

Remainder and Factor Theorems

Question 1.

Find, in each case, the remainder when:

(i) $x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.

(ii) $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$.

(iii) $x^4 + 1$ is divided by $x + 1$.

Solution:

By remainder theorem we know that when a polynomial $f(x)$ is divided by $x - a$, then the remainder is $f(a)$.

$$(i) f(x) = x^4 - 3x^2 + 2x + 1$$

$$\text{Remainder} = f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$$

$$(ii) f(x) = x^3 + 3x^2 - 12x + 4$$

$$\begin{aligned}\text{Remainder} &= f(2) = (2)^3 + 3(2)^2 - 12(2) + 4 \\ &= 8 + 12 - 24 + 4 \\ &= 0\end{aligned}$$

$$(iii) f(x) = x^4 + 1$$

$$\text{Remainder} = f(-1) = (-1)^4 + 1 = 1 + 1 = 2$$

Question 2.

Show that:

(i) $x - 2$ is a factor of $5x^2 + 15x - 50$.

(ii) $3x + 2$ is a factor of $3x^2 - x - 2$.

Solution:

$(x - a)$ is a factor of a polynomial $f(x)$ if the remainder, when $f(x)$ is divided by $(x - a)$, is

0, i.e., if $f(a) = 0$.

$$(i) f(x) = 5x^2 + 15x - 50$$

$$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$$

Hence, $x - 2$ is a factor of $5x^2 + 15x - 50$.

$$(ii) f(x) = 3x^2 - x - 2$$

$$f\left(\frac{-2}{3}\right) = 3\left(\frac{-2}{3}\right)^2 - \left(\frac{-2}{3}\right) - 2 = \frac{4}{3} + \frac{2}{3} - 2 = 2 - 2 = 0$$

Hence, $3x + 2$ is a factor of $3x^2 - x - 2$.

Question 3.

Use the Remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.

(i) $x + 1$

(ii) $2x - 1$

(iii) $x + 2$

Solution:

By remainder theorem we know that when a polynomial $f(x)$ is divided by $x - a$, then the remainder is $f(a)$.

$$\text{Let } f(x) = 2x^3 + 3x^2 - 5x - 6$$

$$(i) f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$$

Thus, $(x + 1)$ is a factor of the polynomial $f(x)$.

(ii)

$$\begin{aligned} f\left(\frac{1}{2}\right) &= 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) - 6 \\ &= \frac{1}{4} + \frac{3}{4} - \frac{5}{2} - 6 \\ &= -\frac{5}{2} - 5 = \frac{-15}{2} \neq 0 \end{aligned}$$

Thus, $(2x - 1)$ is not a factor of the polynomial $f(x)$.

$$(iii) f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$$

Thus, $(x + 2)$ is a factor of the polynomial $f(x)$.

Question 4.

- (i) If $2x + 1$ is a factor of $2x^2 + ax - 3$, find the value of a .
(ii) Find the value of k , if $3x - 4$ is a factor of expression $3x^2 + 2x - k$.

Solution:

(i) $2x + 1$ is a factor of $f(x) = 2x^2 + ax - 3$.

$$\therefore f\left(\frac{-1}{2}\right) = 0$$

$$\Rightarrow 2\left(\frac{-1}{2}\right)^2 + a\left(\frac{-1}{2}\right) - 3 = 0$$

$$\Rightarrow \frac{1}{2} - \frac{a}{2} = 3$$

$$\Rightarrow 1 - a = 6$$

$$\Rightarrow a = -5$$

(ii) $3x - 4$ is a factor of $g(x) = 3x^2 + 2x - k$.

$$\therefore f\left(\frac{4}{3}\right) = 0$$

$$\Rightarrow 3\left(\frac{4}{3}\right)^2 + 2\left(\frac{4}{3}\right) - k = 0$$

$$\Rightarrow \frac{16}{3} + \frac{8}{3} - k = 0$$

$$\Rightarrow \frac{24}{3} = k$$

$$\Rightarrow k = 8$$

Question 5.

Find the values of constants a and b when $x - 2$ and $x + 3$ both are the factors of expression $x^3 + ax^2 + bx - 12$.

Solution:

$$\text{Let } f(x) = x^3 + ax^2 + bx - 12$$

$$x - 2 = 0 \Rightarrow x = 2$$

$x - 2$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (2)^3 + a(2)^2 + b(2) - 12 = 0$$

$$\Rightarrow 8 + 4a + 2b - 12 = 0$$

$$\Rightarrow 4a + 2b - 4 = 0$$

$$\Rightarrow 2a + b - 2 = 0 \quad \dots(1)$$

$$x + 3 = 0 \Rightarrow x = -3$$

$x + 3$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (-3)^3 + a(-3)^2 + b(-3) - 12 = 0$$

$$\Rightarrow -27 + 9a - 3b - 12 = 0$$

$$\Rightarrow 9a - 3b - 39 = 0$$

$$\Rightarrow 3a - b - 13 = 0 \quad \dots(2)$$

Adding (1) and (2), we get,

$$5a - 15 = 0$$

$$\Rightarrow a = 3$$

Putting the value of a in (1), we get,

$$6 + b - 2 = 0$$

$$\Rightarrow b = -4$$

Question 6.

find the value of k , if $2x + 1$ is a factor of $(3k + 2)x^3 + (k - 1)$.

Solution:

$$\text{Let } f(x) = (3k + 2)x^3 + (k - 1)$$

$$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$$

Since, $2x + 1$ is a factor of $f(x)$, remainder is 0.

$$\therefore (3k + 2)\left(-\frac{1}{2}\right)^3 + (k - 1) = 0$$

$$\Rightarrow \frac{-(3k + 2)}{8} + (k - 1) = 0$$

$$\Rightarrow \frac{-3k - 2 + 8k - 8}{8} = 0$$

$$\Rightarrow 5k - 10 = 0$$

$$\Rightarrow k = 2$$

Question 7.

Find the value of a , if $x - 2$ is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.

Solution:

$$f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$$

$$x - 2 = 0 \Rightarrow x = 2$$

Since, $x - 2$ is a factor of $f(x)$, remainder = 0.

$$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$$

$$64 - 96 - 16a + 24a + 8a + 8 = 0$$

$$-24 + 16a = 0$$

$$16a = 24$$

$$a = 1.5$$

Question 8.

Find the values of m and n so that $x - 1$ and $x + 2$ both are factors of $x^3 + (3m + 1)x^2 + nx - 18$.

Solution:

$$\text{Let } f(x) = x^3 + (3m + 1)x^2 + nx - 18$$

$$x - 1 = 0 \Rightarrow x = 1$$

$x - 1$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (1)^3 + (3m + 1)(1)^2 + n(1) - 18 = 0$$

$$\Rightarrow 1 + 3m + 1 + n - 18 = 0$$

$$\Rightarrow 3m + n - 16 = 0 \quad \dots(1)$$

$$x + 2 = 0 \Rightarrow x = -2$$

$x + 2$ is a factor of $f(x)$. So, remainder = 0

$$\therefore (-2)^3 + (3m + 1)(-2)^2 + n(-2) - 18 = 0$$

$$\Rightarrow -8 + 12m + 4 - 2n - 18 = 0$$

$$\Rightarrow 12m - 2n - 22 = 0$$

$$\Rightarrow 6m - n - 11 = 0 \quad \dots(2)$$

Adding (1) and (2), we get,

$$9m - 27 = 0$$

$$m = 3$$

Putting the value of m in (1), we get,

$$3(3) + n - 16 = 0$$

$$9 + n - 16 = 0$$

$$n = 7$$

Question 9.

When $x^3 + 2x^2 - kx + 4$ is divided by $x - 2$, the remainder is k . Find the value of constant k .

Solution:

$$\text{Let } f(x) = x^3 + 2x^2 - kx + 4$$

$$x - 2 = 0 \Rightarrow x = 2$$

On dividing $f(x)$ by $x - 2$, it leaves a remainder k .

$$\therefore f(2) = k$$

$$(2)^3 + 2(2)^2 - k(2) + 4 = k$$

$$8 + 8 - 2k + 4 = k$$

$$20 = 3k$$

$$k = \frac{20}{3} = 6\frac{2}{3}$$

Question 10.

Find the value of a , if the division of $ax^3 + 9x^2 + 4x - 10$ by $x + 3$ leaves a remainder 5.

Solution:

$$\text{Let } f(x) = ax^3 + 9x^2 + 4x - 10$$

$$x + 3 = 0 \Rightarrow x = -3$$

On dividing $f(x)$ by $x + 3$, it leaves a remainder 5.

$$\therefore f(-3) = 5$$

$$a(-3)^3 + 9(-3)^2 + 4(-3) - 10 = 5$$

$$-27a + 81 - 12 - 10 = 5$$

$$54 = 27a$$

$$a = 2$$

Question 11.

If $x^3 + ax^2 + bx + 6$ has $x - 2$ as a factor and leaves a remainder 3 when divided by $x - 3$, find the values of a and b .

Solution:

$$\text{Let } f(x) = x^3 + ax^2 + bx + 6$$

$$x - 2 = 0 \Rightarrow x = 2$$

Since, $x - 2$ is a factor, remainder = 0

$$\therefore f(2) = 0$$

$$(2)^3 + a(2)^2 + b(2) + 6 = 0$$

$$8 + 4a + 2b + 6 = 0$$

$$2a + b + 7 = 0 \quad \dots(i)$$

$$x - 3 = 0 \Rightarrow x = 3$$

On dividing $f(x)$ by $x - 3$, it leaves a remainder 3.

$$\therefore f(3) = 3$$

$$(3)^3 + a(3)^2 + b(3) + 6 = 3$$

$$27 + 9a + 3b + 6 = 3$$

$$3a + b + 10 = 0 \quad \dots(ii)$$

Subtracting (i) from (ii), we get,

$$a + 3 = 0$$

$$a = -3$$

Substituting the value of a in (i), we get,

$$-6 + b + 7 = 0$$

$$b = -1$$

Question 12.

The expression $2x^3 + ax^2 + bx - 2$ leaves remainder 7 and 0 when divided by $2x - 3$ and $x + 2$ respectively. Calculate the values of a and b .

Solution:

$$\text{Let } f(x) = 2x^3 + ax^2 + bx - 2$$

$$2x - 3 = 0 \Rightarrow x = \frac{3}{2}$$

On dividing $f(x)$ by $2x - 3$, it leaves a remainder 7.

$$\therefore 2\left(\frac{3}{2}\right)^3 + a\left(\frac{3}{2}\right)^2 + b\left(\frac{3}{2}\right) - 2 = 7$$

$$\frac{27}{4} + \frac{9a}{4} + \frac{3b}{2} = 9$$

$$\frac{27 + 9a + 6b}{4} = 9$$

$$27 + 9a + 6b = 36$$

$$9a + 6b - 9 = 0$$

$$3a + 2b - 3 = 0 \quad \dots(i)$$

$$x + 2 = 0 \Rightarrow x = -2$$

On dividing $f(x)$ by $x + 2$, it leaves a remainder 0.

$$\therefore 2(-2)^3 + a(-2)^2 + b(-2) - 2 = 0$$

$$-16 + 4a - 2b - 2 = 0$$

$$4a - 2b - 18 = 0 \quad \dots(ii)$$

Adding (i) and (ii), we get,

$$7a - 21 = 0$$

$$a = 3$$

Substituting the value of a in (i), we get,

$$3(3) + 2b - 3 = 0$$

$$9 + 2b - 3 = 0$$

$$2b = -6$$

$$b = -3$$

Question 13.

What number should be added to $3x^3 - 5x^2 + 6x$ so that when resulting polynomial is divided by $x - 3$, the remainder is 8?

Solution:

Let the number k be added and the resulting polynomial be $f(x)$.

$$\text{So, } f(x) = 3x^3 - 5x^2 + 6x + k$$

It is given that when $f(x)$ is divided by $(x - 3)$, the remainder is 8.

$$\therefore f(3) = 8$$

$$3(3)^3 - 5(3)^2 + 6(3) + k = 8$$

$$81 - 45 + 18 + k = 8$$

$$54 + k = 8$$

$$k = -46$$

Thus, the required number is -46.

Question 14.

What number should be subtracted from $x^3 + 3x^2 - 8x + 14$ so that on dividing it with $x - 2$, the remainder is 10.

Solution:

Let the number to be subtracted be k and the resulting polynomial be $f(x)$.

$$\text{So, } f(x) = x^3 + 3x^2 - 8x + 14 - k$$

It is given that when $f(x)$ is divided by $(x - 2)$, the remainder is 10.

$$\therefore f(2) = 10$$

$$(2)^3 + 3(2)^2 - 8(2) + 14 - k = 10$$

$$8 + 12 - 16 + 14 - k = 10$$

$$18 - k = 10$$

$$k = 8$$

Thus, the required number is 8.

Question 15.

The polynomials $2x^3 - 7x^2 + ax - 6$ and $x^3 - 8x^2 + (2a + 1)x - 16$ leaves the same remainder when divided by $x - 2$. Find the value of 'a'.

Solution:

$$\text{Let } f(x) = 2x^3 - 7x^2 + ax - 6$$

$$x - 2 = 0 \Rightarrow x = 2$$

When $f(x)$ is divided by $(x - 2)$, remainder = $f(2)$

$$\therefore f(2) = 2(2)^3 - 7(2)^2 + a(2) - 6$$

$$= 16 - 28 + 2a - 6$$

$$= 2a - 18$$

$$\text{Let } g(x) = x^3 - 8x^2 + (2a + 1)x - 16$$

When $g(x)$ is divided by $(x - 2)$, remainder = $g(2)$

$$\therefore g(2) = (2)^3 - 8(2)^2 + (2a + 1)(2) - 16$$

$$= 8 - 32 + 4a + 2 - 16$$

$$= 4a - 38$$

By the given condition, we have:

$$f(2) = g(2)$$

$$2a - 18 = 4a - 38$$

$$4a - 2a = 38 - 18$$

$$2a = 20$$

$$a = 10$$

Thus, the value of a is 10.

Question 16.

If $(x - 2)$ is a factor of the expression $2x^3 + ax^2 + bx - 14$ and when the expression is divided by $(x - 3)$, it leaves a remainder 52, find the values of a and b

Solution:

Since $(x - 2)$ is a factor of polynomial $2x^3 + ax^2 + bx - 14$, we have

$$2(2)^3 + a(2)^2 + b(2) - 14 = 0$$

$$\Rightarrow 16 + 4a + 2b - 14 = 0$$

$$\Rightarrow 4a + 2b + 2 = 0$$

$$\Rightarrow 2a + b + 1 = 0$$

$$\Rightarrow 2a + b = -1 \quad \dots(i)$$

On dividing by $(x - 3)$, the polynomial $2x^3 + ax^2 + bx - 14$ leaves remainder 52,

$$\Rightarrow 2(3)^3 + a(3)^2 + b(3) - 14 = 52$$

$$\Rightarrow 54 + 9a + 3b - 14 = 52$$

$$\Rightarrow 9a + 3b + 40 = 52$$

$$\Rightarrow 9a + 3b = 12$$

$$\Rightarrow 3a + b = 4 \quad \dots(ii)$$

Subtracting (i) from (ii), we get

$$a = 5$$

Substituting $a = 5$ in (i), we get

$$2 \times 5 + b = -1$$

$$\Rightarrow 10 + b = -1$$

$$\Rightarrow b = -11$$

Hence, $a = 5$ and $b = -11$.

Question 17.

Find 'a' if the two polynomials $ax^3 + 3x^2 - 9$ and $2x^3 + 4x + a$, leave the same remainder when divided by $x + 3$.

Solution:

$$x + 3 = 0 \Rightarrow x = -3$$

Since, the given polynomials leave the same remainder when divided by $(x + 3)$,

Value of polynomial $ax^3 + 3x^2 - 9$ at $x = -3$ is same as value of polynomial $2x^3 + 4x + a$ at $x = -3$.

$$\begin{aligned}
&\Rightarrow a(-3)^3 + 3(-3)^2 - 9 = 2(-3)^3 + 4(-3) + a \\
&\Rightarrow -27a + 27 - 9 = -54 - 12 + a \\
&\Rightarrow -27a + 18 = -66 + a \\
&\Rightarrow 28a = 84 \\
&\Rightarrow a = \frac{84}{28} \\
&\Rightarrow a = 3
\end{aligned}$$

Exercise 8B

Question 1.

Using the Factor Theorem, show that:

- (i) $(x - 2)$ is a factor of $x^3 - 2x^2 - 9x + 18$. Hence, factorise the expression $x^3 - 2x^2 - 9x + 18$ completely.
- (ii) $(x + 5)$ is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.
- (iii) $(3x + 2)$ is a factor of $3x^3 + 2x^2 - 3x - 2$. Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.

Solution:

$$\begin{aligned}
&\text{(i) Let } f(x) = x^3 - 2x^2 - 9x + 18 \\
&x - 2 = 0 \Rightarrow x = 2
\end{aligned}$$

$$\begin{aligned}
&\therefore \text{Remainder} = f(2) \\
&= (2)^3 - 2(2)^2 - 9(2) + 18 \\
&= 8 - 8 - 18 + 18 \\
&= 0
\end{aligned}$$

Hence, $(x - 2)$ is a factor of $f(x)$.

Now, we have:

$$\begin{array}{r}
 \overline{x^2 - 9} \\
x-2 \overline{) x^3 - 2x^2 - 9x + 18} \\
\underline{x^3 - 2x^2} \\
 - 9x + 18 \\
\underline{ - 9x + 18} \\
 0
\end{array}$$

$$\therefore x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$$

$$\begin{aligned}
&\text{(ii) Let } f(x) = 2x^3 + 5x^2 - 28x - 15 \\
&x + 5 = 0 \Rightarrow x = -5
\end{aligned}$$

$$\begin{aligned}
 \therefore \text{Remainder} &= f(-5) \\
 &= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15 \\
 &= -250 + 125 + 140 - 15 \\
 &= -265 + 265 \\
 &= 0 \\
 \text{Hence, } (x + 5) &\text{ is a factor of } f(x).
 \end{aligned}$$

Now, we have:

$$\begin{array}{r}
 2x^2 - 5x - 3 \\
 x+5 \overline{) 2x^3 + 5x^2 - 28x - 15} \\
 \underline{2x^3 + 10x^2} \\
 - 5x^2 - 28x \\
 \underline{- 5x^2 - 25x} \\
 - 3x - 15 \\
 \underline{- 3x - 15} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 2x^3 + 5x^2 - 28x - 15 &= (x + 5)(2x^2 - 5x - 3) \\
 &= (x + 5)[2x^2 - 6x + x - 3] \\
 &= (x + 5)[2x(x - 3) + 1(x - 3)] \\
 &= (x + 5)(2x + 1)(x - 3)
 \end{aligned}$$

$$(iii) \text{ Let } f(x) = 3x^3 + 2x^2 - 3x - 2$$

$$3x + 2 = 0 \Rightarrow x = \frac{-2}{3}$$

$$\begin{aligned}
 \therefore \text{Remainder} &= f\left(\frac{-2}{3}\right) \\
 &= 3\left(\frac{-2}{3}\right)^3 + 2\left(\frac{-2}{3}\right)^2 - 3\left(\frac{-2}{3}\right) - 2 \\
 &= \frac{-8}{9} + \frac{8}{9} + 2 - 2 \\
 &= 0
 \end{aligned}$$

Hence, $(3x + 2)$ is a factor of $f(x)$.

Now, we have:

$$\begin{array}{r}
 \overline{) 3x^3 + 2x^2 - 3x - 2} \\
 \underline{3x^3 + 2x^2} \\
 - 3x - 2 \\
 \underline{ - 3x - 2} \\
 0
 \end{array}$$

$$\therefore 3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1) = (3x + 2)(x + 1)(x - 1)$$

Question 2.

Using the Remainder Theorem, factorise each of the following completely.

(i) $3x^3 + 2x^2 - 19x + 6$

(ii) $2x^3 + x^2 - 13x + 6$

(iii) $3x^3 + 2x^2 - 23x - 30$

(iv) $4x^3 + 7x^2 - 36x - 63$

(v) $x^3 + x^2 - 4x - 4$

Solution:

(i)

For $x = 2$, the value of the given

expression $3x^3 + 2x^2 - 19x + 6$

$$= 3(2)^3 + 2(2)^2 - 19(2) + 6$$

$$= 24 + 8 - 38 + 6$$

$$= 0$$

$\Rightarrow x - 2$ is a factor of $3x^3 + 2x^2 - 19x + 6$

Now let us do long division.

$$\begin{array}{r}
 \overline{) 3x^3 + 2x^2 - 19x + 6} \\
 \underline{3x^3 - 6x^2} \\
 8x^2 - 19x \\
 \underline{ 8x^2 - 16x} \\
 - 3x + 6 \\
 \underline{ - 3x + 6} \\
 0
 \end{array}$$

Thus we have,

$$\begin{aligned}3x^3 + 2x^2 - 19x + 6 &= (x - 2)(3x^2 + 8x - 3) \\&= (x - 2)(3x^2 + 9x - x - 3) \\&= (x - 2)(3x(x + 3) - (x + 3)) \\&= (x - 2)(3x - 1)(x + 3)\end{aligned}$$

(ii) Let $f(x) = 2x^3 + x^2 - 13x + 6$

For $x = 2$,

$$f(x) = f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$$

Hence, $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r} \overline{2x^2+5x-3} \\ x-2 \overline{) 2x^3 + x^2 - 13x + 6} \\ \underline{2x^3 - 4x^2} \\ 5x^2 - 13x \\ \underline{5x^2 - 10x} \\ - 3x + 6 \\ \underline{- 3x + 6} \\ 0 \end{array}$$

$$\begin{aligned}\therefore 2x^3 + x^2 - 13x + 6 &= (x - 2)(2x^2 + 5x - 3) \\&= (x - 2)(2x^2 + 6x - x - 3) \\&= (x - 2)[2x(x + 3) - (x + 3)] \\&= (x - 2)(x + 3)(2x - 1)\end{aligned}$$

(iii) $f(x) = 3x^3 + 2x^2 - 23x - 30$

For $x = -2$,

$$\begin{aligned}f(x) &= f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30 \\&= -24 + 8 + 46 - 30 = -54 + 54 = 0\end{aligned}$$

Hence, $(x + 2)$ is a factor of $f(x)$.

$$\begin{array}{r}
 3x^2 - 4x - 15 \\
 x + 2 \overline{) 3x^3 + 2x^2 - 23x - 30} \\
 \underline{3x^3 + 6x^2} \\
 -4x^2 - 23x \\
 \underline{-4x^2 - 8x} \\
 -15x - 30 \\
 \underline{-15x - 30} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 3x^3 + 2x^2 - 23x - 30 &= (x + 2)(3x^2 - 4x - 15) \\
 &= (x + 2)(3x^2 + 5x - 9x - 15) \\
 &= (x + 2)[x(3x + 5) - 3(3x + 5)] \\
 &= (x + 2)(3x + 5)(x - 3)
 \end{aligned}$$

$$(iv) f(x) = 4x^3 + 7x^2 - 36x - 63$$

For $x = 3$,

$$\begin{aligned}
 f(x) &= f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63 \\
 &= 108 + 63 - 108 - 63 = 0
 \end{aligned}$$

Hence, $(x + 3)$ is a factor of $f(x)$.

$$\begin{array}{r}
 4x^2 - 5x - 21 \\
 x + 3 \overline{) 4x^3 + 7x^2 - 36x - 63} \\
 \underline{4x^3 + 12x^2} \\
 -5x^2 - 36x \\
 \underline{-5x^2 - 15x} \\
 -21x - 63 \\
 \underline{-21x - 63} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 4x^3 + 7x^2 - 36x - 63 &= (x + 3)(4x^2 - 5x - 21) \\
 &= (x + 3)(4x^2 - 12x + 7x - 21) \\
 &= (x + 3)[4x(x - 3) + 7(x - 3)] \\
 &= (x + 3)(4x + 7)(x - 3)
 \end{aligned}$$

$$(v) f(x) = x^3 + x^2 - 4x - 4$$

For $x = -1$,

$$f(x) = f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4 \\ = -1 + 1 + 4 - 4 = 0$$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r} \overline{) - 4x - 4} \\ x+1 \overline{) - 4x - 4} \\ \underline{x^3 + x^2} \\ - 4x - 4 \\ \underline{- 4x - 4} \\ 0 \end{array}$$

$$\therefore x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4) \\ = (x + 1)(x + 2)(x - 2)$$

Question 3.

Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$.

Solution:

$$\text{Let } f(x) = 3x^3 + 10x^2 + x - 6$$

For $x = -1$,

$$f(x) = f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r} \overline{) + x - 6} \\ x+1 \overline{) + x - 6} \\ \underline{3x^3 + 3x^2} \\ 7x^2 + x \\ \underline{7x^2 + 7x} \\ - 6x - 6 \\ \underline{- 6x - 6} \\ 0 \end{array}$$

$$\begin{aligned}
 \therefore 3x^3 + 10x^2 + x - 6 &= (x+1)(3x^2 + 7x - 6) \\
 &= (x+1)(3x^2 + 9x - 2x - 6) \\
 &= (x+1)[3x(x+3) - 2(x+3)] \\
 &= (x+1)(x+3)(3x-2)
 \end{aligned}$$

$$\begin{aligned}
 \text{Now, } 3x^3 + 10x^2 + x - 6 &= 0 \\
 \Rightarrow (x+1)(x+3)(3x-2) &= 0 \\
 \Rightarrow x &= -1, -3, \frac{2}{3}
 \end{aligned}$$

Question 4.

Factorise the expression $f(x) = 2x^3 - 7x^2 - 3x + 18$. Hence, find all possible values of x for which $f(x) = 0$.

Solution:

$$f(x) = 2x^3 - 7x^2 - 3x + 18$$

For $x = 2$,

$$\begin{aligned}
 f(x) &= f(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18 \\
 &= 16 - 28 - 6 + 18 = 0
 \end{aligned}$$

Hence, $(x-2)$ is a factor of $f(x)$.

$$\begin{array}{r}
 2x^2 - 3x - 9 \\
 x-2 \overline{) 2x^3 - 7x^2 - 3x + 18} \\
 \underline{2x^3 - 4x^2} \\
 -3x^2 - 3x \\
 \underline{-3x^2 + 6x} \\
 -9x + 18 \\
 \underline{-9x + 18} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 2x^3 - 7x^2 - 3x + 18 &= (x-2)(2x^2 - 3x - 9) \\
 &= (x-2)(2x^2 - 6x + 3x - 9) \\
 &= (x-2)[2x(x-3) + 3(x-3)] \\
 &= (x-2)(x-3)(2x+3)
 \end{aligned}$$

Now, $f(x) = 0$

$$\begin{aligned}
 \Rightarrow 2x^3 - 7x^2 - 3x + 18 &= 0 \\
 \Rightarrow (x-2)(x-3)(2x+3) &= 0 \\
 \Rightarrow x &= 2, 3, -\frac{3}{2}
 \end{aligned}$$

Question 5.

Given that $x - 2$ and $x + 1$ are factors of $f(x) = x^3 + 3x^2 + ax + b$; calculate the values of a and b . Hence, find all the factors of $f(x)$.

Solution:

$$f(x) = x^3 + 3x^2 + ax + b$$

Since, $(x - 2)$ is a factor of $f(x)$, $f(2) = 0$

$$\Rightarrow (2)^3 + 3(2)^2 + a(2) + b = 0$$

$$\Rightarrow 8 + 12 + 2a + b = 0$$

$$\Rightarrow 2a + b + 20 = 0 \dots(i)$$

Since, $(x + 1)$ is a factor of $f(x)$, $f(-1) = 0$

$$\Rightarrow (-1)^3 + 3(-1)^2 + a(-1) + b = 0$$

$$\Rightarrow -1 + 3 - a + b = 0$$

$$\Rightarrow -a + b + 2 = 0 \dots(ii)$$

Subtracting (ii) from (i), we get,

$$3a + 18 = 0$$

$$\Rightarrow a = -6$$

Substituting the value of a in (ii), we get,

$$b = a - 2 = -6 - 2 = -8$$

$$\therefore f(x) = x^3 + 3x^2 - 6x - 8$$

Now, for $x = -1$,

$$f(x) = f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r} x^2 + 2x - 8 \\ x + 1 \overline{) x^3 + 3x^2 - 6x - 8} \\ \underline{x^3 + x^2} \\ 2x^2 - 6x \\ \underline{2x^2 + 2x} \\ -8x - 8 \\ \underline{-8x - 8} \\ 0 \end{array}$$

$$\begin{aligned} \therefore x^3 + 3x^2 - 6x - 8 &= (x + 1)(x^2 + 2x - 8) \\ &= (x + 1)(x^2 + 4x - 2x - 8) \\ &= (x + 1)[x(x + 4) - 2(x + 4)] \\ &= (x + 1)(x + 4)(x - 2) \end{aligned}$$

Question 6.

The expression $4x^3 - bx^2 + x - c$ leaves remainders 0 and 30 when divided by $x + 1$ and $2x - 3$ respectively. Calculate the values of b and c . Hence, factorise the expression completely.

Solution:

$$\text{Let } f(x) = 4x^3 - bx^2 + x - c$$

It is given that when $f(x)$ is divided by $(x + 1)$, the remainder is 0.

$$\therefore f(-1) = 0$$

$$4(-1)^3 - b(-1)^2 + (-1) - c = 0$$

$$-4 - b - 1 - c = 0$$

$$b + c + 5 = 0 \dots (i)$$

It is given that when $f(x)$ is divided by $(2x - 3)$, the remainder is 30.

$$\therefore f\left(\frac{3}{2}\right) = 30$$

$$4\left(\frac{3}{2}\right)^3 - b\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right) - c = 30$$

$$\frac{27}{2} - \frac{9b}{4} + \frac{3}{2} - c = 30$$

$$54 - 9b + 6 - 4c - 120 = 0$$

$$9b + 4c + 60 = 0 \dots (ii)$$

Multiplying (i) by 4 and subtracting it from (ii), we get,

$$5b + 40 = 0$$

$$b = -8$$

Substituting the value of b in (i), we get,

$$c = -5 + 8 = 3$$

$$\text{Therefore, } f(x) = 4x^3 + 8x^2 + x - 3$$

Now, for $x = -1$, we get,

$$f(x) = f(-1) = 4(-1)^3 + 8(-1)^2 + (-1) - 3 = -4 + 8 - 1 - 3 = 0$$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r}
 4x^2 + 4x - 3 \\
 x + 1 \overline{) 4x^3 + 8x^2 + x - 3} \\
 \underline{4x^3 + 4x^2} \\
 4x^2 + x \\
 \underline{4x^2 + 4x} \\
 -3x - 3 \\
 \underline{-3x - 3} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore 4x^3 + 8x^2 + x - 3 &= (x + 1)(4x^2 + 4x - 3) \\
 &= (x + 1)(4x^2 + 6x - 2x - 3) \\
 &= (x + 1)[2x(2x + 3) - (2x + 3)] \\
 &= (x + 1)(2x + 3)(2x - 1)
 \end{aligned}$$

Question 7.

If $x + a$ is a common factor of expressions $f(x) = x^2 + px + q$ and $g(x) = x^2 + mx + n$;

show that: $a = \frac{n - q}{m - p}$

Solution:

$$f(x) = x^2 + px + q$$

It is given that $(x + a)$ is a factor of $f(x)$.

$$\therefore f(-a) = 0$$

$$\Rightarrow (-a)^2 + p(-a) + q = 0$$

$$\Rightarrow a^2 - pa + q = 0$$

$$\Rightarrow a^2 = pa - q \quad \dots(i)$$

$$g(x) = x^2 + mx + n$$

It is given that $(x + a)$ is a factor of $g(x)$.

$$\therefore g(-a) = 0$$

$$\Rightarrow (-a)^2 + m(-a) + n = 0$$

$$\Rightarrow a^2 - ma + n = 0$$

$$\Rightarrow a^2 = ma - n \quad \dots(ii)$$

From (i) and (ii), we get,

$$\begin{aligned}
 pa - q &= ma - n \\
 n - q &= a(m - p) \\
 a &= \frac{n - q}{m - p}
 \end{aligned}$$

Hence, proved.

Question 8.

The polynomials $ax^3 + 3x^2 - 3$ and $2x^3 - 5x + a$, when divided by $x - 4$, leave the same remainder in each case. Find the value of a .

Solution:

$$\text{Let } f(x) = ax^3 + 3x^2 - 3$$

When $f(x)$ is divided by $(x - 4)$, remainder = $f(4)$

$$f(4) = a(4)^3 + 3(4)^2 - 3 = 64a + 45$$

$$\text{Let } g(x) = 2x^3 - 5x + a$$

When $g(x)$ is divided by $(x - 4)$, remainder = $g(4)$

$$g(4) = 2(4)^3 - 5(4) + a = a + 108$$

It is given that $f(4) = g(4)$

$$64a + 45 = a + 108$$

$$63a = 63$$

$$a = 1$$

Question 9.

Find the value of ' a ', if $(x - a)$ is a factor of $x^3 - ax^2 + x + 2$.

Solution:

$$\text{Let } f(x) = x^3 - ax^2 + x + 2$$

It is given that $(x - a)$ is a factor of $f(x)$.

$$\text{Remainder} = f(a) = 0$$

$$a^3 - a^3 + a + 2 = 0$$

$$a + 2 = 0$$

$$a = -2$$

Question 10.

Find the number that must be subtracted from the polynomial $3y^3 + y^2 - 22y + 15$, so that the resulting polynomial is completely divisible by $y + 3$.

Solution:

Let the number to be subtracted from the given polynomial be k .

$$\text{Let } f(y) = 3y^3 + y^2 - 22y + 15 - k$$

It is given that $f(y)$ is divisible by $(y + 3)$.

$$\text{Remainder} = f(-3) = 0$$

$$3(-3)^3 + (-3)^2 - 22(-3) + 15 - k = 0$$

$$-81 + 9 + 66 + 15 - k = 0$$

$$9 - k = 0$$

$$k = 9$$

Exercise 8C

Question 1.

Show that $(x - 1)$ is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the given expression.

Solution:

$$\text{Let } f(x) = x^3 - 7x^2 + 14x - 8$$

$$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$$

Hence, $(x - 1)$ is a factor of $f(x)$.

$$\begin{array}{r} \overline{x^2 - 6x + 8} \\ x-1 \overline{) x^3 - 7x^2 + 14x - 8} \\ \underline{x^3 - x^2} \\ \overline{-6x^2 + 14x} \\ \underline{-6x^2 + 6x} \\ \phantom{\overline{}}\overline{8x - 8} \\ \phantom{\overline{}}\underline{8x - 8} \\ \phantom{\overline{}}\overline{0} \end{array}$$

$$\begin{aligned} \therefore x^3 - 7x^2 + 14x - 8 &= (x - 1)(x^2 - 6x + 8) \\ &= (x - 1)(x^2 - 2x - 4x + 8) \\ &= (x - 1)[x(x - 2) - 4(x - 2)] \\ &= (x - 1)(x - 2)(x - 4) \end{aligned}$$

Question 2.

Using Remainder Theorem, factorise:

$x^3 + 10x^2 - 37x + 26$ completely.

Solution:

By Remainder Theorem,

For $x = 1$, the value of the given expression is the remainder.

$$x^3 + 10x^2 - 37x + 26$$

$$= (1)^3 + 10(1)^2 - 37(1) + 26$$

$$= 1 + 10 - 37 + 26$$

$$= 37 - 37$$

$$= 0$$

$\Rightarrow x - 1$ is a factor of $x^3 + 10x^2 - 37x + 26$.

$$\begin{array}{r} x^2 + 11x - 26 \\ x - 1 \overline{) x^3 + 10x^2 - 37x + 26} \\ \underline{x^3 - x^2} \\ 11x^2 - 37x \\ \underline{11x^2 - 11x} \\ -26x + 26 \\ \underline{-26x + 26} \\ 0 \end{array}$$

$$\therefore x^3 + 10x^2 - 37x + 26 = (x - 1)(x^2 + 11x - 26)$$

$$= (x - 1)(x^2 + 13x - 2x - 26)$$

$$= (x - 1)[x(x + 13) - 2(x + 13)]$$

$$\therefore x^3 + 10x^2 - 37x + 26 = (x - 1)(x + 13)(x - 2)$$

Question 3.

When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m .

Solution:

$$\text{Let } f(x) = x^3 + 3x^2 - mx + 4$$

According to the given information,

$$f(2) = m + 3$$

$$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$$

$$8 + 12 - 2m + 4 = m + 3$$

$$24 - 3 = m + 2m$$

$$3m = 21$$

$$m = 7$$

Question 4.

What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?

Solution:

Let the required number be k .

Let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$

According to the given information,

$$f(-2) = 0$$

$$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$$

$$-24 - 32 - 8 - 3 - k = 0$$

$$-67 - k = 0$$

$$k = -67$$

Thus, the required number is -67 .

Question 5.

If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b . And then, factorise the given expression completely.

Solution:

$$\text{Let } f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$$

Since, $(x + 1)$ is a factor of $f(x)$.

$$\text{Remainder} = f(-1) = 0$$

$$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$$

$$-1 + (a + 1) + (b - 2) - 6 = 0$$

$$a + b - 8 = 0 \dots(i)$$

Since, $(x - 2)$ is a factor of $f(x)$.

$$\text{Remainder} = f(2) = 0$$

$$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$$

$$8 + 4a + 4 - 2b + 4 - 6 = 0$$

$$4a - 2b + 10 = 0$$

$$2a - b + 5 = 0 \dots(ii)$$

Adding (i) and (ii), we get,

$$3a - 3 = 0$$

$$a = 1$$

Substituting the value of a in (i), we get,

$$1 + b - 8 = 0$$

$$b = 7$$

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Now, $(x + 1)$ and $(x - 2)$ are factors of $f(x)$. Hence, $(x + 1)(x - 2) = x^2 - x - 2$ is a factor

of $f(x)$.

$$\begin{array}{r} \overline{x^3 + 2x^2 - 5x - 6} \\ x^2 - x - 2 \overline{) } \\ \underline{x^3 - x^2 - 2x} \\ 3x^2 - 3x - 6 \\ \underline{3x^2 - 3x - 6} \\ 0 \end{array}$$

$$f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$$

Question 6.

If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b .

Solution:

$$\text{Let } f(x) = x^2 + ax + b$$

Since, $(x - 2)$ is a factor of $f(x)$.

$$\text{Remainder} = f(2) = 0$$

$$(2)^2 + a(2) + b = 0$$

$$4 + 2a + b = 0$$

$$2a + b = -4 \dots (i)$$

It is given that:

$$a + b = 1 \dots (ii)$$

Subtracting (ii) from (i), we get,

$$a = -5$$

Substituting the value of a in (ii), we get,

$$b = 1 - (-5) = 6$$

Question 7.

Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.

Solution:

$$\text{Let } f(x) = x^3 + 6x^2 + 11x + 6$$

$$\text{For } x = -1$$

$$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$$

$$= -1 + 6 - 11 + 6 = 12 - 12 = 0$$

Hence, $(x + 1)$ is a factor of $f(x)$.

$$\begin{array}{r}
 x^2 + 5x + 6 \\
 x + 1 \overline{) x^3 + 6x^2 + 11x + 6} \\
 \underline{x^3 + x^2} \\
 5x^2 + 11x \\
 \underline{5x^2 + 5x} \\
 6x + 6 \\
 \underline{6x + 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore x^3 + 6x^2 + 11x + 6 &= (x + 1)(x^2 + 5x + 6) \\
 &= (x + 1)(x^2 + 2x + 3x + 6) \\
 &= (x + 1)[x(x + 2) + 3(x + 2)] \\
 &= (x + 1)(x + 2)(x + 3)
 \end{aligned}$$

Question 8.

Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by $x - 2$.

Solution:

$$\text{Let } f(x) = mx^3 + 2x^2 - 3$$

$$g(x) = x^2 - mx + 4$$

It is given that $f(x)$ and $g(x)$ leave the same remainder when divided by $(x - 2)$.

Therefore, we have:

$$f(2) = g(2)$$

$$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$$

$$8m + 8 - 3 = 4 - 2m + 4$$

$$10m = 3$$

$$m = 3/10$$

Question 9.

The polynomial $px^3 + 4x^2 - 3x + q$ is completely divisible by $x^2 - 1$; find the values of p and q . Also, for these values of p and q factorize the given polynomial completely.

Solution:

$$\text{Let } f(x) = px^3 + 4x^2 - 3x + q$$

It is given that $f(x)$ is completely divisible by $(x^2 - 1) = (x + 1)(x - 1)$.

Therefore, $f(1) = 0$ and $f(-1) = 0$

$$f(1) = p(1)^3 + 4(1)^2 - 3(1) + q = 0$$

$$p + q + 1 = 0 \dots(i)$$

$$f(-1) = p(-1)^3 + 4(-1)^2 - 3(-1) + q = 0$$

$$-p + q + 7 = 0 \dots(ii)$$

Adding (i) and (ii), we get,

$$2q + 8 = 0$$

$$q = -4$$

Substituting the value of q in (i), we get,

$$p = -q - 1 = 4 - 1 = 3$$

$$f(x) = 3x^3 + 4x^2 - 3x - 4$$

Given that $f(x)$ is completely divisible by $(x^2 - 1)$.

$$\begin{array}{r} \overline{3x + 4} \\ x^2 - 1 \overline{) 3x^3 + 4x^2 - 3x - 4} \\ \underline{3x^3} \\ 4x^2 - 4 \\ \underline{4x^2} - 4 \\ 0 \end{array}$$

$$\therefore 3x^3 + 4x^2 - 3x - 4 = (x^2 - 1)(3x + 4)$$

$$= (x - 1)(x + 1)(3x + 4)$$

Question 10.

Find the number which should be added to $x^2 + x + 3$ so that the resulting polynomial is completely divisible by $(x + 3)$.

Solution:

Let the required number be k.

$$\text{Let } f(x) = x^2 + x + 3 + k$$

It is given that $f(x)$ is divisible by $(x + 3)$.

$$\text{Remainder} = 0$$

$$f(-3) = 0$$

$$(-3)^2 + (-3) + 3 + k = 0$$

$$9 - 3 + 3 + k = 0$$

$$9 + k = 0$$

$$k = -9$$

Thus, the required number is -9.

Question 11.

When the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A and when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B. Find the value of 'a' if $2A + B = 0$.

Solution:

It is given that when the polynomial $x^3 + 2x^2 - 5ax - 7$ is divided by $(x - 1)$, the remainder is A.

$$(1)^3 + 2(1)^2 - 5a(1) - 7 = A$$

$$1 + 2 - 5a - 7 = A$$

$$-5a - 4 = A \dots (i)$$

It is also given that when the polynomial $x^3 + ax^2 - 12x + 16$ is divided by $(x + 2)$, the remainder is B.

$$x^3 + ax^2 - 12x + 16 = B$$

$$(-2)^3 + a(-2)^2 - 12(-2) + 16 = B$$

$$-8 + 4a + 24 + 16 = B$$

$$4a + 32 = B \dots (ii)$$

It is also given that $2A + B = 0$

Using (i) and (ii), we get,

$$2(-5a - 4) + 4a + 32 = 0$$

$$-10a - 8 + 4a + 32 = 0$$

$$-6a + 24 = 0$$

$$6a = 24$$

$$a = 4$$

Question 12.

$(3x + 5)$ is a factor of the polynomial $(a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$. Find the value of 'a', factorise the given polynomial completely.

Solution:

$$\text{Let } f(x) = (a - 1)x^3 + (a + 1)x^2 - (2a + 1)x - 15$$

It is given that $(3x + 5)$ is a factor of $f(x)$.

$$\therefore \text{Remainder} = 0$$

$$f\left(\frac{-5}{3}\right) = 0$$

$$(a - 1)\left(\frac{-5}{3}\right)^3 + (a + 1)\left(\frac{-5}{3}\right)^2 - (2a + 1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$(a - 1)\left(\frac{-125}{27}\right) + (a + 1)\left(\frac{25}{9}\right) - (2a + 1)\left(\frac{-5}{3}\right) - 15 = 0$$

$$\frac{-125(a-1) + 75(a+1) + 45(2a+1) - 405}{27} = 0$$

$$-125a + 125 + 75a + 75 + 90a + 45 - 405 = 0$$

$$40a - 160 = 0$$

$$40a = 160$$

$$a = 4$$

$$\therefore f(x) = (a-1)x^3 + (a+1)x^2 - (2a+1)x - 15$$

$$= 3x^3 + 5x^2 - 9x - 15$$

$$\begin{array}{r} \overline{) 3x^3 + 5x^2 - 9x - 15} \\ \underline{3x^3 + 5x^2} \\ - 9x - 15 \\ \underline{- 9x - 15} \\ 0 \end{array}$$

$$\begin{aligned} \therefore 3x^3 + 5x^2 - 9x - 15 &= (3x+5)(x^2-3) \\ &= (3x+5)(x+\sqrt{3})(x-\sqrt{3}) \end{aligned}$$

Question 13.

When divided by $x-3$ the polynomials $x^3 - px^2 + x + 6$ and $2x^3 - x^2 - (p+3)x - 6$ leave the same remainder. Find the value of 'p'.

Solution:

If $(x-3)$ divides $f(x) = x^3 - px^2 + x + 6$, then,

$$\text{Remainder} = f(3) = 3^3 - p(3)^2 + 3 + 6 = 36 - 9p$$

If $(x-3)$ divides $g(x) = 2x^3 - x^2 - (p+3)x - 6$, then

$$\text{Remainder} = g(3) = 2(3)^3 - (3)^2 - (p+3)(3) - 6 = 30 - 3p$$

$$\text{Now, } f(3) = g(3)$$

$$\Rightarrow 36 - 9p = 30 - 3p$$

$$\Rightarrow -6p = -6$$

$$\Rightarrow p = 1$$

Question 14.

Use the Remainder Theorem to factorise the following expression:

$$2x^3 + x^2 - 13x + 6$$

Solution:

$$f(x) = 2x^3 + x^2 - 13x + 6$$

Factors of constant term 6 are $\pm 1, \pm 2, \pm 3, \pm 6$.

Putting $x = 2$, we have:

$$f(2) = 2(2)^3 + 2^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$$

Hence $(x - 2)$ is a factor of $f(x)$.

$$\begin{array}{r}
 2x^2 + 5x - 3 \\
 x - 2 \overline{) 2x^3 + x^2 - 13x + 6} \\
 \underline{2x^3 - 4x^2} \\
 5x^2 - 13x \\
 \underline{5x^2 - 10x} \\
 -3x + 6 \\
 \underline{-3x + 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 2x^3 + x^2 - 13x + 6 &= (x - 2)(2x^2 + 5x - 3) \\
 &= (x - 2)(2x^2 + 6x - x - 3) \\
 &= (x - 2)(2x(x + 3) - 1(x + 3)) \\
 &= (x - 2)(2x - 1)(x + 3)
 \end{aligned}$$

Question 15.

Using remainder theorem, find the value of k if on dividing $2x^3 + 3x^2 - kx + 5$ by $x - 2$, leaves a remainder 7.

Solution:

$$\text{Let } f(x) = 2x^3 + 3x^2 - kx + 5$$

Using Remainder Theorem, we have

$$f(2) = 7$$

$$\therefore 2(2)^3 + 3(2)^2 - k(2) + 5 = 7$$

$$\therefore 16 + 12 - 2k + 5 = 7$$

$$\therefore 33 - 2k = 7$$

$$\therefore 2k = 26$$

$$\therefore k = 13$$

Question 16.

What must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor?

Solution:

Here, $f(x) = 16x^3 - 8x^2 + 4x + 7$

Let the number subtracted be k from the given polynomial $f(x)$.

Given that $2x + 1$ is a factor of $f(x)$.

$$\therefore f\left(-\frac{1}{2}\right) = 0$$

$$\Rightarrow 16\left(-\frac{1}{2}\right)^3 - 8\left(-\frac{1}{2}\right)^2 + 4\left(-\frac{1}{2}\right) + 7 - k = 0$$

$$\Rightarrow 16 \times \left(-\frac{1}{8}\right) - 8 \times \frac{1}{4} - 2 + 7 - k = 0$$

$$\Rightarrow -2 - 2 - 2 + 7 - k = 0$$

$$\Rightarrow -6 + 7 - k = 0$$

$$\Rightarrow k = 1$$

Therefore 1 must be subtracted from $16x^3 - 8x^2 + 4x + 7$ so that the resulting expression has $2x + 1$ as a factor.