

Ex 10.1

Chapter 10 Differentiability Ex 10.1 Q1

$$\begin{aligned}f(x) &= |x - 3| \\&= \begin{cases} -(x - 3), & \text{if } x < 3 \\ x - 3, & \text{if } x \geq 3 \end{cases}\end{aligned}$$

$$f(3) = 3 - 3 = 0$$

$$\begin{aligned}\text{LHL} &= \lim_{x \rightarrow 3^-} f(x) \\&= \lim_{h \rightarrow 0} f(3 - h) \\&= \lim_{h \rightarrow 0} 3 - (3 - h) \\&= \lim_{h \rightarrow 0} 0\end{aligned}$$

$$\begin{aligned}\text{RHL} &= \lim_{x \rightarrow 3^+} f(x) \\&= \lim_{h \rightarrow 0} f(3 + h) \\&= \lim_{h \rightarrow 0} 3 + h - 3 \\&= 0\end{aligned}$$

$$\text{LHL} = f(3) = \text{RHL}$$

$\therefore f(x)$ is continuous at $x = 3$

$$\begin{aligned}(\text{LHD at } x = 3) &= \lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3} \\&= \lim_{h \rightarrow 0} \frac{f(3 - h) - f(3)}{3 - h - 3} \\&= \lim_{h \rightarrow 0} \frac{3 - (3 - h) - 0}{-h} \\&= \lim_{h \rightarrow 0} \frac{h}{-h} \\&= -1\end{aligned}$$

$$\begin{aligned}(\text{RHD at } x = 3) &= \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} \\&= \lim_{h \rightarrow 0} \frac{f(3 + h) - f(3)}{3 + h - 3} \\&= \lim_{h \rightarrow 0} \frac{3 + h - 3 - 0}{h} \\&= \lim_{h \rightarrow 0} \frac{h}{h} \\&= 1\end{aligned}$$

$$(\text{LHD at } x = 3) \neq (\text{RHD at } x = 3)$$

$\therefore f(x)$ is continuous but not differentiable at $x = 3$.

Chapter 10 Differentiability Ex 10.1 Q2

$$f(x) = x^{\frac{1}{3}}$$

$$\begin{aligned} (\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{0 - h - 0} \\ &= \lim_{h \rightarrow 0} \frac{(-h)^{\frac{1}{3}} - 0}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(-h)^{\frac{1}{3}}}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(-1)^{\frac{1}{3}} h^{\frac{1}{3}}}{(-1)h} \\ &= \lim_{h \rightarrow 0} (-1)^{\frac{-2}{3}} h^{\frac{-2}{3}} \\ &= \text{Not defined} \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 0) &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{0 + h - 0} \\ &= \lim_{h \rightarrow 0} \frac{h^{\frac{1}{3}} - 0}{h} \\ &= \lim_{h \rightarrow 0} h^{\frac{-2}{3}} \\ &= \text{Not defined} \end{aligned}$$

Since,

LHD and RHD does not exists at $x = 0$

$\therefore f(x)$ is not differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q3

$$f(x) = \begin{cases} 12x - 13, & \text{if } x \leq 3 \\ 2x^2 + 5, & \text{if } x > 3 \end{cases}$$

$$\begin{aligned} (\text{LHD at } x = 3) &= \lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3} \\ &= \lim_{h \rightarrow 0} \frac{f(3 - h) - f(3)}{3 - h - 3} \\ &= \lim_{h \rightarrow 0} \frac{[12(3 - h) - 13] - [12(3) - 13]}{-h} \\ &= \lim_{h \rightarrow 0} \frac{36 - 12h - 13 - 36 + 13}{-h} \\ &= \lim_{h \rightarrow 0} \frac{-12h}{-h} \\ &= 12 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 3) &= \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3} \\ &= \lim_{h \rightarrow 0} \frac{f(3 + h) - f(3)}{3 + h - 3} \\ &= \lim_{h \rightarrow 0} \frac{[2(3 + h)^2 + 5] - [12(3) - 13]}{h} \\ &= \lim_{h \rightarrow 0} \frac{18 + 12h + 2h^2 + 5 - 36 + 13}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h^2 + 12h}{h} \\ &= \lim_{h \rightarrow 0} \frac{h(2h + 12)}{h} \\ &= 12 \end{aligned}$$

Now,

$$(\text{LHD at } x = 3) = (\text{RHD at } x = 3)$$

$\therefore f(x)$ is differentiable at $x = 3$

$$f'(x) = 12$$

Chapter 10 Differentiability Ex 10.1 Q4

$$f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 - x, & 1 < x \leq 2 \\ 5x - 4, & x > 2 \end{cases}$$

$$f(2) = 2(2)^2 - 2 \\ = 8 - 2 = 6$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 2^-} f(x) \\ &= \lim_{h \rightarrow 0} f(2 - h) \\ &= \lim_{h \rightarrow 0} [2(2 - h)^2 - (2 - h)] \\ &= 8 - 2 \\ &= 6 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 2^+} f(x) \\ &= \lim_{h \rightarrow 0} f(2 + h) \\ &= \lim_{h \rightarrow 0} 5(2 + h) - 4 \\ &= 6 \end{aligned}$$

$$\text{LHL} = f(2) = \text{RHL}$$

$f(x)$ is continuous at $x = 2$

$$\begin{aligned} (\text{LHD at } x = 2) &= \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{h \rightarrow 0} \frac{f(2 - h) - f(2)}{2 - h - 2} \\ &= \lim_{h \rightarrow 0} \frac{[2(2 - h)^2 - (2 - h)] - [8 - 2]}{-h} \\ &= \lim_{h \rightarrow 0} \frac{8 - 8h + 2h^2 - h - 6}{-h} \\ &= \lim_{h \rightarrow 0} \frac{2h^2 - 6h}{-h} \\ &= \lim_{h \rightarrow 0} \frac{h(2h - 6)}{-h} \\ &= \lim_{h \rightarrow 0} (6 - 2h) \\ &= 6 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 2) &= \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x - 2} \\ &= \lim_{h \rightarrow 0} \frac{f(2 + h) - f(2)}{2 + h - 2} \\ &= \lim_{h \rightarrow 0} \frac{[5(2 + h) - 4] - [8 - 2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{10 + 5h - 4 - 6}{h} \\ &= 5 \end{aligned}$$

Chapter 10 Differentiability Ex 10.1 Q5

$f(x) = |x| + |x-1|$ in the interval $(-1, 2)$.

$$f(x) = \begin{cases} x + x + 1 & -1 < x < 0 \\ 1 & 0 \leq x \leq 1 \\ -x - x + 1 & 1 < x < 2 \end{cases}$$

$$f(x) = \begin{cases} 2x + 1 & -1 < x < 0 \\ 1 & 0 \leq x \leq 1 \\ -2x + 1 & 1 < x < 2 \end{cases}$$

We know that a polynomial and a constant function is continuous and differentiable everywhere.
So, $f(x)$ is continuous and differentiable for $x \in (-1, 0)$, $x \in (0, 1)$ and $(1, 2)$.

We need to check continuity and differentiability at $x = 0$ and $x = 1$.

Continuity at $x = 0$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 2x + 1 = 1$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} 1 = 1$$

$$f(0) = 1$$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$\therefore f(x)$ is continuous at $x = 0$.

Continuity at $x = 1$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} 1 = 1$$

$$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} 1 = 1$$

$$f(1) = 1$$

$$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^+} f(x) = f(1)$$

$\therefore f(x)$ is continuous at $x = 1$.

Differentiability at $x = 0$

$$(\text{LHD at } x = 0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{2x + 1 - 1}{x - 0} = \lim_{x \rightarrow 0^-} \frac{2x}{x} = 2$$

$$(\text{RHD at } x = 0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{1 - 1}{x} = \lim_{x \rightarrow 0^+} \frac{0}{x} = 0$$

$$\therefore (\text{LHD at } x = 0) \neq (\text{RHD at } x = 0)$$

So, $f(x)$ is not differentiable at $x = 0$.

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{1 - 1}{x - 1} = 0$$

$$(\text{RHD at } x = 1) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{-2x + 1 - 1}{x - 1} \rightarrow \infty$$

$$\therefore (\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$$

So, $f(x)$ is not differentiable at $x = 1$.

So, $f(x)$ is continuous on $(-1, 2)$ but not differentiable at $x = 0, 1$.

$$f(x) = \begin{cases} x, & x \leq 1 \\ 2-x, & 1 \leq x \leq 2 \\ -2+3x-x^2, & x > 2 \end{cases}$$

Differentiability at $x = 1$

$$(\text{LHD at } x = 1) = \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^-} \frac{x - 1}{x - 1} = 1$$

$$(\text{RHD at } x = 1) = \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} = \lim_{x \rightarrow 1^+} \frac{2 - x - 1}{x - 1} = \lim_{x \rightarrow 1^+} \frac{1 - x}{x - 1} = -1$$

$$\therefore (\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$$

So, $f(x)$ is not differentiable at $x = 1$.

Differentiability at $x = 2$

$$(\text{LHD at } x = 2) = \lim_{x \rightarrow 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^-} \frac{2 - x - 0}{x - 2} = \lim_{x \rightarrow 2^-} \frac{2 - x}{x - 2} = -1$$

$$(\text{RHD at } x = 2) = \lim_{x \rightarrow 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2^+} \frac{-2 + 3x - x^2 - 0}{x - 2} = \lim_{x \rightarrow 2^+} \frac{(1 - x)(x - 2)}{x - 2} = -1$$

$$\therefore (\text{LHD at } x = 2) = (\text{RHD at } x = 2)$$

So, $f(x)$ is differentiable at $x = 2$.

Chapter 10 Differentiability Ex 10.1 Q7(i)

$$f(x) = \begin{cases} x^m \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$\begin{aligned} (\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 - h) - f(0)}{(0 - h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{(0 - h)^m \sin\left(\frac{1}{-h}\right) - 0}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\ &= \lim_{h \rightarrow 0} (-h)^{m-1} \sin\left(-\frac{1}{h}\right) \\ &= \lim_{h \rightarrow 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right) \\ &= 0 \times k \quad [\text{When } -1 \leq k \leq 1] \\ &= 0 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 0) &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{(0 + h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right) - 0}{h} \\ &= \lim_{h \rightarrow 0} \left(h^{m-1}\right) \sin\left(\frac{1}{h}\right) \\ &= 0 \times k' \quad [\text{Since } -1 \leq k' \leq 1] \\ &= 0 \end{aligned}$$

$$(\text{LHD at } x = 0) = (\text{RHD at } x = 0)$$

$\therefore f(x)$ is differentiable at $x = 0$

Chapter 10 Differentiability Ex 10.1 Q7(ii)

$$\begin{aligned}
\text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\
&= \lim_{h \rightarrow 0} f(0 - h) \\
&= \lim_{h \rightarrow 0} (-h)^m \sin\left(-\frac{1}{h}\right) \\
&= - \lim_{h \rightarrow 0} (-h)^m \sin\left(\frac{1}{h}\right) \\
&= 0 \times k & [\text{When } -1 \leq k \leq 1] \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\
&= \lim_{h \rightarrow 0} f(0 + h) \\
&= \lim_{h \rightarrow 0} (0 + h)^m \sin \frac{1}{(0 + h)} \\
&= \lim_{h \rightarrow 0} h^m \sin\left(\frac{1}{h}\right) \\
&= 0 \times k' & [\text{Where } -1 \leq k' \leq 1] \\
&= 0
\end{aligned}$$

$$\text{LHL} = f(0) = \text{RHL}$$

$\therefore f(x)$ is continuous at $x = 0$
For differentiability at $x = 0$

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{(0 - h) - f(0)}{(0 - h) - 0} \\
&= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\
&= \lim_{h \rightarrow 0} (-h)^{m-1} \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined} & [\text{Since } 0 < m < 1]
\end{aligned}$$

$$\begin{aligned}
(\text{RHD at } x = 0) &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{0 + h - 0} \\
&= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h}
\end{aligned}$$

Chapter 10 Differentiability Ex 10.1 Q7(iii)

$$\begin{aligned}
\text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\
&= \lim_{h \rightarrow 0} f(0 - h) \\
&= \lim_{h \rightarrow 0} (-h)^m \sin\left(-\frac{1}{h}\right) \\
&= \text{Not defined as } m \leq 0 \\
\text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\
&= \lim_{h \rightarrow 0} f(0 + h) \\
&= \lim_{h \rightarrow 0} h^m \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0
\end{aligned}$$

Since RHL and LHL are not defined, so $f(x)$ is not continuous

Let $x = 0$ for $m \leq 0$.

Now,

$$\begin{aligned}
(\text{LHD at } x = 0) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0 - h) - 0}{0 - h - 0} \\
&= \lim_{h \rightarrow 0} \frac{(-h)^m \sin\left(-\frac{1}{h}\right)}{-h} \\
&= \lim_{h \rightarrow 0} -(-h)^{m-1} \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0 \\
\text{RHD} &= \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\
&= \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{0 + h - 0} \\
&= \lim_{h \rightarrow 0} \frac{h^m \sin\left(\frac{1}{h}\right)}{h} \\
&= \lim_{h \rightarrow 0} (h^{m-1}) \sin\left(\frac{1}{h}\right) \\
&= \text{Not defined, as } m \leq 0
\end{aligned}$$

Thus,

$f(x)$ is neither continuous nor differentiable at $x = 0$ for $m \leq 0$.

Chapter 10 Differentiability Ex 10.1 Q8

$$f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \leq 1 \\ bx + 2, & \text{if } x > 1 \end{cases}$$

$$\begin{aligned} (\text{LHD at } x = 1) &= \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} \\ &= \lim_{h \rightarrow 0} \frac{f(1-h) - f(1)}{1-h-1} \\ &= \lim_{h \rightarrow 0} \frac{[(1-h)^2 + 3(1-h) + a] - [1+3+a]}{-h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 - 5h}{-h} \\ &= -5 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 1) &= \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} \\ &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{1+h-1} \\ &= \lim_{h \rightarrow 0} \frac{[b(1+h) + 2] - (b+2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{b + bh + 2 - b - 2}{h} \\ &= b \end{aligned}$$

Since $f(x)$ is differentiable, so

$$\begin{aligned} (\text{LHD at } x = 1) &= (\text{RHD at } x = 1) \\ 5 &= b \end{aligned}$$

$$\begin{aligned} f(1) &= 1 + 3 + a \\ &= 4 + a \end{aligned}$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) \\ &= \lim_{h \rightarrow 0} f(1-h) \\ &= \lim_{h \rightarrow 0} (1-h)^2 + 3(1-h) + a \\ &= 4 + a \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) \\ &= \lim_{h \rightarrow 0} f(1+h) \\ &= \lim_{h \rightarrow 0} b(1+h) + 2 \\ &= b + 2 \end{aligned}$$

Chapter 10 Differentiability Ex 10.1 Q9

$$f(x) = \begin{cases} |2x - 3| [x], & x \geq 1 \\ \sin\left(\frac{\pi x}{2}\right) & , \quad x < 1 \end{cases}$$

$$f(x) = \begin{cases} (2x - 3)[x], & x \geq \frac{3}{2} \\ -(2x - 3) & , \quad 1 \leq x \leq \frac{3}{2} \\ \sin\left(\frac{\pi x}{2}\right) & , \quad x < 1 \end{cases}$$

For continuity at $x = 1$

$$f(1) = -(2 \cdot 1 - 3) = 1$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) \\ &= \lim_{h \rightarrow 0} f(1 - h) \\ &= \lim_{h \rightarrow 0} \sin\left(\frac{\pi(1 - h)}{2}\right) \\ &= \sin \frac{\pi}{2} \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) \\ &= \lim_{h \rightarrow 0} f(1 + h) \\ &= \lim_{h \rightarrow 0} -(2(1 + h) - 3) \\ &= -1(-1) \\ &= 1 \end{aligned}$$

$$\text{LHL} = f(1) = \text{RHL}$$

So, $f(x)$ is continuous at $x = 1$

For differentiability at $x = 1$

$$\begin{aligned} (\text{LHD at } x = 1) &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(1)}{x - 1} \\ &= \lim_{h \rightarrow 0} \frac{f(1-h) - 1}{1-h-1} \\ &= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi(1-h)}{2}\right) - 1}{-h} \\ &= \lim_{h \rightarrow 0} \frac{\sin\left(\frac{\pi}{2} - \frac{\pi}{2}h\right) - 1}{-h} \\ &= \lim_{h \rightarrow 0} \frac{\cos\left(\frac{\pi}{2}h\right) - 1}{-\frac{h}{2}} \\ &= \lim_{h \rightarrow 0} \frac{2 \sin^2\left(\frac{\pi}{4}h\right)}{h} \times \frac{\left(\frac{\pi}{4}h\right)^2}{\left(\frac{\pi}{4}h\right)^2} \\ &= 0 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = 1) &= \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} \\ &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{1+h-1} \\ &= \lim_{h \rightarrow 0} \frac{-[2(1+h) - 3] - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2 - 2h + 3 - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2h}{h} \\ &= -2 \end{aligned}$$

$$(\text{LHD at } x = 1) \neq (\text{RHD at } x = 1)$$

$\therefore f(x)$ is continuous but differentiable at $x = 1$.

Chapter 10 Differentiability Ex 10.1 Q10

$$\text{Here, } f(x) = \begin{cases} ax^2 - b & , \text{ if } |x| < 1 \\ \frac{1}{|x|} & , \text{ if } |x| \geq 1 \end{cases}$$

$$= \begin{cases} -\frac{1}{x} & , \text{ if } x \leq -1 \\ ax^2 - b & , \text{ if } -1 < x < 1 \\ \frac{1}{x} & , \text{ if } x \geq 1 \end{cases}$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 1^-} f(x) \\ &= \lim_{h \rightarrow 0} f(1-h) \\ &= \lim_{h \rightarrow 0} a(1-h)^2 - b \\ &= a - b \end{aligned}$$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 1^+} f(x) \\ &= \lim_{h \rightarrow 0} f(1+h) \\ &= \lim_{h \rightarrow 0} \frac{1}{1+h} \\ &= 1 \end{aligned}$$

Since, $f(x)$ is continuous, so

$$\text{LHS} = \text{RHS}$$

$$a - b = 1$$

---(i)

$$\begin{aligned}
 (\text{LHD at } x = 1) &= \lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1} \\
 &= \lim_{h \rightarrow 0} \frac{f(1-h) - 1}{1-h-1} \\
 &= \lim_{h \rightarrow 0} \frac{a(1-h)^2 - b - 1}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{a(1-h)^2 - (a-1) - 1}{-h}
 \end{aligned}$$

Using equation (i),

$$\begin{aligned}
 &= \lim_{h \rightarrow 0} \frac{a + ah^2 - 2ah - a + 1 - 1}{-h} \\
 &= \lim_{h \rightarrow 0} \frac{ah^2 - 2ah}{-h} \\
 &= \lim_{h \rightarrow 0} (2a - ah) \\
 &= 2a
 \end{aligned}$$

$$\begin{aligned}
 (\text{RHD at } x = 1) &= \lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1} \\
 &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{1+h-1} \\
 &= \lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - 1}{h} \\
 &= \lim_{h \rightarrow 0} \frac{1 - 1 - h}{(1+h)h} \\
 &= \lim_{h \rightarrow 0} \frac{-1}{1+h} \\
 &= -1
 \end{aligned}$$

Since $f(x)$ is differentiable at $x = 1$,

$$(\text{LHD at } x = 1) = (\text{RHD at } x = 1)$$

$$2a = -1$$

$$a = \frac{-1}{2}$$

Put $a = \frac{-1}{2}$ in equation (i),

$$a - b = 1$$

$$\left(\frac{-1}{2}\right) - b = 1$$

$$b = \frac{-1}{2} - 1$$

$$b = \frac{-3}{2}$$

$$a = \frac{-1}{2}$$

Ex 10.2

Differentiability Ex 10.2 Q1

Here, $f(x) = x^2$ is a polynomial function so, it is differentiable at $x = 2$.

$$\begin{aligned} f'(2) &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(2+h)^2 - (2)^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{4 + h^2 + 4h - 4}{h} \\ &= \lim_{h \rightarrow 0} \frac{4h + h^2}{h} \\ &= \lim_{h \rightarrow 0} (4 + h) \\ &= 4 \end{aligned}$$

$$\therefore f'(2) = 4$$

Chapter 10 Differentiability Ex 10.2 Q2

$f(x) = x^2 - 4x + 7$ is a polynomial function, So it is differentiable everywhere.

$$\begin{aligned} f'(5) &= \lim_{h \rightarrow 0} \frac{f(5+h) - f(5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left\{ (5+h)^2 - 4(5+h) + 7 \right\} - [25 - 20 + 7]}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 + 25 + 10h - 20 - 4h + 7 - 12}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 + 6h}{h} \\ &= \lim_{h \rightarrow 0} (h + 6) \\ &= 6 \end{aligned}$$

$$\begin{aligned} f'\left(\frac{7}{2}\right) &= \lim_{h \rightarrow 0} \frac{f\left(\frac{7}{2}+h\right) - f\left(\frac{7}{2}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left[\left(\frac{7}{2}+h\right)^2 - 4\left(\frac{7}{2}+h\right) + 7\right] - \left[\left(\frac{7}{2}\right)^2 - 4\left(\frac{7}{2}\right) + 7\right]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\left[\frac{49}{2} + h^2 + 7h - 14 - 4h + 7\right] - \left[\frac{49}{4} - 14 + 7\right]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{49}{4} + h^2 + 7h - 14 - 4h + 7 - \frac{49}{4} + 14 - 7}{h} \\ &= \lim_{h \rightarrow 0} \frac{h^2 + 3h}{h} \\ &= \lim_{h \rightarrow 0} (h + 3) \\ &= 3 \end{aligned}$$

Now,

$$\begin{aligned} f'(5) &= 6 \\ &= 2(3) \\ f'(5) &= 2f'\left(\frac{7}{2}\right) \end{aligned}$$

Chapter 10 Differentiability Ex 10.2 Q3

We know that, $f(x) = 2x^3 - 9x^2 + 12x + 9$ is a polynomial function. So, it is differentiable every where. For $x = 1$

$$\begin{aligned}
 f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[2(1+h)^3 - 9(1+h)^2 + 12(1+h) + 9] - [2 - 9 + 12 + 9]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[2(1+h^3 + 3h^2 + 3h) - 9(1+h^2 + 2h) + 12 + 12h + 9] - [14]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[2 + 2h^3 + 6h^2 + 6h - 9 - 9h^2 - 18h + 12 + 12h + 9 - 14]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2h^3 - 3h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h^2(2h - 3)}{h} \\
 &= \lim_{h \rightarrow 0} h(2h - 3) \\
 f'(1) &= 0 \quad \text{---(i)}
 \end{aligned}$$

For $x = 2$

$$\begin{aligned}
 f'(2) &= \lim_{h \rightarrow 0} \frac{[2(2+h)^3 - 9(2+h)^2 + 12(2+h) + 9] - [16 - 36 + 24 + 9]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[2(8 + h^3 + 12h + 6h^2) - 9(4 + h^2 + 4h) + 24 + 12h + 9] - [13]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[16 + 2h^3 + 24h + 12h^2 - 36 - 9h^2 - 36h + 33 + 12h - 13]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{2h^3 + 3h^2}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h^2(2h + 3)}{h} \\
 &= \lim_{h \rightarrow 0} h(2h + 3) \\
 f'(2) &= 0 \quad \text{---(ii)}
 \end{aligned}$$

From equation (i) and (ii),

$$f'(1) = f'(2)$$

Chapter 10 Differentiability Ex 10.2 Q4

$$\begin{aligned}
 \Phi(x) &= \lambda x^2 + 7x - 4 \text{ and } \Phi'(5) = 97 \\
 \Phi'(5) &= \lim_{h \rightarrow 0} \frac{[\lambda(5+h)^2 + 7(5+h) - 4] - [25\lambda + 35 - 4]}{h} \\
 97 &= \lim_{h \rightarrow 0} \frac{\lambda(25 + h^2 + 10h) + 35 + 7h - 4 - 25\lambda - 35 + 4}{h} \\
 &= \lim_{h \rightarrow 0} \frac{25\lambda + \lambda h^2 + 10\lambda h - 25\lambda + 7h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{\lambda h^2 + h(10\lambda + 7)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(\lambda h + 10\lambda + 7)}{h} \\
 97 &= 10\lambda + 7 \\
 10\lambda &= 97 + 7 \\
 \lambda &= \frac{90}{10} \\
 \lambda &= 9
 \end{aligned}$$

Chapter 10 Differentiability Ex 10.2 Q5

$f(x) = x^3 + 7x^2 + 8x - 9$ is a polynomial function. So, it is differentiable every where.

$$\begin{aligned}
 f'(4) &= \lim_{h \rightarrow 0} \frac{f(4+h) - f(4)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[(4+h)^3 + 7(4+h)^2 + 8(4+h) - 9] - [64 + 112 + 32 - 9]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{[64 + h^3 + 48h + 12h^2 + 112 + 7h^2 + 56h + 32 + 8h - 9] - [210 - 9]}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h^3 + 19h^2 + 112h + 210 - 9 - 210 + 9}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h^3 + 19h^2 + 112h}{h} \\
 &= \lim_{h \rightarrow 0} \frac{h(h^2 + 19h + 112)}{h} \\
 f'(4) &= 112
 \end{aligned}$$

Chapter 10 Differentiability Ex 10.2 Q6

$$\begin{aligned}
 f(x) &= mx + c \\
 f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(mh + c) - (m \times 0 + c)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{mh + c - c}{h} \\
 &= \lim_{h \rightarrow 0} \frac{mh}{h} \\
 &= m \\
 f'(0) &= m
 \end{aligned}$$

Chapter 10 Differentiability Ex 10.2 Q7

$$f(x) = \begin{cases} 2x + 3, & \text{if } -3 \leq x < -2 \\ x + 1, & \text{if } -2 \leq x < 0 \\ x + 2, & \text{if } 0 \leq x \leq 1 \end{cases}$$

We know that polynomial functions are continuous and differentiable everywhere.

So $f(x)$ is differentiable on $x \in [-3, -2)$, $x \in (-2, 0)$ and $x \in (0, 1]$.

We need to check the differentiability at $x = -2$ and $x = 0$.

Differentiability at $x = -2$

$$(\text{LHD at } x = -2) = \lim_{x \rightarrow -2^-} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \rightarrow -2^-} \frac{2x + 3 + 1}{x + 2} = \lim_{x \rightarrow -2^-} \frac{2(x + 2)}{x + 2} = 2$$

$$(\text{RHD at } x = -2) = \lim_{x \rightarrow -2^+} \frac{f(x) - f(-2)}{x - (-2)} = \lim_{x \rightarrow -2^+} \frac{x + 1 + 1}{x + 2} = \lim_{x \rightarrow -2^+} \frac{x + 2}{x + 2} = 1$$

$$\therefore (\text{LHD at } x = -2) \neq (\text{RHD at } x = -2)$$

So, $f(x)$ is not differentiable at $x = -2$.

Differentiability at $x = 0$

$$(\text{LHD at } x = 0) = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{x + 1 - 2}{x} = \lim_{x \rightarrow 0^-} \frac{x - 1}{x} \rightarrow \infty$$

$$(\text{RHD at } x = 0) = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x + 2 - 2}{x} = \lim_{x \rightarrow 0^+} \frac{x}{x} = 1$$

$$\therefore (\text{LHD at } x = 0) \neq (\text{RHD at } x = 0)$$

So, $f(x)$ is not differentiable at $x = 0$.

Chapter 10 Differentiability Ex 10.2 Q8

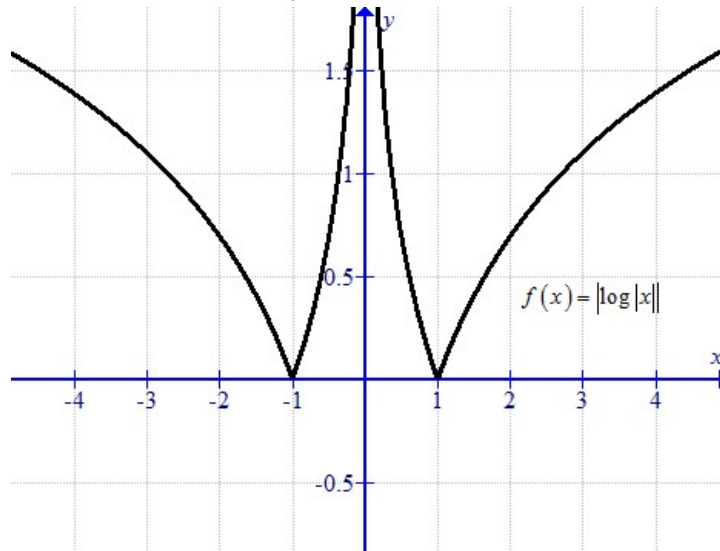
We know that, modulus function

$f(x) = |x|$ is continuous but not differentiable at $x = 0$,

So,

$f(x) = |x| + |x - 1| + |x - 2| + |x - 3| + |x - 4|$ is continuous but not differentiable
 $x = 0, 1, 2, 3, 4$.

Chapter 10 Differentiability Ex 10.2 Q9



$$f(x) = |\log|x||$$

Since, it is an absolute function. So, it is continuous function.

The graph of the function is as below:-

Chapter 10 Differentiability Ex 10.2 Q10

$$f(x) = e^{|x|}$$

$$f(x) = \begin{cases} e^{-x} & , x < 0 \\ e^x & , x \geq 0 \end{cases}$$

For continuity at $x = 0$

$$\begin{aligned} \text{RHL} &= \lim_{x \rightarrow 0^+} f(x) \\ &= \lim_{h \rightarrow 0} f(0+h) \\ &= \lim_{h \rightarrow 0} e^{(0+h)} \\ &= \lim_{h \rightarrow 0} e^h \\ &= e^0 \end{aligned}$$

$$\text{RHL} = 1$$

$$\begin{aligned} \text{LHL} &= \lim_{x \rightarrow 0^-} f(x) \\ &= \lim_{h \rightarrow 0} f(0-h) \\ &= \lim_{h \rightarrow 0} e^{-(0-h)} \\ &= \lim_{h \rightarrow 0} e^h \end{aligned}$$

$$\text{LHL} = 1$$

$$\begin{aligned} f(0) &= e^0 \\ &= 1 \end{aligned}$$

Now,

$$\text{LHL} = f(0) = \text{RHL}$$

So, $f(x)$ is continuous at $x = 0$

For differentiability at $x = 0$

$$\begin{aligned} \text{LHD} &= \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0-h) - e^0}{(0-h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{e^{-(0-h)} - 1}{-h} \\ &= \lim_{h \rightarrow 0} \frac{e^h - 1}{-h} \\ &= 1 \end{aligned}$$

$$\left[\text{Since } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right]$$

$$\begin{aligned} \text{RHD} &= \lim_{h \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} \\ &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{(0+h) - 0} \\ &= \lim_{h \rightarrow 0} \frac{e^x - e^0}{h} \\ &= \lim_{h \rightarrow 0} \frac{e^h - 1}{h} \\ &= 1 \end{aligned}$$

$$\left[\text{Since } \lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1 \right]$$

Clearly,

$$\text{LHD} \neq \text{RHD}$$

So,

$f(x)$ is not differentiable at $x = 0$.

Differentiability Ex 10.2 Q11

$$f(x) = \begin{cases} (x-c) \cos \frac{1}{(x-c)} & , x \neq c \\ 0 & , x = c \end{cases}$$

$$\begin{aligned} (\text{LHL at } x=c) &= \lim_{x \rightarrow c^-} f(x) \\ &= \lim_{h \rightarrow 0} f(-h) \\ &= \lim_{h \rightarrow 0} (c-h-c) \cos \left(\frac{1}{c-h-c} \right) \\ &= \lim_{h \rightarrow 0} -h \cos \left(-\frac{1}{h} \right) \\ &= \lim_{h \rightarrow 0} -h \cos \left(\frac{1}{h} \right) \\ &= 0 \end{aligned}$$

$$\begin{aligned} (\text{RHL at } x=c) &= \lim_{x \rightarrow c^+} f(x) \\ &= \lim_{h \rightarrow 0} f(c+h) \\ &= \lim_{h \rightarrow 0} (c+h-c) \cos \left(\frac{1}{c+h-c} \right) \\ &= \lim_{h \rightarrow 0} h \cos \left(\frac{1}{h} \right) \\ &= 0 \end{aligned}$$

$$f(c) = 0$$

Since, LHL = $f(x)$ = RHL at $x=c$

$\Rightarrow f(x)$ is continuous at $x=c$

$$\begin{aligned} (\text{LHD at } x=c) &= \lim_{h \rightarrow 0} \frac{f(c-h) - f(c)}{-h} \\ &= \lim_{h \rightarrow 0} \frac{(c-h-c) \cos \left(\frac{1}{c-h-c} \right) - 0}{-h} \\ &= \lim_{h \rightarrow 0} \cos \left(-\frac{1}{h} \right) \\ &= \lim_{h \rightarrow 0} \cos \left(\frac{1}{h} \right) \\ &= k \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x=c) &= \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(c+h-c) \cos \left(\frac{1}{c+h-c} \right) - 0}{h} \\ &= \lim_{h \rightarrow 0} \frac{h \cos \left(\frac{1}{h} \right)}{h} \\ &= \lim_{h \rightarrow 0} \cos \left(\frac{1}{h} \right) \\ &= k \end{aligned}$$

$$(\text{LHD at } x=c) = (\text{RHD at } x=c)$$

So,

$f(x)$ is differentiable and continuous at $x=c$.

Differentiability Ex 10.2 Q12

$$f(x) = |\sin x| = \begin{cases} -\sin x, & x < n\pi \\ \sin x, & x \geq n\pi \end{cases}$$

For $x = n\pi$ (n even)

$$\begin{aligned} (\text{LHD at } x = n\pi) &= \lim_{x \rightarrow n\pi^-} \frac{f(x) - f(n\pi)}{x - n\pi} \\ &= \lim_{h \rightarrow 0} \frac{-\sin(n\pi - h) - \sin n\pi}{n\pi - h - n\pi} \\ &= \lim_{h \rightarrow 0} \frac{\sin h - 0}{-h} \\ &= -1 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = n\pi) &= \lim_{h \rightarrow 0} \frac{\sin(n\pi + h) - \sin n\pi}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin h}{h} \\ &= 1 \end{aligned}$$

$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$

For $x = n\pi$ (n is odd)

$$\begin{aligned} (\text{LHD at } x = n\pi) &= \lim_{h \rightarrow 0} \frac{-\sin(n\pi - h) - \sin n\pi}{-h} \\ &= \lim_{h \rightarrow 0} \frac{-\sin h}{-h} \\ &= 1 \end{aligned}$$

$$\begin{aligned} (\text{RHD at } x = n\pi) &= \lim_{h \rightarrow 0} \frac{\sin(n\pi + h) - \sin n\pi}{h} \\ &= \lim_{h \rightarrow 0} \frac{-\sin h - 0}{h} \\ &= -1 \end{aligned}$$

$(\text{LHD at } x = n\pi) \neq (\text{RHD at } x = n\pi)$

Thus,

$f(x) = |\sin x|$ is not differentiable at $x = n\pi$

$$f(x) = \cos|x|$$

Since, $\cos(-x) = \cos x$

$$\Rightarrow f(x) = \cos x$$

$\Rightarrow f(x) = \cos|x|$ is differentiable everywhere.