

# Linear Inequations (in one variable)

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## Exercise 4A

### Question 1.

State, true or false:

(i)  $x < -y \Rightarrow -x > y$

(ii)  $-5x \geq 15 \Rightarrow x \geq -3$

(iii)  $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

(iv)  $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

### Solution:

(i)  $x < -y \Rightarrow -x > y$

The given statement is true.

(ii)  $-5x \geq 15 \Rightarrow \frac{-5x}{5} \geq \frac{15}{5} \Rightarrow x \leq -3$

The given statement is false.

(iii)  $2x \leq -7 \Rightarrow \frac{2x}{-4} \geq \frac{-7}{-4}$

The given statement is true.

(iv)  $7 > 5 \Rightarrow \frac{1}{7} < \frac{1}{5}$

The given statement is true.

### Question 2.

State, whether the following statements are true or false:

(i)  $a < b$ , then  $a - c < b - c$  (ii) If  $a > b$ , then  $a + c > b + c$

(iii) If  $a < b$ , then  $ac > bc$

(iv) If  $a > b$ , then  $\frac{a}{c} < \frac{b}{c}$

(v) If  $a - c > b - d$ , then  $a + d > b + c$

(vi) If  $a < b$ , and  $c > 0$ , then  $a - c > b - c$

Where  $a, b, c$  and  $d$  are real numbers and  $c \neq 0$ .

**Solution:**

(i)  $a < b \Rightarrow a - c < b - c$  The given statement is true.

(ii) If  $a > b \Rightarrow a + c > b + c$

The given statement is true.

(iii) If  $a < b \Rightarrow ac < bc$  The given statement is false.

(iv) If  $a > b \Rightarrow \frac{a}{c} > \frac{b}{c}$

The given statement is false.

(v) If  $a - c > b - d \Rightarrow a + d > b + c$

The given statement is true.

(vi) If  $a < b \Rightarrow a - c < b - c$  (Since,  $c > 0$ )

The given statement is false.

**Question 3.**

If  $x \in \mathbb{N}$ , find the solution set of inequations.

(i)  $5x + 3 \leq 2x + 18$

(ii)  $3x - 2 < 19 - 4x$

**Solution:**

(i)  $5x + 3 \leq 2x + 18$

$5x - 2x \leq 18 - 3$

$3x \leq 15$

$x \leq 5$

Since,  $x \in \mathbb{N}$ , therefore solution set is  $\{1, 2, 3, 4, 5\}$ .

(ii)  $3x - 2 < 19 - 4x$

$3x + 4x < 19 + 2$

$7x < 21$

$x < 3$

Since,  $x \in \mathbb{N}$ , therefore solution set is  $\{1, 2\}$ .

**Question 4.**

If the replacement set is the set of whole numbers, solve:

(i)  $x + 7 \leq 11$

(ii)  $3x - 1 > 8$

(iii)  $8 - x > 5$

(iv)  $7 - 3x \geq -\frac{1}{2}$

(v)  $x - \frac{3}{2} < \frac{3}{2} - x$

(vi)  $18 \leq 3x - 2$

**Solution:**

$$(i) x + 7 \leq 11$$

$$x \leq 11 - 7$$

$$x \leq 4$$

Since, the replacement set = W (set of whole numbers)

$\Rightarrow$  Solution set =  $\{0, 1, 2, 3, 4\}$

$$(ii) 3x - 1 > 8$$

$$3x > 8 + 1$$

$$x > 3$$

Since, the replacement set = W (set of whole numbers)

$\Rightarrow$  Solution set =  $\{4, 5, 6, \dots\}$

$$(iii) 8 - x > 5$$

$$-x > 5 - 8$$

$$-x > -3$$

$$x < 3$$

Since, the replacement set = W (set of whole numbers)

$\Rightarrow$  Solution set =  $\{0, 1, 2\}$

$$(iv) 7 - 3x \geq -\frac{1}{2}$$

$$-3x \geq -\frac{1}{2} - 7$$

$$-3x \geq -\frac{15}{2}$$

$$x \leq \frac{5}{2}$$

Since, the replacement set = W (set of whole numbers)

$\therefore$  Solution set =  $\{0, 1, 2\}$

$$(v) x - \frac{3}{2} < \frac{3}{2} - x$$

$$x + x < \frac{3}{2} + \frac{3}{2}$$

$$2x < 3$$

$$x < \frac{3}{2}$$

Since, the replacement set = W (set of whole numbers)  
 $\therefore$  Solution set =  $\{0, 1\}$

$$\begin{aligned} \text{(vi) } 18 &\leq 3x - 2 \\ 18 + 2 &\leq 3x \\ 20 &\leq 3x \\ x &\geq \frac{20}{3} \end{aligned}$$

Since, the replacement set = W (set of whole numbers)  
 $\therefore$  Solution set =  $\{7, 8, 9, \dots\}$

**Question 5.**

Solve the inequation:  
 $3 - 2x \geq x - 12$  given that  $x \in \mathbb{N}$ .

**Solution:**

$$\begin{aligned} 3 - 2x &\geq x - 12 \\ -2x - x &\geq -12 - 3 \\ -3x &\geq -15 \\ x &\leq 5 \end{aligned}$$

Since,  $x \in \mathbb{N}$ , therefore,  
Solution set =  $\{1, 2, 3, 4, 5\}$

**Question 6.**

If  $25 - 4x \leq 16$ , find:  
(i) the smallest value of  $x$ , when  $x$  is a real number,  
(ii) the smallest value of  $x$ , when  $x$  is an integer.

**Solution:**

$$\begin{aligned} 25 - 4x &\leq 16 \\ -4x &\leq 16 - 25 \\ -4x &\leq -9 \end{aligned}$$

$$\begin{aligned} x &\geq \frac{9}{4} \\ x &\geq 2.25 \end{aligned}$$

- (i) The smallest value of  $x$ , when  $x$  is a real number, is 2.25.  
(ii) The smallest value of  $x$ , when  $x$  is an integer, is 3.

**Question 7.**

If the replacement set is the set of real numbers, solve:

(i)  $-4x \geq -16$

(ii)  $8 - 3x \leq 20$

(iii)  $5 + \frac{x}{4} > \frac{x}{5} + 9$

(iv)  $\frac{x+3}{8} < \frac{x-3}{5}$

**Solution:**

(i)  $-4x \geq -16$

$x \leq 4$

Since, the replacement set of real numbers.

$\therefore$  Solution set =  $\{x: x \in \mathbb{R} \text{ and } x \leq 4\}$

(ii)  $8 - 3x \leq 20$

$-3x \leq 20 - 8$

$-3x \leq 12$

$x \geq -4$

Since, the replacement set of real numbers.

$\therefore$  Solution set =  $\{x: x \in \mathbb{R} \text{ and } x \geq -4\}$

(iii)  $5 + \frac{x}{4} > \frac{x}{5} + 9$

$\frac{x}{4} - \frac{x}{5} > 9 - 5$

$\frac{x}{20} > 4$

$x > 80$

Since, the replacement set of real numbers.

$\therefore$  Solution set =  $\{x: x \in \mathbb{R} \text{ and } x > 80\}$

(iv)  $\frac{x+3}{8} < \frac{x-3}{5}$

$5x + 15 < 8x - 24$

$5x - 8x < -24 - 15$

$-3x < -39$

$x > 13$

Since, the replacement set of real numbers.

$\therefore$  Solution set =  $\{x: x \in \mathbb{R} \text{ and } x > 13\}$

**Question 8.**

Find the smallest value of  $x$  for which  $5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$ , where  $x$  is an integer.

**Solution:**

$$5 - 2x < 5\frac{1}{2} - \frac{5}{3}x$$

$$-2x + \frac{5}{3}x < \frac{11}{2} - 5$$

$$\frac{-x}{3} < \frac{1}{2}$$

$$-x < \frac{3}{2}$$

$$x > \frac{-3}{2}$$

$$x > -1.5$$

Thus, the required smallest value of  $x$  is -1.

**Question 9.**

Find the largest value of  $x$  for which  $2(x - 1) \leq 9 - x$  and  $x \in W$ .

**Solution:**

$$2(x - 1) \leq 9 - x$$

$$2x - 2 \leq 9 - x$$

$$2x + x \leq 9 + 2$$

$$3x \leq 11$$

$$x \leq \frac{11}{3}$$

$$x \leq 3.67$$

Since,  $x \in W$ , thus the required largest value of  $x$  is 3.

**Question 10.**

Solve the inequation:  $12 + 1\frac{5}{6}x \leq 5 + 3x$  and  $x \in R$ .

**Solution:**

$$12 + 1\frac{5}{6}x \leq 5 + 3x$$

$$\frac{11}{6}x - 3x \leq 5 - 12$$

$$\frac{-7}{6}x \leq -7$$

$$x \geq 6$$

$\therefore$  Solution set =  $\{x: x \in \mathbb{R} \text{ and } x \geq 6\}$

**Question 11.**

Given  $x \in \{\text{integers}\}$ , find the solution set of:

$$-5 \leq 2x - 3 < x + 2$$

**Solution:**

$$-5 \leq 2x - 3 < x + 2$$

$$\Rightarrow -5 \leq 2x - 3 \quad \text{and} \quad 2x - 3 < x + 2$$

$$\Rightarrow -5 + 3 \leq 2x \quad \text{and} \quad 2x - x < 2 + 3$$

$$\Rightarrow -2 \leq 2x \quad \text{and} \quad x < 5$$

$$\Rightarrow x \geq -1 \quad \text{and} \quad x < 5$$

Since,  $x \in \{\text{integers}\}$

$\therefore$  Solution set =  $\{-1, 0, 1, 2, 3, 4\}$

**Question 12.**

Given  $x \in \{\text{whole numbers}\}$ , find the solution set of:

$$-1 \leq 3 + 4x < 23$$

**Solution:**

$$-1 \leq 3 + 4x < 23$$

$$\Rightarrow -1 \leq 3 + 4x \quad \text{and} \quad 3 + 4x < 23$$

$$\Rightarrow -4 \leq 4x \quad \text{and} \quad 4x < 20$$

$$\Rightarrow x \geq -1 \quad \text{and} \quad x < 5$$

Since,  $x \in \{\text{whole numbers}\}$

$\therefore$  Solution set =  $\{0, 1, 2, 3, 4\}$

## Exercise 4B

### Question 1.

Represent the following inequalities on real number lines:

(i)  $2x - 1 < 5$

(ii)  $3x + 1 \geq -5$

(iii)  $2(2x - 3) \leq 6$

(iv)  $-4 < x < 4$

(v)  $-2 \leq x < 5$

(vi)  $8 \geq x > -3$

(vii)  $-5 < x \leq -1$

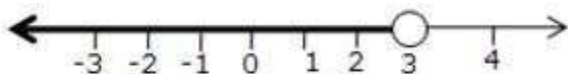
### Solution:

(i)  $2x - 1 < 5$

$2x < 6$

$x < 3$

Solution on number line is:

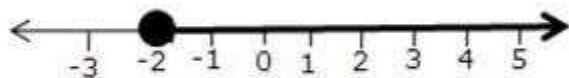


(ii)  $3x + 1 \geq -5$

$3x \geq -6$

$x \geq -2$

Solution on number line is:



(iii)  $2(2x - 3) \leq 6$

$2x - 3 \leq 3$

$2x \leq 6$

$x \leq 3$

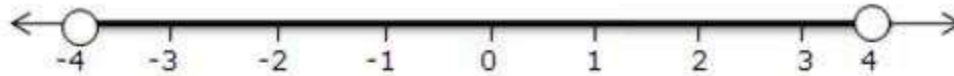
Solution on number line is:





(iv)  $-4 < x < 4$

Solution on number line is:



(v)  $-2 \leq x < 5$

Solution on number line is:



(vi)  $8 \geq x > -3$

Solution on number line is:



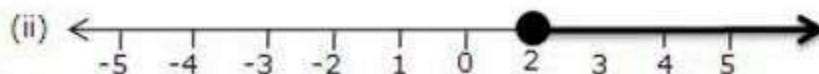
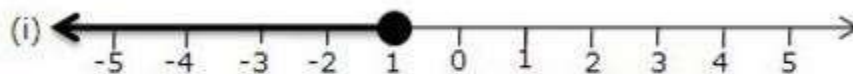
(vii)  $-5 < x \leq -1$

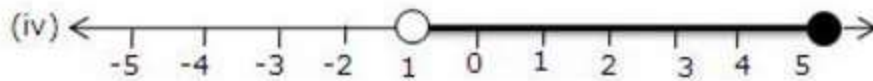
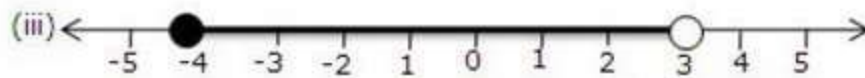
Solution on number line is:



## Question 2.

For each graph given, write an inequation taking  $x$  as the variable:





**Solution:**

(i)  $x \leq -1, x \in \mathbb{R}$

(ii)  $x \geq 2, x \in \mathbb{R}$

(iii)  $-4 \leq x < 3, x \in \mathbb{R}$

(iv)  $-1 < x \leq 5, x \in \mathbb{R}$

**Question 3.**

For the following inequations, graph the solution set on the real number line:

(i)  $-4 \leq 3x - 1 < 8$

(ii)  $x - 1 < 3 - x \leq 5$

**Solution:**

(i)  $-4 \leq 3x - 1 < 8$

$-4 \leq 3x - 1$  and  $3x - 1 < 8$

$-1 \leq x$  and  $x < 3$

The solution set on the real number line is:



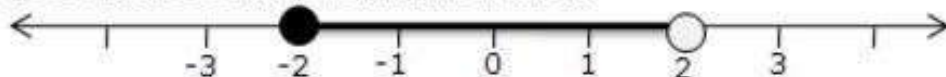
(ii)  $x - 1 < 3 - x \leq 5$

$x - 1 < 3 - x$  and  $3 - x \leq 5$

$2x < 4$  and  $-x \leq 2$

$x < 2$  and  $x \geq -2$

The solution set on the real number line is:



#### Question 4.

Represent the solution of each of the following inequalities on the real number line:

(i)  $4x - 1 > x + 11$

(ii)  $7 - x \leq 2 - 6x$

(iii)  $x + 3 \leq 2x + 9$

(iv)  $2 - 3x > 7 - 5x$

(v)  $1 + x \geq 5x - 11$

(vi)  $\frac{2x+5}{3} > 3x-3$

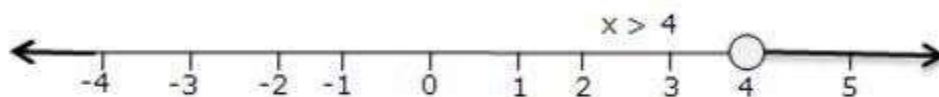
**Solution:**

(i)  $4x - 1 > x + 11$

$$3x > 12$$

$$x > 4$$

The solution on number line is:

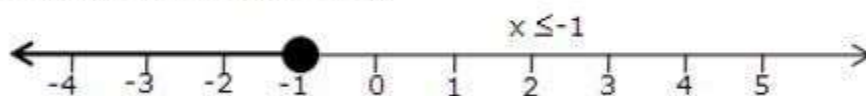


(ii)  $7 - x \leq 2 - 6x$

$$5x \leq -5$$

$$x \leq -1$$

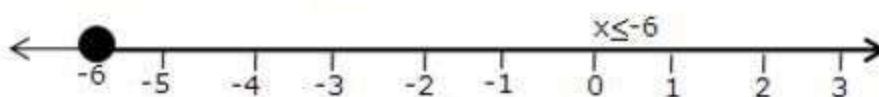
The solution on number line is:



(iii)  $x + 3 \leq 2x + 9$

$$-6 \leq x$$

The solution on number line is:



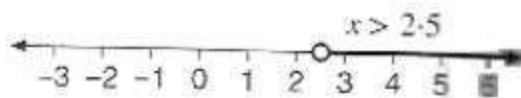
$$(iv) 2 - 3x > 7 - 5x$$

$$2x > 5$$

$$x > \frac{5}{2}$$

$$x > 2.5$$

The solution on number line is:

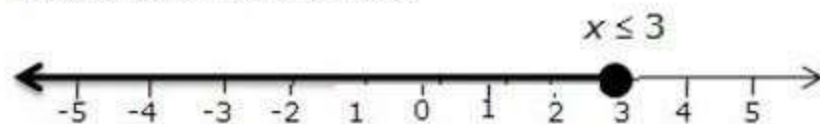


$$(v) 1 + x \geq 5x - 11$$

$$12 \geq 4x$$

$$3 \geq x$$

The solution on number line is:



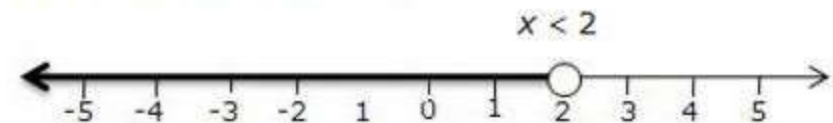
$$(vi) \frac{2x+5}{3} > 3x-3$$

$$2x+5 > 9x-9$$

$$-7x > -14$$

$$x < 2$$

The solution on number line is:



### Question 5.

$x \in \{\text{real numbers}\}$  and  $-1 < 3 - 2x \leq 7$ , evaluate  $x$  and represent it on a number line.

**Solution:**

$$-1 < 3 - 2x \leq 7$$

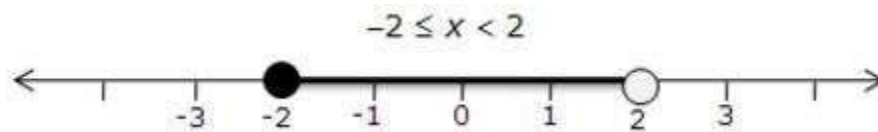
$$-1 < 3 - 2x \text{ and } 3 - 2x \leq 7$$

$$2x < 4 \text{ and } -2x \leq 4$$

$$x < 2 \text{ and } x \geq -2$$

$$\text{Solution set} = \{-2 \leq x < 2, x \in \mathbb{R}\}$$

Thus, the solution can be represented on a number line as:



### Question 6.

List the elements of the solution set of the inequation

$$-3 < x - 2 \leq 9 - 2x; x \in \mathbb{N}.$$

**Solution:**

$$-3 < x - 2 \leq 9 - 2x$$

$$-3 < x - 2 \text{ and } x - 2 \leq 9 - 2x$$

$$-1 < x \text{ and } 3x \leq 11$$

$$-1 < x \leq \frac{11}{3}$$

Since,  $x \in \mathbb{N}$

$$\therefore \text{Solution set} = \{1, 2, 3\}$$

### Question 7.

Find the range of values of  $x$  which satisfies

$$-2\frac{2}{3} \leq x + \frac{1}{3} < 3\frac{1}{3}; x \in \mathbb{R}.$$

Graph these values of  $x$  on the number line.

**Solution:**

$$-2\frac{2}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < 3\frac{1}{3}$$

$$\Rightarrow -\frac{8}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < \frac{10}{3}$$

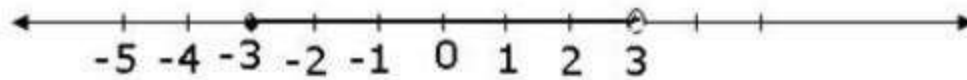
$$\Rightarrow -\frac{8}{3} - \frac{1}{3} \leq x \text{ and } x < \frac{10}{3} - \frac{1}{3}$$

$$\Rightarrow -\frac{9}{3} \leq x \text{ and } x < \frac{9}{3}$$

$$\Rightarrow -3 \leq x \text{ and } x < 3$$

$$\therefore -3 \leq x < 3$$

The required graph of the solution set is:



### Question 8.

Find the values of  $x$ , which satisfy the inequation:

$$-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}; x \in \mathbb{N}.$$

Graph the solution on the number line.

**Solution:**

$$-2 \leq \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}$$

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \quad \text{and} \quad \frac{1}{2} - \frac{2x}{3} < 1\frac{5}{6}$$

$$-\frac{5}{2} \leq -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} < \frac{8}{6}$$

$$\frac{15}{4} \geq x \quad \text{and} \quad x > -2$$

$$3.75 \geq x \quad \text{and} \quad x > -2$$

Since,  $x \in \mathbb{N}$

$$\therefore \text{Solution set} = \{1, 2, 3\}$$

The required graph of the solution set is:



### Question 9.

Given  $x \in \{\text{real numbers}\}$ , find the range of values of  $x$  for which  $-5 \leq 2x - 3 < x + 2$  and represent it on a number line.

**Solution:**

$$-5 \leq 2x - 3 < x + 2$$

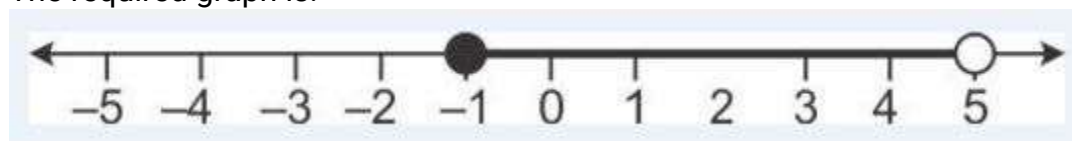
$$-5 \leq 2x - 3 \text{ and } 2x - 3 < x + 2$$

$$-2 \leq 2x \text{ and } x < 5$$

$$-1 \leq x \text{ and } x < 5$$

Required range is  $-1 \leq x < 5$ .

The required graph is:



**Question 10.**

If  $5x - 3 \leq 5 + 3x \leq 4x + 2$ , express it as  $a \leq x \leq b$  and then state the values of  $a$  and  $b$ .

**Solution:**

$$5x - 3 \leq 5 + 3x \leq 4x + 2$$

$$5x - 3 \leq 5 + 3x \text{ and } 5 + 3x \leq 4x + 2$$

$$2x \leq 8 \text{ and } -x \leq -3$$

$$x \leq 4 \text{ and } x \geq 3$$

Thus,  $3 \leq x \leq 4$ .

Hence,  $a = 3$  and  $b = 4$ .

**Question 11.**

Solve the following inequation and graph the solution set on the number line:

$$2x - 3 < x + 2 \leq 3x + 5, x \in \mathbb{R}.$$

**Solution:**

$$2x - 3 < x + 2 \leq 3x + 5$$

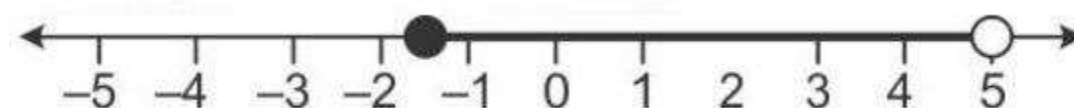
$$2x - 3 < x + 2 \text{ and } x + 2 \leq 3x + 5$$

$$x < 5 \text{ and } -3 \leq 2x$$

$$x < 5 \text{ and } -1.5 \leq x$$

$$\text{Solution set} = \{-1.5 \leq x < 5\}$$

The solution set can be graphed on the number line as:



**Question 12.**

Solve and graph the solution set of:

(i)  $2x - 9 < 7$  and  $3x + 9 \leq 25, x \in \mathbb{R}$  (ii)  $2x - 9 \leq 7$  and  $3x + 9 > 25, x \in \mathbb{I}$

(iii)  $x + 5 \geq 4(x - 1)$  and  $3 - 2x < -7, x \in \mathbb{R}$

**Solution:**

(i)  $2x - 9 < 7$  and  $3x + 9 \leq 25$

$2x < 16$  and  $3x \leq 16$

$x < 8$  and  $x \leq 5\frac{1}{3}$

$\therefore$  Solution set =  $\{x \leq 5\frac{1}{3}, x \in \mathbb{R}\}$

The required graph on number line is:



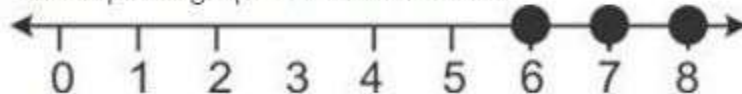
(ii)  $2x - 9 \leq 7$  and  $3x + 9 > 25$

$2x \leq 16$  and  $3x > 16$

$x \leq 8$  and  $x > 5\frac{1}{3}$

$\therefore$  Solution set =  $\{5\frac{1}{3} < x \leq 8, x \in \mathbb{I}\} = \{6, 7, 8\}$

The required graph on number line is:



(iii)  $x + 5 \geq 4(x - 1)$  and  $3 - 2x < -7$

$9 \geq 3x$  and  $-2x < -10$

$3 \geq x$  and  $x > 5$

$\therefore$  Solution set = Empty set

**Question 13.**

Solve and graph the solution set of:

(i)  $3x - 2 > 19$  or  $3 - 2x \geq -7, x \in \mathbb{R}$

(ii)  $5 > p - 1 > 2$  or  $7 \leq 2p - 1 \leq 17, p \in \mathbb{R}$

**Solution:**

(i)  $3x - 2 > 19$  or  $3 - 2x \geq -7$

$3x > 21$  or  $-2x \geq -10$

$x > 7$  or  $x \leq 5$

Graph of solution set of  $x > 7$  or  $x \leq 5$  = Graph of points which belong to  $x > 7$  or  $x \leq 5$  or both.

Thus, the graph of the solution set is:





(ii)  $5 > p - 1 > 2$  or  $7 \leq 2p - 1 \leq 17$

$6 > p > 3$  or  $8 \leq 2p \leq 18$

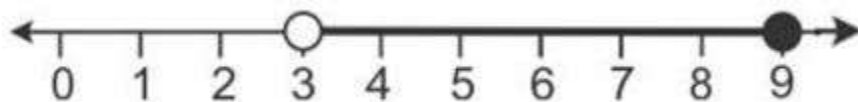
$6 > p > 3$  or  $4 \leq p \leq 9$

Graph of solution set of  $6 > p > 3$  or  $4 \leq p \leq 9$

= Graph of points which belong to  $6 > p > 3$  or  $4 \leq p \leq 9$  or both

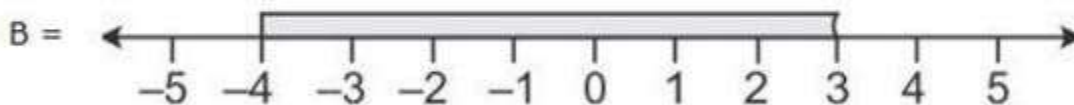
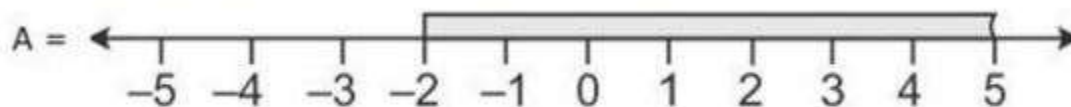
= Graph of points which belong to  $3 < p \leq 9$

Thus, the graph of the solution set is:



#### Question 14.

The diagram represents two inequations A and B on real number lines:



(i) Write down A and B in set builder notation.

(ii) Represent  $A \cup B$  and  $A \cap B'$  on two different number lines.

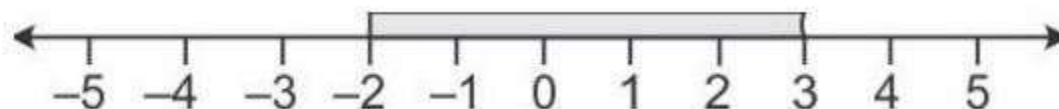
#### Solution:

(i)  $A = \{x \in \mathbb{R} : -2 \leq x < 5\}$

$B = \{x \in \mathbb{R} : -4 \leq x < 3\}$

(ii)  $A \cap B = \{x \in \mathbb{R} : -2 \leq x < 3\}$

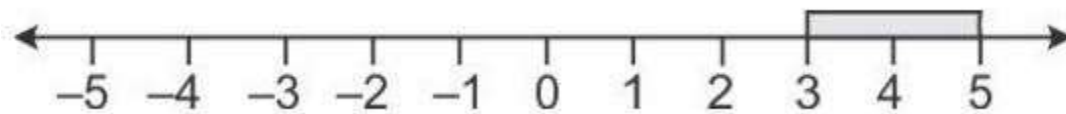
It can be represented on number line as:



$B' = \{x \in \mathbb{R} : 3 < x \leq -4\}$

$A \cap B' = \{x \in \mathbb{R} : 3 \leq x < 5\}$

It can be represented on number line as:



**Question 15.**

Use real number line to find the range of values of  $x$  for which:

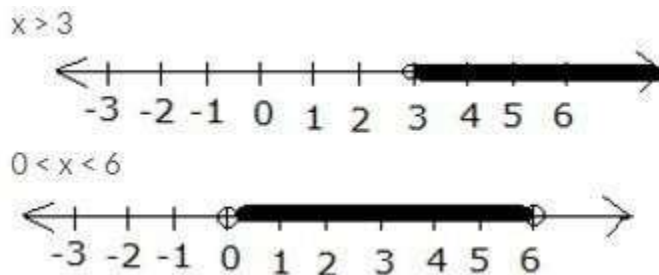
- (i)  $x > 3$  and  $0 < x < 6$
- (ii)  $x < 0$  and  $-3 \leq x < 1$
- (iii)  $-1 < x \leq 6$  and  $-2 \leq x \leq 3$

**Solution:**

- (i)  $x > 3$  and  $0 < x < 6$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

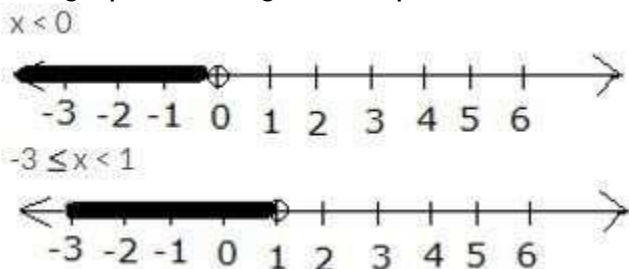


From both graphs, it is clear that their common range is  $3 < x < 6$

- (ii)  $x < 0$  and  $-3 \leq x < 1$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:

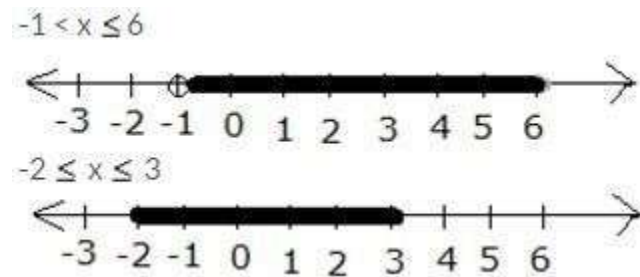


From both graphs, it is clear that their common range is  
 $-3 \leq x < 0$

(iii)  $-1 < x \leq 6$  and  $-2 \leq x \leq 3$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

The graphs of the given inequations can be drawn as:



From both graphs, it is clear that their common range is  
 $-1 < x \leq 3$

#### Question 16.

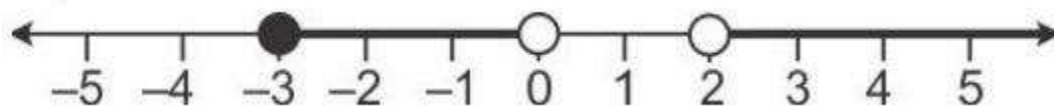
Illustrate the set  $\{x: -3 \leq x < 0 \text{ or } x > 2, x \in \mathbb{R}\}$  on the real number line.

#### Solution:

Graph of solution set of  $-3 \leq x < 0$  or  $x > 2$

= Graph of points which belong to  $-3 \leq x < 0$  or  $x > 2$  or both

Thus, the required graph is:



#### Question 17.

Given  $A = \{x: -1 < x \leq 5, x \in \mathbb{R}\}$  and  $B = \{x: -4 \leq x < 3, x \in \mathbb{R}\}$

Represent on different number lines:

(i)  $A \cap B$

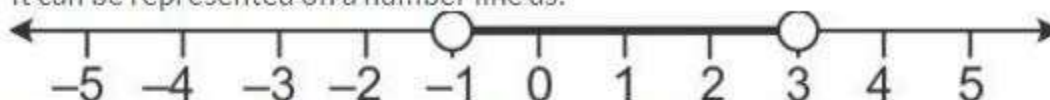
(ii)  $A' \cap B$

(iii)  $A - B$

**Solution:**

(i)  $A \cap B = \{x: -1 < x < 3, x \in \mathbb{R}\}$

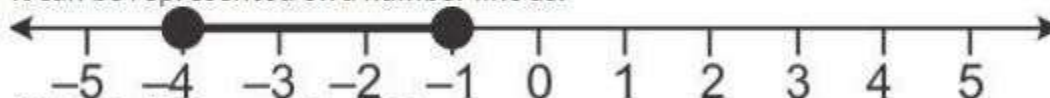
It can be represented on a number line as:



(ii) Numbers which belong to B but do not belong to A = B - A

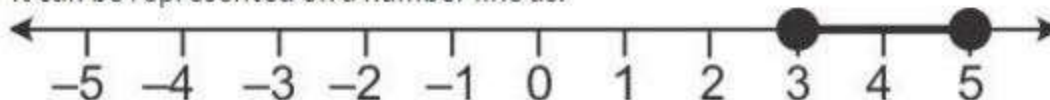
$A' \cap B = \{x: -4 \leq x \leq -1, x \in \mathbb{R}\}$

It can be represented on a number line as:



(iii)  $A - B = \{x: 3 \leq x \leq 5, x \in \mathbb{R}\}$

It can be represented on a number line as:



**Question 18.**

P is the solution set of  $7x - 2 > 4x + 1$  and Q is the solution set of  $9x - 45 \geq 5(x - 5)$ ; where  $x \in \mathbb{R}$ . Represent:

(i)  $P \cap Q$

(ii)  $P - Q$

(iii)  $P \cap Q'$

on different number lines.

**Solution:**

$P = \{x: 7x - 2 > 4x + 1, x \in \mathbb{R}\}$

$7x - 2 > 4x + 1$

$7x - 4x > 1 + 2$

$3x > 3$

$x > 1$

and

$Q = \{x: 9x - 45 \geq 5(x - 5), x \in \mathbb{R}\}$

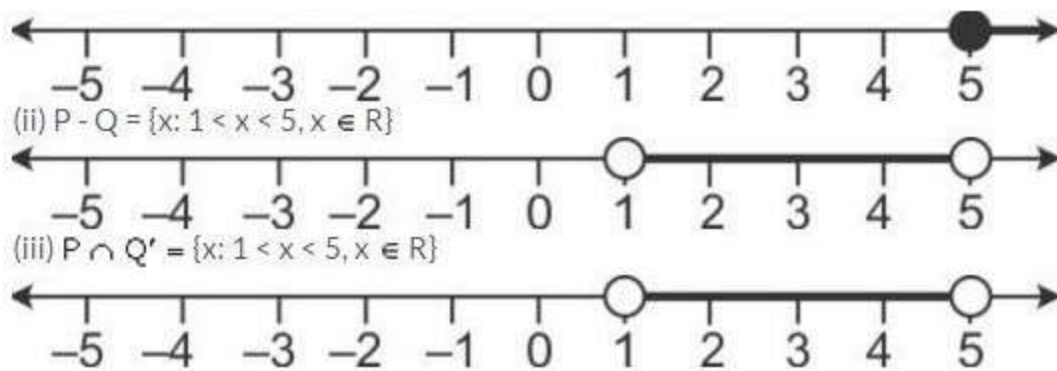
$9x - 45 \geq 5x - 25$

$9x - 5x \geq -25 + 45$

$4x \geq 20$

$x \geq 5$

(i)  $P \cap Q = \{x: x \geq 5, x \in \mathbb{R}\}$



### Question 19.

Find the range of values of  $x$ , which satisfy:

$$-\frac{1}{3} \leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6}$$

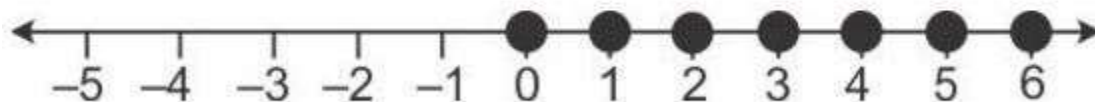
Graph, in each of the following cases, the values of  $x$  on the different real number lines:

(i)  $x \in \mathbb{W}$  (ii)  $x \in \mathbb{Z}$  (iii)  $x \in \mathbb{R}$

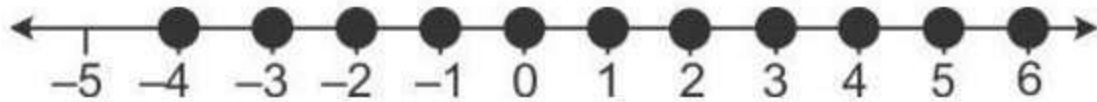
### Solution:

$$\begin{aligned}
 -\frac{1}{3} &\leq \frac{x}{2} + 1\frac{2}{3} < 5\frac{1}{6} \\
 -\frac{1}{3} - \frac{5}{3} &\leq \frac{x}{2} < \frac{31}{6} - \frac{5}{3} \\
 -\frac{6}{3} &\leq \frac{x}{2} < \frac{21}{6} \\
 -4 &\leq x < 7
 \end{aligned}$$

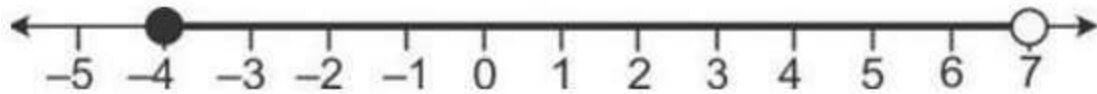
(i) If  $x \in \mathbb{W}$ , range of values of  $x$  is  $\{0, 1, 2, 3, 4, 5, 6\}$ .



(ii) If  $x \in \mathbb{Z}$ , range of values of  $x$  is  $\{-4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$ .



(iii) If  $x \in \mathbb{R}$ , range of values of  $x$  is  $-4 \leq x < 7$ .



### Question 20.

Given:  $A = \{x: -8 < 5x + 2 \leq 17, x \in \mathbb{I}\}$ ,  $B = \{x: -2 \leq 7 + 3x < 17, x \in \mathbb{R}\}$

Where  $\mathbb{R} = \{\text{real numbers}\}$  and  $\mathbb{I} = \{\text{integers}\}$ . Represent  $A$  and  $B$  on two different number lines. Write down the elements of  $A \cap B$ .

**Solution:**

$$A = \{x: -8 < 5x + 2 \leq 17, x \in \mathbb{I}\}$$

$$= \{x: -10 < 5x \leq 15, x \in \mathbb{I}\}$$

$$= \{x: -2 < x \leq 3, x \in \mathbb{I}\}$$

It can be represented on number line as:

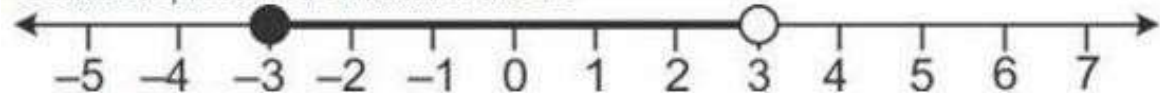


$$B = \{x: -2 \leq 7 + 3x < 17, x \in \mathbb{R}\}$$

$$= \{x: -9 \leq 3x < 10, x \in \mathbb{R}\}$$

$$= \{x: -3 \leq x < 3.33, x \in \mathbb{R}\}$$

It can be represented on number line as:



$$A \cap B = \{-1, 0, 1, 2, 3\}$$

### Question 21.

Solve the following inequation and represent the solution set on the number line  $2x - 5 \leq 5x + 4 < 11$ , where  $x \in \mathbb{I}$

**Solution:**

$$2x - 5 \leq 5x + 4 \text{ and } 5x + 4 < 11$$

$$2x - 5x \leq 4 - 5 \text{ and } 5x < 11 - 4$$

$$3x \leq -1 \text{ and } 5x < 7$$

$$x \geq -1 \text{ and } x < \frac{7}{5}$$

$$x \geq -1 \text{ and } x < 1.4$$

Since  $x \in I$ , the solution set is  $\{-3, -2, -1, 0, 1\}$

And the number line representation is



### Question 22.

Given that  $x \in I$ , solve the inequation and graph the solution on the number line:

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

**Solution:**

$$3 \geq \frac{x-4}{2} + \frac{x}{3} \geq 2$$

$$3 \geq \frac{3x-12+2x}{6} \geq 2$$

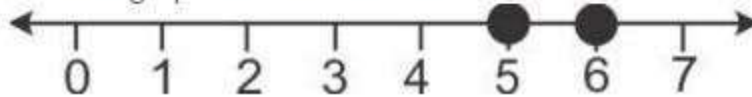
$$18 \geq 5x - 12 \geq 12$$

$$30 \geq 5x \geq 24$$

$$6 \geq x \geq 4.8$$

Solution set =  $\{5, 6\}$

It can be graphed on number line as:



### Question 23.

Given:

$$A = \{x: 11x - 5 > 7x + 3, x \in R\} \text{ and}$$

$$B = \{x: 18x - 9 \geq 15 + 12x, x \in R\}.$$

Find the range of set  $A \cap B$  and represent it on number line.

**Solution:**

$$A = \{x: 11x - 5 > 7x + 3, x \in R\}$$

$$= \{x: 4x > 8, x \in R\}$$

$$= \{x: x > 2, x \in \mathbb{R}\}$$

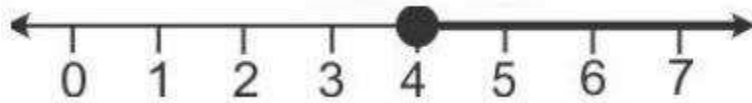
$$B = \{x: 18x - 9 \geq 15 + 12x, x \in \mathbb{R}\}$$

$$= \{x: 6x \geq 24, x \in \mathbb{R}\}$$

$$= \{x: x \geq 4, x \in \mathbb{R}\}$$

$$A \cap B = \{x: x \geq 4, x \in \mathbb{R}\}$$

It can be represented on number line as:



#### Question 24.

Find the set of values of  $x$ , satisfying:

$$7x + 3 \geq 3x - 5 \text{ and } \frac{x}{4} - 5 \leq \frac{5}{4} - x, \text{ where } x \in \mathbb{N}.$$

**Solution:**

$$7x + 3 \geq 3x - 5$$

$$4x \geq -8$$

$$x \geq -2$$

$$\frac{x}{4} - 5 \leq \frac{5}{4} - x$$

$$\frac{x}{4} + x \leq \frac{5}{4} + 5$$

$$\frac{5x}{4} \leq \frac{25}{4}$$

$$x \leq 5$$

Since,  $x \in \mathbb{N}$

$\therefore$  Solution set =  $\{1, 2, 3, 4, 5\}$

#### Question 25.

Solve:

(i)  $\frac{x}{2} + 5 \leq \frac{x}{3} + 6$ , where  $x$  is a positive odd integer.

(ii)  $\frac{2x+3}{3} \geq \frac{3x-1}{4}$ , where  $x$  is a positive even integer.



**Solution:**

$$(i) \frac{x}{2} + 5 \leq \frac{x}{3} + 6$$

$$\frac{x}{2} - \frac{x}{3} \leq 6 - 5$$

$$\frac{x}{6} \leq 1$$

$$x \leq 6$$

Since, x is a positive odd integer

$\therefore$  Solution set = {1, 3, 5}

$$(ii) \frac{2x + 3}{3} \geq \frac{3x - 1}{4}$$

$$8x + 12 \geq 9x - 3$$

$$-x \geq -15$$

$$x \leq 15$$

Since, x is a positive even integer

$\therefore$  Solution set = {2, 4, 6, 8, 10, 12, 14}

### Question 26.

Solve the inequation:

$$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x, x \in \mathbb{W}. \text{ Graph the solution set on the number line.}$$

**Solution:**

$$-2\frac{1}{2} + 2x \leq \frac{4x}{5} \leq \frac{4}{3} + 2x$$

$$-2\frac{1}{2} \leq \frac{4x}{5} - 2x \leq \frac{4}{3}$$

$$-\frac{5}{2} \leq -\frac{6x}{5} \leq \frac{4}{3}$$

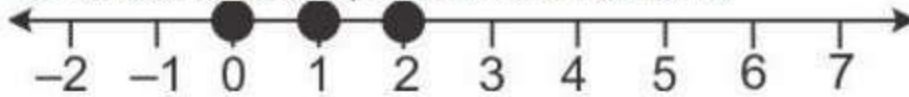
$$\frac{25}{12} \geq x \geq -\frac{10}{9}$$

$$2.083 \geq x \geq -1.111$$

Since,  $x \in W$

$\therefore$  Solution set =  $\{0, 1, 2\}$

The solution set can be represented on number line as:



### Question 27.

Find three consecutive largest positive integers such that the sum of one-third of first, one-fourth of second and one-fifth of third is atmost 20.

### Solution:

Let the required integers be  $x$ ,  $x + 1$  and  $x + 2$ .

According to the given statement,

$$\frac{1}{3}x + \frac{1}{4}(x + 1) + \frac{1}{5}(x + 2) \leq 20$$

$$\frac{20x + 15x + 15 + 12x + 24}{60} \leq 20$$

$$47x + 39 \leq 1200$$

$$47x \leq 1161$$

$$x \leq 24.702$$

Thus, the largest value of the positive integer  $x$  is 24.

Hence, the required integers are 24, 25 and 26.

### Question 28.

Solve the given inequation and graph the solution on the number line.

$$2y - 3 < y + 1 \leq 4y + 7, y \in R$$

### Solution:

$$2y - 3 < y + 1 \leq 4y + 7, y \in R$$

$$\Rightarrow 2y - 3 - y < y + 1 - y \leq 4y + 7 - y$$

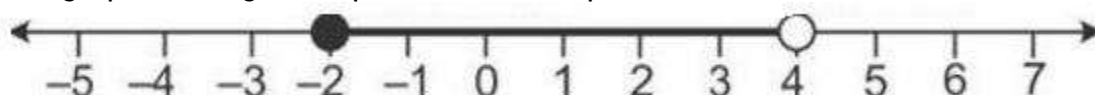
$$\Rightarrow y - 3 < 1 \leq 3y + 7$$

$$\Rightarrow y - 3 < 1 \text{ and } 1 \leq 3y + 7$$

$$\Rightarrow y < 4 \text{ and } 3y \geq 6 \Rightarrow y \geq -2$$

$$\Rightarrow -2 \leq y < 4$$

The graph of the given equation can be represented on a number line as:



**Question 29.**

Solve the inequation:

$$3z - 5 \leq z + 3 < 5z - 9, z \in \mathbb{R}.$$

Graph the solution set on the number line.

**Solution:**

$$3z - 5 \leq z + 3 < 5z - 9$$

$$3z - 5 \leq z + 3 \text{ and } z + 3 < 5z - 9$$

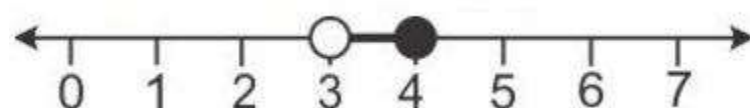
$$2z \leq 8 \text{ and } 12 < 4z$$

$$z \leq 4 \text{ and } 3 < z$$

Since,  $z \in \mathbb{R}$ 

$$\therefore \text{Solution set} = \{3 < z \leq 4, z \in \mathbb{R}\}$$

It can be represented on a number line as:

**Question 30.**

Solve the following inequation and represent the solution set on the number line.

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}, x \in \mathbb{R}$$

**Solution:**

$$-3 < -\frac{1}{2} - \frac{2x}{3} \leq \frac{5}{6}$$

Multiply by 6, we get

$$\Rightarrow -18 < -3 - 4x \leq 5$$

$$\Rightarrow -15 < -4x \leq 8$$

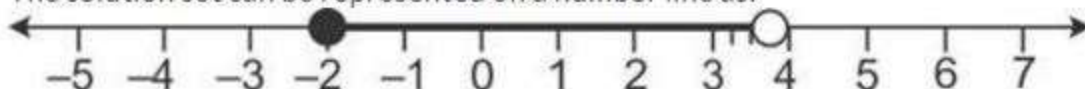
Dividing by  $-4$ , we get

$$\Rightarrow \frac{-15}{-4} > x \geq \frac{8}{-4}$$

$$\Rightarrow -2 \leq x < \frac{15}{4}$$

$$\Rightarrow x \in \left[-2, \frac{15}{4}\right)$$

The solution set can be represented on a number line as:



**Question 31.**

Solve the following inequation and represent the solution set on the number line:

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in \mathbb{R}$$

**Solution:**

Consider the given inequation:

$$4x - 19 < \frac{3x}{5} - 2 \leq \frac{-2}{5} + x, x \in \mathbb{R}$$

$$\Rightarrow 4x - 19 + 2 < \frac{3x}{5} - 2 + 2 \leq \frac{-2}{5} + x + 2, x \in \mathbb{R}$$

$$\Rightarrow 4x - 17 < \frac{3x}{5} \leq x + \frac{8}{5}, x \in \mathbb{R}$$

$$\Rightarrow 4x - \frac{3x}{5} < 17 \text{ and } \frac{-8}{5} \leq x - \frac{3x}{5}, x \in \mathbb{R}$$

$$\Rightarrow \frac{20x - 3x}{5} < 17 \text{ and } \frac{-8}{5} \leq \frac{5x - 3x}{5}, x \in \mathbb{R}$$

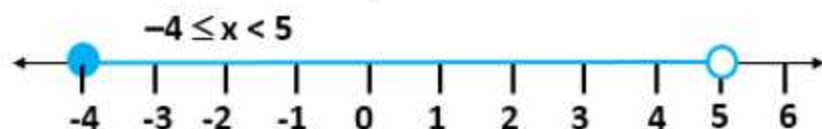
$$\Rightarrow \frac{17x}{5} < 17 \text{ and } \frac{-8}{5} \leq \frac{2x}{5}, x \in \mathbb{R}$$

$$\Rightarrow \frac{x}{5} < 1 \text{ and } -4 \leq x, x \in \mathbb{R}$$

$$\Rightarrow x < 5 \text{ and } -4 \leq x, x \in \mathbb{R}$$

$$\Rightarrow -4 \leq x < 5; \text{ where } x \in \mathbb{R}$$

The solution set can be represented on a number line as follows:

**Question 32.**

Solve the following in equation, write the solution set and represent it on the number line:

$$-\frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}, x \in \mathbb{R}$$

**Solution:**

The given inequation is

$$-\frac{x}{3} \leq \frac{x}{2} - 1 \frac{1}{3} < \frac{1}{6}, x \in \mathbb{R}$$

$$\Rightarrow -\frac{x}{3} \leq \frac{x}{2} - \frac{4}{3} < \frac{1}{6}$$

Now,

$$\begin{array}{ll} -\frac{x}{3} \leq \frac{x}{2} - \frac{4}{3} & \frac{x}{2} - \frac{4}{3} < \frac{1}{6} \\ \Rightarrow -\frac{x}{3} - \frac{x}{2} \leq -\frac{4}{3} & \Rightarrow \frac{x}{2} < \frac{1}{6} + \frac{4}{3} \\ \Rightarrow \frac{2x + 3x}{6} \geq \frac{4}{3} & \Rightarrow \frac{x}{2} < \frac{1 + 4 \times 2}{6} \\ \Rightarrow \frac{5x}{6} \geq \frac{4}{3} & \Rightarrow \frac{x}{2} < \frac{1 + 8}{6} \\ \Rightarrow 5x \geq 8 & \Rightarrow \frac{x}{2} < \frac{9}{6} \\ \Rightarrow x \geq \frac{8}{5} & \Rightarrow \frac{x}{2} < \frac{3}{2} \\ \Rightarrow x \geq 1.6 & \Rightarrow x < 3 \end{array}$$

$$\therefore \text{Solution set} = \{x : 1.6 \leq x < 3\}$$

It can be represented on a number line as follows :

**Question 33.**

Find the values of  $x$ , which satisfy the inequation

$$-2 \frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in \mathbb{W},$$

Graph the solution set on the number line.

**Solution:**

We need to find the values of  $x$ , such that

$x$  satisfies the inequation  $-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2, x \in W$

Consider the given inequation:

$$-2\frac{5}{6} < \frac{1}{2} - \frac{2x}{3} \leq 2$$

$$\Rightarrow \frac{-17}{6} < \frac{3-4x}{6} \leq \frac{12}{6}$$

$$\Rightarrow \frac{17}{6} > \frac{4x-3}{6} \geq \frac{-12}{6}$$

$$\Rightarrow 17 > 4x - 3 \geq -12$$

$$\Rightarrow -12 \leq 4x - 3 < 17$$

$$\Rightarrow -12 + 3 \leq 4x - 3 + 3 < 17 + 3$$

$$\Rightarrow -9 \leq 4x < 20$$

$$\Rightarrow -\frac{9}{4} \leq \frac{4x}{4} < \frac{20}{4}$$

$$\Rightarrow -\frac{9}{4} \leq x < 5$$

Since  $x \in W$ , the values of  $x$  are 0, 1, 2, 3, 4.

And the required line is



#### Question 34.

Solve the following in equation and write the solution set:

$$13x - 5 < 15x + 4 < 7x + 12, x \in R$$

**Solution:**

$$13x - 5 < 15x + 4 < 7x + 12, x \in R$$

We have

$$13x - 5 < 15x + 4 \quad \text{and} \quad 15x + 4 < 7x + 12$$

$$\Rightarrow 13x < 15x + 9 \quad \Rightarrow 15x < 7x + 8$$

$$\Rightarrow 0 < 2x + 9 \quad \Rightarrow 8x < 8$$

$$\Rightarrow -9 < 2x \quad \Rightarrow x < 1$$

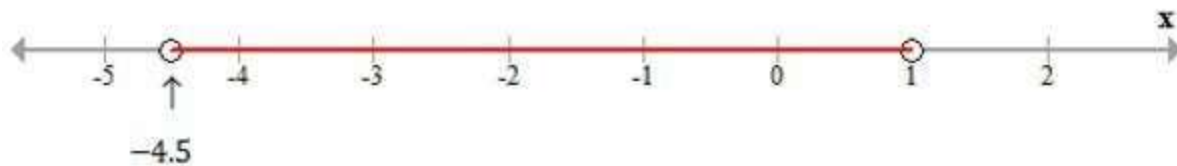
$$\Rightarrow -\frac{9}{2} < x \quad \Rightarrow x < 1$$

$$\therefore -\frac{9}{2} < x < 1$$

$$\text{i.e. } -4.5 < x < 1$$

$$\therefore \text{Solution set} = \{x : x \in \mathbb{R} \text{ and } -4.5 < x < 1\}$$

The required line is



### Question 35.

Solve the following inequation, write the solution set and represent it on the number line.

$$-3(x - 7) \geq 15 - 7x > x + \frac{1}{3}, x \in \mathbb{R}.$$

**Solution:**

$$-3(x - 7) \geq 15 - 7x > \frac{x+1}{3}, x \in \mathbb{R}$$

$$\Rightarrow -3(x - 7) \geq 15 - 7x \text{ and } 15 - 7x > \frac{x+1}{3}$$

$$\Rightarrow -3x + 21 \geq 15 - 7x \text{ and } 45 - 21x > x + 1$$

$$\Rightarrow -3x + 7x \geq 15 - 21 \text{ and } 45 - 1 > x + 21x$$

$$\Rightarrow 4x \geq -6 \text{ and } 44 > 22x$$

$$\Rightarrow x \geq \frac{-3}{2} \text{ and } 2 > x$$

$$\Rightarrow x \geq -1.5 \text{ and } 2 > x$$

$$\therefore \text{The solution set is } \{x : x \in \mathbb{R}, -1.5 \leq x < 2\}.$$

The solution set is represented on number line as follows:



### Question 36.

Solve the following inequation and represent the solution set on a number line.

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, \quad x \in \mathbb{I}$$

**Solution:**

$$-8\frac{1}{2} < -\frac{1}{2} - 4x \leq 7\frac{1}{2}, \quad x \in \mathbb{I}$$

$$-8\frac{1}{2} < -\frac{1}{2} - 4x$$

$$\Rightarrow -\frac{15}{2} < -\frac{1}{2} - 4x$$

$$\Rightarrow -\frac{15}{2} + \frac{1}{2} < -4x$$

$$\Rightarrow -\frac{14}{2} < -4x$$

$$\Rightarrow -7 < -4x$$

$$\Rightarrow 7 > 4x$$

$$\Rightarrow x < \frac{7}{4}$$

$$-\frac{1}{2} - 4x \leq 7\frac{1}{2}$$

$$\Rightarrow -\frac{1}{2} - 4x \leq \frac{15}{2}$$

$$\Rightarrow -4x \leq \frac{15}{2} + \frac{1}{2}$$

$$\Rightarrow -4x \leq 8$$

$$\Rightarrow x \geq -2$$

So,

$$\frac{7}{4} > x \geq -2$$

As,  $x \in \mathbb{I}$

$$x = \{-2, -1, 0, 1\}$$

