1 \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} \\ &= 2(4-1) + 1(-2+1) + 1(1-2) \\ &= 2(3) + 1(-1) + 1(-1) = 6 - 2 = 4 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = +1 & C_{31} = -1 \\ C_{12} = +1 & C_{22} = 3 & C_{32} = +1 \\ C_{13} = -1 & C_{23} = +1 & C_{33} = 3 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} \end{aligned}$$

Now, $A^{-1} = \frac{1}{|A|} \text{adj} A$

$$\therefore A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & \frac{-1}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{1}{4} \\ \frac{-1}{4} & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(iv)

Here, $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Expanding using first column, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} - 0 \begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} - 1 \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix} \\ &= 2(3 - 0) - 0 - 1(5) \\ &= 2(3) - 1(5) = 1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 3 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -15 & C_{22} = 6 & C_{32} = -5 \\ C_{13} = 5 & C_{23} = -2 & C_{33} = 2 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \therefore A^{-1} &= \frac{1}{|A|} \cdot \text{adj } A \\ &= \frac{1}{1} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(v)

Here, $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

Expanding using first column, we get

$$\begin{aligned} |A| &= 0 \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & 4 \\ 3 & 4 \end{vmatrix} - 1 \begin{vmatrix} 4 & -3 \\ 3 & -3 \end{vmatrix} \\ &= 0 - 1(6 - 12) - 1(-12 + 9) \\ &= -1(4) - 1(-3) = -1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 0 & C_{21} = -1 & C_{31} = 1 \\ C_{12} = -4 & C_{22} = 3 & C_{32} = -4 \\ C_{13} = -3 & C_{23} = +3 & C_{33} = -4 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & -4 & -3 \\ -1 & 3 & 3 \\ 1 & -4 & -4 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix} \end{aligned}$$

Now, $A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$

$$A^{-1} = \frac{1}{(-1)} \begin{bmatrix} 0 & -1 & 1 \\ -4 & 3 & -4 \\ -3 & 3 & -4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vi)

Here, $A = \begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$

Expanding using first column, we get

$$\begin{aligned} |A| &= 0 \begin{vmatrix} 4 & 5 \\ -4 & -7 \end{vmatrix} - 0 \begin{vmatrix} 3 & 5 \\ -2 & -7 \end{vmatrix} - 1 \begin{vmatrix} 3 & 4 \\ -2 & -4 \end{vmatrix} \\ &= 0 - 0 - 1(-12 + 8) \\ &= -1(-4) = 4 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -8 & C_{21} = +4 & C_{31} = 4 \\ C_{12} = +11 & C_{22} = -2 & C_{32} = -3 \\ C_{13} = -4 & C_{23} = +0 & C_{33} = 0 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -8 & 11 & -4 \\ 4 & -2 & 0 \\ 4 & -3 & 0 \end{bmatrix}^T \\ &= \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \frac{1}{4} \begin{bmatrix} -8 & 4 & 4 \\ 11 & -2 & -3 \\ -4 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 1 \\ \frac{11}{4} & \frac{-1}{2} & \frac{-3}{4} \\ -1 & 0 & 0 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q8(vii)

Here, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$

Expanding using first column, we get

$$|A| = 1 \begin{vmatrix} \cos \alpha & \sin \alpha \\ \sin \alpha & -\cos \alpha \end{vmatrix} - 0 + 0$$

$$= -\cos^2 \alpha + \sin^2 \alpha$$

$$= -(\cos^2 \alpha + \sin^2 \alpha)$$

$$|A| = -1 \neq 0$$

Therefore, A^{-1} exists

Cofactors of A are:

$$C_{11} = -1 \quad C_{21} = 0 \quad C_{31} = 0$$

$$C_{12} = 0 \quad C_{22} = -\cos \alpha \quad C_{32} = -\sin \alpha$$

$$C_{13} = 0 \quad C_{23} = -\sin \alpha \quad C_{33} = \cos \alpha$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}^T \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj} A$$

$$\therefore A^{-1} = \frac{1}{(-1)} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos \alpha & -\sin \alpha \\ 0 & -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & \sin \alpha \\ 0 & \sin \alpha & -\cos \alpha \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q9(i)

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Expanding using 1st row, we get

$$\begin{aligned} |A| &= 1 \begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} - 3 \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} + 3 \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} \\ &= 1(16 - 9) - 3(4 - 3) + 3(3 - 4) \\ &= 7 - 3(1) + 3(-1) \\ &= 7 - 3 - 3 = +1 = 1 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 7 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = -1 & C_{22} = 1 & C_{32} = -0 \\ C_{13} = -1 & C_{23} = -0 & C_{33} = 1 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}^T \\ &= \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$\therefore A^{-1} = \frac{1}{1} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Also, } A^{-1} \cdot A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q9(ii)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Expanding 1st row, we get

$$\begin{aligned} |A| &= 2 \begin{vmatrix} 4 & 1 \\ 7 & 2 \end{vmatrix} - 3 \begin{vmatrix} 3 & 1 \\ 3 & 2 \end{vmatrix} + 1 \begin{vmatrix} 3 & 4 \\ 3 & 7 \end{vmatrix} \\ &= 2(8-7) - 3(6-3) + 1(21-12) \\ &= 2 - 3(3) + 1(9) = 2 \neq 0 \end{aligned}$$

Therefore, A^{-1} exists

Cofactors of A are:

$$\begin{array}{lll} C_{11} = 1 & C_{21} = +1 & C_{31} = -1 \\ C_{12} = -3 & C_{22} = 1 & C_{32} = +1 \\ C_{13} = 9 & C_{23} = -5 & C_{33} = -1 \end{array}$$

$$\begin{aligned} \therefore \text{adj} A &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & -3 & 9 \\ 1 & 1 & -5 \\ -1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj} A \Rightarrow A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix}$$

$$\begin{aligned} \text{Also, } A^{-1} \cdot A &= \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ -3 & 1 & 1 \\ 9 & -5 & -1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 1 \\ 3 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2+3-3 & 3+4-7 & 1+1-2 \\ -6+3+3 & -9+4+7 & -3+1+2 \\ 18-15-3 & 27-20-7 & 9-5-2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\therefore A^{-1} \cdot A = I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q10(i)

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \quad \therefore |A| = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} \quad \therefore |B| = -10 \neq 0$$

$$\text{adj } B = \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \Rightarrow B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-10} \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 4 & 6 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 18 & 22 \\ 43 & 52 \end{bmatrix}$$

$$|AB| = 936 - 946 = -10 \neq 0$$

$$\text{adj}(AB) = \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix}$$

$$(AB)^{-1} = \frac{\text{adj}(AB)}{|AB|} = \frac{1}{|AB|} \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{-10} \begin{bmatrix} +52 & -22 \\ -43 & +18 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$B^{-1}.A^{-1} = \frac{-1}{10} \begin{bmatrix} 2 & -6 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} = \frac{-1}{10} \begin{bmatrix} 52 & -22 \\ -43 & 18 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} -52 & 22 \\ 43 & -18 \end{bmatrix}$$

$$\text{Hence, } (AB)^{-1} = B^{-1}.A^{-1}$$

Adjoint and Inverse of Matrix Ex 7.1 Q10(ii)

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \therefore |A| = 1 \neq 0 \text{ and } \text{adj } A = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} \quad \therefore |B| = -1 \neq 0 \text{ and } \text{adj } B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{-1} \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$\text{Also, } AB = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 14 \\ 29 & 27 \end{bmatrix}$$

$$|AB| = 407 - 406 = 1 \neq 0$$

$$\text{and, } \text{adj}(AB) = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$$

$$\begin{aligned} \therefore (AB)^{-1} &= \frac{1}{|AB|} \cdot \text{adj}(AB) \\ &= \frac{1}{1} \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} = \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Again, } B^{-1}.A^{-1} &= \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } (AB)^{-1} = B^{-1}.A^{-1}$$

Adjoint and Inverse of Matrix Ex 7.1 Q11

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \quad \therefore |A| = 1 \neq 0 \text{ and } \text{adj } A = \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{1} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 6 & 7 \\ 7 & 9 \end{bmatrix} \quad \therefore |B| = -2 \neq 0 \text{ and } \text{adj } B = \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

$$\therefore B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{(-2)} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix}$$

Now, $(AB)^{-1} = B^{-1} \cdot A^{-1}$

$$(AB)^{-1} = \frac{1}{(-2)} \begin{bmatrix} 9 & -7 \\ -8 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$(AB)^{-1} = -\frac{1}{2} \begin{bmatrix} 94 & -39 \\ -82 & 34 \end{bmatrix}$$

$$(AB)^{-1} = \begin{bmatrix} -47 & \frac{39}{2} \\ 41 & -17 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q12

$$A = \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} \quad \therefore |A| = 2 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

To show: $2A^{-1} = 9I - A$

$$\text{LHS: } 2A^{-1} = 2 \cdot \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

$$\text{RHS: } 9I - A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$$

Therefore, $2A^{-1} = 9I - A$

Adjoint and Inverse of Matrix Ex 7.1 Q13

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} \quad \therefore |A| = -6 \text{ and } \text{adj } A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix} \quad \therefore A^{-1} = \frac{1}{-6} \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$$

To show: $A - 3I = 2(I + 3A^{-1})$

$$\therefore \text{LHS} = A - 3I = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix}$$

$$\begin{aligned} \text{RHS: } 2(I + 3A^{-1}) &= 2I + 2 \cdot 3 \cdot A^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + 2 \cdot 3 \cdot \frac{1}{-6} \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 5 \\ 2 & -2 \end{bmatrix} \end{aligned}$$

$$\therefore A - 3I = 2(I + 3A^{-1})$$

Adjoint and Inverse of Matrix Ex 7.1 Q14

$$A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$$

$$\Rightarrow |A| = (1+bc) - bc = 1 \neq 0$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{1} \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix}$$

$$\text{Now } aA^{-1} = (a^2 + bc + 1)I - aA$$

$$\text{LHS : } aA^{-1} = a \begin{bmatrix} \frac{1+bc}{a} & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

$$\text{RHS : } (a^2 + bc + 1)I - aA = \begin{bmatrix} a^2 + bc + 1 & 0 \\ 0 & a^2 + bc + 1 \end{bmatrix} - \begin{bmatrix} a^2 & ab \\ ac & 1+bc \end{bmatrix} = \begin{bmatrix} 1+bc & -ab \\ -ac & a^2 \end{bmatrix}$$

Since, $LHS = RHS$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q15

Here

$$(AB)^{-1} = B^{-1}A^{-1}$$

Now we need to find A^{-1} .

We have

$$A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

So,

$$|A| = -5 + 4 = -1$$

Co-factors of A are

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 8 & C_{31} = -12 \\ C_{12} = 0 & C_{22} = 1 & C_{32} = -2 \\ C_{13} = 1 & C_{23} = -10 & C_{33} = 15 \end{array}$$

Therefore,

$$\text{adj}A = \begin{bmatrix} -1 & 8 & -12 \\ 0 & 1 & -2 \\ 1 & -10 & 15 \end{bmatrix}$$

So,

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$$

Hence,

$$\begin{aligned} (AB)^{-1} &= B^{-1}A^{-1} \\ &= \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(i)

$$F(\alpha) = \begin{bmatrix} \cos \alpha & -\sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow |F(\alpha)| = \cos^2 \alpha + \sin^2 \alpha = 1$$

$$\begin{array}{lll} C_{11} = \cos \alpha & C_{21} = +\sin \alpha & C_{31} = 0 \\ C_{12} = -\sin \alpha & C_{22} = \cos \alpha & C_{32} = 0 \\ C_{13} = 0 & C_{23} = 0 & C_{33} = 1 \end{array}$$

$$[F(\alpha)]^{-1} = \frac{\text{adj}(F(\alpha))}{|F(\alpha)|} = \frac{1}{1} \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (1)}$$

Now

$$\begin{aligned} F(-\alpha) &= \begin{bmatrix} \cos(-\alpha) & -\sin(-\alpha) & 0 \\ \sin(-\alpha) & \cos(-\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (2)} \end{aligned}$$

$$\text{From (1) \& (2) } F(-\alpha) = [F(\alpha)]^{-1}$$

Hence, proved

Adjoint and Inverse of Matrix Ex 7.1 Q16(ii)

$$G(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{bmatrix} \Rightarrow |G(\beta)| = \cos^2 \beta + \sin^2 \beta$$

$$\begin{array}{lll} C_{11} = \cos \beta & C_{21} = 0 & C_{31} = \sin \beta \\ C_{12} = +0 & C_{22} = 1 & C_{32} = 0 \\ C_{13} = \sin \beta & C_{23} = 0 & C_{33} = \cos \beta \end{array}$$

$$[G(\beta)]^{-1} = \frac{\text{adj}(G(\beta))}{|G(\beta)|} = \frac{1}{1} \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \quad \text{--- (1)}$$

Now

$$\begin{aligned} G(-\beta) &= \begin{bmatrix} \cos(-\beta) & 0 & \sin(-\beta) \\ 0 & 1 & 0 \\ -\sin(-\beta) & 0 & \cos(-\beta) \end{bmatrix} \\ &= \begin{bmatrix} \cos \beta & 0 & -\sin \beta \\ 0 & 1 & 0 \\ \sin \beta & 0 & \cos \beta \end{bmatrix} \quad \text{--- (2)} \end{aligned}$$

From (1) \& (2)

$$[G(\beta)]^{-1} = G(-\beta)$$

Adjoint and Inverse of Matrix Ex 7.1 Q16(iii)

We have to show that

$$[F(\alpha)G(\beta)]^{-1} = G(-\beta)F(-\alpha)$$

We have already shown that

$$G(-\beta) = [G(\beta)]^{-1}$$

$$\text{and } F(-\alpha) = [F(\alpha)]^{-1}$$

$$\begin{aligned} \therefore \text{LHS} &= [F(\alpha)G(\beta)]^{-1} \\ &= [G(\beta)]^{-1}[F(\alpha)]^{-1} \quad \left[\because (AB)^{-1} = B^{-1}A^{-1} \right] \\ &= G(-\beta) \times F(-\alpha) \\ &= \text{RHS} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q17

$$\text{We have } A^2 = A.A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$\begin{aligned} \text{Hence } A^2 - 4A + I &= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 7-8+1 & 12-12+0 \\ 4-4+0 & 7-8+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O \end{aligned}$$

$$\text{Now, } A^2 - 4A + I = O$$

$$\Rightarrow A.A - 4A = -I$$

Post multiplying both sides by A^{-1} , since $|A| \neq 0$

$$AA(A^{-1}) - 4AA^{-1} = -IA^{-1}$$

$$\Rightarrow A(AA^{-1}) - 4I = -A^{-1}$$

$$\Rightarrow AI - 4I = -A^{-1}$$

$$\text{or } A^{-1} = 4I - A = 4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4-2 & 0-3 \\ 0-1 & 4-2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q18

$$A = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix}$$

$$\text{Now } A^2 + 4A - 42I = 0$$

$$\text{For this } A^2 = \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix}$$

Hence,

$$A^2 + 4A - 42I = \begin{bmatrix} 74 & -20 \\ -8 & 26 \end{bmatrix} + \begin{bmatrix} -32 & 20 \\ 8 & 16 \end{bmatrix} - \begin{bmatrix} 42 & 0 \\ 0 & 42 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence proved.

$$\text{Now, } A^2 + 4A - 42I = 0$$

$$\Rightarrow A^{-1}.A.A + 4A^{-1}.A - 42A^{-1}.I = 0$$

$$\Rightarrow IA + 4I - 42A^{-1} = 0$$

$$\Rightarrow 42A^{-1} = A + 4I$$

$$\Rightarrow A^{-1} = \frac{1}{42}[A + 4I] = \frac{1}{42} \left\{ \begin{bmatrix} -8 & 5 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} \right\} = \frac{1}{42} \begin{bmatrix} -4 & 5 \\ 2 & 8 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q19

Here

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now,

$$A^2 - 5A + 7I = 0$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So,

$$A^2 - 5A + 7I = 0$$

Pre-multiplying with A^{-1}

$$A^{-1}A^2 - 5A^{-1}A + 7IA^{-1} = 0$$

$$A - 5I + 7A^{-1} = 0$$

$$A^{-1} = \frac{1}{7}[5I - A]$$

$$= \frac{1}{7} \left\{ \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \right\}$$

$$= \frac{1}{7} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q20

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

$$\text{Now } A^2 - xA + yI = 0$$

$$\Rightarrow \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - \begin{bmatrix} 4x & 3x \\ 2x & 5x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow 22 - 4x + y = 0 \quad \text{or} \quad 4x - y = 22$$

$$\Rightarrow 18 - 2x = 0 \quad \text{or} \quad x = 9$$

$$\therefore y = 14$$

Again,

$$A^2 - 9A + 14I = 0$$

$$\Rightarrow 9A = A^2 + 14I = 0$$

$$\Rightarrow 9A^{-1}A = A^{-1}A^2 + 14A^{-1}$$

$$\Rightarrow 9I = IA + 14A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{14}\{9I - A\} = \frac{1}{14} \left\{ \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} \right\} = \frac{1}{14} \left\{ \begin{bmatrix} 5 & -3 \\ -2 & 4 \end{bmatrix} \right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q21

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \end{aligned}$$

$$\text{If } A^2 = \lambda A - 2I$$

$$\begin{aligned} \lambda A &= A^2 + 2I \\ &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \end{aligned}$$

$$\lambda \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 3\lambda & -2\lambda \\ 4\lambda & -2\lambda \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$3\lambda = 3$$

$$\lambda = 1$$

$$\text{Ans } \lambda = 1$$

$$A^2 = A - 2I$$

Px multiplying by A^{-1}

$$A^{-1}.AA = A^{-1}.A - 2A^{-1}.I$$

$$A = I - 2A^{-1}$$

$$2A^{-1} = I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} -2 & 2 \\ -4 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q22

$$\text{We have } A = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\text{To prove: } A^2 - 3A - 7 = 0$$

$$\text{Now, } A^2 = AA = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix}$$

$$\text{So, } A^2 - 3A - 7 = \begin{bmatrix} 22 & 9 \\ -3 & 1 \end{bmatrix} - \begin{bmatrix} 15 & 9 \\ -3 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\text{Now, } A^2 - 3A - 7 = 0$$

$$\Rightarrow AA^{-1}.A - 3A^{-1}.A - 7A^{-1} = 0$$

$$\Rightarrow A - 3I - 7A^{-1} = 0$$

$$\Rightarrow A - 3I - 7A^{-1} = 0$$

$$\Rightarrow 7A^{-1} = A - 3I$$

$$\Rightarrow A^{-1} = \frac{1}{7} \left(\begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) = \frac{1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{3}{7} \\ -\frac{1}{7} & -\frac{5}{7} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q23

Show that $A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$ satisfies the equation $x^2 - 12x + I = 0$. Thus, find A^{-1} .

$$A = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix}$$

$$\text{Now } A^2 - 12A + I = \begin{bmatrix} 71 & 60 \\ 84 & 71 \end{bmatrix} - 12 \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A^2 - 12A + I = 0$$

$$\Rightarrow A - 12I + A^{-1} = 0$$

$$\Rightarrow A^{-1} = 12I - A = \left\{ \begin{bmatrix} 12 & 0 \\ 0 & 12 \end{bmatrix} - \begin{bmatrix} 6 & 5 \\ 7 & 6 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} 6 & -5 \\ -7 & 6 \end{bmatrix} \right\}$$

Adjoint and Inverse of Matrix Ex 7.1 Q24

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\begin{aligned} A^3 &= A^2 \cdot A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} \end{aligned}$$

$$\therefore A^3 - 6A^2 + 5A + 11I$$

$$\begin{aligned}
 &= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \\
 &= \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O
 \end{aligned}$$

$$\text{Thus, } A^3 - 6A^2 + 5A + 11I = O.$$

Now,

$$A^3 - 6A^2 + 5A + 11I = O$$

$$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 5AA^{-1} + 11IA^{-1} = 0 \quad [\text{Post-multiplying by } A^{-1} \text{ as } |A| \neq 0]$$

$$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 5(AA^{-1}) = -11(AA^{-1})$$

$$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$$

$$\Rightarrow A^{-1} = -\frac{1}{11}(A^2 - 6A + 5I) \quad \dots(1)$$

Now,

$$A^2 - 6A + 5I$$

$$\begin{aligned}
 &= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} 9 & 2 & 1 \\ -3 & 13 & -14 \\ 7 & -3 & 19 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix}
 \end{aligned}$$

From equation (1), we have:

$$A^{-1} = -\frac{1}{11} \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q25

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix}$$

$$\begin{aligned} \text{Now } A^3 - A^2 - 3A - I_3 &= \begin{bmatrix} -1 & -8 & -10 \\ 0 & 7 & 10 \\ 7 & 12 & 7 \end{bmatrix} - \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 3 & 0 & -6 \\ -6 & -3 & 6 \\ 9 & 12 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

$$\therefore A^3 - A^2 - 3A - I_3 = 0$$

$$\Rightarrow A^2 - A - 3I - A^{-1} = 0$$

$$\Rightarrow A^{-1} = A^2 - A - 3I = \begin{bmatrix} -5 & -8 & -4 \\ 6 & 9 & 4 \\ -2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q26

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now,

$$A^3 - 6A^2 + 9A - 4I$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} - \begin{bmatrix} 40 & -30 & 30 \\ -30 & 40 & -30 \\ 30 & -30 & 40 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore A^3 - 6A^2 + 9A - 4I = O$$

Now,

$$A^3 - 6A^2 + 9A - 4I = O$$

$$\Rightarrow (AAA)A^{-1} - 6(AA)A^{-1} + 9AA^{-1} - 4IA^{-1} = O$$

[Post-multiplying by A^{-1} as $|A| \neq 0$]

$$\Rightarrow AA(AA^{-1}) - 6A(AA^{-1}) + 9(AA^{-1}) = 4(AA^{-1})$$

$$\Rightarrow AAI - 6AI + 9I = 4A^{-1}$$

$$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{4}(A^2 - 6A + 9I) \quad \dots(1)$$

$$A^2 - 6A + 9I$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

From equation (1), we have:

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q27

$$A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix} \text{ and } A^T = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix}$$

$$|A| = \frac{1}{9} [-8(16 + 56) - 1(9) + 4(-36)] = -81$$

$$\begin{array}{lll} C_{11} = 72 & C_{21} = -36 & C_{31} = -9 \\ C_{12} = -9 & C_{22} = -36 & C_{32} = +72 \\ C_{13} = -36 & C_{23} = -63 & C_{33} = -36 \end{array}$$

$$A^{-1} = \frac{1}{-81} \begin{bmatrix} 72 & -36 & -9 \\ -9 & -36 & 72 \\ -36 & -63 & -36 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} -8 & 4 & 1 \\ 1 & 4 & -8 \\ 4 & 7 & 4 \end{bmatrix} = A^T$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q28

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \text{ and } |A| = 3 + 6 - 8 = 1$$

$$\begin{array}{lll} C_{11} = 1 & C_{21} = -1 & C_{31} = 0 \\ C_{12} = -2 & C_{22} = 3 & C_{32} = -4 \\ C_{13} = -2 & C_{23} = +3 & C_{33} = -3 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A = \frac{1}{1} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad \text{--- (1)}$$

Now

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{aligned} A^3 = A^2 A &= \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad \text{--- (2)} \end{aligned}$$

From (1) and (2)

$$A^{-1} = A^3$$

Hence proved.

Adjoint and Inverse of Matrix Ex 7.1 Q29

$$A = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Expanding using 1st row, we get

$$\begin{aligned} |A| &= -1 \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} - 2 \begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} + 0 \\ &= -1(0 - 1) - 2(0) + 0 \\ &= 1 - 0 + 0 \\ |A| &= 1 \end{aligned}$$

$$A^2 = AA = \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Cofactors of A are:

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 0 & C_{31} = 2 \\ C_{12} = 0 & C_{22} = 0 & C_{32} = 1 \\ C_{13} = -1 & C_{23} = +1 & C_{33} = 1 \end{array}$$

$$\therefore \text{adj}A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix} = A^2$$

Hence, $A^2 = A^{-1}$

Adjoint and Inverse of Matrix Ex 7.1 Q30

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} X = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

$$\text{So, } AX = B$$

$$\text{or } X = A^{-1}B \quad \text{--- (i)}$$

$$|A| = 1 \neq 0$$

Cofactors of A are:

$$\begin{array}{ll} C_{11} = 1 & C_{12} = -1 \\ C_{21} = -4 & C_{22} = 5 \end{array}$$

$$\begin{aligned} \therefore \text{adj } A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & -1 \\ -4 & 5 \end{bmatrix}^T \\ &= \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \end{aligned}$$

$$\text{Now, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A$$

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix}$$

So from (i)

$$\begin{aligned} X &= \begin{bmatrix} 1 & -4 \\ -1 & 5 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \\ X &= \begin{bmatrix} -3 & -14 \\ 4 & 17 \end{bmatrix} \end{aligned}$$

Ans.

Adjoint and Inverse of Matrix Ex 7.1 Q31

$$X \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$$

$$\text{Let } B = \begin{bmatrix} 5 & 3 \\ -1 & -2 \end{bmatrix} \text{ and } C = \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix}$$

$$\begin{aligned} \text{So, } XB &= C \\ XBB^{-1} &= CB^{-1} \\ XI &= CB^{-1} \\ X &= CB^{-1} \quad \text{--- (i)} \end{aligned}$$

$$\text{Now, } |B| = -7 \neq 0$$

Cofactors of B are:

$$\begin{aligned} C_{11} &= -2 & C_{12} &= 1 \\ C_{21} &= -3 & C_{22} &= 5 \end{aligned}$$

$$\begin{aligned} \therefore \text{adj}A &= \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}^T \\ &= \begin{bmatrix} -2 & 1 \\ -3 & 5 \end{bmatrix}^T \\ &= \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B^{-1} &= \frac{1}{|B|} \cdot \text{adj}(B) \\ &= \frac{1}{(-7)} \begin{bmatrix} -2 & -3 \\ 1 & 5 \end{bmatrix} = \frac{-1}{7} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} \end{aligned}$$

Now from (i)

$$\begin{aligned} X &= \begin{bmatrix} 14 & 7 \\ 7 & 7 \end{bmatrix} \cdot \frac{1}{7} \cdot \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} \\ &= \frac{7}{7} \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & -5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 1 \\ 1 & -2 \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q32

$$\text{Let } A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

Then the given equation becomes

$$A \times B = C$$

$$\Rightarrow X = A^{-1}CB^{-1}$$

$$\text{Now } |A| = 3 \cdot 5 - 14 = 21$$

$$|B| = -1 + 2 = 1$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj}(B)}{|B|} = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$$

$$\begin{aligned} \therefore X &= A^{-1}CB^{-1} = \frac{1}{21} \begin{bmatrix} 5 & -2 \\ -7 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q33

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

Then the given equation can be written as

$$A \times B = I$$

$$\Rightarrow X = A^{-1}B^{-1}$$

$$\text{Now } |A| = 6 - 5 = 1$$

$$|B| = 10 - 9 = 1$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj}(B)}{|B|} = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 9 & -14 \\ -16 & 25 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q34

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$$

$$\text{Now, } A^2 + 4A - 5I$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Also, } A^2 - 4A + 5I = 0$$

$$A^{-1} \cdot A A - 4A^{-1} \cdot A - 5A^{-1} \cdot I = 0$$

$$A - 4I - 5A^{-1} = 0$$

$$A^{-1} = \frac{1}{5} [A - 4I]$$

$$= \frac{1}{5} \left\{ \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right\}$$

$$= \frac{1}{5} \left(\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \right) = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q35

$$|A \text{ adj } A| = |A|^n$$

$$\text{LHS} = |A \text{ Adj } A|$$

$$= |A| \cdot |\text{Adj } A|$$

$$= |A| \cdot |A|^{n-1}$$

$$= |A|^{n-1+1}$$

$$= |A|^n$$

$$= \text{RHS}$$

Adjoint and Inverse of Matrix Ex 7.1 Q36

Here

$$B^{-1} = \frac{1}{|B|} \text{adj } B$$

Co-factors of B are

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 2 & C_{31} = 6 \\ C_{12} = 1 & C_{22} = 1 & C_{32} = 2 \\ C_{13} = 2 & C_{23} = 2 & C_{33} = 5 \end{array}$$

Therefore,

$$\begin{aligned} \text{adj } B &= \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \end{aligned}$$

Therefore,

$$B^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$

Now,

$$\begin{aligned} (AB)^{-1} &= B^{-1}A^{-1} \\ &= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 61 & -24 & 22 \end{bmatrix} \end{aligned}$$

Adjoint and Inverse of Matrix Ex 7.1 Q37

$$\text{Let } B = A^T = \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{vmatrix} = (-1 - 8) - 0 - 2(-8 + 3) = -9 + 10 = 1 \neq 0$$

So, B is invertible matrix.

$$B_{11} = (-1)^{1+1}(-9) = -9; B_{12} = (-1)^{1+2}(-8) = 8; B_{13} = (-1)^{1+3}(-5) = -5$$

$$B_{21} = (-1)^{2+1}(8) = -8; B_{22} = (-1)^{2+2}(7) = 7; B_{23} = (-1)^{2+3}(4) = -4$$

$$B_{31} = (-1)^{3+1}(-2) = -2; B_{32} = (-1)^{3+2}(-2) = 2; B_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } B = \begin{bmatrix} -9 & 8 & -5 \\ -8 & 7 & -4 \\ -2 & 2 & -1 \end{bmatrix}^T = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{adj } B$$

$$\Rightarrow B^{-1} = \frac{1}{1} \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$\Rightarrow (A^T)^{-1} = \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.1 Q38

$$|A| = \begin{vmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{vmatrix} = -1(1-4) + 2(2+4) - 2(-4-2) = 3 + 12 + 12 = 27$$

$$A_{11} = (-1)^{1+1}(-3) = -3; A_{12} = (-1)^{1+2}(6) = -6; A_{13} = (-1)^{1+3}(-6) = -6$$

$$A_{21} = (-1)^{2+1}(-6) = 6; A_{22} = (-1)^{2+2}(3) = 3; A_{23} = (-1)^{2+3}(6) = -6$$

$$A_{31} = (-1)^{3+1}(6) = 6; A_{32} = (-1)^{3+2}(6) = -6; A_{33} = (-1)^{3+3}(3) = 3$$

$$\text{adj } A = \begin{bmatrix} -3 & -6 & -6 \\ 6 & 3 & -6 \\ 6 & -6 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$A(\text{adj } A) = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{bmatrix}$$

$$\Rightarrow A(\text{adj } A) = \begin{bmatrix} 27 & 0 & 0 \\ 0 & 27 & 0 \\ 0 & 0 & 27 \end{bmatrix}$$

$$\Rightarrow A(\text{adj } A) = 27 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A(\text{adj } A) = |A|I_3$$

Adjoint and Inverse of Matrix Ex 7.1 Q39

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} = 0 - 1(0-1) + 1(1-0) = 0 + 1 + 1 = 2 \neq 0$$

So, A is invertible matrix.

$$A_{11} = (-1)^{1+1}(-1) = -1; A_{12} = (-1)^{1+2}(-1) = 1; A_{13} = (-1)^{1+3}(1) = 1$$

$$A_{21} = (-1)^{2+1}(-1) = 1; A_{22} = (-1)^{2+2}(-1) = -1; A_{23} = (-1)^{2+3}(-1) = 1$$

$$A_{31} = (-1)^{3+1}(1) = 1; A_{32} = (-1)^{3+2}(-1) = 1; A_{33} = (-1)^{3+3}(-1) = -1$$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \dots\dots\dots (i)$$

$$A^2 - 3I = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A^2 - 3I = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} \dots\dots\dots (ii)$$

From (i) and (ii) we can see that,

$$A^{-1} = \frac{1}{2}(A^2 - 3I)$$

Ex 7.2

Adjoint and Inverse of Matrix Ex 7.2 Q1

$$A = \begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{7}R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 4R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & -\frac{25}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ -\frac{4}{7} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \left(-\frac{7}{25}\right)R_2$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{4}{25} & \frac{-7}{25} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{7}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & \frac{-7}{25} \end{bmatrix} A$$

Hence, $I = B A$

So, $B = \begin{bmatrix} \frac{3}{25} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix}$ is the inverse of A .

Adjoint and Inverse of Matrix Ex 7.2 Q2

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

For row- transformation $A = IA$

$$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{5}R_1$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -\frac{2}{5} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 5.R_2$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -2 & 5 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{2}{5}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} A$$

$$I = B.A$$

Hence, $B = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ is the inverse of A .

Adjoint and Inverse of Matrix Ex 7.2 Q3

$$\text{Let } A = \begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$

$$\begin{bmatrix} 1 & 6 \\ 0 & 23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{1}{23}R_2$

$$\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 6R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{23} & \frac{-6}{23} \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$$

$$I = B.A$$

Hence, $B = \frac{1}{23} \begin{bmatrix} 5 & -6 \\ 3 & 1 \end{bmatrix}$ is the inverse of A .

Adjoint and Inverse of Matrix Ex 7.2 Q4

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Now, $IA = I$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 2R_2$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

or $I = BA$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q5

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Now, $IA = I$

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -\frac{2}{3} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 3R_2$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{10}{3}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$I = BA$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q6

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} . A$$

Applying $R_3 \rightarrow \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 + R_3$, $R_2 \rightarrow R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} . A$$

$$I = B.A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q7

$$A = I A$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, $R_2 \rightarrow R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$I = B.A$$

Adjoint and Inverse of Matrix Ex 7.2 Q8

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Here, $A = I A$

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{3}{2}R_2$, $R_3 \rightarrow R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 1 & \frac{-5}{2} & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q9

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Now, $AA = I A$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{-2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & 1 & \frac{-4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_2, R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ \frac{2}{3} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow (-3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{-4}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} A$$

$$R_2 \rightarrow R_2 + \frac{4}{3}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} .A$$

$$I = B.A$$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q10

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} .A$$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} .A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} .A$$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix} .A$$

Applying $R_3 \rightarrow \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} .A$$

Applying $R_1 \rightarrow R_1 + 2R_3$, $R_2 \rightarrow R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix} .A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} \frac{-4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & \frac{-1}{2} & \frac{-1}{6} \\ \frac{5}{6} & \frac{-1}{2} & \frac{1}{6} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q11

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$$\text{Applying } R_1 \rightarrow \frac{R_1}{2}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

$$\text{Applying } R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} . A$$

$$\text{Applying } R_2 \rightarrow \left(\frac{2}{5}\right) R_2$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -\frac{3}{2} & -1 & 1 \end{bmatrix} . A$$

$$\text{Applying } R_1 \rightarrow R_1 + \frac{1}{2} R_2, R_3 \rightarrow R_3 - \frac{5}{2} R_2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix} . A$$

$$\text{Applying } R_3 \rightarrow \frac{R_3}{-6}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} . A$$

$$\text{Applying } R_2 \rightarrow R_2 - R_3, R_1 \rightarrow R_1 - 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & -\frac{2}{15} & -\frac{1}{3} \\ -\frac{11}{30} & \frac{7}{30} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} . A$$

$$[\because I = A^{-1}.A]$$

Ans.

Adjoint and Inverse of Matrix Ex 7.2 Q12

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \rightarrow \frac{R_2}{-2}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{-11}{2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{-7}{2} & \frac{1}{2} & \frac{-2}{11} \end{bmatrix} . A$$

Applying $R_3 \rightarrow R_3 \cdot \left(\frac{-2}{11}\right)$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, $R_2 \rightarrow R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{11} & \frac{5}{11} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{-3}{11} & \frac{5}{11} \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} . A$$

Adjoint and Inverse of Matrix Ex 7.2 Q13

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Now, $IA = I$

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \rightarrow R_2 - 4R_1, R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 0 & 2 & -6 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -2 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \rightarrow \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 0 & 1 & -3 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_2, R_3 \rightarrow R_3 + \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ -2 & \frac{1}{4} & 1 \end{bmatrix} . A$$

Applying $R_3 \rightarrow (-2)R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} . A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_3, R_2 \rightarrow R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & -\frac{1}{2} & -2 \end{bmatrix} . A$$

$$I = B.A$$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q14

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Now, $IA = I$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 9R_3$

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, $R_2 \rightarrow R_2 - \frac{2}{9}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} . A$$

or $I = B.A$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q15

Consider the given matrix:

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

We know that $A = IA$

Thus, we have,

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 3R_1 + R_2$ and $R_3 \rightarrow R_3 - 2R_1$, we have,

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -5 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 3R_2$ and $R_3 \rightarrow R_3 + 5R_2$, we have,

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & \frac{11}{9} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-1}{3} & \frac{5}{9} & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{R_1}{\frac{11}{9}}$ we have,

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + \frac{5}{9}R_3$ and $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

$$\Rightarrow \text{Inverse of the given matrix is } \begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q16

Consider the given matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow (-1)R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 3R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{R_2}{3}$, we have,

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_2$ and $R_3 \rightarrow R_3 - 4R_2$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{R_3}{3}$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, $R_2 \rightarrow R_2 - \frac{5}{3}R_3$, we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$.