

Ex 8.1

Solution of Simultaneous Linear Equations Ex 8.1 Q1(i)

We have,

$$5x + 7y = -2$$

$$4x + 6y = -3$$

The above system of equations can be written in the matrix form as

$$\begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

or $AX = B$

where $A = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$

Now, $|A| = 30 - 28 = +2 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A , then

$$C_{11} = 6$$

$$C_{12} = -4$$

$$C_{21} = -7$$

$$C_{22} = 5$$

Also,

$$\text{adj } A = \begin{bmatrix} 6 & -4 \\ -7 & 5 \end{bmatrix}^T = \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{+2} \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix}$$

$\therefore X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2} \begin{bmatrix} 6 & -7 \\ -4 & 5 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{+1}{2} \begin{bmatrix} -12 & +21 \\ 8 & -15 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{-7}{2} \end{bmatrix}$$

Hence, $x = \frac{9}{2}$, $y = \frac{-7}{2}$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(ii)

The above system can be written in matrix form as

$$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $A X = B$

Where,

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Now, $|A| = 10 - 6 = 4 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A , then

$$C_{11} = 2$$

$$C_{12} = -3$$

$$C_{21} = -2$$

$$C_{22} = 5$$

Also,

$$\text{Adj } A = \begin{bmatrix} 2 & -3 \\ -2 & 5 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$= \frac{1}{4} \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} -4 \\ 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Hence, $x = -1$

$$y = 4$$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

or $AX = B$

Where,

$$A = \begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$$

Now, $|A| = -7 \neq 0$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A , then

$$C_{11} = -1$$

$$C_{12} = -1$$

$$C_{21} = -4$$

$$C_{22} = 3$$

Now,

$$\text{Adj } A = \begin{bmatrix} -1 & -1 \\ -4 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{aligned} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{-1}{7} \begin{bmatrix} -1 & -4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \end{bmatrix} \\ &= \frac{-1}{7} \begin{bmatrix} 7 \\ -14 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} -1 \\ 2 \end{bmatrix} \end{aligned}$$

Hence, $x = -1$

$$y = 2$$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(iv)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

or $AX = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 3 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 19 \\ 23 \end{bmatrix}$$

Now, $|A| = -6 \neq 0$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A , then

$$C_{11} = -1$$

$$C_{12} = -3$$

$$C_{21} = -1$$

$$C_{22} = 3$$

Now,

$$\text{Adj } A = \begin{bmatrix} -1 & -3 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\begin{aligned} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{-1}{6} \begin{bmatrix} -1 & -1 \\ -3 & 3 \end{bmatrix} \begin{bmatrix} 19 \\ 23 \end{bmatrix} \\ &= \frac{-1}{6} \begin{bmatrix} -19 & -23 \\ -57 & +69 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 7 \\ -2 \end{bmatrix} \end{aligned}$$

Hence, $x = 7$

$$y = -2$$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(v)

The above system can be written in matrix form as:

$$\begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

or $A X = B$

where $A = \begin{bmatrix} 3 & 7 \\ 1 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Now,

$$|A| = -1 \neq 0$$

So the above system has a unique solution, given by

$$X = A^{-1}B$$

Now, let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 2$$

$$C_{12} = -1$$

$$C_{21} = -7$$

$$C_{22} = 3$$

$$\text{Adj } A = \begin{bmatrix} 2 & -1 \\ -7 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \frac{1}{(-1)} \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 2 & -7 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 15 \\ -7 \end{bmatrix} = \begin{bmatrix} -15 \\ 7 \end{bmatrix}$$

Hence, $x = -15$

$$y = 7$$

Solution of Simultaneous Linear Equations Ex 8.1 Q1(vi)

The given system can be written in matrix form as:

$$\begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

or $A X = B$

Where,

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

Since, $|A| = 4 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 3$$

$$C_{12} = -5$$

$$C_{21} = -1$$

$$C_{22} = 3$$

$$\text{adj } A = \begin{bmatrix} 3 & -5 \\ -1 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{4} \begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix}$$

Now, $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 3 & -1 \\ -5 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 9 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{9}{4} \\ \frac{1}{4} \end{bmatrix}$$

Hence, $x = \frac{9}{4}$

$$y = \frac{1}{4}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(i)

The given system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

or $A X = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } |A| &= 1 \begin{vmatrix} 3 & 1 \\ -1 & -7 \end{vmatrix} - 1 \begin{vmatrix} 2 & 1 \\ 3 & -7 \end{vmatrix} - 1 \begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} \\ &= (-20) - 1(-17) - 1(-11) \\ &= -20 + 17 + 11 = 8 \neq 0 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -20 & C_{21} = 8 & C_{31} = 4 \\ C_{12} = -(-17) = 17 & C_{22} = -4 & C_{32} = -3 \\ C_{13} = -11 & C_{23} = -(-4) = 4 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -20 & 17 & -11 \\ 8 & -4 & 4 \\ 4 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } X &= A^{-1}B = \frac{1}{8} \begin{bmatrix} -20 & 8 & 4 \\ 17 & -4 & -3 \\ -11 & 4 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 10 \\ 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{8} \begin{bmatrix} 24 \\ 8 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \end{aligned}$$

Hence, $x = 3$

$y = 1$

$z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(ii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

or $A X = B$

Where,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 1 & -3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

Since, $|A| = 14 \neq 0$, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & C_{21} = 4 & C_{31} = 2 \\ C_{12} = 8 & C_{22} = -5 & C_{32} = 1 \\ C_{13} = 4 & C_{23} = 1 & C_{33} = -3 \end{array}$$

$$\text{Adj } A = \begin{bmatrix} 2 & 8 & 4 \\ 4 & -5 & 1 \\ 2 & 1 & -3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{|A|} \times \text{Adj } A \times B$$

$$= \frac{1}{14} \begin{bmatrix} 2 & 4 & 2 \\ 8 & -5 & 1 \\ 4 & 1 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \\ -9 \end{bmatrix}$$

$$= \frac{1}{14} \begin{bmatrix} -16 \\ 20 \\ 38 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{-8}{7} \\ \frac{10}{7} \\ \frac{19}{7} \end{bmatrix}$$

$$\text{Hence, } x = \frac{-8}{7}, y = \frac{10}{7}, z = \frac{19}{7}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iii)

The above system can be written in matrix form as:

$$\begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

or $A X = B$

Where,

$$A = \begin{bmatrix} 6 & -12 & 25 \\ 4 & 15 & -20 \\ 2 & 18 & 15 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 3 \\ 10 \end{bmatrix}$$

Now,

$$\begin{aligned} |A| &= 6(225 + 360) + 12(60 + 40) + 25(72 - 30) \\ &= 6(585) + 1200 + 25(42) \\ &= 3510 + 1200 + 1050 \\ &= 5760 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 585 & C_{21} = -(-180 - 450) = 630 & C_{31} = -135 \\ C_{12} = -100 & C_{22} = 40 & C_{32} = 220 \\ C_{13} = 42 & C_{23} = -132 & C_{33} = 138 \end{array}$$

$$X = A^{-1}B = \frac{1}{|A|} (\text{Adj } A) \times B = \frac{1}{5760} \begin{bmatrix} 585 & 630 & -135 \\ -100 & 40 & 220 \\ 42 & -132 & 138 \end{bmatrix} \begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{5760} \begin{bmatrix} 2880 \\ 1920 \\ 1152 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

$$\text{Hence, } x = \frac{1}{2}$$

$$y = \frac{1}{3}$$

$$z = \frac{1}{5}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(iv)

The above system can be written as

$$\begin{bmatrix} 3 & 4 & 7 \\ 2 & -1 & 3 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix}$$

or $AX = B$

$$\begin{aligned} |A| &= 3(-3) - 4(-9) + 7(5) \\ &= -9 + 36 + 35 \\ &= 62 \neq 0 \end{aligned}$$

So, the above system will have a unique solution, given by

$$X = A^{-1}B$$

Now,
$$\begin{array}{lll} C_{11} = -3 & C_{21} = 26 & C_{31} = 19 \\ C_{12} = 9 & C_{22} = -16 & C_{32} = 5 \\ C_{13} = 5 & C_{23} = -2 & C_{33} = -11 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix}$$

$$\begin{aligned} X &= A^{-1}B = \frac{1}{|A|} (\text{Adj } A) B \\ &= \frac{1}{62} \begin{bmatrix} -3 & 26 & 19 \\ 9 & -16 & 5 \\ 5 & -2 & -11 \end{bmatrix} \begin{bmatrix} 14 \\ 4 \\ 0 \end{bmatrix} \\ &= \frac{1}{62} \begin{bmatrix} -42 + 104 + 0 \\ 126 - 64 + 0 \\ 70 - 8 + 0 \end{bmatrix} = \frac{1}{62} \begin{bmatrix} 62 \\ 62 \\ 62 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \end{aligned}$$

Hence, $x = 1, y = 1, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

The above system can be written as

$$\begin{bmatrix} 2 & 6 & 0 \\ 3 & 0 & -1 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix}$$

Or $AX = B$

$$|A| = 2(-1) - 6(5) + 0(-3) = -32 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = -1 \quad C_{21} = -6 \quad C_{31} = -6$$

$$C_{12} = -5 \quad C_{22} = 2 \quad C_{32} = 2$$

$$C_{13} = -3 \quad C_{23} = 14 \quad C_{33} = -18$$

$$\text{adj}A = \begin{bmatrix} -1 & -5 & -3 \\ -6 & 2 & 14 \\ -6 & 2 & -18 \end{bmatrix}^T = \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix}$$

Now,

$$\begin{aligned} X = A^{-1}B &= \frac{1}{|A|} (\text{Adj}A) \times B \\ &= \frac{1}{-32} \begin{bmatrix} -1 & -6 & -6 \\ -5 & 2 & 2 \\ -3 & 14 & -18 \end{bmatrix} \begin{bmatrix} 2 \\ -8 \\ -3 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{-32} \begin{bmatrix} 64 \\ -32 \\ -64 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -2, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(v)

$$\text{Let } \frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$$

$$2u - 3v + 3w = 10$$

$$u + v + w = 10$$

$$3u - v + 2w = 13$$

Which can be written as

$$\begin{bmatrix} 2 & -3 & 3 \\ 1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(3) + 3(-1) + 3(-4) \\ &= 6 - 3 - 12 = -9 \neq 0 \end{aligned}$$

Hence, the system has a unique solution, given by

$$X = A^{-1} \times B$$

$$C_{11} = 3 \qquad C_{21} = 3 \qquad C_{31} = -6$$

$$C_{12} = 1 \qquad C_{22} = -5 \qquad C_{32} = 1$$

$$C_{13} = -4 \qquad C_{23} = -7 \qquad C_{33} = 5$$

$$X = \frac{1}{|A|} (\text{Adj } A) \times (B)$$

$$= \frac{1}{-9} \begin{bmatrix} 3 & 3 & -6 \\ 1 & -5 & 1 \\ -4 & -7 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 13 \end{bmatrix}$$

$$= \frac{-1}{9} \begin{bmatrix} 30 + 30 - 78 \\ 10 - 50 + 13 \\ -40 - 70 + 65 \end{bmatrix}$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{-1}{9} \begin{bmatrix} -18 \\ -27 \\ -45 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\text{Hence, } x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{5}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vi)

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix}$$

or $A X = B$

$$\begin{aligned} |A| &= \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix} \\ &= 5(-2) - 3(5) + 1(3) \\ &= -10 - 15 + 3 = -22 \neq 0 \end{aligned}$$

Hence, it has a unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -2 & C_{21} = -10 & C_{31} = 8 \\ C_{12} = -5 & C_{22} = 19 & C_{32} = -13 \\ C_{13} = 3 & C_{23} = -7 & C_{33} = -1 \end{array}$$

$$\begin{aligned} X &= A^{-1} \times B = \frac{1}{|A|} (\text{Adj } A) \times B \\ &= \frac{1}{-22} \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix} \begin{bmatrix} 16 \\ 19 \\ 25 \end{bmatrix} \\ &= \frac{-1}{22} \begin{bmatrix} -32 - 190 + 200 \\ -80 + 361 - 325 \\ 48 - 133 - 25 \end{bmatrix} \\ &= \frac{-1}{22} \begin{bmatrix} -22 \\ -44 \\ -110 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \end{aligned}$$

Hence, $x = 1, y = 2, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(vii)

$$\begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}$$

or $A X = B$

$$\begin{aligned} |A| &= 3(6) - 4(3) + 2(-2) \\ &= 18 - 12 - 4 \\ &= 2 \neq 0 \end{aligned}$$

Hence, the system has a unique solution, given by

$$X = A^{-1}B$$

$$\begin{array}{lll} C_{11} = 6 & C_{21} = -28 & C_{31} = -16 \\ C_{12} = -3 & C_{22} = 16 & C_{32} = 9 \\ C_{13} = -2 & C_{23} = 10 & C_{33} = 6 \end{array}$$

$$\begin{aligned} \text{Next, } X &= A^{-1}B = \frac{1}{|A|} (\text{Adj } A) \times B \\ &= \frac{1}{2} \begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ 2 & 10 & 6 \end{bmatrix} \begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 48 - 84 + 32 \\ -24 + 48 - 18 \\ -16 + 30 - 12 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} -4 \\ 6 \\ 2 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} \end{aligned}$$

Hence, $x = -2, y = 3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(viii)

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & -1 \\ 3 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$$

$$\begin{aligned} |A| &= 2(-5) - 1(1) + 1(-8) \\ &= -10 - 1 - 8 = -19 \neq 0 \end{aligned}$$

Hence, the unique solution, given by

$$X = A^{-1} \times B$$

$$\begin{array}{lll} C_{11} = -5 & C_{21} = 3 & C_{31} = -4 \\ C_{12} = -1 & C_{22} = -7 & C_{32} = 3 \\ C_{13} = -8 & C_{23} = 1 & C_{33} = 5 \end{array}$$

$$\begin{aligned} \text{Next, } X &= A^{-1} \times B = \frac{1}{|A|} \begin{bmatrix} -5 & 3 & -4 \\ -1 & -7 & 3 \\ -8 & 1 & 5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix} \\ &= \frac{1}{-19} \begin{bmatrix} -10 + 15 - 24 \\ -2 - 35 + 18 \\ -16 + 5 + 30 \end{bmatrix} \\ &= \frac{-1}{19} \begin{bmatrix} -19 \\ -19 \\ 19 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \end{aligned}$$

Hence, $x = 1, y = 1, z = -1$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(x)

The above system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

or $A X = B$

$$|A| = 1(1) + 1(-2) + 1(4) = 1 - 2 + 4 = 3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 1 & C_{21} = 1 & C_{31} = +1 \\ C_{12} = 2 & C_{22} = -1 & C_{32} = 2 \\ C_{13} = 4 & C_{23} = -2 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & -2 \\ +1 & 2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix}$$

$$\begin{aligned} X &= A^{-1}B = \frac{1}{|A|} (\text{Adj } A) \times B \\ &= \frac{1}{3} \begin{bmatrix} 1 & 1 & +1 \\ 2 & -1 & 2 \\ 4 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{3} \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \end{aligned}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xi)

The above system can be written as

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

or $A X = B$

$$|A| = 8(-1) - 4(1) + 3(3) = -8 - 4 + 9 = -3 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -1 & C_{21} = 2 & C_{31} = 1 \\ C_{12} = -1 & C_{22} = 5 & C_{32} = -2 \\ C_{13} = 3 & C_{23} = -12 & C_{33} = 0 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } X &= A^{-1}B = \frac{1}{|A|} (\text{Adj } A) \times B \\ &= \frac{-1}{3} \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix} \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{-1}{3} \begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \end{aligned}$$

Hence, $x = 1, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xii)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

or $AX = B$

$$|A| = 1(-2) - 1(-5) + 1(1) = -2 + 5 + 1 = 4 \neq 0$$

So, $AX = B$ has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = -2 & C_{21} = 0 & C_{31} = 2 \\ C_{12} = +5 & C_{22} = -2 & C_{32} = -1 \\ C_{13} = 1 & C_{23} = 2 & C_{33} = -1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} -2 & 5 & 1 \\ 0 & -2 & 2 \\ 2 & -1 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix}$$

$$X = A^{-1} \times B = \frac{1}{|A|} (\text{Adj } A) \times B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -2 & 0 & 2 \\ 5 & -2 & -1 \\ 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} -12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $x = -3, y = 1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiii)

Let

$$\frac{1}{x} = u, \frac{1}{y} = v, \frac{1}{z} = w$$

The above system can be written as

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

Or $AX = B$

$$|A| = 2(75) - 3(-110) + 10(72) = 1200 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 75 \quad C_{21} = 150 \quad C_{31} = 75$$

$$C_{12} = 110 \quad C_{22} = -100 \quad C_{32} = 30$$

$$C_{13} = 72 \quad C_{23} = 0 \quad C_{33} = -24$$

$$\text{adj}A = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$\begin{aligned} X &= A^{-1}B = \frac{1}{|A|} (\text{Adj}A) \times B \\ &= \frac{1}{1200} \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \\ \begin{bmatrix} u \\ v \\ w \end{bmatrix} &= \frac{1}{1200} \begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix} \end{aligned}$$

Hence, $x = 2, y = 3, z = 5$

Solution of Simultaneous Linear Equations Ex 8.1 Q2(xiv)

The above system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

Or $AX = B$

$$|A| = 1(7) + 1(19) + 2(-11) = 4 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$C_{11} = 7 \quad C_{21} = 1 \quad C_{31} = -3$$

$$C_{12} = -19 \quad C_{22} = -1 \quad C_{32} = 11$$

$$C_{13} = -11 \quad C_{23} = -1 \quad C_{33} = 7$$

$$\text{adj } A = \begin{bmatrix} 7 & -19 & -11 \\ 1 & -1 & -1 \\ -3 & 11 & 7 \end{bmatrix}^T = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

Now,

$$\begin{aligned} X &= A^{-1}B = \frac{1}{|A|} (\text{Adj } A) \times B \\ &= \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

Hence, $x = 2, y = 1, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(i)

The above system can be written as

$$\begin{bmatrix} 6 & 4 \\ 9 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

or $AX = B$

$$|A| = 36 - 36 = 0$$

So, A is singular. Now, X will be consistent if $(\text{adj } A) \times B = 0$

$$C_{11} = 6$$

$$C_{12} = -9$$

$$C_{21} = -4$$

$$C_{22} = 6$$

$$\text{adj } A = \begin{bmatrix} 6 & -9 \\ -4 & 6 \end{bmatrix}^T = \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix}$$

$$\begin{aligned} (\text{Adj } A) \times B &= \begin{bmatrix} 6 & -4 \\ -9 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} 12 - 12 \\ -18 + 18 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{aligned}$$

Thus, $AX = B$ will have infinite solutions.

Let $y = k$

$$\text{Hence, } 6x = 2 - 4k \quad \text{or} \quad 9x = 3 - 6k$$

$$x = \frac{1 - 2k}{3} \quad \text{or} \quad x = \frac{1 - 2k}{3}$$

$$\text{Hence, } x = \frac{1 - 2k}{3}, y = k$$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(ii)

The system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

or $A X = B$

$$|A| = 18 - 18 = 0$$

So, A is singular. Now the system will be inconsistent if $(\text{adj } A) \times B \neq 0$

$$\begin{array}{ll} C_{11} = 9 & C_{21} = -3 \\ C_{12} = -6 & C_{22} = 2 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$\begin{aligned} (\text{Adj } A) \times B &= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 15 \end{bmatrix} \\ &= \begin{bmatrix} 45 - 45 \\ -30 + 30 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \end{aligned}$$

Since, $(\text{Adj } A \times B) = 0$, the system will have infinite solutions.

Now,

Let $y = k$

$$2x = 5 - 3k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

$$x = 15 - 9k \quad \text{or} \quad x = \frac{5 - 3k}{2}$$

Hence, $x = \frac{5 - 3k}{2}, y = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iii)

This can be written as

$$\begin{bmatrix} 5 & 3 & 7 \\ 3 & 26 & 2 \\ 7 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix}$$

or $AX = B$

$$|A| = 5(256) - 3(16) + 7(6 - 182) \\ = 0$$

So, A is singular. Thus, the given system is either inconsistent or it is consistent with infinitely many solutions according as

$$(\text{Adj } A) \times B \neq 0 \quad \text{or} \quad (\text{Adj } A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 256 & C_{21} = -16 & C_{31} = -176 \\ C_{12} = -16 & C_{22} = 1 & C_{32} = 11 \\ C_{13} = -176 & C_{23} = 11 & C_{33} = 121 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}^T = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix}$$

$$\text{adj } A \times B = \begin{bmatrix} 256 & -16 & -176 \\ -16 & 1 & 11 \\ -176 & 11 & 121 \end{bmatrix} \begin{bmatrix} 4 \\ 9 \\ 5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, let $z = k$

then, $5x + 3y = 4 - 7k$

$$3x + 26y = 9 - 2k$$

Which can be written as

$$\begin{bmatrix} 5 & 3 \\ 3 & 26 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

or $AX = B$

$$|A| = 2$$

$$\text{adj } A = \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\text{Now, } X = A^{-1}B = \frac{1}{|A|} \times \text{adj } A \times B$$

$$= \frac{1}{121} \begin{bmatrix} 26 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} 4 - 7k \\ 9 - 2k \end{bmatrix}$$

$$= \frac{1}{121} \begin{bmatrix} 77 - 176k \\ 11k + 33 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{7 - 16k}{11} \\ \frac{k + 3}{11} \end{bmatrix}$$

There values of x, y, z satisfies the third eq.

$$\text{Hence, } x = \frac{7 - 16k}{11}, y = \frac{k + 3}{11}, z = k$$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(iv)

This above system can be written as

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ -1 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

or $AX = B$

$$\begin{aligned} |A| &= 1(2-2) + 1(4-1) + 1(-3) \\ &= 0 + 3 - 3 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system is either inconsistent or consistent with infinitely many solutions according as

$$(\text{Adj } A) \times B \neq 0 \quad \text{or} \quad (\text{Adj } A) \times B = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 0 & C_{21} = 0 & C_{31} = 0 \\ C_{12} = -3 & C_{22} = 3 & C_{32} = 3 \\ C_{13} = -3 & C_{23} = -3 & C_{33} = 3 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 0 & -3 & -3 \\ 0 & 3 & 3 \\ 0 & 3 & 3 \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 0 & 0 & 0 \\ -3 & 3 & 3 \\ -3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Thus, $AX = B$ has infinite many solutions.

Now, let $z = k$

$$\begin{aligned} \text{So, } x - y &= 3 - k \\ 2x + y &= 2 + k \end{aligned}$$

Which can be written as

$$\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 - k \\ 2 + k \end{bmatrix}$$

or $AX = B$

$$|A| = 1 + 2 = 3 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix}$$

and, $X = A^{-1}B$

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{|A|} \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 - k \\ 2 + k \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 3 - k + 2 + k \\ -6 + 2k + 2 + k \end{bmatrix} \\ \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} \frac{5}{3} \\ \frac{3k - 4}{3} \end{bmatrix} \end{aligned}$$

Hence, $x = \frac{5}{3}, y = k - \frac{4}{3}, z = k$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(v)

This system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

or $AX = B$

$$\begin{aligned} |A| &= 1(2) - 1(4) + 1(2) \\ &= 2 - 4 + 2 \\ &= 0 \end{aligned}$$

So, A is singular. Thus, the given system has either no solution or infinite solutions depending on as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 2 & C_{21} = -3 & C_{31} = 1 \\ C_{12} = -4 & C_{22} = 6 & C_{32} = -2 \\ C_{13} = 2 & C_{23} = -3 & C_{33} = 1 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 2 & -4 & 2 \\ -3 & 6 & -3 \\ 1 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 2 & -3 & 1 \\ -4 & 6 & -2 \\ 2 & -3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix} = \begin{bmatrix} 12 - 42 + 30 \\ -24 + 84 - 60 \\ 12 - 42 + 30 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, let $z = k$

$$\begin{aligned} \text{So, } x + y &= 6 - k \\ x + 2y &= 14 - 3k \end{aligned}$$

Which can be written as

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix}$$

or $AX = B$

$$|A| = 1 \neq 0$$

$$\text{adj } A = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$$

$$X = A^{-1}B = \frac{1}{|A|} \text{adj } A \times B$$

$$\begin{aligned} \begin{bmatrix} x \\ y \end{bmatrix} &= \frac{1}{1} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 6 - k \\ 14 - 3k \end{bmatrix} \\ &= \begin{bmatrix} 12 - 2k - 14 + 3k \\ -6 + k + 14 - 3k \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 + k \\ 8 - 2k \end{bmatrix}$$

Hence, $x = k - 2$

$$y = 8 - 2k$$

$$z = k$$

Solution of Simultaneous Linear Equations Ex 8.1 Q3(vi)

This system can be written as

$$\begin{bmatrix} 2 & 2 & -2 \\ 4 & 4 & -1 \\ 6 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

or $A X = B$

$$|A| = 2(14) - 2(14) - 2(0) = 0$$

So, A is singular and the system has either no solution or infinite solutions according as

$$(\text{Adj } A) \times (B) \neq 0 \quad \text{or} \quad (\text{Adj } A) \times (B) = 0$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{array}{lll} C_{11} = 14 & C_{21} = -16 & C_{31} = 6 \\ C_{12} = -14 & C_{22} = 16 & C_{32} = -6 \\ C_{13} = 0 & C_{23} = 0 & C_{33} = 0 \end{array}$$

$$\text{adj } A = \begin{bmatrix} 14 & -14 & 0 \\ -16 & 16 & 0 \\ 6 & -6 & 0 \end{bmatrix}^T = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} 14 & -16 & 6 \\ -14 & 16 & -6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 14 - 32 + 18 \\ -14 + 32 - 18 \\ 0 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

So, $AX = B$ has infinite solutions.

Now, let $z = k$

So, $2x + 2y = 1 + 2k$

$$4x + 4y = 2 + k$$

Which can be written as

$$\begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 + 2k \\ 2 + k \end{bmatrix}$$

or $A X = B$

$$|A| = 0, \quad z = 0$$

Again,

$$2x + 2y = 1$$

$$4x + 4y = 2$$

Solution of Simultaneous Linear Equations Ex 8.1 Q4(i)

The above system can be written as

$$\begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

or $A X = B$

$$|A| = 0$$

So, A is singular, and the above system will be inconsistent if

$$(\text{adj } A) \times B \neq 0$$

Now, $C_{11} = 15$

$$C_{12} = -6$$

$$C_{21} = -5$$

$$C_{22} = 2$$

$$\text{adj } A = \begin{bmatrix} 15 & -6 \\ -5 & 2 \end{bmatrix}^T = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}$$

$$\begin{aligned} (\text{adj } A) \times B &= \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 13 \end{bmatrix} \\ &= \begin{bmatrix} 105 - 65 \\ -42 + 26 \end{bmatrix} \\ &= \begin{bmatrix} 40 \\ -16 \end{bmatrix} \\ &\neq 0 \end{aligned}$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(ii)

This system can be written as

$$\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

or $A X = B$

$$|A| = 0$$

So, the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$C_{11} = 9$$

$$C_{12} = -6$$

$$C_{21} = -3$$

$$C_{22} = 2$$

$$\text{adj } A = \begin{bmatrix} 9 & -6 \\ -3 & 2 \end{bmatrix}^T = \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix}$$

$$\begin{aligned} (\text{adj } A) \times B &= \begin{bmatrix} 9 & -3 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 10 \end{bmatrix} \\ &= \begin{bmatrix} 45 - 30 \\ -30 + 20 \end{bmatrix} \\ &= \begin{bmatrix} 15 \\ -10 \end{bmatrix} \\ &\neq 0 \end{aligned}$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iii)

This system can be written as

$$\begin{bmatrix} 4 & -2 \\ 6 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

or $A X = B$

$$|A| = -12 + 12 = 0$$

So, A is singular. Now system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$C_{11} = -3$$

$$C_{12} = -6$$

$$C_{21} = 2$$

$$C_{22} = 4$$

$$\text{adj } A = \begin{bmatrix} -3 & -6 \\ 2 & 4 \end{bmatrix}^T = \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix}$$

$$\begin{aligned} (\text{adj } A) \times (B) &= \begin{bmatrix} -3 & 2 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} \\ &= \begin{bmatrix} -9 + 10 \\ -18 + 20 \end{bmatrix} \\ &= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ &\neq 0 \end{aligned}$$

Hence, the above system is inconsistent

Solution of Simultaneous Linear Equations Ex 8.1 Q4(iv)

The above system can be written as

$$\begin{bmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix}$$

or $A X = B$

$$\begin{aligned} |A| &= 4(-36) + 5(36) - 2(18) \\ &= -144 + 180 - 36 \\ &= 0 \end{aligned}$$

So, A is singular and the above system will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$C_{11} = -36 \quad C_{21} = 36 \quad C_{31} = -18$$

$$C_{12} = -36 \quad C_{22} = 36 \quad C_{32} = -18$$

$$C_{13} = 18 \quad C_{23} = -18 \quad C_{33} = 9$$

$$(\text{adj } A) = \begin{bmatrix} -36 & -36 & 18 \\ 36 & 36 & -18 \\ -18 & -18 & 9 \end{bmatrix}^T = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix}$$

$$(\text{adj } A) \times (B) = \begin{bmatrix} -36 & 36 & -18 \\ -36 & 36 & -18 \\ 18 & -18 & 9 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -72 - 72 + 18 \\ -72 - 72 + 18 \\ +36 + 36 - 9 \end{bmatrix} \neq 0$$

Hence, the above system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(v)

The above system can be written as

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

or $A X = B$

$$|A| = 3(-5) + 1(3) - 2(-6) = -15 + 3 + 12 = 0$$

So, A is singular and the above system of equations will be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$\begin{array}{lll} C_{11} = -5 & C_{21} = +10 & C_{31} = 5 \\ C_{12} = 3 & C_{22} = 6 & C_{32} = 3 \\ C_{13} = -6 & C_{23} = 12 & C_{33} = 6 \end{array}$$

$$(\text{adj } A) = \begin{bmatrix} -5 & 3 & -6 \\ 10 & 6 & 12 \\ 5 & 3 & 6 \end{bmatrix}^T = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix}$$

$$(\text{adj } A) \times B = \begin{bmatrix} -5 & 10 & 5 \\ 3 & 6 & 3 \\ -6 & 12 & 6 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -10 - 10 + 15 \\ 6 - 6 + 9 \\ -12 - 12 + 18 \end{bmatrix} \neq 0$$

Hence, the given system of equations is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q4(vi)

The above system can be written as

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$$

or $A X = B$

$$|A| = 1(-3) - 1(-3) - 2(-3) = -3 - 3 + 6 = 0$$

So, A is singular. Now the system can be inconsistent, if

$$(\text{adj } A) \times B \neq 0$$

$$\begin{array}{lll} C_{11} = -3 & C_{21} = -3 & C_{31} = -3 \\ C_{12} = -3 & C_{22} = -3 & C_{32} = -3 \\ C_{13} = -3 & C_{23} = -3 & C_{33} = -3 \end{array}$$

$$(\text{adj } A) = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}^T = \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix}$$

$$\begin{aligned} (\text{adj } A) \times B &= \begin{bmatrix} -3 & -3 & -3 \\ -3 & -3 & -3 \\ -3 & -3 & -3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -15 + 6 - 12 \\ -15 + 6 - 12 \\ -15 + 6 - 12 \end{bmatrix} \\ &= \begin{bmatrix} -21 \\ -21 \\ -21 \end{bmatrix} \\ &\neq 0 \end{aligned}$$

Hence, the given system is inconsistent.

Solution of Simultaneous Linear Equations Ex 8.1 Q5

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2+4+0 & 2-2+0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$AB = 6I$, where I is a 3×3 unit matrix

$$\text{or } A^{-1} = \frac{1}{6}B \quad [\text{By def. of inverse}]$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

Now, the given system of equations can be written as

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\text{or } AX = B$$

$$\text{or } X = A^{-1}B$$

$$= \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

Hence, $x = 2, y = -1, z = 4$

Solution of Simultaneous Linear Equations Ex 8.1 Q6

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = 2(0) + 3(-2) + 5(1) = -1 \neq 0$$

$$\begin{array}{lll} \text{Also, } C_{11} = 0 & C_{21} = -1 & C_{31} = 2 \\ C_{12} = 2 & C_{22} = -9 & C_{32} = 23 \\ C_{13} = 1 & C_{23} = -5 & C_{33} = 13 \end{array}$$

$$(\text{adj } A) = \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{adj } A) = \frac{1}{-1} \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

The given system of equations can be written as

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

or $AX = B$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} -5 + 6 \\ -22 + 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q7

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix}$$

$$|A| = 1(1+3) - 2(-1+2) + 5(-2) = 4 - 2 + 25 = 27 \neq 0$$

$$\begin{array}{lll} C_{11} = 4 & C_{21} = 17 & C_{31} = 3 \\ C_{12} = -1 & C_{22} = -11 & C_{32} = 6 \\ C_{13} = 5 & C_{23} = 1 & C_{33} = -3 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A = \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix}$$

Now, the given set of equations can be represented as

$$x + 2y + 5z = 10$$

$$x - y - z = -2$$

$$2x + 3y - z = -11$$

$$\text{or} \quad \begin{bmatrix} 1 & 2 & 5 \\ 1 & -1 & -1 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix}$$

$$\begin{aligned} \text{or} \quad X &= A^{-1} \times B \\ &= \frac{1}{27} \begin{bmatrix} 4 & 17 & 3 \\ -1 & -11 & 6 \\ 5 & 1 & -3 \end{bmatrix} \begin{bmatrix} 10 \\ -2 \\ -11 \end{bmatrix} \\ &= \frac{1}{27} \begin{bmatrix} 40 - 34 - 33 \\ -10 + 22 - 66 \\ 50 - 2 + 33 \end{bmatrix} = \frac{1}{27} \begin{bmatrix} -27 \\ -54 \\ 81 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 3 \end{bmatrix} \end{aligned}$$

Hence, $x = -1, y = -2, z = 3$

Solution of Simultaneous Linear Equations Ex 8.1 Q8

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(7) + 2(2) = 11$$

$$\begin{array}{lll} C_{11} = 7 & C_{21} = 2 & C_{31} = -6 \\ C_{12} = -2 & C_{22} = 1 & C_{32} = -3 \\ C_{13} = -4 & C_{23} = 2 & C_{33} = 5 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$\begin{array}{l} \text{Now, } x - 2y = 10 \\ 2x + y + 3z = 8 \\ -2y + z = 7 \end{array}$$

$$\text{or } \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{aligned} \text{or } X &= A^{-1} \times B \\ &= \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \\ &= \frac{1}{11} \begin{bmatrix} 70 + 16 - 42 \\ -20 + 8 - 21 \\ -40 + 16 + 35 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} \end{aligned}$$

$$\text{Hence, } x = 4, y = -3, z = 1$$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(ii)

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$|A| = 3(3) + 4(-3) + 2(-3) = -9$$

$$\begin{array}{lll} C_{11} = 3 & C_{21} = 4 & C_{31} = -26 \\ C_{12} = 3 & C_{22} = 1 & C_{32} = -11 \\ C_{13} = -3 & C_{23} = -4 & C_{33} = 17 \end{array}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix}$$

$$\begin{array}{l} \text{Now, } 3x - 4y + 2z = -1 \\ 2x + 3y + 5z = 7 \\ x + z = 2 \end{array}$$

$$\text{Or } \begin{bmatrix} 3 & -4 & 2 \\ 2 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix}$$

$$\begin{aligned} \text{Or } X &= A^{-1} \times B \\ &= \frac{1}{-9} \begin{bmatrix} 3 & 4 & -26 \\ 3 & 1 & -11 \\ -3 & -4 & 17 \end{bmatrix} \begin{bmatrix} -1 \\ 7 \\ 2 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{-9} \begin{bmatrix} -27 \\ -18 \\ 9 \end{bmatrix} \end{aligned}$$

$$\text{Hence } x = 3, y = 2, z = -1$$

Solution of Simultaneous Linear Equations Ex 8.1 Q8(iii)

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$AB = 11I$, where I is a 3×3 unit matrix

$$A^{-1} = \frac{1}{11} B \quad [\text{By def. of inverse}]$$

$$\text{Or } = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$$

Now, the given system of equations can be written as

$$\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\text{Or } AX = B$$

$$X = A^{-1}B$$

$$\text{Or } = \frac{1}{11} \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 44 \\ -33 \\ 11 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$$

Hence, $x = 4, y = -3, z = 1$

Solution of Simultaneous Linear Equations Ex 8.1 Q9

Let the numbers are x, y, z .

$$x + y + z = 2 \quad \text{--- (1)}$$

Also, $2y + (x + z) = 1$

$$x + 2y + z = 1 \quad \text{--- (2)}$$

Again,

$$x + z + 5(x) = 6$$

$$5x + y + z = 6 \quad \text{--- (3)}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 5 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$$

or $AX = B$

$$\begin{aligned} |A| &= 1(1) - 1(-4) + 1(-9) \\ &= 1 + 4 - 9 = -4 \neq 0 \end{aligned}$$

Hence, the unique solutions given by $x = A^{-1}B$

$$\begin{array}{lll} C_{11} = 1 & C_{21} = 0 & C_{31} = -1 \\ C_{12} = 4 & C_{22} = -4 & C_{32} = 0 \\ C_{13} = -9 & C_{23} = 4 & C_{33} = 1 \end{array}$$

$$\begin{aligned} \text{or } X &= A^{-1}B = \frac{1}{|A|} (\text{adj } A) \times B = \frac{1}{-4} \begin{bmatrix} 1 & 0 & -1 \\ 4 & -4 & 0 \\ -9 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} \\ &= \frac{-1}{4} \begin{bmatrix} 2 - 6 \\ 8 - 4 \\ -18 + 4 + 6 \end{bmatrix} = \frac{-1}{4} \begin{bmatrix} -4 \\ 4 \\ -8 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

Hence, $x = 1, y = -1, z = 2$

Solution of Simultaneous Linear Equations Ex 8.1 Q10

Let the three investments are x, y, z

$$x + y + z = 10,000 \quad \dots\dots (1)$$

Also

$$\begin{aligned} \frac{10}{100}x + \frac{12}{100}y + \frac{15}{100}z &= 1310 \\ 0.1x + 0.12y + 0.15z &= 1310 \quad \dots\dots (2) \end{aligned}$$

Also

$$\begin{aligned} \frac{10}{100}x + \frac{12}{100}y - \frac{15}{100}z &= -190 \\ 0.1x + 0.12y - 0.15z &= -190 \quad \dots\dots (3) \end{aligned}$$

The above system can be written as

$$\begin{bmatrix} 1 & 1 & 1 \\ 0.1 & 0.12 & 0.15 \\ 0.1 & 0.12 & -0.15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix}$$

Or $AX = B$

$$|A| = 1(-0.036) - 1(-0.03) + 1(0) = -0.006 \neq 0$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Let C_{ij} be the co-factor of a_{ij} in A

$$\begin{aligned} C_{11} &= -0.036 & C_{21} &= 0.27 & C_{31} &= 0.03 \\ C_{12} &= 0.03 & C_{22} &= -0.25 & C_{32} &= -0.05 \\ C_{13} &= 0 & C_{23} &= -0.02 & C_{33} &= 0.02 \end{aligned}$$

$$\text{adj}A = \begin{bmatrix} -0.036 & 0.03 & 0 \\ 0.27 & -0.25 & -0.02 \\ 0.03 & -0.05 & 0.02 \end{bmatrix}^T = \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix}$$

Now,

$$\begin{aligned} X &= A^{-1}B = \frac{1}{|A|} (\text{Adj}A) \times B \\ &= \frac{1}{-0.006} \begin{bmatrix} -0.036 & 0.27 & 0.03 \\ 0.03 & -0.25 & -0.05 \\ 0 & -0.02 & 0.02 \end{bmatrix} \begin{bmatrix} 10000 \\ 1310 \\ -190 \end{bmatrix} \\ \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \frac{1}{-0.006} \begin{bmatrix} -12 \\ -18 \\ -30 \end{bmatrix} = \begin{bmatrix} 2000 \\ 3000 \\ 5000 \end{bmatrix} \end{aligned}$$

Hence, $x = \text{Rs } 2000, y = \text{Rs } 3000, z = \text{Rs } 5000$

Solution of Simultaneous Linear Equations Ex 8.1 Q11

$$x + y + z = 45 \quad \text{--- (1)}$$

$$z = x + 8 \quad \text{--- (2)}$$

$$x + z = 2y \quad \text{--- (3)}$$

or
$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$$

$$\begin{aligned} |A| &= 1(2) - 1(-2) + 1(2) \\ &= 2 + 2 + 2 = 6 \neq 0 \end{aligned}$$

$$\begin{array}{lll} C_{11} = 2 & C_{21} = -3 & C_{31} = 1 \\ C_{12} = 2 & C_{22} = 0 & C_{32} = -2 \\ C_{13} = 2 & C_{23} = +3 & C_{33} = 1 \end{array}$$

$$\begin{aligned} X &= A^{-1} \times B = \frac{1}{|A|} (\text{adj } A) \times B \\ &= \frac{1}{6} \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} 90 - 24 \\ 90 \\ 90 + 24 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 66 \\ 90 \\ 114 \end{bmatrix} \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$$

Hence, $x = 11, y = 15, z = 19$

Solution of Simultaneous Linear Equations Ex 8.1 Q12

The given problem can be modelled using the following system of equations

$$3x + 5y - 4z = 6000$$

$$2x - 3y + z = 5000$$

$$-x + 4y + 6z = 13000$$

Which can write as $Ax = B$,

Where

$$A = \begin{bmatrix} 3 & 5 & -4 \\ 2 & -3 & 1 \\ -1 & 4 & 6 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix}$$

Now

$$\begin{aligned} |A| &= 3(-18 - 4) - 2(30 + 16) - 1(5 - 12) \\ &= 3(-22) - 2(46) + 7 \\ &= -66 - 92 + 7 \\ &= -151 \neq 0 \end{aligned}$$

$\therefore A^{-1}$ exists.

Now $Ax = B \Rightarrow x = A^{-1}B$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

Cofactors of A are

$$C_{11} = -22 \quad C_{21} = -13 \quad C_{31} = 5$$

$$C_{12} = -46 \quad C_{22} = 14 \quad C_{32} = -17$$

$$C_{13} = -7 \quad C_{23} = -11 \quad C_{33} = -19$$

$$\therefore \text{adj}(A) = \begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix}$$

Hence,

$$\begin{aligned} x &= \frac{1}{|A|} \text{adj}(A)(B) \\ &= \frac{1}{-151} \begin{bmatrix} -22 & -46 & -7 \\ -13 & +14 & -11 \\ 5 & -17 & -19 \end{bmatrix} \begin{bmatrix} 6000 \\ 5000 \\ 13000 \end{bmatrix} \\ &= \frac{1}{-151} \begin{bmatrix} -132000 & -23000 & -91000 \\ -78000 & +70000 & -143000 \\ -3000 & -85000 & -247000 \end{bmatrix} \\ &= \begin{bmatrix} 3000 \\ 1000 \\ 2000 \end{bmatrix} \end{aligned}$$

$\therefore x = 3000, y = 1000$ and $z = 2000$.

Solution of Simultaneous Linear Equations Ex 8.1 Q13

From the given data, we get
the following three equations:

$$x + y + z = 12$$

$$2x + 3y + 3z = 33$$

$$x - 2y + z = 0$$

This system of equations can be written
in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix} \dots (1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(9) - 1(-1) + 1(-7) = 3$$

$$\text{cof}A = \begin{bmatrix} 9 & 1 & -7 \\ -3 & 0 & 3 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\text{adj}A = [\text{cof}A]^T = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 12 \\ 33 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 & -3 & 0 \\ 1 & 0 & -1 \\ -7 & 3 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 36 - 33 + 0 \\ 4 + 0 + 0 \\ -28 + 33 + 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}$$

An award for organising different festivals in the colony
can be included by the management.

Solution of Simultaneous Linear Equations Ex 8.1 Q14

Let X, Y and Z be the cash awards for
Honesty, Regularity and Hard work respectively.
As per the data in the question, we get

$$X + Y + Z = 6000$$

$$X + 3Z = 11000$$

$$X - 2Y + Z = 0$$

The above three simultaneous equations
can be written in the matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 6000 \\ 11000 \\ 0 \end{bmatrix} \dots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 1 & -2 & 1 \end{bmatrix}$$

$$|A| = 1(6) - 1(-2) + 1(-2) = 6$$

$$\text{cof}A = \begin{bmatrix} 6 & 2 & -2 \\ -3 & 0 & 3 \\ 3 & -2 & -1 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q15

Let x, y and z be the prize amount per person for
Resourcefulness, Competence and Determination respectively.
As per the data in the question, we get

$$4x + 3y + 2z = 37000$$

$$5x + 3y + 4z = 47000$$

$$x + y + z = 12000$$

The above three simultaneous equations
can be written in matrix form as

$$\begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix} \dots(1)$$

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 5 & 3 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 4(-1) - 3(1) + 2(2) = -3$$

$$\text{cof}A = \begin{bmatrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 6 & -6 & -3 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{-3} \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -1 & -1 & 6 \\ -1 & 2 & -6 \\ 2 & -1 & -3 \end{bmatrix} \begin{bmatrix} 37000 \\ 47000 \\ 12000 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{1}{-3} \begin{bmatrix} -12000 \\ -15000 \\ -9000 \end{bmatrix} = \begin{bmatrix} 4000 \\ 5000 \\ 3000 \end{bmatrix}$$

The values x, y and z describe the amount of prizes per person for resourcefulness, competence and determination.

Solution of Simultaneous Linear Equations Ex 8.1 Q16

Let x, y and z be the prize amount per person for adaptability, carefulness and calmness respectively.

As per the given data, we get

$$2x + 4y + 3z = 29000$$

$$5x + 2y + 3z = 30500$$

$$x + y + z = 9500$$

The above three simultaneous equations can be written in the matrix form as

$$\begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \dots (1)$$

$$A = \begin{bmatrix} 2 & 4 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 2(-1) - 4(2) + 3(3) = -1$$

$$\text{cof}A = \begin{bmatrix} -1 & -2 & 3 \\ -1 & -1 & 2 \\ 6 & 9 & -16 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} -1 & -1 & 6 \\ -2 & -1 & 9 \\ 3 & 2 & -16 \end{bmatrix}}{-1} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -6 \\ 2 & 1 & -9 \\ -3 & -2 & 16 \end{bmatrix} \begin{bmatrix} 29000 \\ 30500 \\ 9500 \end{bmatrix} \dots (1)$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 2500 \\ 3000 \\ 4000 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q17

Let x , y and z be the prize amount per student for sincerity, truthfulness and helpfulness respectively.

As per the data in the question, we get

$$3x + 2y + z = 1600$$

$$4x + y + 3z = 2300$$

$$x + y + z = 900$$

The above three simultaneous equations can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix} \dots (1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$\text{cof}A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5} \begin{bmatrix} 1600 \\ 2300 \\ 900 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -320 \\ -460 \\ -180 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 200 \\ 300 \\ 400 \end{bmatrix}$$

Excellence in extra-curricular activities should be another value considered for an award.

Solution of Simultaneous Linear Equations Ex 8.1 Q18

x, y and z be prize amount per student for Discipline, Politeness and Punctuality respectively.

As per the data in the question, we get

$$3x+2y+z=1000$$

$$4x+y+3z=1500$$

$$x+y+z=600$$

The above three simultaneous equations can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix} \dots (1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$\text{cof}A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5} \begin{bmatrix} 1000 \\ 1500 \\ 600 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -200 \\ -300 \\ -120 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q19

x, y and z be prize amount per student for
Tolerance, Kindness and Leadership respectively.

As per the data in the question, we get

$$3x+2y+z=2200$$

$$4x+y+3z=3100$$

$$x+y+z=1200$$

The above three simultaneous equations
can be written in matrix form as

$$\begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \dots (1)$$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$|A| = 3(-2) - 2(1) + 1(3) = -5$$

$$\text{cof}A = \begin{bmatrix} -2 & -1 & 3 \\ -1 & 2 & -1 \\ 5 & -5 & -5 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5}$$

From (1)

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \frac{\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix}}{-5} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} -440 \\ -620 \\ -240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 300 \\ 400 \\ 500 \end{bmatrix}$$

Solution of Simultaneous Linear Equations Ex 8.1 Q20

Let the amount deposited be x, y and z respectively.

As per the data in the question, we get

$$x + y + z = 7000$$

$$5\%x + 8\%y + 8.5\%z = 550$$

$$\Rightarrow 5x + 8y + 8.5z = 55000$$

$$x - y = 0$$

The above equations can be written in matrix form as

$$\begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix} \dots(1)$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 8 & 8.5 \\ 1 & -1 & 0 \end{bmatrix}$$

$$|A| = 1(8.5) - 1(-8.5) + 1(-13) = 4$$

$$\text{cof}A = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$\text{adj}A = (\text{cof}A)^T = \begin{bmatrix} 8.5 & 8.5 & -13 \\ -1 & -1 & 2 \\ 0.5 & -3.5 & 3 \end{bmatrix}^T$$

$$= \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{4} \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix}$$

From (1)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 8.5 & -1 & 0.5 \\ 8.5 & -1 & -3.5 \\ -13 & 2 & 3 \end{bmatrix} \begin{bmatrix} 7000 \\ 55000 \\ 0 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 4500 \\ 4500 \\ 19000 \end{bmatrix} = \begin{bmatrix} 1125 \\ 1125 \\ 4750 \end{bmatrix}$$

Hence, the amounts deposited in the three accounts are 1125, 1125 and 4750 respectively.

Ex 8.2

Solution of Simultaneous Linear Equations Ex 8.2 Q1

$$2x - y + z = 0$$

$$3x + 2y - z = 0$$

$$x + 4y + 3z = 0$$

The system can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$A \quad x = 0$$

$$\begin{aligned} \text{Now } |A| &= 2(10) + 1(10) + 1(10) \\ &= 40 \\ &\neq 0 \end{aligned}$$

Since $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous system.

Solution of Simultaneous Linear Equations Ex 8.2 Q2

$$\begin{aligned}2x - y + 2z &= 0 \\5x + 3y - z &= 0 \\x + 5y - 5z &= 0\end{aligned}$$

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or $A \quad x = 0$

$$\begin{aligned}|A| &= 2(-10) + 1(-24) + 2(22) \\&= -20 - 24 + 44 \\&= 0\end{aligned}$$

Hence, the system has infinite solutions.

Let $z = k$

$$\begin{aligned}2x - y &= -2k \\5x + 3y &= k\end{aligned}$$

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

$A \quad x = B$

$$|A| = 6 + 5 = 11 \neq 0 \quad \text{so } A^{-1} \text{ exist}$$

$$\text{Now } \text{adj } A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$$

$$x = A^{-1} \cdot B = \frac{1}{|A|} (\text{adj } A) B = \frac{1}{11} \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} -2k \\ k \end{bmatrix} = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

$$\text{Hence, } x = \frac{-5k}{11}, y = \frac{12k}{11}, z = k$$

Solution of Simultaneous Linear Equations Ex 8.2 Q3

$$\begin{aligned}3x - y + 2z &= 0 \\4x + 3y + 3z &= 0 \\5x + 7y + 4z &= 0\end{aligned}$$

$$|A| = \begin{vmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{vmatrix}$$

$$\begin{aligned}&= 3(-9) + 1(1) + 2(13) = -27 + 1 + 26 = -27 + 27 \\&= 0\end{aligned}$$

Hence, it has infinite solutions.

Let $z = k$

$$\begin{aligned}3x - y &= -2k \\4x + 3y &= -3k\end{aligned}$$

or $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$

or $A \quad x = B$

$$|A| = 9 + 4 = 13 \neq 0 \quad \text{hence } A^{-1} \text{ exists}$$

$$\text{adj } A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$$

$$\text{Now } x = A^{-1} B = \frac{1}{|A|} (\text{adj } A) B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} -2k \\ -3k \end{bmatrix} = \frac{1}{13} \begin{bmatrix} -9k \\ -k \end{bmatrix}$$

$$\text{Hence, } x = \frac{-9k}{13}, y = \frac{-k}{13}, z = k$$

Solution of Simultaneous Linear Equations Ex 8.2 Q4

$$\begin{aligned}x + y - 6z &= 0 \\x - y + 2z &= 0 \\-3x + y + 2z &= 0\end{aligned}$$

$$\begin{aligned}\text{Hence, } |A| &= \begin{vmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{vmatrix} \\&= 1(-4) - 1(8) - 6(-2) \\&= -4 - 8 + 12 \\&= 0\end{aligned}$$

Hence, the system has infinite solutions.

$$\begin{aligned}\text{Let } z &= k \\x + y &= 6k \\x - y &= -2k\end{aligned}$$

$$\begin{aligned}\text{or } \begin{bmatrix} 1 & +1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} 6k \\ -2k \end{bmatrix} \\ \text{or } A \quad x &= B\end{aligned}$$

$$|A| = -1 - 1 = -2 \neq 0 \quad \text{hence } A^{-1} \text{ exists.}$$

$$\text{adj } A = \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{|A|}(\text{adj } A)B = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix} \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix} = \left(\frac{1}{-2} \right) \begin{bmatrix} -4k \\ -8k \end{bmatrix} = \frac{-1}{2} \begin{bmatrix} -4k \\ -8k \end{bmatrix} = \begin{bmatrix} 2k \\ 4k \end{bmatrix}$$

$$\text{Hence, } x = 2k, \quad y = 4k, \quad z = k$$

Solution of Simultaneous Linear Equations Ex 8.2 Q5

$$\begin{aligned}x + y + z &= 0 \\x - y - 5z &= 0 \\x + 2y + 4z &= 0\end{aligned}$$

$$\begin{aligned}|A| &= \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{vmatrix} \\&= 1(6) - 1(9) + 1(3) = 9 - 9 = 0\end{aligned}$$

Hence, the system has infinite solutions.

$$\begin{aligned}\text{Let } z &= k \\x + y &= -k \\x - y &= 5k\end{aligned}$$

$$\begin{aligned}\text{or } \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} -k \\ 5k \end{bmatrix} \\ \text{or } A \quad x &= B\end{aligned}$$

$$|A| = -2 \neq 0, \quad \text{hence } A^{-1} \text{ exists.}$$

$$\text{adj } A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\text{so, } x = A^{-1}B = \frac{1}{|A|}(\text{adj } A)B = \frac{1}{-2} \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -k \\ 5k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{1}{-2} \right) \begin{bmatrix} k - 5k \\ k + 5k \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \end{bmatrix}$$

$$x = 2k, \quad y = -3k, \quad z = k$$

Solution of Simultaneous Linear Equations Ex 8.2 Q6

$$\begin{aligned}x + y - z &= 0 \\x - 2y + z &= 0 \\3x + 6y - 5z &= 0\end{aligned}$$

$$\begin{aligned}\text{Hence, } |A| &= \begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{vmatrix} \\&= 1(4) - 1(-8) - 1(12) \\&= 4 + 8 - 12 = 0\end{aligned}$$

Hence, the system will have infinite solutions.

$$\begin{aligned}\text{Let } z &= k \\x + y &= -k \\x - 2y &= -k\end{aligned}$$

$$\begin{aligned}\text{or } \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} k \\ -k \end{bmatrix} \\ \text{or } A x &= B\end{aligned}$$

$|A| = -3 \neq 0$, hence A^{-1} exists.

$$\text{Now, } \text{adj } A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$$

Next $x = A^{-1}B$

$$\begin{aligned}&= \frac{1}{|A|} (\text{adj } A)(B) = \frac{1}{-3} \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} k \\ -k \end{bmatrix} \\&= \frac{-1}{3} \begin{bmatrix} -2k + k \\ -2k \end{bmatrix} \\&= \frac{-1}{3} \begin{bmatrix} -k \\ -2k \end{bmatrix} = \begin{bmatrix} \frac{k}{3} \\ \frac{2k}{3} \end{bmatrix}\end{aligned}$$

$$\begin{aligned}\text{Hence, } x &= \frac{k}{3}, y = \frac{2k}{3}, z = k \\ \text{or } x &= k, y = 2k, z = 3k\end{aligned}$$

Solution of Simultaneous Linear Equations Ex 8.2 Q7

$$\begin{aligned}3x + y - 2z &= 0 \\x + y + z &= 0 \\x - 2y + z &= 0\end{aligned}$$

$$\begin{aligned}\text{Hence, } |A| &= \begin{vmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} \\|A| &= 3(1+2) - 1(1-1) - 2(-3) \\&= 9 - 0 + 6 \\&= 15 \neq 0\end{aligned}$$

Hence, the given system has only trivial solutions given by $x = y = z = 0$

Solution of Simultaneous Linear Equations Ex 8.2 Q8

$$2x + 3y - z = 0$$

$$x - y - 2z = 0$$

$$3x + y + 3z = 0$$

$$\text{Hence, } A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{vmatrix}$$

$$= 2(-3+2) - 3(3+6) - 1(4)$$

$$= -2 - 27 - 4$$

$$\neq 0$$

Hence, the system has only trivial solutions given by $x = y = z = 0$