Mathematics

(Chapter – 12) (Introduction to Three Dimensional Geometry)

(Class – XI)

Exercise 12.1

Question 1:

A point is on the *x*-axis. What are its *y*-coordinates and *z*-coordinates?

Answer 1:

If a point is on the *x*-axis, then its *y*-coordinates and *z*-coordinates are zero.

Question 2:

A point is in the XZ-plane. What can you say about its *y*-coordinate?

Answer 2:

If a point is in the XZ plane, then its *y*-coordinate is zero.

Question 3:

Name the octants in which the following points lie:

(1, 2, 3), (4, −2, 3), (4, −2, −5), (4, 2, −5), (−4, 2, −5), (−4, 2, 5), (−3, −1, 6), (2, −4, −7)

Answer 3:

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (1, 2, 3) are all positive. Therefore, this point lies in octant **I**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, −2, 3) are positive, negative, and positive respectively. Therefore, this point lies in octant **IV**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, −2, −5) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (4, 2, −5) are positive, positive, and negative respectively. Therefore, this point lies in octant **V**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (−4, 2, −5) are negative, positive, and negative respectively. Therefore, this point lies in octant **VI**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (−4, 2, 5) are negative, positive, and positive respectively. Therefore, this point lies in octant **II**.

The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (–3, –1, 6) are negative, negative, and positive respectively. Therefore, this point lies in octant **III**.
The *x*-coordinate, *y*-coordinate, and *z*-coordinate of point (2, –4, –7) are positive, negative, and negative respectively. Therefore, this point lies in octant **VIII**.

Question 4:

Fill in the blanks:

(i) The *x*-axis and *y*-axis taken together determine a plane known as________.

(ii) The coordinates of points in the XY-plane are of the form ________.

(iii) Coordinate planes divide the space into _______ octants.

Answer 4:

(i) The *x*-axis and *y*-axis taken together determine a plane known as **xy - plane**.
(ii) The coordinates of points in the XY-plane are of the form **(x, y, 0).**
(iii) Coordinate planes divide the space into **eight** octants.

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Exercise 12.2

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3, 5) and (4, 3, 1) (ii) (−3, 7, 2) and (2, 4, −1)

(iii) (−1, 3, −4) and (1, −3, 4) (iv) (2, −1, 3) and (−2, 1, 3)

Answer 1:

The distance between points P(x_1, y_1, z_1) and P(x_2, y_2, z_2) is given by

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(i) Distance between points (2, 3, 5) and (4, 3, 1)

$$= \sqrt{(4-2)^2 + (3-3)^2 + (1-5)^2}$$
$$= \sqrt{(2)^2 + (0)^2 + (-4)^2}$$
$$= \sqrt{4+16}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

(ii) Distance between points (−3, 7, 2) and (2, 4, −1)

$$= \sqrt{(2+3)^2 + (4-7)^2 + (-1-2)^2}$$
$$= \sqrt{(5)^2 + (-3)^2 + (-3)^2}$$
$$= \sqrt{25+9+9}$$
$$= \sqrt{43}$$

(iii) Distance between points (−1, 3, −4) and (1, −3, 4)

$$= \sqrt{(1+1)^2 + (-3-3)^2 + (4+4)^2}$$
$$= \sqrt{(2)^2 + (-6)^3 + (8)^2}$$
$$= \sqrt{4+36+64} = \sqrt{104} = 2\sqrt{26}$$

(iv) Distance between points (2, −1, 3) and (−2, 1, 3)

$$= \sqrt{(-2-2)^2+(1+1)^2+(3-3)^2}$$
$$= \sqrt{(-4)^2+(2)^2+(0)^2}$$
$$= \sqrt{16+4}$$
$$= \sqrt{20}$$
$$= 2\sqrt{5}$$

Question 2:

Show that the points (−2, 3, 5), (1, 2, 3) and (7, 0, −1) are collinear.

Answer 2:

Let points (−2, 3, 5), (1, 2, 3), and (7, 0, −1) be denoted by P, Q, and R respectively.
Points P, Q, and R are collinear if they lie on a line.

$$PQ = \sqrt{(1+2)^2+(2-3)^2+(3-5)^2}$$
$$= \sqrt{(3)^2+(-1)^2+(-2)^2}$$
$$= \sqrt{9+1+4}$$
$$= \sqrt{14}$$

$$QR = \sqrt{(7-1)^2+(0-2)^2+(-1-3)^2}$$
$$= \sqrt{(6)^2+(-2)^2+(-4)^2}$$
$$= \sqrt{36+4+16}$$
$$= \sqrt{56}$$
$$= 2\sqrt{14}$$

$$PR = \sqrt{(7+2)^2+(0-3)^2+(-1-5)^2}$$
$$= \sqrt{(9)^2+(-3)^2+(-6)^2}$$
$$= \sqrt{81+9+36}$$
$$= \sqrt{126}$$
$$= 3\sqrt{14}$$

Here, PQ + QR $= \sqrt{14} + 2\sqrt{14} = 3\sqrt{14}$ = PR

Hence, points P(−2, 3, 5), Q(1, 2, 3), and R(7, 0, −1) are collinear.

Question 3:

Verify the following:

(i) (0, 7, −10), (1, 6, −6) and (4, 9, −6) are the vertices of an isosceles triangle.

(ii) (0, 7, 10), (−1, 6, 6) and (−4, 9, 6) are the vertices of a right angled triangle.

(iii) (−1, 2, 1), (1, −2, 5), (4, −7, 8) and (2, −3, 4) are the vertices of a parallelogram.

Answer 3:

(i) Let points (0, 7, −10), (1, 6, −6), and (4, 9, −6) be denoted by A, B, and C respectively.

$$\begin{aligned} AB &= \sqrt{(1-0)^2 + (6-7)^2 + (-6+10)^2} \\ &= \sqrt{(1)^2 + (-1)^2 + (4)^2} \\ &= \sqrt{1+1+16} \\ &= \sqrt{18} \\ &= 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(4-1)^2 + (9-6)^2 + (-6+6)^2} \\ &= \sqrt{(3)^2 + (3)^2} \\ &= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{(0-4)^2 + (7-9)^2 + (-10+6)^2} \\ &= \sqrt{(-4)^2 + (-2)^2 + (-4)^2} \\ &= \sqrt{16+4+16} = \sqrt{36} = 6 \end{aligned}$$

Here, AB = BC ≠ CA

Thus, the given points are the vertices of an isosceles triangle.

(ii) Let (0, 7, 10), (–1, 6, 6), and (–4, 9, 6) be denoted by A, B, and C respectively.

$$\begin{aligned}AB &= \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2}\\ &= \sqrt{(-1)^2+(-1)^2+(-4)^2}\\ &= \sqrt{1+1+16} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2}\\ &= \sqrt{(-3)^2+(3)^2+(0)^2}\\ &= \sqrt{9+9} = \sqrt{18}\\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}CA &= \sqrt{(0+4)^2+(7-9)^2+(10-6)^2}\\ &= \sqrt{(4)^2+(-2)^2+(4)^2}\\ &= \sqrt{16+4+16}\\ &= \sqrt{36}\\ &= 6\end{aligned}$$

$$\text{Now, } AB^2+BC^2 = \left(3\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2 = 18+18 = 36 = AC^2$$

Therefore, by Pythagoras theorem, ABC is a right triangle.

Hence, the given points are the vertices of a right-angled triangle.

(iii) Let (–1, 2, 1), (1, –2, 5), (4, –7, 8), and (2, –3, 4) be denoted by A, B, C, and D respectively.

$$AB = \sqrt{(1+1)^2 + (-2-2)^2 + (5-1)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$BC = \sqrt{(4-1)^2 + (-7+2)^2 + (8-5)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

$$CD = \sqrt{(2-4)^2 + (-3+7)^2 + (4-8)^2}$$
$$= \sqrt{4+16+16}$$
$$= \sqrt{36}$$
$$= 6$$

$$DA = \sqrt{(-1-2)^2 + (2+3)^2 + (1-4)^2}$$
$$= \sqrt{9+25+9} = \sqrt{43}$$

Here, AB = CD = 6, BC = AD = $\sqrt{43}$

Hence, the opposite sides of quadrilateral ABCD, whose vertices are taken in order, are equal.

Therefore, ABCD is a parallelogram.

Hence, the given points are the vertices of a parallelogram.

Question 4:

Find the equation of the set of points which are equidistant from the points (1, 2, 3) and (3, 2, −1).

Answer 4:

Let P (x, y, z) be the point that is equidistant from points A(1, 2, 3) and B(3, 2, −1).
Accordingly, PA = PB

$$\Rightarrow PA^2 = PB^2$$
$$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2 = (x-3)^2 + (y-2)^2 + (z+1)^2$$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9 = x^2 - 6x + 9 + y^2 - 4y + 4 + z^2 + 2z + 1$

$\Rightarrow -2x - 4y - 6z + 14 = -6x - 4y + 2z + 14$

$\Rightarrow -2x - 6z + 6x - 2z = 0$

$\Rightarrow 4x - 8z = 0$

$\Rightarrow x - 2z = 0$

Thus, the required equation is $x - 2z = 0$.

Question 5:

Find the equation of the set of points P, the sum of whose distances from A (4, 0, 0) and B (−4, 0, 0) is equal to 10.

Answer 5:

Let the coordinates of P be (x, y, z).

The coordinates of points A and B are (4, 0, 0) and (−4, 0, 0) respectively.

It is given that PA + PB = 10.

$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} + \sqrt{(x+4)^2 + y^2 + z^2} = 10$$
$$\Rightarrow \sqrt{(x-4)^2 + y^2 + z^2} = 10 - \sqrt{(x+4)^2 + y^2 + z^2}$$

On squaring both sides, we obtain

$$\Rightarrow (x-4)^2 + y^2 + z^2 = 100 - 20\sqrt{(x+4)^2 + y^2 + z^2} + (x+4)^2 + y^2 + z^2$$
$$\Rightarrow x^2 - 8x + 16 + y^2 + z^2 = 100 - 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} + x^2 + 8x + 16 + y^2 + z^2$$
$$\Rightarrow 20\sqrt{x^2 + 8x + 16 + y^2 + z^2} = 100 + 16x$$
$$\Rightarrow 5\sqrt{x^2 + 8x + 16 + y^2 + z^2} = (25 + 4x)$$

On squaring both sides again, we obtain

$25(x^2 + 8x + 16 + y^2 + z^2) = 625 + 16x^2 + 200x$

$\Rightarrow 25x^2 + 200x + 400 + 25y^2 + 25z^2 = 625 + 16x^2 + 200x$

$\Rightarrow 9x^2 + 25y^2 + 25z^2 - 225 = 0$

Thus, the required equation is $9x^2 + 25y^2 + 25z^2 - 225 = 0$.

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Exercise 12.3

Question 1:
Find the coordinates of the point which divides the line segment joining the points (−2, 3, 5) and (1, −4, 6) in the ratio **(i)** 2:3 internally, **(ii)** 2:3 externally.

Answer 1:
(i) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) internally in the ratio *m*: *n* are

$$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}, \frac{mz_2+nz_1}{m+n}\right).$$

Let R (*x*, *y*, *z*) be the point that divides the line segment joining points (−2, 3, 5) and (1, −4, 6) internally in the ratio 2:3

$$x=\frac{2(1)+3(-2)}{2+3}, y=\frac{2(-4)+3(3)}{2+3}, \text{ and } z=\frac{2(6)+3(5)}{2+3}$$
$$\text{i.e., } x=\frac{-4}{5}, y=\frac{1}{5}, \text{ and } z=\frac{27}{5}$$

Thus, the coordinates of the required point are $\left(-\frac{4}{5},\frac{1}{5},\frac{27}{5}\right)$.

(ii) The coordinates of point R that divides the line segment joining points P (x_1, y_1, z_1) and Q (x_2, y_2, z_2) externally in the ratio *m*: *n* are

$$\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}, \frac{mz_2-nz_1}{m-n}\right).$$

Let R (*x*, *y*, *z*) be the point that divides the line segment joining points (−2, 3, 5) and (1, −4, 6) externally in the ratio 2:3

$$x=\frac{2(1)-3(-2)}{2-3}, y=\frac{2(-4)-3(3)}{2-3}, \text{ and } z=\frac{2(6)-3(5)}{2-3}$$
$$\text{i.e., } x=-8, y=17, \text{ and } z=3$$

Thus, the coordinates of the required point are (−8, 17, 3).

Question 2:
Given that P (3, 2, − 4), Q (5, 4, −6) and R (9, 8, −10) are collinear. Find the ratio in which Q divides PR.

Answer 2:
Let point Q (5, 4, −6) divide the line segment joining points P (3, 2, −4) and R (9, 8, −10) in the ratio k:1.

Therefore, by section formula,

$$(5,4,-6)=\left(\frac{k(9)+3}{k+1},\frac{k(8)+2}{k+1},\frac{k(-10)-4}{k+1}\right)$$

$$\Rightarrow \frac{9k+3}{k+1}=5$$
$$\Rightarrow 9k+3=5k+5$$
$$\Rightarrow 4k=2$$
$$\Rightarrow k=\frac{2}{4}=\frac{1}{2}$$

Thus, point Q divides PR in the ratio 1:2.

Question 3:
Find the ratio in which the YZ-plane divides the line segment formed by joining the points (−2, 4, 7) and (3, −5, 8).

Answer 3:
Let the YZ plane divide the line segment joining points (−2, 4, 7) and (3, −5, 8) in the ratio k:1.

Hence, by section formula, the coordinates of point of intersection are given by

$$\left(\frac{k(3)-2}{k+1},\frac{k(-5)+4}{k+1},\frac{k(8)+7}{k+1}\right)$$

On the YZ plane, the x-coordinate of any point is zero.

$$\frac{3k-2}{k+1}=0$$
$$\Rightarrow 3k-2=0$$
$$\Rightarrow k=\frac{2}{3}$$

Thus, the YZ plane divides the line segment formed by joining the given points in the ratio 2:3.

Question 4:

Using section formula, show that the points A (2, −3, 4), B (−1, 2, 1) and $C\left(0,\frac{1}{3},2\right)$are collinear.

Answer 4:

The given points are A (2, −3, 4), B (−1, 2, 1), and $C\left(0,\frac{1}{3},2\right)$.

Let P be a point that divides AB in the ratio *k*:1.

Hence, by section formula, the coordinates of P are given by

$$\left(\frac{k(-1)+2}{k+1},\frac{k(2)-3}{k+1},\frac{k(1)+4}{k+1}\right)$$

Now, we find the value of *k* at which point P coincides with point C.

By taking $\frac{-k+2}{k+1}=0$, we obtain *k* = 2.

For *k* = 2, the coordinates of point P are $\left(0,\frac{1}{3},2\right)$.

i.e., $C\left(0,\frac{1}{3},2\right)$ is a point that divides AB externally in the ratio 2:1 and is the same as point P.

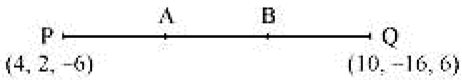
Hence, points A, B, and C are collinear.

Question 5:

Find the coordinates of the points which trisect the line segment joining the points P (4, 2, −6) and Q (10, −16, 6).

Answer 5:

Let A and B be the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6)



Point A divides PQ in the ratio 1:2. Therefore, by section formula, the coordinates of point A are given by

$$\left(\frac{1(10)+2(4)}{1+2},\frac{1(-16)+2(2)}{1+2},\frac{1(6)+2(-6)}{1+2}\right)=(6,-4,-2)$$

Point B divides PQ in the ratio 2:1. Therefore, by section formula, the coordinates of point B are given by

$$\left(\frac{2(10)+1(4)}{2+1},\frac{2(-16)+1(2)}{2+1},\frac{2(6)-1(6)}{2+1}\right)=(8,-10,2)$$

Thus, (6, −4, −2) and (8, −10, 2) are the points that trisect the line segment joining points P (4, 2, −6) and Q (10, −16, 6).

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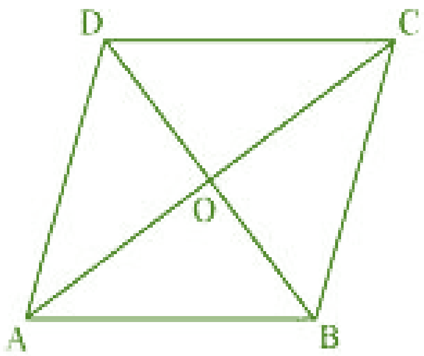
Miscellaneous Exercise on Chapter 12

Question 1:
Three vertices of a parallelogram ABCD are A (3, −1, 2), B (1, 2, −4) and C (−1, 1, 2). Find the coordinates of the fourth vertex.

Answer 1:
The three vertices of a parallelogram ABCD are given as A (3, −1, 2), B (1, 2, −4), and C (−1, 1, 2). Let the coordinates of the fourth vertex be D (x, y, z).



We know that the diagonals of a parallelogram bisect each other.

Therefore, in parallelogram ABCD, AC and BD bisect each other.

∴ Mid-point of AC = Mid-point of BD

$$\Rightarrow \left(\frac{3-1}{2}, \frac{-1+1}{2}, \frac{2+2}{2}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

$$\Rightarrow (1, 0, 2) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z-4}{2}\right)$$

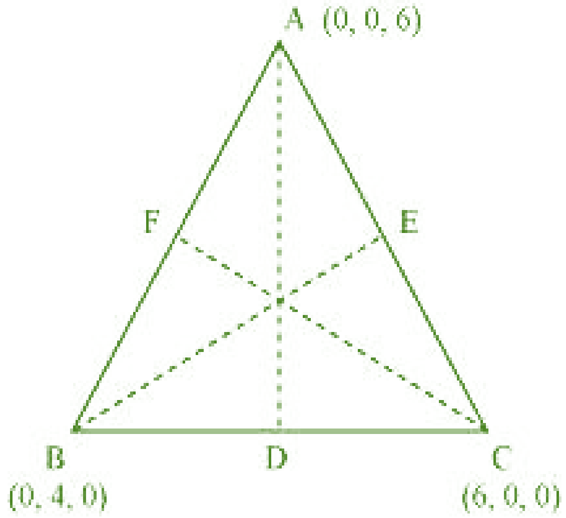
$$\Rightarrow \frac{x+1}{2} = 1, \frac{y+2}{2} = 0, \text{ and } \frac{z-4}{2} = 2$$

⇒ $x = 1$, $y = -2$, and $z = 8$

Thus, the coordinates of the fourth vertex are (1, −2, 8).

Question 2:
Find the lengths of the medians of the triangle with vertices A (0, 0, 6), B (0, 4, 0) and (6, 0, 0).

Answer 2:
Let AD, BE, and CF be the medians of the given triangle ABC.



Since AD is the median, D is the mid-point of BC.

∴Coordinates of point D = $\left(\frac{0+6}{2},\frac{4+0}{2},\frac{0+0}{2}\right)$ = (3, 2, 0)

$$AD=\sqrt{(0-3)^2+(0-2)^2+(6-0)^2}=\sqrt{9+4+36}=\sqrt{49}=7$$

Since BE is the median, E is the mid-point of AC.

$\therefore$ Coordinates of point E $=\left(\frac{0+6}{2},\frac{0+0}{2},\frac{6+0}{2}\right)=(3,0,3)$

$$BE=\sqrt{(3-0)^2+(0-4)^2+(3-0)^2}=\sqrt{9+16+9}=\sqrt{34}$$

Since CF is the median, F is the mid-point of AB.

$\therefore$ Coordinates of point F $=\left(\frac{0+0}{2},\frac{0+4}{2},\frac{6+0}{2}\right)=(0,2,3)$

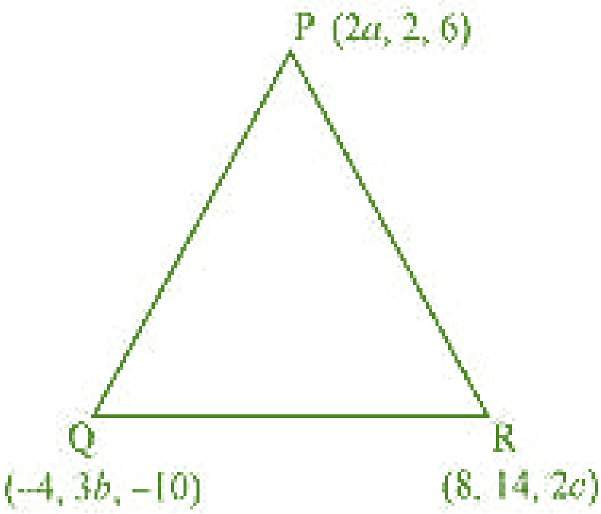
$$\text{Length of } CF=\sqrt{(6-0)^2+(0-2)^2+(0-3)^2}=\sqrt{36+4+9}=\sqrt{49}=7$$

Thus, the lengths of the medians of ΔABC are $7, \sqrt{34}$, and 7.

Question 3:

If the origin is the centroid of the triangle PQR with vertices P (2*a*, 2, 6), Q (−4, 3*b*, −10) and R (8, 14, 2*c*), then find the values of *a*, *b* and *c*.

Answer 3:



It is known that the coordinates of the centroid of the triangle, whose vertices are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3), are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$.

Therefore, coordinates of the centroid of ΔPQR

$$=\left(\frac{2a-4+8}{3}, \frac{2+3b+14}{3}, \frac{6-10+2c}{3}\right)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

It is given that origin is the centroid of ΔPQR.

$$\therefore (0,0,0)=\left(\frac{2a+4}{3}, \frac{3b+16}{3}, \frac{2c-4}{3}\right)$$

$$\Rightarrow \frac{2a+4}{3}=0, \frac{3b+16}{3}=0 \text{ and } \frac{2c-4}{3}=0$$

$$\Rightarrow a=-2, b=-\frac{16}{3} \text{ and } c=2$$

Thus, the respective values of *a*, *b*, and *c* are $-2, -\frac{16}{3}$, and 2.

Question 4:
Find the coordinates of a point on *y*-axis which are at a distance of $5\sqrt{2}$ from the point P (3, −2, 5).

Answer 4:
If a point is on the *y*-axis, then *x*-coordinate and the *z*-coordinate of the point are zero.
Let A (0, *b*, 0) be the point on the *y*-axis at a distance of $5\sqrt{2}$ from point P (3, −2, 5).
Accordingly, AP = $5\sqrt{2}$

$$\therefore AP^2 = 50$$
$$\Rightarrow (3-0)^2 + (-2-b)^2 + (5-0)^2 = 50$$
$$\Rightarrow 9 + 4 + b^2 + 4b + 25 = 50$$
$$\Rightarrow b^2 + 4b - 12 = 0$$
$$\Rightarrow b^2 + 6b - 2b - 12 = 0$$
$$\Rightarrow (b+6)(b-2) = 0$$
$$\Rightarrow b = -6 \text{ or } 2$$

Thus, the coordinates of the required points are (0, 2, 0) and (0, −6, 0).

Question 5:
A point R with *x*-coordinate 4 lies on the line segment joining the pointsP (2, −3, 4) and Q (8, 0, 10). Find the coordinates of the point R.
[**Hint** suppose R divides PQ in the ratio *k*: 1. The coordinates of the point R are given by $\left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$]

Answer 5:
The coordinates of points P and Q are given as P (2, −3, 4) and Q (8, 0, 10).
Let R divide line segment PQ in the ratio *k*:1.

Hence, by section formula, the coordinates of point R are given by

$$\left(\frac{k(8)+2}{k+1}, \frac{k(0)-3}{k+1}, \frac{k(10)+4}{k+1}\right) = \left(\frac{8k+2}{k+1}, \frac{-3}{k+1}, \frac{10k+4}{k+1}\right)$$

It is given that the *x*-coordinate of point R is 4.

$$\therefore \frac{8k+2}{k+1}=4$$
$$\Rightarrow 8k+2=4k+4$$
$$\Rightarrow 4k=2$$
$$\Rightarrow k=\frac{1}{2}$$

Therefore, the coordinates of point R are $\left(4, \frac{-3}{\frac{1}{2}+1}, \frac{10\left(\frac{1}{2}\right)+4}{\frac{1}{2}+1}\right)=(4,-2,6)$

Question 6:

If A and B be the points (3, 4, 5) and (−1, 3, −7), respectively, find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where k is a constant.

Answer 6:

The coordinates of points A and B are given as (3, 4, 5) and (−1, 3, −7) respectively.
Let the coordinates of point P be (x, y, z).

On using distance formula, we obtain

$$PA^2=(x-3)^2+(y-4)^2+(z-5)^2$$
$$=x^2+9-6x+y^2+16-8y+z^2+25-10z$$
$$=x^2-6x+y^2-8y+z^2-10z+50$$
$$PB^2=(x+1)^2+(y-3)^2+(z+7)^2$$
$$=x^2+2x+y^2-6y+z^2+14z+59$$

Now, if $PA^2 + PB^2 = k^2$, then

$$\left(x^2-6x+y^2-8y+z^2-10z+50\right)+\left(x^2+2x+y^2-6y+z^2+14z+59\right)=k^2$$
$$\Rightarrow 2x^2+2y^2+2z^2-4x-14y+4z+109=k^2$$
$$\Rightarrow 2\left(x^2+y^2+z^2-2x-7y+2z\right)=k^2-109$$
$$\Rightarrow x^2+y^2+z^2-2x-7y+2z=\frac{k^2-109}{2}$$

Thus, the required equation is $x^2+y^2+z^2-2x-7y+2z=\frac{k^2-109}{2}$